

An $\widehat{su}(8)_1$ theory of the SM quarks and leptons

an endeavor to the SM flavor puzzle

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talk at Liaoning Normal University, 2024.11.02



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References

Earlier works:

- a “Towards a Grand Unified Theory of Flavor”, Nucl.Phys.B 156 (1979) 126, Howard Georgi.
- b “Possible $SU(4) \times SU(3) \times U(1)$ model”, Phys. Rev. D 23 (1981) 2690, Chong-Shou Gao, Kuang-Chao Chou.
- c “An $SU(8)$ Grand Unified Model Accommodating Three Generations With $V - A$ Coupling”, Phys. Lett. B 100 (1981) 399-402, Z.-q Ma, T.-s Tu, P.-y Xue, X.-j Zhou.
- d “Gauge Model of Generation Nonuniversality”, Phys. Rev. Lett. 47 (1981) 1788, X. Li and E. Ma.
- e “Doubly Lopsided Mass Matrices from Unitary Unification”, Phys. Rev. D 78 (2008) 075001, 0804.1356, Stephen Barr.

References

Recent papers:

- α “The global $B - L$ symmetry in the flavor-unified $SU(N)$ theories”, JHEP 04 (2024) 046, 2307.07921, **NC**, Ying-nan Mao, Zhaolong Teng.
- β “The Standard Model quark/lepton masses and the Cabibbo-Kobayashi-Maskawa mixing in an $SU(8)$ theory”, 2402.10471, **NC**, Ying-nan Mao, Zhaolong Teng.
- γ “The gauge coupling evolutions of an $SU(8)$ theory with the maximally symmetry breaking pattern”, JHEP 10 (2024) 149, 2406.09970, **NC**, Zhanpeng Hou, Ying-nan Mao, Zhaolong Teng.
- ρ “Further study of the maximally symmetry breaking patterns in an $SU(8)$ theory”, 2409.03172, **NC**, Zhiyuan Chen, Zhanpeng Hou, Ying-nan Mao, Zhaolong Teng.
- δ “The unification in an $\widehat{\mathfrak{su}}(8)_1$ affine Lie algebra”, in preparation, **NC**, Zhanpeng Hou, Zhaolong Teng.

Historical reviews

GUTs were proposed in terms of the simple gauge groups of $SU(5)$ with $3 \times [\overline{\mathbf{5}}_{\mathbf{F}} \oplus \mathbf{10}_{\mathbf{F}}]$ by ['74, Georgi-Glashow] (GG), and $SO(10)$ with $3 \times \mathbf{16}_{\mathbf{F}}$ by ['75, Fritzsch-Minkowski]. The main ingredients are

- i The gauge symmetries $\mathcal{G}_{\text{SM}} \equiv SU(3)_c \otimes SU(2)_W \otimes U(1)_Y \subset \mathcal{G}_{\text{GUT}}$ can be unified based on the AF of the QCD ['73, Gross, Wilczek, Politzer]. The SUSY extension to the $SU(5)$ can unify three SM gauge couplings at $\mu \sim 10^{16}$ GeV ['81, Dimopoulos-Georgi], with sparticle masses $\sim \mathcal{O}(1)$ TeV.
- ii The two-generational chiral fermions (with charm quark theorized in '70 by Glashow-Iliopoulos-Maiani, and discovered in late '74),

$$\overline{\mathbf{5}}_{\mathbf{F}} = \underbrace{(\overline{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})_{\mathbf{F}}}_{d_{R^c}} \oplus \underbrace{(\mathbf{1}, \overline{\mathbf{2}}, -\frac{1}{2})_{\mathbf{F}}}_{\ell_L} \text{ and}$$

$$\mathbf{10}_{\mathbf{F}} = \underbrace{(\mathbf{3}, \mathbf{2}, +\frac{1}{6})_{\mathbf{F}}}_{q_L} \oplus \underbrace{(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})_{\mathbf{F}}}_{u_{R^c}} \oplus \underbrace{(\mathbf{1}, \mathbf{1}, +1)_{\mathbf{F}}}_{e_{R^c}}.$$

Historical reviews

- Besides of the well-acknowledged challenges within the *minimal* GUTs, there are two longstanding problems within the SM that have never been solved with/without the susy extension, which are: (i) the SM flavor puzzle (quarks+leptons+neutrinos), (ii) the PQ quality problem of the QCD axion.
- My understanding: the flavor sector is the “rough water” sector of the SM where every BSM builder must confront with in your preferred framework, according to S. Weinberg’s lesson two.
- The formulation of the QM solved several fundamental puzzles in the late 19th century: (i) blackbody radiation [1900, Max Planck], (ii) photoelectric effects [1905, Albert Einstein], (iii) hydrogen spectrum [1913, Niels Bohr], by hypothesizing the quantized energies/angular momenta of particles.

The flavor puzzle: origin

- The SM flavor puzzle: (i) inter-generational mass hierarchies, (ii) intra-generational mass hierarchies with non-universal splitting patterns, and (iii) the CKM mixing pattern of the quarks and the PMNS mixing pattern of the neutrinos.
- Why/how $n_g = 3$? Both the SM and the *minimal* GUTs exhibit the simple repetitive structure in terms of their chiral irreducible anomaly-free fermion sets (IRAFFSs).

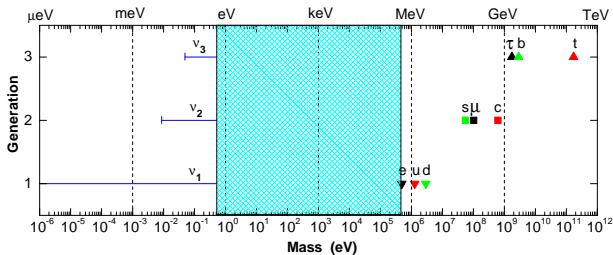


Figure: The SM fermion mass spectrum, [1909.09610, Z.Z. Xing].

