B 介子两体非轻衰变研究进展

focus on the pQCD factorization approach

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Overview

B physics and CP Violation

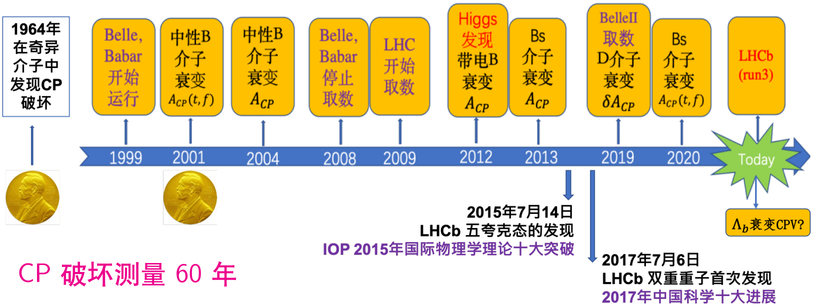
- "Matter $\neq$ Antimatter" indicates the interaction with CPV
- Heavy flavour physics (HFP) provides many processes with CPV
  although it is inadequate and new mechanism of CPV is imminent

Timeline of B physics

- 1973, CKM mechanism is proposed (4 parameters)
- 1977, CFS-E288 at FermiLab discovered Υ ($b\bar{b}$), *Lederman*
- 1983, CLEO found B^0 meson in $\Upsilon(4S)$ decay
- 1986, *Bigi* & *Santa* expectated large CPV in $B^0 \to J/\Psi K_S^0$
- 1987, ARGU measured the $B^0 - \bar{B}^0$ oscillation $\Rightarrow A_{CP}(t)$
- 1987, *Oddone* proposed the construction of B factories to study CPV
- 1999, BABAR, Belle started running 2001(04), $A_{CP}(t,f)(A_{CP})$ in B^0 decays
- 2009, LHCb played in to the game with 3-5 order more events
  2013(20), $A_{CP}(A_{CP}(t,f))$ in B_s decays, 2012, A_{CP} in B^+ decays
  2019, δA_{CP} in D decays; 2024 A_{CP} in Λ_b decays ?
- Anomalies: $b \to s l^+ l^-$ see Qi-dong's talk, 11/3 , $B_s \to \mu\mu$, $|V_{ub}|$, $|V_{cb}|$

Experimental promotion

Successful running of B factories(1999-2008) and LHC(2009-2026)



CP 破坏测量 60 年

- SuperKEKB(2018-2026) [E. Kou et al. [Belle-II], 2019]

  - Belle II has collected 531 fb^{-1} data so far with record peak luminosity $4.7\times10^{34} cm^{-2}s^{-1}$
  - Goal: 50 ab^{-1} data and peak luminosity at $6.5\times10^{35} cm^{-2}s^{-1}$
  - $|V_{ub}|$ to 1.2% in $B\to\phi, \rho l\nu$, δA_{CP} in $B\to K^*\pi, K\rho, K^*\rho$, $B\to VV$, α from $B\to\pi^0\pi^0, \cdots$

  † $B^+\to\rho^+\rho^0, B^0\to K^0\pi^0$ [Belle-II, 2021]

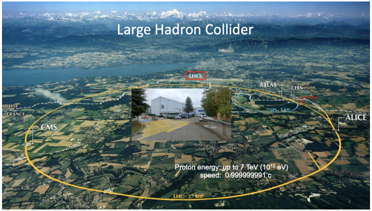
  † First measurement of CP asymmetry parameters in $B^0\to K_S^0\pi^0, \omega\omega$ [Belle-(II), 2023,04].

- HL-LHC(2030-2033) [CERN Yellow Rep. Monogr, 2019]

  - $\mathcal{L} = 23(300)\mathrm{fb}^{-1}$ in phase 1(2), 2 order larger than LHC, $2\times10^{33(34)}\mathrm{cm}^{-2}\mathrm{s}^{-1}$
  - $|V_{ub}|$ to 0.7%(0.4%) in $B\to\pi\pi, \pi\rho, \rho\rho$, $C_{\pi^+\pi^-}, S_{\pi^+\pi^-}$ (one order improvement), α from $B\to\rho\rho, \rho\pi, \cdots$.

  † $A_{\pi\pi}^{\mathrm{dir}} = (2.32\pm0.61)\times10^{-3}$ [LHCb, 2023]
  Belle/Belle II result see Long-ke's talk, 11/3

  † $\delta A_{CP}(\Lambda_b^0\to\Lambda K^+K^-) = 0.083\pm0.028$ [LHCb-Paper-2024-043]



Three-scale factorization

i low energy effective hamiltonian
ii three-scale factorization

Low energy effective hamiltonian

New physics: $\mathcal{L}_{NP}$

$\downarrow$

Electroweak scale (m_W): $\mathcal{L}_{EW} + \mathcal{L}_{D>4}$

$\downarrow$

Heavy quark scale (m_b): $\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{\rm CKM} \sum_i C_i(\mu) O_i(\mu) + \mathcal{L}_{{\rm eff},D>6}$

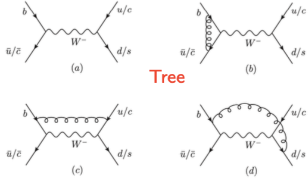
$\downarrow$

Hadron scale ($\Lambda_{\rm QCD}$): LCDAs, PDF, PDA

- derive the effective Hamiltonian by integrating over m_W [Buchalla 1996]
  - product of two charged currents $\sim$ a series of local operators O_i with the weighted coefficients C_i
- dynamics at the scale $\mathcal{O}(m_W)$ is absorbed into Wilson coefficients $C_i(\mu)$
  - $C_i \sim$ match the $\mathcal{L}_{\rm eff}$ with the full theory of weak decays [Ma 80, Inami&Lim 81, Clements 83]
- the rest go into the four fermion effective operators $O_i(\mu)$
- the key is to calculate the hadron matrix element $\langle M_1 M_2 | O_i | B \rangle$

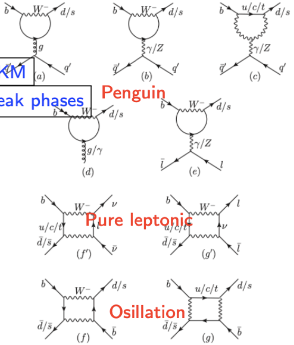
definitely a QCD problem

Low energy effective hamiltonian



Different CKM

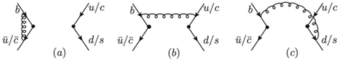
Different weak phases



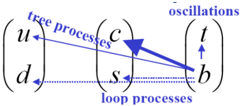
$\Downarrow$ Diagrams at scale $\mathcal{O}(m_W)$

