G2HDM和左右对称弱电等标准模型拓展中的线算子谱

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Outline

- The generalized symmetry and line operators
- The line operator in the Standard Model
- The line operator in models beyond Standard model
- The spontaneously symmetry breaking
- The θ angle
- Conclusion

The conserved current and charge

- The Noether current for a symmetry is a 1-form

$$J = J_\mu dx^\mu$$

- The Hodge dual

$$*J = \frac{1}{(D-1)!} J_\mu \epsilon^\mu_{\mu_1 \cdots \mu_{D-1}} dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_{D-1}}$$

  Is a closed (D-1)-form
- The conservation law

$$d*J = \partial_\mu J^\mu dx^0 \wedge \cdots \wedge dx^{D-1} = 0$$

- The Noether charge is an integration over a $(D-1)$ submanifold Σ

$$Q(\Sigma) = \int_\Sigma *J = \int_\Sigma \frac{1}{(D-1)!} J_\mu \epsilon^\mu_{\mu_1 \cdots \mu_{D-1}} dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_{D-1}}$$

Higher form symmetry

- **From 0-form to 1-form**
- 0-form global symmetry: the charged object is point supported, parameter ξ closed 0-form, $d\xi = 0$; conservation law $d * J = 0$ by localization ξ, $\delta S = \int_{\mathcal{M}^{(\mathcal{D})}} * J \wedge d\xi$
- 1-form global symmetry: parameter $\xi_1 = \xi_\mu dx^\mu$ closed 1-form; localization ξ_1 ; conservation law $d * J = 0$ by $\delta S = \int_{\mathcal{M}^{(\mathcal{D})}} * J \wedge d\xi_1$; $* J$ is (D-2)-form, J is a 2-form; $\partial_\mu J^{\mu\nu} = 0, J^{\mu\nu} = -J^{\nu\mu}$
- The conserved charge $Q(\Sigma_{D-2}) \equiv \int_{\Sigma_{D-2}} * J$ for the closed $D - 2$ dim submanifold
- The charged object has non-trivial link with Σ_{D-2}

The charged operator for 1-form symmetry

- The charged object for 1-form symmetry is the operator supported along line, the line operator
- The transformation parameter is $\int_{\mathcal{C}} \ \xi_i(\Sigma_{D-2}) = \int_{\mathcal{C}} \ \xi_i dx^i$ for a operator supported along line $\mathcal{C}$
- The infinitesimal transformation for the line operator : $W[C] \to W[C]' = W[C] + \int_C \ \xi_i(\Sigma_{d-1})\, \delta W[C]$、
- Ward Identity for line defect $\langle \partial_\mu J^{\mu\nu}(x) W[C] \rangle \ = -i \int_C \ dy^\nu \delta^{(D)}(x-y) \langle \delta W[C] \rangle$

Dirac quantum condition

- **The magnetic monopole**

$$B = \frac{g\boldsymbol{r}}{4\pi r^3} \Rightarrow \int d\boldsymbol{S} \cdot \boldsymbol{B} = g$$

- **Dirac quantum condition**

$$eg = 2\pi\hbar\mathbb{Z}, \quad e_1 g_2 - e_2 g_1 = 2\pi\hbar\mathbb{Z}$$

- Theta term

$$S_\theta = \frac{\theta e^2}{4\pi^2\hbar}\int d^4x \frac{1}{4} * F^{\mu\nu}F_{\mu\nu} = -\frac{\theta e^2}{4\pi^2\hbar c}\int d^4x \boldsymbol{E}\cdot\boldsymbol{B}$$

- Theta term is a total derivative

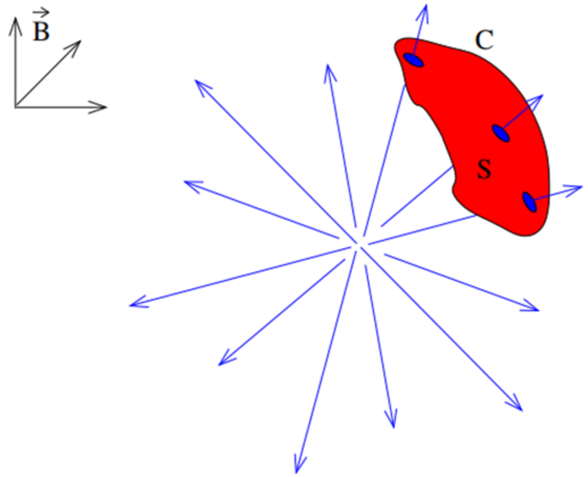
$$S_\theta = \frac{\theta e^2}{8\pi^2\hbar}\int d^4x \partial_\mu\left(\epsilon^{\mu\nu\rho\sigma}A_\nu\partial_\rho A_\sigma\right)$$

The magnetic monopole

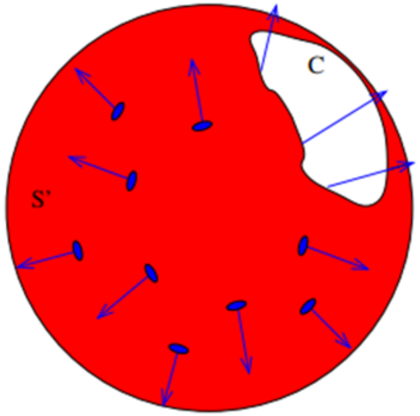
- For a magnetic monopole

$$\boldsymbol{B} = \frac{g\hat{\boldsymbol{r}}}{4\pi r^3} \Rightarrow \int d\boldsymbol{S} \cdot \boldsymbol{B} = g$$

- The charged particle with charge e moves along a closed path with solid angle Ω one round, the charged particle acquires a phase factor



$$e^{\mathrm{i}e\alpha/\hbar}, \quad \alpha = \int_S d\boldsymbol{S} \cdot \boldsymbol{B} = \frac{\Omega g}{4\pi}$$



Dirac quantum condition

- The path also the boundary of surface with solid angle $\Omega' = 4\pi - \Omega$, the acquired phase should be $\alpha' = -\frac{(4\pi - \Omega)g}{4\pi}$
- The two phase factors must be the same, $e^{\mathrm{i}e\alpha/\hbar} = e^{\mathrm{i}e\alpha\prime/\hbar}$, gives the Dirac quantum condition