The flavor puzzle: Yukawa couplings

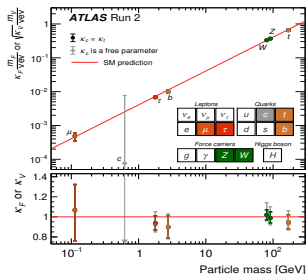


Figure: The LHC measurements of the SM Higgs boson, [2207.00092].

- The hierarchical/non-universal Yukawa couplings of the *single* SM Higgs boson $y_f = \sqrt{2}m_f/v_{EW}$ for all SM quarks/leptons.
- Symmetry dictates interactions [‘80, Chen-Ning Yang]: flavor non-universality in the BSM.

The origin of generations

- The main conjecture: the flavor non-universality in the UV theories can be naturally realized w.o. the simply repetitive structure, à la the generations, such as in the $SU(N > 5)$ GUT [‘79, Georgi, ‘80, Nanopolous].
- The anti-symmetric chiral fermions of

$$\{f_L\}_{SU(N)} = \sum_k n_k [N, k]_{\mathbf{F}}, \quad n_k \in \mathbb{Z}. \quad (1)$$

No exotic fermions in the spectrum with the $[N, k]_{\mathbf{F}}$.

- The anomaly-free condition can be expressed in terms of a Diophantine equation

$$\sum_k n_k \text{Anom}([N, k]_{\mathbf{F}}) = 0, \quad (2)$$

$$\text{Anom}([N, k]_{\mathbf{F}}) = \frac{(N - 2k)(N - 3)!}{(N - k - 1)!(k - 1)!}. \quad (3)$$

Georgi's counting of the SM generations

- To decompose the $SU(N)$ irreps under the $SU(5)$, e.g., $\mathbf{N}_F = (N - 5) \times \mathbf{1}_F \oplus \mathbf{5}_F$. Decompositions of other irreps can be obtained by tensor products, [79, Georgi].
- All fermion irreps in Eq. (1) can be decomposed into the $SU(5)$ irreps of $(\mathbf{1}_F, \mathbf{5}_F, \mathbf{10}_F, \overline{\mathbf{10}}_F, \overline{\mathbf{5}}_F)$, and we denote their multiplicities as $(\nu_{\mathbf{1}_F}, \nu_{\mathbf{5}_F}, \nu_{\mathbf{10}_F}, \nu_{\overline{\mathbf{10}}_F}, \nu_{\overline{\mathbf{5}}_F})$.
- Their multiplicities should satisfy $\nu_{\mathbf{5}_F} + \nu_{\mathbf{10}_F} = \nu_{\overline{\mathbf{5}}_F} + \nu_{\overline{\mathbf{10}}_F}$ from the anomaly-free condition.
- The total SM fermion generations are determined by the net $\overline{\mathbf{5}}_F$'s or net $\mathbf{10}_F$'s

$$n_g = \nu_{\overline{\mathbf{5}}_F} - \nu_{\mathbf{5}_F} = \nu_{\mathbf{10}_F} - \nu_{\overline{\mathbf{10}}_F}. \quad (4)$$

- The SM $\overline{\mathbf{5}}_F$'s are from the $[\overline{N}, 1]_F$, and the SM $\mathbf{10}_F$'s are from the $[N, k \geq 2]_F$.

Georgi's counting of the SM generations in GUTs

- Georgi's "third law" of GUT [79]: no repetition of a particular irrep of $SU(N)$, i.e., $n_k = 0$ or $n_k = 1$ in Eq. (1). Georgi's minimal solution is

$$\{f_L\}_{SU(11)} = [11, 4]_{\mathbf{F}} \oplus [11, 8]_{\mathbf{F}} \oplus [11, 9]_{\mathbf{F}} \oplus [11, 10]_{\mathbf{F}}, \quad (5)$$

with $\dim_{\mathbf{F}} = 561$.

- The $[\overline{5}_{\mathbf{F}} \oplus 10_{\mathbf{F}}]$ is one chiral irreducible anomaly-free fermion set (IRAFFS) of the $SU(5)$, so do one-generational SM fermions.
- A chiral IRAFFS is a set of left-handed anti-symmetric fermions of $\sum_{\mathcal{R}} m_{\mathcal{R}} \mathcal{F}_L(\mathcal{R})$, with the anomaly-free condition of $\sum_{\mathcal{R}} m_{\mathcal{R}} \text{Anom}(\mathcal{F}_L(\mathcal{R})) = 0$. We also demand that
 - A chiral IRAFFS can no longer be partitioned into smaller chiral IRAFFS.
 - No singlet, self-conjugate, or adjoint fermions in a chiral IRAFFS.
- My "third law" [2307.07921]: *only distinctive chiral IRAFFSs without simple repetitions and can lead to $n_g = 3$ at the EW scale are allowed in an $SU(N)$ theory.*

The SU(8) theory

- The SU(8) theory with rank-2 and rank-3 chiral IRAFFSs of

$$\{f_L\}_{\text{SU}(8)}^{n_g=3} = \left[\overline{\mathbf{8}_F}^\omega \oplus \mathbf{28}_F \right] \bigoplus \left[\overline{\mathbf{8}_F}^{\dot{\omega}} \oplus \mathbf{56}_F \right], \quad \dim_{\mathbf{F}} = 156,$$

$$\Omega = (\omega, \dot{\omega}), \quad \omega = (3, \text{IV}, \text{V}, \text{VI}), \quad \dot{\omega} = (\dot{1}, \dot{2}, \text{VII}, \text{VIII}, \text{IX}). \quad (6)$$

- There are no simply repetitive “generations” in the UV, and the generations only emerge when the theory flows to the IR.
- Georgi's decompositions:

$$\begin{aligned} \overline{\mathbf{8}_F}^\Omega &= 3 \times \mathbf{1}_F^\Omega \oplus \overline{\mathbf{5}_F}^\Omega, \\ \mathbf{28}_F &= 3 \times \mathbf{1}_F \oplus 3 \times \mathbf{5}_F \oplus \mathbf{10}_F, \\ \mathbf{56}_F &= \mathbf{1}_F \oplus 3 \times \mathbf{5}_F \oplus 3 \times \mathbf{10}_F \oplus \overline{\mathbf{10}_F}. \end{aligned} \quad (7)$$

Six $(\mathbf{5}_F, \overline{\mathbf{5}_F})$ pairs, one $(\mathbf{10}_F, \overline{\mathbf{10}_F})$ pair from the $\mathbf{56}_F$, and $3 \times [\overline{\mathbf{5}_F} \oplus \mathbf{10}_F]_{\text{SM}}$.