\alpha_s

Effective tree, bullent denotes O_i



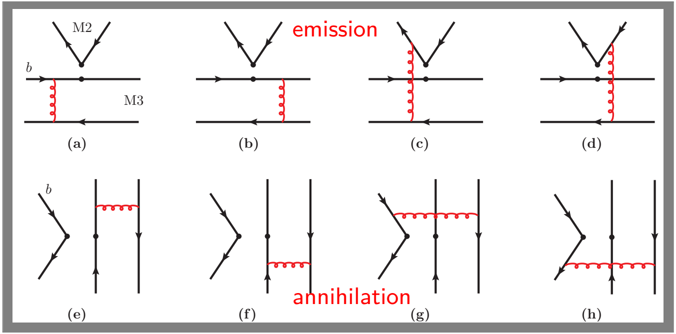
- integrating over the m_W and m_b
- weak phase difference between charged and FCNC of b decays



Low energy effective hamiltonian

- diagrams at scales $\mathcal{O}(\Lambda_{\rm QCD}) - \mathcal{O}(m_b)$, Hadron matrix element $\langle M_1 M_2 | O_i | B \rangle$

different strong phases



- factorization: detach the hard kernel $\mathcal{H}$ [O_i at scale $\mathcal{O}(m_b)$]

  from the hadron wave function Φs [B, M_1, M_2 mesons at scale $\mathcal{O}(\Lambda_{\rm QCD})$]
- $\mathcal{H}$ is calculated perturbatively order by order, Φs are universal

Low energy effective hamiltonian

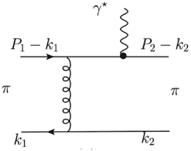
- high precision calculation of Hadronic matrix element $\langle M_1 M_2 | O_i | B \rangle$
  - naive factorization: $\sim F_{B\to M_2} \otimes f_{M_1}$ [Bauer&Stech&Wirbel 1985,87]
  - generalized factorization: QCD corrections from $\mathrm{O}_{i=1,\cdots,10}$ [Ali&Kramer&Lü 1998,99]
  - QCDF: VC to $\mathcal{M}_{t,p}$ + corrections to spectator scattering, full NNLO $(\mathcal{O}(\alpha_s^2))$ [Benele 2010, Bell 15, 20, Huber 16, Beneke 06,07, Jain 07]
  - SCET: introduces different fields in different energy regions, simple kinematics but complicated dynamics [Bauer 2001, Chay 04, Becher 15], QCDF/SCET [Beneke 2015]
  - $B \to \pi\pi$ decay is studied from LCSRs [Khodjamirian 2001,03,05]
  - high order & power corrections to $B \to P, V$ form factors LQCD [HPQCD 2013] [Bharucha 2016, Wang 15,16,20, Lü 19, Beneke 17, Gubernari 19, Cheng 17,19]

- the transversal momentum is picked up in the hard scattering amplitudes to regulate the end-point singularity PQCD
  - $B \to M$ FFs and the annihilation amp. are both calculable [Keum 2001, Lü 01]
  - LO $(\mathcal{O}(\alpha_s))$ $B \to PP, PV, VV$ decays [Xiao 2007; Lü 02; Li 05, Li 06, Zou 15], [Hua 2021]
  - partially NLO $(\mathcal{O}(\alpha_s^2))$: factorizable amplitudes [Cheng & Xiao 2021]
    effective operators [Mishima 2003, Li 05]
    hard scattering [Li 2012, Cheng 14], [Li 13, Cheng 15,15, Hua 18], [Li 14, Liu 15,16]

Three scale factorization

- end-point singularities appear in exclusive QCD processes

  † $m_{1,2}^2 \ll Q^2$, light-cone coordinate $p_2 = (\frac{Q}{\sqrt{2}}, 0, 0_{\rm T})$, $p_3 = (0, \frac{Q}{\sqrt{2}}, 0_{\rm T})$,
  (anti-)valence quarks: $k_2 = x_2 p_2$, $\bar{k}_2 = \bar{x}_2 p_2$



$\phi \propto u(1-u)$, $\quad m_0^\pi \phi^{P,\sigma} \propto m_0^\pi$

$$\propto \sum_t \int du_1 du_2 \kappa_t(u_i) \frac{\alpha_s(\mu)\phi_1^t(u_1)\phi_2^t(u_2)}{u_1 u_2 Q^2 u_2 Q^2}$$

- pick up k_T in the internal propagators

$$\mathcal{M} \propto \sum_{t=2,3,4} \int du_1 du_2 dk_{1T} dk_{2T} \kappa_t(u_i) \frac{\alpha_s(\mu)\phi_1^t(u_1)\phi_2^t(u_2)}{u_1 u_2 Q^2 - (k_{1T} - k_{2T})^2}$$

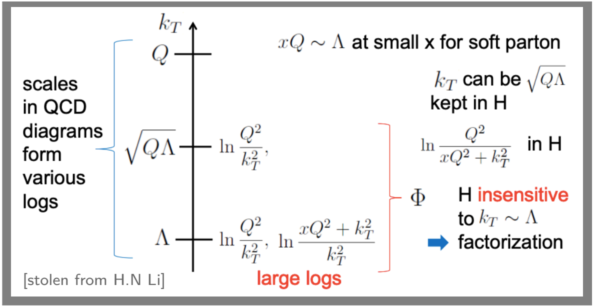
- end-point singularity at leading and subleading powers

$$\mathcal{H} \propto \frac{\alpha_s(\mu)}{u_1 u_2 Q^2 - k_T^2} \sim \frac{\alpha_s(\mu)}{u_1 u_2 Q^2} - \frac{\alpha_s(\mu) k_T^2}{(u_1 u_2 Q^2)^2} + \cdots$$

- the power suppressed TMD terms becomes important at the end-points

Three scale factorization

- introduce k_T to regularize the end-point singularity [Huang 1991]
- scales of transversal momentum and the large logarithms



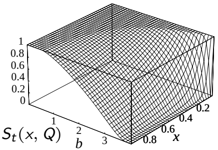
- multiple scales, hence large single logarithms in $\mathcal{H}/\Phi$ by QCD correction
- double logs in the soft-collinear regions $\alpha_s(\mu)\ln^2(k_T^2/m_B^2)$

Three scale factorization

- in order to repair the perturbative expansion, do **resummation**
- k_T resummation for $\mathcal{H}$ to obtain $S(x_i, b_i, Q)$ [Botts 1989, Li 92]
  - † decreases the inverse power of the q^2 in the divergence amplitude
  - † exhibits high suppression for large transversal distances (small k_T) interactions
- integrating over k_T, large log $\ln^2(x_i)$ when intermediate gluon is on shell
- threshold resummation for Φ to obtain $S_t(x_i, Q)$ [Li 1999]
  - † suppresses the small x_i regions
  - † repairs the self-consistency between $\alpha_s(t)$ and hard log $\ln(x_1 x_3 Q^2/t^2)$

‡ dynamics with $k_T < \sqrt{Q\Lambda}$ is organized into $S(x, b, Q)$

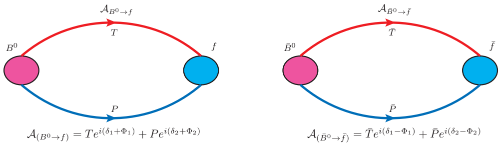
‡ dynamics in small x is suppressed by $S_t(x, Q)$



$$\mathcal{M} = \sum_t \phi^t(u_1, b_1) \otimes \mathcal{H}_i(t, b) \otimes \phi^t(u_2, b_2)\, Exp\left[-s(p^+, b) - \int_{1/b}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_\phi(\alpha_s(\bar{\mu}))\right]$$

