



轻子：新物理的门户

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Nov. 3rd, 2024, Liaoning Normal University, Dalian

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致谢：上海交通大学——许金祥，北京大学——李强、张策等，南开大学——李佟、Lorenzo Calibbi，南华大学——郑波、李小华等



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- Review of the past experience
- Motivations for muon physics
- Conceptual design of MACE
- Local laboratory: SMOOTH



这些年，这些事儿

拔尖人才培养

- 本科：以赛促学、以赛促研
- 研究生：培育卓越科学家
- 建设“一流”课程

走出去和引进来

- 中德合作：JUNO实验
- 中日合作：COMET实验
- 欧盟合作：缪子对撞机预研



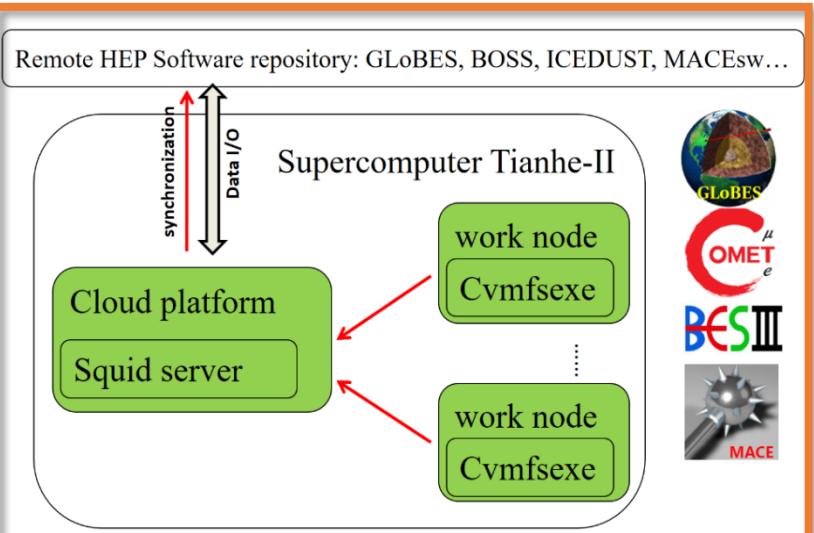
大数据模拟和物理分析

- 拓展目标：JUNO、MACE实验
- 满足需求：快速部署、及时更新、百万核时级别的数据量
- 服务各类型前沿科学实验

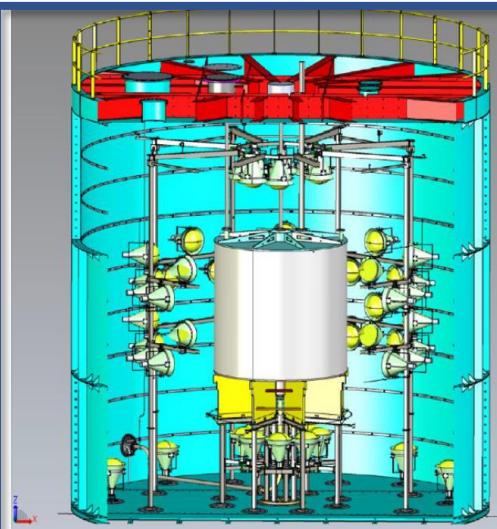
缪子径迹探测技术及其应用

- 分辨率：时间—纳秒级、空间—毫米级
- 加速器缪子源：COMET、MACE实验
缪子对撞机计划
- 宇生缪子源：MuGrid及其多学科应用

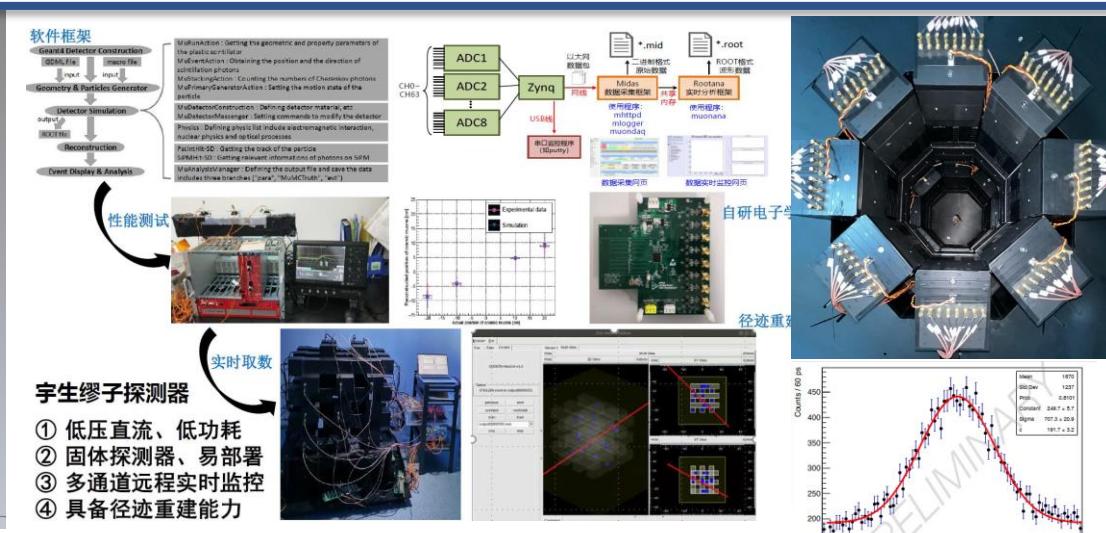
以轻子为探针寻找超越标准模型新物理



依托超级计算机“天河二号”
建设粒子物理实验大数据平台



参与国际大科学实验JUNO
突破低本底探测器关键技术



建设缪子前沿科学与技术应用实验室
积累探测器核心技术面向多学科应用

鼓励探索，突出原创；聚焦前沿，独辟蹊径；需求牵引，突破瓶颈；共性导向，交叉融通。



工欲善其事必先利其器，软件平台和硬件研发同步推进





SMOOTH团队情况



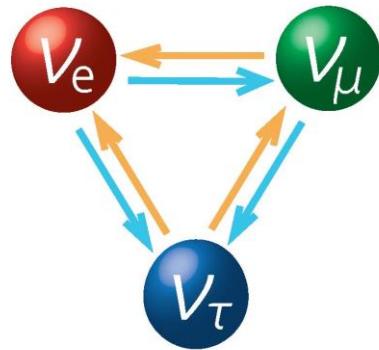
- 当前团队成员：博士后1名，博士生4名，在读硕士生6名，本科生科研项目学生10+名，电子学工程师1名，超算平台维护1名.....
- 校内合作伙伴：物理实验中心，测试中心，超算中心，材料科学与工程学院等
- 校外合作伙伴：中科大电子学实验室，中科院近代物理研究所，清华大学，中国散裂中子源等
- 国际合作伙伴：德国Mainz大学，日本Osaka大学和KEK，意大利INFN-Padova等



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Do charged leptons "oscillate"?



