第三届高能物理理论与实验融合发展研讨会, Nov 1-4 2024

Flavor physics in the continuum using CLQCD ensembles

Yi-Bo Yang

中国科学院理论物理研究所
INSTITUTE OF THEORETICAL PHYSICS, CHINESE ACADEMY OF SCIENCES
1978

中国科学院大学
University of Chinese Academy of Sciences

ICTP-AP
International Centre for Theoretical Physics Asia-Pacific
国际理论物理中心-亚太地区

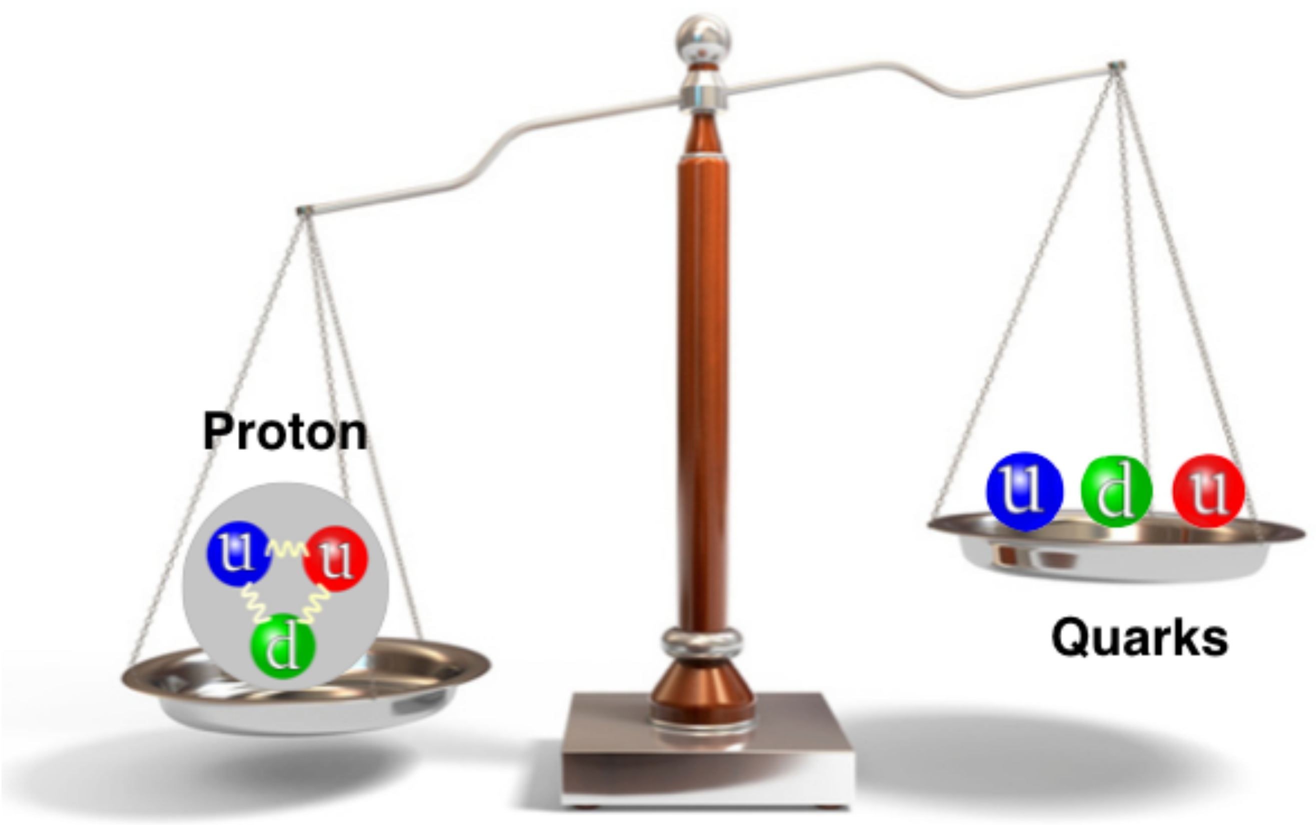
With Bo-Lun Hu, et.al.,

For CLQCD collaboration

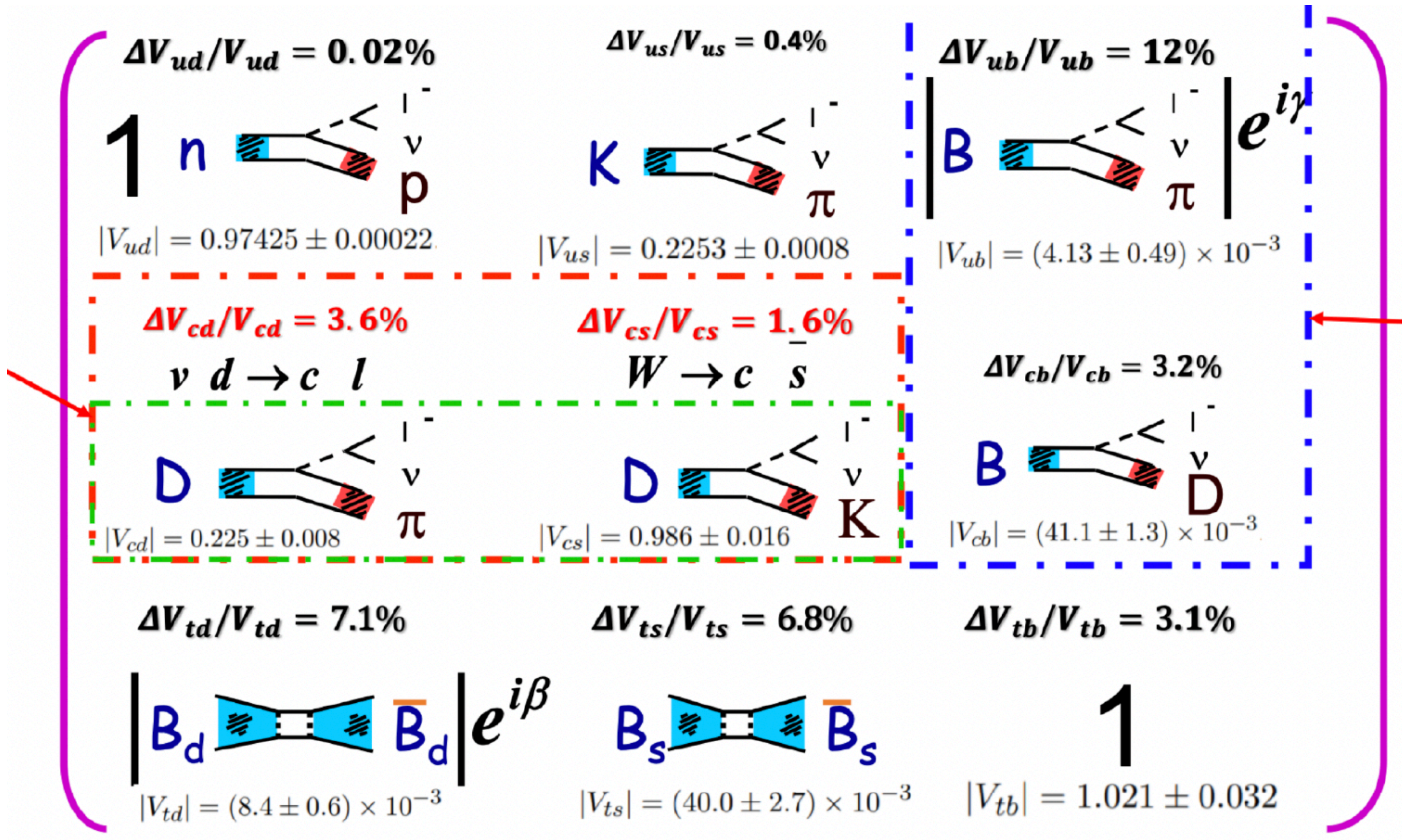
CLQCD

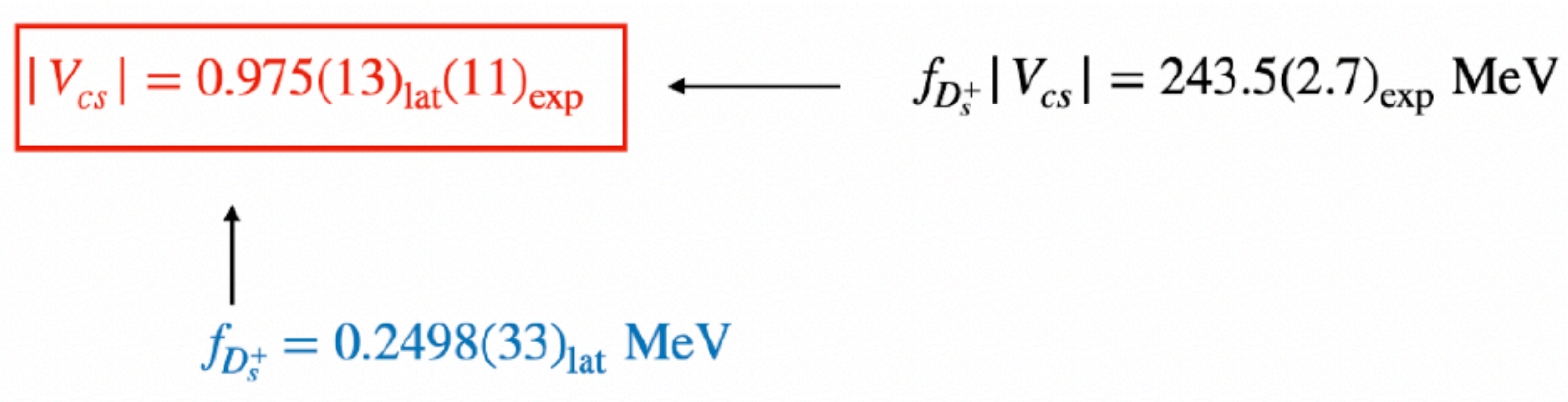
Quark mass

and CKM matrix elements

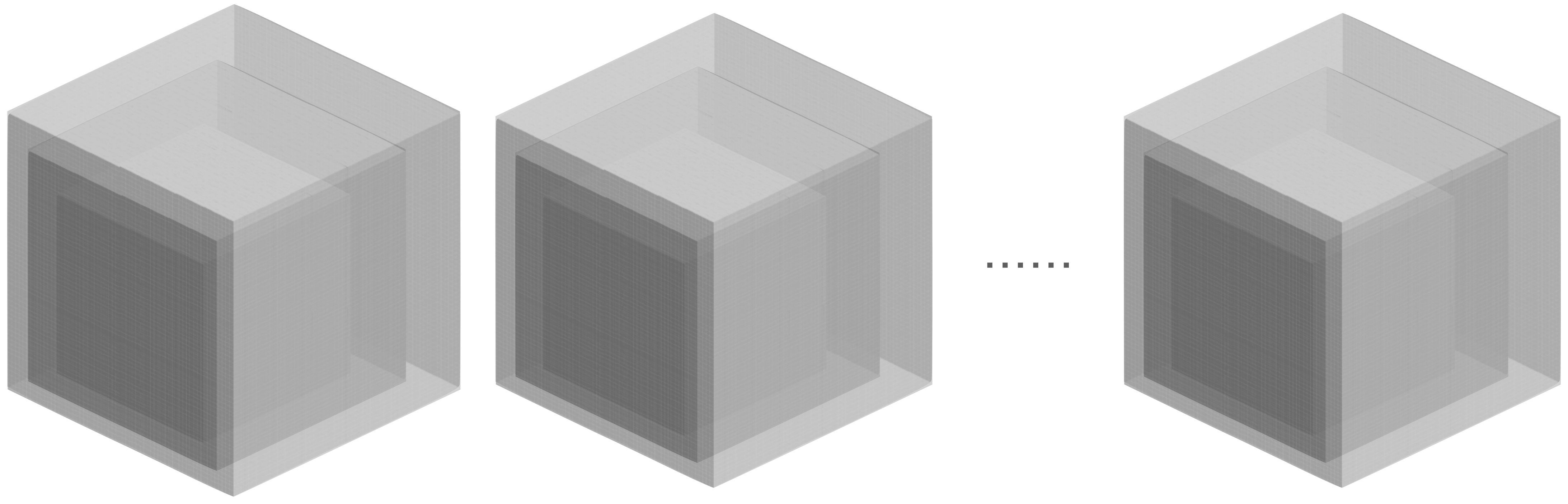


- Higgs boson provides very different coupling to six quark flavors;
- Light quark mass can not be determined by experiment directly;
- Weak decay between different flavors also requires inputs from strong interaction.



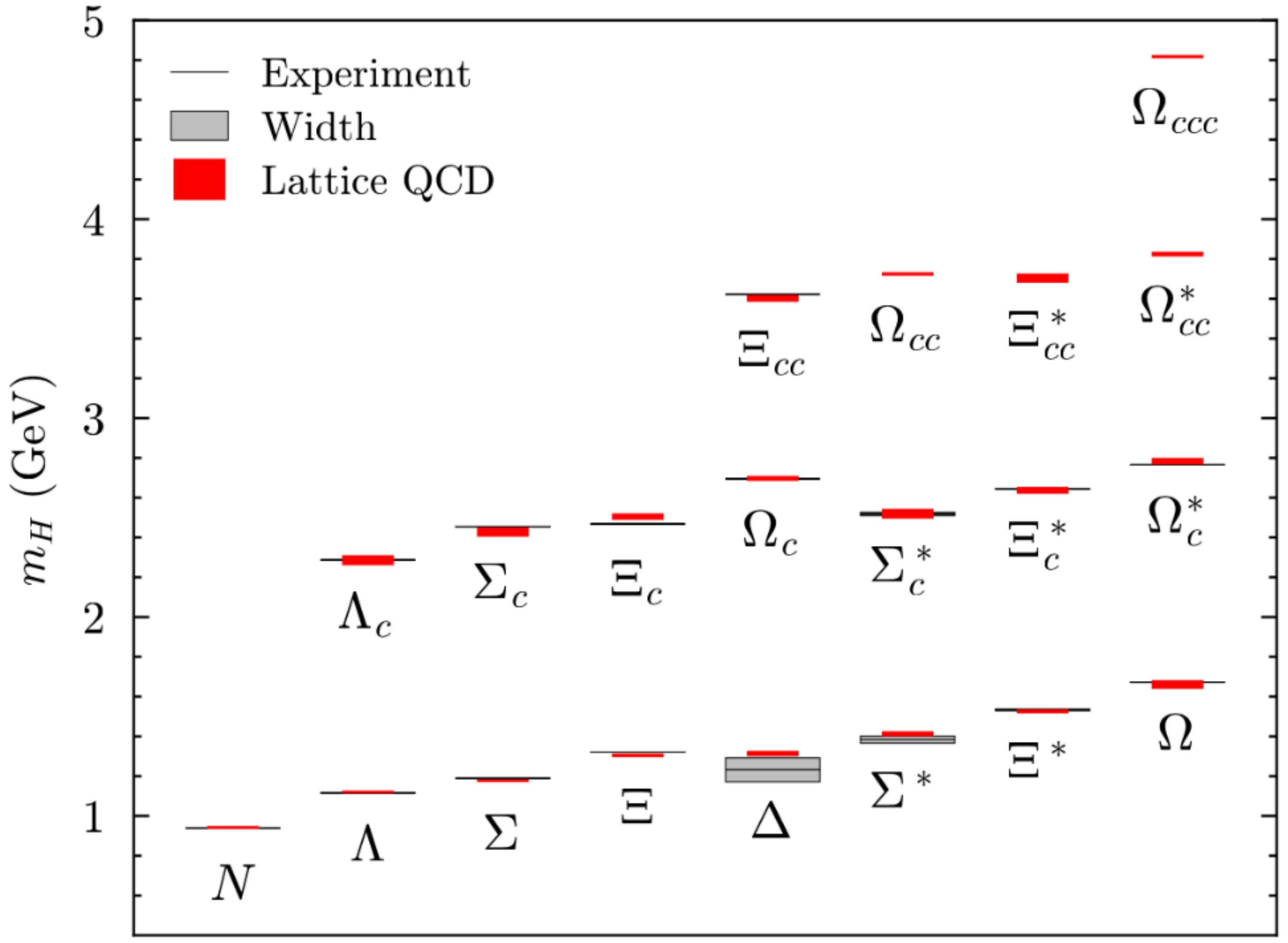


Lattice QCD



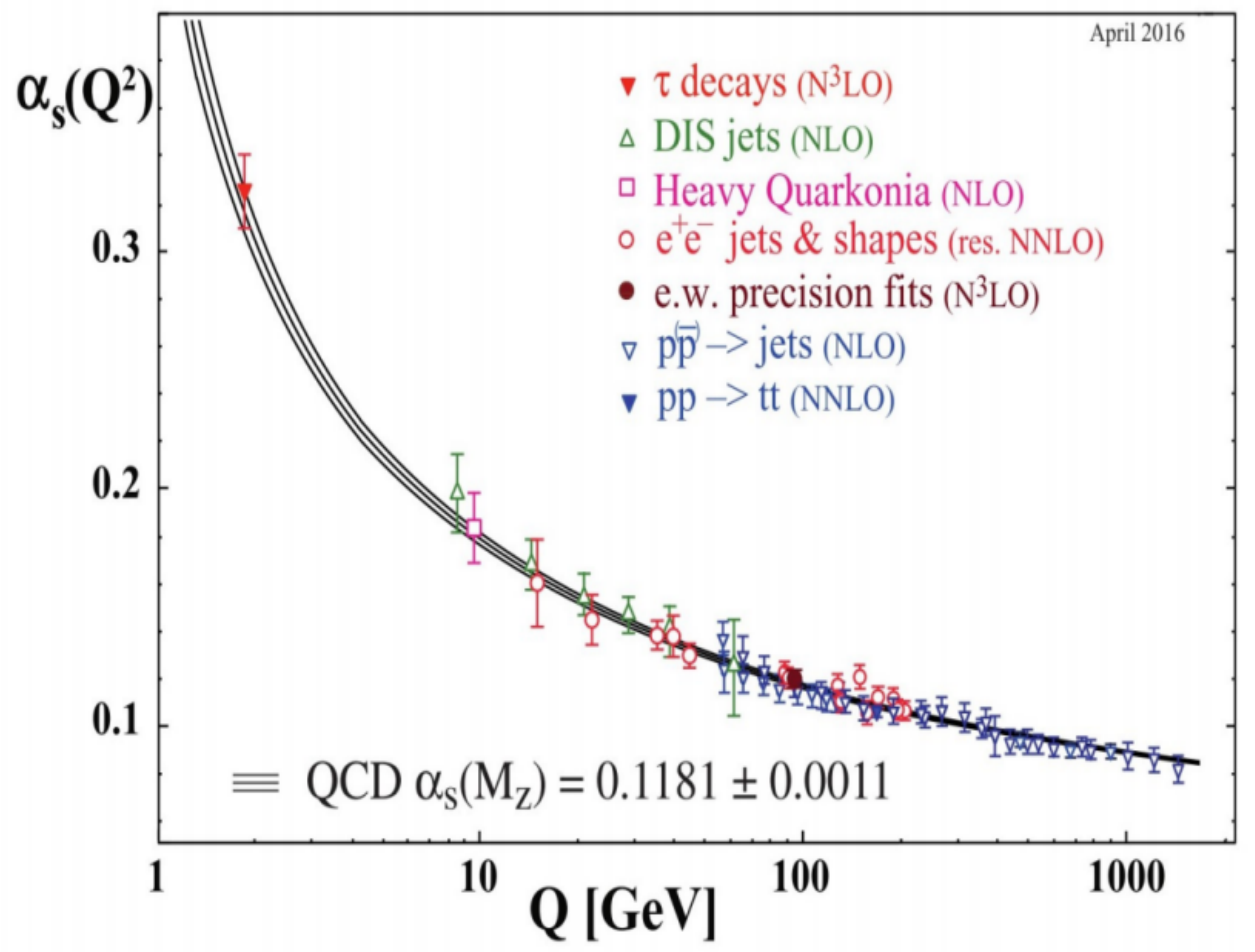
$$\langle \mathcal{O} \rangle = \frac{\int [\mathrm{d}A\mathrm{d}\psi]\mathcal{O}(A,\psi)e^{-\int \mathrm{d}^4x\mathcal{L}(A(x),\psi(x))}}{\int [\mathrm{d}A\mathrm{d}\psi]e^{-\int \mathrm{d}^4x\mathcal{L}(A(x),\psi(x))}}$$

$$= \frac{\sum_{i=1}^{N}\mathcal{O}(A_i, S_\psi(A_i))}{N} + \mathcal{O}(\frac{1}{\sqrt{N}})$$



