Flavor Structures in Modular Flavor Flipped SU(5) GUT Model

Fei Wang

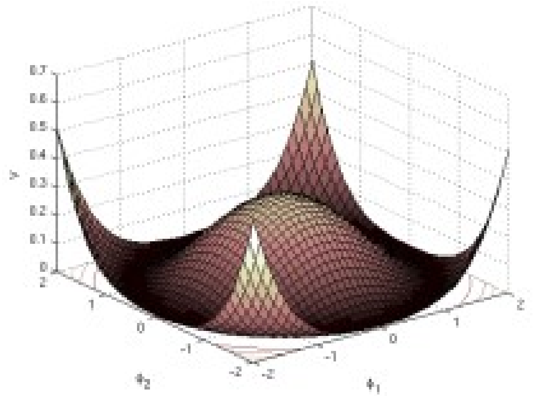
Zhengzhou University

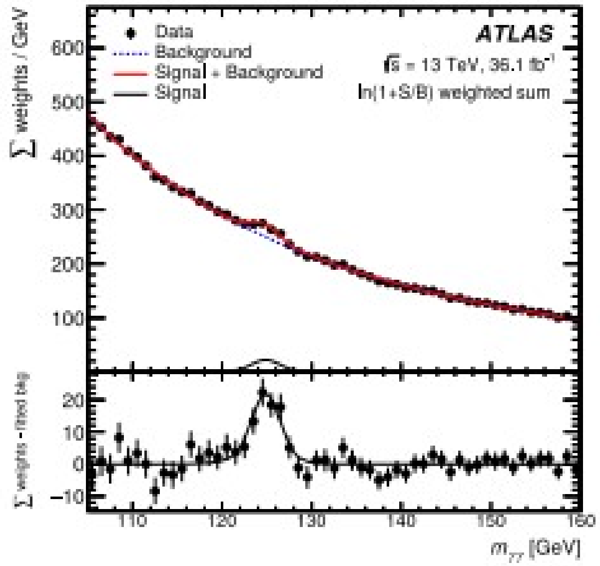
Nov 2, 2024

Based on JHEP(2023), Du & Wang

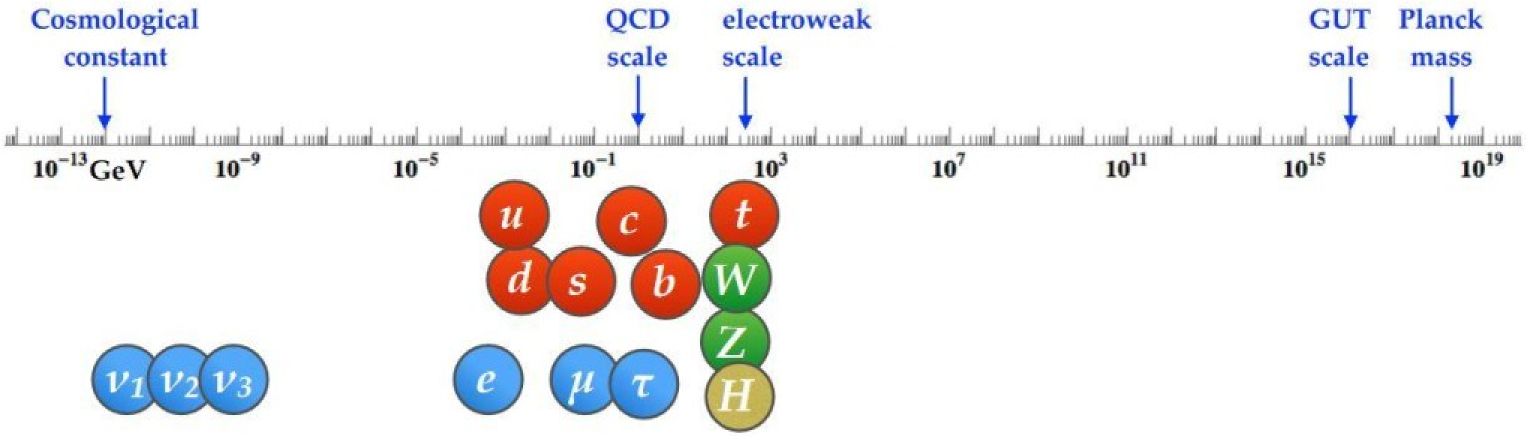
• Particle physics

Name	Label	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$	Spin
Quarks	$Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$	$(\mathbf{3}, \mathbf{2}, +\frac{1}{6})$	$\frac{1}{2}$
	u_R^i	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3})$	$\frac{1}{2}$
	d_R^i	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3})$	$\frac{1}{2}$
Leptons	$L_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$\frac{1}{2}$
	e_R^i	$(\mathbf{1}, \mathbf{1}, -1)$	$\frac{1}{2}$
	ν_R^{i*}	$(\mathbf{1}, \mathbf{1}, 0)$	$\frac{1}{2}$
Higgs	H	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$	0
Gluons	g_α	$(\mathbf{8}, \mathbf{1}, 0)$	1
W/Z-Bosons	$W^\pm$, Z^0	$(\mathbf{1}, \mathbf{3}, 0)$	1
Photon	γ	$(\mathbf{1}, \mathbf{1}, 0)$	1
Graviton*	$h_{\mu\nu}$	$(\mathbf{1}, \mathbf{1}, 0)$	2





Triumph of gauge field theories and effective field theories (EFT) !



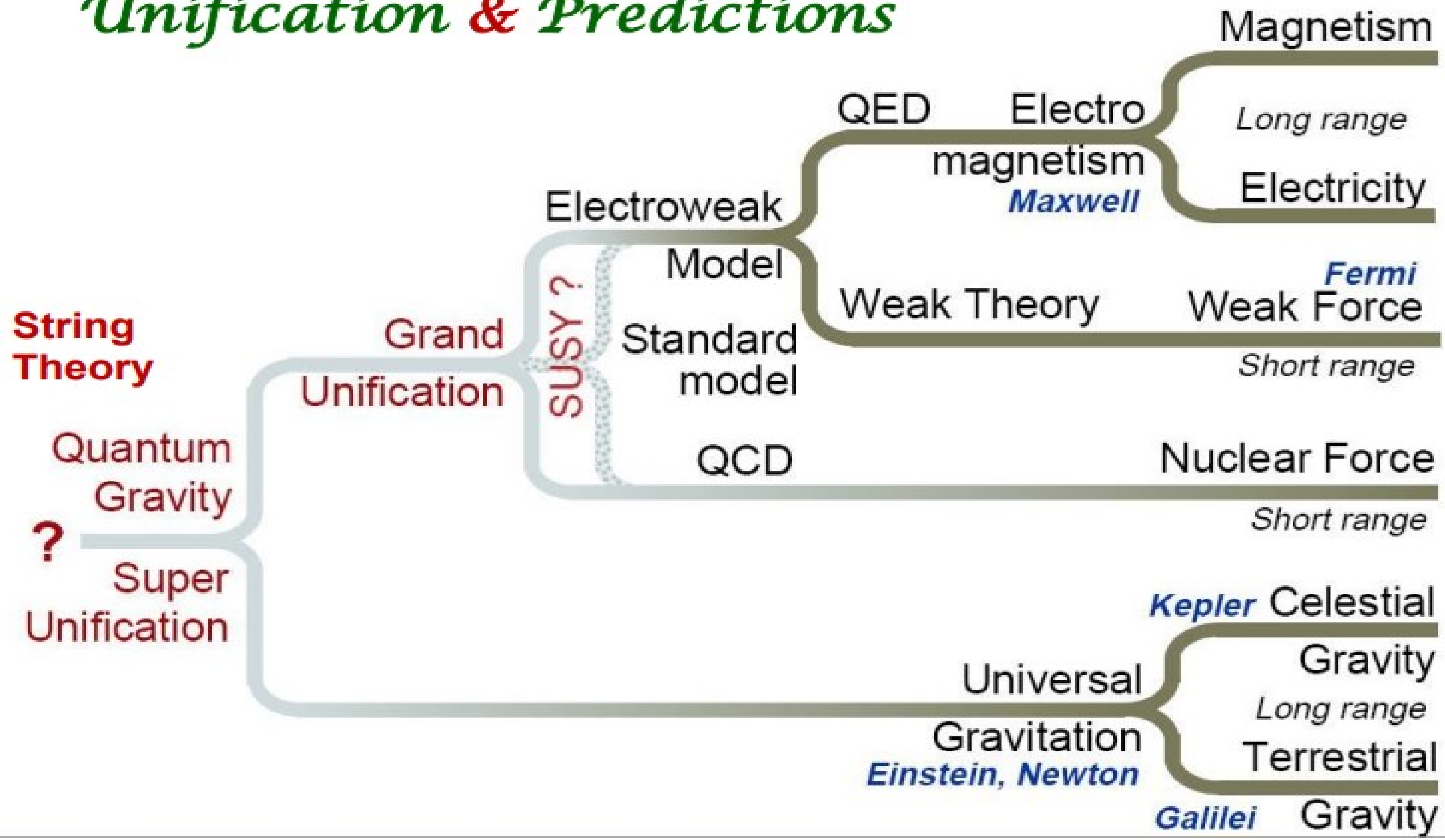
- Quark mixings are small [Particle Data Group 2022]

$$V_{\rm CKM} = \begin{pmatrix} 0.97401 \pm 0.00011 & 0.22650 \pm 0.00048 & 0.00361^{+0.00011}_{-0.00009} \\ 0.22636 \pm 0.00048 & 0.97320 \pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.000024}_{-0.000035} \end{pmatrix}$$