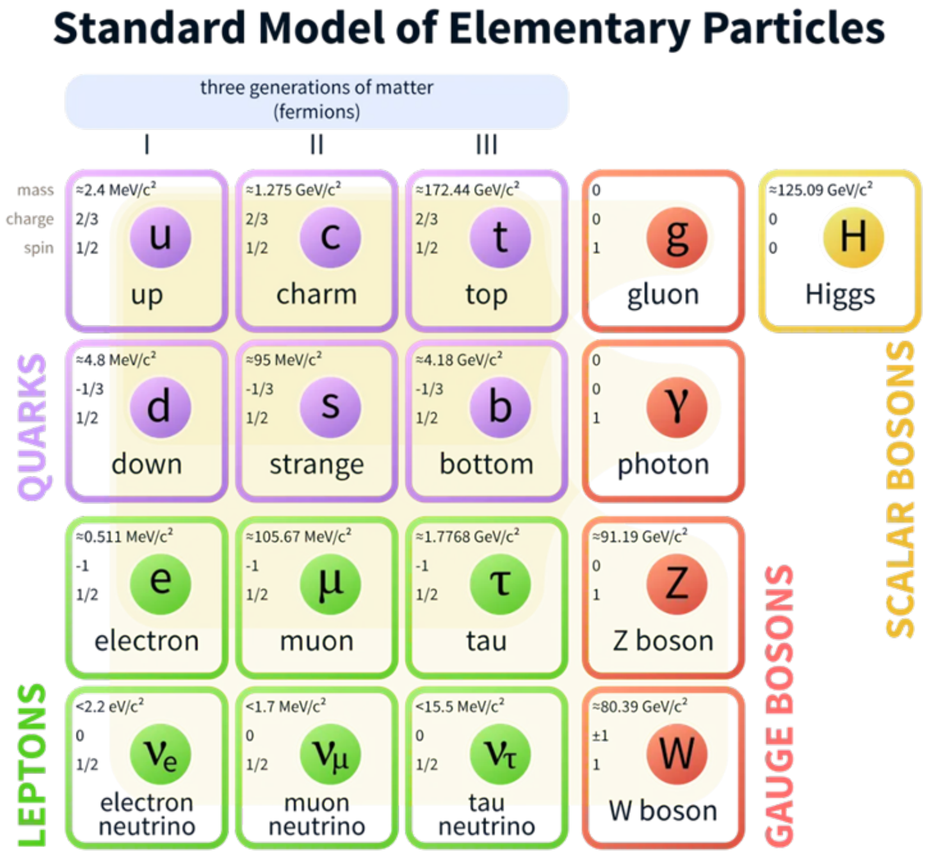
$$eg = 2\pi\hbar n, \quad n \in \boldsymbol{Z}$$

- Similarly, for two dyon carrying electric charge and magnetic charge, (e_1, g_1) and (e_2, g_2), Dirac quantum condition is extended as

$$e_1 g_2 - e_2 g_1 \in 2\pi\hbar\boldsymbol{Z}$$

The Standard Model

- The gauge group for Standard Model is $\tilde{G} = SU(3)_C \times SU(2)_L \times U(1)_Y$



The particle content of the Standard Model

Matter Fields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Q_L	3	2	1
U_R	3	1	4
D_R	3	1	–2
L_L	1	2	–3
E_R	1	1	–6
H	1	2	3

The gauge group of the Standard Model

- If the gauge group of the Standard Model is taken as one of $G = \tilde{G}/\Gamma$, where $\Gamma = \mathbf{1}, \boldsymbol{Z}_2, \boldsymbol{Z}_3, \boldsymbol{Z}_6$, is the center or the subgroups of the center of $\tilde{G}$, the dynamics of the gauge theory will be the same

- Different choice of Γ, the global property of the gauge theory is different., with different line operator spectrum (Wilson line,'t Hooft line)

Line operators

- the gauge group of the Standard Model $G = \tilde{G}/\Gamma$, $\Gamma = \mathbf{1}, \boldsymbol{Z}_2, \boldsymbol{Z}_3, \boldsymbol{Z}_6$ the subgroups of the center of $\tilde{G}$, the same local dynamics
- Different Γ, different global property of the gauge theory, different line operator spectrum (Wilson line,'t Hooft line)
- Wilson line: $W[C] = Tr_{R_e} P e^{\mathrm{i} \oint a}$, exist for all representations R_e, labelled by points in the lattice Λ_w/W, W the Weyl group of $\tilde{G}$
- Representations of $G = \tilde{G}/\Gamma$ labelled by some sublattice Λ_w/W
- Similarly, 't Hooft line: $T[C] = Tr_{R_m} P e^{\mathrm{i} \oint \tilde{a}}$, labelled by some sublattice of Λ_{mw}/W , Λ_{mw} dual of the root lattice

Extended Dirac quantization condition

- Simplest way to introduce 't Hooft line: line extended along time direction
- On two charts to cover S^2, The magnetic monopole penitential
- $a_r^N = a_\theta^N = 0,\ a_\phi^N = \frac{Q}{4\pi r}\frac{(1-\cos\theta)}{\sin\theta}$ and $a_r^S = a_\theta^S = 0,\ a_\phi^S = -\frac{Q}{4\pi r}\frac{(1+\cos\theta)}{\sin\theta}$
- A Wilson line along the overlapping region $\left(\theta = \frac{\pi}{2}\right)$, a closed path should lead to the same result using either a^N or a^S
- $\mathrm{Tr}\, P\exp\left(i\int_0^{2\pi} a_\phi^N\right) = \mathrm{Tr}\, P\exp\left(i\int_0^{2\pi} a_\phi^S\right) \Rightarrow \mathrm{Tr}\, e^{\frac{i\vec{m}\cdot\vec{H}}{2}} = \mathrm{Tr}\, e^{\frac{-i\vec{m}\cdot\vec{H}}{2}}$
- the magnetic charge $Q = \vec{m}\cdot\vec{H}$, the Cantan generator, satisfies $\vec{m}\cdot\vec{\mu} = 2\pi\mathbb{Z}$ or $\vec{m} \equiv 2\pi\frac{2\vec{\alpha}}{\vec{\alpha}^2}$
- t' Hoot lines must lie in the adjoint representation

