Determining heavy meson LCDAs from lattice QCD

Based on 2403.17492, 2410.18654 *et al.*

In collaboration with LPC members and J. Xu, S. Zhao, *et al.*

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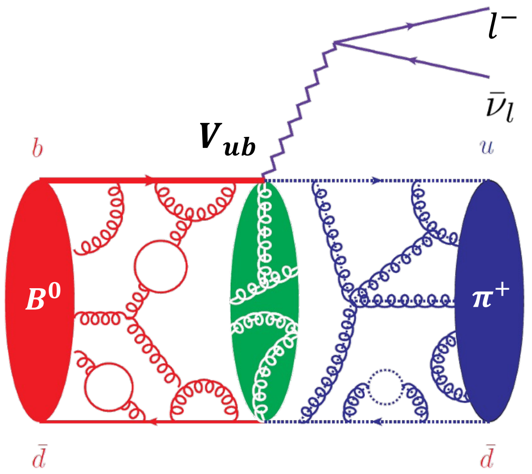
Outline

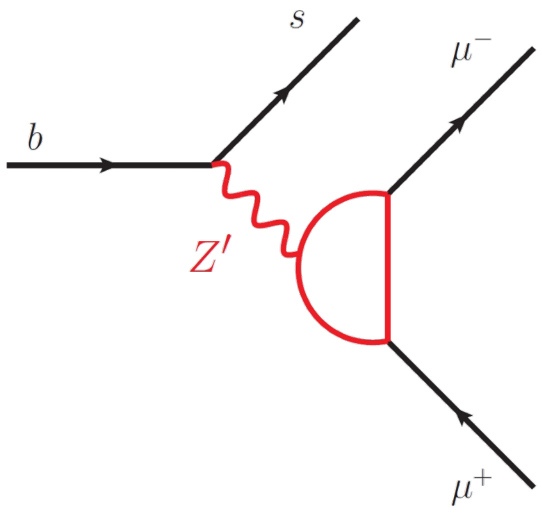
- Motivation
- Theoretical framework for the two-step factorization method
- Lattice QCD verification
- Phenomenological discussions
- Summary and prospect

Motivation

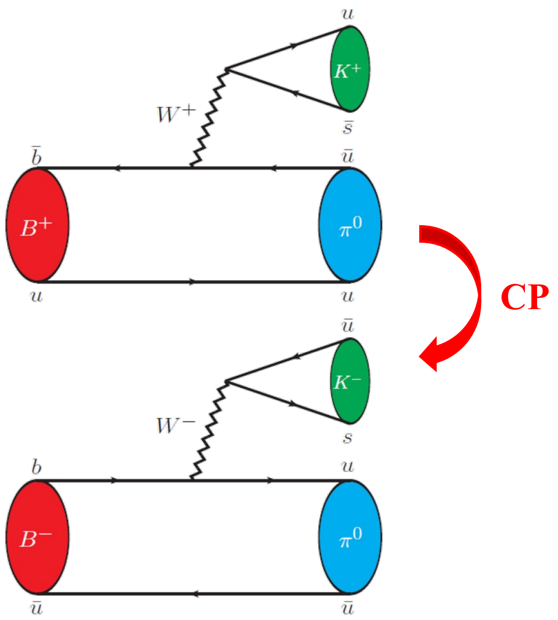
Heavy flavor physics is one of the frontier topics in particle physics:



Precisely testing standard model



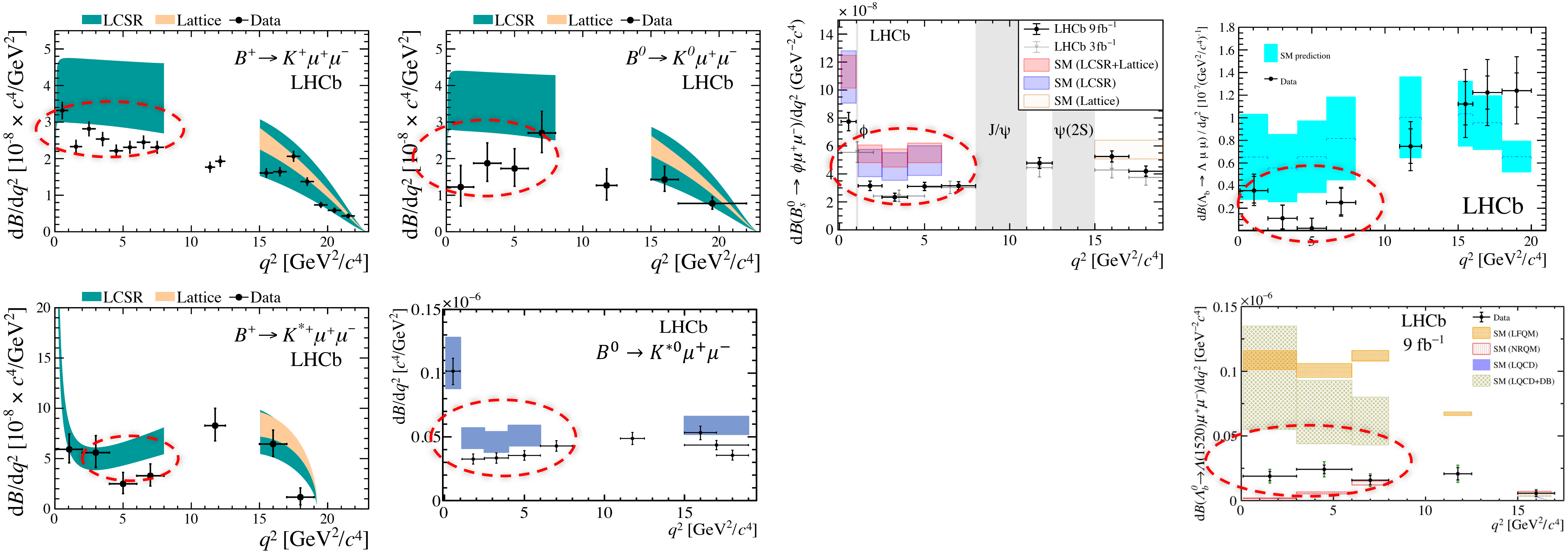
Indirect search for new physics



Study on CP violation

Motivation

Current experimental results show deviations from theoretical prediction…



High precision calculations play a crucial role in the search for new physics!

Motivation

Uncertainties originating from *B* meson LCDAs dominate the primary errors in theoretical calculation.

- For example: $B \to \pi,\ K^*$ form factors from LCSRs:

Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035

Cui, Huang, Shen, Wang, JHEP 03 (2023) 140

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

$$f^+_{B\to\pi}(0) = 0.122 \times \Big[1 \pm 0.07\Big|_{S_0^\pi} \pm 0.11\Big|_{\Lambda_q} \pm 0.02\Big|_{\lambda_E^2/\lambda_H^2}{}^{+0.05}_{-0.06}\Big|_{M^2} \pm 0.05\Big|_{2\lambda_E^2+\lambda_H^2}$$
$$\qquad {}^{+0.06}_{-0.10}\Big|_{\mu_h} \pm 0.04\Big|_{\mu}{}^{+1.36}_{-0.56}\Big|_{\lambda_B}{}^{+0.25}_{-0.43}\Big|_{\sigma_1,\sigma_2}\Big].$$

λ_B and σ_n: the first inverse and inverse-log moments,

$\varphi_B^\pm$: uncertainties from different parameterizations of the *B* meson LCDA.

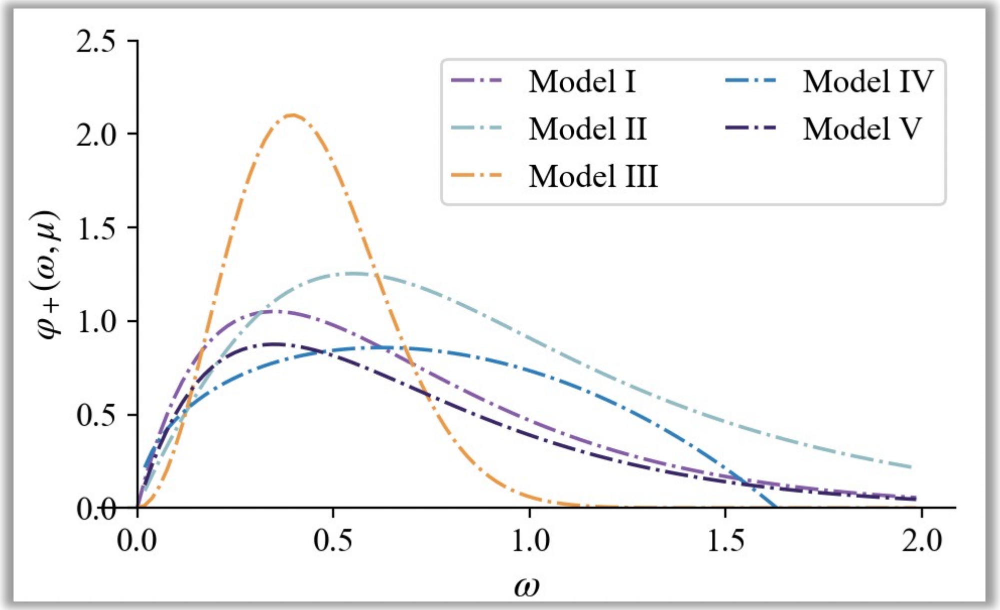
Without reliable *B* LCDA, it is impossible to discuss precision calculation!

