第三届高能物理理论与实验融合发展研讨会

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Abelian and non-Abelian domain walls

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摘要

- 引言 ——Z_2 畴壁的简介
- 一般的Abel 畴壁 ——考虑 Z_N $(N > 2)$ 破缺产生的畴壁
- 非 Abel 畴壁 ——以立方体群 S_4 的破缺为例
- 畴壁引力波 ——利用畴壁引力波检验分立味对称性

[1] *Gravitational wave signatures from discrete flavor symmetries,* G. Gelmini, S. Pascoli, E. Vitagliano, YLZ, 2009.01903

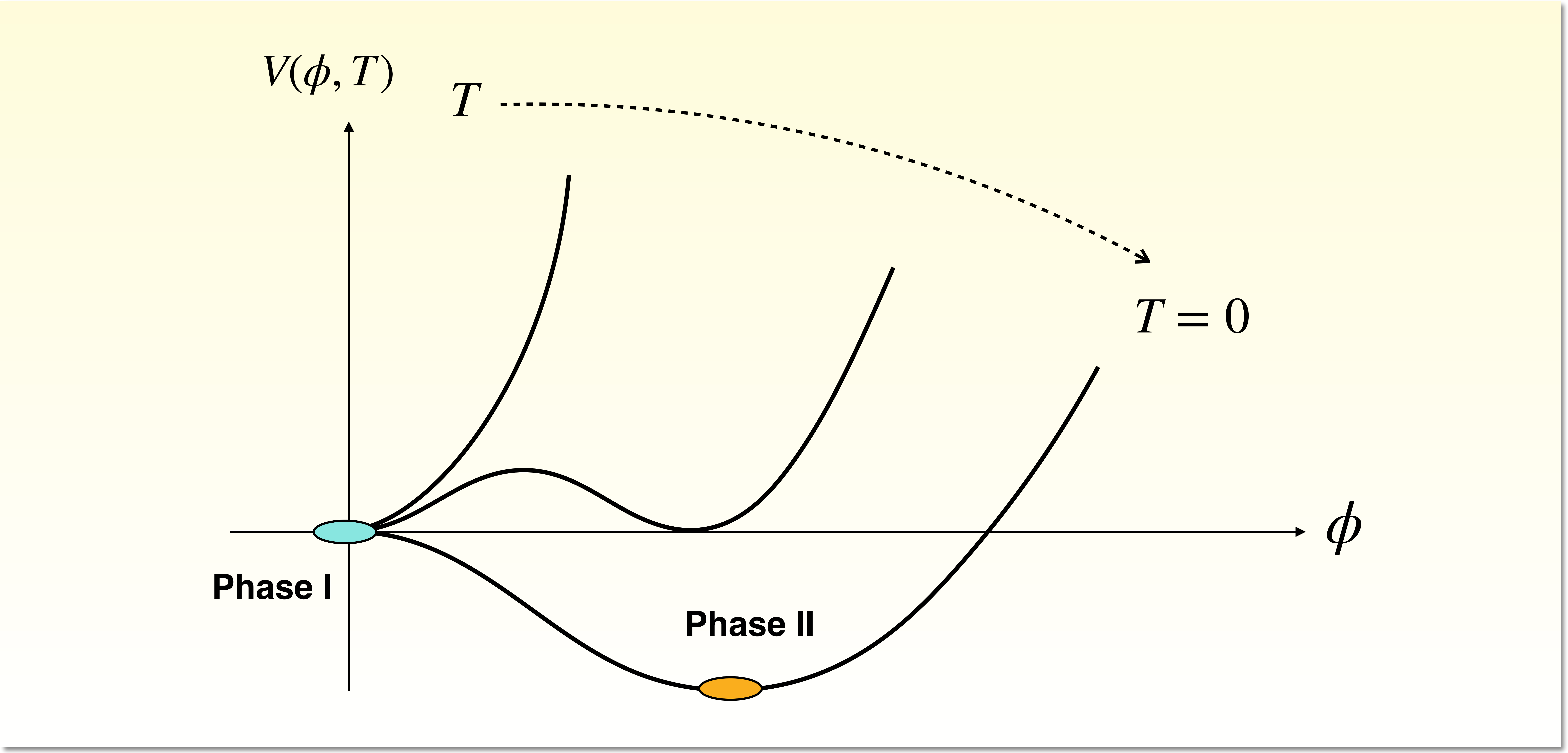
[2] *Collapsing domain walls beyond Z2,* 吴永成, 谢柯盼, YLZ, 2204.04374

[3] *Classification of Abelian domain walls,* 吴永成, 谢柯盼, YLZ, 2205.11529

[4] *Non-Abelian domain walls,* 付博文, S. King, L. Marsili, S. Pascoli, J. Turner, YLZ, 2409.16359

相变的一般过程 (假设为一阶相变)

有限温度框架下的有效势 $V(\phi, T) \approx D(T^2 - T_0^2)\phi^2 - \tilde{\mu}_T \phi^3 + \frac{\lambda_T}{4}\phi^4$



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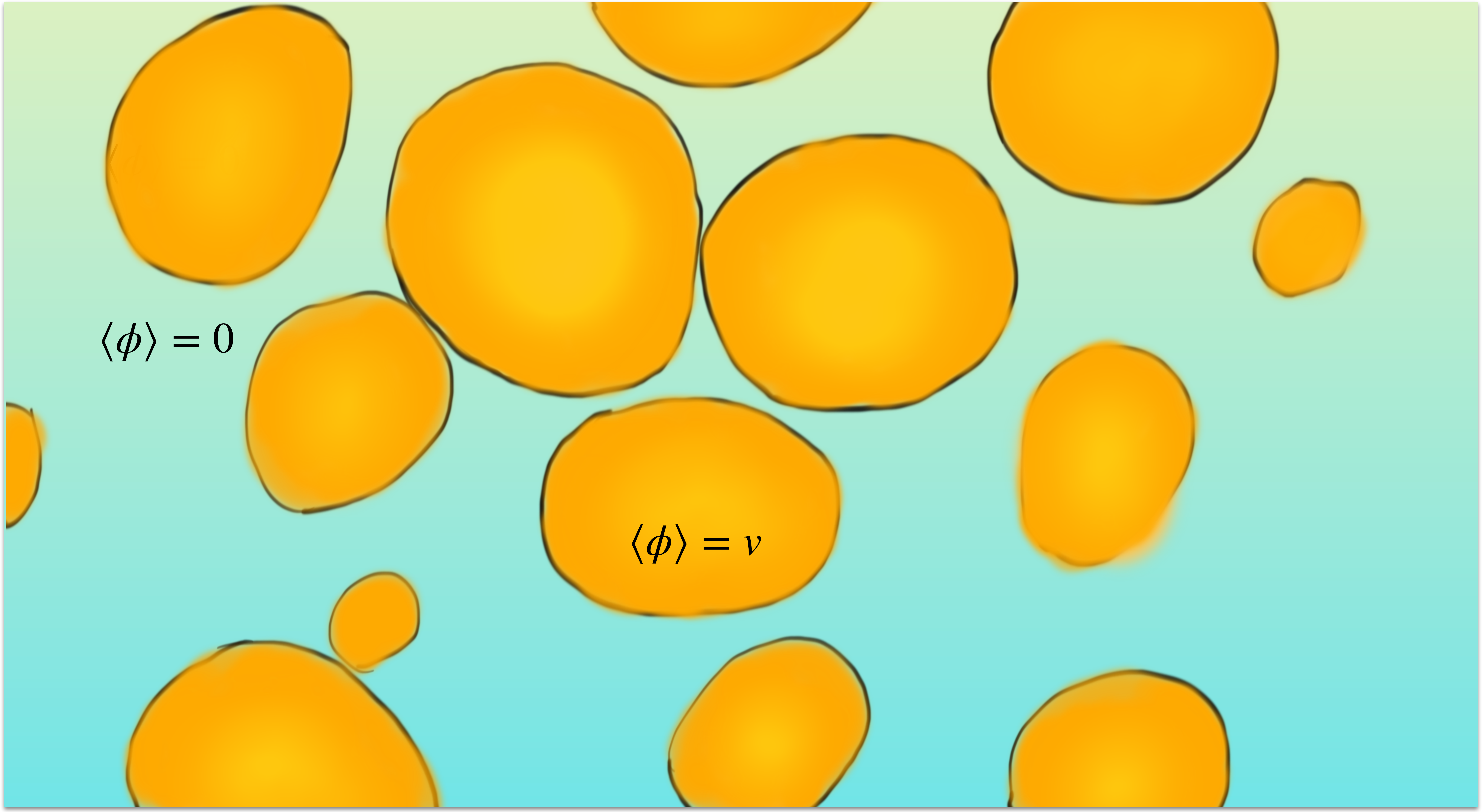

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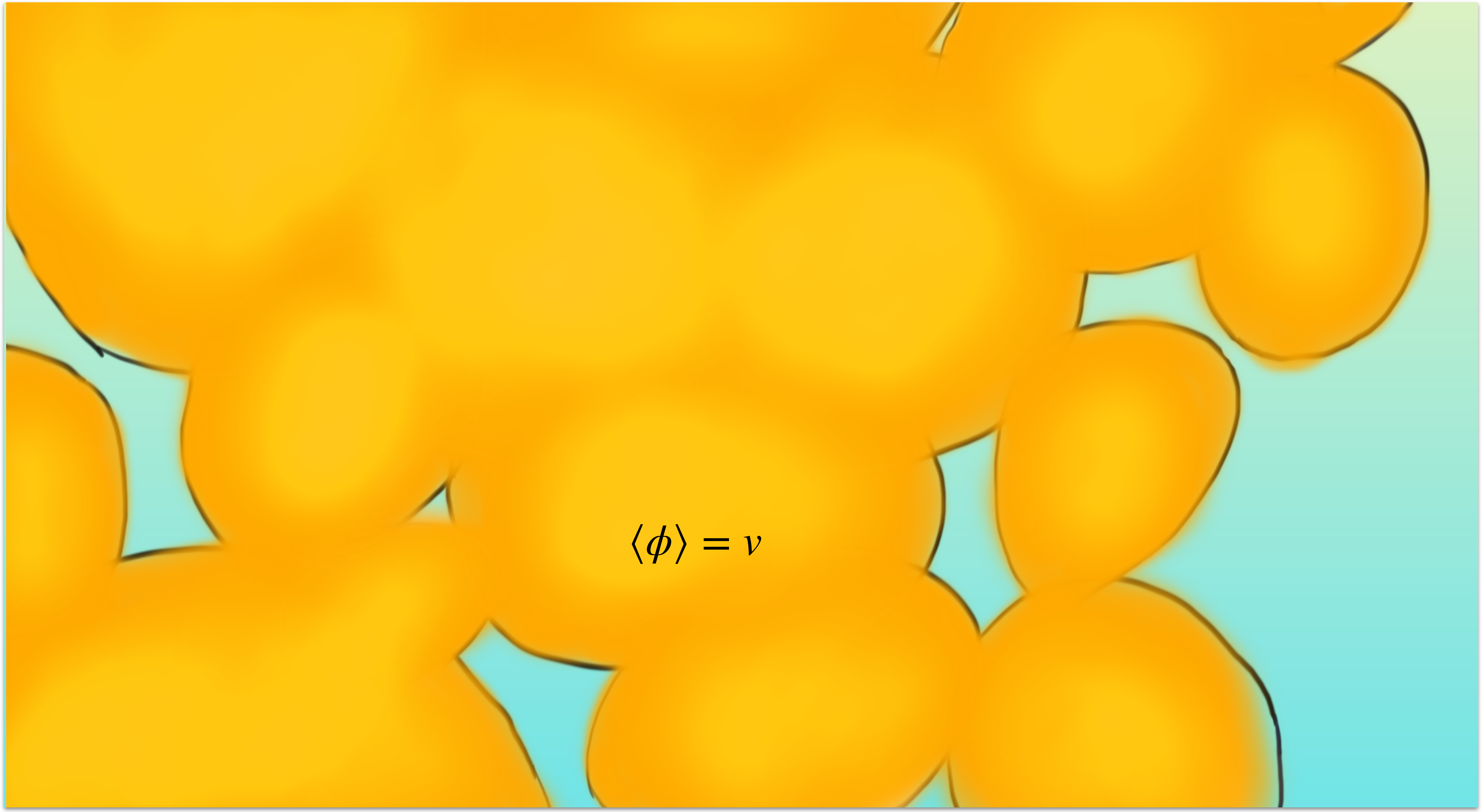
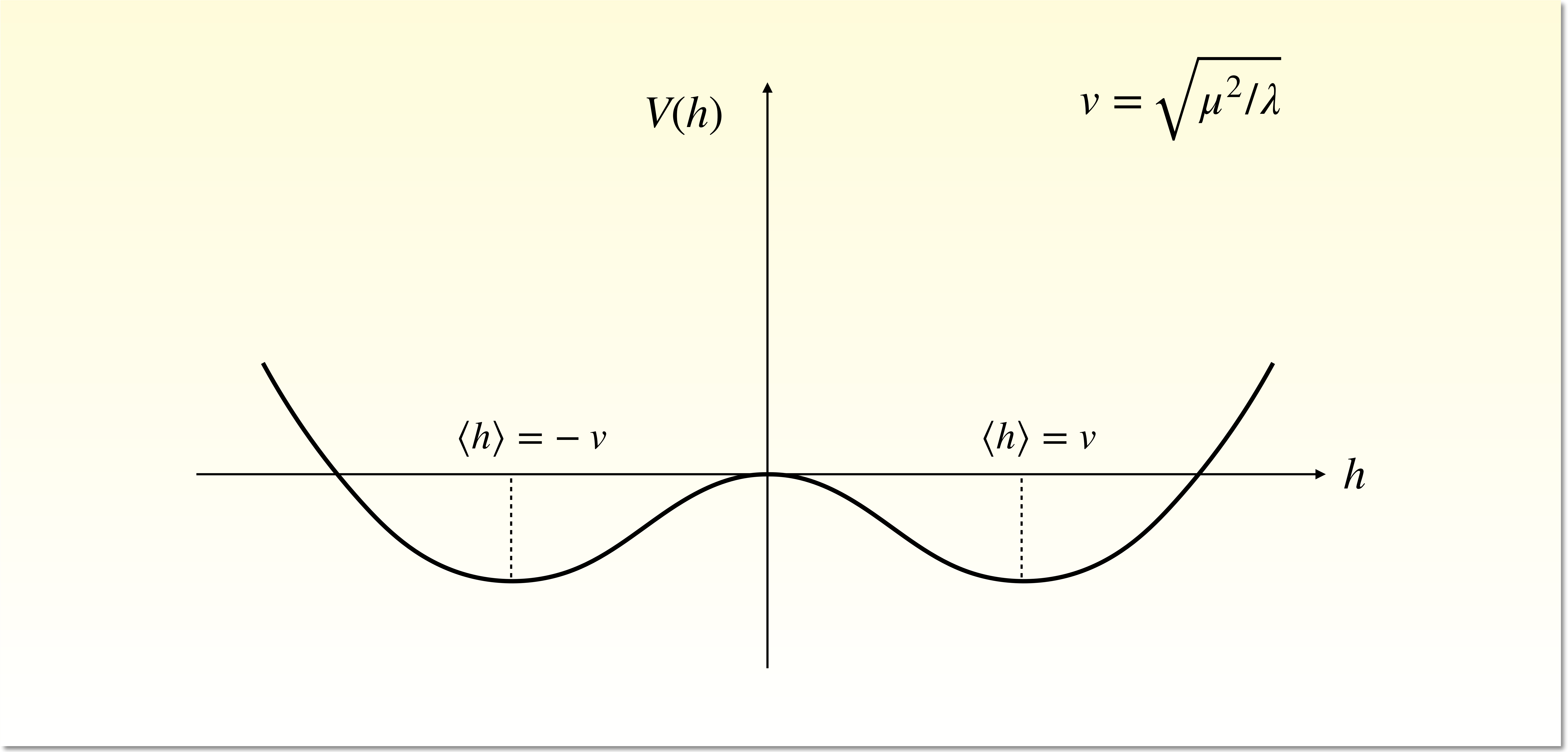


伴随着Z_2对称性自发破缺的相变

Z_2对称性下实标量场 h 的势能（树图阶） $$V(h) = -\frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4}h^4$$