- Lepton mixings are large [NuFIT 5.2 (2022)]

$$|U|_{3\sigma}^{\text{w/o SK-atm}} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.143 \to 0.156 \\ 0.232 \to 0.507 & 0.459 \to 0.694 & 0.629 \to 0.779 \\ 0.260 \to 0.526 & 0.470 \to 0.702 & 0.609 \to 0.763 \end{pmatrix}$$

Unification & Predictions

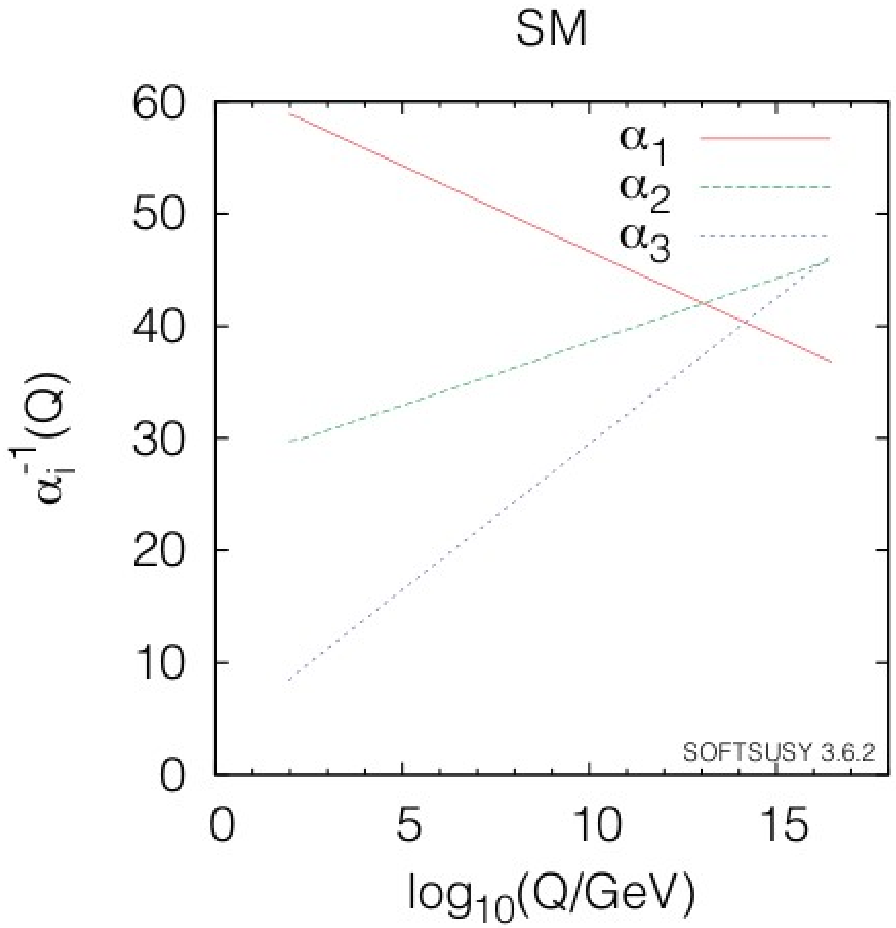


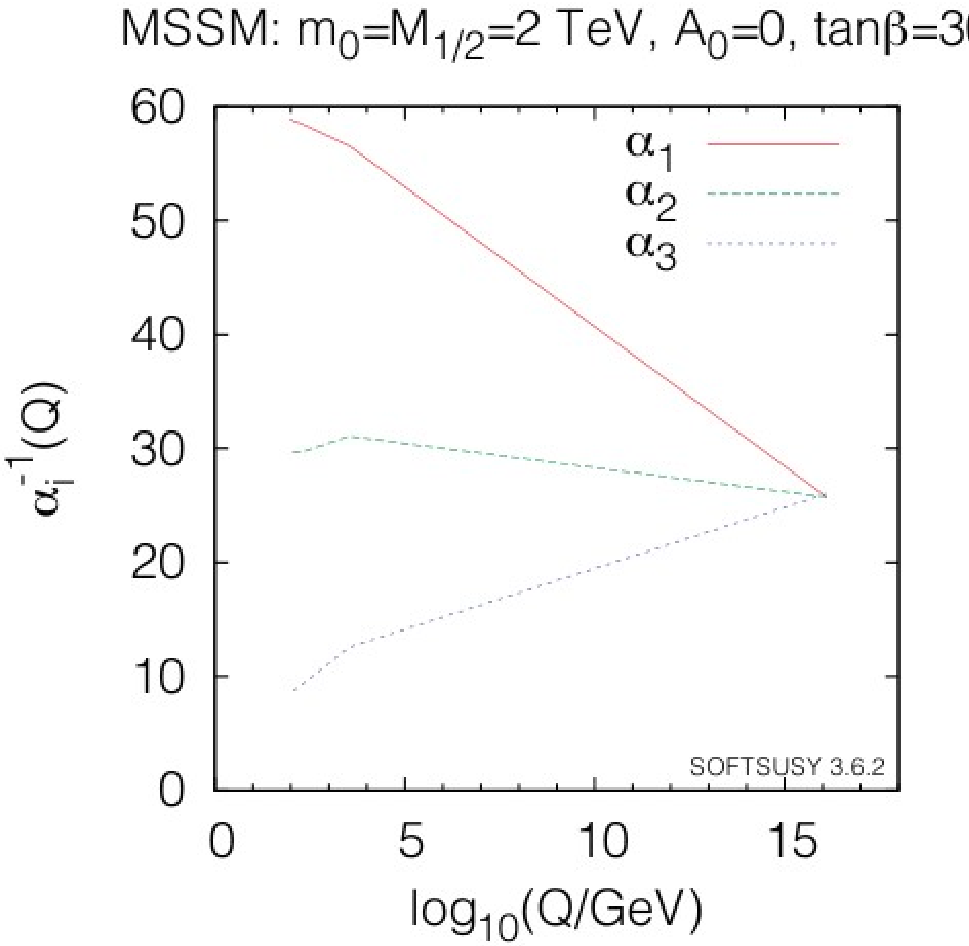
Motivations to GUT

- Why the charges of the proton and positron are exactly the same.
- Why there is apparent disparity between gauge couplings of strong interactions and electroweak interactions at low-energies
- The origin of so many low energy Yukawa couplings. (13 parameters associated with the Yukawa couplings. In addition, Majorana neutrinos introduce 3 more masses and 6 mixing angles and phases.)
- The origin of Baryon and Lepton Number Violation.
- Is proton absolutely stable?
- Why there are distinctions between quarks and leptons?
- Any possible explanations to quark-lepton complementarity?

$$\theta_{12}^{quark} + \theta_{12}^{lepton} = \pi/4$$

$$\theta_{23}^{quark} + \theta_{23}^{lepton} = \pi/4$$





Neutrino mass + GUT without SUSY?

SUSY SU(5) GUT Model

G_{SM} has rank four

rank-four group $SU(5)$ is the minimal choice for unification in a simple group

$$\mathbf{10}: \begin{pmatrix} 0 & u_b^c & -u_g^c & u_r & d_r \\ -u_b^c & 0 & u_r^c & u_g & d_g \\ u_g^c & -u_r^c & 0 & u_b & d_b \\ -u_r & -u_g & -u_b & 0 & e^c \\ -d_r & -d_g & -d_b & -e^c & 0 \end{pmatrix} \quad \text{and} \quad \bar{\mathbf{5}}: \begin{pmatrix} d_r^c \\ d_g^c \\ d_b^c \\ e \\ -\nu_e \end{pmatrix}.$$

$$\sin^2(\theta_W) = \frac{g'^2}{g^2+g'^2}$$

Predicts $\sin^2(\theta_W^{GUT}) = \frac{3}{8}$

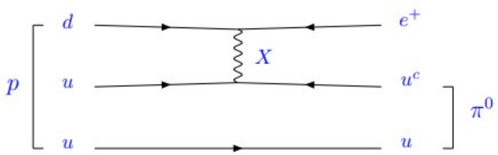
Proton Decay in the Supersymmetry SU(5)

dimension-four operators in $(\mathbf{10}\ \bar{\mathbf{5}}\ \bar{\mathbf{5}})$ eliminated by requiring R parity

D type gauge boson exchange

dimension-six operators

$p \to e^+ \pi^0$ $(n \to e^+ \pi^-)$

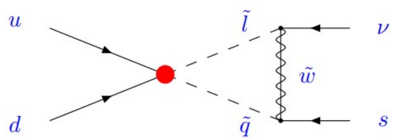


$$\frac{1}{M_U^2}\int d^4\theta \Phi^+\Phi\Phi^+\Phi,$$

F type triplet Higgs exchange

dimension-five operators

$p \to K^+ \bar{\nu}$ $(n \to K^0 \bar{\nu})$



$$\frac{1}{M_U}\int d^2\theta \Phi^4.$$

Problem of SU(5)

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad \begin{pmatrix} \bar{3} \\ \text{other} \end{pmatrix} \quad \begin{pmatrix} 3 \\ \text{other} \end{pmatrix} \quad \begin{pmatrix} \bar{3} \\ \bar{2} \end{pmatrix}$$

$$\| \qquad \|$$

$$5_H \quad \overline{50}_H \quad 50_H \quad \bar{5}_H$$

- Doublet-Triplet splitting

in $SU(5)$ one requires (at least) the set $5 + \bar{5} + 50 + \overline{50} + 75$

the **75** uniquely breaks $SU(5)$ down to $SU(3) \otimes SU(2) \otimes U(1)$

In the $50(\overline{50})$ there is a color $3(\bar{3})$ but no weak $2(\bar{2})$

$$\lambda 5_H \overline{50}_H \langle 75_H \rangle + \lambda' \bar{5}_H 50_H \langle 75_H \rangle$$

give mass to the triplets in $5_H + \bar{5}_H$ but not to the doublets.