Motivation

- HQET field h_v and the associated cusp divergence hinder the first-principle calculation of B meson LCDA.

$$i\tilde{f}_H(\mu)m_H\varphi^+(\omega,\mu) = \int_{-\infty}^{+\infty}\frac{dt}{2\pi}e^{i\omega n_+\cdot vt} \times \langle 0|\bar{q}\,(tn_+)\, \not{n}_+\gamma_5 W_c\,(tn_+,0)\, h_v(0)|H(v)\rangle$$

Grozin, Neubert, PRD 55, 272 (1997)
Braun, Ivanov, Korchemsky, PRD 69, 034014 (2004)
Beneke, Braun, Ji, Wei, JHEP 07, 154 (2018)



- Only available through model parametrizations now, predictions from different models vary significantly.

- This model dependence contribute to the largest theoretical uncertainties in $B \to K^*$ form factor:

Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035

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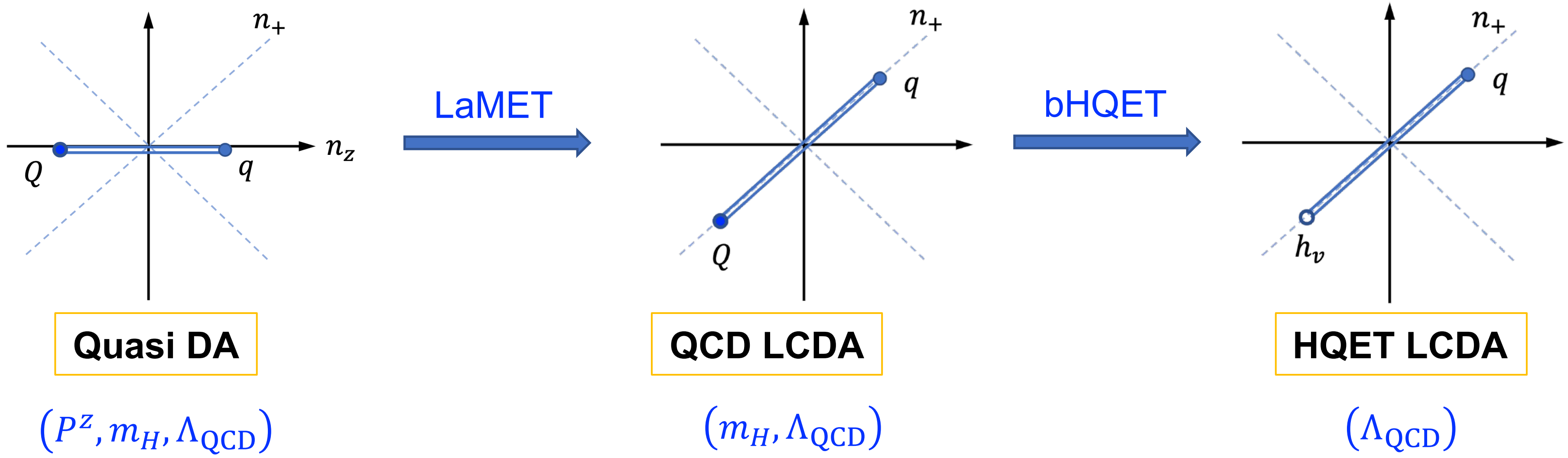
A two-step factorization method

We propose a **chained factorization formula** to determine both heavy meson QCD LCDA and HQET LCDA from the first-principle:



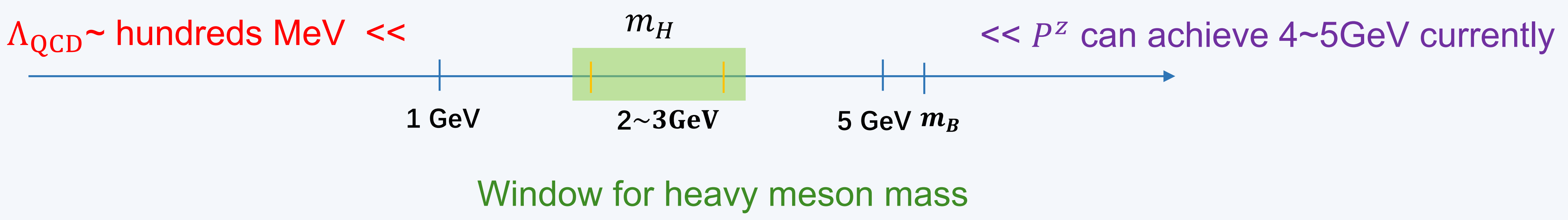
- LaMET: $\Lambda_{\text{QCD}}, m_H \ll P^z$ and integrate out P^z
- bHQET: $\Lambda_{\text{QCD}} \ll m_H$ and integrate out m_H

$\Rightarrow$ Introduce a hierarchy $\boldsymbol{\Lambda_{\text{QCD}} \ll m_H \ll P^z}$

A two-step factorization method

The hierarchy $\Lambda_{\text{QCD}} \ll m_H \ll P^z$ imposes limitations on lattice calculation:

- Currently, direct simulation of *B* meson is not practical.



- *D* meson can be realized on the lattice;
- Heavy quark flavor symmetry ensures that the HQET LCDA is independent of heavy quark mass;
- m_H (m_D or m_B) only contributes to the power corrections.