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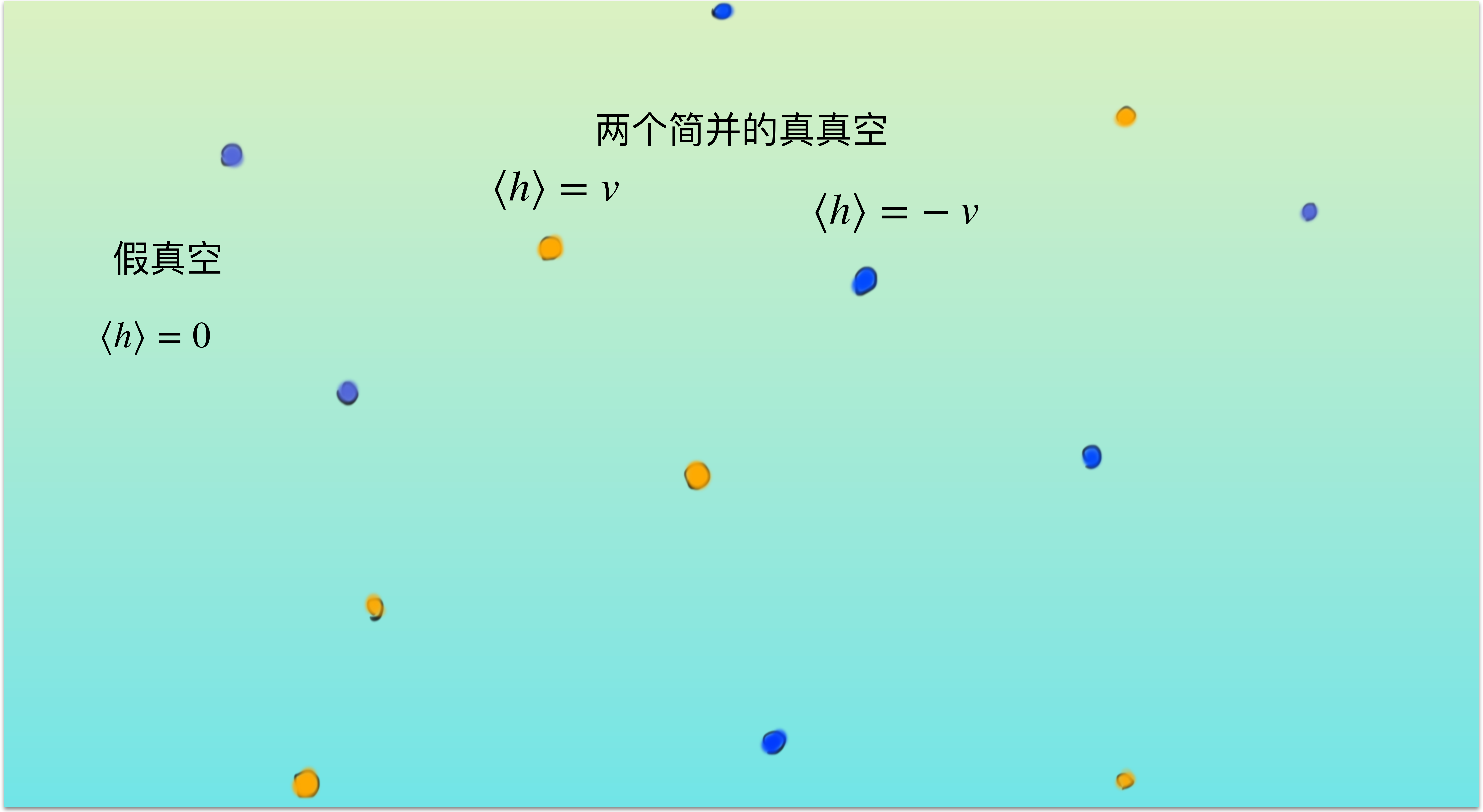

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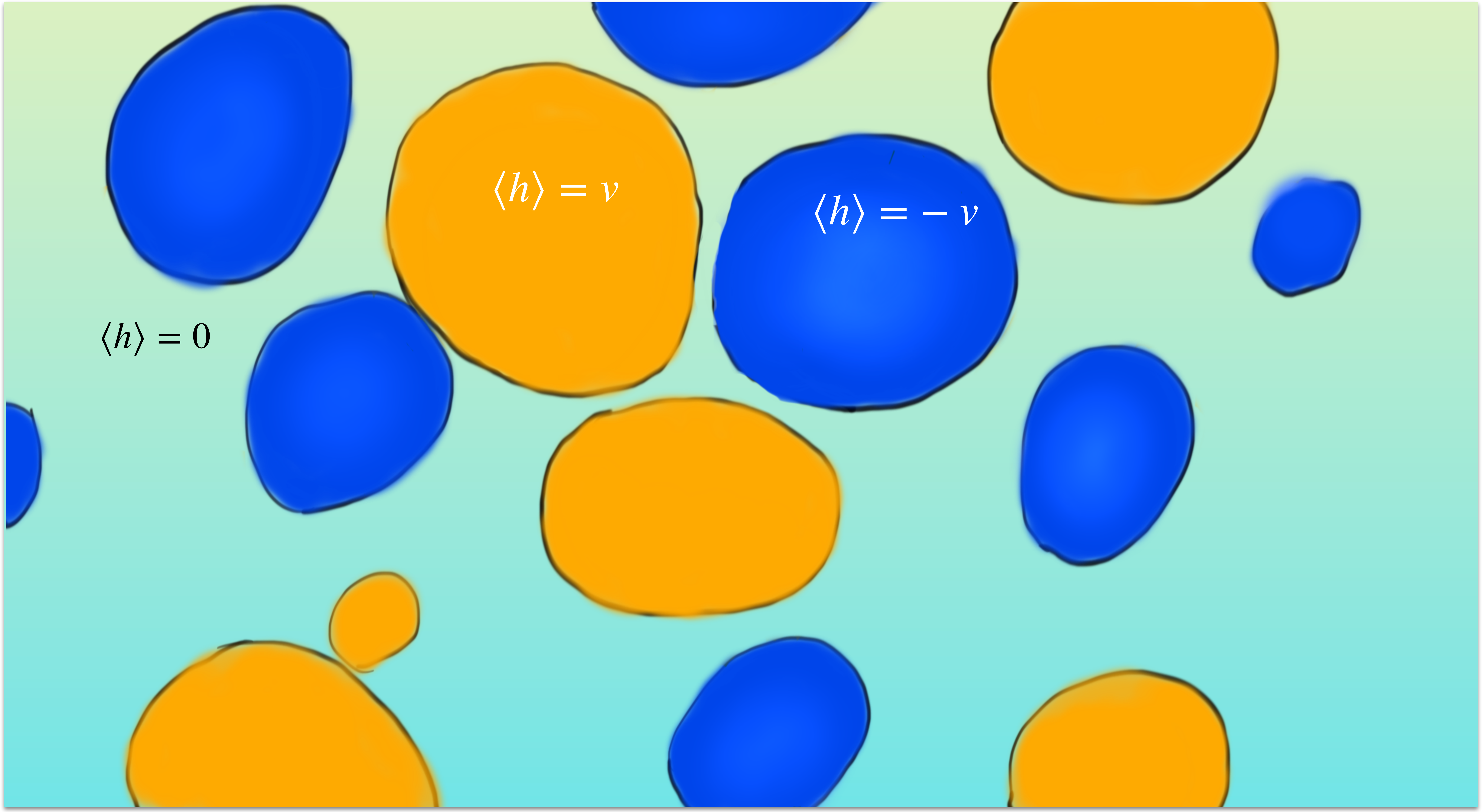
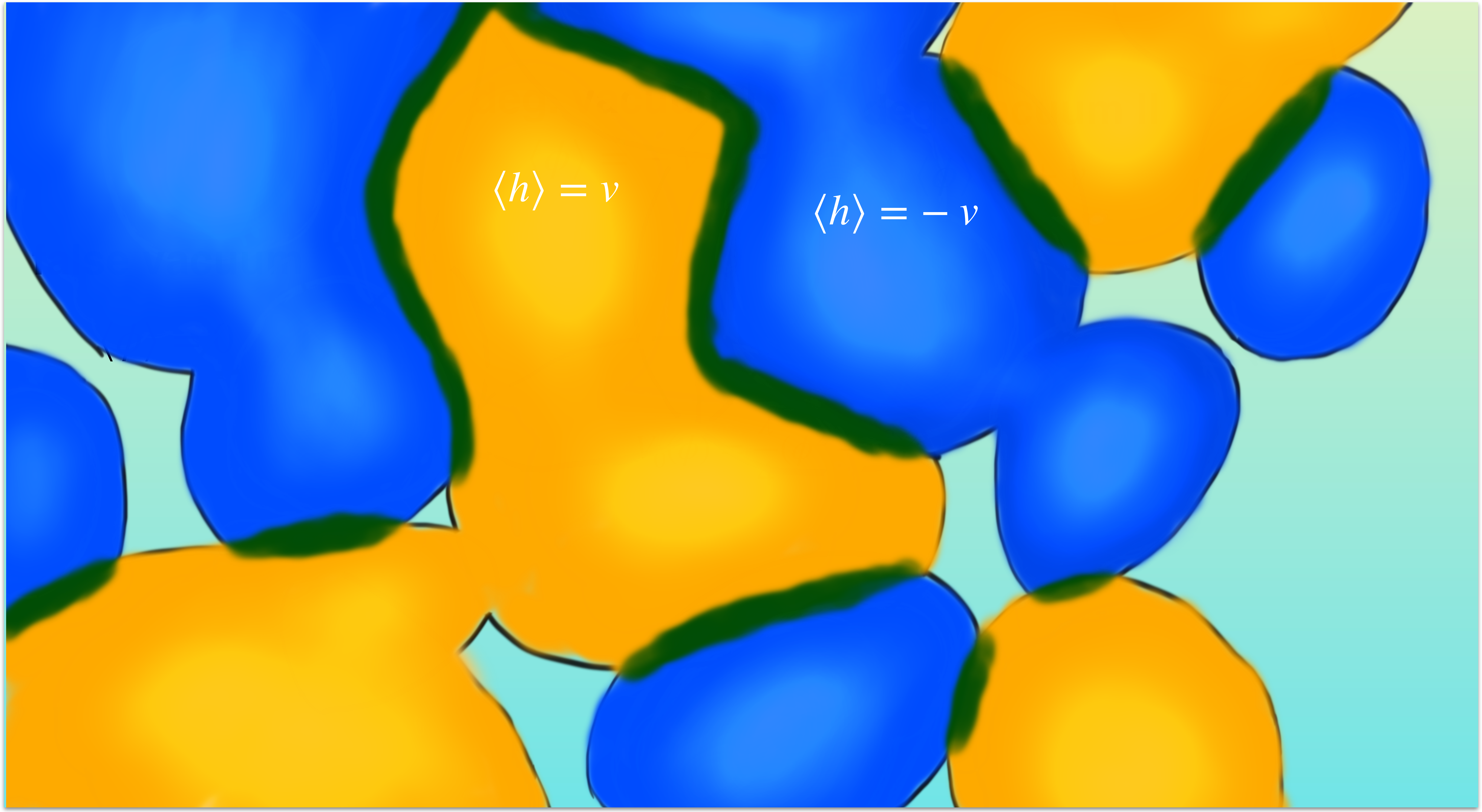
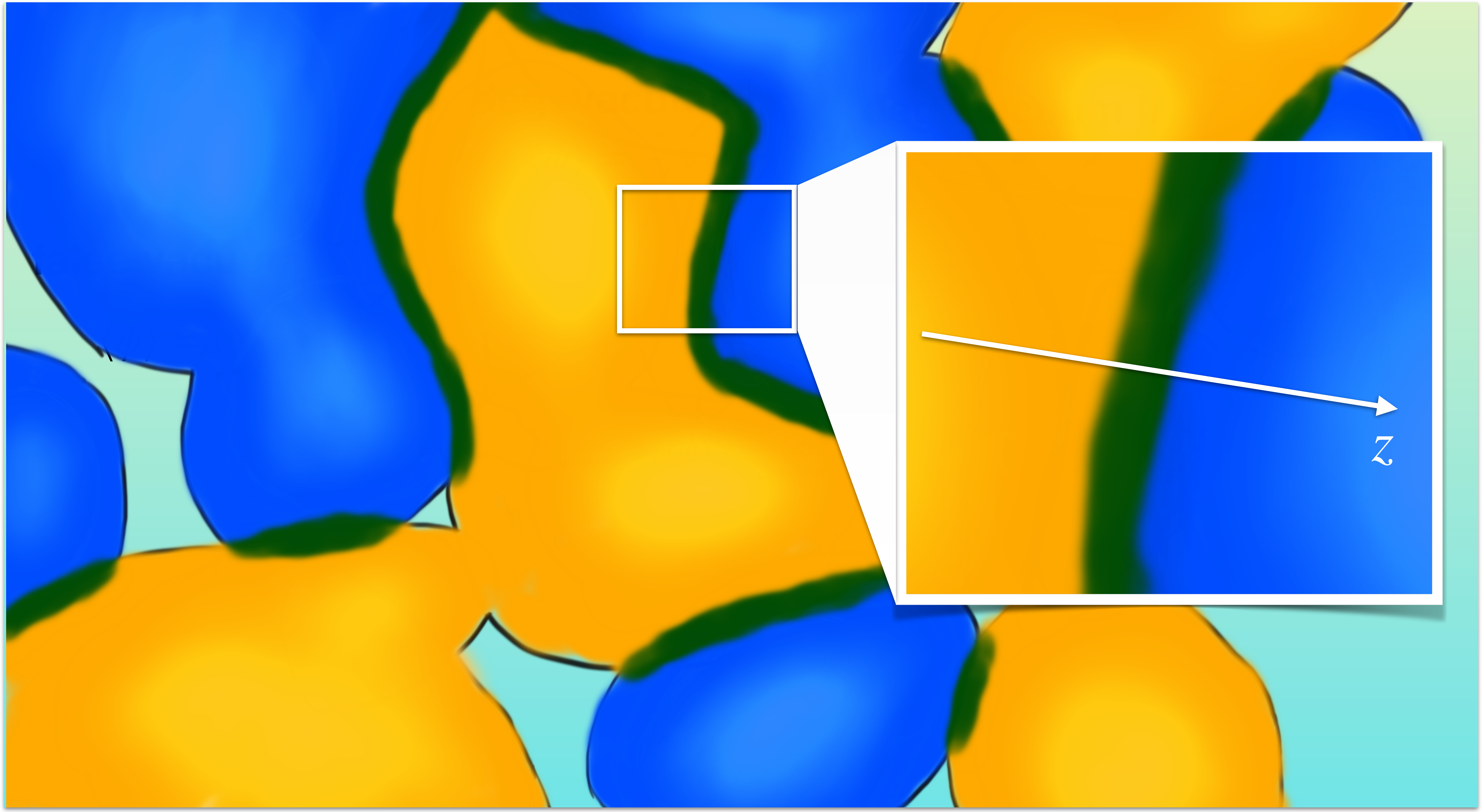


伴随着Z_2对称性自发破缺的相变



Domain walls（畴壁）： static solution of classic field in 1D



Z_2 畴壁 —— 最简单的畴壁

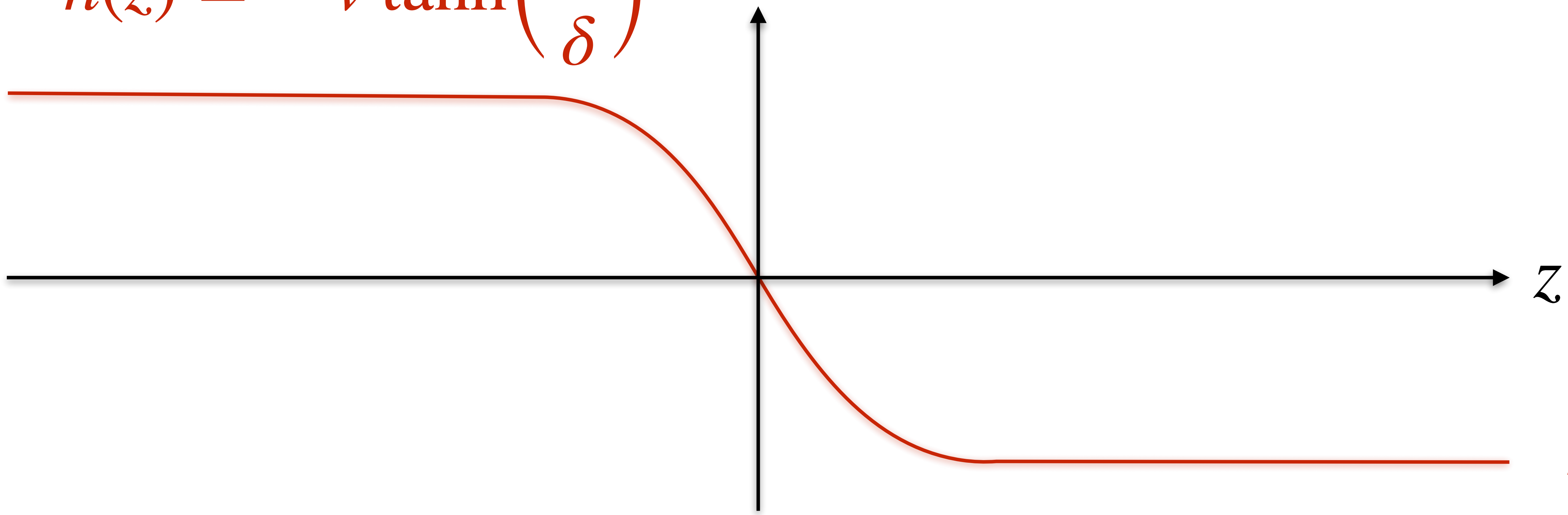
Z_2对称性下，一个实标量场的势能 $$V = -\frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4}h^4$$

场的运动方程 $$\partial^2 h + \frac{\partial V(h)}{\partial h} = 0 \Rightarrow \frac{d^2}{dz^2}h(z) = \lambda h(h^2 - v^2) \qquad v = \sqrt{\frac{\mu^2}{\lambda}}$$

时间无关、沿着z方向的经典场方程的解（一种孤子解）

$$h(z) = -v\tanh\left(\frac{z}{\delta}\right)$$

$$h|_{z\to-\infty} = +v$$



$$h|_{z\to+\infty} = -v$$

Vilenkin, Phys. Rept.121 (1985) 263

Z_2 畴壁 —— 最简单的畴壁

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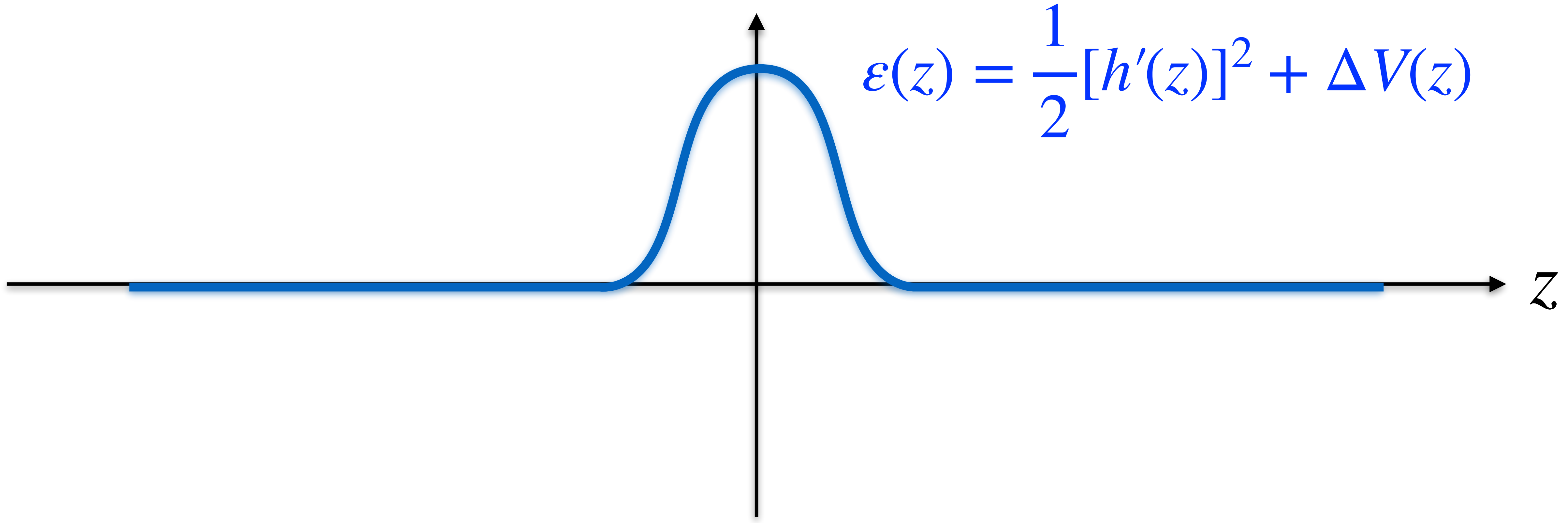
Tension / 畴壁的表面能 $\sigma = \frac{4}{3}\sqrt{\frac{\lambda}{2}}v_0^3$ $\sigma = \int_{-\infty}^{+\infty} \varepsilon(z)dz$