B.-L. Hu, et. al., CLQCD, in preparation

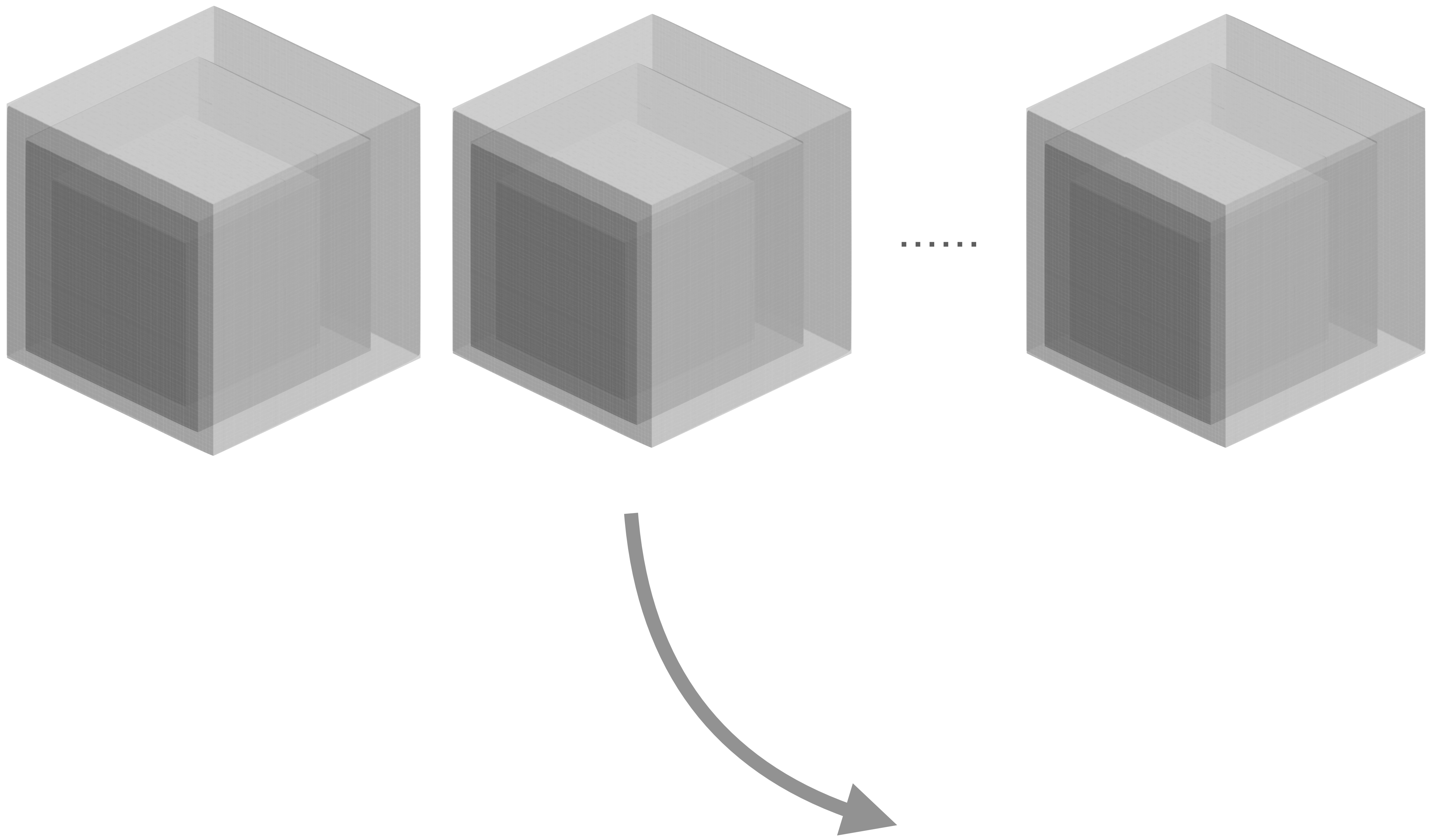
and QCD



- QCD is non-perturbative at the hadron scale;
- Lattice QCD can provide accurate first principle predictions on the QCD features of hadron.

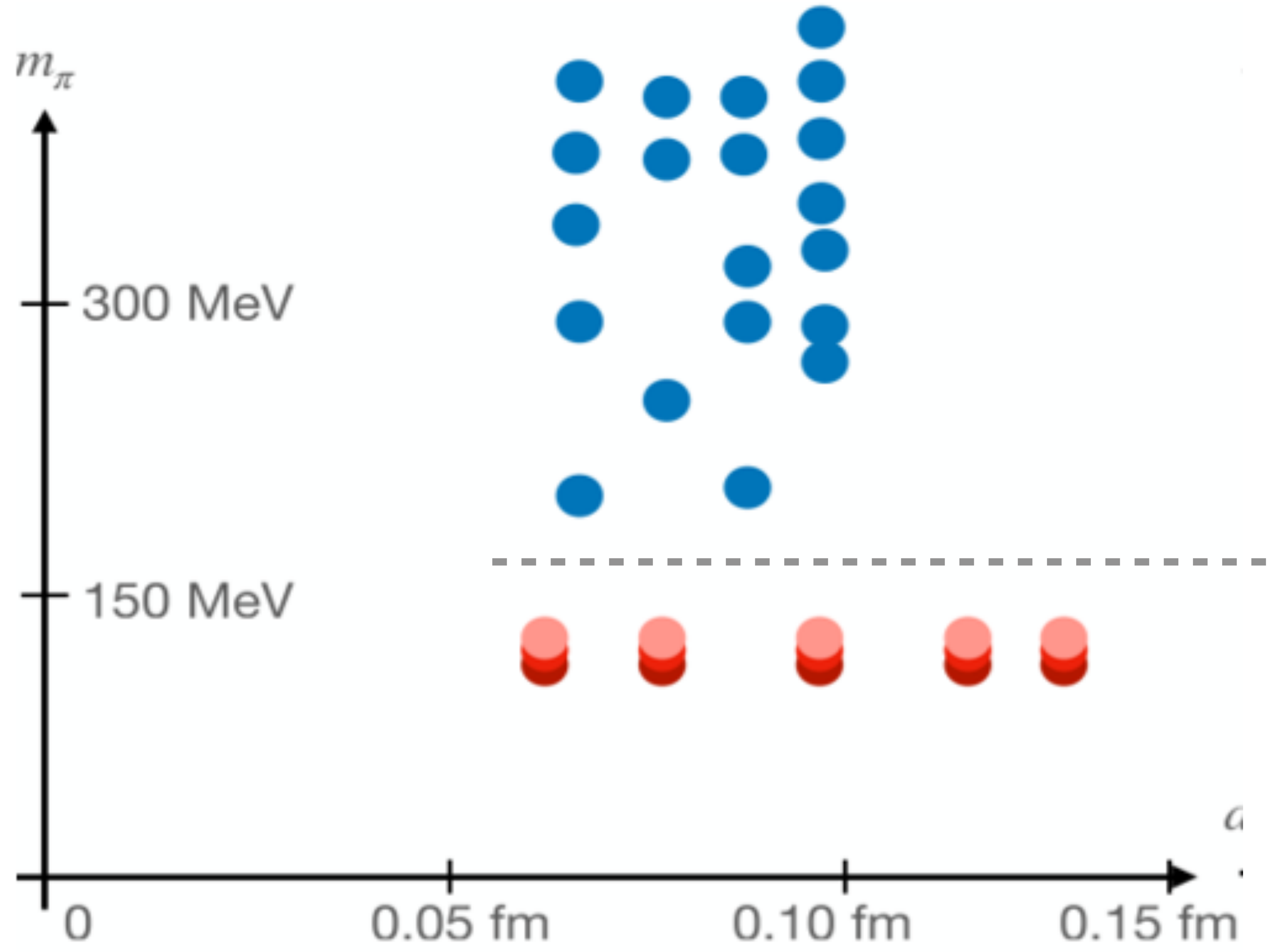
Lattice QCD

and gauge configurations

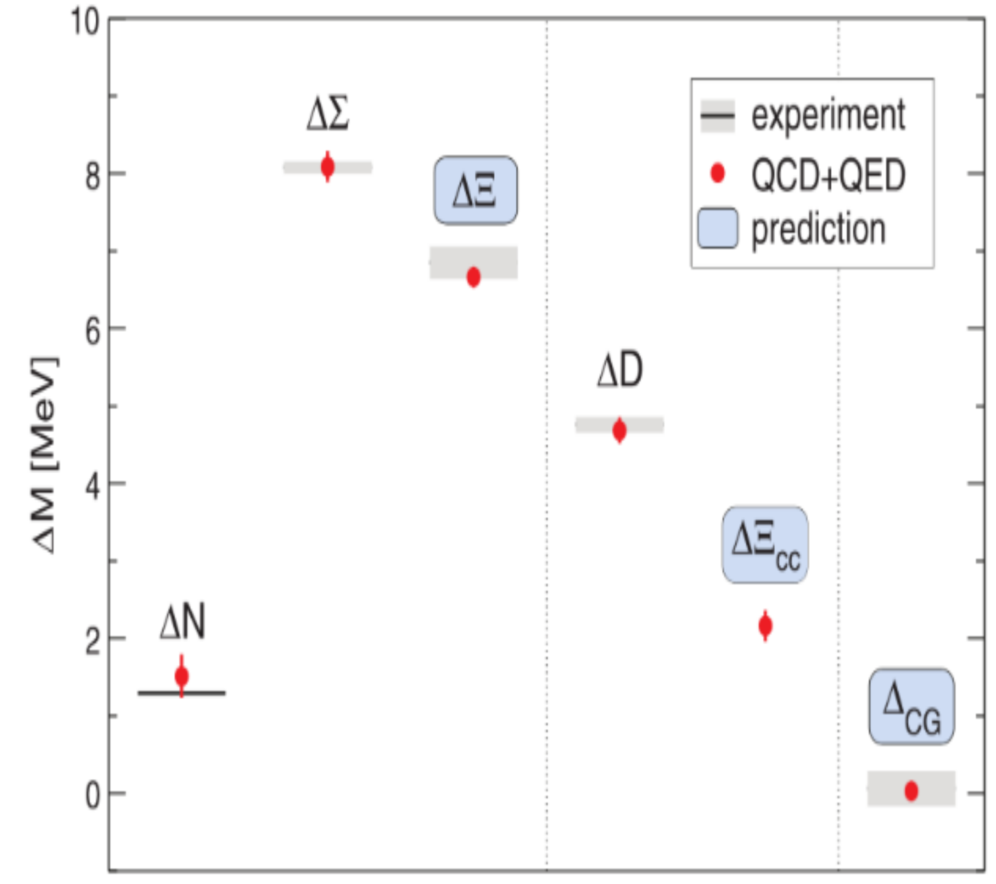


- Accurate Lattice QCD prediction requires thousands gauge configurations on many ensembles with different lattice spacing, quark masses, and volume to control the systematic uncertainties.

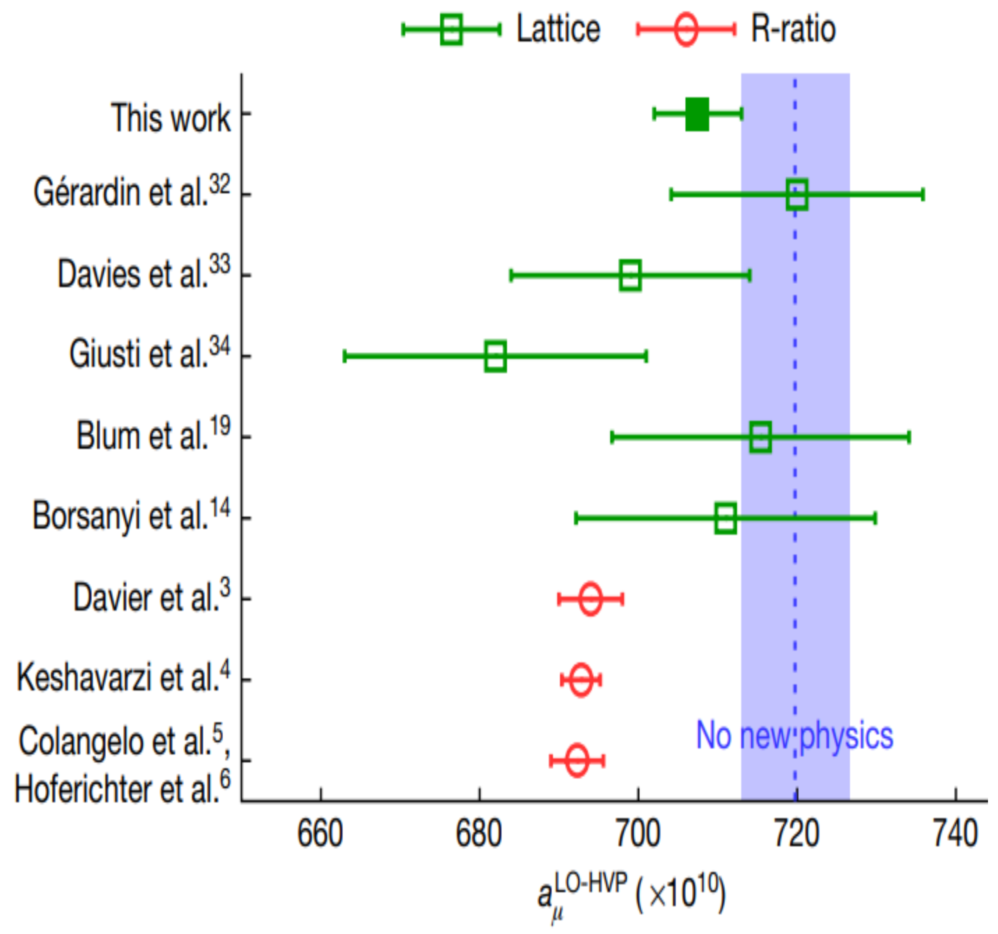
- Generate those ensembles will take several years and spend millions US dollars.