The SU(8) theory

- The global symmetries of the SU(8) theory:

$$\begin{aligned}\tilde{\mathcal{G}}_{\text{flavor}} [\text{SU}(8)] &= \left[\widetilde{\text{SU}}(4)_{\omega} \otimes \tilde{\text{U}}(1)_{T_2} \otimes \tilde{\text{U}}(1)_{\text{PQ}_2} \right] \\ &\otimes \left[\widetilde{\text{SU}}(5)_{\dot{\omega}} \otimes \tilde{\text{U}}(1)_{T_3} \otimes \tilde{\text{U}}(1)_{\text{PQ}_3} \right], \\ [\text{SU}(8)]^2 \cdot \tilde{\text{U}}(1)_{T_{2,3}} &= 0, \quad [\text{SU}(8)]^2 \cdot \tilde{\text{U}}(1)_{\text{PQ}_{2,3}} \neq 0.\end{aligned}\quad (8)$$

- The Higgs fields and the Yukawa couplings:

$$\begin{aligned}-\mathcal{L}_Y &= Y_B \overline{\mathbf{8}_F}^{\omega} \mathbf{28}_F \overline{\mathbf{8}_H}_{,\omega} + Y_T \mathbf{28}_F \mathbf{28}_F \mathbf{70}_H \\ &+ Y_D \overline{\mathbf{8}_F}^{\dot{\omega}} \mathbf{56}_F \overline{\mathbf{28}_H}_{,\dot{\omega}} + \frac{c_4}{M_{\text{pl}}} \mathbf{56}_F \mathbf{56}_F \overline{\mathbf{28}_H}^{\dagger}_{,\dot{\omega}} \mathbf{63}_H + H.c..\end{aligned}\quad (9)$$

NB: $\mathbf{56}_F \mathbf{56}_F \mathbf{28}_H = 0$ ['08, S. Barr], $d = 5$ operator suppressed by $1/M_{\text{pl}}$ is possible, with $M_{\text{pl}} = (8\pi G_N)^{1/2} = 2.4 \times 10^{18}$ GeV.

- Gravity breaks global symmetries.*

Global symmetries in the SU(8) theory

Fermions	$\overline{\mathbf{8}}_{\mathbf{F}}^{\Omega=\omega, \dot{\omega}}$	$\mathbf{28}_{\mathbf{F}}$	$\mathbf{56}_{\mathbf{F}}$	
$\widetilde{\mathbf{U}}(1)_T$	$-3t$	$+2t$	$+t$	
$\widetilde{\mathbf{U}}(1)_{\text{PQ}}$	p	q_2	q_3	
Higgs	$\overline{\mathbf{8}}_{\mathbf{H}, \omega}$	$\overline{\mathbf{28}}_{\mathbf{H}, \dot{\omega}}$	$\mathbf{70}_{\mathbf{H}}$	$\mathbf{63}_{\mathbf{H}}$
$\widetilde{\mathbf{U}}(1)_T$	$+t$	$+2t$	$-4t$	0
$\widetilde{\mathbf{U}}(1)_{\text{PQ}}$	$-(p + q_2)$	$-(p + q_3)$	$-2q_2$	0

Table: The $\widetilde{\mathbf{U}}(1)_T$ and the $\widetilde{\mathbf{U}}(1)_{\text{PQ}}$ charges, $p : q_2 \neq -3 : +2$ and $p : q_3 \neq -3 : +1$.

- The symmetry breaking pattern [‘74, L.F.Li] of $\text{SU}(8) \rightarrow \mathcal{G}_{441} \rightarrow \mathcal{G}_{341} \rightarrow \mathcal{G}_{331} \rightarrow \mathcal{G}_{\text{SM}} \rightarrow \text{SU}(3)_c \otimes \text{U}(1)_{\text{EM}}$. Other patterns are also likely with the SUSY extension [‘81, Witten].

Global symmetries in the SU(8) theory

- The global $\tilde{U}(1)_T$ symmetries at different stages

$$\begin{aligned}
 \text{SU}(8) &\rightarrow \mathcal{G}_{441_{X_0}} : \mathcal{T}' = \mathcal{T} - 4t\mathcal{X}_0, \\
 \mathcal{G}_{441_{X_0}} &\rightarrow \mathcal{G}_{341_{X_1}} : \mathcal{T}'' = \mathcal{T}' + 8t\mathcal{X}_1, \\
 \mathcal{G}_{341_{X_1}} &\rightarrow \mathcal{G}_{331_{X_2}} : \mathcal{T}''' = \mathcal{T}'' , \quad \mathcal{G}_{331_{X_2}} \rightarrow \mathcal{G}_{\text{SM}} : \mathcal{B} - \mathcal{L} = \mathcal{T}''' . \quad (10)
 \end{aligned}$$

Consistent relations of $(\mathcal{B} - \mathcal{L})(q_L) = \frac{4}{3}t$, $(\mathcal{B} - \mathcal{L})(\ell_L) = -4t$, and etc.

Higgs	$\mathcal{G}_{441} \rightarrow \mathcal{G}_{341}$	$\mathcal{G}_{341} \rightarrow \mathcal{G}_{331}$	$\mathcal{G}_{331} \rightarrow \mathcal{G}_{\text{SM}}$	$\mathcal{G}_{\text{SM}} \rightarrow$ $\text{SU}(3)_c \otimes \text{U}(1)_{\text{EM}}$
$\overline{8}_{\text{H},\omega}$	✓	✓	✓	✓
$28_{\text{H},\dot{\omega}}$	✗	✓	✓	✓
70_{H}	✗	✗	✗	✓

Table: The ✓ and ✗ represent possible and impossible symmetry breaking directions for a given Higgs field in the SU(8) theory.