Labelling of the line operators

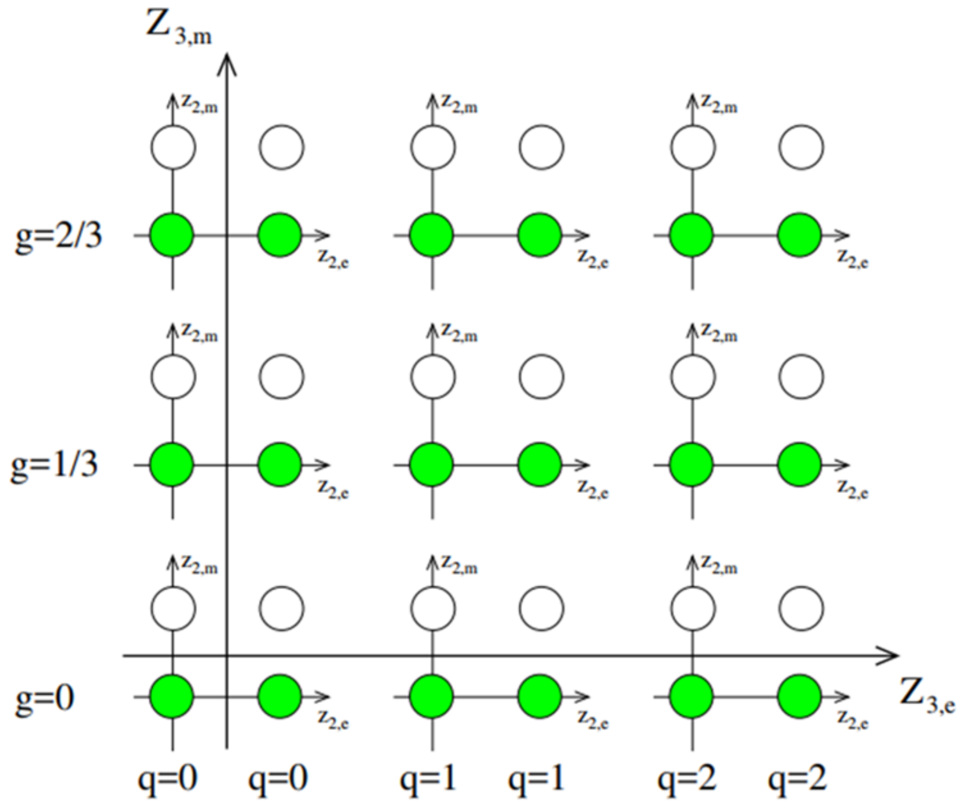
- The line operators as heavy electric and magnetic test particles
- the Wilson lines characterized by $\Lambda_w/\Lambda_r = Z(\tilde{G})$
- 't Hooft lines characterized by $\Lambda_{\mathrm{mw}}/\Lambda_{\mathrm{cr}} = Z(\tilde{G})$
- Generally, dyonic Wilson-'t Hooft line, as $(\lambda^e, \lambda^m) \in \Lambda_w \times \Lambda_{\mathrm{mw}}$, with $(\lambda^e, \lambda^m) \sim (w\lambda^e, w\lambda^m)$, $w \in W$
- line operators labelled by $(z^e, z^m) \in Z(\tilde{G}) \times Z(\tilde{G})$
- For $Z(\tilde{G}) = \mathbf{Z}_N$, both z^e and z^m integers mod N. Two lines (z^e, z^m) and (z'^e, z'^m) can both exist as operators only if
- $z^e z'^m - z^m z'^e = 0 \mathrm{mod} N$

The line operator of the Standard Model

- The generator of Γ can be written as: $e^{2\mathrm{i}\pi\frac{q}{6}}e^{\mathrm{i}\pi z_2^e}e^{\frac{2}{3}\mathrm{i}\pi z_3^e}$
- Wilson line of Standard model labelled by $(\mathrm{z}_2^e, \mathrm{z}_3^e, q)$， ‘t Hooft line by $(\mathrm{z}_2^m, \mathrm{z}_3^m, g)$ satisfying Dirac quantum condition: $3z_2^e z_2^m + 2z_3^e z_3^m - 6gq \in 6\boldsymbol{Z}$
- For $G = \tilde{G}/\Gamma$，Wilson line is invariant under element of Γ (e.g. for $\Gamma = \boldsymbol{Z}_3$，generator is $e^{\frac{2}{3}\mathrm{i}\pi q}e^{\frac{2}{3}\mathrm{i}\pi z_3^e}$=$e^{\frac{2}{3}\mathrm{i}\pi(q+z_3^e)}$，charges satisfying $e^{\frac{2}{3}i\pi(q+\mathrm{z}_3^e)} = e^{\frac{4}{3}i\pi(q+\mathrm{z}_3^e)} = 1$)constrain on electric charges: $q = z_3^e \mathrm{mod}3$
- Solving Dirac quantum condition，gives the constrain on magnetic charge：$3g = z_3^m \mathrm{mod}3$

The spectrum of line operator in the Standard Model

- E.g. for $\Gamma = Z_3$



The Theta term of the Standard Model

- The θ term for $U(1)\times SU(N)$ is $S_\theta = \frac{\theta_N}{16\pi^2}\int tr(\,^*f_N f_N) + \frac{\widetilde{\theta}}{16\pi^2}\int \,^*ff$
- For $\Gamma = 1, \theta \in [0,2\pi)$, $\theta = 0, \pi$ theory is time reversal invariant
- $\Gamma = \boldsymbol{Z}_N, \theta$ term is $S_\theta = \frac{\theta_N}{16\pi^2}\int tr(\,^*gg) + \frac{\widetilde{\theta}-N\theta_N}{16\pi^2N^2}\int \,^*(trg)(trg)$
- The range for θ changes as Γ; so does the time reversal invariant θ
- For example: $\Gamma = \boldsymbol{Z}_2$, $\tilde{\theta} \in [0,8\pi)$, the time reversal invariant $\tilde{\theta} = 0,4\pi$
- The line spectrum also depends on θ by the Witten effect