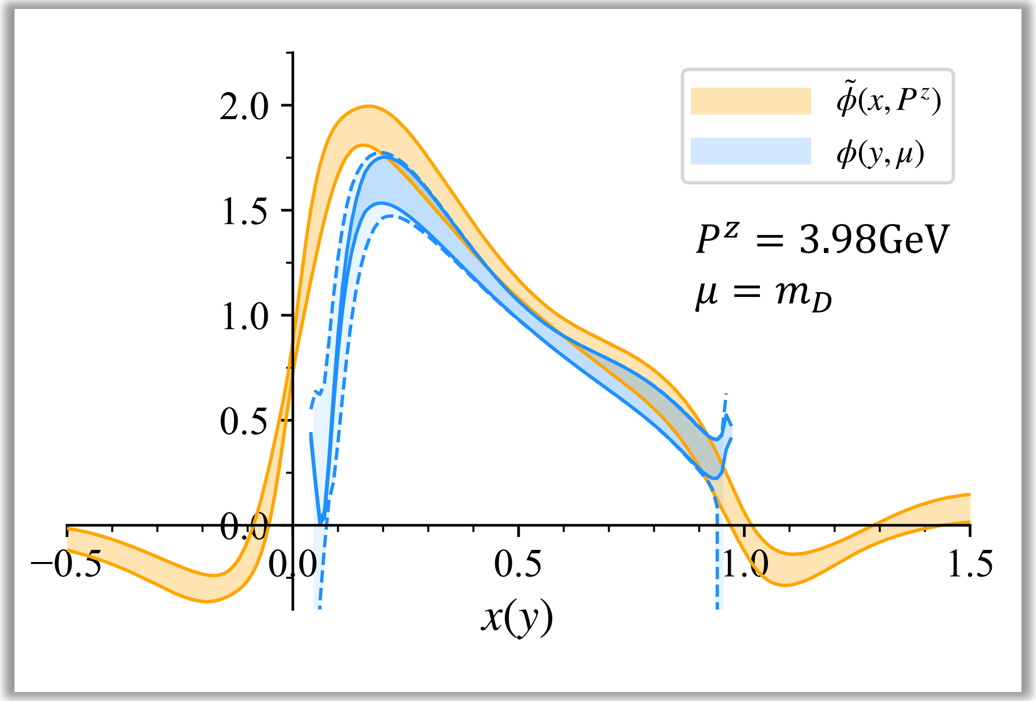
Lattice QCD verification

- A numerical simulation on the finest CLQCD ensemble ($a = 0.05187$ fm);

  CLQCD Collaboration, PRD 109, 054507 (2024)

- Simulate the *D* meson quasi DA with $m_D \simeq 1.92\text{GeV}$, up to $P^z \simeq 3.98\text{GeV}$;

$$\tilde{\phi}(x, P^z) = \int \frac{dz}{2\pi} e^{-ixP^z z} \tilde{M}(z, P^z)$$



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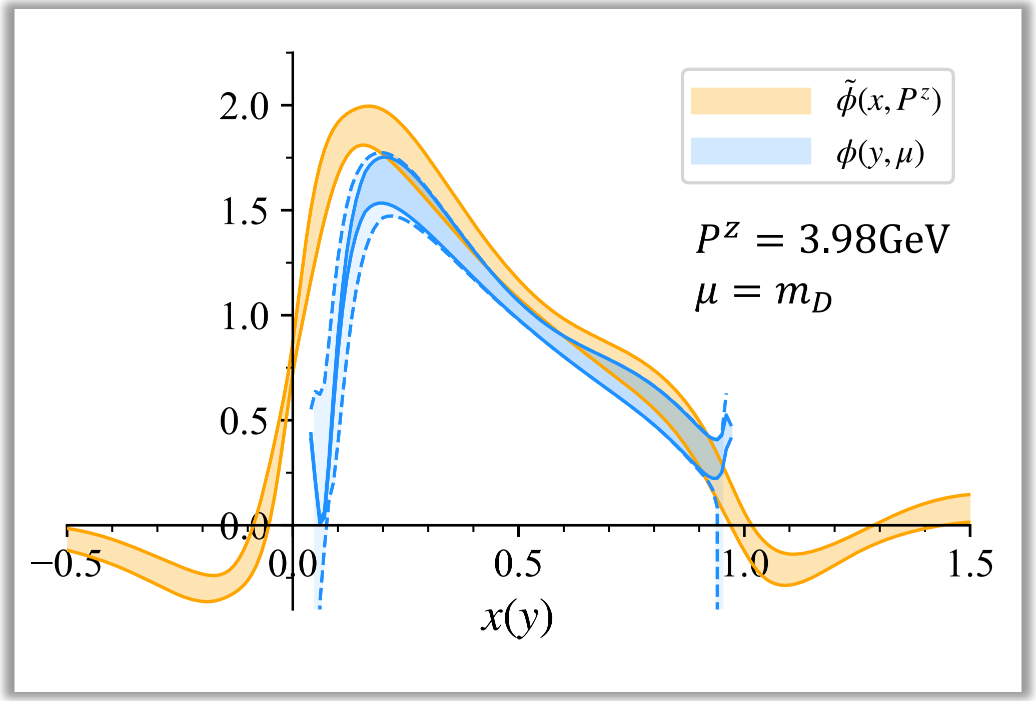
- Matching formula in LaMET:

$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\text{QCD}}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

  Matching kernel at NLO in α_s

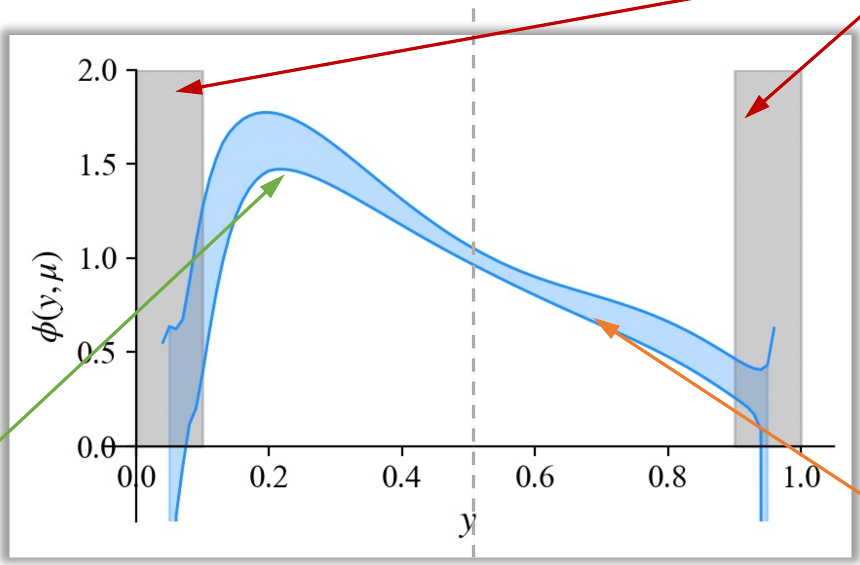
  Liu, Wang, Xu, QAZ, Zhao, PRD 99, 094036 (2019)

  Han, Hua, Ji, Lu, Wang, Xu, QAZ, Zhao, 2410.18654



Lattice QCD verification

QCD LCDA of D meson



End-point region:

- LaMET matching kernel suffer large power corrections.
- Lattice QCD predictions fail

Peak region: $y \sim \frac{\Lambda_{\mathrm{QCD}}}{m_H}$

- Light quark carries small momentum fraction;
- Related to the HQET LCDA.

Ishaq, Jia, Xiong, Yang, PRL125(2020)132001
Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

Tail region: $y \sim 1$

- Contain only hard-collinear physics, perturbative calculable;
- Suppressed in LCDA.

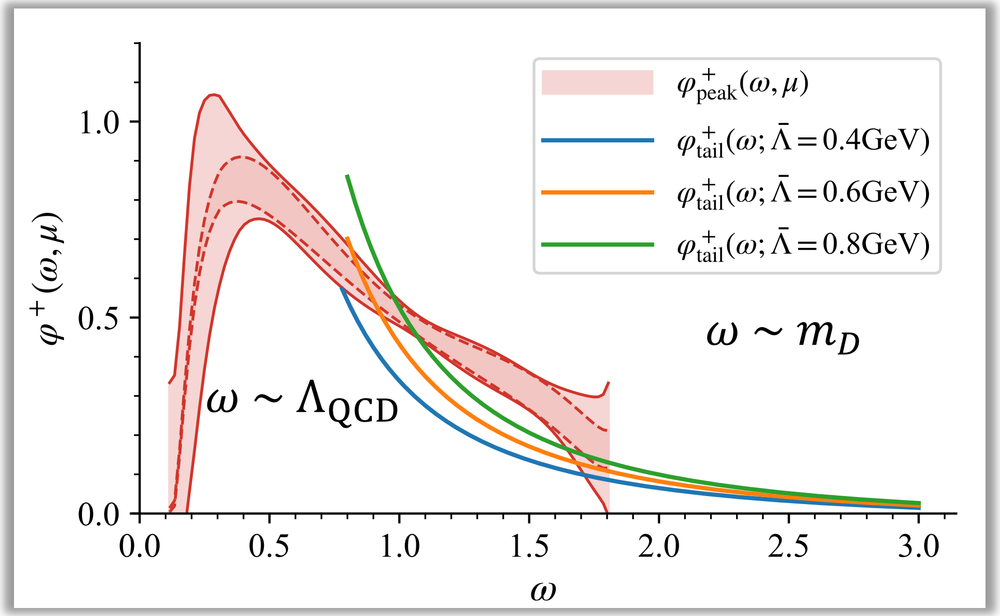
Lattice QCD verification

- Peak region:
  - A multiplicative factorization from QCD LCDA to HQET LCDA:

$$\varphi^+_{\rm peak}(\omega,\mu) = \frac{f_H}{\widetilde{f}_H}\frac{1}{\mathcal{J}_{\rm peak}}\phi(y,\mu;m_H) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_H}\right)$$

Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

  - Nonperturbative, determined from lattice QCD.