BMW collaboration

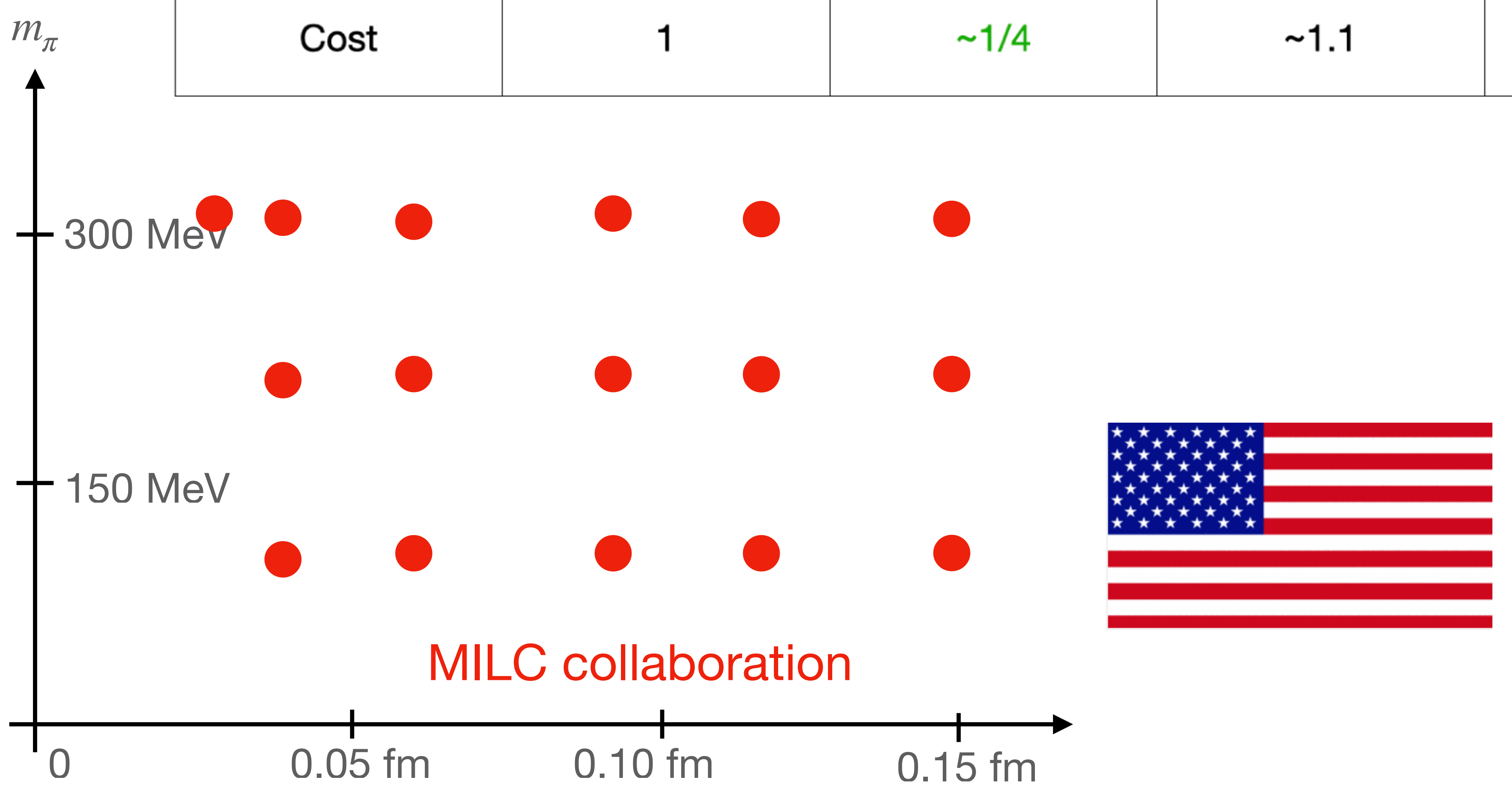


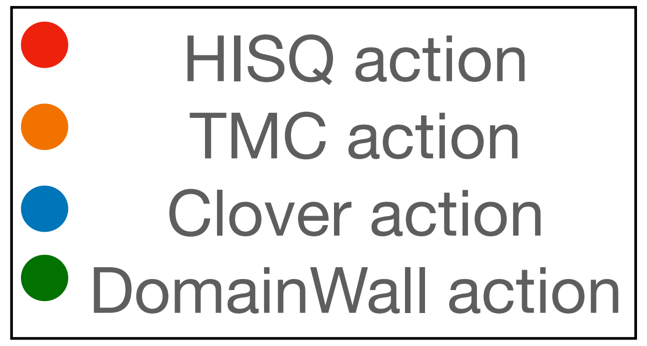
Science 347 1452-1455(2015)

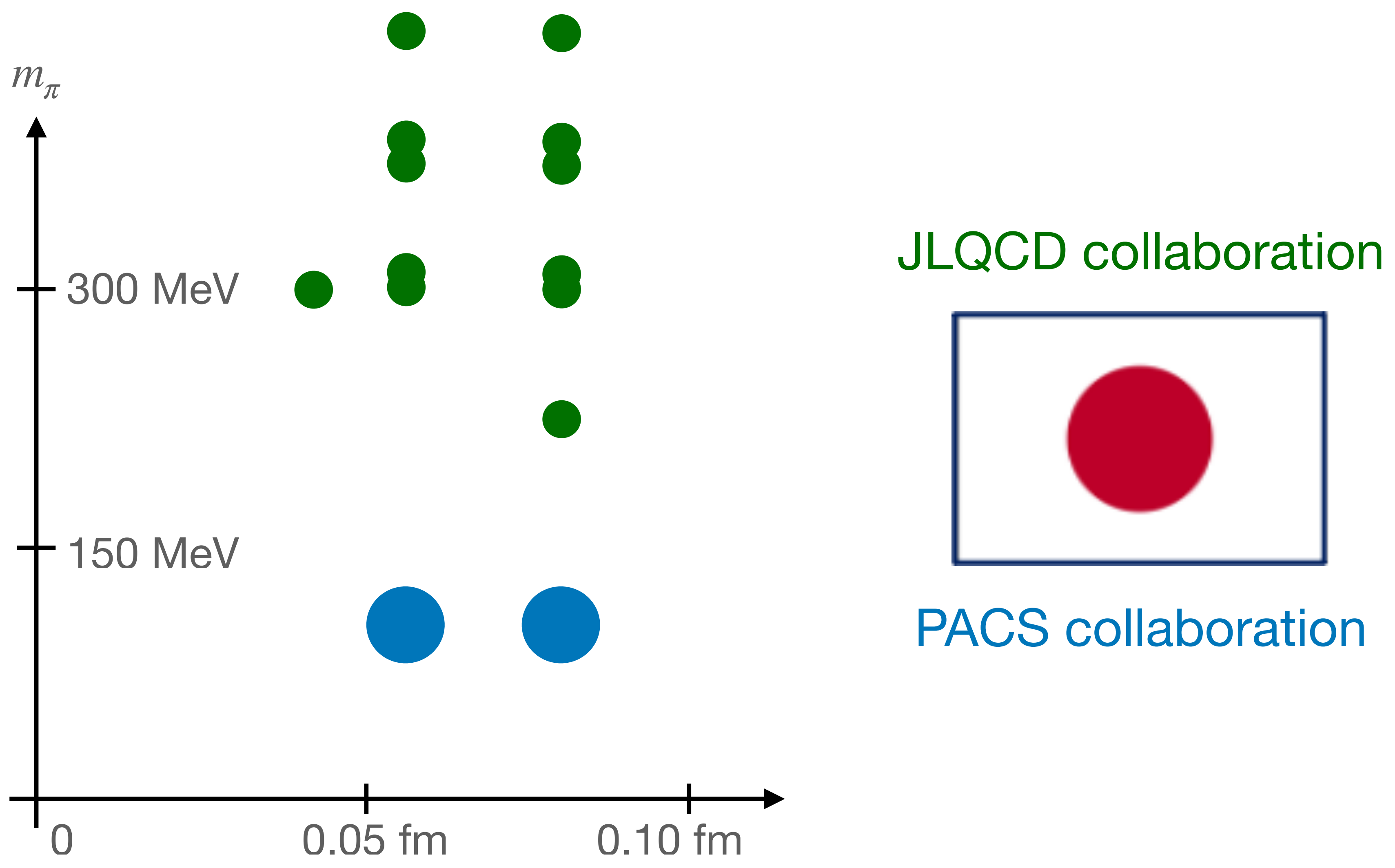


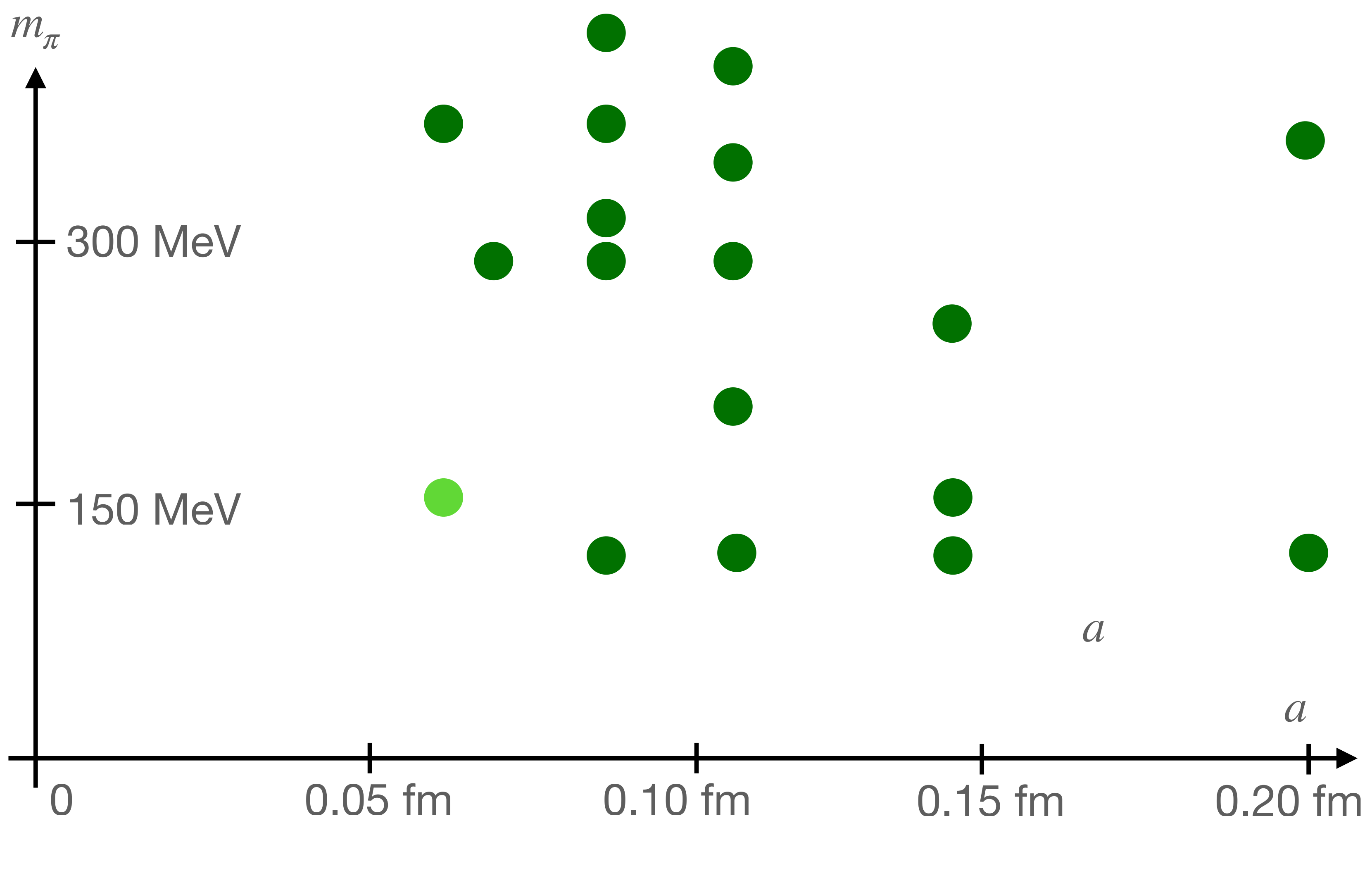
Nature 593 7857, 51-55 (2021)

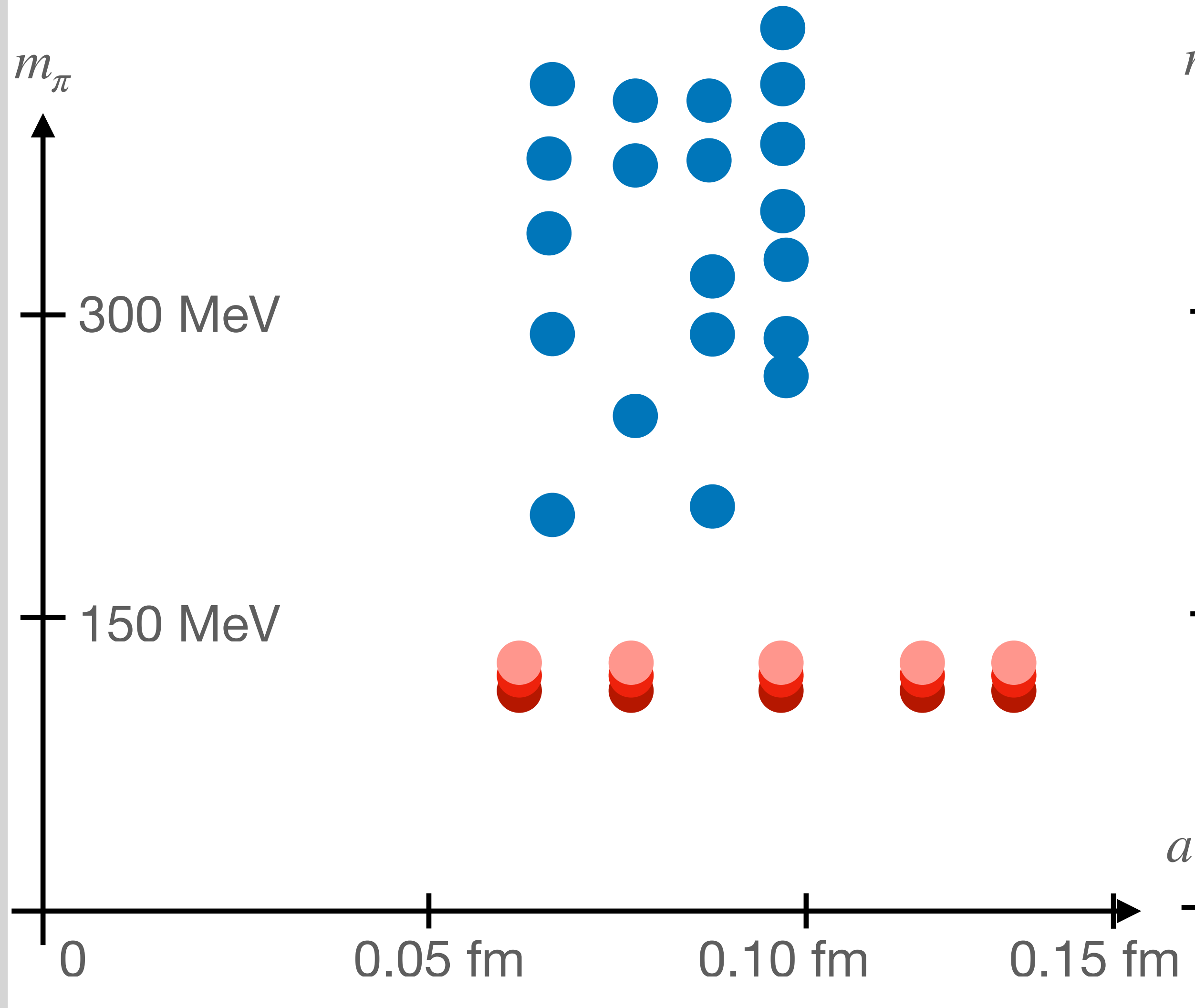
	Naive	Staggered/HISQ	Wilson/Clover	Twisted-mass	Overlap/Domain wall
Form	$D^{\text{naive}} = \gamma_\mu(\delta_{x,x+\mu} - \delta_{x,x-\mu})$	$D^{\text{st}} = \gamma_\mu^{\text{st}}(x)(\delta_{x,x+\mu} - \delta_{x,x-\mu})$	$D^{\text{clv}} = D + aD^2 + ac_{sw}F_{\mu\nu}\sigma^{\mu\nu}$	$D^{\text{tm}} = D^{\text{clv}} + \mathrm{i}\tau_3 m$	$D^{\text{ov}} = [1 + \gamma_5 D(-\rho)/\sqrt{D^\dagger(-\rho)D(-\rho)}]/\rho$
Fermion copies	16	4	1	1	1
Chiral symmetry breaking	N/A	$\mathcal{O}(a^4)$	$\mathcal{O}(\alpha_s/a)$	$\mathcal{O}(\alpha_s)$	N/A
Cost	1	~1/4	~1.1	~1.1	~10-100