Three scale factorization

Sources of strong phase (differences) $\delta_{1,2}$ to generate $\not{C}\not{P}$



- different topologies: emission (real, $\delta_1 = 0$) and annihilation (plural, $\delta_2 \neq 0$)

$$\frac{1}{k_T^2 - xm_B^2 - i\epsilon} = \mathcal{P}\left(\frac{1}{k_T^2 - xm_B^2}\right) + i\delta(k_T^2 - xm_B^2)$$

- Sudakov expanent (NLO)
  - center of mass scattering angle and angular distribution of scattering hadrons
  - important in baryon decays but not in B meson decays
- NLO corrections to spectator emission amplitude from Glauber gluon
  - only supplies a sizable phase to the pion final state
  - modifies the interactions between different topological amplitudes
- on shell charm quark loop correction (NLO), leading source in QCDF

State-of-the-art PQCD prediction

$B \to PP, PV, VV$ decays: amplitudes

- general decomposition of Wilson coefficients for certain effective weak vertex

spectator meson M_3

$\uparrow$

$[q_1, q_2, q_3]$

$\downarrow$

emission meson M_2

Weak vertex	Typical amplitudes	Wilson coefficients
$[s,s,s],[d,d,d]$	$\mathcal{E}^{\rm LL}/\mathcal{A}^{\rm LL}, \mathcal{E}^{\rm LL}_{NF}/\mathcal{A}^{\rm LL}_{NF}$	$a_3+a_4-\frac{a_9+a_{10}}{2}, \quad C_3+C_4-\frac{C_9+C_{10}}{2}$
	$\mathcal{E}^{\rm LR}/\mathcal{A}^{\rm LR}, \quad \mathcal{E}^{\rm LR}_{NF}/\mathcal{A}^{\rm LR}_{NF}$	$a_5-\frac{a_7}{2}, \quad C_5-\frac{C_7}{2}$
	$\mathcal{E}^{\rm SP}/\mathcal{A}^{\rm SP}, \quad \mathcal{E}^{\rm SP}_{NF}/\mathcal{A}^{\rm SP}_{NF}$	$a_6-\frac{a_8}{2}, \quad C_6-\frac{C_8}{2}$
$[d,s,s], \quad [s,d,d]$	$\mathcal{E}^{\rm LL}/\mathcal{A}^{\rm LL}, \quad \mathcal{E}^{\rm LL}_{NF}/\mathcal{A}^{\rm LL}_{NF}$	$a_4-\frac{a_{10}}{2}, \quad C_3-\frac{C_9}{2}$
	$\mathcal{E}^{\rm LR}/\mathcal{A}^{\rm LR}, \quad \mathcal{E}^{\rm LR}_{NF}/\mathcal{A}^{\rm LR}_{NF}$	$a_6-\frac{a_8}{2}, \quad C_5-\frac{C_7}{2}$
$[s,s,d], \quad [d,d,s]$	$\mathcal{E}^{\rm LL}/\mathcal{A}^{\rm LL}, \quad \mathcal{E}^{\rm LL}_{NF}/\mathcal{A}^{\rm LL}_{NF}$	$a_3-\frac{a_9}{2}, \quad C_4-\frac{C_{10}}{2}$
	$\mathcal{E}^{\rm LR}/\mathcal{A}^{\rm LR}, \quad \mathcal{E}^{\rm LR}_{NF}/\mathcal{A}^{\rm LR}_{NF}$	$a_5-\frac{a_7}{2}, \quad C_6-\frac{C_8}{2}$
$[u,u,s], \quad [u,u,d]$	$\mathcal{E}^{\rm LL}/\mathcal{A}^{\rm LL}, \quad \mathcal{E}^{\rm LL}_{NF}/\mathcal{A}^{\rm LL}_{NF}$	$a_2, \quad C_2$
	$\mathcal{E}^{\rm LR}/\mathcal{A}^{\rm LR}, \quad \mathcal{E}^{\rm LR}_{NF}/\mathcal{A}^{\rm LR}_{NF}$	$a_3+a_9, \quad C_4+C_{10}$
	$\mathcal{E}^{\rm SP}/\mathcal{A}^{\rm SP}, \quad \mathcal{E}^{\rm SP}_{NF}/\mathcal{A}^{\rm SP}_{NF}$	$a_5+a_7, \quad C_6+C_8$
$[s,u,u], \quad [d,u,u]$	$\mathcal{E}^{\rm LL}/\mathcal{A}^{\rm LL}, \quad \mathcal{E}^{\rm LL}_{NF}/\mathcal{A}^{\rm LL}_{NF}$	$a_1, \quad C_1$
	$\mathcal{E}^{\rm LR}/\mathcal{A}^{\rm LR}, \quad \mathcal{E}^{\rm LR}_{NF}/\mathcal{A}^{\rm LR}_{NF}$	$a_4+a_{10}, \quad C_3+C_9$
	$\mathcal{E}^{\rm SP}/\mathcal{A}^{\rm SP}, \quad \mathcal{E}^{\rm SP}_{NF}/\mathcal{A}^{\rm SP}_{NF}$	$a_6+a_8, \quad C_5+C_7$

- ie. decay amplitude of $B^+ \to \pi^+ K^0$ at NLO

$[s, d, d]$

$$\mathcal{M}(B^+\to\pi^+K^0) = \frac{G_F}{\sqrt{2}}V^*_{ub}V_{us}\left[a_1\mathcal{A}^{\rm LL}_\pi + C_1\mathcal{A}^{\rm LL}_{NF,\pi} + \mathcal{M}^{\rm (ql,u)}_{B\to K^0\pi^+}\right] + \frac{G_F}{\sqrt{2}}V^*_{cb}V_{cs}\mathcal{M}^{\rm (ql,c)}_{B\to K^0\pi^+} - \frac{G_F}{\sqrt{2}}V^*_{tb}V_{ts}\Bigg[\left(a_4-\frac{a_{10}}{2}\right)\mathcal{E}^{\rm LL}_\pi$$
$$+\left(a_6-\frac{a_8}{2}\right)\mathcal{E}^{\rm SP}_\pi + \left(C_3-\frac{C_9}{2}\right)\mathcal{E}^{\rm LL}_{NF,\pi} + \left(C_5-\frac{C_7}{2}\right)\mathcal{E}^{\rm LR}_{NF,\pi} + (a_4+a_{10})\mathcal{A}^{\rm LL}_\pi + (a_6+a_8)\mathcal{A}^{\rm SP}_\pi$$
$$+(C_3+C_9)\mathcal{A}^{\rm LL}_{NF,\pi} + (C_5+C_7)\mathcal{A}^{\rm LR}_{NF,\pi} + \mathcal{M}^{\rm (ql,t)}_{B\to K^0\pi^+} + \mathcal{M}^{\rm (mp)}_{B\to K^0\pi^+}\Bigg],$$

△ the glauber gluon corrections and TMD wave functions are not taken into account in this work