The SU(8) Higgs fields

- Decompositions of $\overline{\mathbf{8}}_{\mathbf{H},\omega}/\overline{\mathbf{28}}_{\mathbf{H},\dot{\omega}}$

$$\begin{aligned}\overline{\mathbf{8}}_{\mathbf{H},\omega} &\supset \underline{(\overline{\mathbf{4}}, \mathbf{1}, +\frac{1}{4})_{\mathbf{H},\omega}} \oplus \underline{(\mathbf{1}, \overline{\mathbf{4}}, -\frac{1}{4})_{\mathbf{H},\omega}} \supset \underline{(\mathbf{1}, \overline{\mathbf{3}}, -\frac{1}{3})_{\mathbf{H},\omega}}, \\ \overline{\mathbf{28}}_{\mathbf{H},\dot{\omega}} &\supset \underline{(\mathbf{1}, \mathbf{6}, -\frac{1}{2})_{\mathbf{H},\dot{\omega}}} \oplus \underline{(\mathbf{1}, \overline{\mathbf{4}}, -\frac{1}{4})_{\mathbf{H},\dot{\omega}}} \\ &\supset \left[\underline{(\mathbf{1}, \overline{\mathbf{3}}, -\frac{1}{3})'_{\mathbf{H},\dot{\omega}}} \oplus \underline{(\mathbf{1}, \mathbf{3}, -\frac{2}{3})_{\mathbf{H},\dot{\omega}}} \right] \oplus \underline{(\mathbf{1}, \overline{\mathbf{3}}, -\frac{1}{3})_{\mathbf{H},\dot{\omega}}}. \quad (11)\end{aligned}$$

- The global $\widetilde{\text{U}}(1)_{B-L}$ according to Eq. (10)

$$\begin{aligned}\mathbf{70}_{\mathbf{H}} &\supset \underline{(4, \overline{\mathbf{4}}, +\frac{1}{2})_{\mathbf{H}}} \oplus (\overline{\mathbf{4}}, \mathbf{4}, -\frac{1}{2})_{\mathbf{H}} \supset \dots \\ &\supset \underbrace{(\mathbf{1}, \overline{\mathbf{2}}, +\frac{1}{2})'''_{\mathbf{H}}}_{B-L=0} \oplus \underbrace{(\mathbf{1}, \mathbf{2}, -\frac{1}{2})'''_{\mathbf{H}}}_{B-L=-8t}. \quad (12)\end{aligned}$$

Conjecture: the $(\mathbf{1}, \overline{\mathbf{2}}, +\frac{1}{2})'''_{\mathbf{H}} \subset \mathbf{70}_{\mathbf{H}}$ is the unique SM Higgs doublet.

The SU(8) Higgs fields

- The VEV assignments

$$\mathcal{G}_{441} \rightarrow \mathcal{G}_{341} : \langle (\bar{\mathbf{4}}, \mathbf{1}, +\frac{1}{4})_{\mathbf{H}, \text{IV}} \rangle \equiv \frac{1}{\sqrt{2}} W_{\bar{\mathbf{4}}, \text{IV}}, \quad \zeta_1 \equiv \frac{W_{\bar{\mathbf{4}}}}{M_{\text{pl}}} \quad (13a)$$

$$\mathcal{G}_{341} \rightarrow \mathcal{G}_{331} : \langle (\mathbf{1}, \bar{\mathbf{4}}, -\frac{1}{4})_{\mathbf{H}, \text{V}, \mathbf{i}, \text{VII}} \rangle \equiv \frac{1}{\sqrt{2}} w_{\bar{\mathbf{4}}, \text{V}, \mathbf{i}, \text{VII}}, \quad \zeta_2 \equiv \frac{w_{\bar{\mathbf{4}}}}{M_{\text{pl}}} \quad (13b)$$

$$\mathcal{G}_{331} \rightarrow \mathcal{G}_{\text{SM}} : \langle (\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{\mathbf{H}, \mathbf{3}, \text{VI}, \text{IX}} \rangle \equiv \frac{1}{\sqrt{2}} V_{\bar{\mathbf{3}}, \mathbf{3}, \text{VI}, \text{IX}}$$

$$\langle (\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})'_{\mathbf{H}, \mathbf{2}, \text{VIII}} \rangle \equiv \frac{1}{\sqrt{2}} V'_{\bar{\mathbf{3}}, \mathbf{2}, \text{VIII}}, \quad \zeta_3 \equiv \frac{V_{\bar{\mathbf{3}}}}{M_{\text{pl}}} \quad (13c)$$

$$\text{EWSB} : \langle (\mathbf{1}, \bar{\mathbf{2}}, +\frac{1}{2})'''_{\mathbf{H}} \rangle \equiv \frac{1}{\sqrt{2}} v_{\text{EW}}. \quad (13d)$$

The VEVs in black are the minimal set to integrate out the massive vectorlike fermions. The VEVs in red are necessary for the (d^i, ℓ^i) masses.