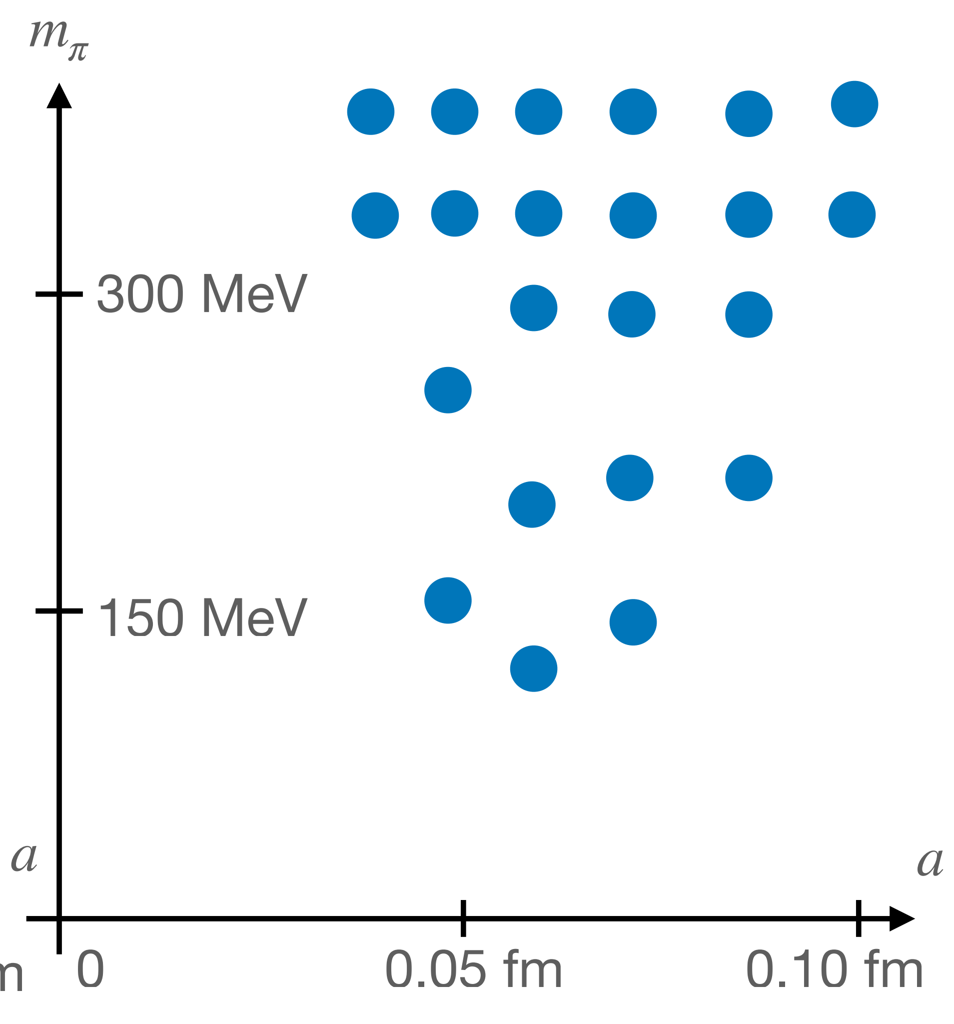


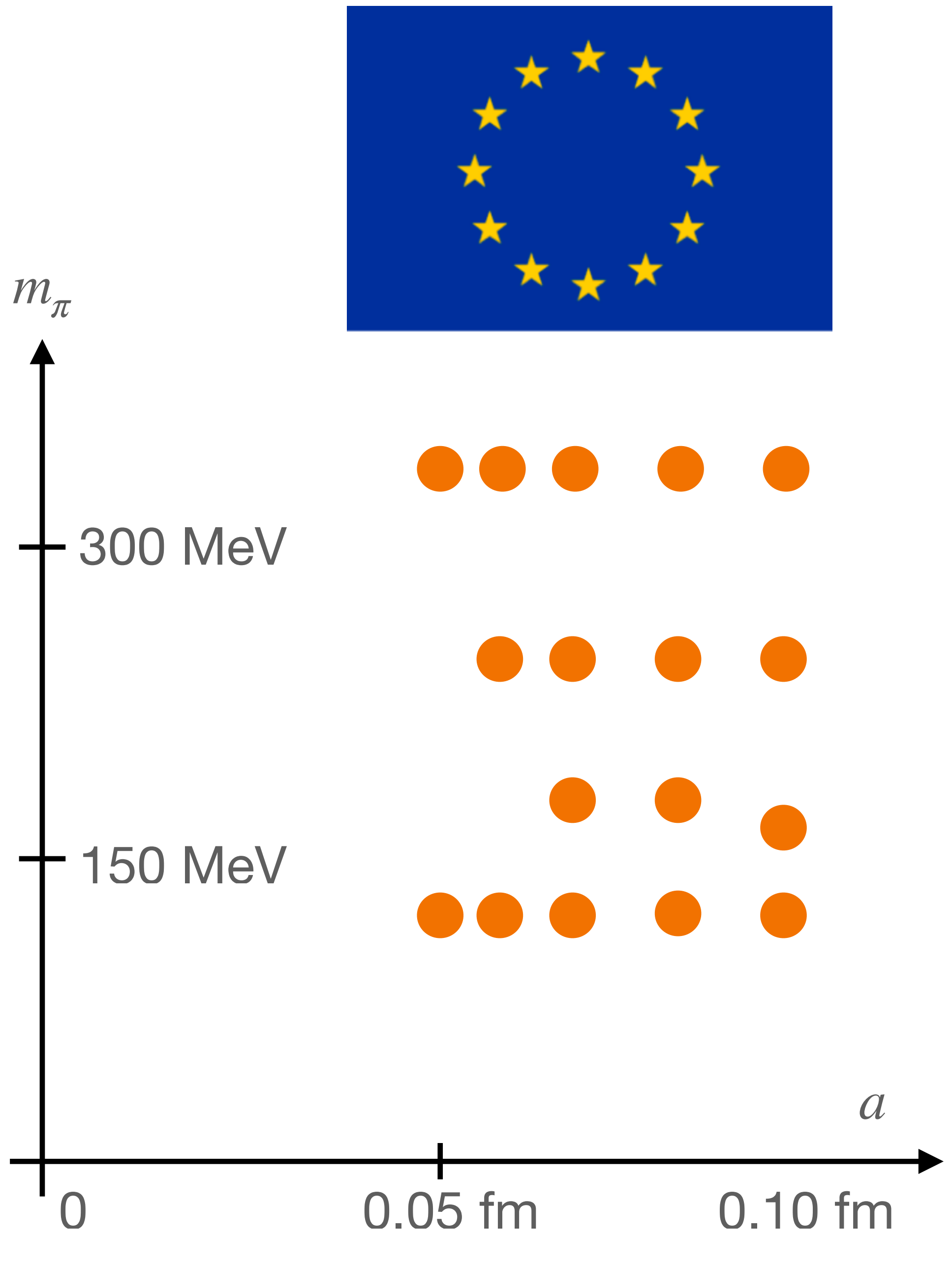




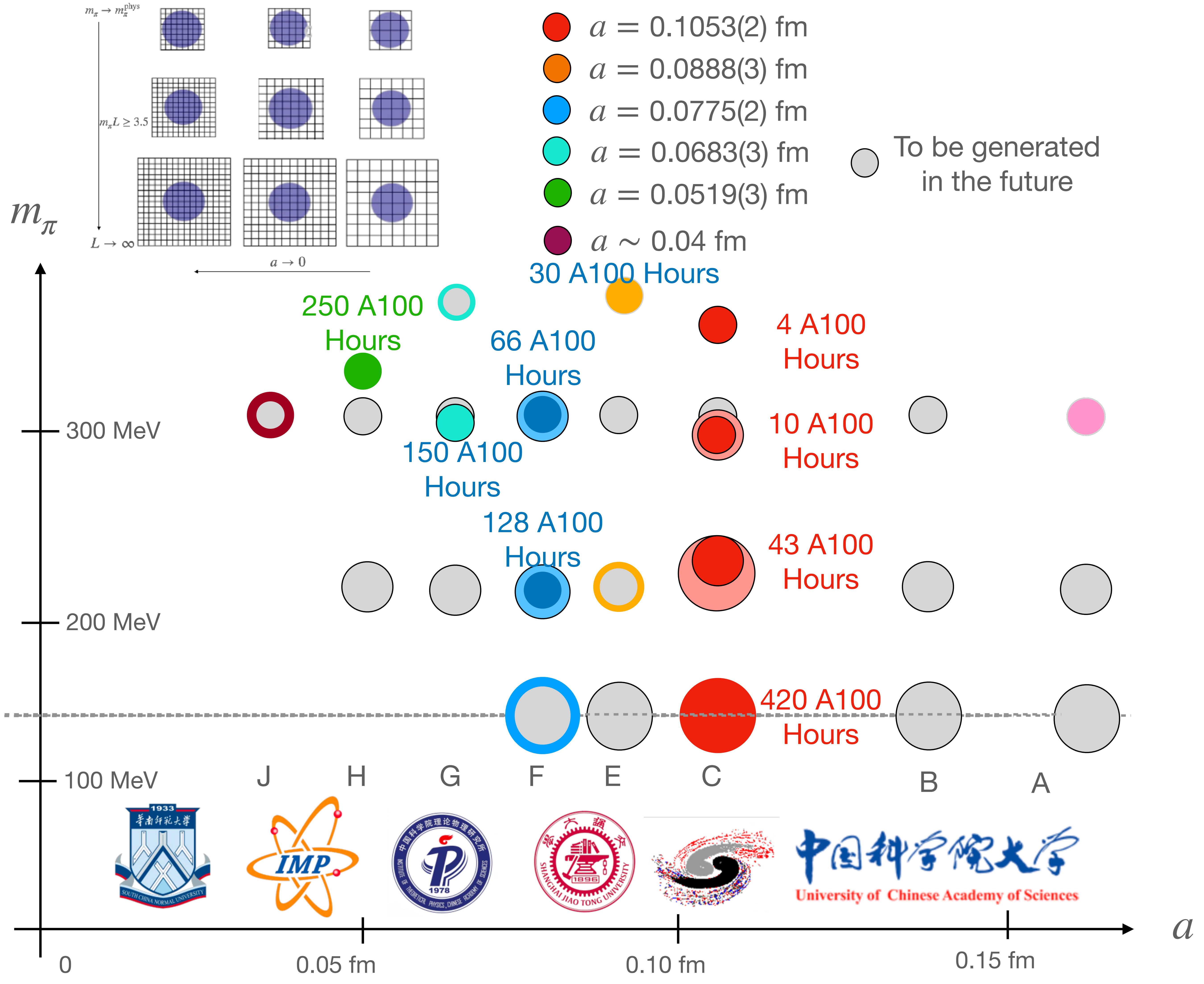








Gauge ensembles



Z.-H. Hu, B.-L. Hu, J.-H. Wang, et. al., CLQCD, PRD109(2024) 054507

CLQCD ensembles

14 ensembles with more than 5,000 configurations in total:

- 5 lattice spacings from 0.05 fm to 0.11 fm;
- 7 pion masses from 130 MeV to 350 MeV;
- 4 Volumes from 2.5 fm to 5.0fm.
- More ensembles are in production;

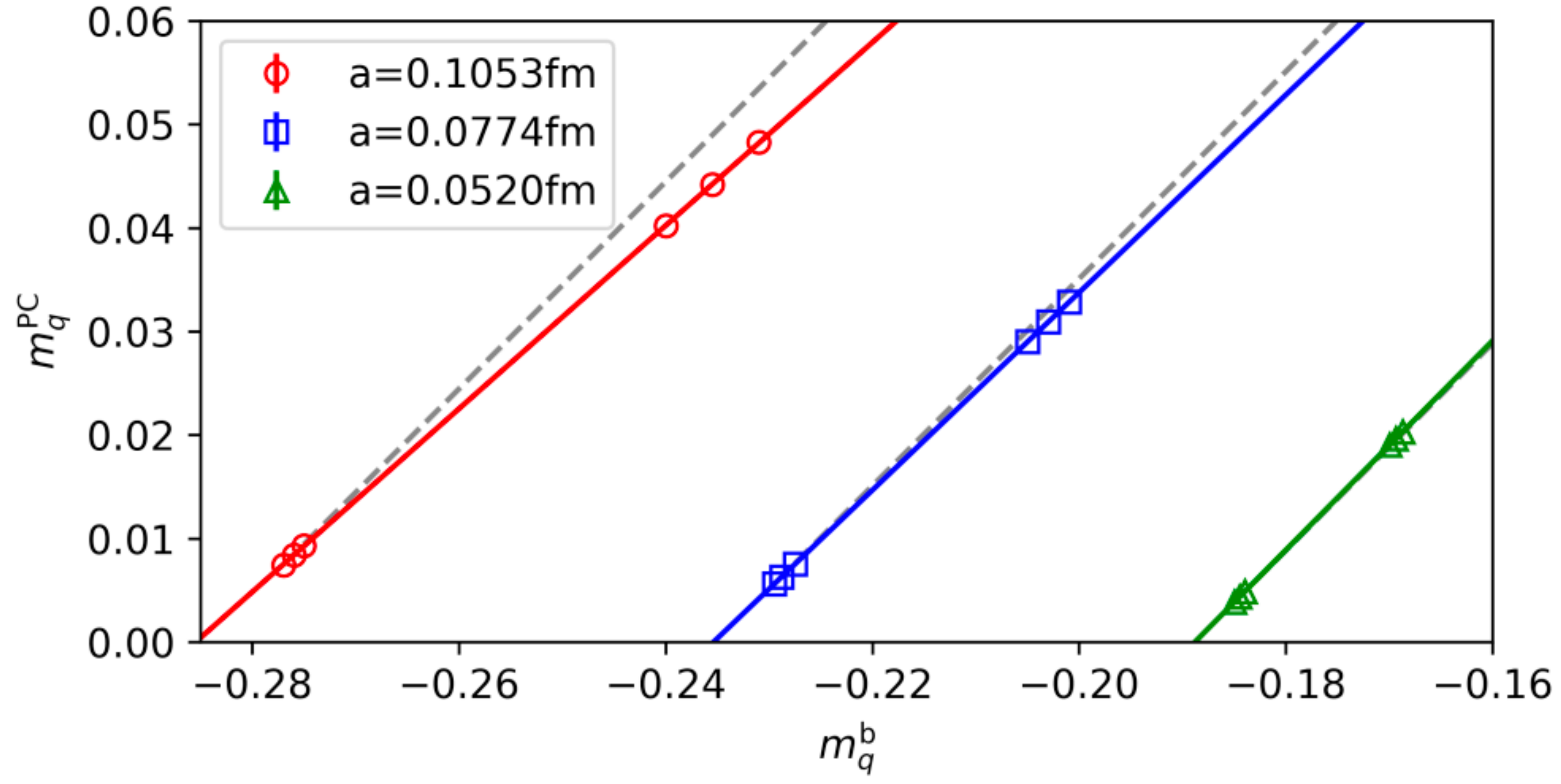
Work published with CLQCD ensembles:

- HB Yan et al PRL accepted **(2024)**
- Y Meng et al Phys.Rev.D 7, 074510 (**2024**)
- ZC Hu et al Phys.Rev.D 109.054507 (**2024**)
- Y Meng et al Phys.Rev.D 109 7, 074511(**2024**)
- H Liu et al Sci.China Phys.Mech.Astron. 67 1, 211011(**2024**)
- H Liu et al Phys.Rev.D 109 3, 036037(**2024**)

- H Liu et al Phys.Lett.B 841 137941(2023)
- DJ Zhao et al Phys.Rev.D 107 9, L091501(2023)

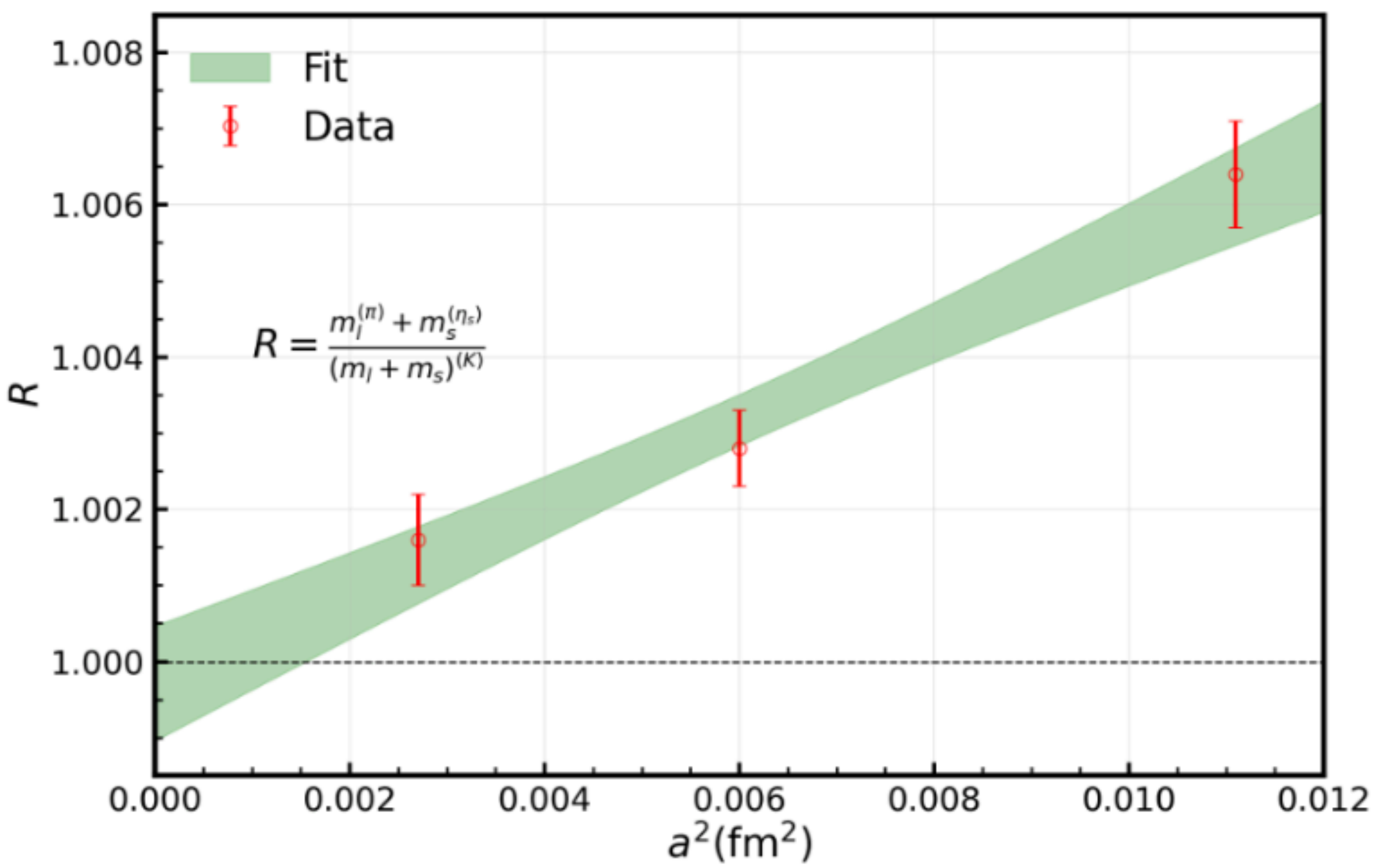
- QA Zhang et al Chin.Phys.C 46 (**2022**)