$B \to PP, PV, VV$ decays: amplitudes

- operator decomposition of $B \to PP$ decays

Tree/color-favoured tree emission

QCD **P**enguin

Color-suppressed tree emission

$\mathrm{P_{em}}$: Electroweak penguin

E: tree annihilation amplitude

Topology	Channel
$\{\mathbf{P}, \mathrm{T}, \mathrm{C}, \mathrm{E}, \mathrm{P_{ew}}\}$	$\pi^0 K^+$, $\eta_q K^+$
$\{\mathbf{T}, \mathrm{P}, \mathrm{C}, \mathrm{E}, \mathrm{P_{ew}}\}$	$\pi^+ \eta_q$
$\{\mathbf{T}, \mathrm{C}, \mathrm{P_{ew}}\}$	$\pi^+ \pi^0$
$\{\mathbf{P}, \mathrm{E}, \mathrm{P_{ew}}\}$	$\pi^+ K^0$, $\eta_s K^+$, $K^+ \bar{K}^0$
$\{\mathbf{P}, \mathrm{P_{ew}}\}$	$\pi^+ \eta_s$
$\{\mathbf{T}, \mathrm{P}, \mathrm{E}, \mathrm{P_{ew}}\}$	$\pi^+ \pi^-$
$\{\mathbf{P}, \mathrm{T}, \mathrm{P_{ew}}\}$	$\pi^- K^+$
$\{\mathbf{C}, \mathbf{E}, \mathbf{P}, \mathrm{P_{ew}}\}$	$\pi^0 \pi^0$, $\pi^0 \eta_q$, $\eta_q \eta_q$
$\{\mathbf{P}, \mathrm{C}, \mathrm{P_{ew}}\}$	$\pi^0 K^0$, $\eta_q K^0$
$\{\mathbf{P}, \mathrm{P_{ew}}\}$	$\eta_s K^0$, $K^0 \bar{K}^0$, $\pi^0 \eta_s$, $\eta_s \eta_s$, $\eta_q \eta_s$
$\{\mathbf{E}, \mathrm{P}, \mathrm{P_{ew}}\}$	$K^+ K^-$

$B \to PP, PV, VV$ decays: numerics

- main uncertainties of PQCD calculation: high order QCD corrections & LCDAs

  $\downarrow$

  † characterized by the variation in the factorization scale

  † minimized by setting μ_t as the largest virtuality in hard scattering

  † two-loop expression for the strong coupling

- input parameters of meson LCDAs

Meson	$\pi^\pm/\pi^0$	$K^\pm/K^0$	η_q	η_s
m/GeV [108]	0.140/0.135	0.494/0.498	0.104	0.705
f/GeV	0.130 [108]	0.156 [108]	0.125 [114]	0.177 [114]
m_0/GeV	1.400	1.892 [112]	1.087	1.990
a_1	0	0.076 ± 0.004 [113]	0	0
a_2	0.270 ± 0.047[14]	0.221 ± 0.082 [113]	0.250 ± 0.150 [115]	0.250 ± 0.150 [115]
Meson	$\rho^\pm/\rho^0$	$K^{*\pm}/K^{*0}$	ω	ϕ
m/GeV [108]	0.775	0.892	0.783	1.019
$f^\parallel$/GeV [9]	0.210/0.213	0.204	0.197	0.233
$f^\perp$/GeV	0.144/0.146 [116]	0.159 [9]	0.162 [9]	0.191 [9]
$a_1^\parallel$	0	0.060 ± 0.040 [117]	0	0
$a_1^\perp$	0	0.040 ± 0.030 [117]	0	0
$a_2^\parallel$	0.180 ± 0.037 [116]	0.160 ± 0.090 [117]	0.150 ± 0.120 [117]	0.230 ± 0.080 [117]
$a_2^\perp$	0.137 ± 0.030 [116]	0.100 ± 0.080 [117]	0.140 ± 0.120 [117]	0.140 ± 0.070 [117]

default scale $1\,\mathrm{GeV}$

$B \to PP, PV, VV$ decays: numerics

- anatomy of NLO corrections to $\mathcal{B}$ and $\mathcal{A}_{\rm CP}$ of $\pi\pi, \pi K$ modes

Mode	LO	+VC	+QL	+MP	$+\mathcal{F}^{\rm NLO}$	PDG [108]
$\mathcal{B}(B^+ \to \pi^+\pi^0)$	3.58	3.89	···	···	$4.18^{+1.32}_{-0.97}$	5.5 ± 0.4
$\mathcal{A}_{CP}$	−0.05	0.09	···	···	$0.08^{+0.09}_{-0.09}$	3 ± 4
$\mathcal{B}(B^0 \to \pi^+\pi^-)$	6.97	6.82	6.92	6.76	$7.31^{+2.38}_{-1.72}$	5.12 ± 0.19
$C_{\pi^+\pi^-}$	−23.4	−27.6	−13.8	−13.3	$-12.8^{+3.5}_{-3.3}$	-32 ± 4
$S_{\pi^+\pi^-}$	−31.1	−35.5	−46.4	−37.0	$-36.4^{+1.5}_{-1.5}$	-65 ± 4
$\mathcal{B}(B^0 \to \pi^0\pi^0)$	0.14	0.29	0.30	0.22	$0.23^{+0.07}_{-0.05}$	1.59 ± 0.26
$C_{\pi^0\pi^0}$	−3.1	60.1	73.6	77.6	$80.2^{+5.2}_{-6.7}$	33 ± 22
$\mathcal{B}(B^+ \to \pi^+K^0)$	17.0	20.8	28.0	19.4	$20.3^{+6.3}_{-4.4}$	23.7 ± 0.8
$\mathcal{A}_{CP}$	−1.19	−0.95	−0.06	−0.08	$-0.08^{+0.08}_{-0.09}$	-1.7 ± 1.6
$\mathcal{B}(B^+ \to \pi^0K^+)$	10.0	12.75	16.76	11.92	$12.3^{+3.8}_{-2.7}$	12.9 ± 0.5
$\mathcal{A}_{CP}$	−10.9	−5.20	2.26	2.48	$2.28^{+1.61}_{-1.74}$	3.7 ± 2.1
$\mathcal{B}(B^0 \to \pi^-K^+)$	14.3	18.0	23.9	16.4	$17.1^{+5.2}_{-3.7}$	19.6 ± 0.5
$\mathcal{A}_{CP}$	−15.2	−14.2	−4.16	−5.42	$-5.43^{+2.24}_{-2.34}$	-8.3 ± 0.4
$\mathcal{B}(B^0 \to \pi^0K^0)$	5.90	8.12	10.4	6.99	$7.38^{+2.11}_{-1.50}$	9.9 ± 0.5
$C_{\pi^0K^0}$	−2.62	−7.31	−6.57	−7.97	$-7.70^{+0.21}_{-0.13}$	0 ± 13
$S_{\pi^0K^0}$	70.1	73.5	71.6	71.9	$71.9^{+0.6}_{-0.6}$	58 ± 17

  - $\mathcal{B}$: QL cancels with MP corrections, not sensitive to VC and NLO ffs
  - color-suppressed modes $(\pi^0\pi^0, \pi^0K^0)$ are more sensitive to NLO corrections
  - asymmetry parameters is more sensitive to the NLO corrections
  - VC (QL) flips the sign of the direct CPV of $\pi^+\pi^0$ and $\pi^0\pi^0$ (π^0K^+) modes
    $\mathcal{A}_{CP}(B^+ \to K^+\pi^0) - \mathcal{A}_{CP}(B^+ \to K^+\pi^-) = 7.71^{+2.74}_{-2.92}(\text{PQCD}) \quad \text{VS} \quad 12.0 \pm 2.4(\text{Data})$
  - LO PQCD predicted a large direct CPV in $\pi^-K^+, \pi^+\pi^-$ (2000), which were confirmed subsequently by the BABRA and Belle collaborations.