The G2HDM Model

- Gauge group for G2HDM： $\tilde{G} = SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_H \times U(1)_X$

Matter Fields	$\mathrm{SU}(3)_C$	$\mathrm{SU}(2)_L$	$\mathrm{SU}(2)_H$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_X$	h-parity
$Q_L = (u_L \ \ d_L)^{\mathrm{T}}$	3	2	1	1/6	0	$+\,+$
$U_R = \left(u_R \ \ u_R^H\right)^{\mathrm{T}}$	3	1	2	2/3	1	$+\,-$
$D_R = \left(d_R^H \ \ d_R\right)^{\mathrm{T}}$	3	1	2	$-1/3$	-1	$-\,+$
u_L^H	3	1	1	2/3	0	$-$
d_L^H	3	1	1	$-1/3$	0	$-$
$L_L = (\nu_L \ \ e_L)^{\mathrm{T}}$	1	2	1	$-1/2$	0	$+\,+$
$N_R = \left(\nu_R \ \ \nu_R^H\right)^{\mathrm{T}}$	1	1	2	0	1	$+\,-$
$E_R = \left(e_R^H \ \ e_R\right)^{\mathrm{T}}$	1	1	2	-1	-1	$-\,+$
ν_L^H	1	1	1	0	0	$-$
e_L^H	1	1	1	-1	0	$-$
$H = (H_1 \ \ H_2)^{\mathrm{T}}$	1	2	2	1/2	1	$+\,-$
$\Phi_H = (\Phi_1 \ \ \Phi_2)^{\mathrm{T}}$	1	1	2	0	1	$-\,+$
$\mathcal{S}$	1	1	1	0	0	$+$

The choices of gauge groups for G2HDM

$$G=\frac{U(1)_Y\times SU(2)_L\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_H\times U(1)_X}{\boldsymbol{Z}_m}(1)=\frac{U(1)_Y\times SU(2)_H\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_L\times U(1)_X}{\boldsymbol{Z}_m}(2)$$

$$=\frac{U(1)_X\times SU(2)_L\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_H\times U(1)_Y}{\boldsymbol{Z}_m}(3)=\frac{U(1)_X\times SU(2)_H\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_L\times U(1)_Y}{\boldsymbol{Z}_m}(4)$$

$$=\frac{U(1)_Y\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_H\times U(1)_X}{\boldsymbol{Z}_m}\times\frac{SU(2)_L}{\boldsymbol{Z}_n}(5)=\frac{U(1)_Y\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_L\times U(1)_X}{\boldsymbol{Z}_m}\times\frac{SU(2)_H}{\boldsymbol{Z}_n}(6)$$

$$=\frac{U(1)_X\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_H\times U(1)_Y}{\boldsymbol{Z}_m}\times\frac{SU(2)_L}{\boldsymbol{Z}_n}(7)=\frac{U(1)_X\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_L\times U(1)_X}{\boldsymbol{Z}_m}\times\frac{SU(2)_Y}{\boldsymbol{Z}_n}(8)$$

$$=\frac{U(1)_Y\times SU(2)_L}{\boldsymbol{Z}_p}\times\frac{SU(2)_H\times U(1)_X}{\boldsymbol{Z}_m}\times\frac{SU(3)_C}{\boldsymbol{Z}_n}(9)=\frac{U(1)_Y\times SU(2)_H}{\boldsymbol{Z}_p}\times\frac{SU(2)_L\times U(1)_X}{\boldsymbol{Z}_m}\times\frac{SU(3)_C}{\boldsymbol{Z}_n}(10)$$

The choices of gauge groups for G2HDM

$$=\frac{U(1)_V\times SU(2)_L\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_H\times U(1)_A}{\boldsymbol{Z}_m}(11)=\frac{U(1)_V\times SU(2)_H\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_L\times U(1)_A}{\boldsymbol{Z}_m}(12)$$

$$=\frac{U(1)_A\times SU(2)_L\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_H\times U(1)_V}{\boldsymbol{Z}_m}(13)=\frac{U(1)_A\times SU(2)_H\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_L\times U(1)_V}{\boldsymbol{Z}_m}(14)$$

$$=\frac{U(1)_V\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_H\times U(1)_A}{\boldsymbol{Z}_m}\times\frac{SU(2)_L}{\boldsymbol{Z}_n}(15)=\frac{U(1)_V\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_L\times U(1)_A}{\boldsymbol{Z}_m}\times\frac{SU(2)_H}{\boldsymbol{Z}_n}(16)$$

$$=\frac{U(1)_A\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_H\times U(1)_V}{\boldsymbol{Z}_m}\times\frac{SU(2)_L}{\boldsymbol{Z}_n}(17)=\frac{U(1)_A\times SU(3)_C}{\boldsymbol{Z}_p}\times\frac{SU(2)_L\times U(1)_V}{\boldsymbol{Z}_m}\times\frac{SU(2)_H}{\boldsymbol{Z}_n}(18)$$

$$=\frac{U(1)_V\times SU(2)_L}{\boldsymbol{Z}_p}\times\frac{SU(2)_H\times U(1)_A}{\boldsymbol{Z}_m}\times\frac{SU(3)_C}{\boldsymbol{Z}_n}(19)=\frac{U(1)_V\times SU(2)_H}{\boldsymbol{Z}_p}\times\frac{SU(2)_L\times U(1)_A}{\boldsymbol{Z}_m}\times\frac{SU(3)_C}{\boldsymbol{Z}_n}(20)$$

Dirac quantum condition for G2HDM

- Dirac quantum condition : $-6gq + 3z_2^e z_2^m + 2z_3^e z_3^m + 3x_2^e x_2^m - 6kh = 0 \mathrm{mod} 6$

- Take $G = \frac{U(1)_Y \times SU(2)_L \times SU(3)_C}{\boldsymbol{Z}_p} \times \frac{SU(2)_H \times U(1)_X}{\boldsymbol{Z}_m}$ as example