- Undesirable mass relations? $m_\tau = m_b$ ☑
- Neutrino mass $\frac{m_e}{m_\mu} = \frac{m_d}{m_s}$, ☒
- Minimal version of SUSY SU(5) rule out ?

include a $\{45\}$-dimensional H^P_{qs}

$$\langle H^i_{i5} \rangle = -\tfrac{1}{3} \langle H^4_{45} \rangle$$

$$\begin{array}{c} \\ e \\ \mu \end{array} \begin{array}{c} e \quad \mu \\ \begin{pmatrix} 0 & a \\ a & 3b \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ d \\ s \end{array} \begin{array}{c} d \quad s \\ \begin{pmatrix} 0 & a \\ a & b \end{pmatrix} \end{array}$$

$$\frac{m_e}{m\mu} = \frac{1}{9} \frac{m_d}{m_s}$$

FLIPPED $SU(5)$ UNIFICATION

$$\begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ \nu_e \end{pmatrix}_L \quad ; \quad \left(\begin{pmatrix} u \\ d \end{pmatrix}_L \quad d_L^c \quad e_L^c \right) \quad ; \quad \boxed{\nu_L^c}$$

$\bar{\mathbf{5}}$ $\qquad$ $\mathbf{10}$ $\qquad$ $\boxed{\mathbf{1}}$

$$u_L^c \Leftrightarrow d_L^c$$
$$\nu_L^c \Leftrightarrow e_L^c$$

$$f_{\bar{\mathbf{5}}} = \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \\ e \\ \nu_e \end{pmatrix}_L \quad ; \quad F_{\mathbf{10}} = \left(\begin{pmatrix} u \\ d \end{pmatrix}_L \quad d_L^c \quad \nu_L^c \right) \quad ; \quad l_{\mathbf{1}} = e_L^c \, ,$$

$SU(5) \times U(1)$

$\downarrow$ scale M_{32} $\qquad \dfrac{25}{\alpha_1'} = \dfrac{1}{\alpha_5} + \dfrac{24}{\alpha_1}$

$SU(3)_c \times SU(2)_L \times U(1)_Y$

Basic GUT tests	$SU(5)$	Flipped $SU(5)$
$\sin^2\theta_W \Rightarrow \alpha_3(M_Z)$	$\times$	$\surd$
Proton decay	$\{p \to \bar{\nu}K^+\}$ $\times$	$\{p \to (e^+/\mu^+)\pi^0\}$
Doublet-triplet splitting	$\times$	$\surd$
Neutrino masses	$\times$	$\surd$
Baryogenesis	$\times$	$\surd$

Neutrio mass generation

$$\mathcal{W} \supseteq Y_{ij}^U F_i \bar{f}_j \bar{h} + Y_{ij}^S \overline{H} S_i F_j + \frac{M_{SS;ij}}{2} S_i S_j$$

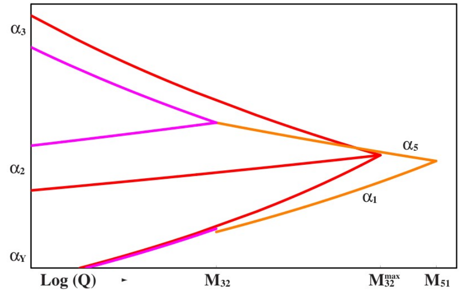
$$\mathcal{W}_{low} \supseteq Y_{ij}^N L_{L;i} N_{L;j}^c H_U + Y_{ij}^S S_i N_{L;j}^c M_H + \frac{M_{SS;ij}}{2} S_i S_j \, ,$$

the neutrino mass matrix can be obtained to

$$\mathcal{M} = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M_{SN} \\ 0 & M_{SN} & M_{SS} \end{pmatrix}$$

FLIPPED $SU(5)$ UNIFICATION

The breaking of the GUT group in 4 dimension while solving doublet-triplet splitting problem without using large GUT representations



$$h_{\mathbf{5}} = \{H_{\mathbf{2}}, H_{\mathbf{3}}\} \quad ; \quad h_{\bar{\mathbf{5}}} = \{H_{\bar{\mathbf{2}}}, H_{\bar{\mathbf{3}}}\}$$

$$W_G = HHh + \bar{H}\bar{H}\bar{h} + F\bar{H}\Phi + \mu h\bar{h} + S(\overline{H}H - M_{\rm H}^2)$$

missing partner

$$HHh \quad \rightarrow \quad d_H^c \langle \nu_H^c \rangle H_{\mathbf{3}}$$

$$\bar{H}\bar{H}\bar{h} \quad \rightarrow \quad \bar{d}_H^c \langle \bar{\nu}_H^c \rangle H_{\bar{\mathbf{3}}} \ ,$$

Nucleon decay

exchanges of SU(5) gauge bosons; dim-5 suppressed

$$K_{\rm gauge} = \sqrt{2} g_5 \big(-\epsilon_{\alpha\beta} (u_a^c)^\dagger X_a^\alpha U_l^T L^\beta + \epsilon^{abc} (Q^{a\alpha})^\dagger X_b^\alpha V P^\dagger d_c^c + \epsilon_{\alpha\beta} (\nu^c)^\dagger U_{\nu^c}^\dagger X_a^\alpha Q^{a\beta} + {\rm h.c.} \big) \ ,$$

$$\tau(p \rightarrow \pi^0 e^+)_{\rm flipped} \simeq 7.9 \times 10^{35} \times |(U_l)_{11}|^{-2} \left(\frac{M_X}{10^{16}\ {\rm GeV}} \right)^4 \left(\frac{0.0378}{\alpha_5} \right)^2 \ {\rm yrs}\, .$$

Modular Flavor Symmetry

- Flavor symmetries are interesting approaches to attack the origin of fermion mass hierarchies and mixing angles.
- The modular symmetry is a geometrical symmetry of T^2 and T^2/Z_2, and corresponds to change of their cycle basis.
- Matter modes transform non-trivially under the modular symmetry. That is, the modular symmetry is a flavor symmetry.
- Texture zeros of the fermion mass matrix provide an attractive approach to understand the flavor mixing. Those can be possibly related with modular flavor symmetries.
- The modular flavor symmetry also control higher dimensional operators.Allowed couplings are controlled by stringy symmetries and n-point couplings are written by products of 3-point couplings.

Modular group

See Feruglio arXiv:1706.08749
Gui-jun Ding& King arXiv: 2311.09282

modular group $\Gamma \equiv \mathrm{SL}(2,Z)$ can act on the upper half plane

$$\gamma : \tau \mapsto \frac{a\tau+b}{c\tau+d}, \quad \text{for } \gamma \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,Z), \quad \mathtt{Im}(\tau) > 0\,.$$

inhomogeneous modular group is $\bar{\Gamma} = \Gamma/\{I,-I\}$

The group $\overline{\Gamma}$ is generated by S and T

$$S^2 = \mathbb{1} \;\; , \;\; (ST)^3 = \mathbb{1}$$

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$S : \tau \to -\frac{1}{\tau} \qquad T : \tau \to \tau + 1$$

infinite normal subgroups

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,Z) \;\middle|\; \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} (\mathtt{mod}\ N) \right\}$$

inhomogeneous finite modular group Γ_N

$$\Gamma_N \cong \overline{\Gamma}/\overline{\Gamma}(N) \cong PSL(2,Z_N) = SL(2,Z_N)/\{e,-e\}$$

$\Gamma_2 \simeq S_3$, $\Gamma_3 \simeq A_4$, $\Gamma_4 \simeq S_4$ and $\Gamma_5 \simeq A_5$

Modular forms of weight k and level N

$$f(\gamma\tau) = (c\tau+d)^k f(\tau)\,, \quad \gamma \in \Gamma(N)$$

k an even and non-negative integer

Can be more general!

holomorphic functions $f(\tau)$ transforming under the action of $\Gamma(N)$

Modular forms of weight $2k$ and level N form a linear space $\mathcal{M}_{2k}(\Gamma(N))$ finite dimension $d_{2k}(\Gamma(N))$

$$f_i(\gamma\tau) = (c\tau+d)^k \rho(\gamma)_{ij} f_j(\tau)\,, \quad \gamma \in \bar{\Gamma}$$

$\rho(\gamma)_{ij}$ is a unitary matrix under Γ_N

Modular flavor models for SUSY Theories

See Feruglio arXiv:1706.08749
Gui-jun Ding arXiv: 2311.09282

$$\mathcal{S} = \int d^4x d^2\theta d^2\bar{\theta}\; K(\Phi, \bar{\Phi}) + \int d^4x d^2\theta\; w(\Phi) + h.c.$$

the supermultiplets $\varphi^{(I)}$ transform in representation $\rho^{(I)}$ of a quotient group Γ_N

$$\begin{cases} \tau \to \dfrac{a\tau + b}{c\tau + d} \\ \varphi^{(I)} \to (c\tau + d)^{-k_I} \rho^{(I)}(\gamma) \varphi^{(I)} \end{cases}$$

γ an element of Γ_N.