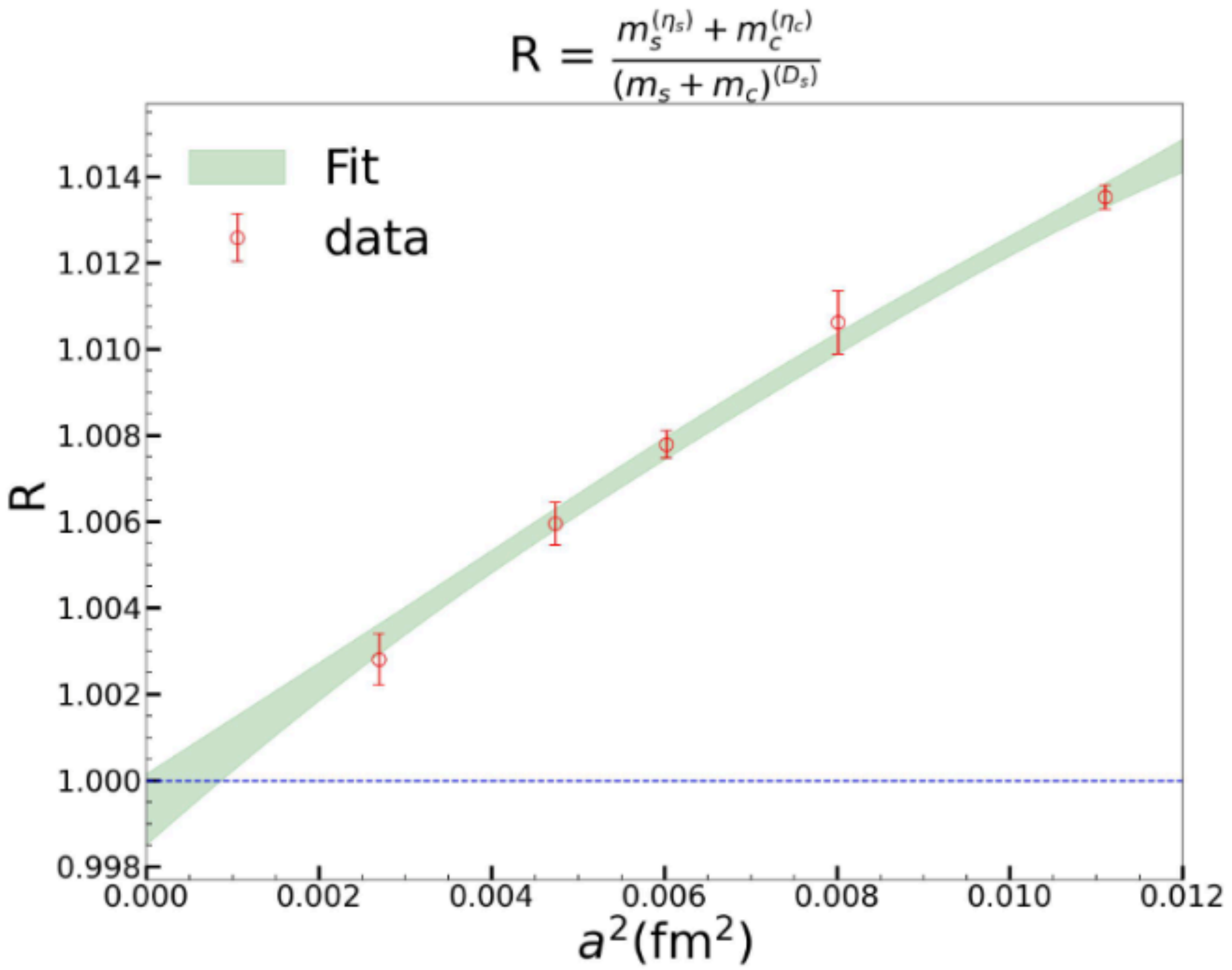
PCAC quark masses



$$m_q^{PC} = \frac{\langle 0|\partial_4 A_4|PS\rangle}{2\langle 0|P|PS\rangle} = \frac{m_{PS}\sum_{\vec{x}}\langle A_4(\vec{x},t)P^\dagger(\vec{0},0)\rangle}{2\sum_{\vec{x}}\langle P(\vec{x},t)P^\dagger(\vec{0},0)\rangle}\Big|_{t\to\infty}$$

$$= \frac{m_\pi^2}{2\Sigma/F^2}\left(1+\mathcal{O}\left(\frac{m_\pi^2}{16\pi^2F^2}\right)\right) \sim \frac{m_\pi^2}{5\ \mathrm{GeV}}$$





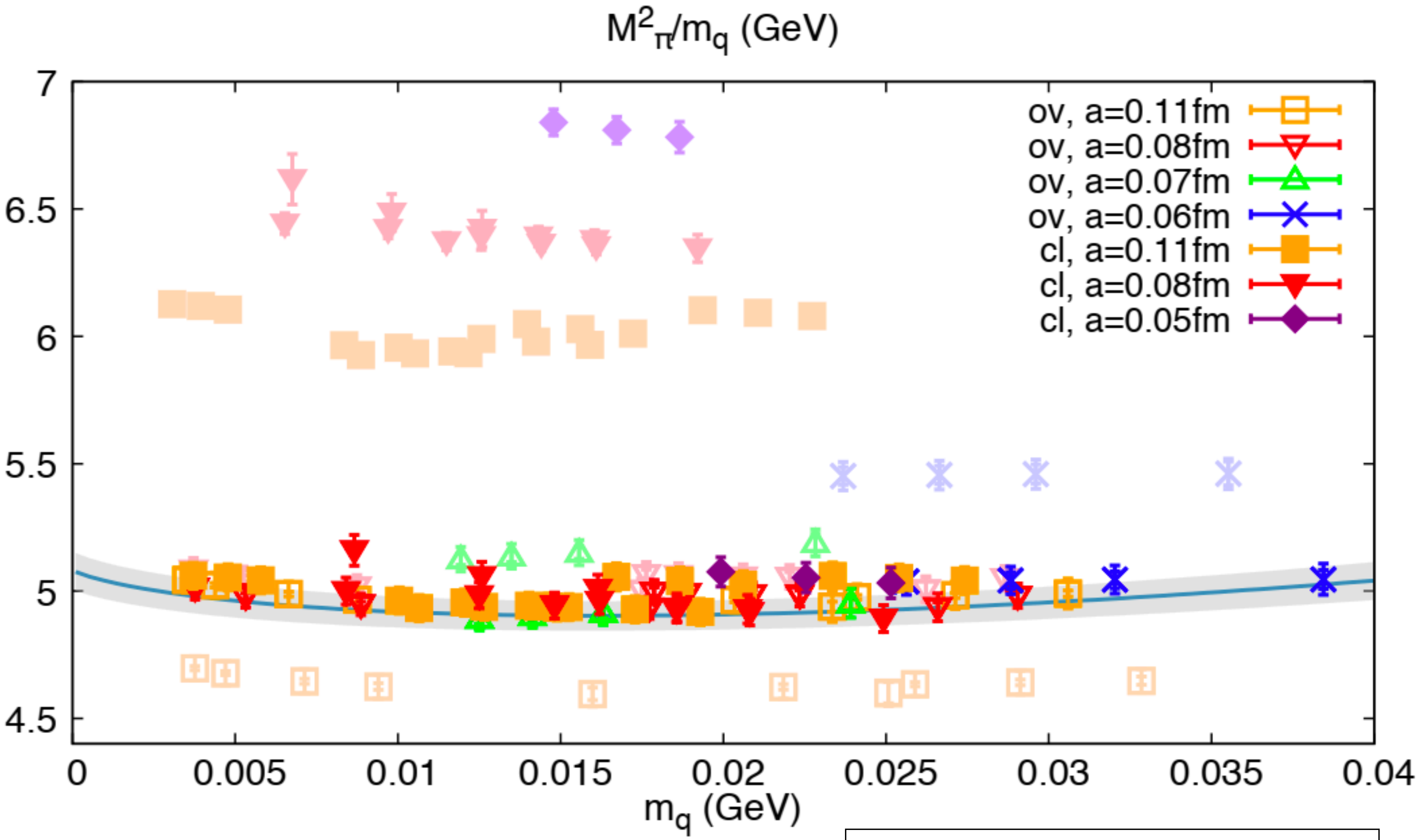
Definition

Defining quark mass from the PCAC relation can avoid the additive renormalization of the clover fermion action:

- PCAC quark mass defined from different PS hadron can differ by ~1% at the coarsest lattice spacing ~0.11 fm;
- And becomes consistent with each other at 0.1% level after the continuum extrapolation.

Renormalized quark masses

Impact of the renormalization



Dian-Jun Zhao, et.al., χQCD, in preparation

- $m_\pi^2/m_q \sim \Sigma/F^2$ which is insensitive to the quark mass, with the partially quenching effect subtracted;
- The PCAC mass $m_q^{PC} = \frac{\langle 0|\partial_4 A_4|PS\rangle}{2\langle 0|P|PS\rangle}$ has obvious 1/a and action dependences:

1. Smaller with large intrinsic scale 1/a;
2. Very sensitive to the fermion action.

- RI/MOM renormalization eliminates both the dependences and makes $m_\pi^2/m_q^{\overline{MS}}$ of all the ensembles on a similar curve.

Quark mass

Light and strange quark masses

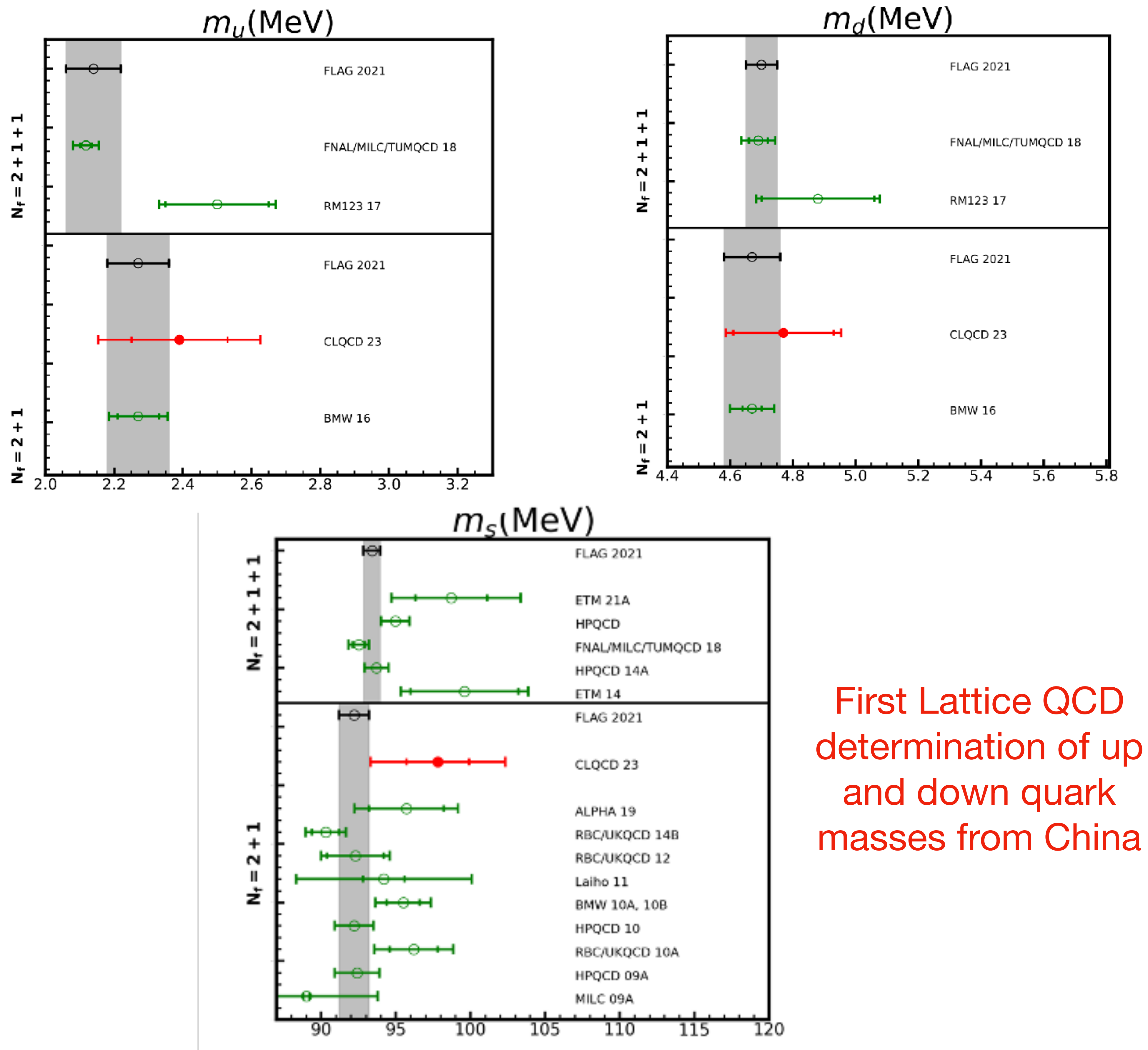
P.Zyla et,al, PTEP(2020)083C01 (PDG2020):

- $m_p = 938.27\ \mathrm{MeV} = m_{p,\mathrm{QCD}} + 1.00(16)\ \mathrm{MeV} + \ldots;$
- $m_n = 939.57\ \mathrm{MeV};$
- $m_\pi^0 = 134.98\ \mathrm{MeV};$
- $m_\pi^+ = 139.57\ \mathrm{MeV} = m_\pi^0 + 4.53(6)\ \mathrm{MeV} + \ldots;$

X. Feng, et,al. Phys.Rev.Lett.128(2022)062003

- $m_K^0 = 497.61(1)\ \mathrm{MeV} = m_{K,\mathrm{QCD}}^0 + 0.17(02)\ \mathrm{MeV} + \ldots;$
- $m_K^+ = 493.68(2)\ \mathrm{MeV} = m_{K,\mathrm{QCD}}^+ + 2.24(15)\ \mathrm{MeV} + \ldots$

D. Giusti, et,al. PRD95(2017)114504

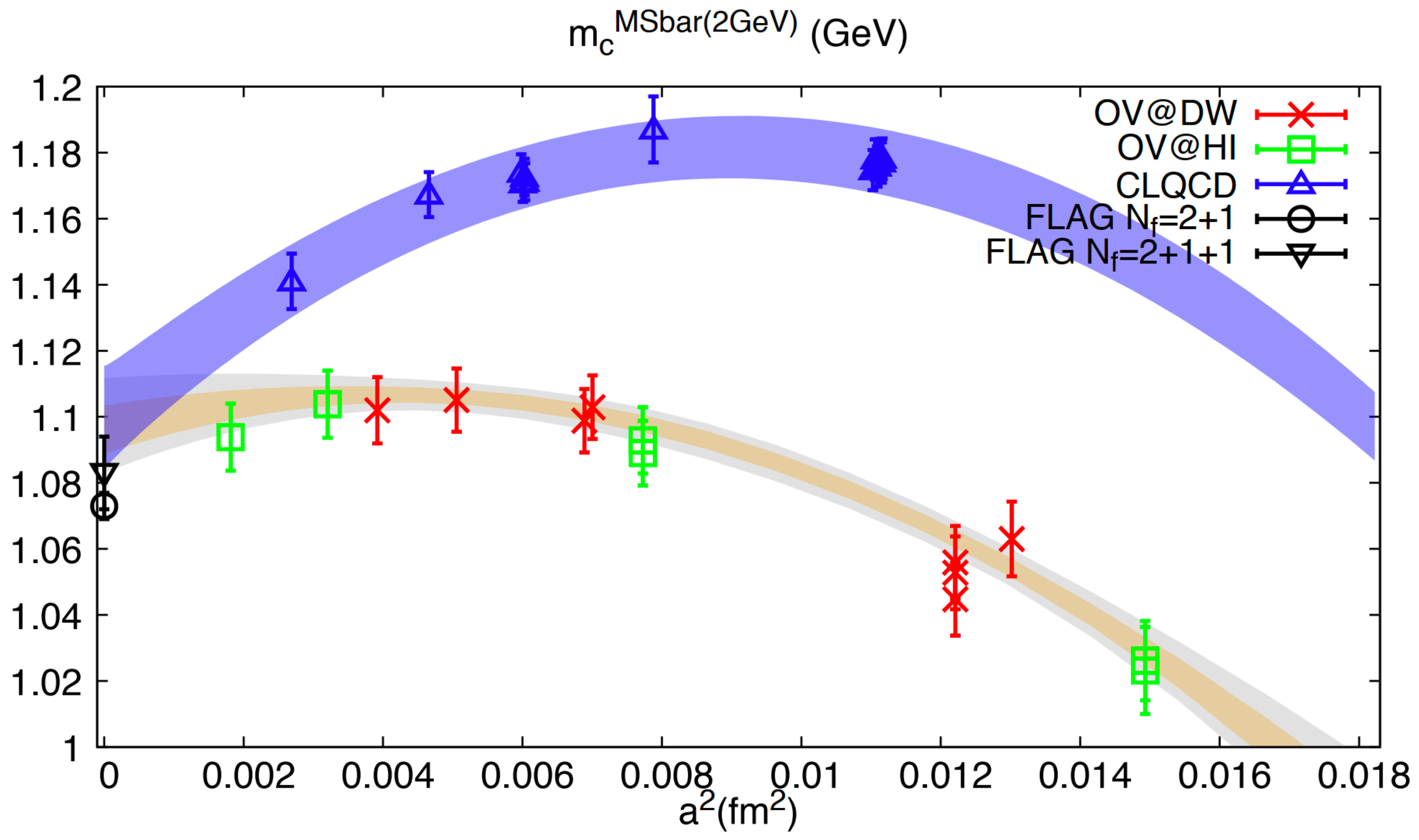


First Lattice QCD determination of up and down quark masses from China

Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, PRD109(2024) 054507

Renormalized quark masses

Charm quark mass



Hai-Yang Du, B.L. Hu, et. al., CLQCD, 2408.03548

D.J. Zhao, et. al., χQCD, in preparation

Based on the $a^2 + a^4$ extrapolation:

- The impact of unphysical light and strange quark masses have been corrected based on the global fit.
- Such a value is similar to the current lattice averages within ~2%.