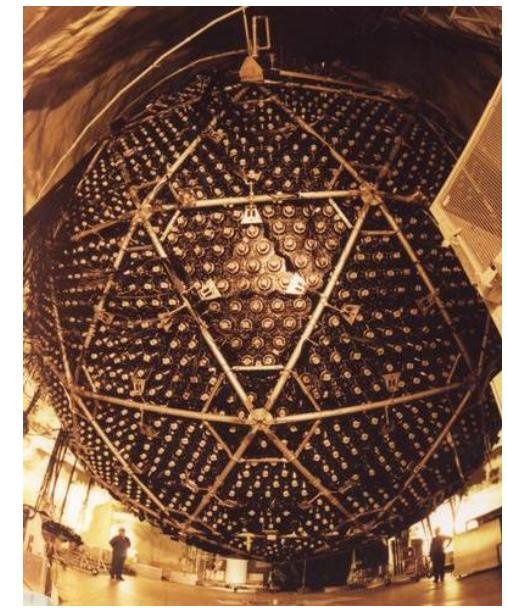
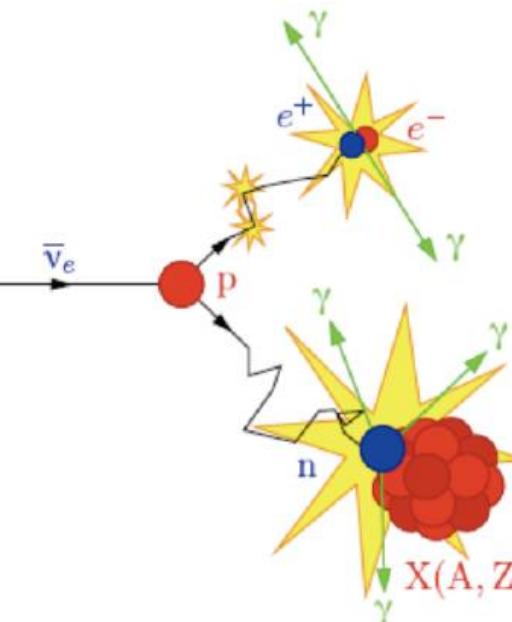
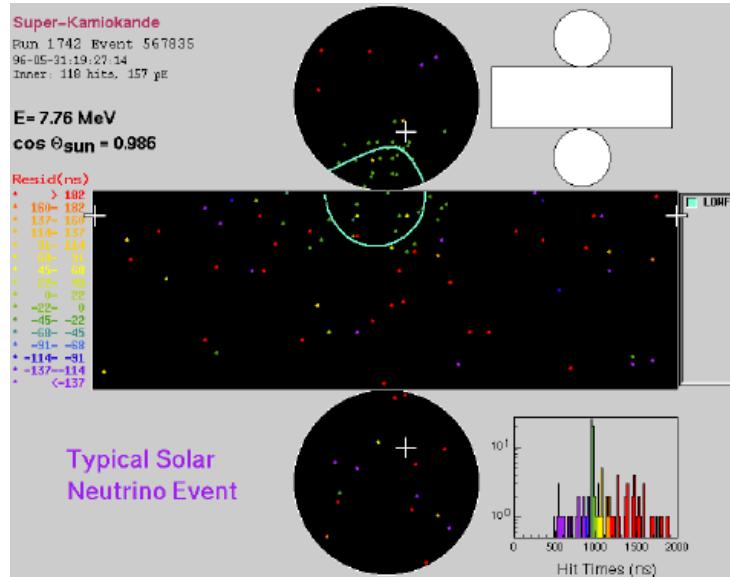
"For the greatest benefit to mankind"
Alfred Nobel
2015 NOBEL PRIZE IN PHYSICS

Takaaki Kajita
Arthur B. McDonald



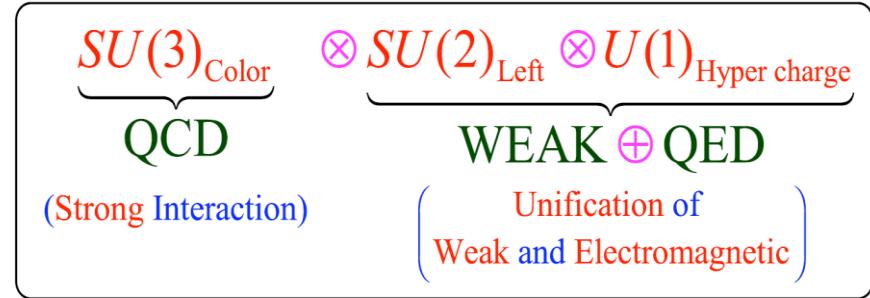
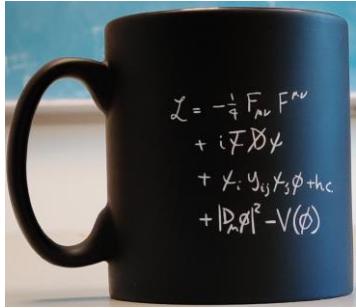
"For the discovery of neutrino oscillations,
which shows that neutrinos have mass"

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$





Symmetries of SM



$$\mathcal{G}_{\text{local}}^{\text{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$\mathcal{G}_{\text{local}}^{\text{SM}} \rightarrow SU(3)_c \times U(1)_{\text{EM}}$$

$$Q_L^i \sim (3, 2)_{1/6}, \quad U_R^i \sim (3, 1)_{2/3},$$

$$D_R^i \sim (3, 1)_{-1/3}, \quad L_L^i \sim (1, 2)_{-1/2},$$

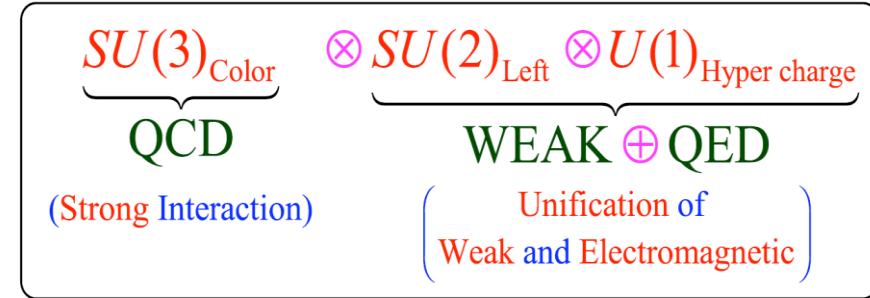
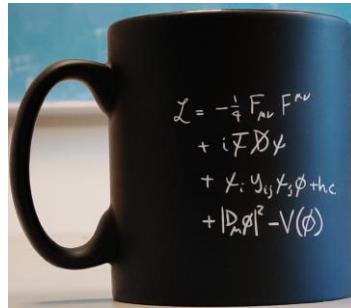
$$\phi \sim (1, 2)_{1/2}, \quad \langle \phi^0 \rangle \equiv \frac{v}{\sqrt{2}} \simeq 174 \text{GeV}$$

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{kinetic}}^{\text{SM}} + \mathcal{L}_{\text{EWSB}}^{\text{SM}} + \mathcal{L}_{\text{Yukawa}}^{\text{SM}}$$

- simple and symmetric (g, g', g_s)
- EWSB, 2 params
- SM flavour dynamics, flavour parameters



Symmetries of SM



- Flavor physics
 - Within SM: weak and Yukawa interactions
- Flavor parameters in the quark sector
 - Within SM: 9 masses of charged fermions & 4 mixing parameters (3 angle + 1 CP phase)
- Flavor universal (flavor blind)
 - Within SM: QCD & QED
- Flavor diagonal
 - Within SM: Yukawa interactions

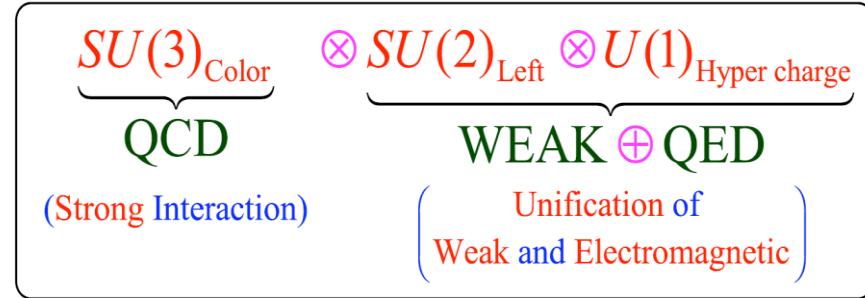
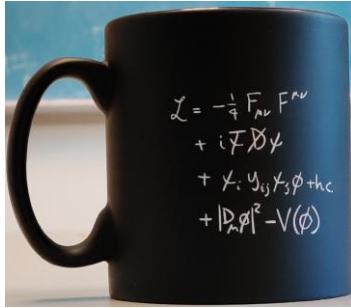
	Lepton number	Lepton family number (lepton flavor)		
		L_e	L_μ	L_τ
$e^- & \nu_e$	1	1	0	0
$\mu^- & \nu_\mu$	1	0	1	0
$\tau^- & \nu_\tau$	1	0	0	1