Thickness / 畴壁的厚度 $\delta = \sqrt{\frac{2}{\lambda v_0^2}}$



$\varepsilon(z) = \frac{1}{2}[h'(z)]^2 + \Delta V(z)$ $\Delta V = V - V_{\min}$

Vilenkin, Phys. Rept.121 (1985) 263

引入 bias term 的必要性

- 稳定的畴壁会导致宇宙学问题

$$\rho_{\rm DW} \sim \sigma H$$ (scaling solution) $$\Longrightarrow \quad \frac{\rho_{\rm DW}}{\rho_c} \sim \frac{\sigma G}{H} \sim \frac{\lambda^{1/2} v^3}{M_{\rm pl} T^2} \qquad \rho_c = \frac{3H^2}{8\pi G}$$

- 在粒子理论中，没有某个基本原理可以保证整体对称性是一个严格的对称性

- 引力、手征反常等效应会导致整体对称性的精确破坏

- Bias term: $\delta V = \epsilon v h \left(\frac{1}{3}h^2 - v^2\right)$ Hiramatsu, Kawasaki, Saikawa, 1002.1555

$\Longrightarrow$ 真空的劈裂 $(V_{\rm bias})_{10} = V|_{+v} - V|_{-v} = -\frac{4}{3}\epsilon v^4$

◆ 得足够小，使得不同（近）简并真空能同时产生，并使畴壁可以存在一段时间

◆ 又不能太小，畴壁不能太稳定，它们得在BBN之前崩塌，避免跟宇宙观测相矛盾

- 畴壁会产生引力波，频谱的峰值对应畴壁的崩塌（由bias导致）

超越 Z_2 畴壁 —— 动机

- 强CP问题，$U(1)_{\rm PQ}$ 破缺后可能会导致 Z_N

P. Sikivie, 1982

- 超对称中可能引入的分立对称性

比如 NMSSM中的 Z_3 对称性

Review in Chung, Everett, Kane, King, Lykken, Wang, 0312378

- Abel的分立味对称性 Z_N
- 非Abel的分立味对称性 $A_4, S_4, \ldots$
- ……

Reviews e.g.,
Altarelli, Feruglio, 1002.0211
King, Luhn, 1301.1340;
Xing, 1909.09610;
Feruglio, Romanino, 1912.06028
……

$Z_N\ (N>2)$ 及其真空结构

Z_N对称性下，一个 复标量场 $\phi=(h+ia)/\sqrt{2}$ 的势能

$$V=-\mu^2|\phi|^2+\lambda_1|\phi|^4-\lambda_2\mu^{4-N}(\phi^N+\phi^{*N})$$

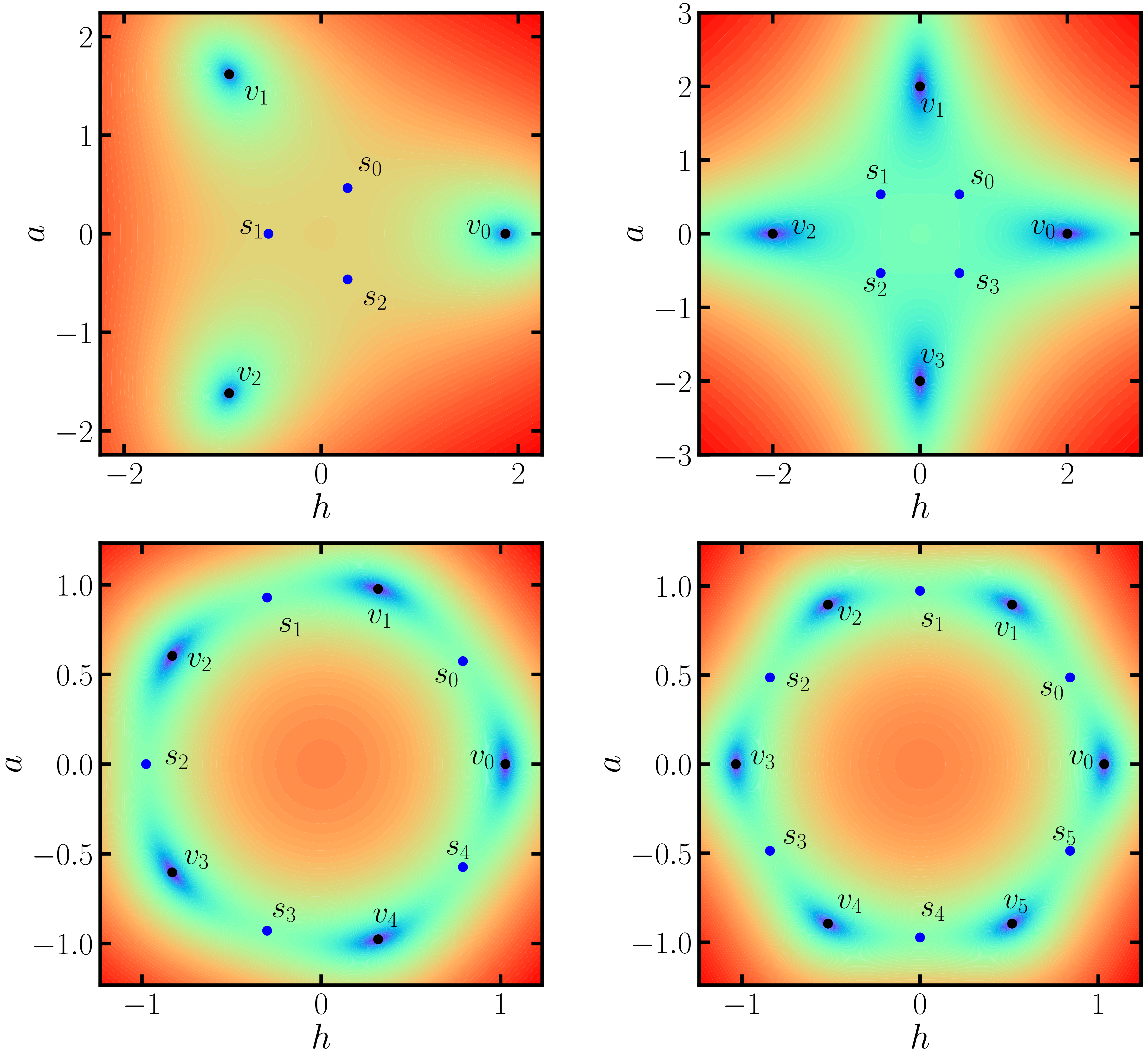
(为了简化讨论，假设没有CP破坏)

N 个简并真空:

$$v_k=v_0e^{i2\pi\frac{k}{N}}$$

$$k=0,1,\ldots,N-1$$

吴永成, 谢柯盼,
YLZ, 2205.11529

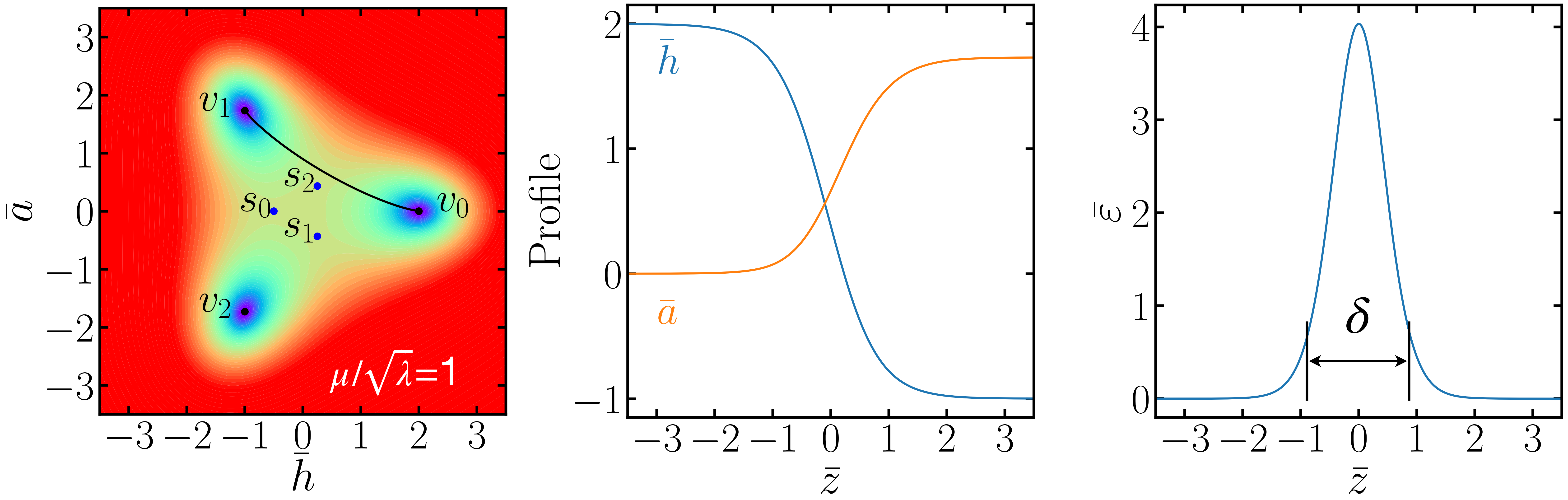


Z_3 畴壁

Z_3 对称性的势能 $V = -\mu^2|\phi|^2 + \lambda_1|\phi|^4 - \lambda_2\mu(\phi^3 + \phi^{*3})$ $\beta = 3\lambda_2/\sqrt{8\lambda_1} > 0$

3 个真空 $v_k = v_0\, e^{i2\pi k/3}$ $k = 0,1,2$ $v_0 = \dfrac{\mu}{\sqrt{2\lambda_1}}(\beta + \sqrt{1+\beta^2})$

$\beta = 3/4$



v_0	v_1

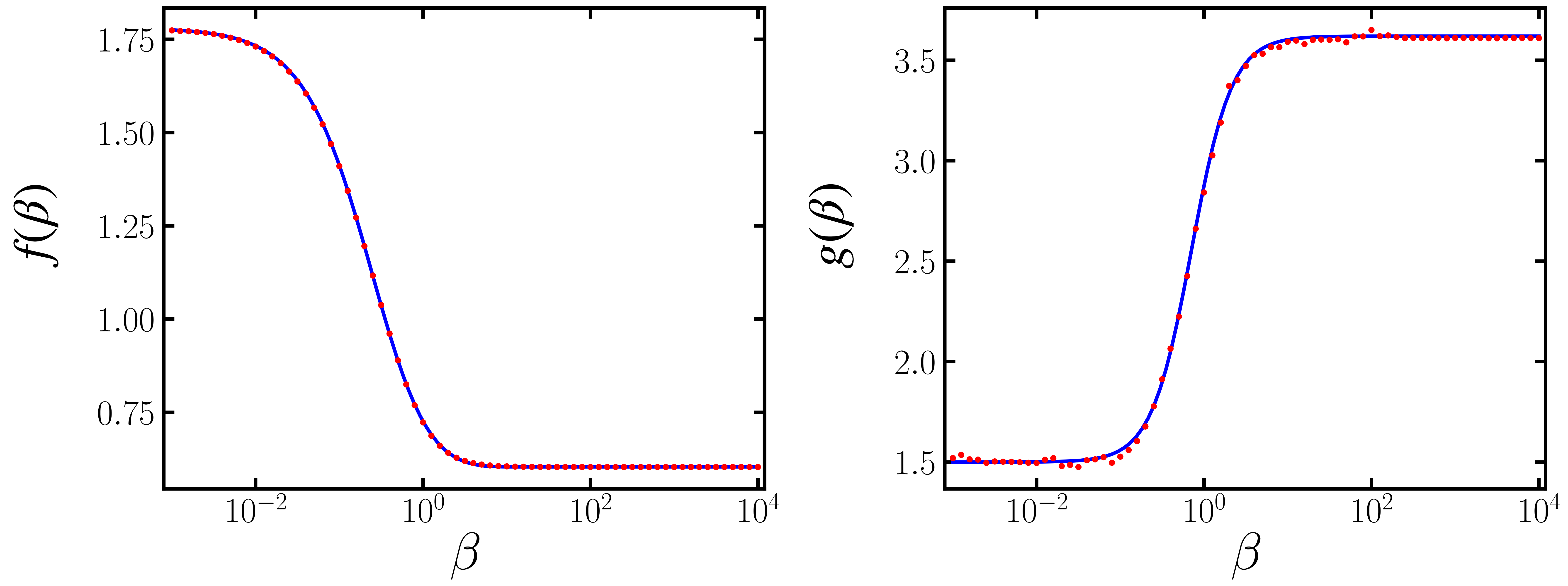
$$\varepsilon(z) = \frac{1}{2}\{[h'(z)]^2 + [a'(z)]^2\} + \Delta V(z)$$

表面能 $\Rightarrow$ $\sigma = \int_{-\infty}^{+\infty} \varepsilon(z)dz$ 厚度 $\Rightarrow$ $\int_{-\delta/2}^{\delta/2} dz\varepsilon(z) = 64\% \times \sigma\,.$

Z_3 畴壁

Tension $\sigma = m_a v_0^2 f(\beta)$

Thickness $\delta = m_a^{-1} g(\beta)$



$$f(\beta) = 0.604 + \frac{0.234}{e^{0.826\beta} + 0.435\beta^2 - 0.801}$$

$$g(\beta) = 3.62 - \frac{2.12}{1 + 1.85\beta^{1.81}}$$

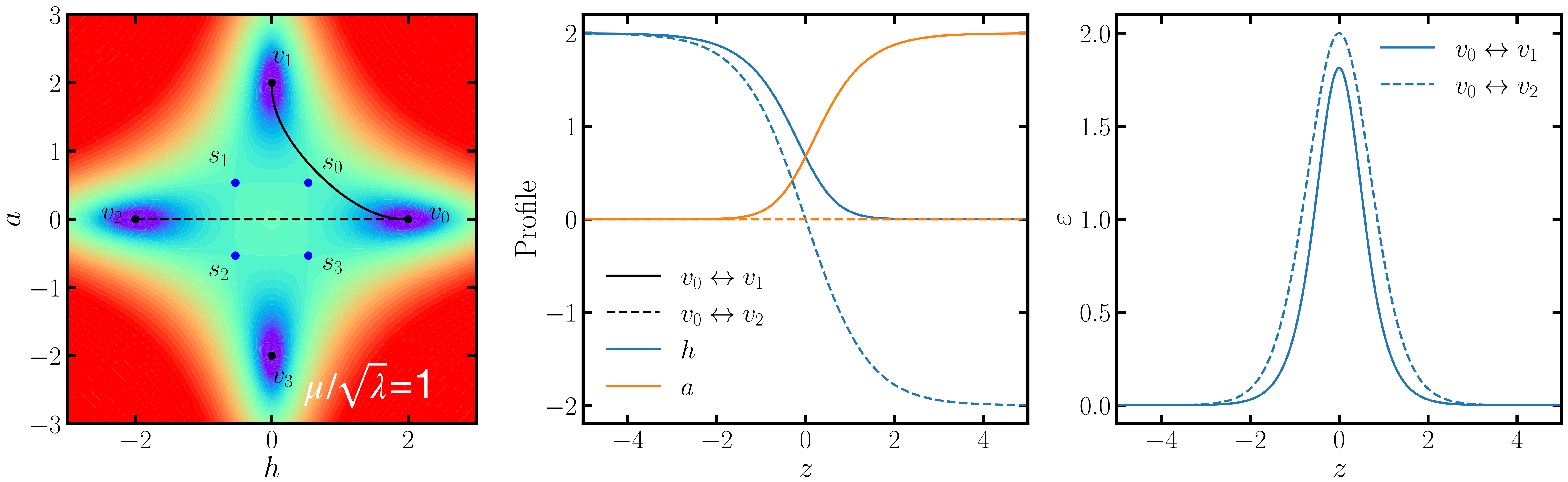
m_a mass of pseudo Nambu-Goldstone boson

Z_4 畴壁

Z4-invariant potential $V = -\mu^2|\phi|^2 + \lambda_1|\phi|^4 - \lambda_2(\phi^4 + \phi^{*4})$ $\beta \equiv 2\lambda_2/\lambda_1$

$v_k = v_0 e^{i\frac{2\pi}{4}k}$ $k = 0,1,2,3$ $v_0 = \dfrac{\mu}{\sqrt{2\lambda_1(1-\beta)}}$