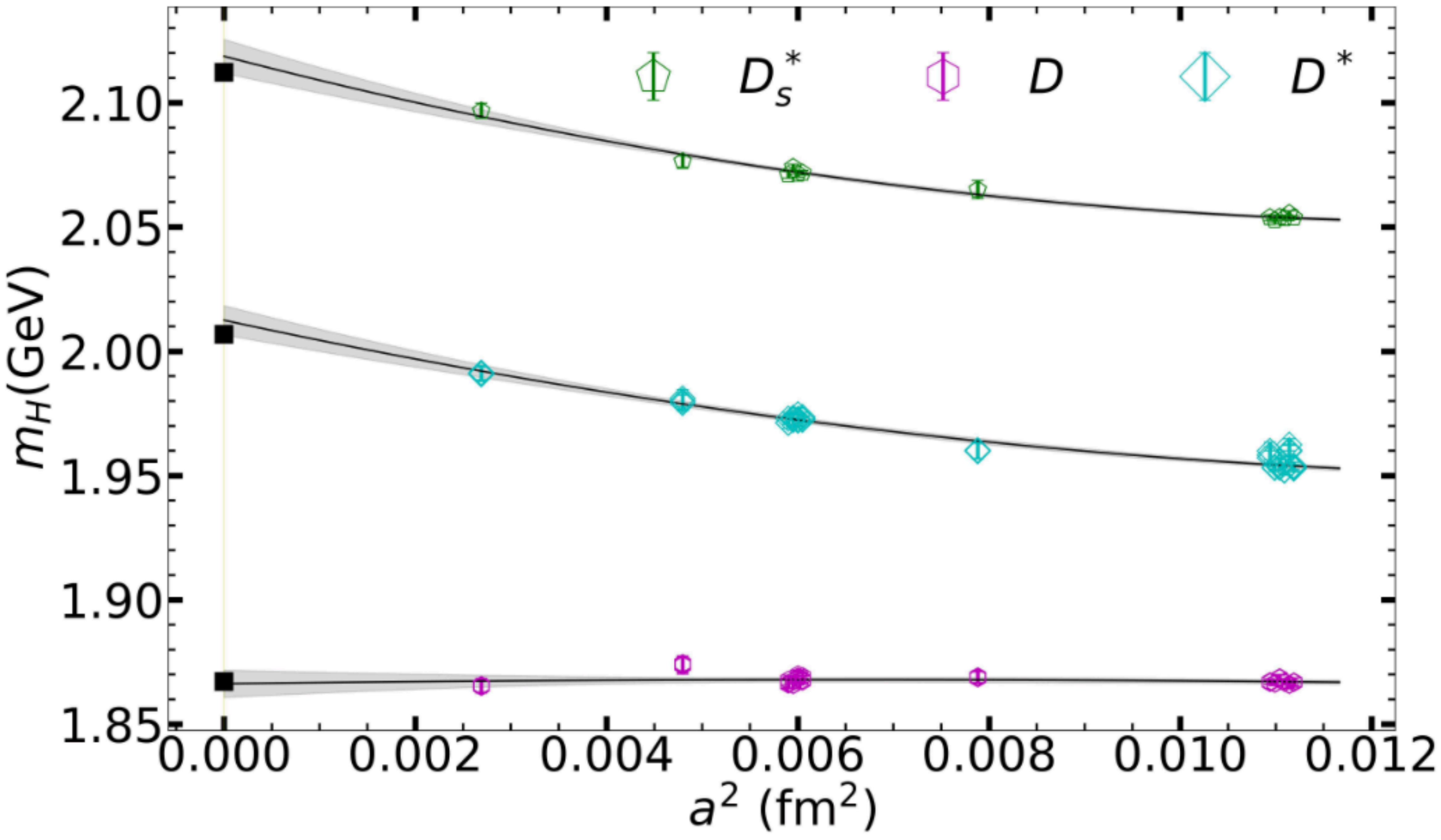
Charmed meson spectrum

Open charm cases

$$m_{D_s}^{\rm QCD} = m_{D_s}^{\rm phys} - \Delta^{\rm QED} m_{D_s} = 1966.7(1.5)\ {\rm MeV}\,.$$

RM123, Phys.Rev.D100 (2019) 034514

Input to determine the charm quark mass



- m_D is almost constant at different lattice spacing, with
  $m_D^{\pm} - m_D^0 = 2.9(2)_{\rm QCD} + 2.4(5)_{\rm QED} = 5.3(2)(5)\ {\rm MeV}$;
  RM123, Phys.Rev.D95(2017) 114504
- Agree with the PDG value 4.8(1) MeV well.
- Both m_D^* and $m_{D_s}^*$ have obvious lattice spacing dependence and the continuum extrapolated values agree with PDG well.

Hai-Yang Du, B.L. Hu, et. al., CLQCD, 2408.03548

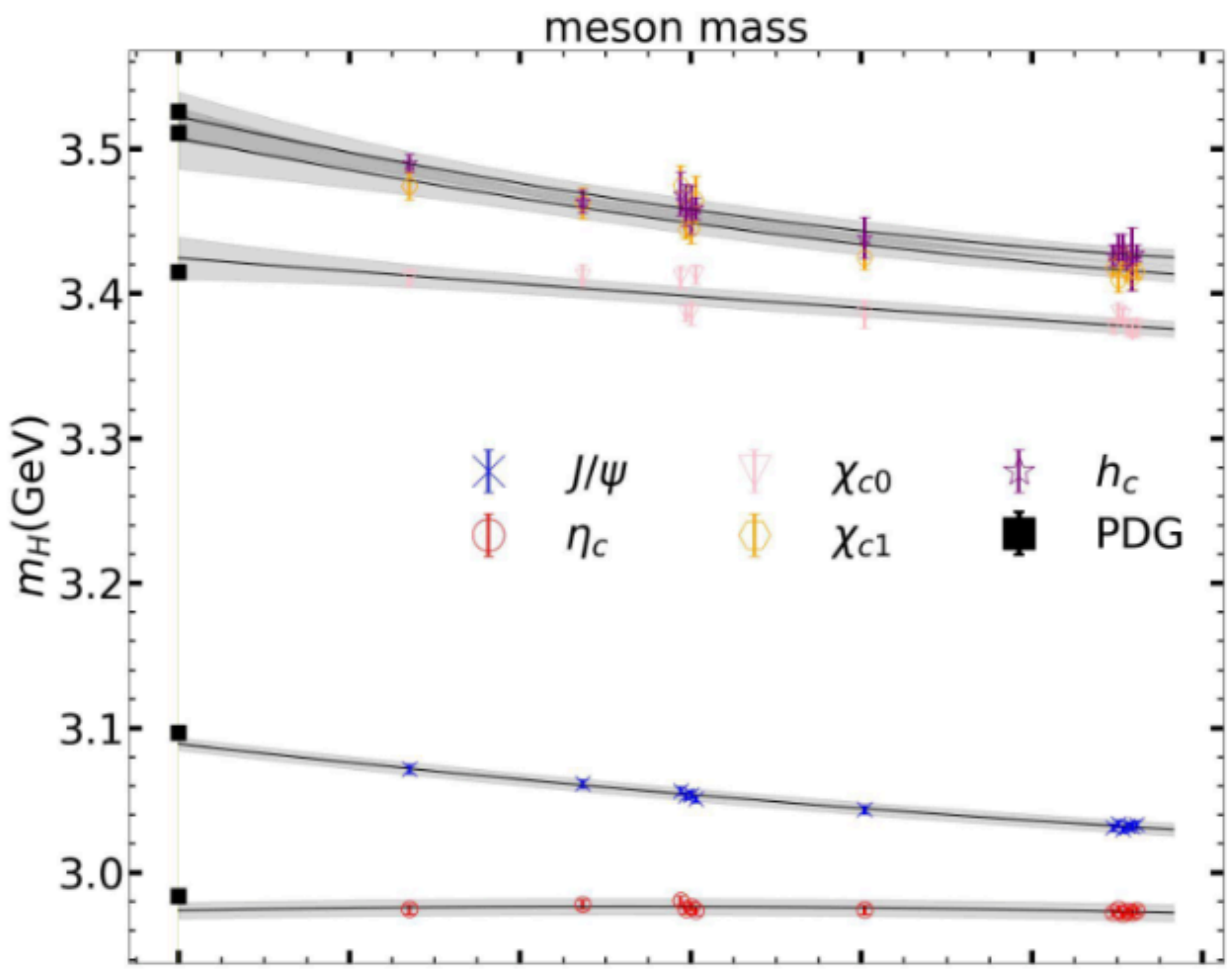
Charmed meson spectrum

charmonium cases

$$m_{D_s}^{\text{QCD}} = m_{D_s}^{\text{phys}} - \Delta^{\text{QED}} m_{D_s} = 1966.7(1.5)\ \text{MeV}\,.$$

RM123, Phys.Rev.D100 (2019) 034514

Input to determine the charm quark mass



- $m_{J/\psi}$ agrees with PDG well but m_{η_c} is a few MeV lower;
- $m_{J/\psi} - m_{\eta_c} = 116(3)$ MeV agree with previous HPQCD pure QCD prediction 119(1) MeV.
- P-wave charmonium masses also agree with PDG well, with $m_{1P} - m_{1S} = 461(19)$ MeV.

Hai-Yang Du, B.L. Hu, et. al., CLQCD, 2408.03548

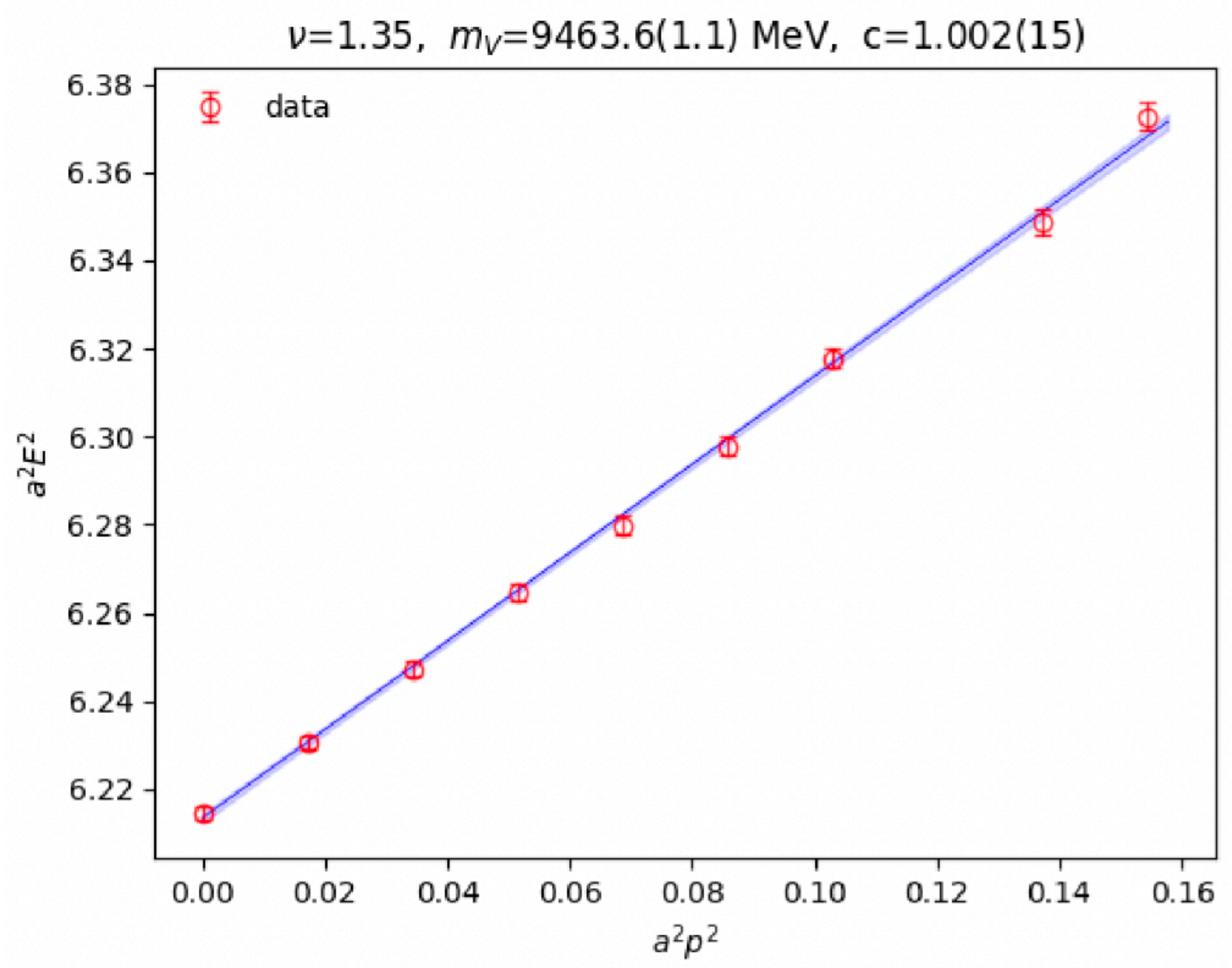
Toward the bottom physics

anisotropic action

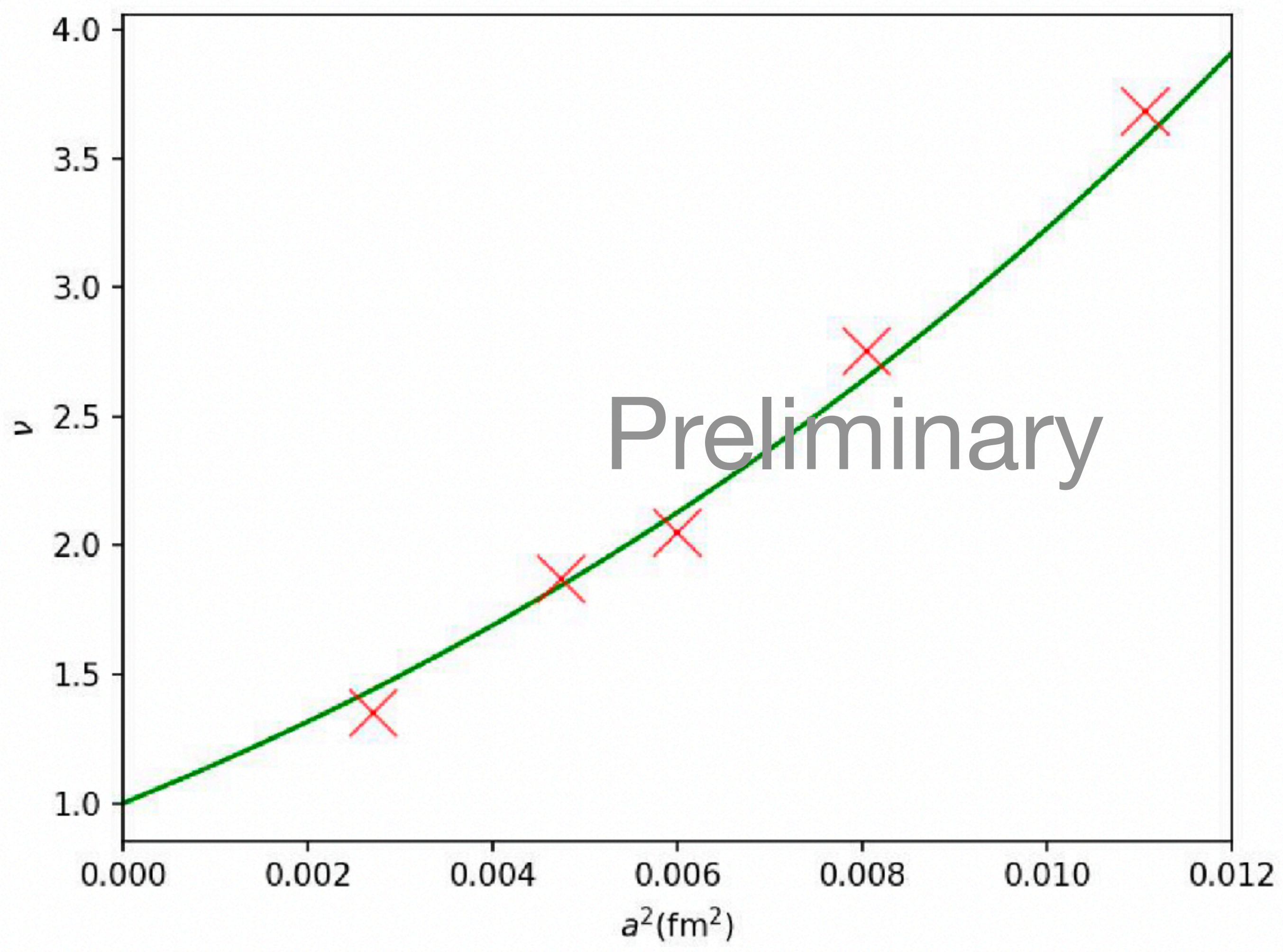
$$S_Q = a^4 \sum_x \bar{Q}\mathcal{M}Q,\ \mathcal{M} = \left[m_Q + \gamma_4\nabla_4 - \frac{a}{2}\nabla_4^2 + \nu\sum_{i=1}^{3}(\gamma_i\nabla_i - \frac{a}{2}\nabla_i^2) - \frac{1+\nu}{4u_0^3}a\sum_{i=1}^{3}\sigma_{i4}F_{i4} - \frac{\nu}{4u_0^3}a\sum_{i,j=1}^{3}\sigma_{ij}F_{ij}\right]$$

L. Liu, et. al., PRD81(2010)094505
Z. S. Brown, et. al., PRD90(2014)094507

Ensemble	a(fm)	$\tilde{L}^3 \times \tilde{T}$	m_π(MeV)	$N_{\rm cfg} \times N_{src}$	ν	m_Q	m_Υ(MeV)	c
C24P29	0.10521(11)	$24^3 \times 72$	292.3(1.0)	25×3	3.68	7.42	9460.8(1.6)	1.008(24)
E28P35	0.08970(26)	$28^3 \times 64$	351.4(1.4)	24×4	2.75	4.87	9466.2(1.9)	1.002(26)
F32P30	0.07751(14)	$32^3 \times 96$	300.4(1.2)	24×3	2.05	3.48	9462.7(1.5)	0.998(19)
G36P29	0.06884(18)	$36^3 \times 108$	297.2(0.9)	25×4	1.87	2.64	9457.2(1.5)	1.006(18)
H48P32	0.05198(20)	$48^3 \times 144$	316.6(1.0)	25×3	1.35	1.52	9463.6(1.1)	1.002(15)



- Determine the bare bottom quark mass using the physical Υ mass using the anisotropic action;
- The anisotropic rate ν is determined by requiring the dispersion relation of Υ to be the same as that in the continuum;
- ν approaches 1 in the continuum limit with $\mathcal{O}(a^2)$ corrections, while careful estimate of its uncertainty is in progress.