Change the sign for all anti-leptons

Lepton sector complements quark sector in flavor physics



Symmetries of SM



- Flavor physics
 - Within SM: weak and Yukawa interactions
- Flavor parameters in the quark sector
 - Within SM: 9 masses of charged fermions & 4 mixing parameters (3 angle + 1 CP phase)
- Flavor universal (flavor blind)
 - Within SM: QCD & QED
- Flavor diagonal
 - Within SM: Yukawa interactions

- Rephasing lepton and quark fields:

$$\begin{aligned}
 & U(1)_B \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau} \\
 & = \\
 & \cancel{U(1)_{B+L}} \times U(1)_{B-L} \times U(1)_{L_\mu-L_\tau} \times U(1)_{L_\mu+L_\tau-2L_e} .
 \end{aligned}$$
- Broken non-perturbatively, but unobservable. [t Hooft, PRL '76]
- True accidental global symmetry:

$$U(1)_{B-L} \times U(1)_{L_\mu-L_\tau} \times U(1)_{L_\mu+L_\tau-2L_e} .$$

Lepton flavor conservation!
Prediction of Standard Model.

cLFV offers a chance of new physics discovery

Neutrino oscillation = cLFV?

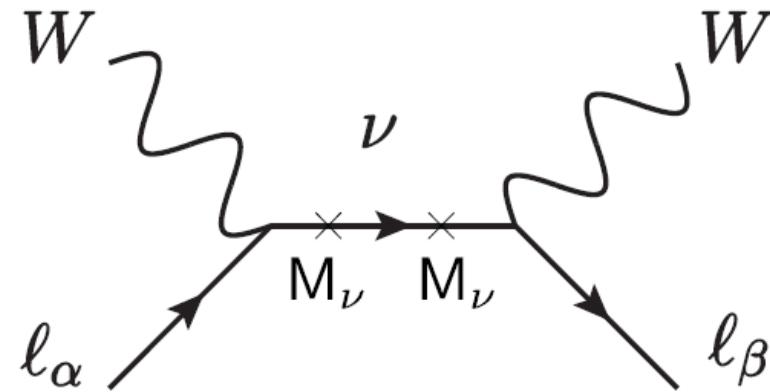
- Neutrino oscillation $\rightarrow M_\nu \neq 0$
- cLFV should exist, but we don't see it

$$U(1)_{L_\mu} - \cancel{U(1)_{L_\tau}} \times U(1)_{L_\mu + L_\tau - 2L_e}$$

- Tiny neutrino masses \rightarrow suppressed cLFV in vSM

$$\mathcal{A}(\ell_\alpha^- \rightarrow \ell_\beta^-) \propto \frac{(M_\nu M_\nu^\dagger)_{\alpha\beta}}{M_W^2} < 10^{-24}$$

- Neutrino mass models, e.g. seesaw, predict scalable cLFV!



Right-handed neutrino acquire a lepton number violating mass, leaving an $SU(2)_L \times U(1)$ subgroup unbroken. Consequence for the decay $\mu \rightarrow e\gamma$ are studied. Now called Type-I seesaw model.

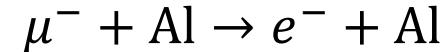
Peter Minkowski, Phys.Lett.B 67 (1977) 421-428

cLFV offers a probe to the origin of neutrino mass

High-intensity/-precision frontier

- Experiments search for cLFV:

 - Mu2e (Fermilab)



 - COMET (J-PARC)

 - MEG (PSI)



 - Mu3e (PSI)



- Precision measurements of muon properties:

 - MuLan & FAST at PSI: Muon lifetime.

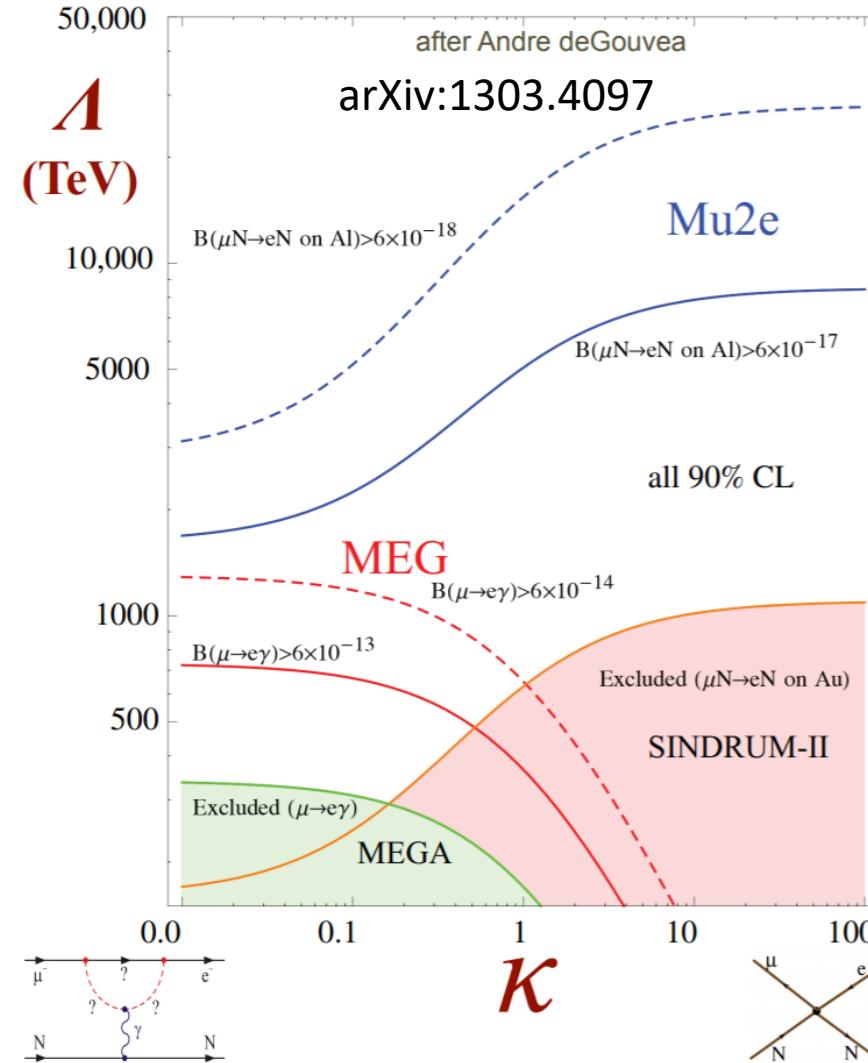
 - MuCap in PSI: Muon capture coupling constant.

 - MuSun: Muon Electroweak interactions and muon polarization.

 - TWIST at TRIUMF: Muon decay Michel parameters.

 - Fermi lab muon g-2 and J-PARC muon g-2.

 - MUSEUM: Muonium hyperfine structure.