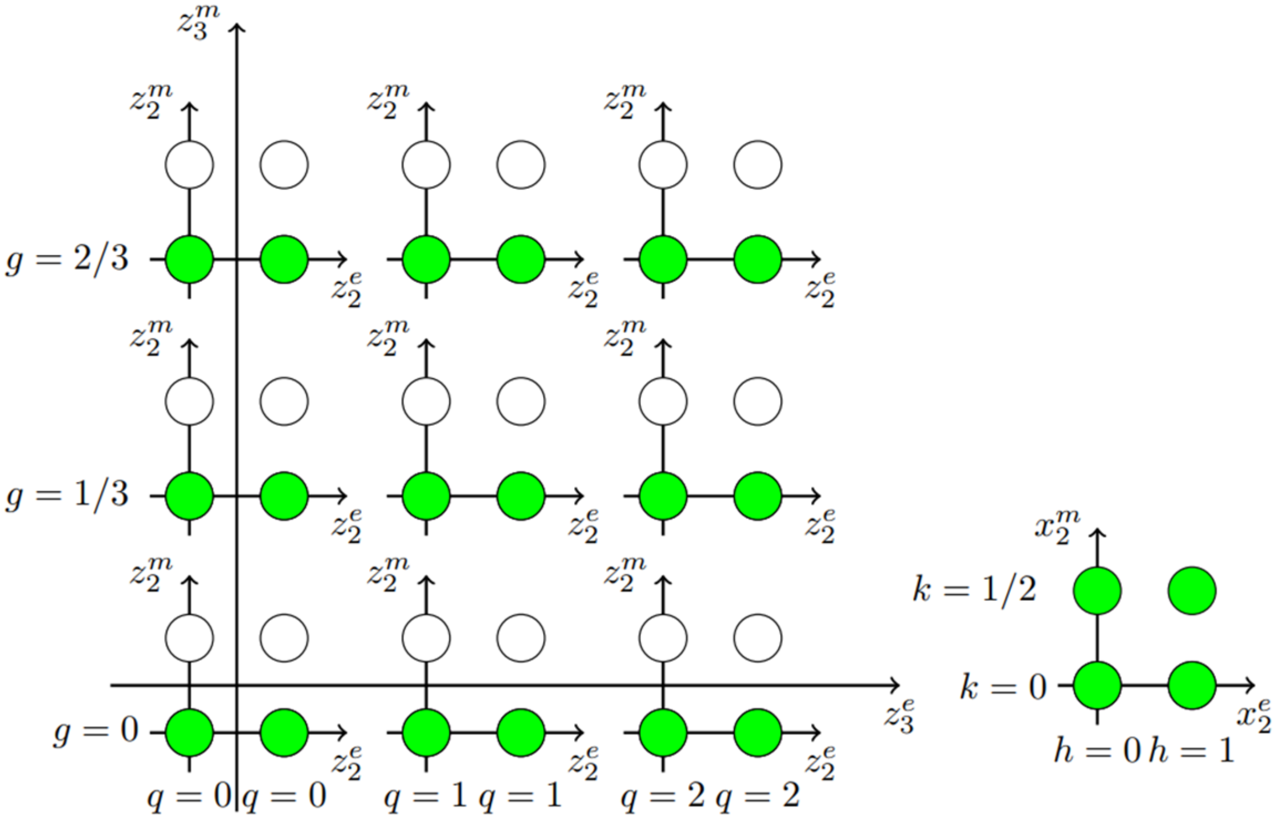
- 8 choices of center $\Gamma = \boldsymbol{1}, \boldsymbol{Z}_{2L}, \boldsymbol{Z}_{2H}, \boldsymbol{Z}_3, \boldsymbol{Z}_{2L} \times \boldsymbol{Z}_{2H}, \boldsymbol{Z}_{6L}, \boldsymbol{Z}_3 \times \boldsymbol{Z}_{2H}, \boldsymbol{Z}_{6L} \times \boldsymbol{Z}_{2H}$

- Two corresponding generators $e^{2\mathrm{i}\pi\frac{q}{6}} e^{\mathrm{i}\pi z_2^e} e^{\frac{2}{3}\mathrm{i}\pi z_3^e}$, $e^{2\mathrm{i}\pi\frac{h}{2}} e^{\mathrm{i}\pi x_2^e}$

Spectrum of line operator for G2HDM

- E.g. $\Gamma = \boldsymbol{Z}_3 \times \boldsymbol{Z}_{2H}$, two generators are $e^{\frac{2}{3}\mathrm{i}\pi q} e^{\frac{2}{3}\mathrm{i}\pi z_3^e} = e^{\frac{2}{3}\mathrm{i}\pi(q+z_3^e)}$， $e^{\mathrm{i}\pi x_2^e} e^{\mathrm{i}\pi h} = e^{\frac{1}{3}\mathrm{i}\pi(3x_2^e+3h)}$
- Constrain on electric charges, $q = z_3^e \mathrm{mod} 3$， $h = x_2^e \mathrm{mod} 2$
- Corresponding constrain on magnetic charges, $3g = z_3^m \mathrm{mod} 3$， $2k = x_2^m \mathrm{mod} 2$
- $\tilde{\theta}_Y \in [0,18\pi)$, $\tilde{\theta}_X \in [0,8\pi)$. The angles that makes the theory invariant under time reversal are $\begin{cases} \theta_3 = 0 \to \tilde{\theta}_Y = 0, 9\pi \\ \theta_3 = \pi \to \tilde{\theta}_Y = 3\pi, 12\pi \end{cases}$; $\begin{cases} \theta_{2H} = 0 \to \tilde{\theta}_X = 0, 4\pi \\ \theta_{2H} = \pi \to \tilde{\theta}_X = 2\pi, 6\pi \end{cases}$

Spectrum of line operator for G2HDM



$\Gamma = \boldsymbol{Z}_{2H} \times \boldsymbol{Z}_{3C}$

Symmetry breaking for G2HDM

- Two steps breaking： first step breaks the dark sector; second step breaks the electroweak
- The Gellman-Nishijima: $Q = \frac{q}{6} + \frac{1}{2} z_2^e$， $Q_D = \frac{h}{2} + \frac{1}{2} x_2^e$
- Higgs$(2,2,1)_{3,1}$ and $(1,2,1)_{0,1}$ determine the deconfined 't Hooft line: $6g = z_2^m mod 2$， $2k = x_2^m mod 2$
- The magnetic charges for deconfined 't Hooft line: $G = 6g$， $G_D = 2k$