The invariance of the action $\mathcal{S}$ requires

$$\begin{cases} w(\Phi) \to w(\Phi) \\ K(\Phi, \bar{\Phi}) \to K(\Phi, \bar{\Phi}) + f(\Phi) + f(\overline{\Phi}) \end{cases}$$

invariance of the Kahler potential

$$K(\Phi, \bar{\Phi}) = -h \log(-i\tau + i\bar{\tau}) + \sum_I (-i\tau + i\bar{\tau})^{-k_I} |\varphi^{(I)}|^2$$

invariance of the superpotential $w(\Phi)$ under the modular group

$$w(\Phi) = \sum_n Y_{I_1 \ldots I_n}(\tau)\; \varphi^{(I_1)} \ldots \varphi^{(I_n)}$$

the functions $Y_{I_1 \ldots I_n}(\tau)$ modular forms of weight $k_Y(n)$ transforming in the representation ρ of Γ_N

$$k_Y(n) = k_{I_1} + \ldots + k_{I_n}$$

The product $\rho \times \rho^{I_1} \times \ldots \times \rho^{I_n}$ contains an invariant singlet

Multiple modular symmetries

Here is a simplest factorized tori extension!
Non-factorized-automorphic form--see Ding 2010.07952

finite modular transformations $\gamma_1, \ldots, \gamma_M$ in $\Gamma^1_{N_1} \times \Gamma^2_{N_2} \times \cdots \times \Gamma^M_{N_M}$

$$\gamma_J : \tau_J \to \gamma_J \tau_J = \frac{a_J \tau_J + b_J}{c_J \tau_J + d_J} \,,$$

$$\phi_i(\tau_1, \ldots, \tau_M) \to \phi_i(\gamma_1 \tau_1, \ldots, \gamma_M \tau_M) = \prod_{J=1,\ldots,M} (c_J \tau_J + d_J)^{-2k_{i,J}} \bigotimes_{J=1,\ldots,M} \rho_{I_{i,J}}(\gamma_J) \phi_i(\tau_1, \tau_2, \ldots, \tau_M)$$

$$Y_{(I_{Y,1},\ldots,I_{Y,M})}(\tau_1, \ldots, \tau_M) \to Y_{(I_{Y,1},\ldots,I_{Y,M})}(\gamma_1 \tau_1, \ldots, \gamma_M \tau_M) = \prod_{J=1,\ldots,M} (c_J \tau_J + d_J)^{2k_{Y,J}} \bigotimes_{J=1,\ldots,M} \rho_{I_{Y,J}}(\gamma_J) Y_{(I_{Y,1},\ldots,I_{Y,M})}(\tau_1, \ldots, \tau_M) \,.$$

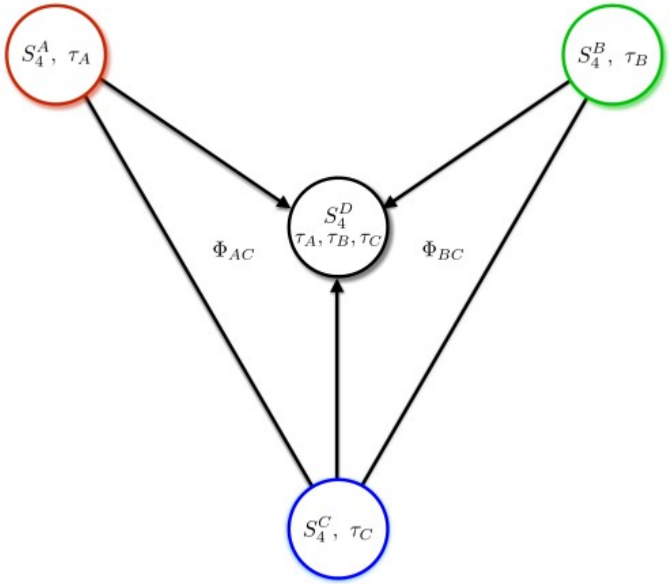
$$K(\phi_i, \overline{\phi}_i; \tau_1, \ldots, \tau_M, \overline{\tau}_1, \ldots, \overline{\tau}_M) = -\sum_{J=1,\ldots,M} h_J \log(-i\tau_J + i\overline{\tau}_J) + \sum_i \frac{\overline{\phi}_i \phi_i}{\prod_{J=1,\ldots,M} (-i\tau_J + i\overline{\tau}_J)^{2k_{i,J}}} \,,$$

$$W(\phi_i; \tau_1, \ldots, \tau_M) = \sum_n \sum_{\{i_I, \cdots, i_n\}} \left(Y_{(I_{Y,1},\ldots,I_{Y,M})} \phi_{i_1} \cdots \phi_{i_n} \right)_{\mathbf{1}} \,,$$

Break the Multiple Modular Symmetries via Higgs Fields

See Ye-Ling & King's paper 1906.02208

Illustration of the breaking of $S_4^A \times S_4^B \times S_4^C \to S_4^D$



Vacuum alignments

for the bi-triplet scalars Φ_{AC} and Φ_{BC}

Fields	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$
χ_{AC}	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	0	0	0
χ_{BC}	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	0	0	0
χ_A	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	0	0	0
χ_B	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	0	0	0

$$\begin{aligned} w_d &= \Phi_{AC}\Phi_{AC}\chi_{AC} + \mathrm{M}_A\Phi_{AC}\chi_{AC} + \Phi_{AC}\Phi_{AC}\chi_A\,, \\ &+ \Phi_{BC}\Phi_{BC}\chi_{BC} + \mathrm{M}_B\Phi_{BC}\chi_{BC} + \Phi_{BC}\Phi_{BC}\chi_B\,, \end{aligned}$$

$$\sum_{j,k=1,2,3;}\ \sum_{\beta,\gamma=1,2,3} |\epsilon_{ijk}||\epsilon_{\alpha\beta\gamma}|(\tilde{\Phi}_{AC})_{j\beta}(\tilde{\Phi}_{AC})_{kc} + \mathrm{M}_A(\tilde{\Phi}_{AC})_{i\alpha} = 0$$

24 solutions $\quad \langle\Phi_{AC}\rangle = \rho_{\mathbf{3}}(\gamma)P_{23}v_{AC}$

$$S_4^A \times S_4^C \to S_4^D$$

$$\sum_{j,k=1,2,3;}\ \sum_{\alpha=1,2,3} |\epsilon_{ijk}|(\tilde{\Phi}_{AC})_{j\alpha}(\tilde{\Phi}_{AC})_{k\alpha} = 0$$

$\langle\Phi_{AC}\rangle_{i\alpha} = (P_{23})_{i\alpha}v_{AC}$ corresponding to $\langle\tilde{\Phi}_{AC}\rangle_{i\alpha} \propto \delta_{i\alpha}$

Reduce the Multiple Modular Symmetries via Boundary Conditions

It is possible to break the multiple $A_4^Q \times A_4^L$ by BCs to diagonal A_4^D, which is then identified to be the (single) modular A_4 symmetry in the low energy effective theory.

choices with $\Phi_{i\alpha}^{(++)}$ for fixed $'i'$ (or $'\alpha'$) corresponds to the breaking of $A_4^Q \times A_4^L$ to A_4^Q (or A_4^L), respectively.

We can introduce bi-triplets to reduce the modular symmetries into the diagonal one.

assign the following BCs for the bi-triplet $(\mathbf{3}, \mathbf{3})$ fields $\Phi_{i\alpha}$ of $A_4^Q \times A_4^L$,

i, α the indices for A_4^Q and A_4^L, respectively.

$A_4^Q \times A_4^L$ to the diagonal A_4^D

$\gamma^Q \in A_4^Q$ and $\gamma^L \in A_4^L$ being associated to the $\gamma^D \in A_4^D$ by $\gamma^Q = \gamma^L = \gamma^D$.

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3_s} \oplus \mathbf{3_a}, \qquad \Phi_\kappa^{\mathbf{r}} = \sum C_{i\alpha;\kappa}^{\mathbf{r}} \Phi_{i\alpha} \,,$$

$$(\Phi_\kappa^{\mathbf{r}})^{++} \to J\left(\gamma_D, \tau_Q\right)^{-2k_Q} J\left(\gamma_D, \tau_L\right)^{-2k_L} \left(\rho_{r_Q}(\gamma_D) \otimes \rho_{r_L}(\gamma_D)\right)_{\mathbf{r}} (\Phi_\kappa^{\mathbf{r}})^{++} \,,$$

For example: $$\Phi = (\Phi^{\mathbf{1}})^{(++)} \oplus (\Phi^{\mathbf{1}'})^{(+-)} \oplus (\Phi^{\mathbf{1}''})^{(+-)} \oplus (\Phi_\kappa^{\mathbf{3_s}})^{(+-)} \oplus (\Phi_\kappa^{\mathbf{3_a}})^{(+-)} \,.$$

$$\Phi^{\mathbf{1}} = \frac{1}{\sqrt{3}} \left(\Phi_{11} + \Phi_{23} + \Phi_{32}\right) \,,$$ only the zero modes of the singlet (that is, $\Phi^{\mathbf{1}}$) survives.

any combination of the representations can be allowed to act as the BCs that break the $A_4^Q \times A_4^L$ to A_4^D

Modular GUT for flipped SU(5)

In modular GUT framework, the superfields within a multiplet of GUT gauge group should transform identically with the same modulus field.