Top quark mass in the SU(8) theory

- The natural top quark mass from the tree level

$$\begin{aligned}
 Y_{\mathcal{T}} \mathbf{28}_F \mathbf{28}_F \mathbf{70}_H &\supset Y_{\mathcal{T}} (\mathbf{6}, \mathbf{1}, -\frac{1}{2})_F \otimes (\mathbf{4}, \mathbf{4}, 0)_F \otimes (\mathbf{4}, \bar{\mathbf{4}}, +\frac{1}{2})_H \\
 &\supset \dots \supset Y_{\mathcal{T}} (\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})_F \otimes (\mathbf{3}, \mathbf{2}, +\frac{1}{6})_F \otimes (\mathbf{1}, \bar{\mathbf{2}}, +\frac{1}{2})_H''' \\
 &\Rightarrow \frac{1}{\sqrt{2}} Y_{\mathcal{T}} t_L t_R^c v_{EW}.
 \end{aligned} \tag{14}$$

- With $(\mathbf{3}, \mathbf{2}, +\frac{1}{6})_F \equiv (t_L, b_L)^T$ and $(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})_F \equiv t_R^c$ within the $\mathbf{28}_F$, it is straightforward to infer that $(\mathbf{1}, \mathbf{1}, +1)_F \equiv \tau_R^c$ also lives in the $\mathbf{28}_F$. This explains why do the third-generational SM $\mathbf{10}_F$ reside in the $\mathbf{28}_F$, while the first- and second-generational SM $\mathbf{10}_F$'s must reside in the $\mathbf{56}_F$.
- Top quark mass conjecture: a rank-2 chiral IRAFFS is necessary so that only the top quark obtains mass with the natural Yukawa coupling at the EW scale. The SU(9) with $9 \times \overline{\mathbf{9}}_F \oplus \mathbf{84}_F$ is ruled out [2307.07921].

The SU(8) fermions

SU(8)	\mathcal{G}_{441}	\mathcal{G}_{341}	\mathcal{G}_{331}	\mathcal{G}_{SM}
$8_{\mathbf{F}}^{\Omega}$	$(\bar{4}, 1, +\frac{1}{4})_{\mathbf{F}}^{\Omega}$ $(1, \bar{4}, -\frac{1}{4})_{\mathbf{F}}^{\Omega}$	$(\bar{3}, 1, +\frac{1}{3})_{\mathbf{F}}^{\Omega}$ $(1, 1, 0)_{\mathbf{F}}^{\Omega}$ $(1, \bar{4}, -\frac{1}{4})_{\mathbf{F}}^{\Omega}$	$(\bar{3}, 1, +\frac{1}{3})_{\mathbf{F}}^{\Omega}$ $(1, 1, 0)_{\mathbf{F}}^{\Omega}$ $(1, \bar{3}, -\frac{1}{3})_{\mathbf{F}}^{\Omega}$ $(1, 1, 0)_{\mathbf{F}}^{\Omega''}$	$(\bar{3}, 1, +\frac{1}{3})_{\mathbf{F}}^{\Omega} : \mathcal{D}_R^{\Omega c}$ $(1, 1, 0)_{\mathbf{F}}^{\Omega} : \tilde{\mathcal{N}}_L^{\Omega}$ $(1, \bar{2}, -\frac{1}{2})_{\mathbf{F}}^{\Omega} : \mathcal{L}_L^{\Omega} = (\mathcal{E}_L^{\Omega}, -\mathcal{N}_L^{\Omega})^T$ $(1, 1, 0)_{\mathbf{F}}^{\Omega'} : \tilde{\mathcal{N}}_L^{\Omega'}$ $(1, 1, 0)_{\mathbf{F}}^{\Omega''} : \tilde{\mathcal{N}}_L^{\Omega''}$

SU(8)	\mathcal{G}_{441}	\mathcal{G}_{341}	\mathcal{G}_{331}	\mathcal{G}_{SM}
$28_{\mathbf{F}}$	$(6, 1, -\frac{1}{2})_{\mathbf{F}}$ $(1, 6, +\frac{1}{2})_{\mathbf{F}}$ $(4, 4, 0)_{\mathbf{F}}$	$(3, 1, -\frac{1}{3})_{\mathbf{F}}$ $(\bar{3}, 1, -\frac{2}{3})_{\mathbf{F}}$ $(1, 6, +\frac{1}{2})_{\mathbf{F}}$ $(3, 4, -\frac{1}{12})_{\mathbf{F}}$ $(1, 4, +\frac{1}{4})_{\mathbf{F}}$	$(3, 1, -\frac{1}{3})_{\mathbf{F}}$ $(\bar{3}, 1, -\frac{2}{3})_{\mathbf{F}}$ $(1, 3, +\frac{1}{3})_{\mathbf{F}}$ $(1, \bar{3}, +\frac{2}{3})_{\mathbf{F}}$ $(3, 3, 0)_{\mathbf{F}}$ $(3, 1, -\frac{1}{3})_{\mathbf{F}}''$ $(1, 3, +\frac{1}{3})_{\mathbf{F}}''$ $(1, 1, 0)_{\mathbf{F}}''$	$(3, 1, -\frac{1}{3})_{\mathbf{F}} : \mathcal{D}_L$ $(\bar{3}, 1, -\frac{2}{3})_{\mathbf{F}} : t_R^c$ $(1, 2, +\frac{1}{2})_{\mathbf{F}} : (\epsilon_R^c, n_R^c)^T$ $(1, 1, 0)_{\mathbf{F}} : \bar{n}_R^c$ $(1, 2, +\frac{1}{2})_{\mathbf{F}}' : (n_R^c, -\epsilon_R^c)^T$ $(1, 1, +1)_{\mathbf{F}} : \tau_R^c$ $(3, 2, +\frac{1}{6})_{\mathbf{F}} : (t_L, b_L)^T$ $(3, 1, -\frac{1}{3})_{\mathbf{F}}' : \mathcal{D}'_L$ $(3, 1, -\frac{1}{3})_{\mathbf{F}}'' : \mathcal{D}''_L$ $(1, 2, +\frac{1}{2})_{\mathbf{F}}'' : (\epsilon_R^c, n_R^c)^T$ $(1, 1, 0)_{\mathbf{F}}' : \bar{n}'_R$ $(1, 1, 0)_{\mathbf{F}}'' : \bar{n}''_R$