$\beta = 3/4$



Adjacent walls: separating adjacent walls in the field space

e.g., that separating v_0 and v_1

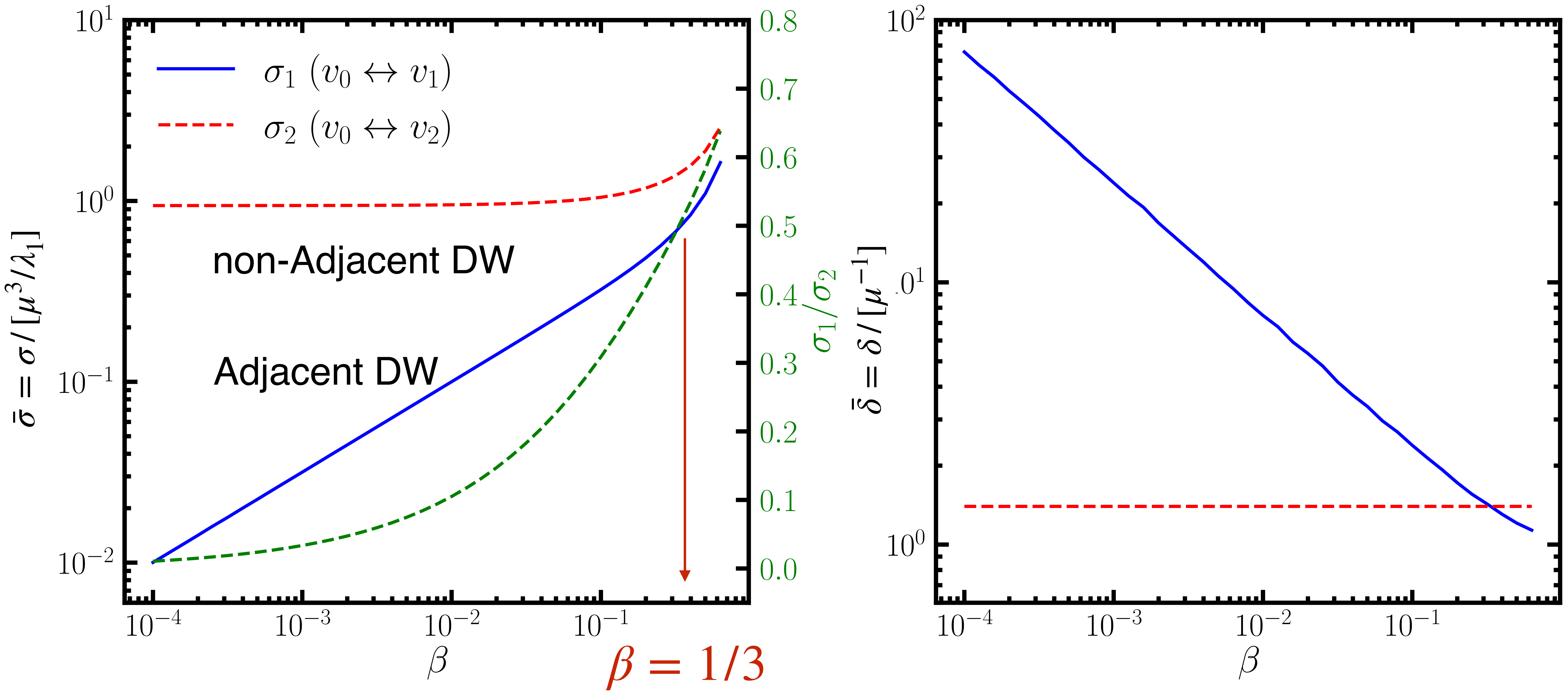
v_0	v_1

Non-adjacent walls： separating non-adjacent walls

e.g., that separating v_0 and v_2

v_0	v_2

Z_4 畴壁



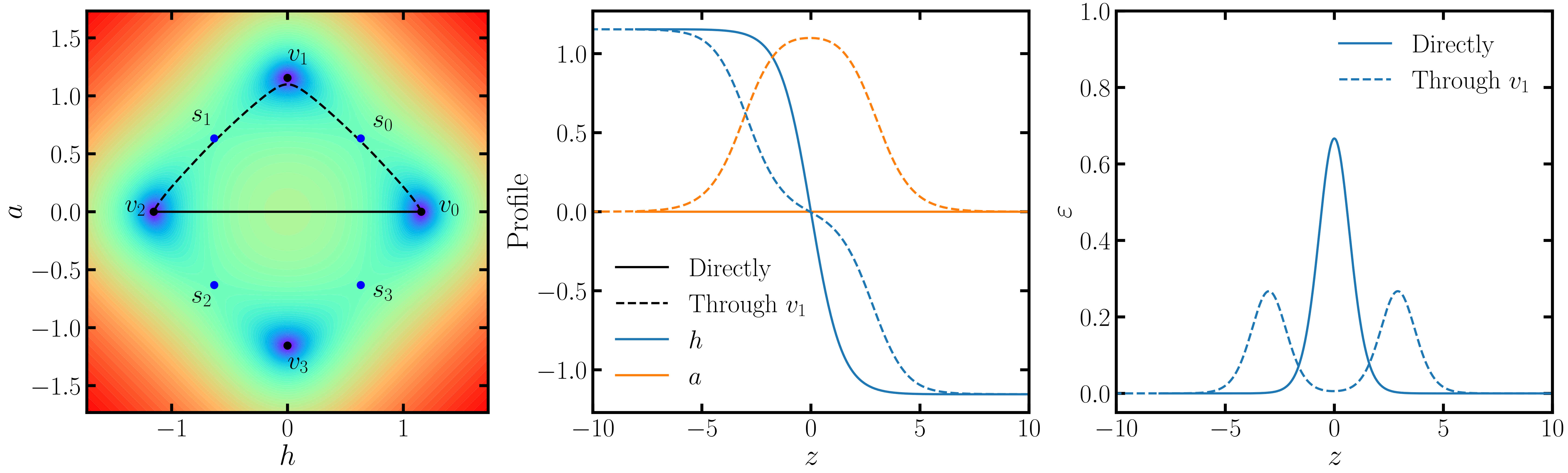
$\beta = 1/3$

$\Downarrow$

$$\bar{\sigma}_{\text{non-adj}} = 2\,\bar{\sigma}_{\text{adj}} = \sqrt{2}$$

Z_4 畴壁

$\beta = 1/4$

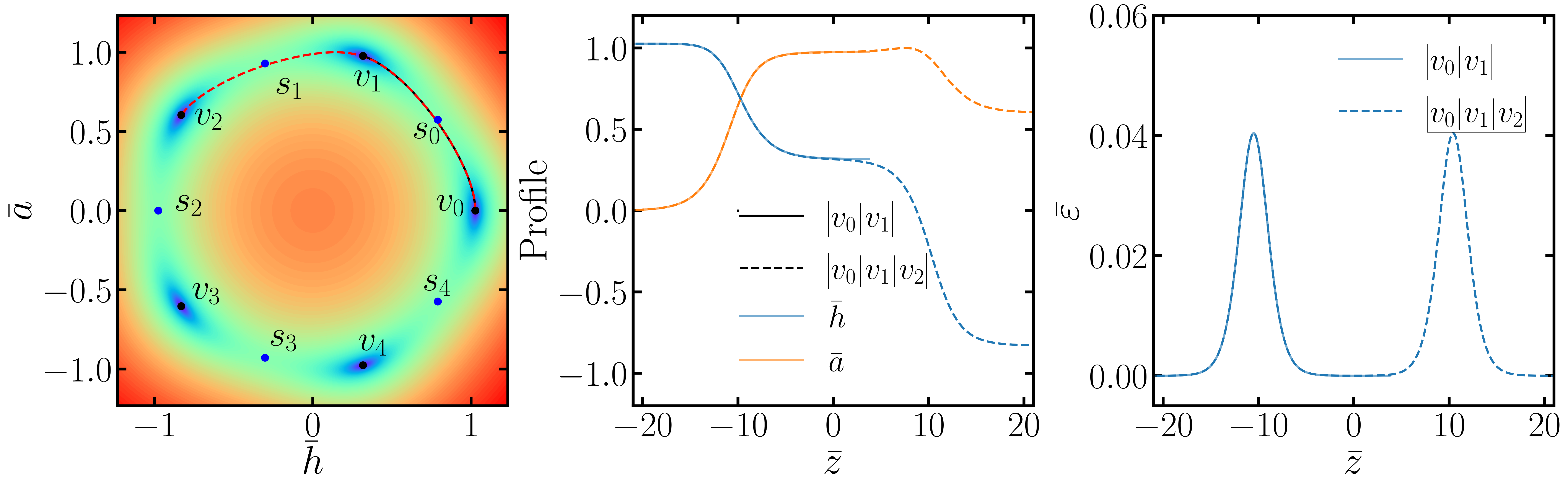


For $\beta < 1/3$, $\sigma_2 > 2\sigma_1$, non-Adjacent DWs are unstable, decaying to two adjacent DWs

$$\boxed{v_0 \mid v_2} \Rightarrow \boxed{v_0 \mid v_1 \mid v_2} \Rightarrow \boxed{v_0 \mid v_1} + \boxed{v_1 \mid v_2}$$

更复杂的 Z_N 畴壁 （$N > 4$）

$\phi^N (N > 4)$ 不可重整



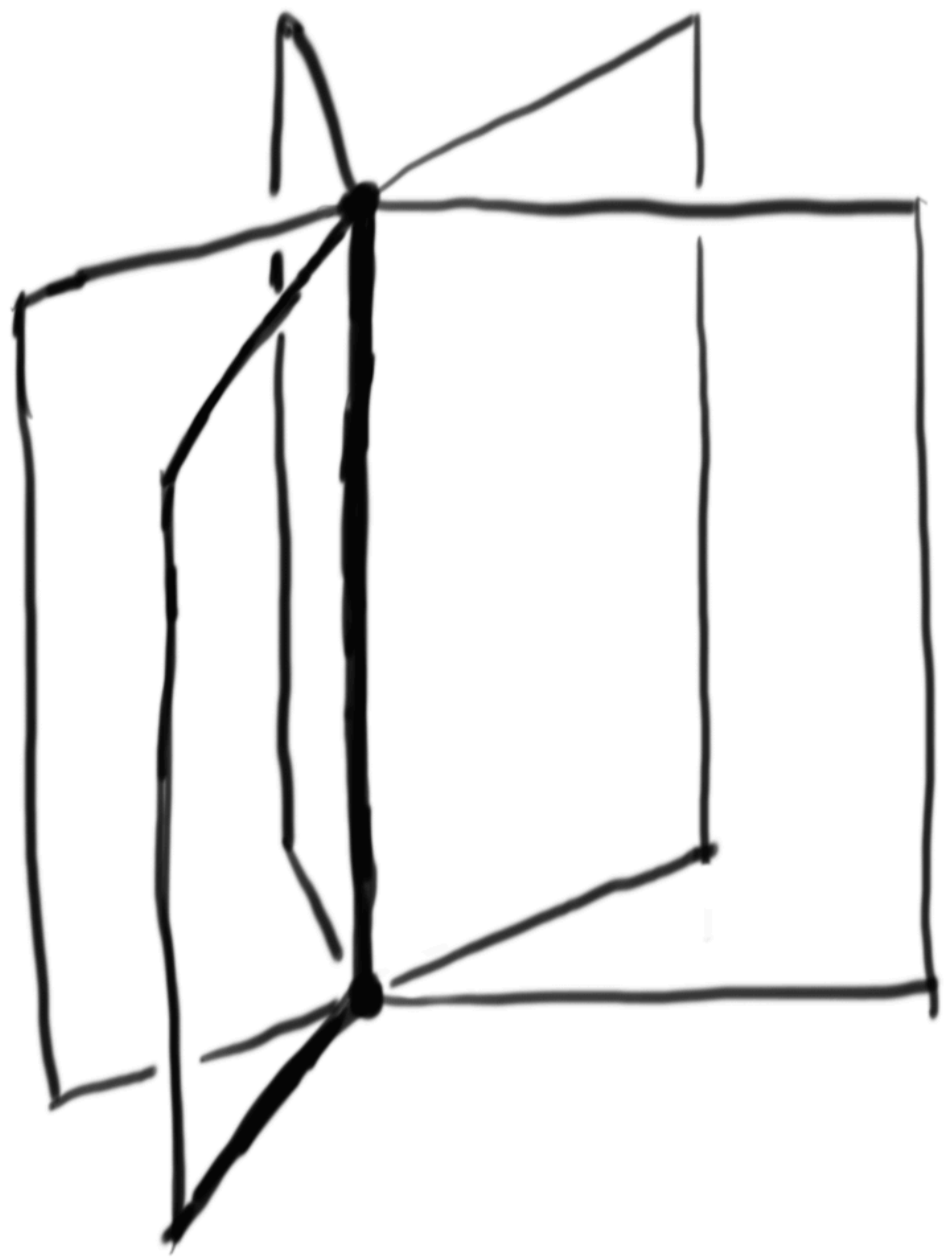
Approx U(1) $\phi = |\phi| e^{i\theta}$

$\downarrow \langle |\phi| \rangle \approx v$

Z_N

$\downarrow \langle \theta \rangle = 2\pi k / N$

1



String-bounded walls

Similar to axion domain walls, e.g., Hiramatsu, Kawasaki, Saikawa, Sekiguchi, 1207.3166; 1412.0789

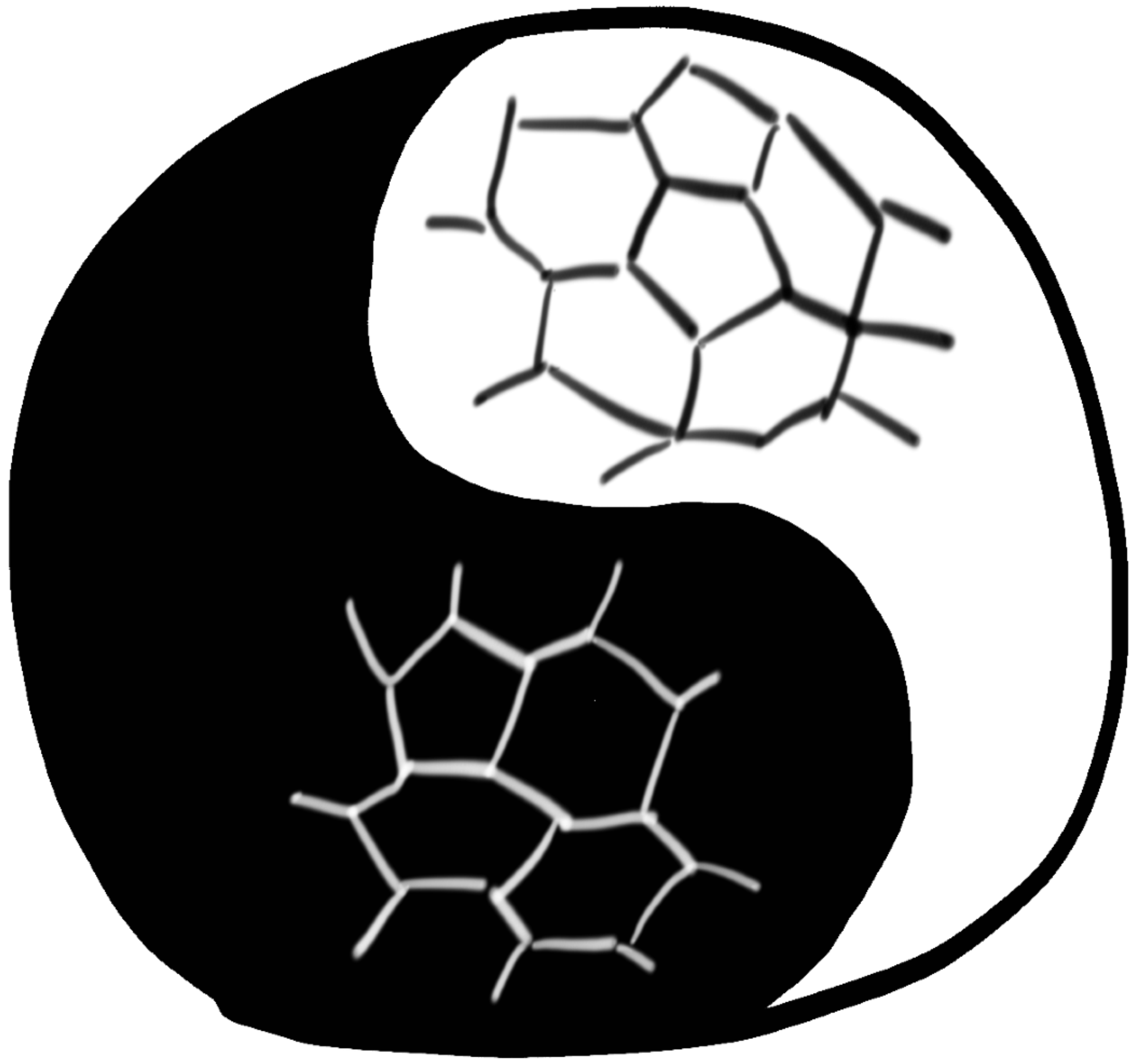
Z_N 畴壁与多标量场

比如 Z_6 对称性下的双标量场

Scalar	ϕ	ξ
Z_6 charge	1	3

$$V = -\mu^2|\phi|^2 + \lambda_1|\phi|^4$$
$$-\frac{1}{2}\mu_\xi^2\xi^2 + \frac{1}{4}\lambda_\xi\xi^4$$
$$-\lambda_{\phi\xi}(\phi^3 + \phi^{*3})\xi$$

$$Z_6 \xrightarrow{\langle\xi\rangle = \pm\sqrt{\mu_\xi^2/2\lambda_\xi}} Z_3 \xrightarrow{\langle\phi\rangle = \frac{\mu(\beta+\sqrt{1+\beta^2})}{\sqrt{2\lambda_1}} e^{\pm i2\pi k/3}} 1$$



Walls wrapped by walls

Abel畴壁的分类

不完整的清单

吴永成, 谢柯盼, YLZ, 2205.11529

Potential forms		breaking chains	textures of domain walls	
single scalar	large ϕ^N	$Z_N \to 1$	adj. walls	non-adj. walls ($N \geqslant 4$)
	small ϕ^N	appr. $U(1) \to Z_N \to 1$	string-bounded adj. walls	
multiscalar (ϕ, ξ with charges q_ϕ, q_ξ)	C1	appr. $U(1) \to Z_N \to 1$	string-bounded adj. walls	
	C2	$Z_N \to Z_{\gcd(q_\xi,N)} \to 1$	walls wrapped by walls	
	C3	$Z_N \to \begin{cases} Z_{\gcd(q_\xi,N)} \\ Z_{\gcd(q_\phi,N)} \end{cases}$	walls blind among diff. types	

(gcd: 最大公约数

C1) q_ϕ 和 q_ξ 均与 N 互质, i.e., gcd(q_ξ, N) = gcd(q_ϕ, N) = 1.