It seems that the unification of matter contents will be spoiled if different values of modulus are assigned separately for quarks and leptons.

We propose to reconcile such an inconsistency in the orbifold GUT 5D $\mathcal{M}_4 \times S^1/Z_2$ orbifold.

Compactification on S^1/Z_2 is obtained by identifying the fifth coordinate y under the two operations

$$Z : y \to -y, \qquad T : y \to y + 2\pi R.$$

5D $N = 1$ SUSY (corresponding to 4D $N = 2$ SUSY) to 4D $N = 1$ SUSY by proper boundary conditions

$$S = \int d^5x \frac{1}{kg^2} \mathrm{Tr}\left[\frac{1}{4}\int d^2\theta \left(W^\alpha W_\alpha + \text{h.c.}\right) + \int d^4\theta \left((\sqrt{2}\partial_5 + \bar{\Sigma})e^{-V}(-\sqrt{2}\partial_5 + \Sigma)e^V + \partial_5 e^{-V}\partial_5 e^V\right)\right] + \int d^5x \left[\int d^4\theta \left(\Phi^c e^V \bar{\Phi}^c + \bar{\Phi} e^{-V}\Phi\right) + \int d^2\theta \left(\Phi^c(\partial_5 - \frac{1}{\sqrt{2}}\Sigma)\Phi + \text{h.c.}\right)\right]$$

vector multiplet

$$V(x^\mu, y) \to V(x^\mu, -y) = PV(x^\mu, y)P^{-1},$$
$$\Sigma(x^\mu, y) \to \Sigma(x^\mu, -y) = -P\Sigma(x^\mu, y)P^{-1}$$

hypermultiplet fundamental representations,

$$\Phi(x^\mu, y) \to \Phi(x^\mu, -y) = \eta_\Phi P\Phi(x^\mu, y),$$
$$\Phi^c(x^\mu, y) \to \Phi^c(x^\mu, -y) = -\eta_\Phi P\Phi^c(x^\mu, y)$$

Modular GUT for flipped SU(5)

$$P_{O(y=0)} = \mathrm{diag}\,(\ +1,\ \ +1,\ \ +1,\ \ +1,\ \ +1)\,,$$
$$P_{O'(y=\pi R)} = \mathrm{diag}\,(\ +1,\ \ +1,\ \ +1,\ \ -1,\ \ -1)\,,$$

and proper brane mass terms

For Yukawa couplings $W \supseteq y^D_{ij} F_i F_j h + y^U_{ij} F_i \bar{f}_j \bar{h} + y^E_{ij} \bar{f}_i E_j h\,,$

For Higgs sector $W \supseteq HHh + \overline{H}\overline{H}\bar{h} + X(\overline{H}H - M_H^2)$

introduce additional neutral superfield $S_i(1,0)$ for neutrino sector

inverse seesaw mechanism $\mathcal{W} \supseteq Y^U_{ij} F_i \bar{f}_j \bar{h} + Y^S_{ij} \overline{H} S_i F_j + \frac{M_{SS;ij}}{2} S_i S_j$

$$m_\nu \approx m_D^T M_{SN}^{-1} M_{SS} (M_{SN}^T)^{-1} m_D$$

neutrino mass matrix

$$\mathcal{M} = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M_{SN} \\ 0 & M_{SN} & M_{SS} \end{pmatrix}$$

$$m_D \sim Y_2 v_u \qquad M_{SN} \sim Y_S M_H$$

$$T_F^a(\mathbf{10_1}) = Q_L^a(\mathbf{3},\mathbf{2})^{(+,-)}_{(\mathbf{1},\mathbf{1/6})} \oplus D_L^{c,a}(\bar{\mathbf{3}},1)^{(+,-)}_{(\mathbf{1},-\mathbf{2/3})} \oplus N_L^{c,a}(1,1)^{(+,+)}_{(\mathbf{1},\mathbf{1})}\,,$$
$$T_F^{\prime,a}(\mathbf{10_1}) = Q_L^a(\mathbf{3},\mathbf{2})^{(+,-)}_{(\mathbf{1},\mathbf{1/6})} \oplus D_L^{c,a}(\bar{\mathbf{3}},1)^{(+,+)}_{(\mathbf{1},-\mathbf{2/3})} \oplus N_L^{c,a}(1,1)^{(+,-)}_{(\mathbf{1},\mathbf{1})}$$
$$T_F^{\prime\prime,a}(\mathbf{10_1}) = Q_L^a(\mathbf{3},\mathbf{2})^{(+,+)}_{(\mathbf{1},\mathbf{1/6})} \oplus D_L^{c,a}(\bar{\mathbf{3}},1)^{(+,-)}_{(\mathbf{1},-\mathbf{2/3})} \oplus N_L^{c,a}(1,1)^{+,-}_{(\mathbf{1},\mathbf{1})}\,,$$
$$F_{\bar{f}}^a(\bar{\mathbf{5}}_{-\mathbf{3}}) = U_L^{c,a}(\bar{\mathbf{3}},1)^{(+,+)}_{(-\mathbf{3},\mathbf{1/3})} \oplus L_L^a(\mathbf{1},\mathbf{2})^{(+,-)}_{(-\mathbf{3},-\mathbf{1/2})}\,,$$
$$F_{\bar{f}}^{\prime,a}(\bar{\mathbf{5}}_{-\mathbf{3}}) = U_L^{c,a}(\bar{\mathbf{3}},1)^{(+,-)}_{(-\mathbf{3},\mathbf{1/3})} \oplus L_L^a(\mathbf{1},\mathbf{2})^{(+,+)}_{(-\mathbf{3},-\mathbf{1/2})}\,,$$
$$O_E^a(\mathbf{1_{-5}}) = E_L^{c,a}(\mathbf{1},\mathbf{1})^{(+,+)}_{(-\mathbf{5},\mathbf{0})}\,, \quad O_S^a(\mathbf{1_0}) = S^a(\mathbf{1},\mathbf{1})^{(+,+)}_{(\mathbf{0},\mathbf{0})}\,,$$

$$h(\mathbf{5_{-2}}) = H_T(\mathbf{3},\mathbf{1})^{(+,-)}_{(-2,-1/3)} \oplus H_D(\mathbf{1},\mathbf{2})^{(+,+)}_{(-2,1/2)}\,,$$
$$\bar{h}(\bar{\mathbf{5}}_{\mathbf{2}}) = H_T'(\mathbf{3},\mathbf{1})^{(+,-)}_{(2,1/3)} \oplus H_{\mathrm{U}}(\mathbf{1},\mathbf{2})^{(+,+)}_{(2,-1/2)}\,,$$
$$H(\mathbf{10_1}) = H_{TQ}(\mathbf{3},\mathbf{2})^{(+,-)}_{(\mathbf{1},\mathbf{1/6})} \oplus H_{TD}(\bar{\mathbf{3}},1)^{(+,-)}_{(\mathbf{1},-\mathbf{2/3})} \oplus H_N(1,1)^{(+,+)}_{(\mathbf{1},\mathbf{1})}\,,$$
$$\overline{H}(\overline{\mathbf{10}}_{-1}) = \overline{H}_{TQ}(\bar{\mathbf{3}},\bar{\mathbf{2}})^{(+,-)}_{(-1,-1/6)} \oplus \overline{H}_{TD}(\bar{\mathbf{3}},1)^{(+,-)}_{(\mathbf{1},-\mathbf{2/3})} \oplus \overline{H}_N(1,1)^{(+,+)}_{(-\mathbf{1},-\mathbf{1})}$$
$$O_X(\mathbf{1}_0) = X(\mathbf{1},\mathbf{1})^{(+,+)}_{(\mathbf{0},\mathbf{0})}\,.$$

Further breaking[1] of $\mathrm{U}(1)_X \times \mathrm{U}(1)_{Y'}$ into $\mathrm{U}(1)_Y$ triggered via proper Higgs field

$(\mathcal{Z},\mathcal{T})$	KK modes	4D masses
$(+,+)$	$\cos[ny/R]$	n/R
$(+,-)$	$\cos[(n+1/2)y/R]$	$(n+1/2)/R$
$(-,+)$	$\sin[(n+1)y/R]$	$(n+1)/R$
$(-,-)$	$\sin[(n+1/2)y/R]$	$(n+1/2)/R$

three families	$A_4^Q\times A_4^L$
$T_F^{\prime,a}(\mathbf{10_1}), T_F^{\prime\prime,a}(\mathbf{10_1})$	$(3,1)$
$T_F^a(\mathbf{10_1})$	$(1,3)$
$F_{\bar{f}}^a(\bar{\mathbf{5}}_{-\mathbf{3}})$	$(3,1)$
$F_{\bar{f}}^{\prime,a}(\bar{\mathbf{5}}_{-\mathbf{3}})$,$O_E^a(\mathbf{1_{-5}})$	$(1,3)$
$O_S^a(\mathbf{1_0})$	$(1,3)$

The fittings of the SM plus neutrino flavor structure are sometimes not good enough with a single modulus field, which on the other hand prefer multiple values of modulus VEVs for various sectors.

zero modes for the quarks and leptons transform as $(3,1)$ and $(1,3)$ under $A_4^Q\times A_4^L$, respectively.