Spontaneous symmetry breaking for G2HDM

- E.g. $G = \frac{U(1)_Y \times SU(2)_L \times SU(3)_C}{\boldsymbol{Z}_3} \times \frac{SU(2)_H \times U(1)_X}{\boldsymbol{Z}_{2H}}$
- Constraint on X hypercharge : $h = x_2^e \text{mod} 2$, smallest $Q_D = \frac{2}{2} + 0 = 1$

  corresponding Wilson line $(\boldsymbol{1}, \boldsymbol{1}, \boldsymbol{1})_{0,2}$
- Constraint on dark magnetic charge : $2k = x_2^m \text{mod} 2$, the smallest $G_D = 1$
- Constraint on X hypercharge : $q = z_3^e \text{mod} 3$, the smallest $Q = \frac{1}{6} + 0 = \frac{1}{6}$

  corresponding Wilson line $(\boldsymbol{1}, \boldsymbol{3})_1$
- Constraint on dark magnetic charge : $3g = z_3^m \text{mod} 3$, the smallest $G = 2$

The cases hypercharges mixing

- E.g. $G = \frac{U(1)_V \times SU(2)_L \times SU(3)_C}{\boldsymbol{Z}_p} \times \frac{SU(2)_H \times U(1)_A}{\boldsymbol{Z}_m}$
- The generators: $\xi = e^{2\mathrm{i}\pi\frac{q+3h}{6}} \otimes \eta \otimes \omega = e^{2\mathrm{i}\pi\frac{q+3h}{6}} e^{\mathrm{i}\pi z_2^e} e^{\frac{2}{3}\mathrm{i}\pi z_3^e}$, $\chi = e^{2\mathrm{i}\pi\frac{q-3h}{2}} \otimes \rho = e^{2\mathrm{i}\pi\frac{q-3h}{2}} e^{\mathrm{i}\pi x_2^e}$
- Dirac quantization condition: $-6g_+(q+3h) + 3z_2^e z_2^m + 2z_3^e z_3^m + 3x_2^e x_2^m - 6g_-(q-3h) = 0 \mathrm{mod} 6$
- Gell-Mann–Nishijima formula: $Q = \frac{q}{6} + \frac{1}{2} z_2^e$, $Q_D = \frac{h}{2} + \frac{1}{2} x_2^e$

SSB for hypercharges mixing

- Case of $\Gamma = \boldsymbol{Z}_3 \times \boldsymbol{Z}_{2H}$:
- $(q - 3h) = x_2^e \mathrm{mod}2$, $(q + 3h) = z_3^e \mathrm{mod}3$
- $2g_- = x_2^m \mathrm{mod}2$, $3g_+ = z_3^m \mathrm{mod}3$
- The smallest dark electric charge: $Q_D = \frac{1}{2} + 0 = \frac{1}{2}$, Wilson line $(\mathbf{1}, \mathbf{1}, \mathbf{1})_{0,1}$; the smallest dark magnetic charge: $G_D = 1$
- The smallest electric charge: $Q = \frac{1}{6} + 0 = \frac{1}{6}$, Wilson line $(\mathbf{1}, \mathbf{3})_1$; the smallest dark magnetic charge: $G = 2$

The left-right symmetric model

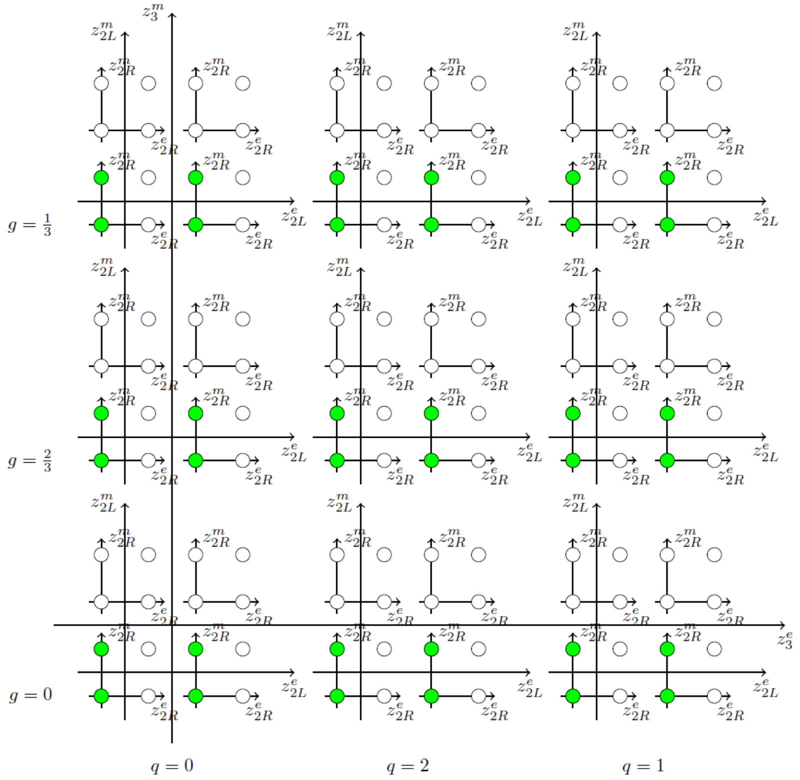
- beyond the Standard Model by adding a right-handed $SU(2)_R$
- $\tilde{G} = SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
- $G = \frac{\tilde{G}}{\Gamma}$, where $\Gamma = 1, Z_{2L}, Z_{2R}, Z_{2L} \times Z_{2R}, Z_3, Z_{2L} \times Z_3, Z_{2R} \times Z_3,$ $Z_{2L} \times Z_{2R} \times Z_3$
- different combinations of the two $SU(2)$ groups with other gauge groups will lead to different spectrum of line operators
- take $\frac{q}{3} = B - L$, $q \in Z$ be the electric charge of $U(1)_{B-L}$, and g be the magnetic charge of $U(1)_{B-L}$
- Dirac quantum condition: $2z_3^e z_3^m + 3z_{2L}^e z_{2L}^m + 3z_{2R}^e z_{2R}^m - 6qg \in 6Z$