$B \to PP, PV, VV$ decays: numerics

- updated PQCD results for $\mathcal{B}(B \to PP)$ (in units of 10^{-6})

η_q-η_s mixing

✓ η_q-η_s-η_g mixing
[Fan 2012]

✓ Glauber gluon effect
[Liu 2014]

Mode	PQCD	SCET1 [125]	SCET2 [125]	QCDF [127]	PDG [108]
$B^+ \to \pi^+ K^0$	$20.3^{+6.3+0.1}_{-4.4-0.1}$	···	···	$21.7^{+13.4}_{-9.1}$	23.7 ± 0.8
$B^+ \to \pi^0 K^+$	$12.3^{+3.8+0.1}_{-2.7-0.1}$	···	···	$12.5^{+6.8}_{-4.8}$	12.9 ± 0.5
$B^+ \to \eta' K^+$	$52.0^{+15.0+2.1}_{-10.8-0.7}$	69.5 ± 28.4	69.3 ± 27.7	$74.5^{+63.6}_{-31.6}$	70.4 ± 2.5
$B^+ \to \eta K^+$	$6.68^{+2.26+1.85}_{-1.60-0.96}$	2.7 ± 4.8	2.3 ± 4.5	$2.2^{+2.0}_{-1.3}$	2.4 ± 0.4
$B^+ \to K^+ \bar{K}^0$	$1.56^{+0.48+0.02}_{-0.34-0.02}$	···	···	$1.8^{+1.1}_{-0.7}$	1.31 ± 0.17
$B^+ \to \pi^0 \pi^+$	$4.18^{+1.30+0.22}_{-0.94-0.22}$ 4.45	···	···	$5.9^{+2.6}_{-1.6}$	5.5 ± 0.4
$B^+ \to \pi^+ \eta'$	$2.00^{+0.57+0.36}_{-0.42-0.31}$	2.4 ± 1.3	2.8 ± 1.3	$3.8^{+1.6}_{-0.8}$	2.7 ± 0.9
$B^+ \to \pi^+ \eta$	$2.62^{+0.78+0.45}_{-0.57-0.40}$	4.9 ± 2.0	5.0 ± 2.1	$5.0^{+1.5}_{-0.9}$	4.02 ± 0.27
$B^0 \to \pi^- K^+$	$17.1^{+5.2+0.1}_{-3.7-0.1}$	···	···	$19.3^{+11.4}_{-7.8}$	19.6 ± 0.5
$B^0 \to \pi^0 K^0$	$7.38^{+2.11+0.03}_{-1.50-0.04}$	···	···	$8.6^{+5.4}_{-3.6}$	9.9 ± 0.5
$B^0 \to \eta' K^0$	$52.3^{+14.9+2.1}_{-10.8-0.3}$	63.2 ± 26.3	62.2 ± 25.4	$70.9^{+59.1}_{-29.8}$	66 ± 4
$B^0 \to \eta K^0$	$4.63^{+1.57+1.51}_{-1.09-0.79}$	2.4 ± 4.4	2.3 ± 4.4	$1.5^{+1.7}_{-1.1}$	$1.23^{+0.27}_{-0.24}$
$B^0 \to K^0 \bar{K}^0$	$1.48^{+0.47+0.01}_{-0.33-0.00}$	···	···	$2.1^{+1.3}_{-0.8}$	1.21 ± 0.16
$B^0 \to K^+ K^-$	$0.046^{+0.058+0.009}_{-0.039-0.008}$	···	···	0.1 ± 0.04	0.078 ± 0.015
$B^0 \to \pi^+ \pi^-$	$7.31^{+2.35+0.38}_{-1.68-0.36}$ 5.35	···	···	$7.0^{+0.8}_{-1.0}$	5.12 ± 0.19
$B^0 \to \pi^0 \pi^0$	$0.23^{+0.07+0.01}_{-0.05-0.01}$ 0.61	···	···	$1.1^{+1.2}_{-0.5}$	1.59 ± 0.26
$B^0 \to \pi^0 \eta'$	$0.20^{+0.05+0.02}_{-0.03-0.01}$	2.3 ± 2.8	1.3 ± 0.6	$0.42^{+0.28}_{-0.15}$	1.2 ± 0.6
$B^0 \to \pi^0 \eta$	$0.20^{+0.06+0.02}_{-0.04-0.01}$	0.88 ± 0.68	0.68 ± 0.62	$0.36^{+0.13}_{-0.11}$	0.41 ± 0.17
$B^0 \to \eta\eta$	$0.37^{+0.09+0.08}_{-0.07-0.07}$	0.69 ± 0.71	1.0 ± 1.5	$0.32^{+0.15}_{-0.08}$	< 1
$B^0 \to \eta\eta'$	$0.29^{+0.07+0.06}_{-0.05-0.06}$	1.0 ± 1.6	2.2 ± 5.5	$0.36^{+0.27}_{-0.13}$	< 1.2
$B^0 \to \eta'\eta'$	$0.42^{+0.09+0.13}_{-0.07-0.11}$	0.57 ± 0.73	1.2 ± 3.7	$0.22^{+0.16}_{-0.08}$	< 1.7

- NLO corrections are important in penguin dominated models $\pi K, \eta' K$ and pure annihilation mode $K^0 K^0$
- LO PQCD $\mathcal{B}(B_s \to \pi^+\pi^-) \sim 6 \times 10^{-6}$ (2007), confirmed by CDF (2011)

$B \to PP, PV, VV$ decays: numerics

- updated PQCD results for the CPV of $B \to PP$ decays (in units of 10^{-2})

η_q-η_s mixing

✓ η_q-η_s-η_g mixing [Fan 2012]

Belle II, PRL.131.111803(2023)
$-0.04 \pm 0.15 \pm 0.05$
$0.75 \pm 0.20 \pm 0.04$