The superpotential in the SU(5) preserving $O(y=0)$ brane can be written as

$$\mathcal{L}\supseteq\delta(y)\int d^2\theta\left[Y^D_{ab;23}T_F^{a;2}T_F^{b;3}h+Y^N_{ab;12}T_F^{a;1}F_{\bar{f}}^{b;2}\bar{h}+Y^U_{ab;31}T_F^{a;3}F_{\bar{f}}^{b;1}\bar{h}+Y^E_{ab;1}F_{\bar{f}}^{a;1}E^b h\right.$$
$$\left.+\;Y^S_{ab;1}\overline{H}O_S^aT_F^{b;1}+\frac{M_{SS;ab}}{2}O_S^aO_S^b+O_X(\overline{H}H-M_H^2)\right]\ .$$

to break the multiple $A_4^Q\times A_4^L$ by BCs to diagonal A_4^D

Classification according to the choice of representation and modulaı weights

Up-type quark sector

- $\rho_{\bar{f}} = \mathbf{3},\ \rho_F = \mathbf{1}, \mathbf{1}', \mathbf{1}''.$
- $\rho_{\bar{f}} = \mathbf{1}, \mathbf{1}', \mathbf{1}'',\ \rho_F = \mathbf{3}.$
- $\rho_{\bar{f}} = \mathbf{1}, \mathbf{1}', \mathbf{1}'',\ \rho_F = \mathbf{1}, \mathbf{1}', \mathbf{1}''.$

Down-type quark sector

- $\rho_F = \mathbf{3}.$
- $\rho_F = \mathbf{1}, \mathbf{1}', \mathbf{1}''.$

Charged lepton sector

- $\rho_{\bar{f}} = \mathbf{3}, \rho_E = \mathbf{3}.$
- $\rho_{\bar{f}} = \mathbf{3}, \rho_E = \mathbf{1}, \mathbf{1}', \mathbf{1}''.$
- $\rho_{\bar{f}} = \mathbf{1}, \mathbf{1}', \mathbf{1}'', \rho_E = \mathbf{3}.$
- $\rho_{\bar{f}} = \mathbf{1}, \mathbf{1}', \mathbf{1}'', \rho_E = \mathbf{1}, \mathbf{1}', \mathbf{1}''.$

Neutrino sector

- $\rho_F = \mathbf{3}, \rho_S = \mathbf{3}.$
- $\rho_F = \mathbf{3}, \rho_S = \mathbf{1}, \mathbf{1}', \mathbf{1}''.$
- $\rho_F = \mathbf{1}, \mathbf{1}', \mathbf{1}'', \rho_S = \mathbf{3}.$
- $\rho_F = \mathbf{1}, \mathbf{1}', \mathbf{1}'', \rho_S = \mathbf{1}, \mathbf{1}', \mathbf{1}''.$

Form of the mass matrix for representations

$$\boldsymbol{\rho_{\bar{f}} = 3,\ \rho_F = 3,\ \rho_E = 3,\ \rho_S = 3}$$

$$W \supseteq \left[\alpha_1 Y_{\mathbf{3}}^{(4)} (FF)_{\mathbf{3}S} h + \alpha_2 Y_{\mathbf{1}}^{(4)} (FF)_{\mathbf{1}} h + \alpha_3 Y_{\mathbf{1}'}^{(4)} (FF)_{\mathbf{1}''} h\right]$$
$$+ \left[\beta_1 Y_{\mathbf{3}}^{(2)} (F\bar{f})_{\mathbf{3}S} \bar{h} + \beta_2 Y_{\mathbf{3}}^{(2)} (F\bar{f})_{\mathbf{3}A} \bar{h}\right]$$
$$+ \gamma_1 (\bar{f} E)_{\mathbf{1}} h + \left[\lambda_1 Y_{\mathbf{3}}^{(2)} (SF)_{\mathbf{3}S} \overline{H} + \lambda_2 Y_{\mathbf{3}}^{(2)} (SF)_{\mathbf{3}A} \overline{H}\right] + \frac{\kappa_1}{2} \Lambda_2 (SS)_{\mathbf{1}}$$

- $(k_{\bar{f}}, k_F, k_E, k_S) = (0, 2, 0, 0).$

$$\begin{aligned}
\mathcal{M}_U / v_u \equiv (y_U)_{ij} &= \beta_1 S_{\mathbf{3}}^{(2)}(\tau) + \beta_2 A_{\mathbf{3}}^{(2)}(\tau)\,, \\
\mathcal{M}_D / v_d \equiv (y_D)_{ij} &= \alpha_1 S_{\mathbf{3}}^{(4)} + \alpha_2 S_{\mathbf{1}}^{(4)} + \alpha_3 S_{\mathbf{1}'}^{(4)}\,, \\
\mathcal{M}_E / v_d \equiv (y_E)_{ij} &= \gamma_1 S_{\mathbf{1}}^{0}(\tau)\,, \\
\mathcal{M}_N^{Dirac} / v_u \equiv (y_N)_{ij}^{Dirac} &= (y_U)^T\,, \\
\mathcal{M}_{ij}^{SN} &= \lambda_1 \Lambda_1 S_{\mathbf{3}}^{(2)}(\tau) + \lambda_2 \Lambda_1 A_{\mathbf{3}}^{(2)}(\tau)\,, \\
\mathcal{M}_{ij}^{SS} &= \kappa_1 \Lambda_2 S_{\mathbf{1}}^{0}(\tau)\,,
\end{aligned}$$

Numerical fitting

To keep the predictive power
we concentrate on the scenarios in which at least one of
the $\bar{f}, F, E, S$ superfields transforms as the triplet of A_4 modular group.
The GUT-scale flavor structures of quarks and leptons predicted by our models need
to be evolved to the EW scale with the renormalization group equation (RGE) be
the implement of the χ^2 fit to the experimental data of SM and neutrino
we use two-loop RGE to evolve our predicted

observable	Value2
$y_u/10^{-6}$	6.644
$y_c/10^{-3}$	3.445
y_t	0.868
$y_d/10^{-5}$	1.323
$y_s/10^{-4}$	1.841
$y_b/10^{-2}$	1.395
θ^q_{12}	0.22737
$\theta^q_{13}/10^{-3}$	3.716
$\theta^q_{23}/10^{-2}$	4.296
δ^q_{CP}	1.194
χ^2_q	46.122

$y_e/10^{-6}$	2.848
$y_\mu/10^{-4}$	6.104
$y_\tau/10^{-2}$	1.022
$\Delta m^2_{21}/10^{-5}/\mathrm{eV}^{22}$	7.419
$\Delta m^2_{31}/10^{-3}/\mathrm{eV}^2$	2.516
$\sin^2\theta^l_{12}$	0.457
$\sin^2\theta^l_{13}/10^{-2}$	2.304
$\sin^2\theta^l_{23}$	0.806
χ^2_l	236.259

To obtain the best-fit parameters in our fitting, we scan randomly the allowed parameter regions to find good seeds for further MCMC scanning. In practice, we try to find the best-fit points for the quark sector first, and then perform the numerical fitting for the lepton sector with the best-fit value of τ in the quark sector. However, it is sometimes difficult to obtain good fittings with a common τ for both sectors. Multiple τ values (for quark and lepton sectors, respectively) are then used in the fitting if the single modulus scenarios do not work well.

$\rho_{\bar f} = \rho_F = \rho_E = \rho_S = \mathbf{3}$ and $k_{\bar f} = k_F = 4, k_E = k_S = 2.$

observable	Value2
$y_u/10^{-6}$	6.644
$y_c/10^{-3}$	3.445
y_t	0.868
$y_d/10^{-5}$	1.323
$y_s/10^{-4}$	1.841
$y_b/10^{-2}$	1.395
θ^q_{12}	0.22737
$\theta^q_{13}/10^{-3}$	3.716
$\theta^q_{23}/10^{-2}$	4.296
δ^q_{CP}	1.194
χ^2_q	46.122

$y_e/10^{-6}$	2.848
$y_\mu/10^{-4}$	6.104
$y_\tau/10^{-2}$	1.022
$\Delta m^2_{21}/10^{-5}/\mathrm{eV}^{22}$	7.419
$\Delta m^2_{31}/10^{-3}/\mathrm{eV}^2$	2.516
$\sin^2\theta^l_{12}$	0.457
$\sin^2\theta^l_{13}/10^{-2}$	2.304
$\sin^2\theta^l_{23}$	0.806
χ^2_l	236.259

Parameter	Value1
$\beta_1/10^{-2}$	5.292
$\beta_2/10^{-4}$	$1588.315 - 6.410i$
$\beta_3/10^{-2}$	$-3.704 + 10.670i$
$\beta_4/10^{-3}$	$-5.817 + 2.049i$
$\beta_5/10^{-3}$	$-1058.838 - 1.620i$
$\beta_6/10^{-2}$	$2.055 + 9.933i$
$\beta_7/10^{-1}$	$1.710 - 1.460i$
$\alpha_1/10^{-4}$	3.774
$\alpha_2/10^{-3}$	$-2.821 - 273.122i$
$\alpha_3/10^{-5}$	$-70.375 + 1.104i$
$\alpha_4/10^{-4}$	$-3.691 + 2707.307i$
$\alpha_5/10^{-4}$	$-411.085 - 1.680i$
τ	$1.198 + 2.830i$