C2) q_ξ 与 N 有非平庸公约数， q_ϕ 仍与 N 互质, i.e., gcd(q_ξ, N) > 1 and gcd(q_ϕ, N) = 1.

C3) q_ϕ 和 q_ξ 都与 N 有非平庸的公约数, i.e., gcd(q_ϕ, N), gcd(q_ξ, N) > 1.

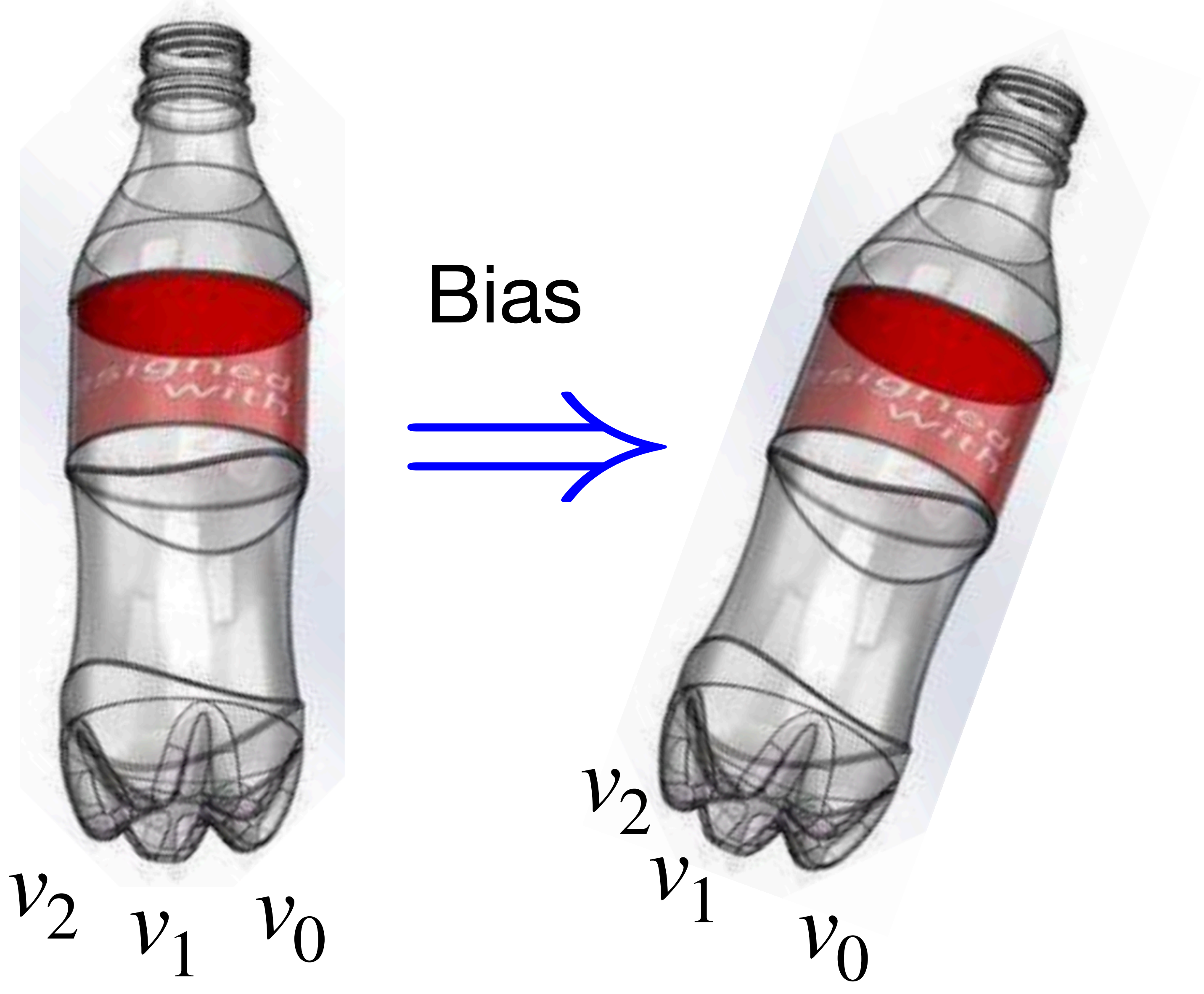
Bias term 与引力波：以 Z_3 为例

GW spectrum with broken power laws based on Saikawa [1703.02576]

$$\delta V = \frac{2e^{i\alpha}}{3\sqrt{3}}\epsilon\phi\left(\frac{1}{4}\phi^3 - v_0^3\right) + \mathrm{h.c.}$$

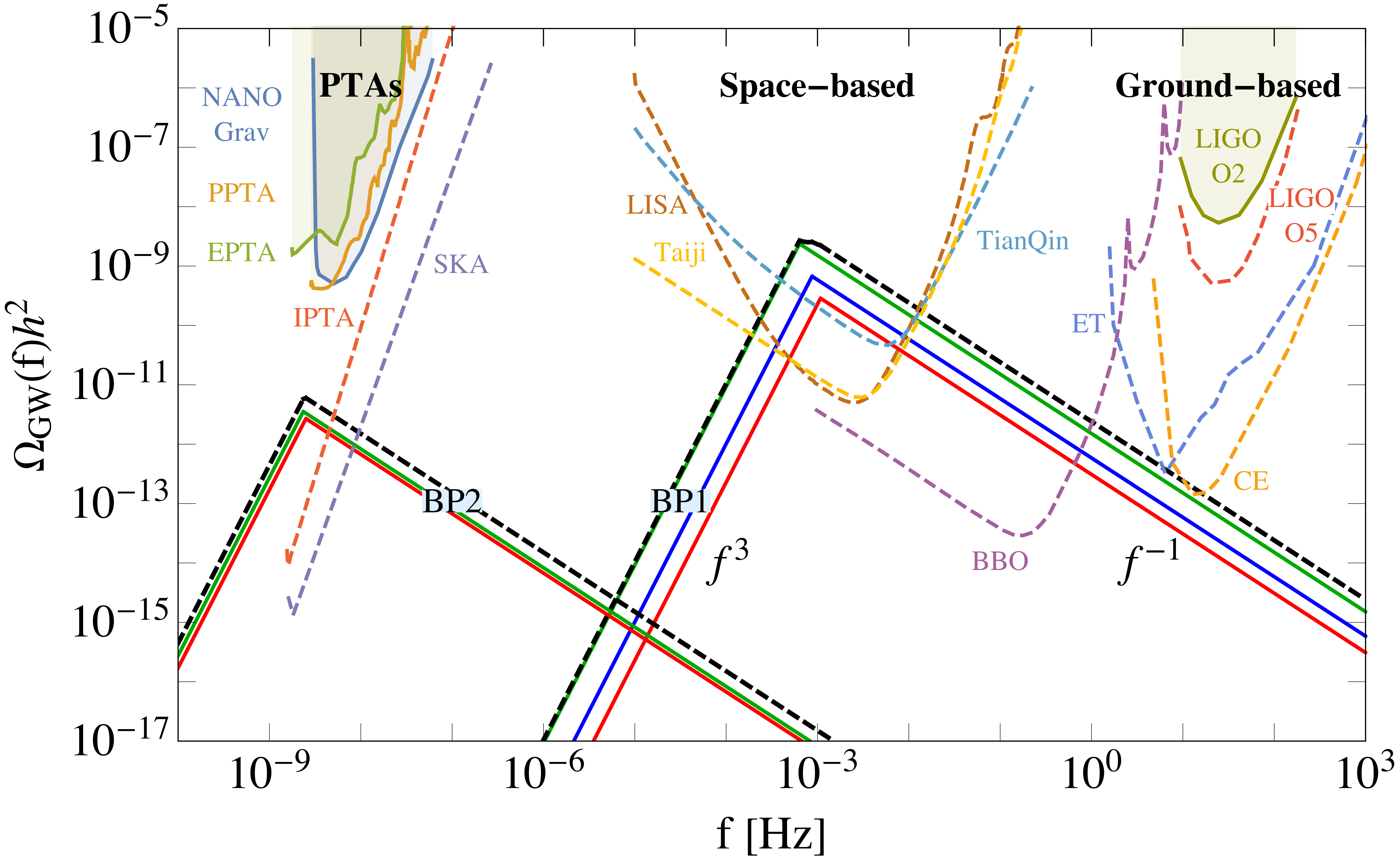
$$(V_{\rm bias})_{10} = V|_{v_1} - V|_{v_0} = \epsilon v_0^4 \cos\left(\alpha + \frac{\pi}{6}\right)$$

$$(V_{\rm bias})_{20} = V|_{v_2} - V|_{v_0} = \epsilon v_0^4 \cos\left(\alpha - \frac{\pi}{6}\right)$$



简并真空

简并真空劈裂为三个“能级”

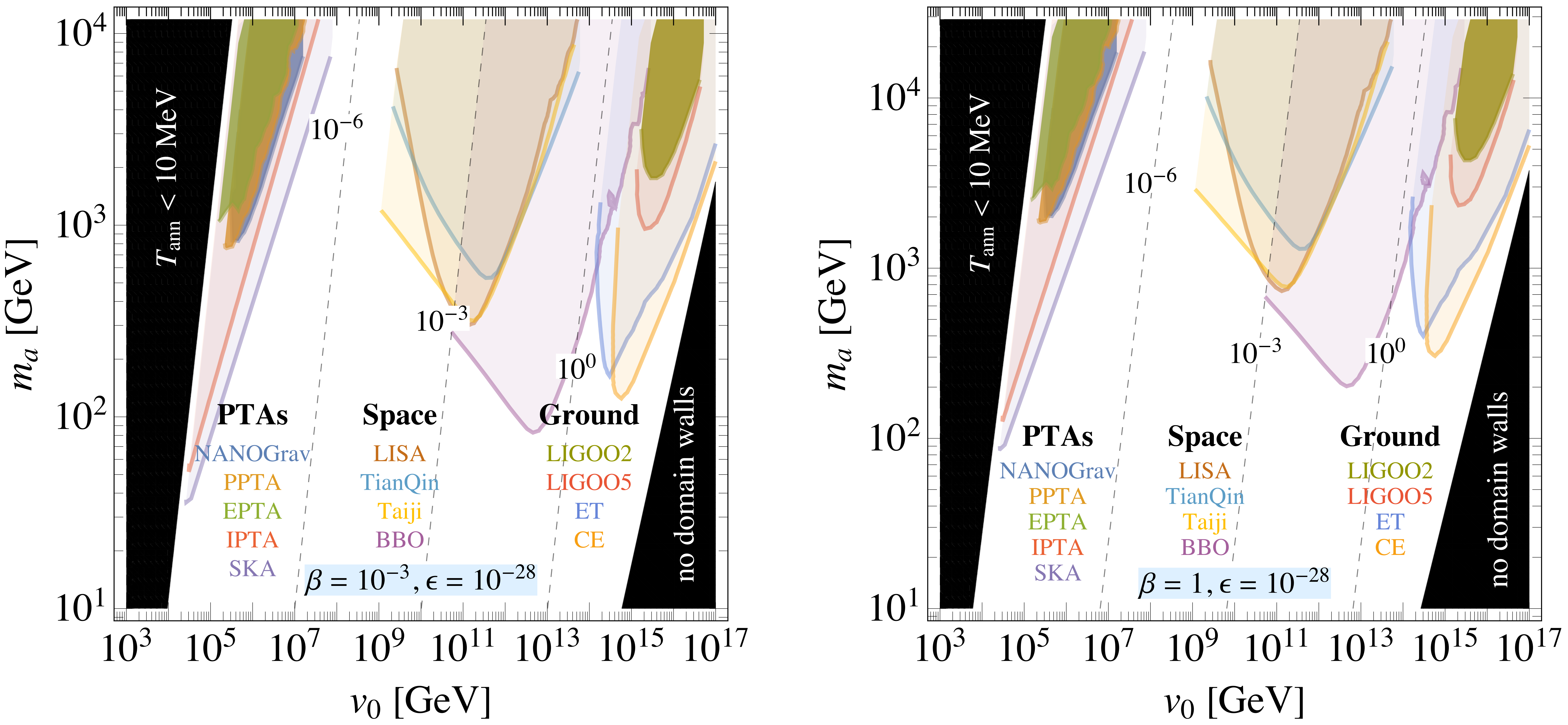


$$\mathrm{BP1}: v_0 = 10^{11}\ \mathrm{GeV},\quad m_a = 2\ \mathrm{TeV},\quad \alpha = \frac{2\pi}{9};$$

$$\mathrm{BP2}: v_0 = 10^{5}\ \mathrm{GeV},\quad m_a = 500\ \mathrm{GeV},\quad \alpha = \frac{\pi}{27}.$$

吴永成, 谢柯盼, YLZ, 2204.04374

Z_N 畴壁在引力波实验中的可检验性



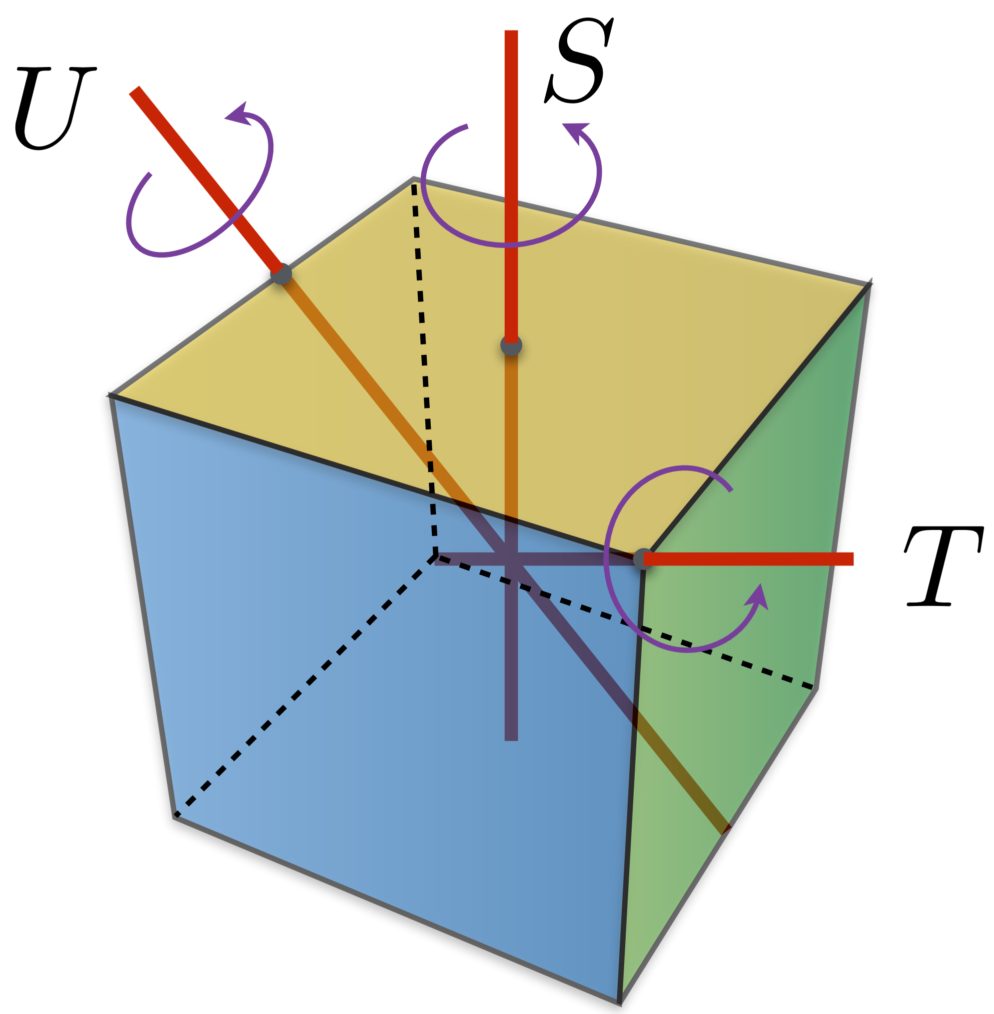
- 考虑到 Z_3 畴壁与 Z_2 畴壁动力学的不同，我们预期可能存在一个不同的引力波频谱
- 一个定量的引力波频谱需要做格点模拟，目前还没有这方面的结果