Mode	PQCD	SCET1 [125]	SCET2 [125]	QCDF [127]	PDG [108]
$B^+ \to \pi^+ K_S^0$	$-0.08^{+0.08+0.02}_{-0.09-0.02}$	⋯	⋯	$0.28^{+0.09}_{-0.10}$	-1.7 ± 1.6
$B^+ \to \pi^0 K^+$	$2.28^{+1.53+0.50}_{-1.65-0.57}$	⋯	⋯	$4.9^{+6.3}_{-5.8}$	3.0 ± 2.1
$B^+ \to \eta' K^+$	$-1.83^{+0.40+0.77}_{-0.40-1.03}$	-1 ± 1	7 ± 1	$0.45^{+1.4}_{-1.1}$	0.4 ± 1.1
$B^+ \to \eta K^+$	$-7.75^{+1.06+0.81}_{-0.99-0.43}$	33 ± 31	-33 ± 40	$-14.5^{+18.6}_{-28.1}$	-37 ± 8
$B^+ \to K^+ K_S^0$	$1.83^{+1.93+0.14}_{-1.87-0.18}$	⋯	⋯	-6.4 ± 2.0	14
$B^+ \to \pi^0 \pi^+$	$0.08^{+0.06+0.07}_{-0.06-0.04}$	⋯	⋯	$-0.11^{+0.06}_{-0.03}$	3 ± 4
$B^+ \to \pi^+ \eta'$	$68.9^{+2.4+1.0}_{-2.4-0.9}$	21 ± 21	2 ± 18	$1.6^{+10.6}_{-13.8}$	6 ± 16
$B^+ \to \pi^+ \eta$	$24.8^{+3.6+0.8}_{-3.3-0.7}$	5 ± 29	37 ± 29	$-5.0^{+8.7}_{-10.8}$	-14 ± 7
$B^0 \to \pi^- K^+$	$-5.43^{+1.86+1.26}_{-1.92-1.34}$	⋯	⋯	$-7.4^{+4.6}_{-5.0}$	-8.3 ± 0.4
$B^0 \to \pi^0 K_S^0$	$-7.70^{+0.17+0.12}_{-0.09-0.09}$	⋯	⋯	$-10.6^{+6.2}_{-5.7}$	$C_{\pi^0 K^0} = 0 \pm 13$
	$71.9^{+0.3+0.5}_{-0.3-0.5}$	⋯	⋯	$79.0^{+7.2}_{-5.7}$	$S_{\pi^0 K^0} = 58 \pm 17$
$B^0 \to \eta' K_S^0$	$-2.65^{+0.10+0.07}_{-0.10-0.11}$	1.1 ± 1.4	-2.7 ± 1.2	$3.0^{+1.0}_{-0.9}$	$C_{\eta' K^0} = -6 \pm 4$
	$69.8^{+0.1+0.1}_{-0.1-0.1}$	70.6	71.5	67.0 ± 1.4	$S_{\eta' K^0} = 63 \pm 6$
$B^0 \to \eta K_S^0$	$-7.88^{+0.14+0.06}_{-0.10-0.02}$	21 ± 21	-18 ± 23.2	$-23.6^{+16.0}_{-29.0}$	⋯
	$70.0^{+0.2+0.2}_{-0.3-0.1}$	69	79	$79.0^{+8.9}_{-8.5}$	⋯
$B^0 \to K_S^0 K_S^0$	$-17.3^{+0.6+0.4}_{-0.4-0.3}$	⋯	⋯	$-10.0^{+1.2}_{-2.0}$	$C_{K_S^0 K_S^0} = 0 \pm 40$
	$5.34^{+1.05+0.53}_{-1.06-0.49}$	⋯	⋯	⋯	$S_{K_S^0 K_S^0} = -80 \pm 50$
$B^0 \to \pi^+ \pi^-$	$-12.8^{+3.3+1.1}_{-3.1-1.1}$	⋯	⋯	$17.0^{+4.5}_{-8.8}$	$C_{\pi^+ \pi^-} = -32 \pm 4$
	$-36.4^{+0.5+1.4}_{-0.4-1.4}$	⋯	⋯	$-69^{+20.6}_{-13.5}$	$S_{\pi^+ \pi^-} = -65 \pm 4$
$B^0 \to \pi^0 \pi^0$	$-80.2^{+5.2+0.4}_{-6.7-0.2}$	⋯	⋯	$57.2^{+33.7}_{-40.4}$	$C_{\pi^0 \pi^0} = -33 \pm 22$
	$53.5^{+8.7+3.1}_{-8.4-3.0}$	⋯	⋯	⋯	⋯

$B \to PP$, PV, VV decays: numerics

- updated PQCD results for the CPV of $B^+ \to PV$ decays (in units of 10^{-2})

Mode	PQCD	SCET1 [128]	SCET2 [128]	QCDF [127]	PDG [108]
$B^+ \to \eta' K^{*+}$	$1.54^{+9.05+14.9}_{-8.16-9.74}$	$2.7^{+27.4}_{-19.5}$	$2.6^{+26.7}_{-32.9}$	$65.5^{+35.7}_{-63.9}$	-26 ± 27
$B^+ \to \eta K^{*+}$	$-34.5^{+2.5+0.9}_{-2.4-0.8}$	$-2.6^{+5.4}_{-5.5}$	$-1.9^{+3.4}_{-3.6}$	$-9.7^{+7.3}_{-8.0}$	2 ± 6
$B^+ \to K^+ \omega$	$31.5^{+0.6+0.1}_{-1.1-0.7}$	$11.6^{+18.2}_{-20.4}$	$12.3^{+16.6}_{-17.3}$	$22.1^{+19.6}_{-18.2}$	-2 ± 4
$B^+ \to \pi^+ K^{*0}$	$-0.94^{+0.26+0.04}_{-0.29-0.03}$	0	0	$0.4^{+4.5}_{-4.2}$	-4 ± 9
$B^+ \to \pi^0 K^{*+}$	$-0.01^{+4.40+1.12}_{-4.87-1.26}$	$-17.8^{+30.4}_{-24.7}$	$-12.9^{+12.0}_{-12.2}$	$1.6^{+11.5}_{-4.2}$	-39 ± 21
$B^+ \to K^+ \rho^0$	$58.7^{+4.3+3.2}_{-4.0-2.8}$	$9.2^{+15.2}_{-16.1}$	$16.0^{+20.5}_{-22.5}$	$45.4^{+36.1}_{-30.2}$	37 ± 10
$B^+ \to K^0 \rho^+$	$0.99^{+0.01+0.13}_{-0.01-0.18}$	0	0	$0.3^{+0.5}_{-0.3}$	-3 ± 15
$B^+ \to K^+ \bar{K}^{*0}$	$21.3^{+6.2+1.2}_{-5.7-1.4}$	$-3.6^{+6.1}_{-5.3}$	$-4.4^{+4.1}_{-4.1}$	$-8.9^{+3.0}_{-2.6}$	12 ± 10
$B^+ \to K^+ \phi$	$-1.93^{+0.66+0.66}_{-0.60-0.42}$	0	0	$0.6^{+0.1}_{-0.1}$	2.4 ± 2.8
$B^+ \to \pi^+ \phi$	0.0	$\cdots$	$\cdots$	0.0	1 ± 5
$B^+ \to \pi^+ \omega$	$-29.8^{+0.5+1.1}_{-0.4-0.8}$	$0.5^{+19.1}_{-19.6}$	$2.3^{+13.4}_{-13.2}$	$-13.2^{+12.4}_{-10.9}$	-4 ± 5
$B^+ \to \pi^+ \rho^0$	$14.9^{+0.4+0.5}_{-0.4-0.6}$	$-10.8^{+13.1}_{-12.7}$	$-19.2^{+15.6}_{-13.5}$	$-9.8^{+11.9}_{-10.5}$	0.9 ± 1.9
$B^+ \to \pi^0 \rho^+$	$-7.31^{+0.06+0.07}_{-0.02-0.03}$	$15.5^{+17.0}_{-19.0}$	$12.3^{+9.4}_{-10}$	$9.7^{+8.3}_{-10.8}$	2 ± 11
$B^+ \to \eta' \rho^+$	$29.0^{+0.4+0.0}_{-0.4-0.1}$	$-19.8^{+66.6}_{-37.6}$	$-21.7^{+135.9}_{-24.3}$	$1.4^{+14.0}_{-11.9}$	26 ± 17
$B^+ \to \eta \rho^+$	$-13.0^{+0.1+0.1}_{-0.1-1.5}$	$-6.6^{+21.5}_{-21.3}$	$-9.1^{+16.7}_{-15.8}$	$-8.5^{+6.5}_{-5.3}$	11 ± 11