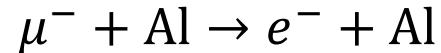


Low-energy cLFV experiments complement high-energy frontier

High-intensity/-precision frontier

- Experiments search for cLFV:

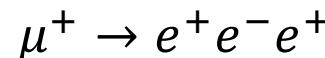
- Mu2e (Fermilab)



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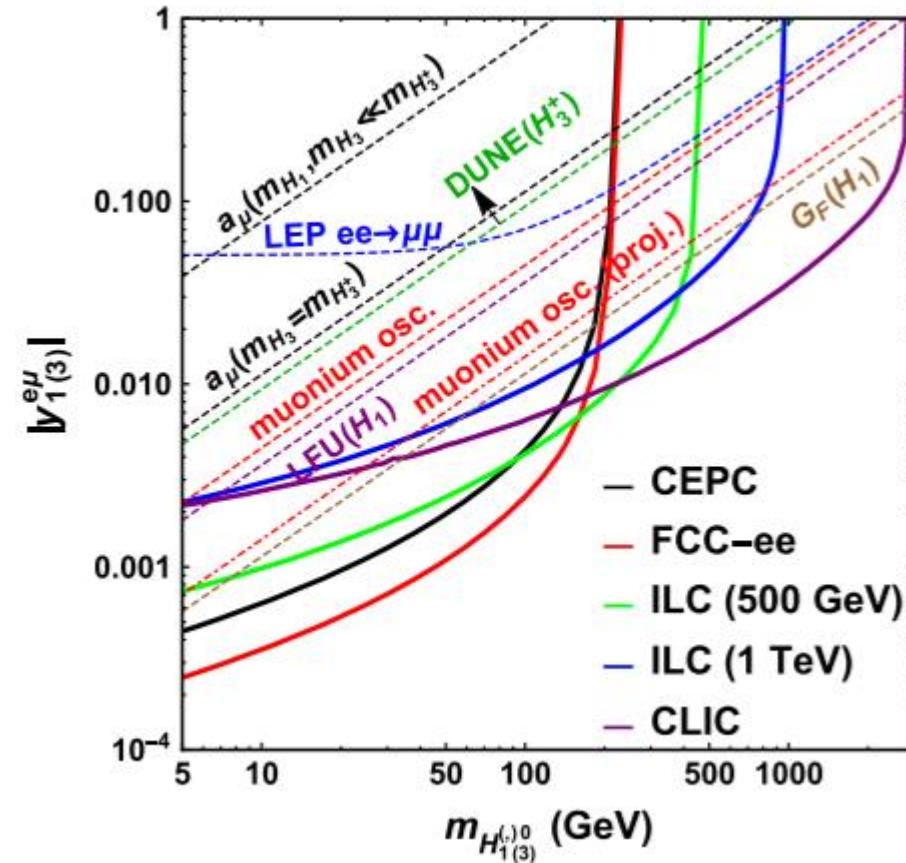
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REF: Tong Li, Michael A. Schmidt. Phys.Rev.D 100 (2019) 11, 115007

- Low-energy cLFV experiments complement high-energy frontier
- cLFV complement neutrino physics

High-intensity/-precision frontier

- Experiments search for cLFV:

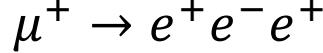
➤ Mu2e (Fermilab)



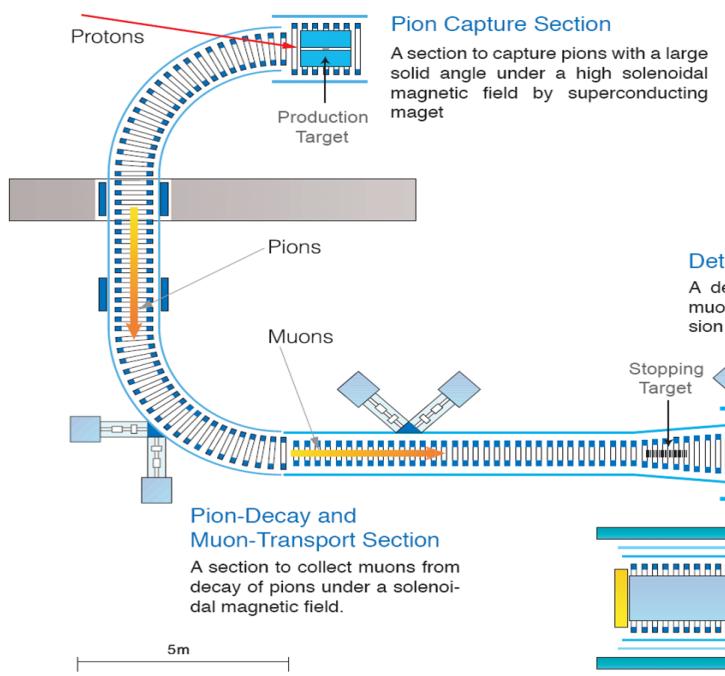
➤ COMET (J-PARC)



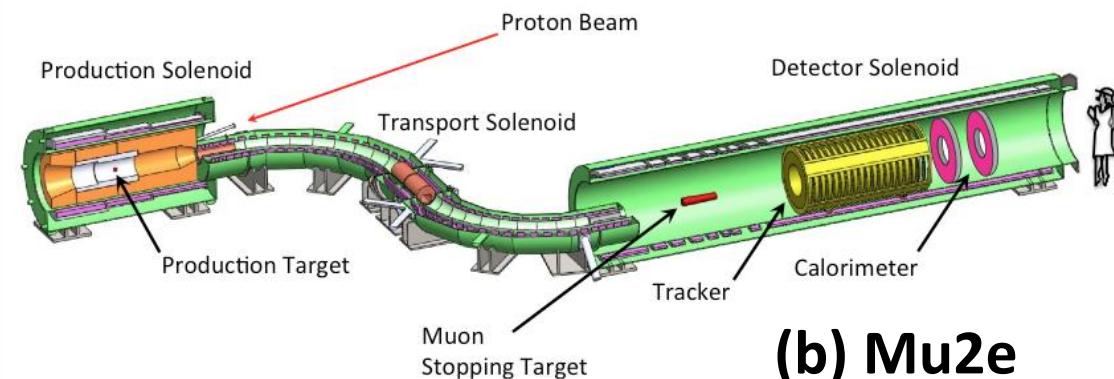
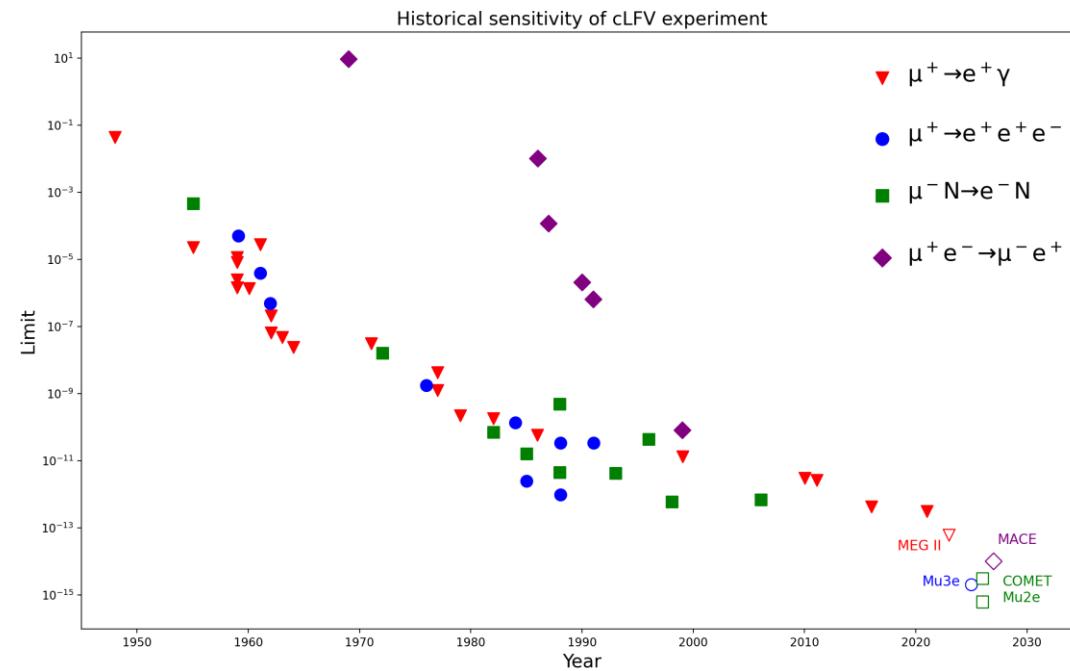
➤ MEG (PSI)