$y_e/10^{-6}$	2.847
$y_\mu/10^{-4}$	6.109
$y_\tau/10^{-2}$	1.022
$\Delta m^2_{21}/10^{-5}/\mathrm{eV}^2$	7.405
$\Delta m^2_{31}/10^{-3}/\mathrm{eV}^2$	2.511
$\sin^2\theta^l_{12}$	0.384
$\sin^2\theta^l_{13}/10^{-2}$	2.260
$\sin^2\theta^l_{23}$	0.642
χ^2_l	48.806

low energy predictions of the best-fit point

$$\chi^2_{q,l} \equiv \sum_{i_{q,l}} \chi^2_{i_{q,l}} \,,$$

$\gamma_1/10^{-2}$	1.203
$\gamma_2/10^{-4}$	$3.536 - 9.922i$
$\gamma_3/10^{-3}$	$1.713 - 3.086i$
$\gamma_4/10^{-4}$	$-6.732 + 105.230i$
$\gamma_5/10^{-4}$	$123.915 - 9.913i$
$\Lambda_1/(10^9\ \mathrm{GeV})$	1.207
$\Lambda_2/(10^2\ \mathrm{GeV})$	2.006
$\lambda_1/10^{-3}$	8.678
$\lambda_2/10^{-3}$	$3.249 - 81.540i$
$\lambda_3/10^{-4}$	$9.383 - 154.099i$
$\lambda_4/10^{-2}$	$8.095 + 18.176i$
$\lambda_5/10^{-2}$	$1.193 - 8.156i$
$\kappa_1/10^{-2}$	-4.752
$\kappa_2/10^{-2}$	$-2.809 + 2.805i$
$\kappa_3/10^{-2}$	$-1.245 - 1.635i$

$\gamma_1/10^{-2}$	1.204
$\gamma_2/10^{-4}$	$3.522 - 9.722i$
$\gamma_3/10^{-3}$	$1.710 - 3.078i$
$\gamma_4/10^{-4}$	$-6.723 + 105.127i$
$\gamma_5/10^{-4}$	$123.760 - 9.751i$
$\Lambda_1/(10^9\ \mathrm{GeV})$	1.298
$\Lambda_2/(10^2\ \mathrm{GeV})$	1.833
$\lambda_1/10^{-3}$	8.674
$\lambda_2/10^{-3}$	$3.246 - 81.548i$
$\lambda_3/10^{-4}$	$9.385 - 154.042i$
$\lambda_4/10^{-2}$	$8.095 + 18.164i$
$\lambda_5/10^{-2}$	$1.194 - 8.155i$
$\kappa_1/10^{-2}$	-4.753
$\kappa_2/10^{-2}$	$-2.808 + 2.804i$
$\kappa_3/10^{-2}$	$-1.244 - 1.635i$
τ_l	$1.180 + 2.711i$

Changing in the lepton sector will feed back into the quark sector by only affecting slightly their RGE evolutions. The best fit point for the quark sector is almost insensitive to the changes in the lepton sector So, we keep fixed the best fit point for the quark sector while best fit the lepton sector with multiple τ.

Numerical fitting

	the same τ	multiple
II: $\rho_{\bar{f}} \in \{\mathbf{1}, \mathbf{1'}, \mathbf{1''}\}$, $\rho_F = \mathbf{3}$, $\rho_E = \mathbf{3}$, $\rho_S = \mathbf{3}$.	$\tau/10^{-2}$	$\tau/10^{-2}$
– **IX**: $\rho_{\bar{f}_{1,2,3}} = (\mathbf{1'}, \mathbf{1''}, \mathbf{1''})$ with modular weights $k_{\bar{f}_{1,2,3}} = (2, 0, 2)$; $k_F = 4$ and $k_E = k_S = 2$.	$3.310 + 359.899i$ $\chi_q^2 = 16.408$ $\chi_l^2 = 486.036$	$3.310 + 359.899i$ $1.744 + 13.565i$ $\chi_l^2 = 30.215$
– **X**: $\rho_{\bar{f}_{1,2,3}} = (\mathbf{1}, \mathbf{1'}, \mathbf{1''})$ with modular weights $k_{\bar{f}_{1,2,3}} = (4, 2, 4)$; $k_F = 4$ and $k_E = k_S = 2$.	$1.198 + 2.835i$ $\chi_q^2 = 69.216$ $\chi_l^2 = 208.261$	$1.198 + 2.835i$ $1.326 + 1.317i$ $\chi_l^2 = 0.878.$
III: $\rho_{\bar{f}} = \mathbf{3}$, $\rho_F \in \{\mathbf{1}, \mathbf{1'}, \mathbf{1''}\}$, $\rho_E = \mathbf{3}$, $\rho_S = \mathbf{3}$.	$\tau/10^{-3}$	$\tau/10^{-3}$
– **IX′**: $\rho_{F_{1,2,3}} = (\mathbf{1'}, \mathbf{1''}, \mathbf{1''})$ with modular weights $k_{F_{1,2,3}} = (2, 4, 2)$; $k_{\bar{f}} = k_E = k_S = 2$.	$3.040 + 719.434i$ $\chi_q^2 = 1.221$ $\chi_l^2 = 0.358$	$3.040 + 719.434$
– **X′**: $\rho_{F_{1,2,3}} = (\mathbf{1}, \mathbf{1'}, \mathbf{1''})$ with modular weights $k_{F_{1,2,3}} = (0, 2, 4)$; $k_{\bar{f}} = k_E = k_S = 2$.	$1.017 + 1.913i$ $\chi_q^2 = 50.511$ $\chi_l^2 = 232.993$	$1.017 + 1.913i$ $5.435 + 9.0761i$ $\chi_l^2 = 58.908$

Conclusions

- Accommodate multiple modulus for a GUT "multiplet" via orbifolding.
- Explain the flavor structures of the Standard Model plus neutrinos in the framework of flipped SU(5) GUT with A_4 modular flavor symmetry.
- Reduce the multiple modular symmetries to a single modular symmetry in the low energy effective theory with proper boundary conditions.
- Classify all possible scenarios in this scheme according to the assignments of the modular A_4 representations for matter superfields.
- Predictions of many scenarios can fit nicely to the experimental data both with one modulus value and multiple modulus values.

Thanks

Petcov, arXiv: 2311.04185

Natural Hierarchy of Flavor Structures In Modular Flavor Models

"Local" Normalisation at $\tau = \tau_v$

modular forms are determined up to a constant

$$N_Y^{(K)} = \left(\sum_i |Y_i^{(K)}(\tau)|^2 (2Im(\tau))^K\right)^{\frac{1}{2}}$$

modular invariant under the modular transformation

$$\sum_i |Y_{\mathbf{r}i}^{(K)}|^2 \to \sum_i |Y_{\mathbf{r}i}^{(K)}|^2 |(c\tau + d)|^{2K} ,$$

$$(2Im(\tau))^K \to (2Im(\tau))^K |(c\tau + d)|^{-2K} .$$

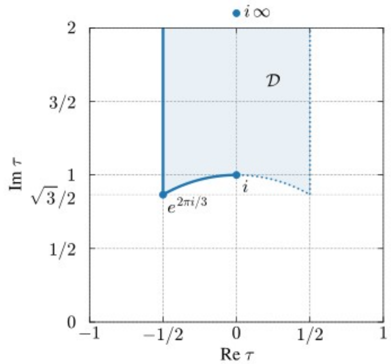
$$K(\tau, \overline{\tau}, \psi_I, \overline{\psi_I}) = -\Lambda_0^2 \log(-i\tau + i\overline{\tau}) + \sum_I \frac{|\psi_I|^2}{(-i\tau + i\overline{\tau})^{k_I}}$$

$$\psi_I \to \sqrt{(2Im(\tau_v))^{k_I}}\,\psi_I$$

$$\psi^c\, Y_{\mathbf{r}}^{(K)}(\tau)\,\psi \to \sqrt{(2Im(\tau_v))^{k_\psi + k_\psi^c}}\,\psi^c\, Y_{\mathbf{r}}^{(K)}(\tau_v)\,\psi / N_Y^{(K)}(\tau_v) = \psi^c\, R_{\mathrm{am}}\, \frac{Y_{\mathbf{r}}^{(K)}(\tau)}{\sqrt{\sum_i |Y_i^{(K)}|^2}}\,\psi ,$$

$$R_{\mathrm{am}} = \frac{\sqrt{(2Im(\tau_v))^{k_\psi + k_\psi^c}}}{\sqrt{(2Im(\tau_v))^K}} = \frac{1}{\sqrt{(2Im(\tau_v))^{k_H}}}$$

$$-\frac{1}{2} \le \mathrm{Re}(\tau) \le \frac{1}{2}, \qquad |\tau| > 1,$$



Mass Hierarchies from Weighton mechanism

arXiv:2002.00969
King& King

- Analogous to the Froggatt-Nielsen mechanism.
- Without requiring any Abelian symmetry, nor any SM singlet flavon to break it.
- The modular weights play the role of FN charges
- SM singlet fields with non-zero modular weight called weightons play the role of flavons.