The SU(8) fermions

SU(8)	\mathcal{G}_{441}	\mathcal{G}_{341}	\mathcal{G}_{331}	\mathcal{G}_{SM}
56_F	$(\mathbf{1}, \mathbf{4}, +\frac{3}{4})_F$	$(\mathbf{1}, \mathbf{4}, +\frac{3}{4})_F$	$(\mathbf{1}, \mathbf{3}, +\frac{2}{3})'_F$	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})'''_F : (\mathbf{n}'''_R, -\mathbf{e}'''_R)^T$
			$(\mathbf{1}, \mathbf{1}, +1)''_F$	$(\mathbf{1}, \mathbf{1}, +1)_F : \mu_R^c$
	$(\bar{\mathbf{4}}, \mathbf{1}, -\frac{3}{4})_F$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})'_F$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})'_F$	$(\mathbf{1}, \mathbf{1}, +1)''_F : \mathbf{e}_R^c$
		$(\mathbf{1}, \mathbf{1}, -1)_F$	$(\mathbf{1}, \mathbf{1}, -1)_F$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})'_F : u_R^c$
	$(\mathbf{4}, \mathbf{6}, +\frac{1}{4})_F$	$(\mathbf{3}, \mathbf{6}, +\frac{1}{6})_F$	$(\mathbf{3}, \mathbf{3}, 0)_F$	$(\mathbf{1}, \mathbf{1}, -1)_F : \mathbf{e}_L$
			$(\mathbf{3}, \bar{\mathbf{3}}, +\frac{1}{3})_F$	$(\mathbf{3}, \mathbf{2}, +\frac{1}{6})_F : (c_L, s_L)^T$
				$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})'''_F : \mathfrak{D}'_L$
				$(\mathbf{3}, \bar{\mathbf{2}}, +\frac{1}{6})''_F : (\mathfrak{d}_L, -u_L)^T$
				$(\mathbf{3}, \mathbf{1}, +\frac{2}{3})_F : \mathfrak{U}_L$
		$(\mathbf{1}, \mathbf{6}, +\frac{1}{2})'_F$	$(\mathbf{1}, \mathbf{3}, +\frac{1}{3})'_F$	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})''''_F : (\mathbf{e}''''_R, \mathbf{n}''''_R)^T$
			$(\mathbf{1}, \bar{\mathbf{3}}, +\frac{2}{3})''_F$	$(\mathbf{1}, \mathbf{1}, 0)'''_F : \mathbf{n}'''_R^c$
				$(\mathbf{1}, \bar{\mathbf{2}}, +\frac{1}{2})''''_F : (\mathbf{n}''''_R, -\mathbf{e}''''_R)^T$
	$(\mathbf{6}, \mathbf{4}, -\frac{1}{4})_F$	$(\mathbf{3}, \mathbf{4}, -\frac{1}{12})'_F$	$(\mathbf{1}, \mathbf{1}, +1)''_F : e_R^c$	
		$(\mathbf{3}, \mathbf{3}, 0)''_F$	$(\mathbf{3}, \mathbf{2}, +\frac{1}{6})'''_F : (u_L, d_L)^T$	
		$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})''''_F$	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})''''_F : \mathfrak{D}''''_L$	
		$(\bar{\mathbf{3}}, \mathbf{3}, -\frac{1}{3})_F$	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})''''_F : \mathfrak{D}''''_L$	
	$(\bar{\mathbf{3}}, \mathbf{4}, -\frac{5}{12})_F$		$(\bar{\mathbf{3}}, \mathbf{2}, -\frac{1}{6})_F : (\mathfrak{d}_R^c, u_R^c)^T$	
		$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})''_F$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})'_F : \mathfrak{U}_R^c$	
			$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})''_F : c_R^c$	

SM fermion masses in the SU(8) theory

- To generate lighter SM fermion masses: the gravity breaks the global symmetries in Eq. (8) explicitly.
- The direct Yukawa couplings of $\mathcal{O}_{\mathcal{F}}^{d=5}$:

$$\begin{aligned}
 c_3 \mathcal{O}_{\mathcal{F}}^{(3,2)} &\equiv c_3 \overline{\mathbf{8}_F}^{\dot{\omega}} \mathbf{56}_F \cdot \overline{\mathbf{28}_H}^{\dot{\omega}} \cdot \mathbf{70}_H^\dagger \\
 \Rightarrow c_3 &\left[\dot{\zeta}_3 (s_L \mathcal{D}_R^{\dot{\omega}c} - \mathcal{E}_L^{\dot{\omega}} \mu_R^c) + \dot{\zeta}'_3 (d_L \mathcal{D}_R^{\dot{\omega}c} - \mathcal{E}_L^{\dot{\omega}} e_R^c) \right] v_{EW}, \\
 c_4 \mathcal{O}_{\mathcal{F}}^{(4,1)} &\equiv c_4 \mathbf{56}_F \mathbf{56}_F \cdot \overline{\mathbf{28}_H}^{\dot{\omega}} \cdot \mathbf{70}_H \Rightarrow c_4 \dot{\zeta}_2 (c_L u_R^c + u_L e_R^e) v_{EW}, \\
 c_5 \mathcal{O}_{\mathcal{F}}^{(5,1)} &\equiv c_5 \mathbf{28}_F \mathbf{56}_F \cdot \overline{\mathbf{8}_H}^{\dot{\omega}} \cdot \mathbf{70}_H \\
 \Rightarrow c_5 &[\zeta_1 (u_L t_R^c + t_L u_R^c) + \zeta_2 (c_L t_R^c + t_L c_R^c)] v_{EW}. \tag{15}
 \end{aligned}$$

- All (u, c, t) obtain hierarchical masses, while all (d^i, ℓ^i) are massless.

SM fermion masses in the SU(8) theory

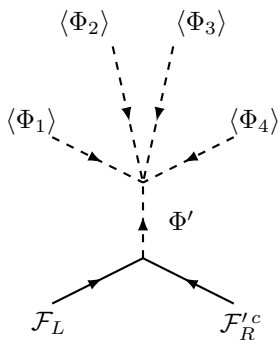


Figure: The indirect Yukawa couplings.