➤ Mu3e (PSI)



(a) COMET



(b) Mu2e

High-intensity/-precision frontier

- Experiments search for cLFV:

➤ Mu2e (Fermi lab)

$$\mu^- + Al \rightarrow e^- + Al$$

➤ COMET (J-PARC)

➤ MEG (PSI)

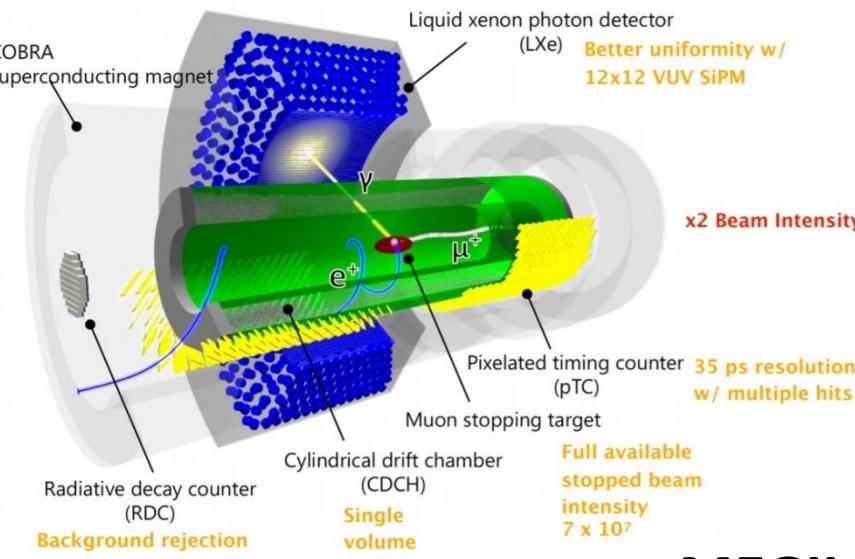
$$\mu^+ \rightarrow e^+ \gamma$$

➤ Mu3e (PSI)

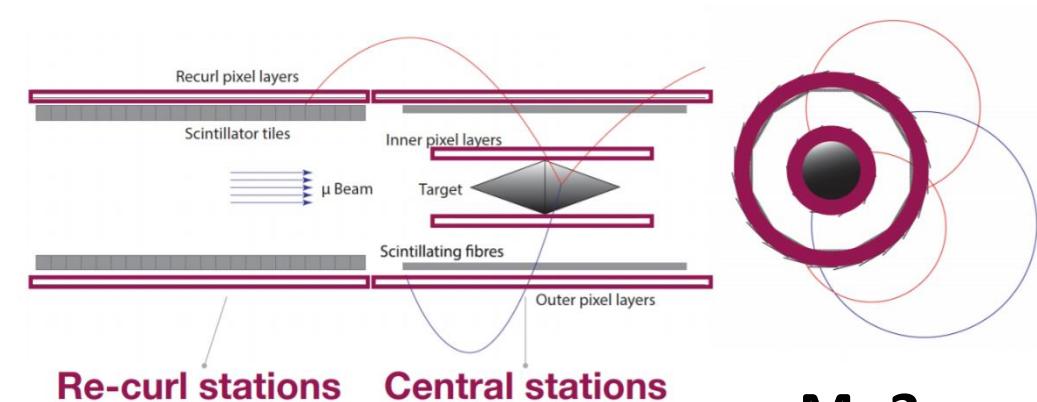
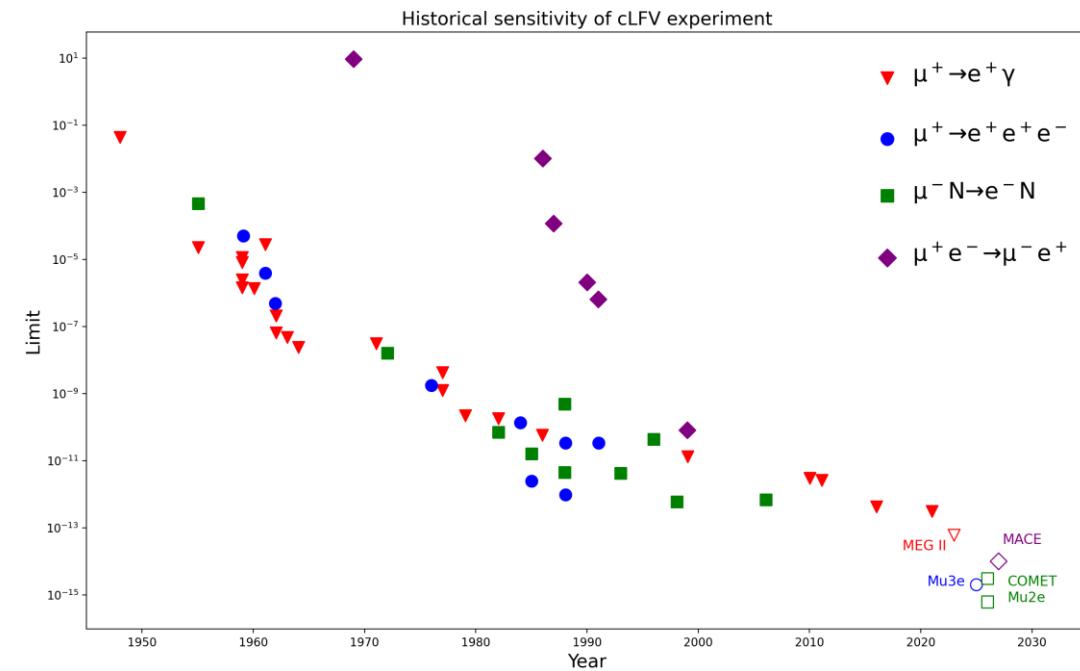
$$\mu^+ \rightarrow e^+ e^- e^+$$

New electronics:
Wavedream

~9000
channels
at 5GSPS



MEGII



Mu3e

Guangdong: a hub for high-intensity accelerators



Ref: Sheng Wang (IHEP, CAS)



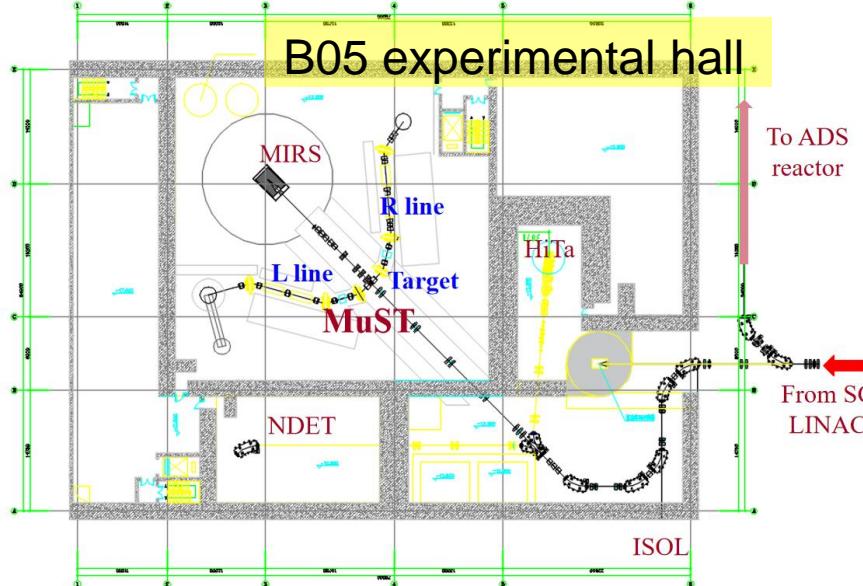
Ref: Wen-Long Zhan (IMP, CAS)

- (1) International muon sources: FNAL (USA), PSI (Switzerland), J-PARC (Japan), ISIS (UK).
- (2) China's first high-intensity muon source to be built in the Greater Bay Area: CSNS/CiADS/HIAF.
- (3) Exploring cutting-edge research with accelerator muon sources?