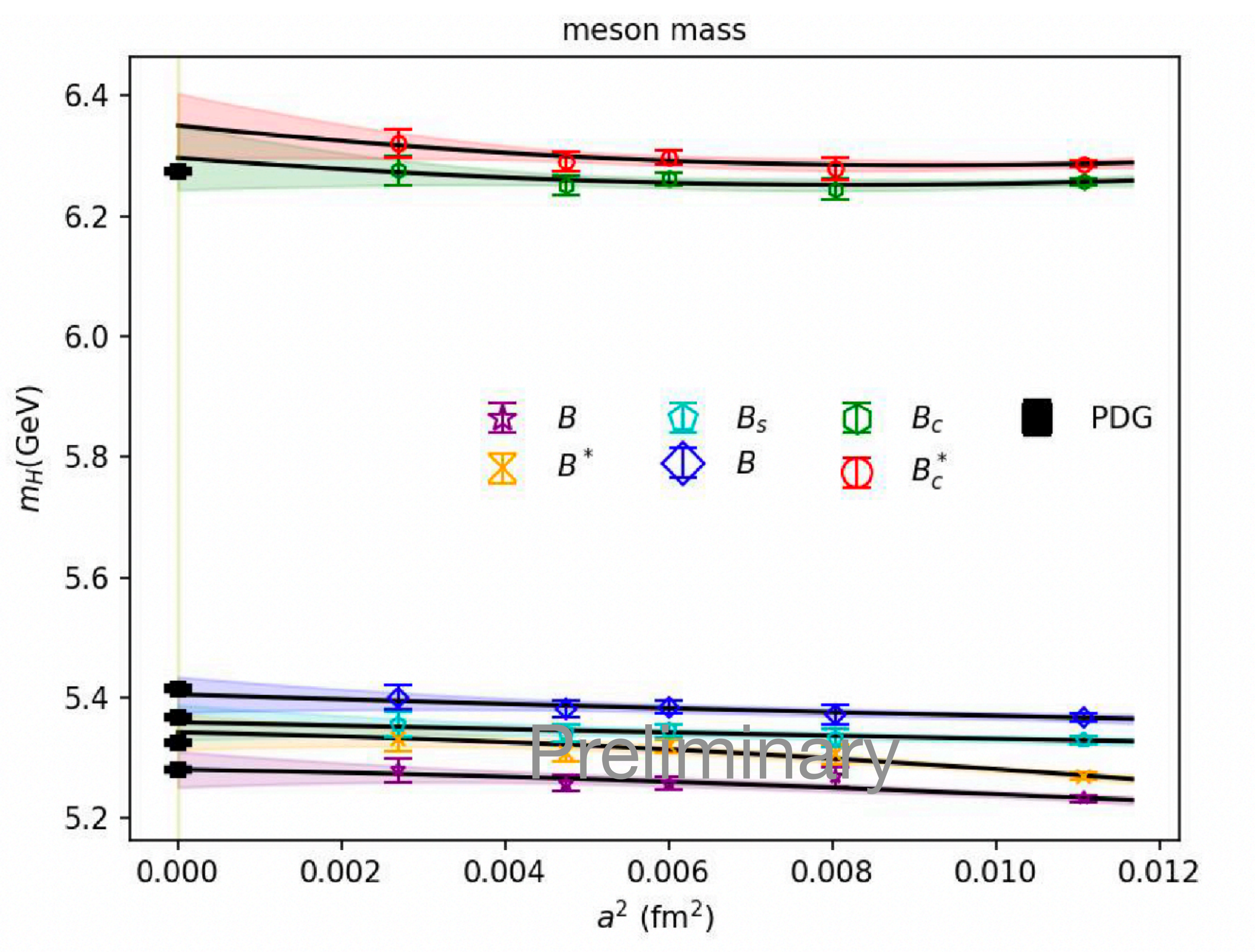
H.-Y. Du, et. al., CLQCD, in preparation

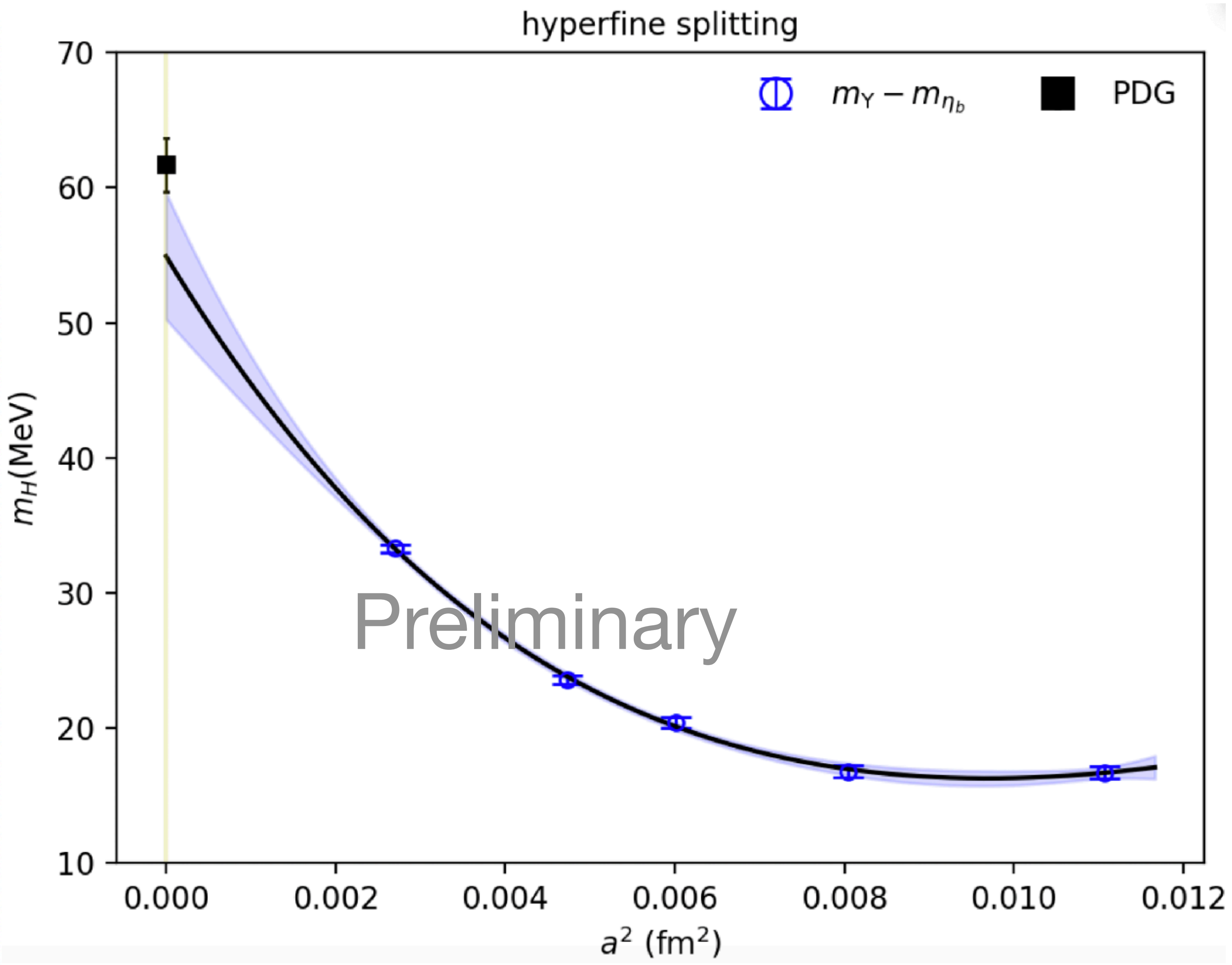
Toward the bottom physics

Hadron spectrum

$$S_Q = a^4 \sum_x \bar{Q}\mathcal{M}Q,\ \mathcal{M} = \left[m_Q + \gamma_4\nabla_4 - \frac{a}{2}\nabla_4^2 + \nu\sum_{i=1}^{3}(\gamma_i\nabla_i - \frac{a}{2}\nabla_i^2) - \frac{1+\nu}{4u_0^3}a\sum_{i=1}^{3}\sigma_{i4}F_{i4} - \frac{\nu}{4u_0^3}a\sum_{i,j=1}^{3}\sigma_{ij}F_{ij}\right]$$

L. Liu, et. al., PRD81(2010)094505
Z. S. Brown, et. al., PRD90(2014)094507

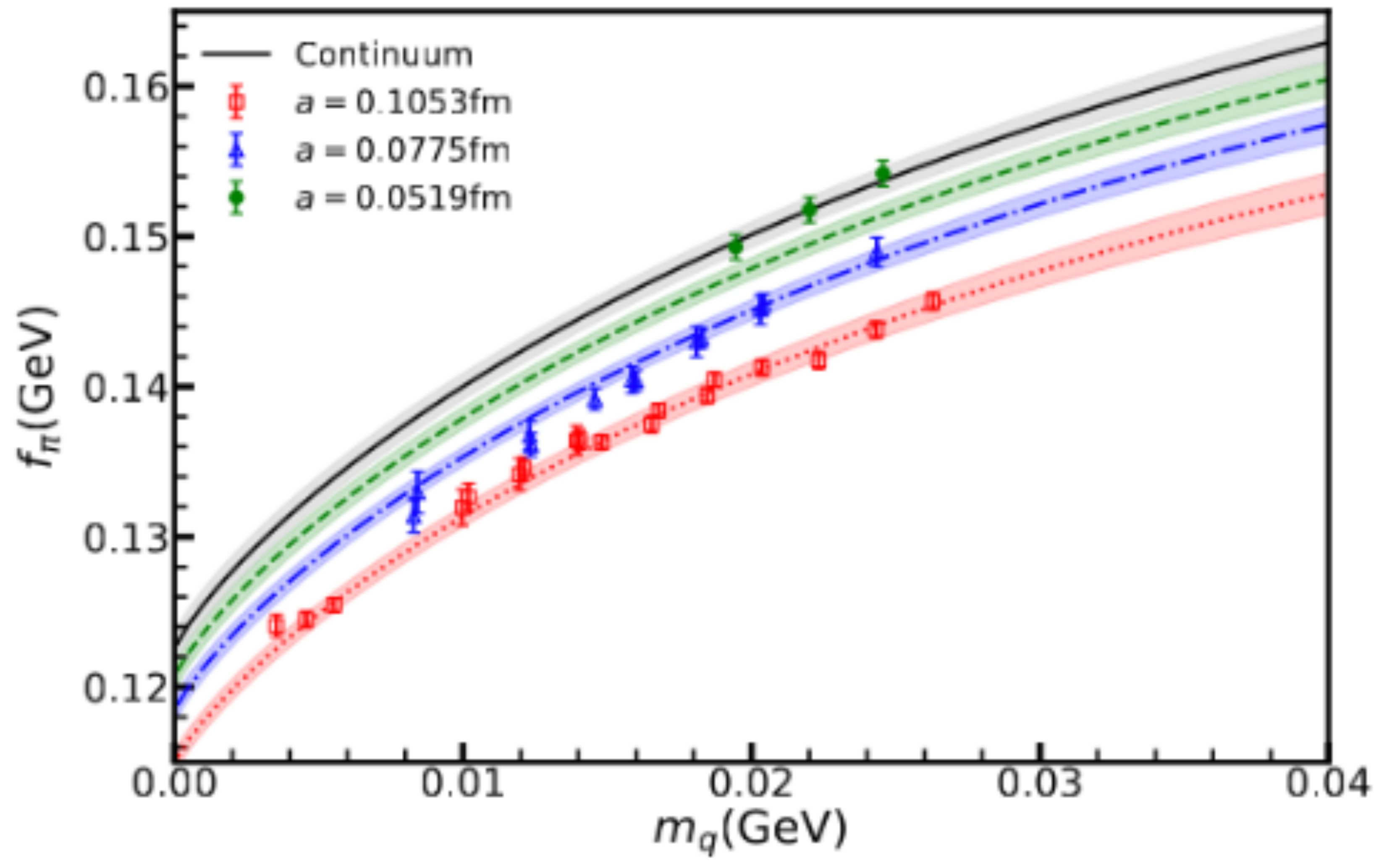


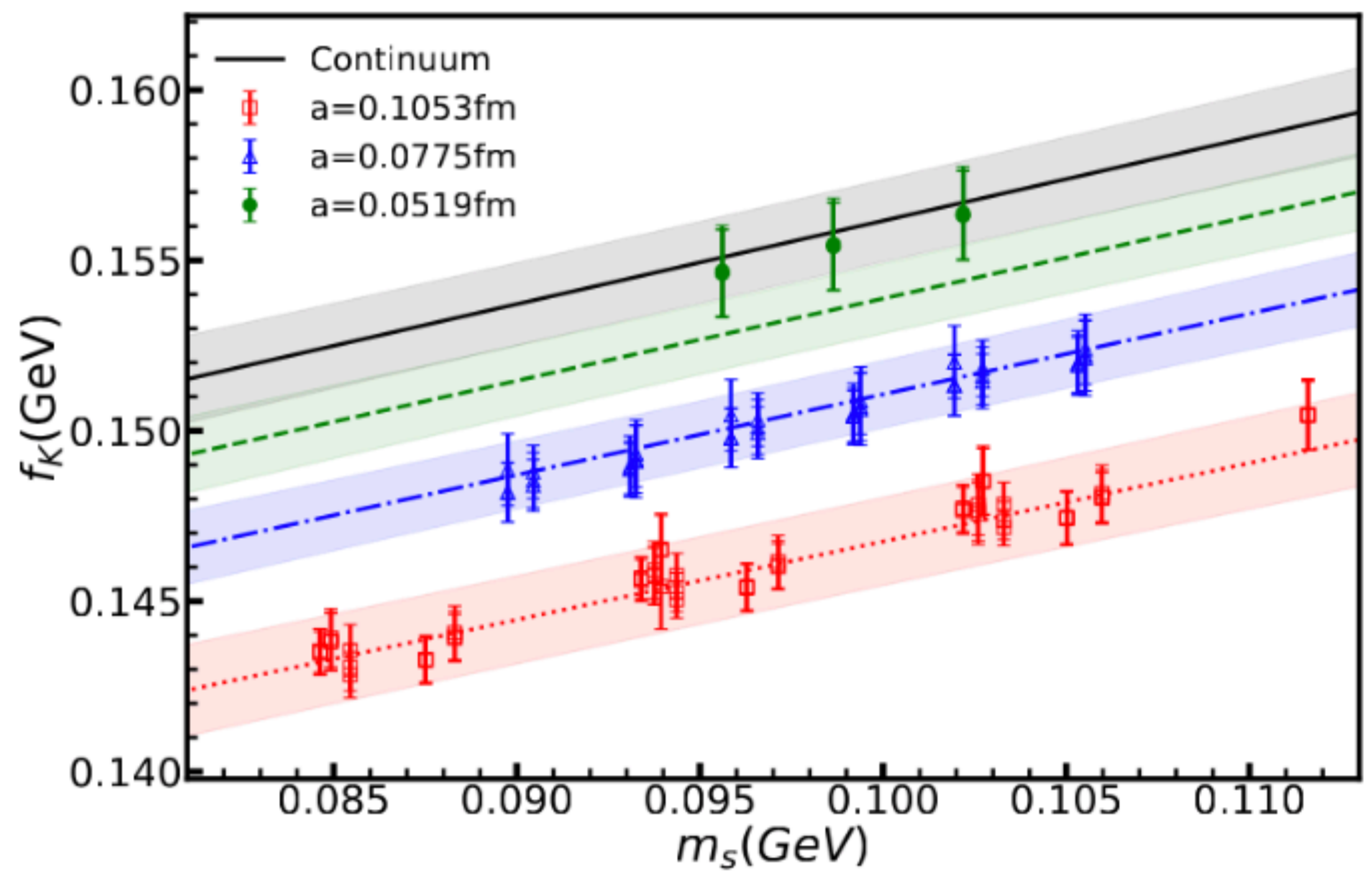


- Based on this action, the $B^{(*)}_{(s/c)}$ masses agree with experiment within sub-percent statistical uncertainty;
- The hyperfine splitting $m_\Upsilon - m_{\eta_b}$ suffers from sizable discretization error and requires input from smaller lattice spacing.