非Abel畴壁

- 对称性：假设为立方体群 S_4 （也是4个客体的置换群）

三维不可约表示$\mathbf{3}'$下的表示矩阵

$$T=\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}\qquad S=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-1\end{pmatrix}\qquad U=\begin{pmatrix}-1&0&0\\0&0&-1\\0&-1&0\end{pmatrix}$$



$$T^3=S^2=(ST)^3=\mathbf{1}$$

$$U^2=(SU)^2=(TU)^2=(STU)^4=\mathbf{1}$$

不可约表示: $\mathbf{1},\mathbf{1}',\mathbf{2},\mathbf{3},\mathbf{3}'$

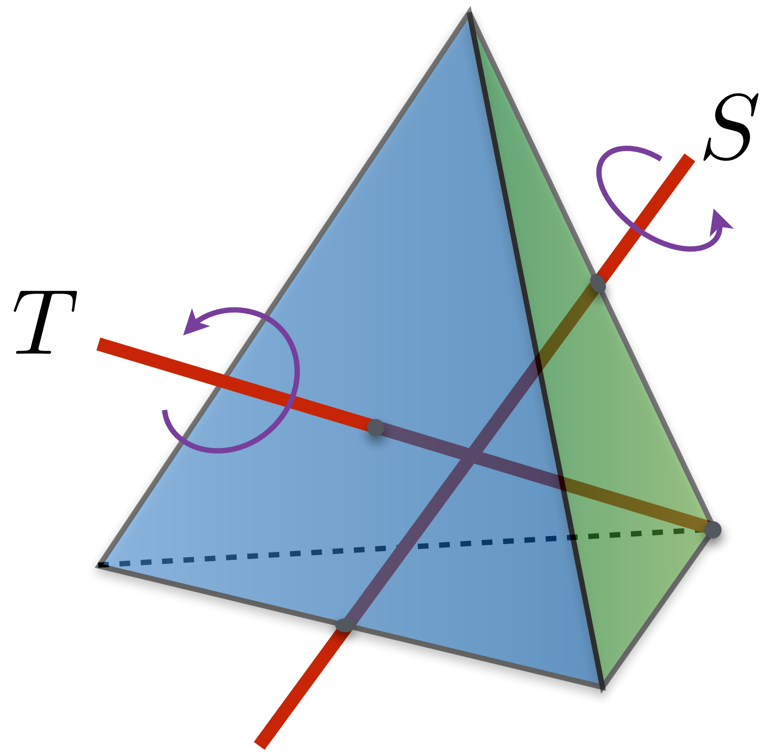
- 可重整的势能

$$V(\phi)=-\frac{\mu^2}{2}I_1+\frac{g_1}{4}I_1^2+\frac{g_2}{2}I_2\qquad g_1>0\quad g_2>-4g_1$$

$$I_1=\phi_1^2+\phi_2^2+\phi_3^2$$

$$I_2=\phi_1^2\phi_2^2+\phi_2^2\phi_3^2+\phi_3^2\phi_1^2$$

也适用于 $A_4\times Z_2^P\ (\phi\leftrightarrow-\phi)$



非Abel畴壁

- S_4 的真空结构

$g_2 > 0$ 保留 Z_2 的剩余对称性

$$v_m \in \left\{ \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}, \begin{pmatrix}-1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\-1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\-1\end{pmatrix} \right\} v$$

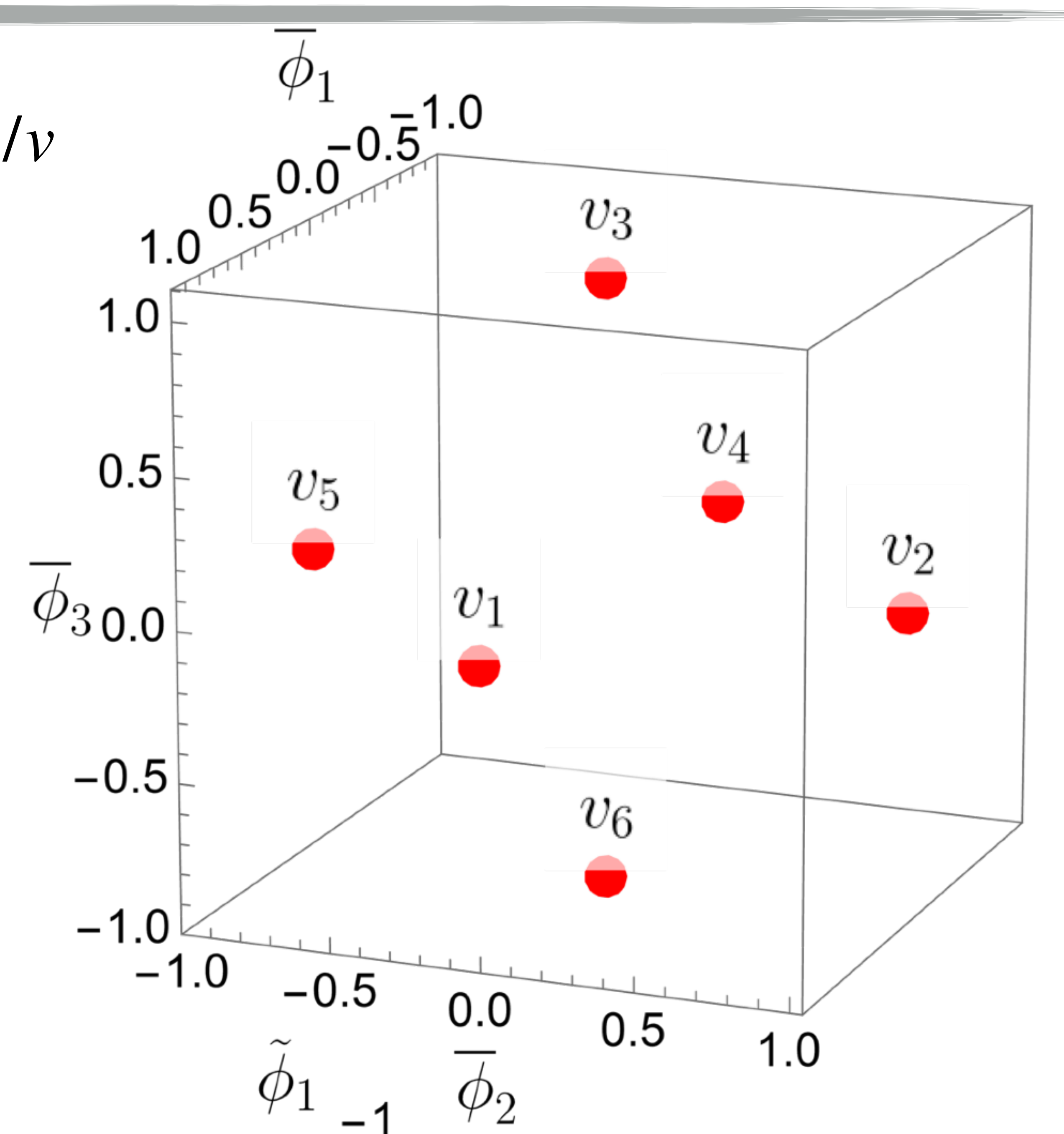
$$m = 1,2,3,4,5,6$$

$g_2 < 0$ 保留 Z_3 的剩余对称性

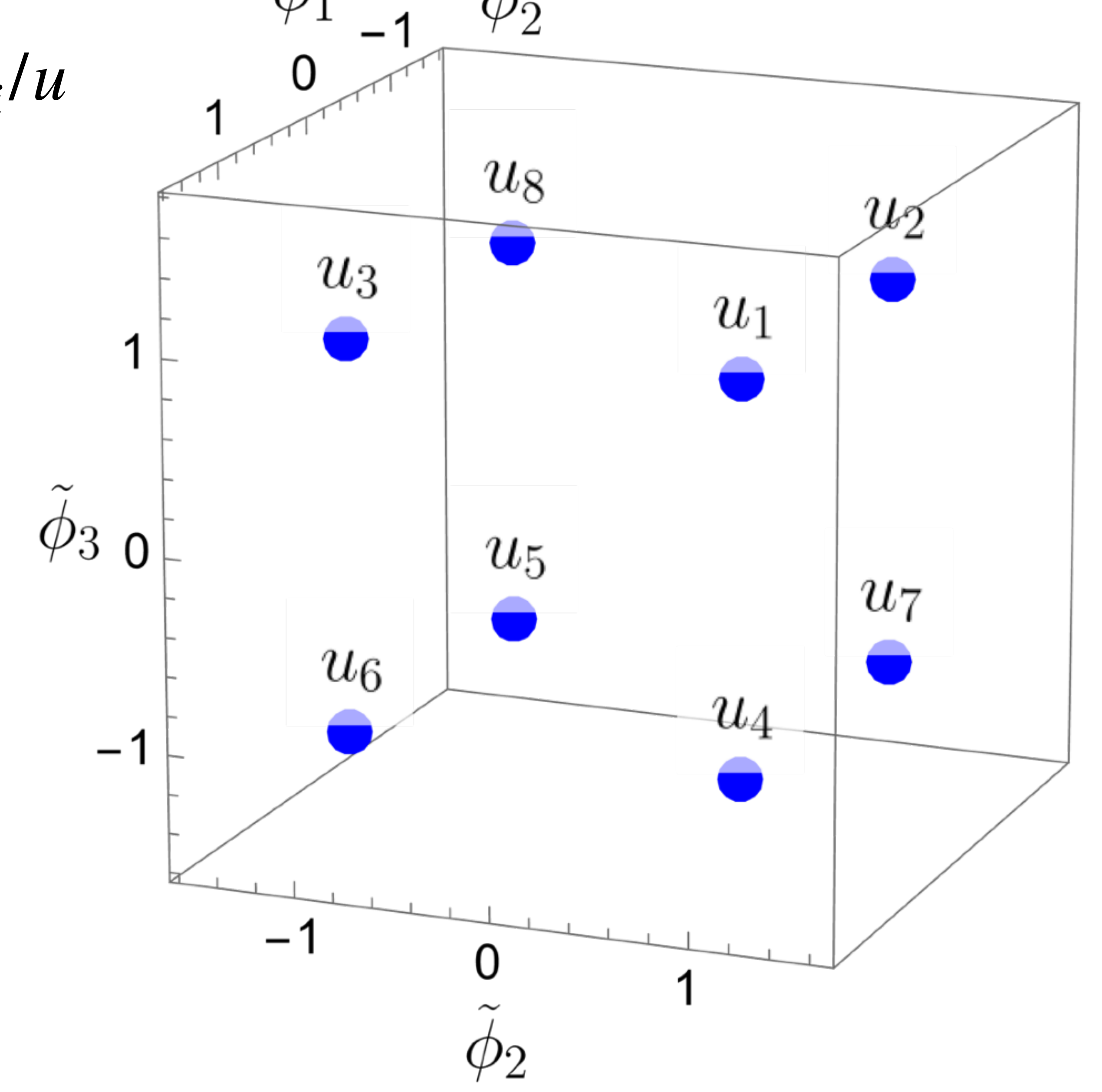
$$u_n = \left\{ \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}-1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\-1\end{pmatrix}, \begin{pmatrix}-1\\-1\\-1\end{pmatrix}, \begin{pmatrix}1\\-1\\-1\end{pmatrix}, \begin{pmatrix}-1\\1\\-1\end{pmatrix}, \begin{pmatrix}-1\\-1\\1\end{pmatrix} \right\} u$$