CiADS muon source

- Muon source is planning in China Initiative Accelerator Driven System (CiADS).



H.-J Cai et al, Phys.Rev.AB 27, 023703, 2024

束流功率 (kW)	靶方案	束线聚焦方案	缪子强度 (μ^+/s)
300	石墨转靶	螺线管+四极铁	>1E8
		全螺线管	>5E8
3000	液态锂靶	螺线管+四极铁	>2E9
		全螺线管	>1E10

2025 ~ 2026

25 kW:
500MeV & 50 μ A

- Accelerator construction
- Proton beam commissioning

2026 ~ 2027

250 kW:
500MeV & 500 μ A

- Beam stability test
- Muon beam commissioning
- One target station, two beamlines

2027~2030

2.5 MW:
500MeV & 5mA

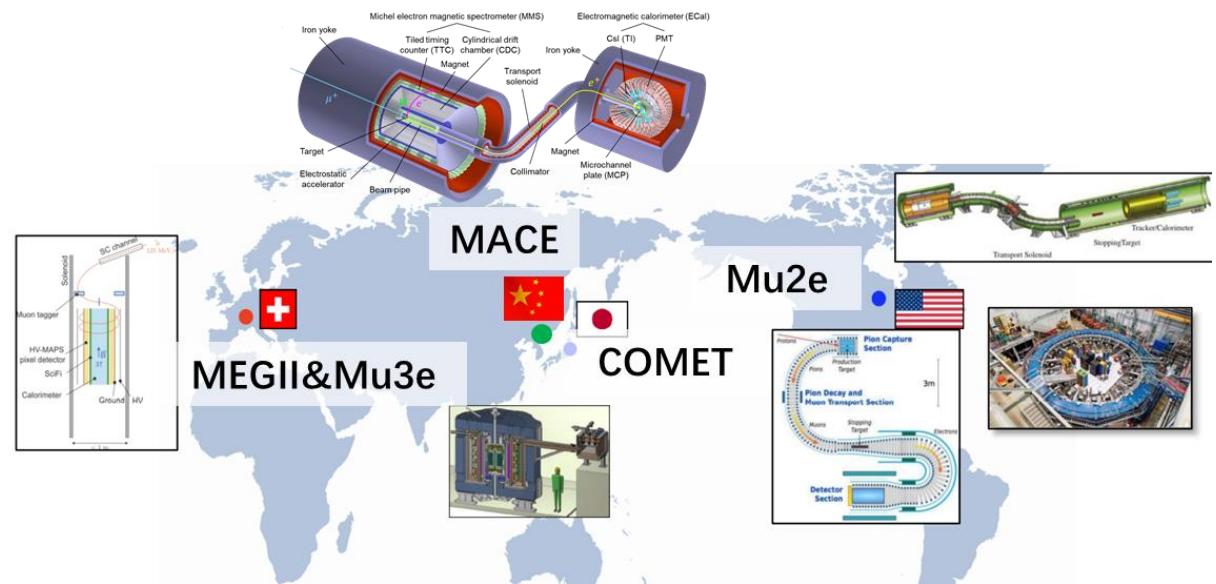
- 2.5 MW commissioning
- 2.5 MW long-term run
- Two target station, two beamlines
- Muon physics and applications



Worldwide cLFV experiments

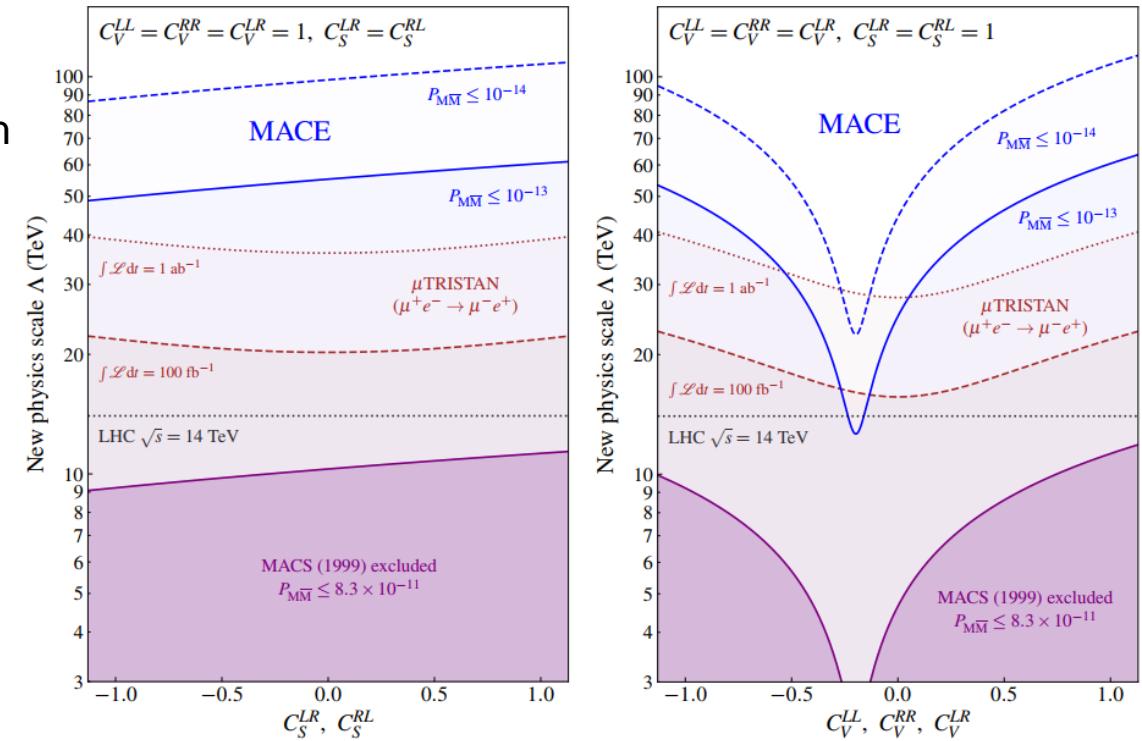
Experiment	Facility	Process	Progress
MEGII	PSI (Switzerland)	$\mu^+ \rightarrow e^+ \gamma$	Data taking
Mu2e	Fermilab (US)	$\mu^- \text{Al} \rightarrow e^- \text{Al}$	Construction
COMET	J-PARC (Japan)	$\mu^- \text{Al} \rightarrow e^- \text{Al}$	Construction
Mu3e	PSI (Switzerland)	$\mu^+ \rightarrow e^+ e^- e^+$	Commissioning
MACS	PSI (Switzerland)	$M \rightarrow \bar{M}$	Finished (1999)

- Muonium conversion is a key cLFV process.
- After PSI set the bound $P < 8.3 \times 10^{-11}$ in 1999, no new experiments were proposed for 20 years.
- With enhanced beam intensity and advances in detector technology, breakthroughs in this field are anticipated.