Lattice QCD verification

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Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

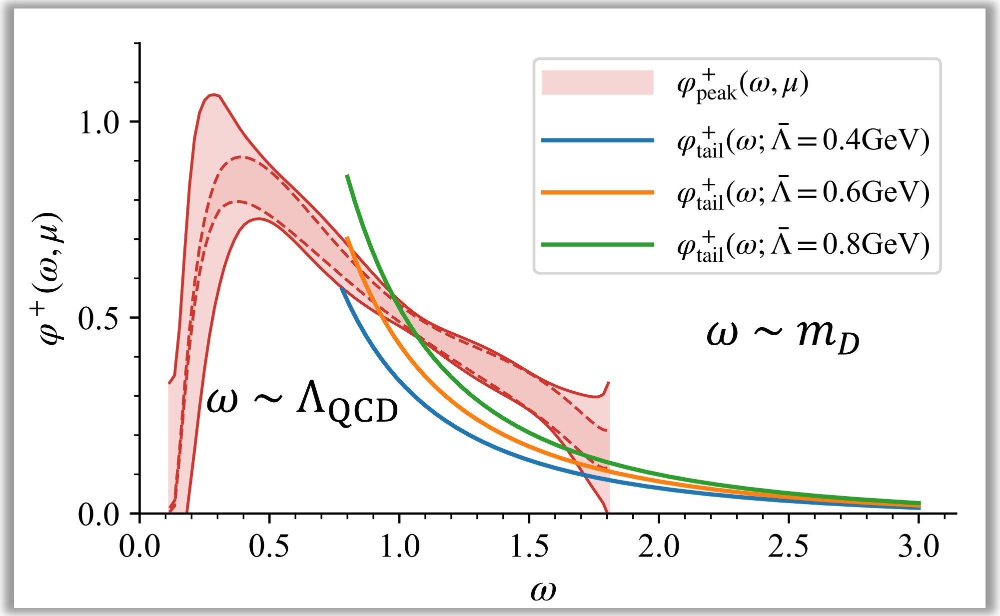
- Nonperturbative, determined from lattice QCD.

➢ Tail region:

- Perturbative result at 1-loop order:

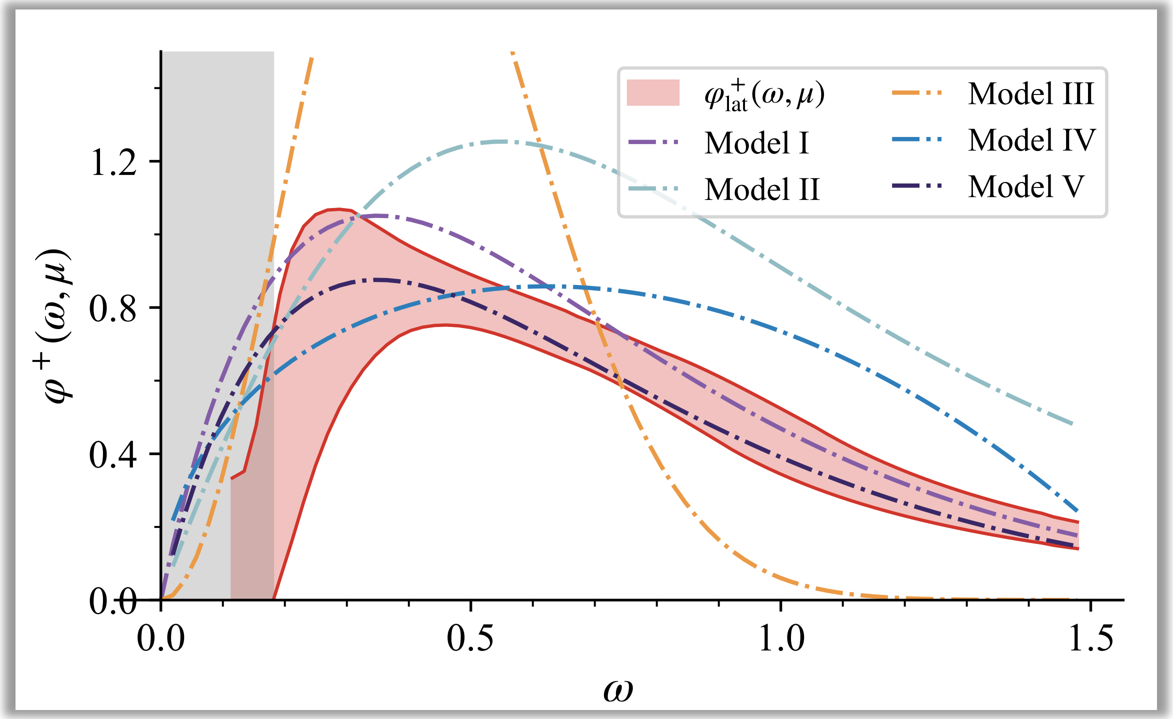
$$\varphi^+_{\rm tail}(\omega,\mu) = \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right]$$

Lee, Neubert, PRD 72, 094028 (2005)



Discussions I: comparison with models

- The model dependence contributes the largest systematic error in the form factors.
- Our result is basically consistent with most of the model estimates, and will also provide a first-principle constrains on the existing models.