$$n = 1,2,3,4,5,6,7,8$$

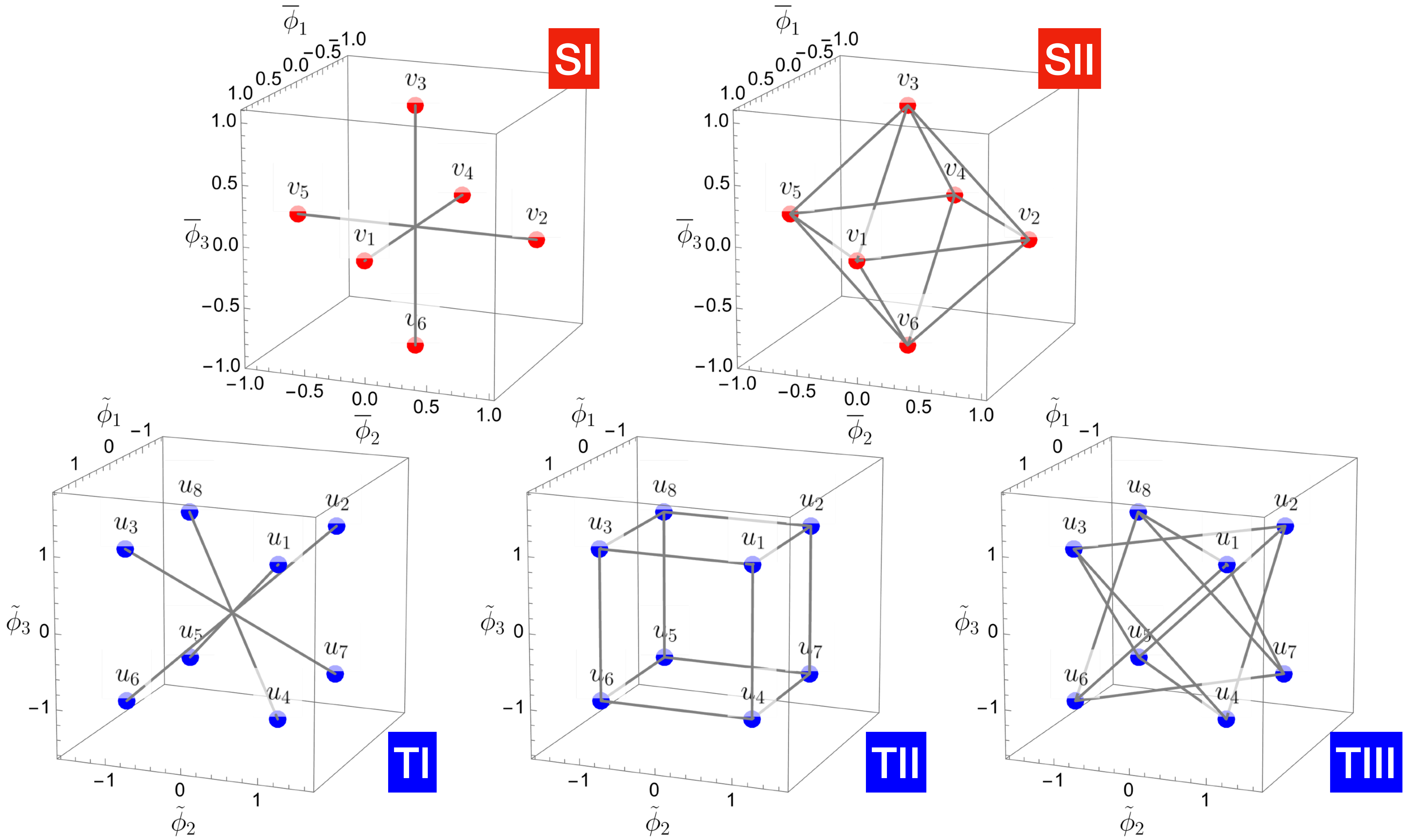
$\bar{\phi}_i = \phi_i / v$



$\tilde{\phi}_i = \phi_i / u$

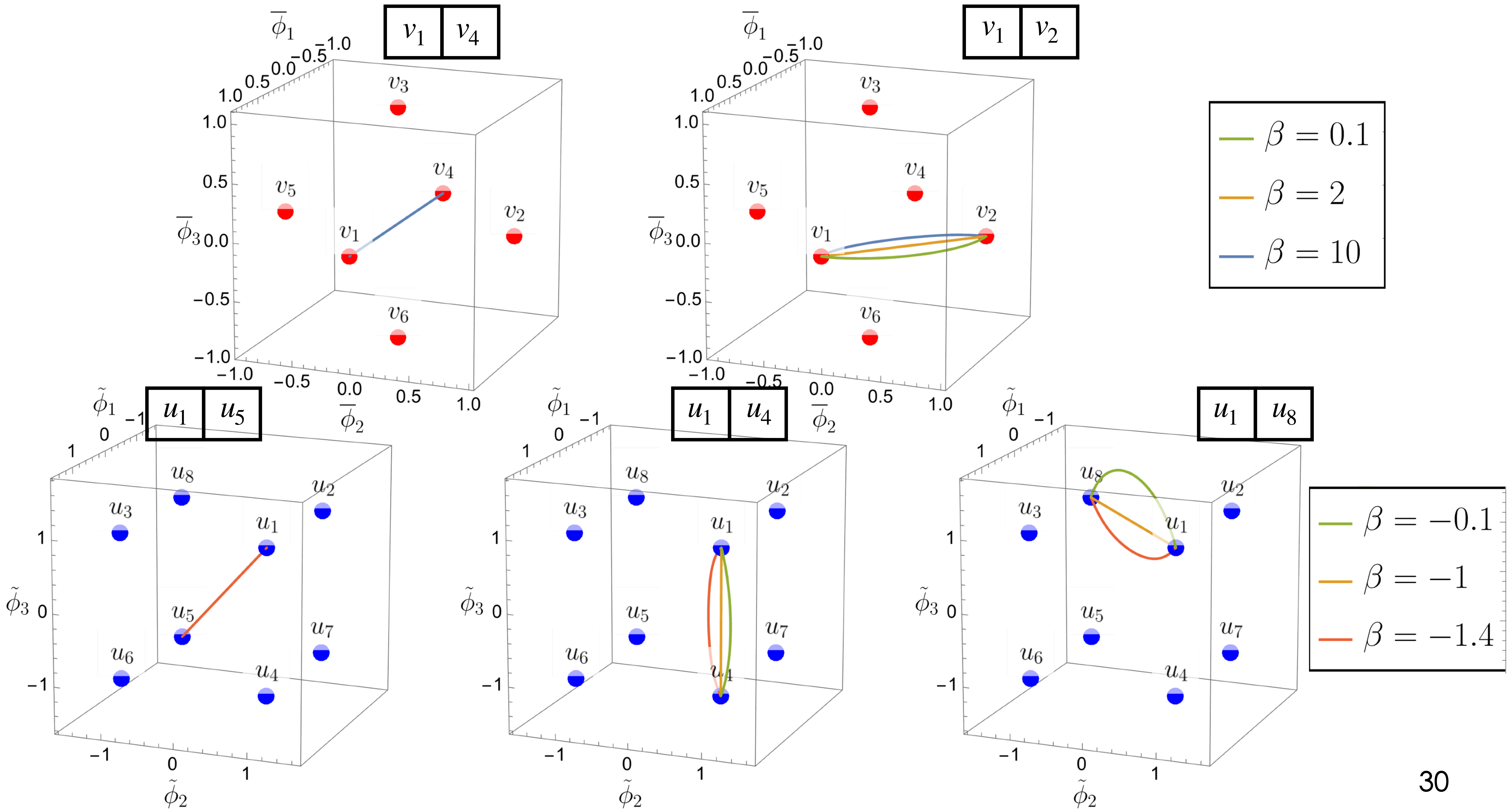


S_4 破缺得到的所有畴壁 $\Rightarrow$ 五大类



畴壁的解

付博文, S. King, L. Marsili, S. Pascoli, J. Turner, YLZ, 2409. 2409.16359



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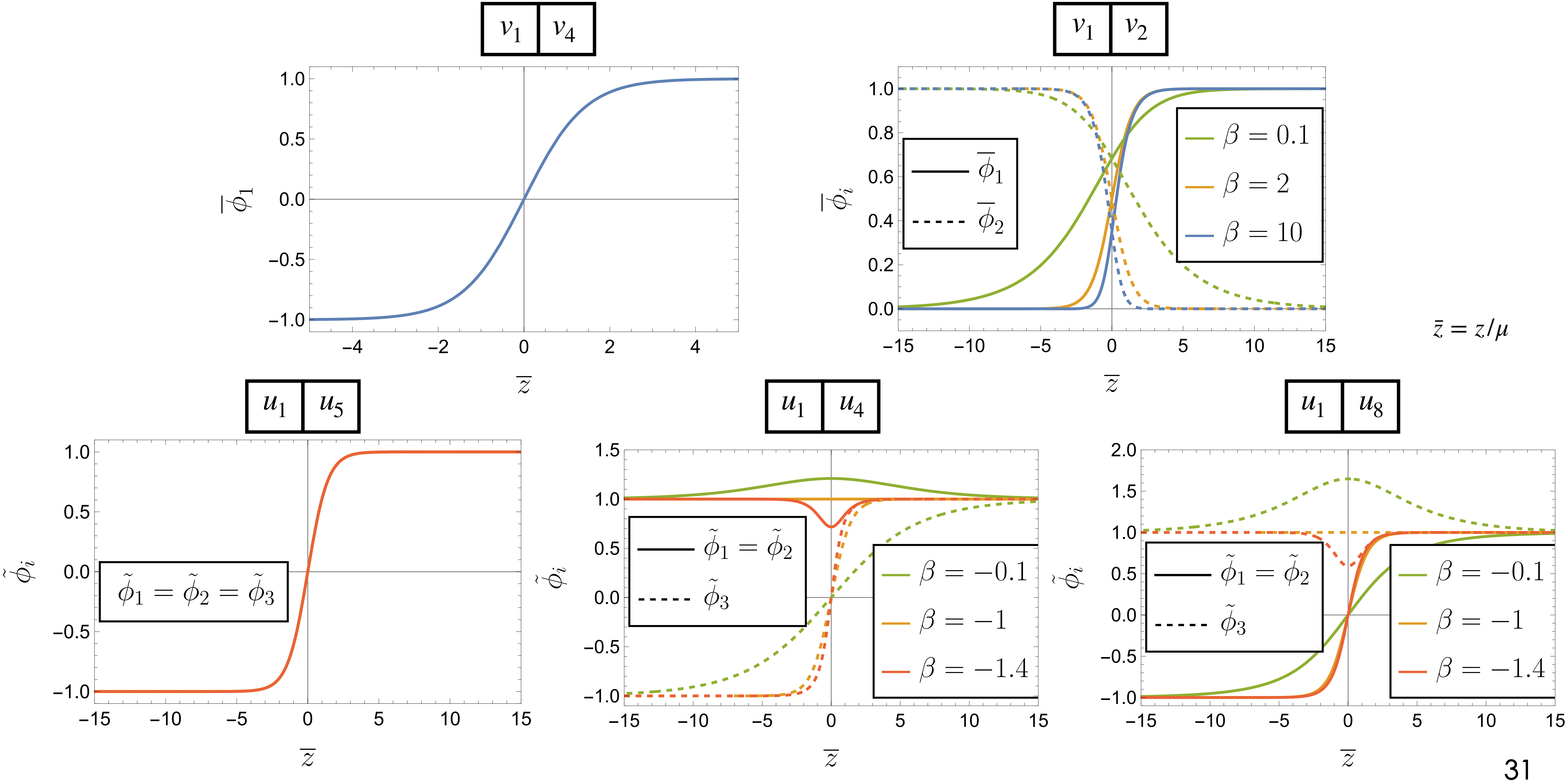
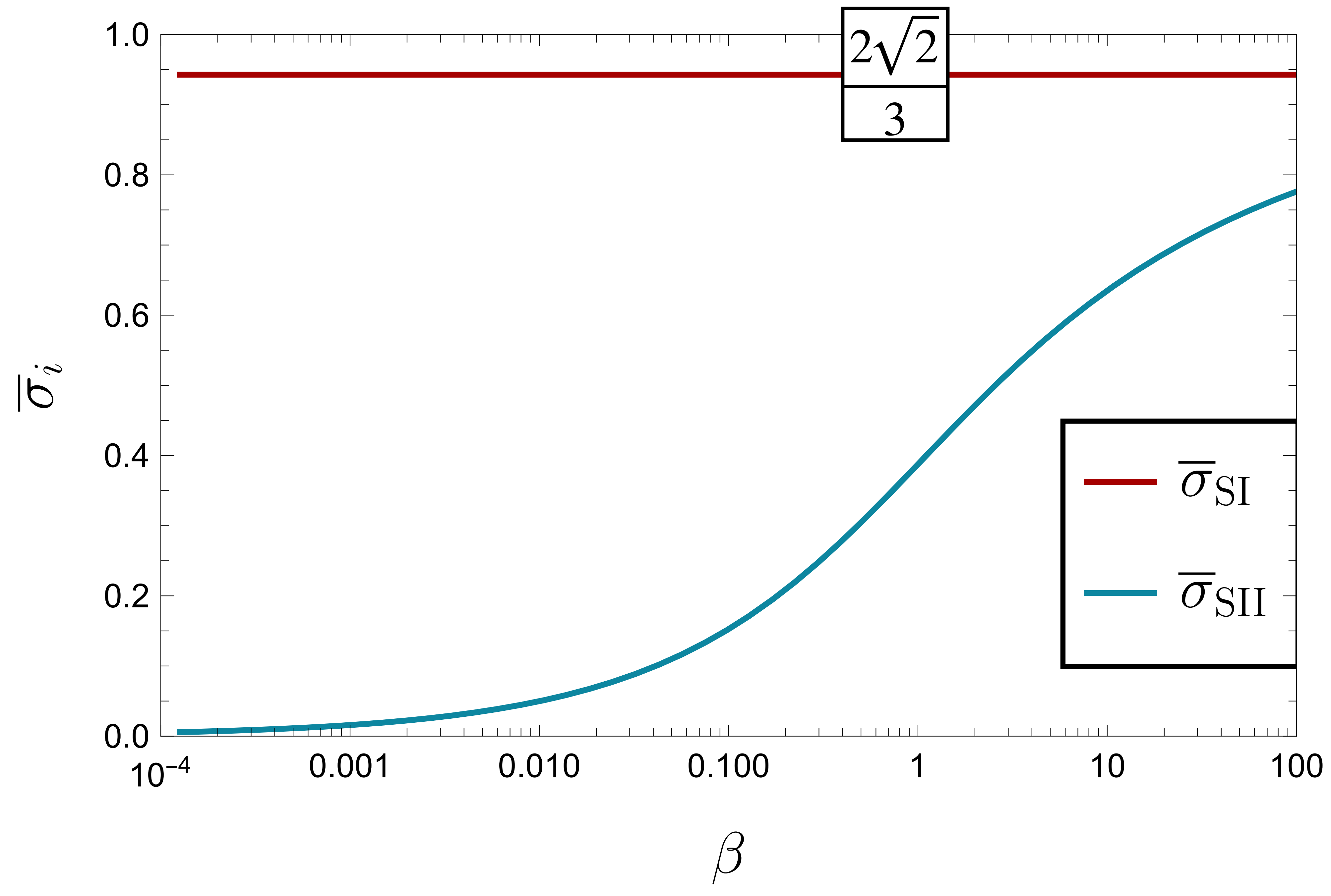


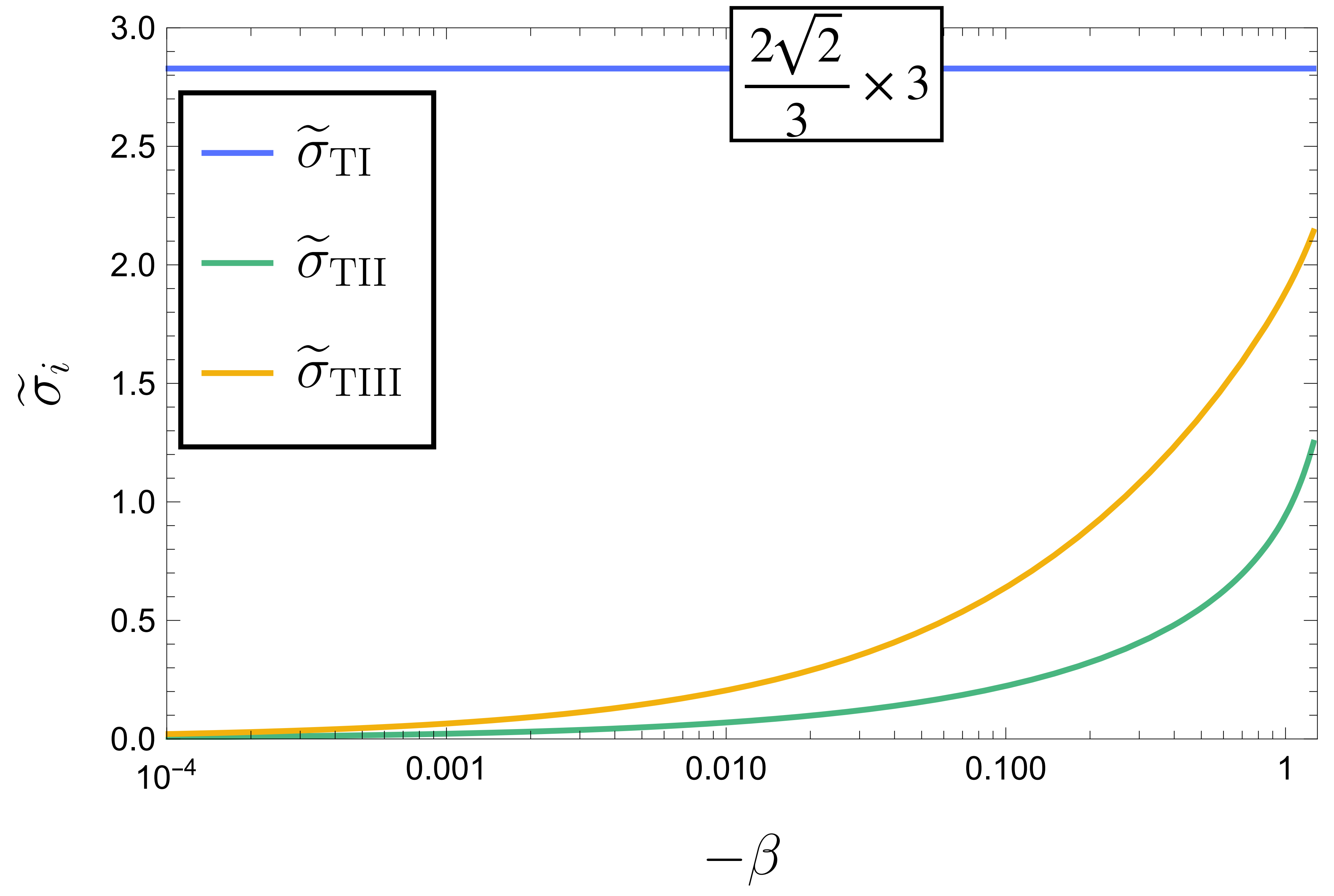
畴壁的解

付博文, S. King, L. Marsili, S. Pascoli, J. Turner, YLZ, 2409. 2409.16359

$$\bar{\sigma}_i = \frac{\sigma_i}{\mu v^2}$$

$$\tilde{\sigma}_i = \frac{\sigma_i}{\mu u^2}$$





$$\bar{\sigma}_{\rm SII}(\beta) \approx \frac{2\sqrt{2}}{3}\frac{1}{1+1.875\beta^{-1/2}}\left[1+0.5\frac{\beta^{1/2}}{1+2\beta}\right]$$

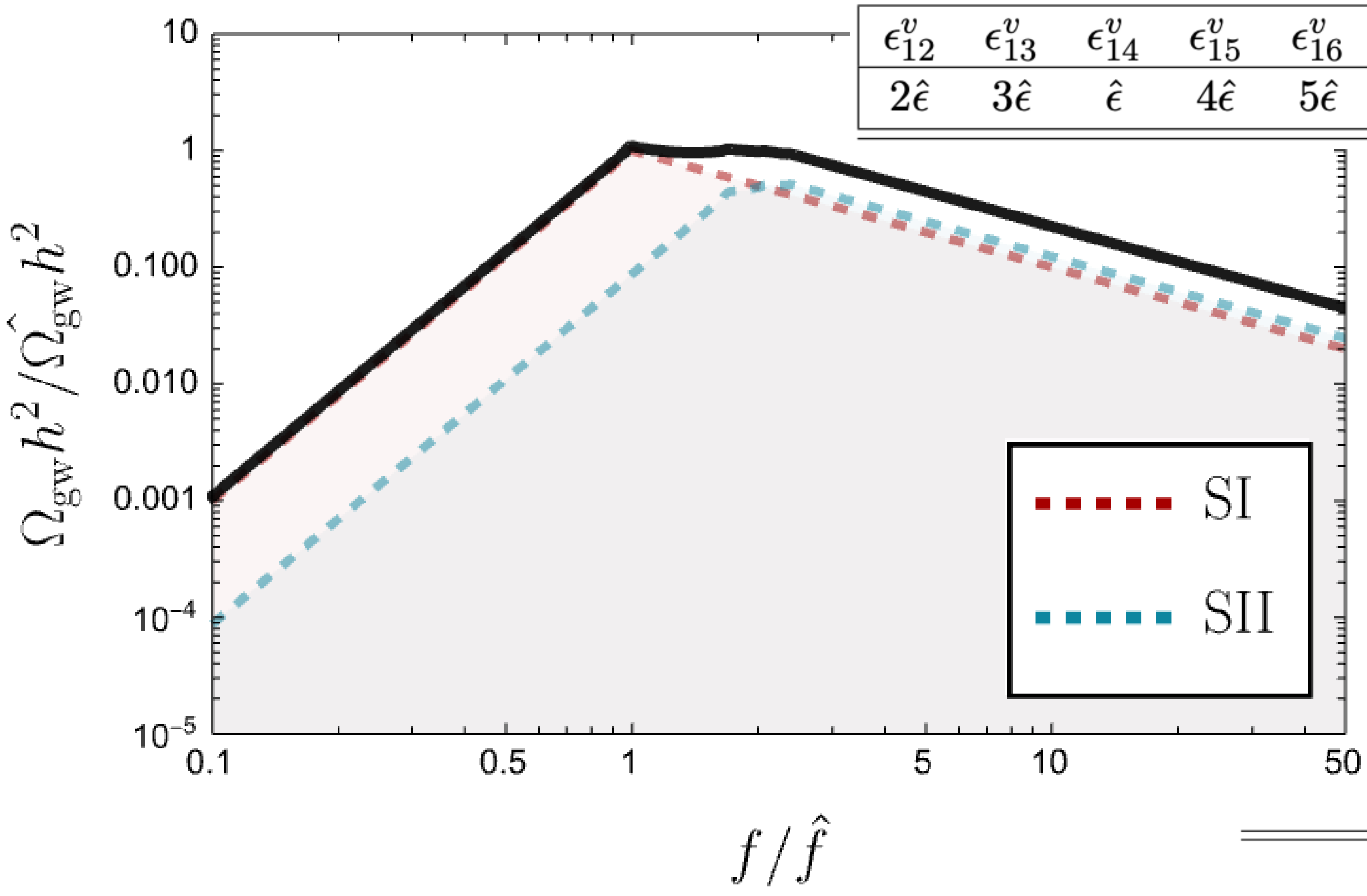
$$\tilde{\sigma}_{\rm TII}(\beta) = \frac{0.77(-\beta)^{0.5}}{(1.5+\beta)^{0.25}} \qquad \tilde{\sigma}_{\rm TIII}(\beta) = \frac{2.06(-\beta)^{0.5}}{1+0.09(-\beta)^{0.6}}$$