(d^i, ℓ^i) obtain masses through the EWSB components in renormalizable Yukawa couplings of $\overline{\mathbf{8}_F}^\omega \mathbf{28}_F \overline{\mathbf{8}_H}_{,\omega}$ and $\overline{\mathbf{8}_F}^{\dot{\omega}} \mathbf{56}_F \overline{\mathbf{28}_H}_{,\dot{\omega}}$ with the $d = 5$ Higgs mixing operators.

SM fermion masses in the SU(8) theory

- There are two indirect Yukawa couplings with the irreducible Higgs mixing operators of $\mathcal{O}_{\mathcal{H}}^{d=5}$:

$$\begin{aligned}\mathcal{O}_{\mathcal{A}}^{d=5} &\equiv \epsilon_{\omega_1\omega_2\omega_3\omega_4} \overline{\mathbf{8}_{\mathbf{H},\omega_1}}^\dagger \overline{\mathbf{8}_{\mathbf{H},\omega_2}}^\dagger \overline{\mathbf{8}_{\mathbf{H},\omega_3}}^\dagger \overline{\mathbf{8}_{\mathbf{H},\omega_4}}^\dagger \mathbf{70}_{\mathbf{H}}^\dagger, \\ \mathcal{PQ} &= 2(2p + 3q_2) \neq 0,\end{aligned}\tag{16a}$$

$$\begin{aligned}\mathcal{O}_{\mathcal{B}}^{d=5} &\supset (\overline{\mathbf{28}_{\mathbf{H},\mathbf{i}}}^\dagger \overline{\mathbf{28}_{\mathbf{H},\text{VII}}}) \cdot \overline{\mathbf{28}_{\mathbf{H},\text{IX}}}^\dagger \overline{\mathbf{28}_{\mathbf{H},\mathbf{i}}}^\dagger \mathbf{70}_{\mathbf{H}}^\dagger, \\ &(\overline{\mathbf{28}_{\mathbf{H},\mathbf{i}}}^\dagger \overline{\mathbf{28}_{\mathbf{H},\text{VII}}}) \cdot \overline{\mathbf{28}_{\mathbf{H},\text{IX}}}^\dagger \overline{\mathbf{28}_{\mathbf{H},\mathbf{2}}}^\dagger \mathbf{70}_{\mathbf{H}}^\dagger, \\ \mathcal{PQ} &= 2(p + q_2 + q_3).\end{aligned}\tag{16b}$$

- Each operator of $\mathcal{O}_{\mathcal{H}}^{d=5}$
 - breaks the global symmetries explicitly;
 - can not be further partitioned into subset of renormalizable operators, among which any of them can be allowed by both the gauge and the global symmetries. This relies on the VEV assignments in Eqs. (13).

SM fermion masses in the SU(8) theory

- The (u, c, t) masses

$$m_u \approx c_4 \frac{\zeta_2 \dot{\zeta}_2}{2\zeta_1} v_{EW}, \quad m_c \approx c_5^2 \frac{\zeta_1^2}{2\sqrt{2}Y_{\mathcal{T}}} v_{EW}, \quad m_t \approx \frac{Y_{\mathcal{T}}}{\sqrt{2}} v_{EW}. \quad (17)$$

- The (d, s, b) and (e, μ, τ) masses

$$m_d = m_e \approx \frac{c_3 \dot{\zeta}_3}{2} |\tan \lambda| v_{EW}, \quad m_s = m_\mu \approx \frac{1}{4} (2c_3 + Y_{\mathcal{D}} d_{\mathcal{B}} \zeta_{23}^{-2}) \dot{\zeta}_3 v_{EW},$$
$$m_b = m_\tau \approx Y_{\mathcal{B}} \frac{d_{\mathcal{A}} \zeta_1 \zeta_2}{4\zeta_3} v_{EW}. \quad (18)$$

- The CKM mixing:

$$\hat{V}_{\text{CKM}} \Big|_{\text{SU}(8)} \approx \begin{pmatrix} 1 - \lambda^2/2 & \lambda & \frac{c_5}{Y_{\mathcal{T}}} \zeta_2 \\ -\lambda & 1 - \lambda^2/2 & -\frac{c_5}{Y_{\mathcal{T}}} \zeta_1 \\ -\frac{c_5}{Y_{\mathcal{T}}} (\lambda \zeta_1 + \zeta_2) & -\frac{c_5}{Y_{\mathcal{T}}} \zeta_1 & 1 \end{pmatrix}. \quad (19)$$

SM fermion masses in the SU(8) theory: benchmark

ζ_1	ζ_2	ζ_3	$Y_{\mathcal{D}}$	$Y_{\mathcal{B}}$	$Y_{\mathcal{T}}$
6.0×10^{-2}	2.0×10^{-3}	2.0×10^{-5}	0.5	0.5	0.8
c_3	c_4	c_5	$d_{\mathcal{A}}$	$d_{\mathcal{B}}$	λ
1.0	0.2	1.0	0.01	0.01	0.22
m_u	m_c	m_t	$m_d = m_e$	$m_s = m_\mu$	$m_b = m_\tau$
1.6×10^{-3}	0.6	139.2	0.5×10^{-3}	6.4×10^{-2}	1.5
$ V_{ud} $	$ V_{us} $	$ V_{ub} $			
0.98	0.22	3.0×10^{-3}			
$ V_{cd} $	$ V_{cs} $	$ V_{cb} $			
0.22	0.98	7.5×10^{-2}			
$ V_{td} $	$ V_{ts} $	$ V_{tb} $			
0.019	7.5×10^{-2}	1			

Table: The parameters of the SU(8) benchmark point and the predicted SM quark/lepton masses (in unit of GeV) as well as the CKM mixings.