- For theoretical calculations, result from first-principles will help to REMOVE the primary uncertainties arising from the model parametrizations.

Discussions II: Inverse and inverse-logarithmic moments

- Significant uncertainties from λ_B and σ_1: Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035

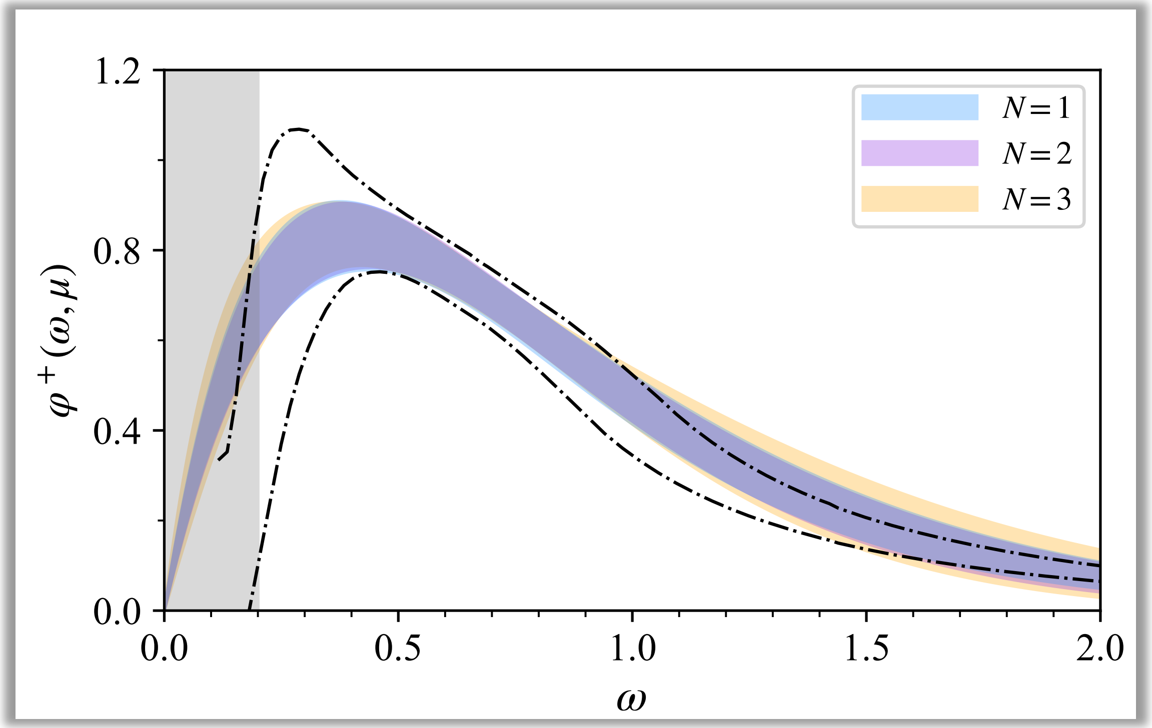
$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

- A model-independent parametrization form:

$$\varphi^+(\omega,\mu) = \sum_{n=1}^{N} c_n \frac{\omega^n}{\omega_0^{n+1}} e^{-\omega/\omega_0}$$
$$= \frac{c_1\omega}{\omega_0^2}\left[1 + c_2'\frac{\omega}{\omega_0} + c_3'\left(\frac{\omega}{\omega_0}\right)^2 + \cdots\right] e^{-\omega/\omega_0},$$

Fit results up to the N-th order:

$$N=1:\ \omega_0 = 0.403(44),\ c_1 = 0.932(73);$$
$$N=2:\ \omega_0 = 0.352(82),\ c_1 = 0.69(37),\ c_2' = 0.17(32);$$
$$N=3:\ \omega_0 = 0.32(15),\ c_1 = 0.63(44),\ c_2' = 0.12(37), c_3' = 0.04(19).$$



Discussions II: Inverse and inverse-logarithmic moments

Numerical results of λ_B and $\sigma_B^{(1)}$ at $\mu = 1\text{GeV}$:

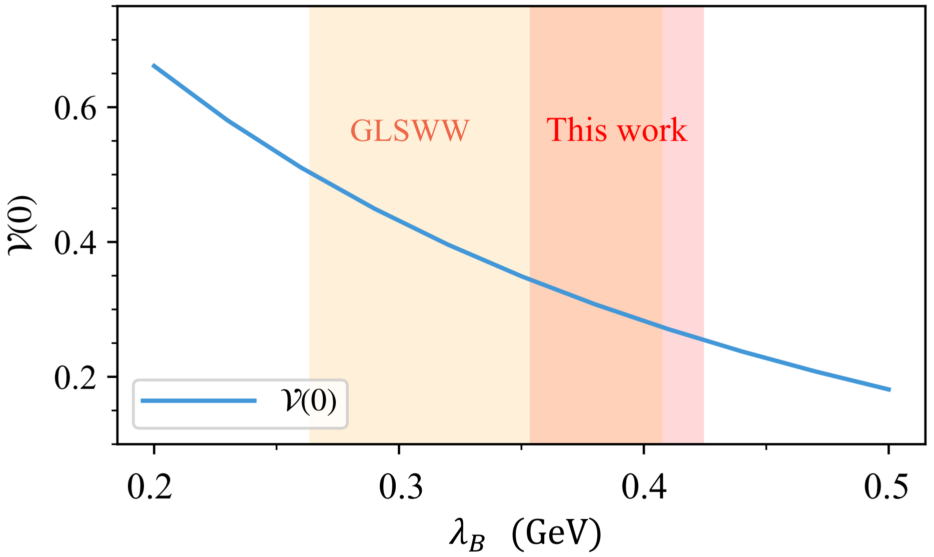
		λ_B (GeV)	$\sigma_B^{(1)}$
Our results	*N=1*	0.389(35)	1.63(8)
	N=2	0.393(37)	1.62(7)
	N=3	0.381(59)	1.63(12)
Experiment	*Belle 2018*	> 0.24	
Other theoretical approach	*Khodjamirian, Mandal, Mannel, 2020*	0.383(153)	
	Gao, Lu, Shen, Wang, Wei, 2020	$0.343^{+0.064}_{-0.079}$	
	Lee, Neubert, 2005	0.48(11)	1.6(2)
	Braun, Ivanov, Korchemsky, 2004	0.46(11)	1.4(4)
	Grozin, Neubert, 1997	0.35(15)	
	Mandal, Nandi, Ray, 2024	0.338(68)	

Discussions III: Impact on $B \to V$ form factors

An accurate λ_B will significantly improve the prediction for the $B \to K^*$ form factors:

	GLSWW		Our result
λ_B:	0.343^{+64}_{-79}	$\rightarrow$	$0.389(35)$
Error of $\mathcal{V}(0)$:	0.23	$\rightarrow$	0.11

Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035



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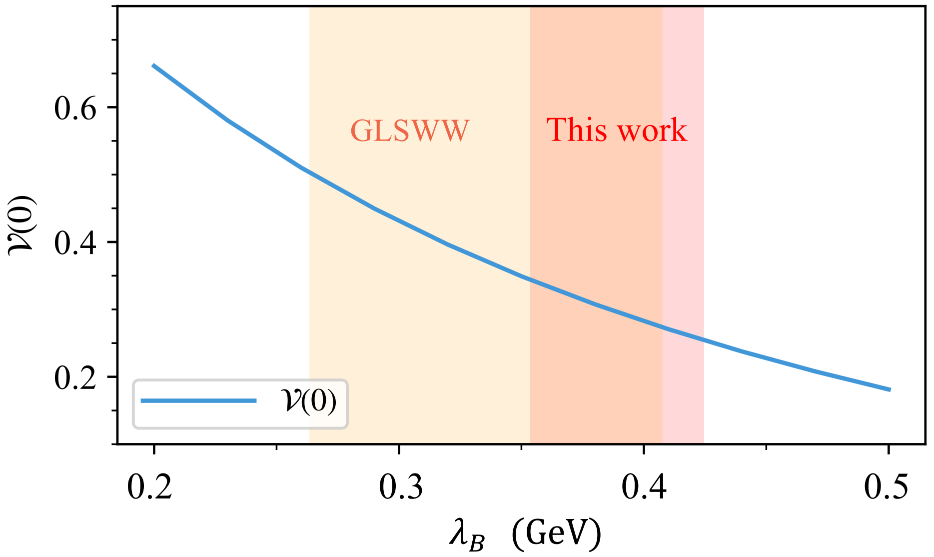
Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035



We are looking forward to a more precise analysis of the form factors and accordingly physical observables.

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

Our results can:

- REDUCE the errors from λ_B and $\sigma_B^{(n)}$;
- REMOVE the errors from model dependence.

Summary and Prospect

✓ We present a first lattice-implementable method to extract the heavy meson LCDA, and implement it on a CLQCD ensemble.

✓ Although the results are preliminary, they can be continually improved.

✓ The phenomenological implications demonstrate that our results will significantly advance the theoretical studies towards the frontier of high precision.

More importantly, improving the reliability of our results for the next stage:

- How to properly control the power corrections within two step factorization?
- More systematic lattice QCD calculations: more a, larger P^z, …

Thanks for your attention!