large CPV predictions

large CPV in rare decay

η_q-η_s mixing

? η_q-η_s-η_g mixing

- measured direct CPV in $B \to PV$ is significantly larger than that in $B \to PP$
- it is hard to measure $B \to PV$ decays precisely $\Leftarrow$ vector meson is not stable
- cascade decay $B \to PV \to PPP$, hard to resolve see Wen-fei's talk, 11/3
- dimeson light-cone distribution amplitudes provide a possible solution
  [Cheng, Khodjomirian, Virto 2017, 2019, Cheng 2021]

$B \to PP, PV, VV$ decays: numerics

- updated PQCD prediction of $\mathcal{B}(B^+ \to VV)$ (in units of 10^{-6})

Mode	PQCD	SCET [130]	QCDF [127,131]	PDG [108]
$B^+ \to \rho^+ K^{*0}$	$9.40^{+1.43+1.05}_{-1.34-0.95}$	8.93 ± 3.18	$9.2^{+3.8}_{-5.5}$	9.2 ± 1.5
$B^+ \to \rho^0 K^{*+}$	$6.25^{+1.12+0.59}_{-0.84-0.53}$	4.64 ± 1.37	$5.5^{+1.4}_{-2.5}$	4.6 ± 1.1
$B^+ \to \omega K^{*+}$	$5.48^{+1.52+0.81}_{-1.36-0.66}$	5.56 ± 1.60	$3.0^{+2.5}_{-1.5}$	< 7.4
$B^+ \to \phi K^{*+}$	$12.3^{+1.7+1.5}_{-1.4-1.4}$	9.86 ± 3.39	$10.0^{+12.4}_{-3.5}$	10.2 ± 2.0
$B^+ \to K^{*+} \bar{K}^{*0}$	$0.66^{+0.12+0.09}_{-0.09-0.08}$	0.52 ± 0.18	$0.6^{+0.3}_{-0.3}$	0.91 ± 0.29
$B^+ \to \rho^0 \rho^+$	$14.0^{+4.1+0.4}_{-3.0-0.4}$	22.1 ± 3.7	$20.06^{+4.5}_{-2.1}$	24.0 ± 1.9
$B^+ \to \rho^+ \omega$	$10.9^{+2.8+1.0}_{-2.1-0.9}$	19.2 ± 3.1	$16.9^{+3.6}_{-1.8}$	15.9 ± 2.1
$B^+ \to \rho^+ \phi$	$0.042^{+0.011+0.004}_{-0.008-0.003}$	0.005 ± 0.001	$\cdots$	< 3.0
$B^0 \to \rho^- K^{*+}$	$8.72^{+1.27+0.97}_{-0.96-0.87}$	10.6 ± 3.2	$8.9^{+4.9}_{-5.6}$	10.3 ± 2.6
$B^0 \to \rho^0 K^{*0}$	$3.37^{+0.38+0.43}_{-0.29-0.39}$	5.87 ± 1.87	$4.6^{+3.6}_{-3.6}$	3.9 ± 1.3
$B^0 \to \omega K^{*0}$	$5.93^{+0.89+1.74}_{-0.73-1.55}$	3.82 ± 1.39	$2.5^{+2.5}_{-1.5}$	2.0 ± 0.5
$B^0 \to \phi K^{*0}$	$11.8^{+1.6+1.5}_{-1.3-1.5}$	9.14 ± 3.14	10.0 ± 0.5	$\cdots$
$B^0 \to K^{*0} \bar{K}^{*0}$	$0.38^{+0.09+0.02}_{-0.06-0.01}$	0.48 ± 0.16	$0.6^{+0.2}_{-0.3}$	0.83 ± 0.24
$B^0 \to K^{*+} K^{*-}$	$0.17^{+0.02+0.05}_{-0.02-0.03}$	$\cdots$	$0.16^{+0.1}_{-0.1}$	< 2.0
$B^0 \to \rho^+ \rho^-$	$22.7^{+6.3+0.6}_{-4.8-0.6}$	27.7 ± 4.1	$25.5^{+2.8}_{-3.0}$	27.7 ± 1.9
$B^0 \to \rho^0 \rho^0$	$0.54^{+0.16+0.04}_{-0.11-0.04}$	1.00 ± 0.29	$0.9^{+1.9}_{-0.5}$	0.96 ± 0.15
$B^0 \to \rho^0 \omega$	$0.76^{+0.15+0.14}_{-0.11-0.12}$	0.59 ± 0.19	$0.08^{+0.36}_{-0.02}$	< 1.6
$B^0 \to \rho^0 \phi$	$0.019^{+0.005+0.002}_{-0.004-0.001}$	~ 0.002	$\cdots$	< 3.3
$B^0 \to \omega \omega$	$1.21^{+0.24+0.31}_{-0.19-0.24}$	0.39 ± 0.13	$0.7^{+1.1}_{-0.4}$	1.2 ± 0.4
$B^0 \to \omega \phi$	$0.018^{+0.005+0.005}_{-0.004-0.005}$	~ 0.002	$\cdots$	< 0.7
$B^0 \to \phi \phi$	$0.029^{+0.002+0.006}_{-0.002-0.006}$	$\cdots$	$\cdots$	< 0.027

isospin symmetry
smallness of $\mathcal{B}(\rho^0\rho^0)$
↓
$\mathcal{B}(\rho^+\rho^-) \sim 2\mathcal{B}(\rho^+\rho^0)$

VS (no new physics violates QCD isospin symmetry)

PQCD: ~ 1.6
Data: ~ 1

PQCD : $\mathcal{B}(\rho^0\rho^0) \sim 2\mathcal{B}(\pi^0\pi^0)$
Data : $\mathcal{B}(\rho^0\rho^0) \sim \mathcal{B}(\pi^0\pi^0)/2$

0.89 Glauber gluon

~~inconsistence between two B factories~~

Belle, PRL.133.1081801(2024)
$1.53 \pm 0.29 \pm 0.17$

- ○ NLO corrections are important in rare modes $\rho^+\phi, \rho^0\rho^0(\omega, \rho), \omega\omega(\phi)$