Summary: motivation of MACE

- Neutrinos are in oscillation; charged leptons?
- Demand for cutting-edge research:
 - cLFV selects neutrino mass mechanism.
 - Charged leptons and neutrinos share Yukawa couplings, with cLFV complementing neutrino physics.
 - Lepton cLFV complements quark flavor physics.
 - Low-energy cLFV experiments complement high-energy frontier research.
 - Muonium conversion experiments have stalled for decades, presenting both opportunities and challenges.
- Opportunities in China national research facilities:
 - China is set to build a high-intensity muon source.
 - What type of physics deserves exploration?
 - An innovative approach: MACE!

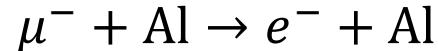


Snowmass2021 - Letter of Interest

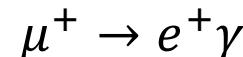


- Experiments search for cLFV:

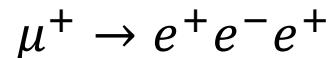
➤ Mu2e (Fermilab)



➤ COMET (J-PARC)



➤ MEG (PSI)



➤ Mu3e (PSI)

- Precision measurements of muon properties:

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➤ MUSEUM: Muonium hyperfine structure.

Snowmass2021 - Letter of Interest

RF5-RF0-126

Search for Muonium to Antimuonium Conversion

RF Topical Groups: (check all that apply)

- (RF1) Weak decays of b and c quarks
- (RF2) Weak decays of strange and light quarks
- (RF3) Fundamental Physics in Small Experiments
- (RF4) Baryon and Lepton Number Violating Processes
- (RF5) Charged Lepton Flavor Violation (electrons, muons and taus)
- (RF6) Dark Sector Studies at High Intensities
- (RF7) Hadron Spectroscopy
- (Other) [Please specify frontier/topical group(s)]



MACE

Contact Information: (authors listed after the text)

Name and Institution: Jian Tang/Sun Yat-sen University

Collaboration: MACE working group

Contact Email: tangjian5@mail.sysu.edu.cn

Abstract: It is puzzling whether there is any charged lepton flavor violation phenomenon beyond standard model. The upcoming Muonium (bound state of $\mu^+ e^-$) to Antimuonium ($\mu^- e^+$) Conversion Experiment (MACE) will serve as a complementary experiment to search for charged lepton flavor violation processes, compared with other on-going experiments like Mu3e ($\mu^+ \rightarrow e^+ e^- e^+$), MEG-II ($\mu^+ \rightarrow e^+ \gamma$) and Mu2e/COMET ($\mu^- N \rightarrow e^- N$). MACE aims at a sensitivity of $P(\mu^+ e^- \rightarrow \mu^- e^+) \sim \mathcal{O}(10^{-13})$, about three orders of magnitude better than the best limit published two decades ago. It is desirable to optimize the slow and ultra-pure μ^+ beam, select high-efficiency muonium formation materials, develop Monte-Carlo simulation tools and design a new magnetic spectrometer to increase S/B.

Yu Chen, Yu-Zhe Mao, Jian Tang, School of Physics, Sun Yat-sen University, China.

Yu Bao, Yu-Kai Chen, Rui-Rui Fan, Zhi-Long Hou, Han-Tao Jing, Hai-Bo Li, Yang Li, Han Miao, Ying-Peng Song, Jing-Yu Tang, Nikolaos Vassilopoulos, Tian-Yu Xing, Ye Yuan, Yao Zhang, Guang Zhao, Luping Zhou, Institute of High-Energy Physics, Beijing, China.

Chen Wu, Research Center of Nuclear Physics (RCNP), Osaka University, Japan.

Probing the doubly charged Higgs boson with a muonium to antimuonium conversion experiment

Chengcheng Han,¹ Da Huang,^{2,3,4,*} Jian Tang,^{1,†} and Yu Zhang,^{5,6}

¹School of Physics, Sun Yat-Sen University, Guangzhou 510275, China

²National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100012, China

³School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study,

University of Chinese Academy of Sciences, Hangzhou 310024, China

⁴International Center for Theoretical Physics Asia-Pacific, Beijing/Hangzhou 10010, China

⁵Institutes of Physical Science and Information Technology, Anhui University, Hefei 230601, China

⁶School of Physics and Materials Science, Anhui University, Hefei 230601, China



Snowmass2021 whitepaper

March 23, 2022

arXiv: 2203.11406

Muonium to antimuonium conversion: Contributed paper for Snowmass 21

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The spontaneous muonium to antimuonium conversion is one of the interesting charged lepton flavor violation processes. It serves as a clear indication of new physics and plays an important role in constraining the parameter space beyond Standard Model. MACE is a proposed experiment to probe such a phenomenon and expected to enhance the sensitivity to the conversion probability by more than two orders of magnitude from the current best upper constraint obtained by the PSI experiment two decades ago. Recent developments in the theoretical and experimental aspects to search for such a rare process are summarized.

International response to Snowmass LOI



A New Charged Lepton Flavor Violation Program at Fermilab

Bertrand Echenard – Caltech

with Robert Bernstein (FNAL) and Jaroslav Pasternak (ICL/RAL SCTF)

Potential Fermilab Muon Campus & Storage Ring Experiments Workshop
May 2021



Snowmass process and contributed papers

Frontier for Rare Processes and Precision Measurements

Alexey A. Petrov
Wayne State University

This effort is part of a global muon program under study within Snowmass

- Muon decays (MEG and Mu3e)
- Muon conversion (Mu2e / COMET and Mu2e II)
- $\Delta L=2$ processes $\mu^- N \rightarrow e^+ N$
- Muonium – antimuonium (MACE)
- General Low Energy Muon Facility (FNAL)
- Light new physics in muon decays (MEG-Fwd)

Bertrand lists MACE as a key next-generation CLFV experiment proposal

A large community committed to muon physics at FNAL and around the world

- Theoretical Letter of Intent

Physics of muonium and muonium oscillations

Alexey A. Petrov¹

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Wayne State University, Detroit, MI 48201, USA

Precision studies of a muonium, the bound state of a muon and an electron, provide access to physics beyond the Standard Model. We propose that extensive theoretical and experimental studies of atomic physics of a muonium, its decays and muonium-antimuonium oscillations could provide an impact on indirect searches for new physics.

Search for Muonium to Antimuonium Conversion

RF Topical Groups: (check all that apply)

- (RF1) Weak decays of b and c quarks
- (RF2) Weak decays of strange and light quarks
- (RF3) Fundamental Physics in Small Experiments
- (RF4) Baryon Lepton Number Violating Processes
- (RF5) Charged Lepton Flavor Violation (electrons, muons and taus)
- (RF6) Dark Sector Studies at High Intensities
- (RF7) Hadron Spectroscopy
- (Other) Please specify frontier topical group(s)

Contact Information: (authors listed after the text)
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Collaboration: MACE working group
Contact Email: tangjian5@mail.sysu.edu.cn

Abstract: It is puzzling whether there is any charged lepton flavor violation phenomenon beyond standard model. The upcoming Muonium (bound state of $\mu^+ e^-$) to Antimuonium ($\mu^- e^+$) Conversion Experiment (MACE) will serve as a complementary experiment to search for charged lepton flavor violation processes, compared with other on-going experiments like Mu3e ($\mu^+ \rightarrow e^+ e^- e^+$), MEG-II ($\mu^+ \rightarrow e^+ \gamma$) and Mu2e/COMET ($\mu^- N \rightarrow e^- N$). MACE aims at a sensitivity of $P(\mu^+ e^- \rightarrow \mu^- e^+) \sim \mathcal{O}(10^{-13})$, about three orders of magnitude better than the best limit published two decades ago. It is desirable to optimize the slow and ultra-pure μ^- beam, select high-efficiency muonium formation materials, develop Monte-Carlo simulation tools and design a new magnetic spectrometer to increase S/B.