Flavor non-universality in the SU(8)

	u	c	t	
$g_f^{V\mathcal{M}}$	$-\frac{1}{24} + \frac{7}{24}s_{\vartheta_C}^2$	$-\frac{1}{24} + \frac{7}{24}s_{\vartheta_C}^2$	$-\frac{1}{24} + \frac{7}{24}s_{\vartheta_C}^2$	
$g_f^{A\mathcal{M}}$	$\frac{1}{8} + \frac{3}{8}s_{\vartheta_C}^2$	$-\frac{1}{8} + \frac{1}{8}s_{\vartheta_C}^2$	$-\frac{1}{8} + \frac{3}{8}s_{\vartheta_C}^2$	
	d	s	b	
$g_f^{V\mathcal{M}}$	$-\frac{1}{24} - \frac{5}{24}s_{\vartheta_C}^2$	$\frac{1}{12} - \frac{1}{12}s_{\vartheta_C}^2$	$\frac{1}{12} - \frac{5}{24}s_{\vartheta_C}^2$	
$g_f^{A\mathcal{M}}$	$\frac{1}{8} - \frac{1}{8}s_{\vartheta_C}^2$	$-\frac{1}{4}s_{\vartheta_C}^2$	$-\frac{1}{8}s_{\vartheta_C}^2$	
	e	μ	τ	$\nu_{eL}/\nu_{\mu L}/\nu_{\tau L}$
$g_f^{V\mathcal{M}}$	$\frac{1}{8} - \frac{3}{8}s_{\vartheta_C}^2$	$-\frac{1}{2}s_{\vartheta_C}^2$	$-\frac{3}{8}s_{\vartheta_C}^2$	$-\frac{1}{8}s_{\vartheta_C}^2$
$g_f^{A\mathcal{M}}$	$\frac{1}{8} - \frac{1}{8}s_{\vartheta_C}^2$	$-\frac{1}{4}s_{\vartheta_C}^2$	$-\frac{1}{8}s_{\vartheta_C}^2$	$\frac{1}{8}s_{\vartheta_C}^2$

Table: The couplings of the neutral currents mediated by the neutral \mathcal{G}_{441} gauge boson M_μ in the $V - A$ basis, $\tan \vartheta_C \equiv g_{X_0}/(\sqrt{6}g_{4s})$. [2406.09970].

The SU(8) RGEs

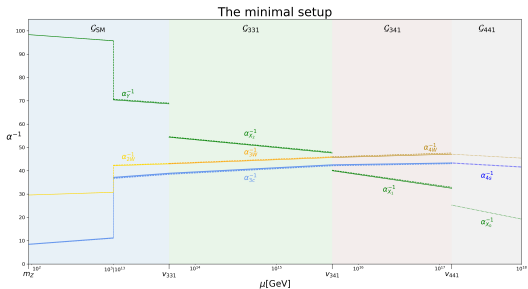
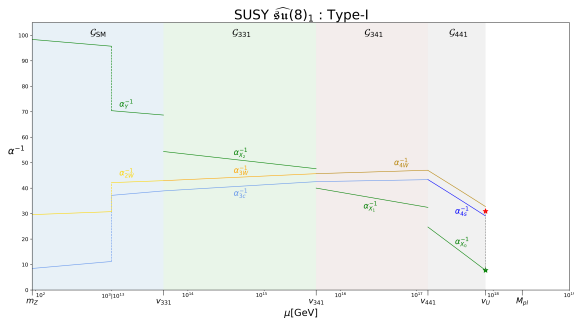


Figure: The minimal non-SUSY SU(8) setup cannot achieve the unification. [2406.09970]

The SUSY $\widehat{\mathfrak{su}}(8)_1$ RGEs



The Type-I SUSY $\widehat{\mathfrak{su}}(8)_1 \supset \widehat{\mathfrak{su}}(4)_{s,1} \oplus \widehat{\mathfrak{su}}(4)_{W,1} \oplus \widehat{\mathfrak{u}}(1)_{X_0,1/4}$ achieves the unification:

$$\{H\}_I = \mathbf{8}_H^\omega \oplus \mathbf{28}_H^\omega, \quad \alpha_{4s} = \alpha_{4W} = \frac{1}{4}\alpha_{X_0}. \quad (20)$$

Summary

- We propose an $SU(8)$ theory to address the SM flavor puzzle. The flavor non-universality of SM quarks/leptons is a built-in in the UV theory.
- Our construction relaxes Georgi's "third law" in 1979, and we avoid the repetitions of one IRAFFS. The global symmetries based on the chiral IRAFFSs are vital to: (i) determine one single SM Higgs doublet through the non-anomalous $\tilde{U}(1)_{B-L}$ symmetry, (ii) count the massless left-handed sterile neutrinos precisely through the 't Hooft anomaly matching, (iii) organize the $d = 5$ operators for the SM fermion mass (mixing) terms.
- The symmetry-breaking pattern of the $SU(8)$ theory is described, and all light SM fermion masses besides of the top quark are due to the inevitable gravitational effects that break the emergent global symmetries explicitly.

Summary and Outlook

- Crucial assumptions: (i) the VEV assignments of three intermediate symmetry-breaking scales, (ii) the SM flavor IDs in the $\mathbf{28_F}$ and $\mathbf{56_F}$, and (iii) the $d = 5$ operators of direct and indirect Yukawa couplings containing the SM Higgs doublet.
- Main results: all SM quark/lepton masses, as well as the CKM mixing pattern can be quantitatively recovered with $\mathcal{O}(0.1) - \mathcal{O}(1)$ direct Yukawa couplings and $\mathcal{O}(0.01)$ Higgs mixing coefficients.
- All SM quark/leptons are flavor non-universal under the extended strong/weak symmetries, while the SM neutrinos $\nu_L \in \overline{\mathbf{8_F}^\Omega}$ are flavor universal.
- The degenerate $m_{di} = m_{\ell i}$ will be further probed based on the RGEs of $\frac{dm_f(\mu)}{d \log \mu} \equiv \gamma_{m_f} m_f(\mu)$, $\gamma_{m_f}(\alpha^\Upsilon) = \frac{\alpha^\Upsilon}{4\pi} \gamma_0(\mathcal{R}_f^\Upsilon)$.
- The gauge coupling unification is achieved in the SUSY $\widehat{\mathfrak{su}}(8)_1$ [in preparation].