$B \to PP, PV, VV$ decays: numerics

- updated PQCD results for the f_L of $B^+ \to VV$ decays (in units of 10^{-2})

Mode	PQCD$_{\rm LO}$ [51]	PQCD	SCET [130]	QCDF [127,131]	HFLAV [134]
$B^+ \to \rho^+ K^{*0}$	70.0 ± 5.0	$76.6^{+1.5}_{-1.4}$	45.0 ± 18.0	$48.0^{+52.0}_{-40.0}$	48 ± 8
$B^+ \to \rho^0 K^{*+}$	$75.0^{+4.0}_{-5.0}$	$80.0^{+1.5}_{-1.5}$	42.0 ± 14.0	$67.0^{+31.0}_{-48.0}$	78 ± 12
$B^+ \to \omega K^{*+}$	64.0 ± 7.0	$77.4^{+0.5}_{-0.9}$	53.0 ± 14.0	$67.0^{+32.0}_{-39.0}$	41 ± 19
$B^+ \to \phi K^{*+}$	$57.0^{+6.3}_{-5.9}$	$68.7^{+1.3}_{-1.5}$	51.0 ± 16.4	$49.0^{+51.0}_{-43.0}$	50 ± 5
$B^+ \to K^{*+} \bar{K}^{*0}$	74.0 ± 7.0	$82.4^{+1.1}_{-1.1}$	50.0 ± 16.0	$45.0^{+55.0}_{-38.0}$	82^{+15}_{-21}
$B^+ \to \rho^0 \rho^+$	98.0 ± 1.0	$96.9^{+0.1}_{-0.1}$	~ 100	96.0 ± 2.0	95 ± 1.6
$B^+ \to \rho^+ \omega$	97.0 ± 1.0	$96.3^{+0.3}_{-0.4}$	97.0 ± 1.0	$96.0^{+2.0}_{-3.0}$	90 ± 6
$B^+ \to \rho^+ \phi$	95.0 ± 1.0	$81.3^{+1.9}_{-1.8}$	~ 100	$\cdots$	$\cdots$
$B^0 \to \rho^- K^{*+}$	$68.0^{+5.0}_{-4.0}$	$75.7^{+1.5}_{-1.4}$	55 ± 14	$53.0^{+45.0}_{-32.0}$	38 ± 13
$B^0 \to \rho^0 K^{*0}$	$65.0^{+4.0}_{-5.0}$	$71.0^{+1.5}_{-1.3}$	61.0 ± 13.0	$39.0^{+60.0}_{-31.0}$	17.3 ± 2.6
$B^0 \to \omega K^{*0}$	65.0 ± 5.0	$77.7^{+0.4}_{-0.9}$	40.0 ± 20.0	$58.0^{+44.0}_{-17.0}$	69 ± 11
$B^0 \to \phi K^{*0}$	$56.5^{+5.8}_{-5.9}$	$69.5^{+1.2}_{-1.5}$	51.0 ± 16.4	$50.0^{+51.0}_{-44.0}$	49.7 ± 1.7
$B^0 \to K^{*0} \bar{K}^{*0}$	58.0 ± 8.0	$68.8^{+5.3}_{-5.3}$	50.0 ± 16.0	$52.0^{+48.0}_{-49.0}$	74 ± 5
$B^0 \to K^{*+} K^{*-}$	~ 100.0	~ 100.0	$\cdots$	~ 100.0	$\cdots$
$B^0 \to \rho^+ \rho^-$	95.0 ± 1.0	$93.8^{+0.1}_{-0.1}$	99.1 ± 0.3	$92.0^{+1.0}_{-3.0}$	$99.0^{+2.1}_{-1.9}$
$B^0 \to \rho^0 \rho^0$	$12.0^{+16.0}_{-2.0}$	$80.9^{+1.9}_{-1.9}$	87.0 ± 5.0	$92.0^{+7.0}_{-37.0}$	71^{+8}_{-9}
$B^0 \to \rho^0 \omega$	$67.0^{+8.0}_{-9.0}$	$74.2^{+0.1}_{-0.1}$	58.0 ± 14.0	$52.0^{+12.0}_{-44.0}$	$\cdots$
$B^0 \to \rho^0 \phi$	95.0 ± 1.0	$81.3^{+1.9}_{-1.8}$	~ 100	$\cdots$	$\cdots$
$B^0 \to \omega\omega$	$66.0^{+10.0}_{-11.0}$	$88.4^{+0.9}_{-0.8}$	64.0 ± 15.0	$94.0^{+4.0}_{-20.0}$	$\cdots$
$B^0 \to \omega\phi$	$94.0^{+2.0}_{-3.0}$	$80.8^{+0.8}_{-1.4}$	~ 100	$\cdots$	$\cdots$
$B^0 \to \phi\phi$	97.0 ± 1.0	$99.9^{+0.0}_{-0.0}$	$\cdots$	$\cdots$	$\cdots$

Belle, PRL.133.1081801(2024)
$0.87 \pm 0.13 \pm 0.13$

† LO PQCD predicted $f_L \sim 0.7$ in penguin dominated $B \to VV$ by annihilation mechanism (2002), before the observation of "polarization puzzle".

Conclusion

- state-of-the-art PQCD calculations with including the current well-known NLO and sub-leading power corrections

  - $K\pi, K\rho, K\omega, K\phi$ and $K^*\rho, K^*\omega, K^*\phi$ channels ✓✓ $K^*\pi, K^*K$ channels ✓
  - f_L in $K^*\rho, K^*\omega, K^*\phi$ channels is still larger than the HFLAV result

    LD effect in $B \to K^*$ transition ? NLO corrections to $B \to V$ form factors ? width effect of the intermediate vector resonant (four-body decays) ?
  - $\eta^{(\prime)}$ involved channels do not consist well with data

    the complicate mixing mechanism η_q-η_s-η_g ?
  - CPV of charged (neutral) B decays is (not) sensitive to the new added two power correction (heavy quark expansion), especially for the channels with at least one $\eta^{(\prime)}$ in the final state

Conclusion

- predictions in $K_S^0\pi^0, \omega\omega$ modes are confirmed
- what's the next ? CPV or $\eta\eta, \eta\eta', \eta'\eta'$?

Our results for $\mathcal{B}$ and f_L agree well with predictions from next-to-leading-order (NLO) perturbative QCD (PQCD) [23], but not from leading-order (LO) PQCD [19]. This indicates that NLO corrections and power-suppressed terms play an important role in color-suppressed $b \to (u, d)$ decays. Such a role would help clarify the puzzle in $B^0 \to \rho^0\rho^0$, where the measured f_L is significantly higher than the LO PQCD prediction [19]. Our result for $\mathcal{B}(B^0 \to \omega\omega)$ is significantly higher than the prediction from soft collinear effective theory [22]. Our result for A_{CP} shows no significant CP violation, consistent within uncertainties with CKM unitarity. Belle, PRL.133.1081801(2024)

[23] Chai, SC, Ju, Yan, Lu, Xiao, CPC 46.12(2022)123103
[19] Zou, Ali, Lu, Liu, Li, PRD 91.054033(2015)

Thank you for your patience.