H.-Y. Du, et. al., CLQCD, in preparation

Decay constants





Pion and Kaon cases

$$\frac{f_K}{f_\pi} = 1.1907(76)(03)$$

$$\downarrow$$

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = 0.27683(29)_{\text{exp}}(20)_{\text{th}} \longrightarrow$$

$$|V_{ud}| = 0.9740(03)_{\text{lat}}(01)_{\text{ph}}$$

$$|V_{us}| = 0.2265(14)_{\text{lat}}(03)_{\text{ph}}$$

$$|V_{us}| = 0.2243(8)_{\text{PDG}}$$

$$\uparrow$$

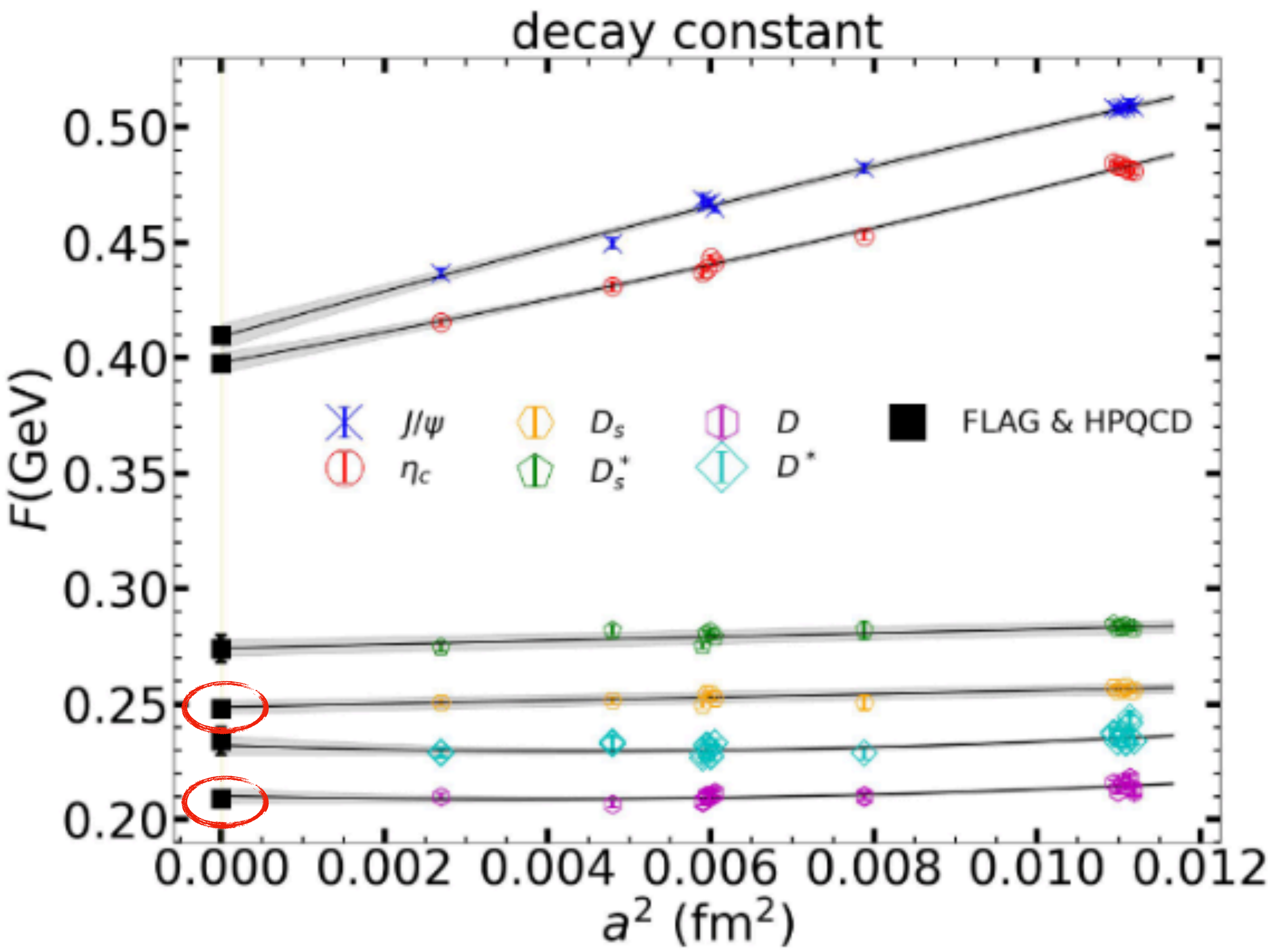
$$1 = |V_{ud}|^2 + |V_{us}^2| + |V_{ub}|^2 = |V_{ud}|^2 + |V_{us}^2| + 0.0035^2$$

- Additional input likes the form factor of the semi-leptonic decay $K^0 \to \pi^- l\nu$ is required to determine $|V_{ud(s)}|$ directly and verify the unitarity of CKM.

Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, PRD109(2024) 054507

Decay constants

Open charm cases



$f_{D^+} = 0.2113(33)_{\text{lat}}$ MeV

↓

$f_{D^+}|V_{cd}| = 45.8(1.1)_{\text{exp}}$ MeV ⟶ $|V_{cd}| = 0.2168(33)_{\text{lat}}(52)_{\text{exp}}$

$f_{D_s^+}|V_{cs}| = 243.5(2.7)_{\text{exp}}$ MeV ⟶ $|V_{cs}| = 0.975(13)_{\text{lat}}(11)_{\text{exp}}$

↑

$f_{D_s^+} = 0.2498(33)_{\text{lat}}$ MeV

- Verified the unitarity of CKM matrix elements involving the charm quark: $|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 0.999(25)(22)$.
- Also provide the most precise f_{D^*} and $f_{D_s^*}$ so far.

Hai-Yang Du, B.L. Hu, et. al., CLQCD, 2408.03548

Summary on CLQCD determinations of Stand model parameters

$m_{\pi^\pm}(m_u, m_d)$

$m_{\pi^0} = 134.98$ MeV

$w_0 = 0.1736(9)$ fm

$(m_u + m_d)^{\overline{\mathrm{MS}}(2\ \mathrm{GeV})}/2 = 3.60(19)$ MeV

$m_{K^0}^{\mathrm{QCD}} = 497.44(02)$ MeV

$m_{K^\pm}^{\mathrm{QCD}} = 491.44(15)$ MeV

$m_u^{\overline{\mathrm{MS}}(2\ \mathrm{GeV})} = 2.45(30)$ MeV

$m_d^{\overline{\mathrm{MS}}(2\ \mathrm{GeV})} = 4.74(14)$ MeV

$m_s^{\overline{\mathrm{MS}}(2\ \mathrm{GeV})} = 98.8(5.5)$ MeV

$m_K(m_u^{val}, m_d^{val}, m_l^{sea}, m_s^{sea})$

$m_{D_s}^{\mathrm{QCD}} = 1966.7(1.5)$ MeV

$m_c^{\overline{\mathrm{MS}}(m_c)} = 1293(13)(11)$ MeV

$m_{D_s}(m_c^{val}, m_s^{val}, m_l^{sea}, m_s^{sea})$

$\frac{f_K}{f_\pi} = 1.1907(76)_{\mathrm{lat}}$

$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = 0.27683(29)_{\mathrm{exp}}(20)_{\mathrm{th}}$

$|V_{ud}| = 0.9740(03)_{\mathrm{lat}}(01)_{\mathrm{ph}}$

$|V_{us}| = 0.2265(14)_{\mathrm{lat}}(03)_{\mathrm{ph}}$

$1 = |V_{ud}|^2 + |V_{us}^2| + |V_{ub}|^2 = |V_{ud}|^2 + |V_{us}^2| + 0.0035^2$

$f_{D^+} = 0.2113(33)_{\mathrm{lat}}$ MeV

$f_{D^+}|V_{cd}| = 45.8(1.1)_{\mathrm{exp}}$ MeV

$|V_{cd}| = 0.2168(33)_{\mathrm{lat}}(52)_{\mathrm{exp}}$

$f_{D_s^+}|V_{cs}| = 243.5(2.7)_{\mathrm{exp}}$ MeV

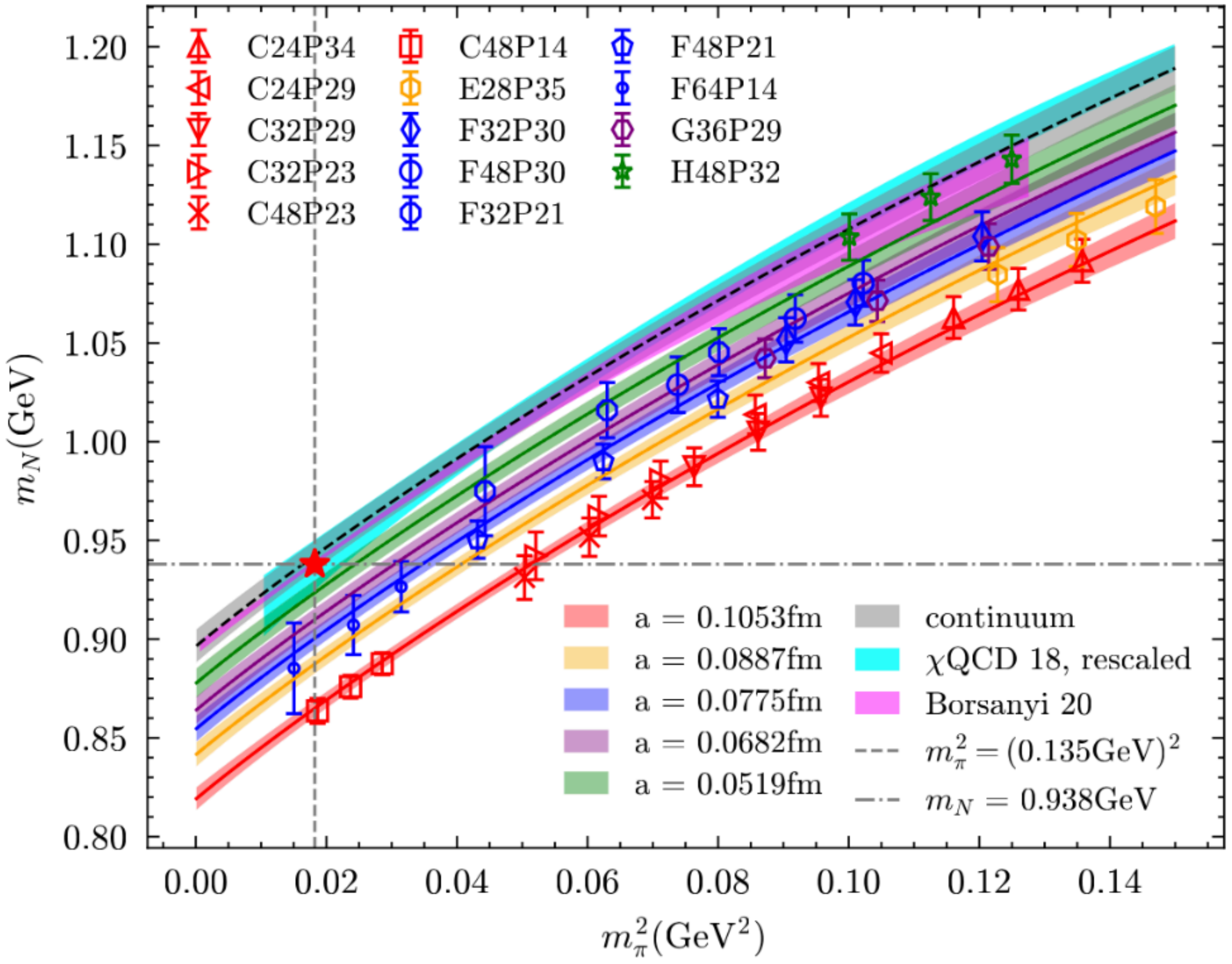
$|V_{cs}| = 0.975(13)_{\mathrm{lat}}(11)_{\mathrm{exp}}$

$f_{D_s^+} = 0.2498(33)_{\mathrm{lat}}$ MeV

Baryon spectrum

Proton case

Symbol	$\tilde{L}^3 \times \tilde{T}$	$\hat{\beta}$	a (fm)	$\tilde{m}_l^{\mathrm{b}}$	$\tilde{m}_s^{\mathrm{b}}$	m_π(MeV)	m_{η_s}(MeV)	$m_\pi L$	n_{cfg}	n_{src}^{l}	$n_{\mathrm{src}}^{s,c}$
C24P34	$24^3 \times 64$	6.20	0.10521(11)(62)	-0.2770	-0.2310	340.2(1.7)	748.61(75)	4.38	200	32	32
C24P29	$24^3 \times 72$	6.20	0.10521(11)(62)	-0.2770	-0.2400	292.3(1.0)	657.83(64)	3.75	760	3	3
C32P29	$32^3 \times 64$			-0.2770	-0.2400	293.1(0.8)	658.80(43)	5.01	489	3	3
C32P23	$32^3 \times 64$			-0.2790	-0.2400	227.9(1.2)	643.93(45)	3.91	400	3	3
C48P23	$48^3 \times 96$			-0.2790	-0.2400	224.1(1.2)	644.08(62)	5.79	62	3	3
C48P14	$48^3 \times 96$			-0.2825	-0.2310	136.4(1.7)	706.55(39)	3.56	188	48	3
E28P35	$28^3 \times 64$	6.308	0.08970(26)(53)	-0.2490	-0.2170	351.4(1.4)	717.94(93)	4.43	147	4	4
F32P30	$32^3 \times 96$	6.41	0.07751(14)(45)	-0.2295	-0.2050	300.4(1.2)	675.98(97)	3.81	250	3	3
F48P30	$48^3 \times 96$			-0.2295	-0.2050	302.7(0.9)	674.76(58)	5.72	99	3	3
F32P21	$32^3 \times 64$			-0.2320	-0.2050	210.3(2.3)	658.79(94)	2.67	194	3	3
F48P21	$48^3 \times 96$			-0.2320	-0.2050	207.5(1.1)	661.94(64)	3.91	82	12	12
F64P14	$64^3 \times 128$			-0.2336	-0.2030	122.8(0.9)	679.9(0.3)	3.09	109	4	4
G36P29	$36^3 \times 108$	6.498	0.06884(18)(41)	-0.2150	-0.1926	297.2(0.9)	693.05(46)	3.68	270	4	4
H48P32	$48^3 \times 144$	6.72	0.05198(20)(31)	-0.1850	-0.1700	316.6(1.0)	691.88(65)	4.06	157	12	12



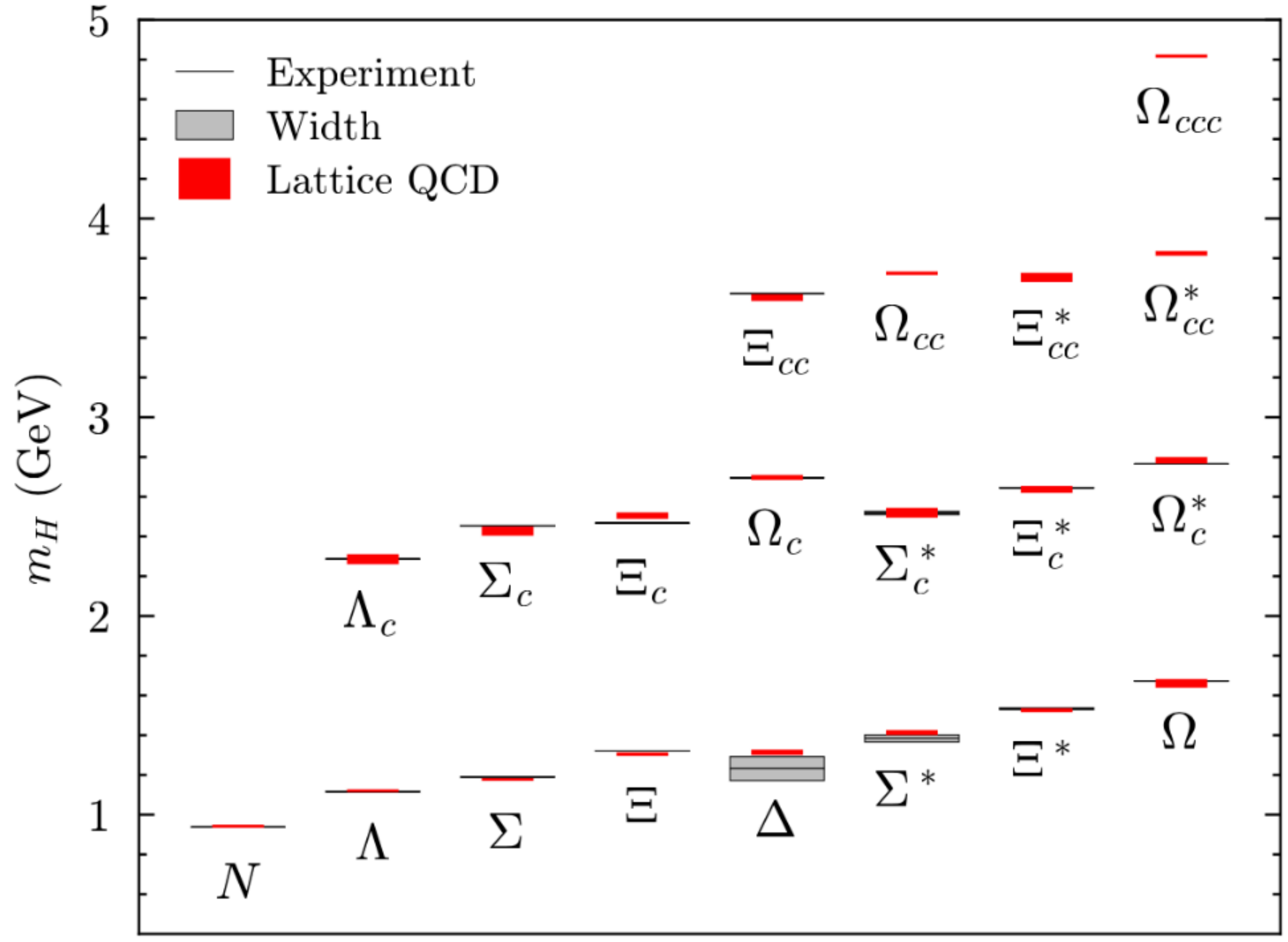
- Used 2,636 configurations with 27,158 measurements in total;
- After the continuum extrapolation, agree with the previous lattice results well.

B.-L. Hu, et. al., CLQCD, in preparation

Baryon spectrum

of four light flavors

- Generally agree with the PDG values at 1% level;
- Decay width is added to the experimental values as an uncertainty of the pole position;
- The mass of baryon with 1 charm quark is around 2.5 GeV;
- The mass of baryon with 2 charm quarks is around 3.8 GeV;
- The mass of baryon with 3 charm quarks is around 5 GeV;
- The missing QED effect will be investigated in the near future.



B.-L. Hu, et. al., CLQCD, in preparation

Summary

- The state-of-the-arts Lattice QCD ensemble should have enough ensembles to approach the continuum, infinite volume and physical quark masses reliably; and the present CLQCD ensembles have been close to this goal.

- Up, down, strange and charm quark masses have been determined at a few percent level;

- The charmed meson and baryon masses are predicted at ~0.3% uncertainty and agree with the experimental values at 1% level.

- More predictions are in progress.