The choice of Γ in left-right symmetric model

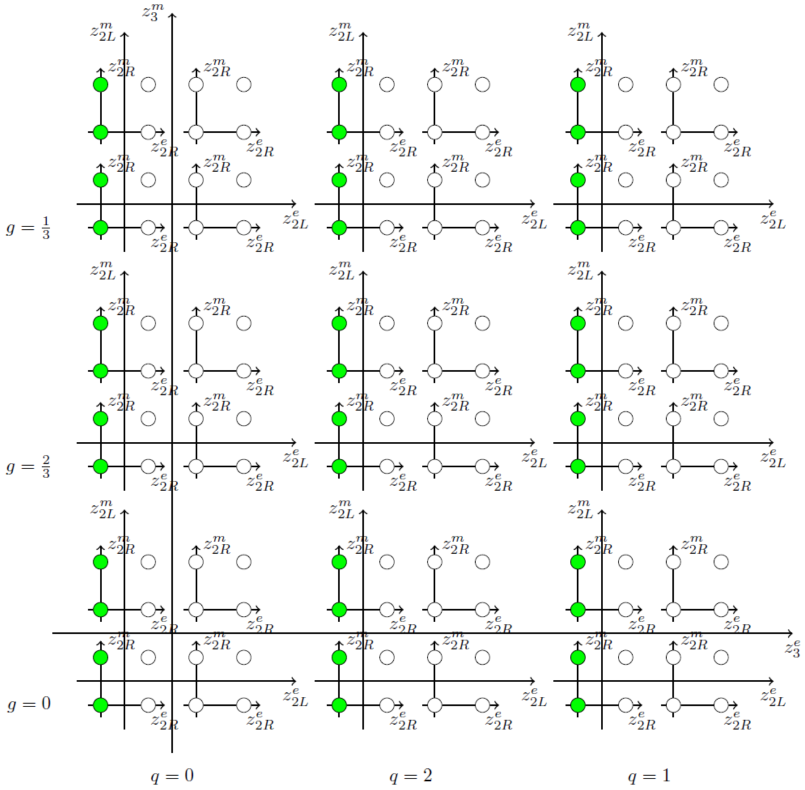
- Choice of Γ, constrain on admissible line spectrum
- E.g. for $\tilde{G} = (SU(3) \times SU(2)_L \times U(1)_{B-L}) \times SU(2)_R$
- Generators of the centre, $\xi = e^{\frac{2}{3}\pi i z_3^e} \otimes e^{\frac{2}{2}\pi i z_{2L}^e} \otimes e^{\frac{2}{3}\pi i q}$, $\chi = e^{\frac{2}{2}\pi i z_{2R}^e}$
- For $\tilde{G} = (SU(3) \times SU(2)_L) \times (SU(2)_R \times U(1)_{B-L})$
- Generators of the centre, $\xi = e^{\frac{2}{3}\pi i z_3^e} \otimes e^{\frac{2}{2}\pi i z_{2L}^e}$, $\chi = e^{\frac{2}{2}\pi i z_{2R}^e} \otimes e^{\frac{2}{3}\pi i q}$

Spectrum of line operator for L-R symmetric model

- $\tilde{G} = (SU(3) \times SU(2)_L \times U(1)_{B-L}) \times SU(2)_R$



$\Gamma = Z_{2R} \times Z_3$



$\Gamma = Z_{2L} \times Z_{2R} \times Z_3$

Symmetry breaking for L-R model

- The Gellman-Nishijima of two steps symmetry breaking:
- $Q_{em} = I_{3L} + \frac{q_Y}{6}, \frac{q_Y}{6} = I_{3R} + \frac{B-L}{2} = \frac{1}{2} z_{2R}^{e} + \frac{q}{6}$
- Higgs $\Delta_R \in (\mathbf{1}_L, \mathbf{3}_R, 2)$ determine the deconfined ' t Hooft line: $6g = z_2^m + n$, $g_Y = g$ in first step
- The magnetic charges for deconfined 't Hooft line: $g_Y = g$ for the first step
- Second step by Higgs triplet $\left(\mathbf{2}_{SU(2)_L}, \mathbf{1}_{SU(3)}, 3_{U(1)_Y}\right)$ with $G_{em} = 6g_Y$

First step of symmetry breaking for $\tilde{G} = (SU(3) \times SU(2)_L \times U(1)_{B-L}) \times (SU(2)_R)$

- $\Gamma = 1, Z_{2R}$: $z_{2L}^e = 0,1; z_3^e = 0,1,2; q_Y \in Z; z_{2L}^m, z_3^m = 0; g_Y \in Z$
- $\Gamma = Z_{2L}, Z_{2L} \times Z_{2R}$: $z_{2L}^e = 0; z_3^e = 0,1,2; q_Y \in Z; z_{2L}^m = 0,1; z_3^m = 0; g_Y \in Z$
- $\Gamma = Z_3, Z_{2R} \times Z_3$: $z_{2L}^e = 0,1; z_3^e = 0,1,2; q_Y = 3n - z_3^e; z_{2L}^m = 0; z_3^m = 0,1,2; g_Y = m - \frac{1}{3} z_3^m$
- $\Gamma = Z_{2L} \times Z_3, Z_{2L} \times Z_{2R} \times Z_3$: $z_{2L}^e = 0; z_3^e = 0,1,2; q_Y = 3n - z_3^e; z_{2L}^m = 0,1; z_3^m = 0,1,2; g_Y = m - \frac{1}{3} z_3^m$

Second step of symmetry breaking for $\tilde{G} = (SU(3) \times SU(2)_L \times U(1)_{B-L}) \times (SU(2)_R)$