"weighton" ϕ which is defined to be a SM and A_4 singlet field $k_\phi = 1$ (i.e. weight -1)

$$W_e = \alpha_e e_1^c \tilde{\phi}^4 (LY_{\mathbf{3}}^{(2)})_{\mathbf{1}} H_d + \beta_e e_2^c \tilde{\phi}^2 (LY_{\mathbf{3}}^{(2)})_{\mathbf{1}'} H_d + \gamma_e e_3^c \tilde{\phi} (LY_{\mathbf{3}}^{(2)})_{\mathbf{1}''} H_d$$
$$= \alpha_e e_1^c \tilde{\phi}^4 (L_1 Y_1 + L_2 Y_3 + L_3 Y_2) H_d + \beta_e e_2^c \tilde{\phi}^2 (L_3 Y_3 + L_1 Y_2 + L_2 Y_1) H_d$$
$$+ \gamma_e e_3^c \tilde{\phi} (L_2 Y_2 + L_3 Y_1 + L_1 Y_3) H_d \,,$$

$$W_{driv} = \chi (Y_{\mathbf{1}}^{(4)} \frac{\phi^4}{M_{fl}^2} - M^2)$$

$$\tilde{\phi} \equiv \frac{\langle \phi \rangle}{M_{fl}},$$

$$\tau_T = i\infty$$

$$Y_e = \begin{pmatrix} \alpha_e \tilde{\phi}^4 Y_1 & \alpha_e \tilde{\phi}^4 Y_3 & \alpha_e \tilde{\phi}^4 Y_2 \\ \beta_e \tilde{\phi}^2 Y_2 & \beta_e \tilde{\phi}^2 Y_1 & \beta_e \tilde{\phi}^2 Y_3 \\ \gamma_e \tilde{\phi} Y_3 & \gamma_e \tilde{\phi} Y_2 & \gamma_e \tilde{\phi} Y_1 \end{pmatrix}$$

$$m_e : m_\mu : m_\tau = \alpha_e \tilde{\phi}^4 : \beta_e \tilde{\phi}^2 : \gamma_e \tilde{\phi}.$$

$$\alpha_e, \beta_e, \gamma_e \sim 1. \qquad \tilde{\phi} \approx 1/15$$

$$m_e/m_\mu = 1/207 \text{ and } m_\mu/m_\tau = 1/17$$

Petcov, arXiv: 2311.04185

Natural Hierarchy of Flavor Structures In Modular Flavor Models

“Local” Normalisation at $\tau = \tau_v$

Modular forms are determined up to a constant

Normalize the modular form $Y_{\mathbf{r}}^{K}(\tau)$ by a factor $N_Y^K(\tau)$

modular invariant under the modular transformation

$$N_Y^{(K)} = \left(\sum_i |Y_i^{(K)}(\tau)|^2 (2Im(\tau))^K\right)^{\frac{1}{2}}$$

$$\sum_i |Y_{\mathbf{r}i}^{(K)}|^2 \to \sum_i |Y_{\mathbf{r}i}^{(K)}|^2 |(c\tau+d)|^{2K} ,$$

$$(2Im(\tau))^K \to (2Im(\tau))^K |(c\tau+d)|^{-2K} .$$

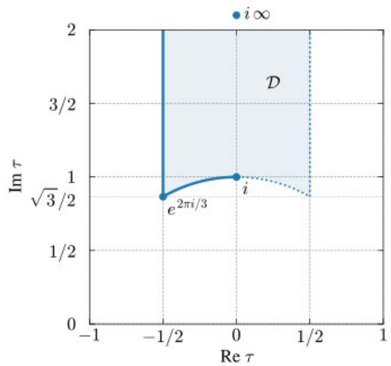
$$K(\tau,\overline{\tau},\psi_I,\overline{\psi_I}) = -\Lambda_0^2 \log(-i\tau+i\overline{\tau}) + \sum_I \frac{|\psi_I|^2}{(-i\tau+i\overline{\tau})^{k_I}}$$

$$\psi_I \to \sqrt{(2Im(\tau_v))^{k_I}}\,\psi_I$$

$$\psi^c\, Y_{\mathbf{r}}^{(K)}(\tau)\, \psi \to \sqrt{(2Im(\tau_v))^{k_\psi+k_\psi^c}}\,\psi^c\, Y_{\mathbf{r}}^{(K)}(\tau_v)\, \psi / N_Y^{(K)}(\tau_v) = \psi^c\, R_{\rm am}\, \frac{Y_{\mathbf{r}}^{(K)}(\tau)}{\sqrt{\sum_i |Y_i^{(K)}|^2}}\, \psi\, ,$$

$$R_{\rm am} = \frac{\sqrt{(2Im(\tau_v))^{k_\psi+k_\psi^c}}}{\sqrt{(2Im(\tau_v))^{K}}} = \frac{1}{\sqrt{(2Im(\tau_v))^{k_H}}}$$

$$-\frac{1}{2} \le {\rm Re}(\tau) \le \frac{1}{2}, \qquad |\tau| > 1,$$



$$
\begin{aligned}
W \supset\ & \alpha \left(LY_{\mathbf{3}}^{(2)}\right)_{\mathbf{1}} H_{d_1} E_1^c + \beta \left(LY_{\mathbf{3}}^{(2)}\right)_{\mathbf{1}'} H_{d_2} E_2^c + \gamma \left(LY_{\mathbf{3}}^{(2)}\right)_{\mathbf{1}''} H_{d_3} E_3^c \\
&= \alpha e_1^c (L_1 Y_1 + L_2 Y_3 + L_3 Y_2) H_{d_1} + \beta e_2^c (L_3 Y_3 + L_1 Y_2 + L_2 Y_1) H_{d_2} \\
&\quad + \gamma e_3^c (L_2 Y_2 + L_3 Y_1 + L_1 Y_3) H_{d_3}
\end{aligned}
$$

$$
\boxed{W_d = \alpha_d d_1^c (QY_{\mathbf{3}}^{(2)})_{\mathbf{1}} H_{d_1} + \beta_d d_2^c (QY_{\mathbf{3}}^{(2)})_{\mathbf{1}'} H_{d_2} + \gamma_d d_3^c (QY_{\mathbf{3}}^{(4)})_{\mathbf{1}''} H_{d_3}}
$$

$$
Y_d^{II} = \begin{pmatrix}
\frac{\alpha_d}{\sqrt{\sum_i |Y_{3;i}^{(2)}(\tau)|^2}} J^{-H_{d_1}} Y_1 & \frac{\alpha_d}{\sqrt{\sum_i |Y_{3;i}^{(2)}(\tau)|^2}} J^{-H_{d_1}} Y_3 & \frac{\alpha_d}{\sqrt{\sum_i |Y_{3;i}^{(2)}(\tau)|^2}} J^{-H_{d_1}} Y_2 \\
\frac{\beta_d}{\sqrt{\sum_i |Y_{3;i}^{(2)}(\tau)|^2}} J^{-H_{d_2}} Y_2 & \frac{\beta_d}{\sqrt{\sum_i |Y_{3;i}^{(2)}(\tau)|^2}} J^{-H_{d_2}} Y_1 & \frac{\beta_d}{\sqrt{\sum_i |Y_{3;i}^{(2)}(\tau)|^2}} J^{-H_{d_2}} Y_3 \\
\frac{\gamma_d}{\sqrt{\sum_i |Y_{3;i}^{(4)}(\tau)|^2}} J^{-H_{d_3}} Y_{\mathbf{3};3}^{(4)} & \frac{\gamma_d}{\sqrt{\sum_i |Y_{3;i}^{(4)}(\tau)|^2}} J^{-H_{d_3}} Y_{\mathbf{3};2}^{(4)} & \frac{\gamma_d}{\sqrt{\sum_i |Y_{3;i}^{(4)}(\tau)|^2}} J^{-H_{d_3}} Y_{\mathbf{3};1}^{(4)}
\end{pmatrix}
$$

$$\mathrm{Re}\langle\tau\rangle = -0.9490362\ 110788305\ , \mathrm{Im}\langle\tau\rangle = 3.48074851\ 04376763\ ,$$

$$\tan\beta = 10, \lambda = 1000000000\ 00, K_{Hu_1} = 6, K_{Hu_2} = 0, K_{Hu_3} = 0,$$

$$K_{Hd_1} = 6, K_{Hd_2} = 0, K_{Hd_3} = 0, \alpha_e = 0.00995733\ 4176341166$$

$$\beta_{\mathrm{e}} = 0.00610265\ 7788897477\ , \gamma_{\mathrm{e}} = 0.10262857\ 237324072$$

$$\alpha_{\mathrm{u}} = 0.00571992\ 995110763,\ \ \beta_u^I = 0.00538587\ 6442593288$$

$$\beta_{\mathrm{u}}^{\mathrm{II}} = 0.52450197\ 54067213,\ \ \gamma_{\mathrm{u}}^{\mathrm{I}} = 0.99973927\ 5925628$$

$$\gamma_u^{II} = 0.92762607\ 30756551,\ \ \alpha_{\mathrm{d}} = 0.09099967\ 098193282$$

$$\beta_{\mathrm{d}} = 0.00537153\ 0083699261\ , \gamma_{\mathrm{d}} = 0.24143006\ 98269463$$

$$\chi_e^2 = 12.790378306057285, \chi_d^2 = 3.71963917459072\mathrm{e}\text{-}15, \chi_u^2 = 6.304823038093906\mathrm{e}-07,$$

$$\chi_{PMNS}^2 = 3235.1099228609064, \chi_{CKM}^2 = 56880.82027869426$$