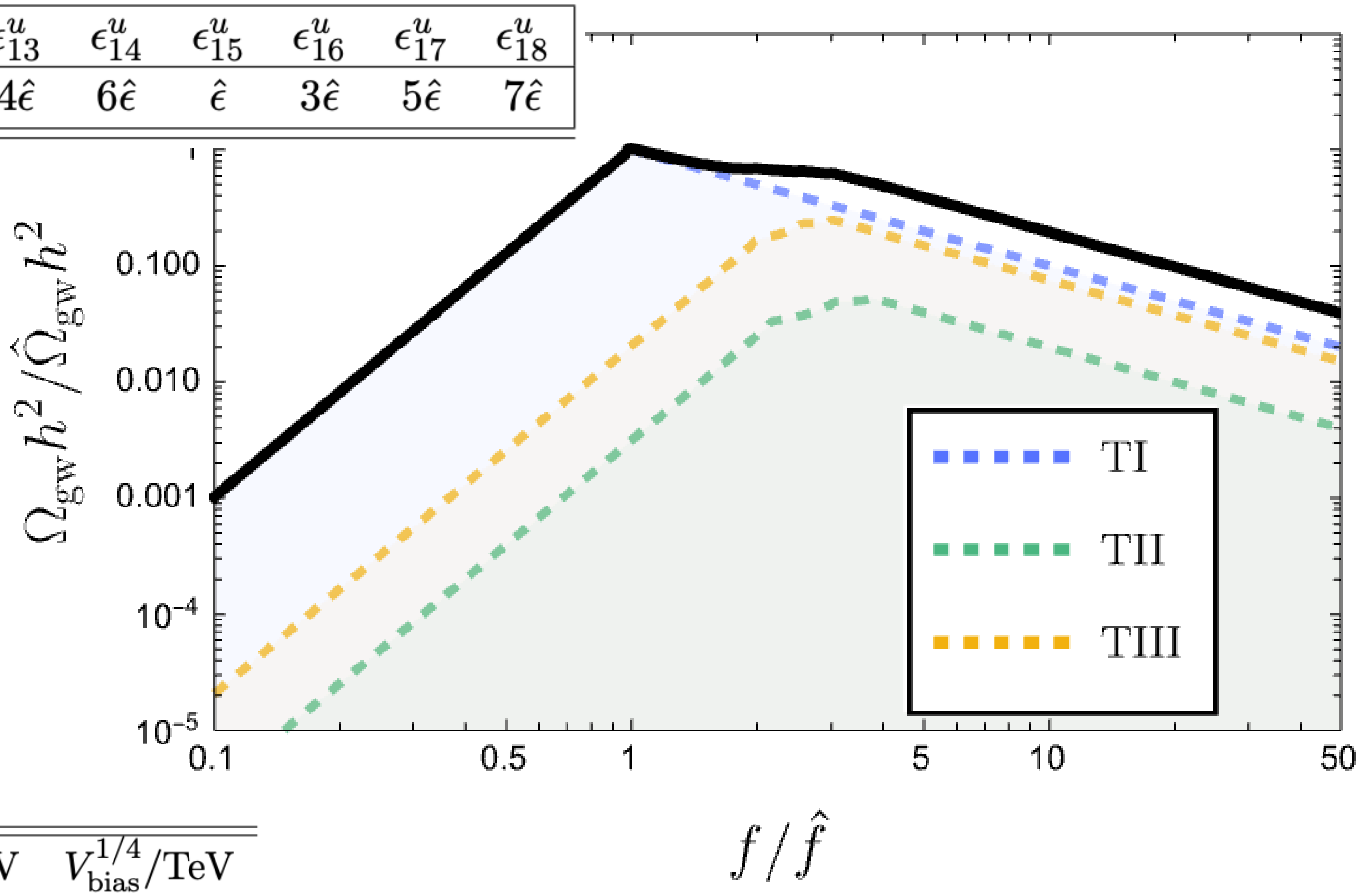
引力波频谱 (for illustration)

bias 导致的能级劈裂，不同真空间的能量密度的差异为 $V^{ij}_{\rm bias} = \epsilon_{ij} v^4$

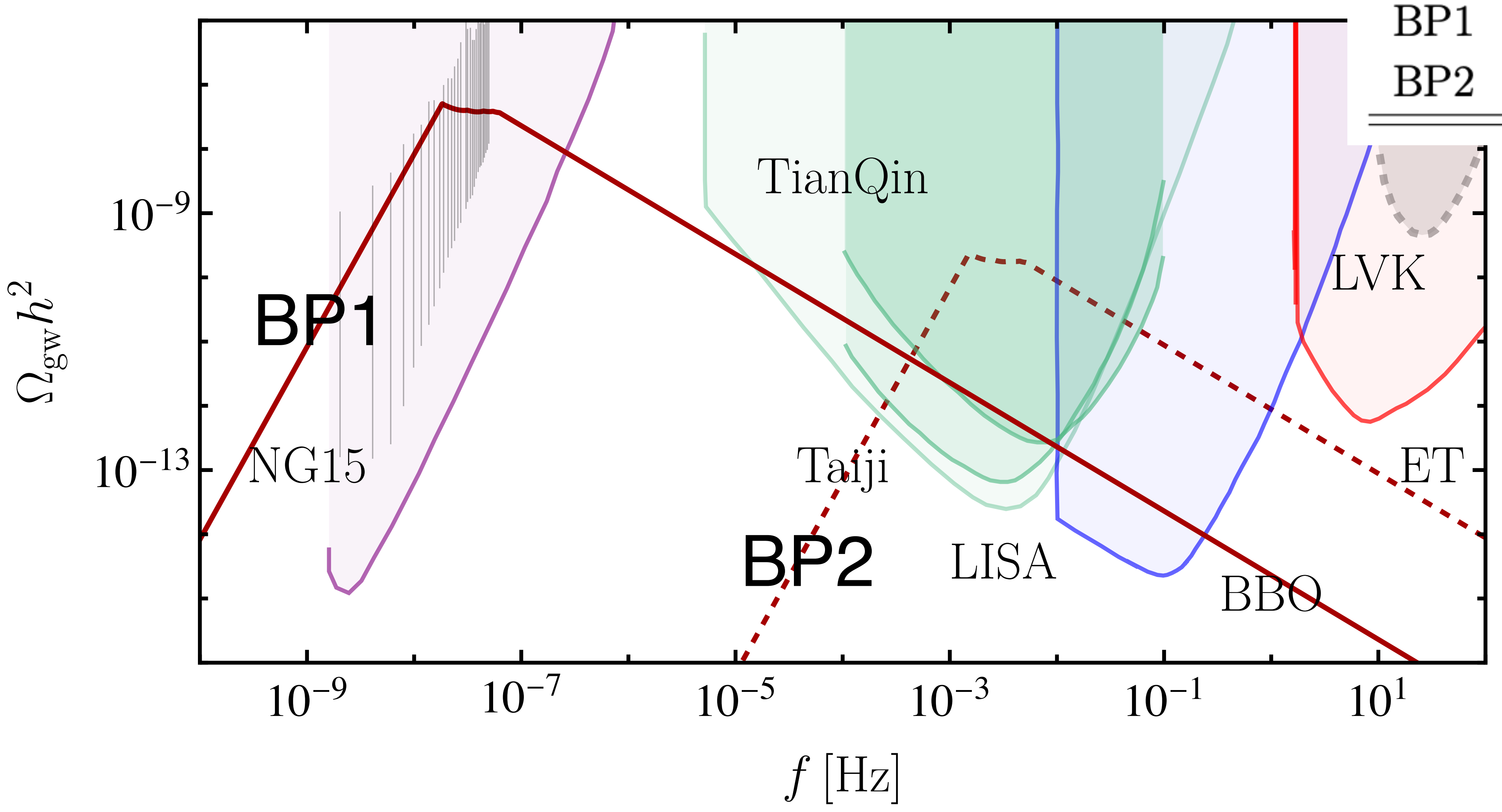
ϵ^v_{12}	ϵ^v_{13}	ϵ^v_{14}	ϵ^v_{15}	ϵ^v_{16}
$2\hat\epsilon$	$3\hat\epsilon$	$\hat\epsilon$	$4\hat\epsilon$	$5\hat\epsilon$

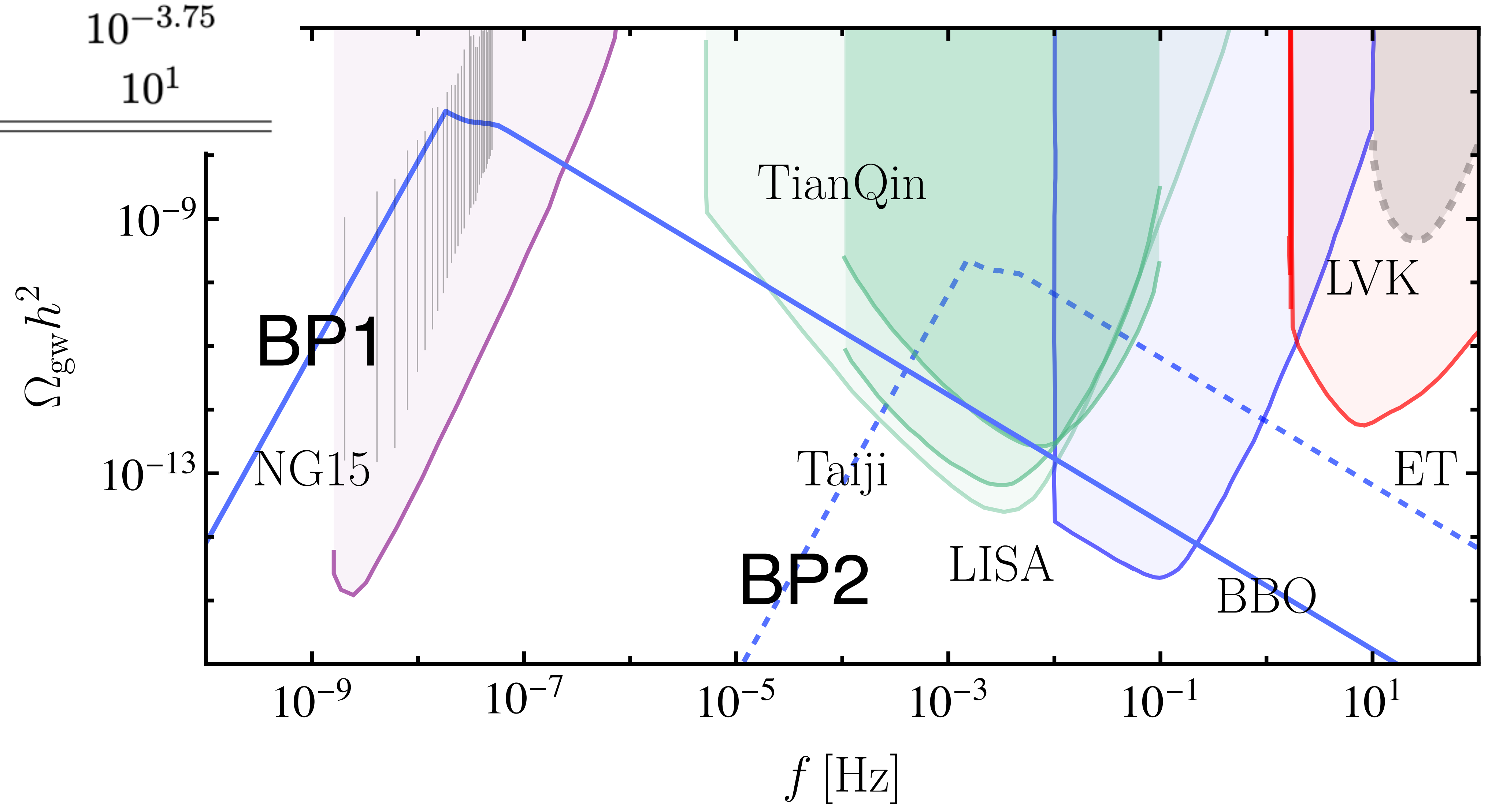
ϵ^u_{12}	ϵ^u_{13}	ϵ^u_{14}	ϵ^u_{15}	ϵ^u_{16}	ϵ^u_{17}	ϵ^u_{18}
$2\hat\epsilon$	$4\hat\epsilon$	$6\hat\epsilon$	$\hat\epsilon$	$3\hat\epsilon$	$5\hat\epsilon$	$7\hat\epsilon$





	$\sigma^{1/3}$/TeV	$V^{1/4}_{\rm bias}$/TeV
BP1	3×10^2	$10^{-3.75}$
BP2	3×10^5	10^1





检验分立味对称性

Gelmini, Pascoli, Vitagliano, YLZ, 2009.01903

A_4 下的一个轻子味混合模型

$$-\mathcal{L}_{l,\nu} \supset y_D \bar{L}_i \tilde{H} N_i + y_N \bar{N}_i N_j^c \chi_k + \frac{1}{2} u \bar{N}_i^c N_i + \frac{\varphi_i}{\Lambda} \bar{L}_i H (y_e e_R + \omega^{1-i} y_\mu \mu_R + \omega^{i-1} y_\tau \tau_R) + \text{h.c.}$$

$i \neq j \neq k \neq i,\ \omega = e^{i2\pi/3}$

+ 精确破坏项

$$-\mathcal{L}_{\cancel{A_4}} = \frac{1}{2} \epsilon_{ij} v_\chi \bar{N}_i^c N_j + \text{h.c.}$$

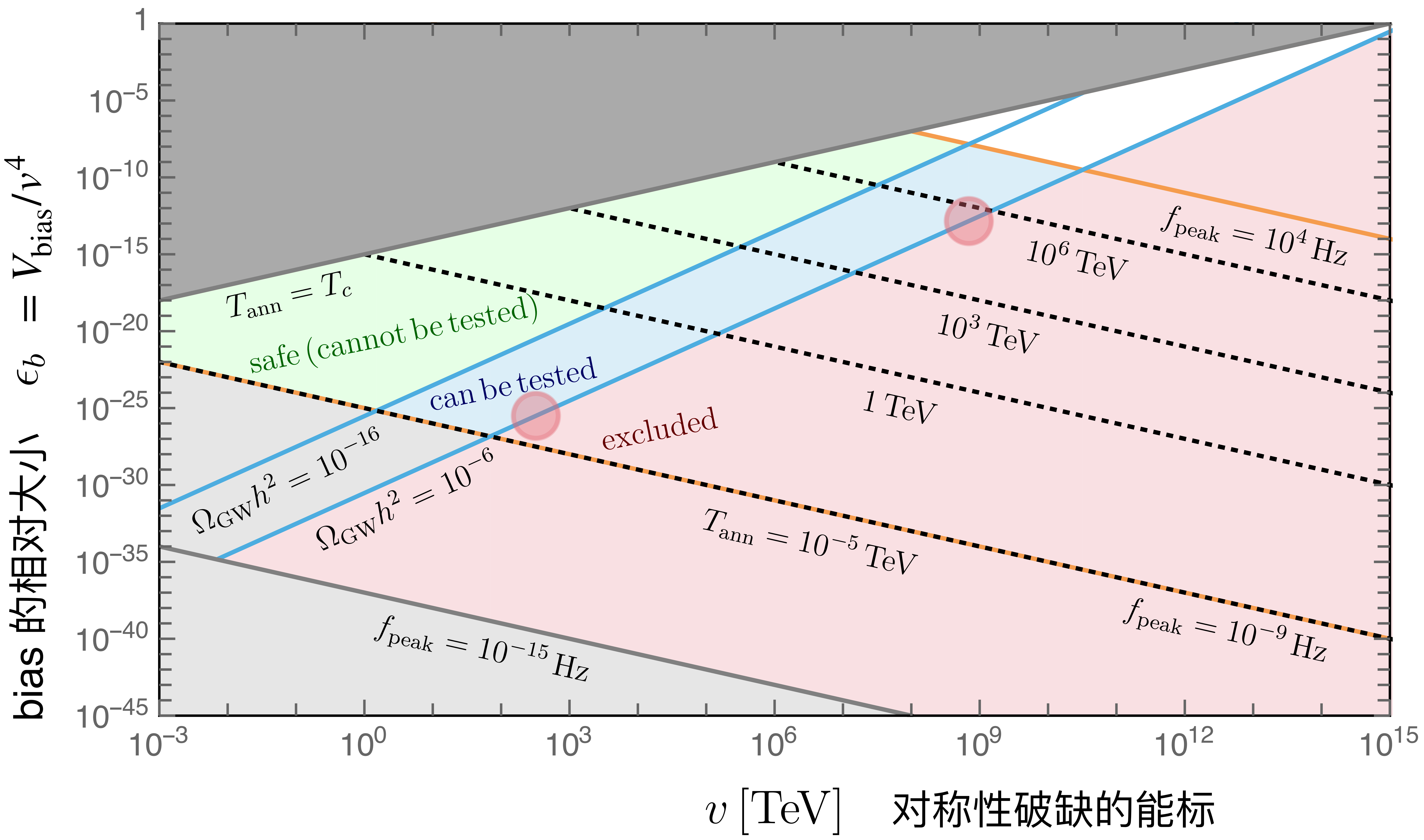
$$\epsilon_{12} = \epsilon_{13} = 0,\ \epsilon_{22} = -\epsilon_{33}$$

$\mu - \tau$ 反演对称性 & TM2 混合

$$|U| = \begin{pmatrix} \frac{2}{\sqrt{6}} \cos\theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} |\sin\theta| \\ \left|\frac{1}{\sqrt{6}} \cos\theta - \frac{i}{\sqrt{2}} \sin\theta\right| & \frac{1}{\sqrt{3}} & \left|\frac{1}{\sqrt{6}} \sin\theta + \frac{i}{\sqrt{2}} \cos\theta\right| \\ \left|\frac{1}{\sqrt{6}} \cos\theta - \frac{i}{\sqrt{2}} \sin\theta\right| & \frac{1}{\sqrt{3}} & \left|\frac{1}{\sqrt{6}} \sin\theta - \frac{i}{\sqrt{2}} \cos\theta\right| \end{pmatrix}$$

$\theta_{23} = 45^\circ$ $\quad \delta = \pm 90^\circ$ $\quad \alpha_{21}, \alpha_{31} = 0, 180^\circ$ $\quad \sin\theta_{12} = \frac{1}{\sqrt{3}\cos\theta_{13}}$ $\quad \sin\theta_{13} \simeq 2\sqrt{\frac{2}{3}} \frac{|\epsilon_{22}| v_\chi M_2}{|\Delta M_{31}^2|}$

检验分立味对称性



bias 的相对大小 $\epsilon_b = V_{\rm bias}/v^4$
v [TeV] 对称性破缺的能标
$T_{\rm ann} = T_c$
safe (cannot be tested)
can be tested
excluded
$\Omega_{\rm GW}h^2 = 10^{-16}$
$\Omega_{\rm GW}h^2 = 10^{-6}$
$T_{\rm ann} = 10^{-5}$ TeV
1 TeV
10^3 TeV
10^6 TeV
$f_{\rm peak} = 10^4$ Hz
$f_{\rm peak} = 10^{-9}$ Hz
$f_{\rm peak} = 10^{-15}$ Hz

总 结 & 展 望

- Abel 畴壁展现出与 Z_2 畴壁很不一样的性质
  - Z_4 畴壁有两种: 临近（adjacent）畴壁 & 非临近 （non-adjacent）畴壁，后者在某些参数空间不稳定
  - Z_N 畴壁可以很复杂：比如，string-bounded DWs, DW-wrapped DWs，我们对 Z_N 畴壁给了不完备的分类
- 非 Abel 畴壁，以 S_4 为例
  - 一共5类不同的畴壁：SI、SII、TI、TII、TIII，前两类隔离 Z_2 真空，后三类隔离 Z_3 真空
  - 在部分参数空间，SI、TI、和 TIII 会不稳定
- bias terms 需要引入进来，以避免宇宙学问题
  - bias 可使简并真空发生劈裂，Abel 畴壁 和 非 Abel 畴壁存在更多的简并真空，劈裂后得到很多能级
- 畴壁崩塌可以导致引力波，通过畴壁引力波的观测可以检验高能标处的分立对称性
  - 引力波的强度和峰值频率取决于分立对称性的破缺能标和bias的大小
  - 由于Abel 畴壁 和 非 Abel 畴壁引入了更多复杂的结构，可以预期它们产生的引力波信号将会与Z_2 畴壁有所不同

非 常 感 谢！