- $\Gamma = 1, Z_{2R}$: $z_3^e = 0,1,2; Q_{em} = \frac{n}{6}; z_3^m = 0; G_{em} = 6m$
- $\Gamma = \Gamma = Z_3, Z_{2L} \times Z_3$: $z_3^e = 0,1,2; \begin{cases} z_3^e = 0 \rightarrow Q_{em} = 0 \\ z_3^e = 1 \rightarrow Q_{em} = \frac{2}{6} \\ z_3^e = 2 \rightarrow Q_{em} = \frac{1}{6} \end{cases};$

$$z_3^m = 0,1,2; \begin{cases} z_3^m = 0 \rightarrow G_{em} = 6 \\ z_3^m = 1 \rightarrow G_{em} = 4 \\ z_3^m = 2 \rightarrow G_{em} = 2 \end{cases}$$

The Theta angle for G2HDM(Similar discussion on L-R symmetric model)

- 5 theta angles, $\tilde{\theta}$,θ_2,θ_3,$\widetilde{\omega}$ and ω_2, for $U(1)_Y$,$SU(2)_L$,$SU(3)_C$,$U(1)_X$ and $SU(2)_H$, respectively.
- the quotient gauge group of the G2HDM: $\frac{U(1)\times\cdots}{Z_N}\times\frac{U(1)\times\cdots}{Z_M}\times\cdots$
- Theta terms of the G2HDM
- $S_\theta = \frac{\widetilde{\theta}}{16\pi^2}\int dx^4\,\widetilde{{}^\star f}\tilde{f} + \frac{3\theta_2}{16\pi^2}\int dx^4 tr({}^\star f_2 f_2) + \frac{2\theta_3}{16\pi^2}\int dx^4 tr({}^\star f_3 f_3) + \frac{\widetilde{\omega}}{16\pi^2}\int dx^4\,\widetilde{{}^\star g}\tilde{g} + \frac{\omega_2}{16\pi^2}\int dx^4 tr({}^\star g g)$
- Where $\tilde{f}$, f_2 , f_3 are the strength of $U(1)_Y$, $SU(2)_L$, $SU(3)_C$ field, respectively. And $\tilde{g}$,g for $U(1)_X$ and $SU(2)_H$, respectively.

The Theta angle for G2HDM

- Theta terms of $\frac{U(1)\times SU(N)}{Z_N}$:
- $S_\theta = \frac{\theta_N}{16\pi^2}\int d^4x \mathrm{tr}(\,^\star FF) + \frac{\tilde{\theta}-N\theta_N}{16\pi^2 N^2}\int d^4x^\star(\mathrm{tr}F)(\mathrm{tr}F)$
- Strength Fderived from field $a + \tilde{a}1_N$. a and $\tilde{a}$ are $U(1)$ and $SU(N)$ gauge fields, respectively. $\theta_N \in [0, 2\pi)$,$\tilde{\theta} \in [0, 2\pi N^2)$,
- For $\frac{U(1)\times SU(N)\times SU(M)}{Z_{N\times M}}$, theta terms of $U(1)\times SU(N)\times SU(M)$ theory are
- $S_\theta = \frac{M\theta_N}{16\pi^2}\int dx^4 tr(\,^\star f_N 1_M f_N 1_M) + \frac{N\theta_M}{16\pi^2}\int dx^4 tr(\,^\star f_M 1_N f_M 1_N) + \frac{\tilde{\theta}}{16\pi^2}\int dx^4 \,\widetilde{^\star f}\tilde{f}$

The Theta angle for G2HDM

- To describe this new $U(N \times M)$ gauge theory, introduce gauge fields $a_N + \tilde{a}1_N$ and $a_M + \tilde{a}1_M$, with the corresponding strengths $F_N = f_N + \tilde{f}1_N$ and $F_M = f_M + \tilde{f}1_M$. Theta terms for $\frac{U(1)\times SU(N)\times SU(M)}{Z_{N\times M}}$ theory:
- $S_\theta = \frac{M\theta_N}{16\pi^2}\int dx^4 tr(\,^\star F_N 1_M F_N 1_M) + \frac{N\theta_M}{16\pi^2}\int dx^4 tr(\,^\star F_M 1_N F_M 1_N) + \frac{\tilde{\theta}-NM^2\theta_N-MN^2\theta_M}{16\pi^2(N\times M)^2}\int dx^4\,^\star tr\left(\tilde{f}1_{N\times M}\right)tr\left(\tilde{f}1_{N\times M}\right)$
- In the last term $tr\left(\tilde{f}1_{N\times M}\right)$ equals to $tr(F_N 1_M)$ or $tr(F_M 1_N)$
- obtain $\theta_N \in [0, 2\pi)$,$\theta_M \in [0, 2\pi)$ while $\tilde{\theta} \in [0, 2\pi N^2 M^2)$

The CP invariant Theta angle for G2HDM

- theta terms vary with respect to quotient groups.
- $G = \frac{U(1)_Y \times SU(2)_L \times SU(3)_C}{Z_N} \times \frac{SU(2)_H \times U(1)_X}{Z_M}$
- for example, $\Gamma = Z_{2L} \times Z_{2H}$
- There are two $U(2)$ θ angles, $\tilde{\theta}$ and $\widetilde{\omega}$
- CP invariant Theta angles: $\theta_3 = 0, \pi$
- $\tilde{\theta} = 0, 4\pi$ for $\theta_2 = 0$; $\tilde{\theta} = 2\pi, 6\pi$ for $\theta_2 = \pi$
- $\widetilde{\omega} = 0, 4\pi$ for $\omega_2 = 0$ and $\widetilde{\omega} = 2\pi, 6\pi$ for $\omega_2 = \pi$
- similar to the theories of $\Gamma = Z_{2L}$ and $\Gamma = Z_{2H}$

Conclusion and outlook

- The spectrum of line operator in G2HDM model is obtained for all the possible cases
- The variation of Theta angles under different center group is presented for all the possible cases
- The spectrum of line operator under spontaneously symmetry breaking is discussed for all the possible cases
- The similar treatment also apply to the ledt-right symmetric model of elctro-weak interaction with $\tilde{G} = SU(3) \times SU(2)_L \times U(1)_{B-L} \times SU(2)_R$
- The inclusion of axion couplings can result the non-invertible symmetry

- THANKS