- Experimental Letter of Intent

Alexey A. Petrov (WSU)

2

Muon Campus Experiments, 24-27 May 2021

International response to Snowmass LOI



Detectors and concepts for future CLFV experiments

Bertrand Echenard
Caltech

NuFact 2021
Cagliari - September 2021



MACE at EMuS

EMuS – new muon facility in China

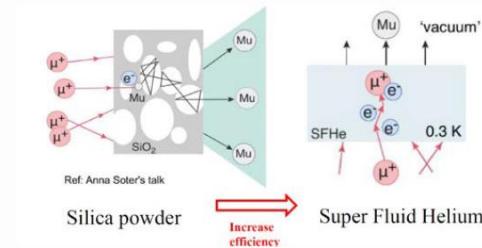
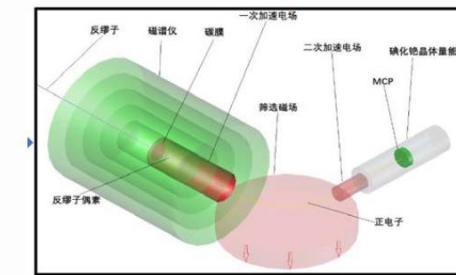


	Proton driver [MW]	Surface muons			Decay muons		
		Intensity [1E6/s]	Polarization [%]	Spread [%]	energy [MeV/c]	Intensity [1E6/s]	Spread [%]
PSI	1.3	420	90	10	85-125	240	3
ISIS	0.16	1.5	95	<15	20-120	0.4	10
RIKEN/RAL	0.16	0.8	95	<15	65-120	1	10
JPARC	1	100	95	15	33-250	10	15
TRIUMF	0.075	1.4	90	7	20-100	0.0014	10
EMuS	0.005	83	50	10	50-450	16	10
Baby EMuS	0.005	1.2	95	10			

×5 CSNS-II upgrade

Jian Tang
(Snowmass 2021 RPP meeting)

MACE concept



On-going physics studies and detector R&D

Bertrand Echenard - Caltech

p. 27

International response to Snowmass LOI



Progress of Muonium-to-Antimuonium Conversion Experiment (MACE)

Workshop on a Future Muon Program at Fermilab



2023-03-28

Shihan Zhao

zhaoshh7@mail2.sysu.edu.cn

Muonium-to-Antimuonium Conversion Experiment

MACE working group: Ai-Yu Bai,¹ Yu Chen,¹ Yukai Chen,² Rui-Rui Fan,² Zhilong Hou,² Han-Tao Jing,² Hai-Bo Li,² Yang Li,² Han Miao,² Huaxing Peng,² Ying-Peng Song,² Jian Tang,¹ Jing-Yu Tang,² Nikolaos Vassilopoulos,² Chen Wu,³ Tian-Yu Xing,² Yu Xu,¹ Ye Yuan,² Yao Zhang,² Guang Zhao,² Shihan Zhao,¹ and Luping Zhou²

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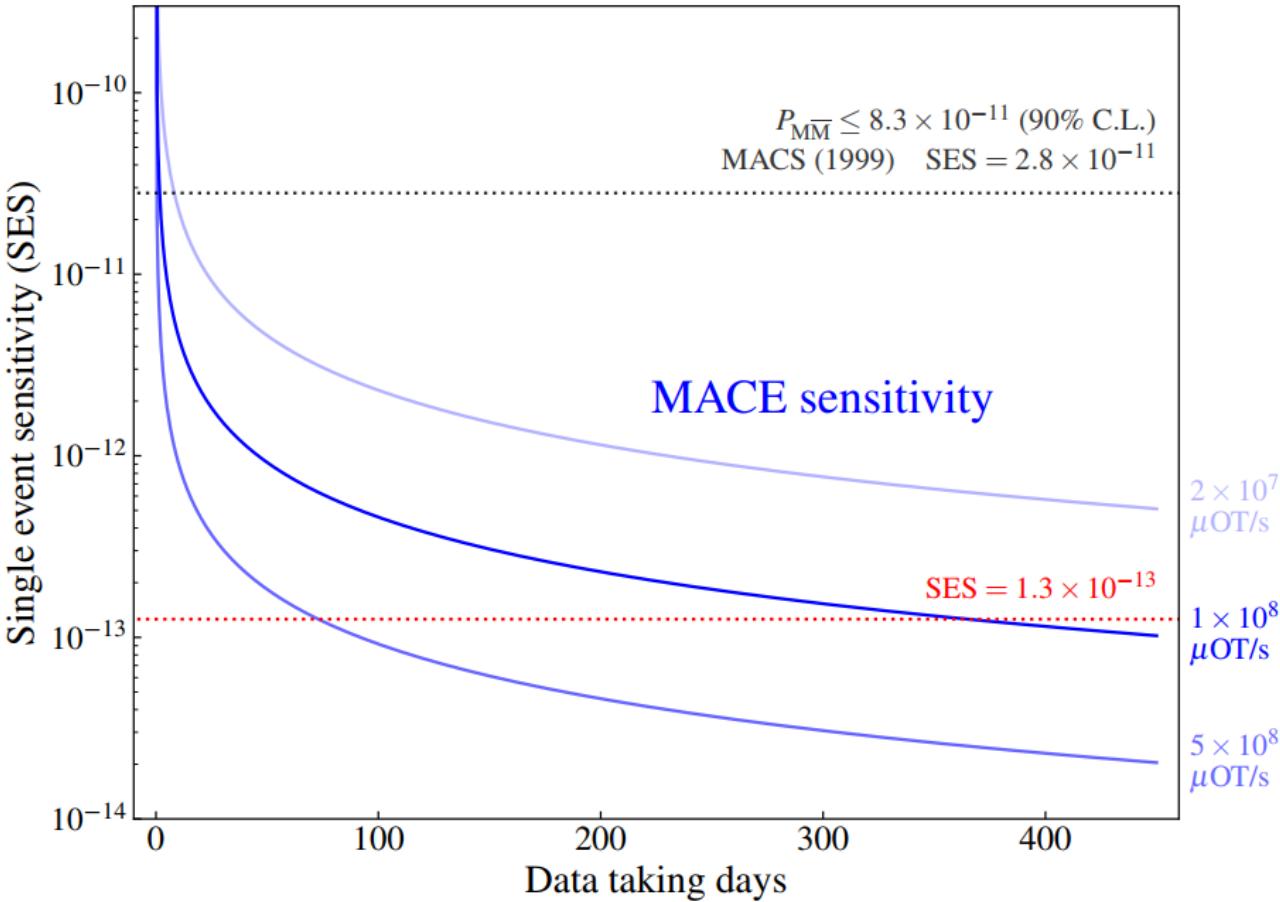
Reference: Snowmass2021 Whitepaper: Muonium to antimuonium conversion, arXiv:2203.11406

- Invited talks at ICHEP and Fermilab workshop, see also conference proceedings
<https://arxiv.org/abs/2309.05933>
- International Advisory Committee at NuFact



Plenary talk at CLFV2023, Heidelberg University

Breakthrough point for fundamental research



- The latest result was obtained by MACS in 1999, with a muon flux of $8 \times 10^6 \mu^+/\text{s}$.
- Requirement: China domestic accelerator muon source to provide $10^8 \mu^+/\text{s}$, surface muon.
- Over 20 years, significant advances in detector technology.
- China's accelerator and particle detection technology have made great strides.
- Currently, there are no ongoing muonium conversion experiments internationally.
- The new generation of experiments is expected to improve sensitivity by over two orders of magnitude compared to the 1999 PSI results!
- MACE is expected to be at the forefront of global research!

MACE: Muonium to Antimuonium Conversion Experiment.



MACE conceptual design report

arXiv:2410.18817v1 [hep-ex] 24 Oct 2024

Conceptual Design of the Muonium-to-Antimuonium Conversion Experiment (MACE)

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(Dated: October 25, 2024)

<https://indico.impcas.ac.cn/event/63/overview>

CDR review - 8/26

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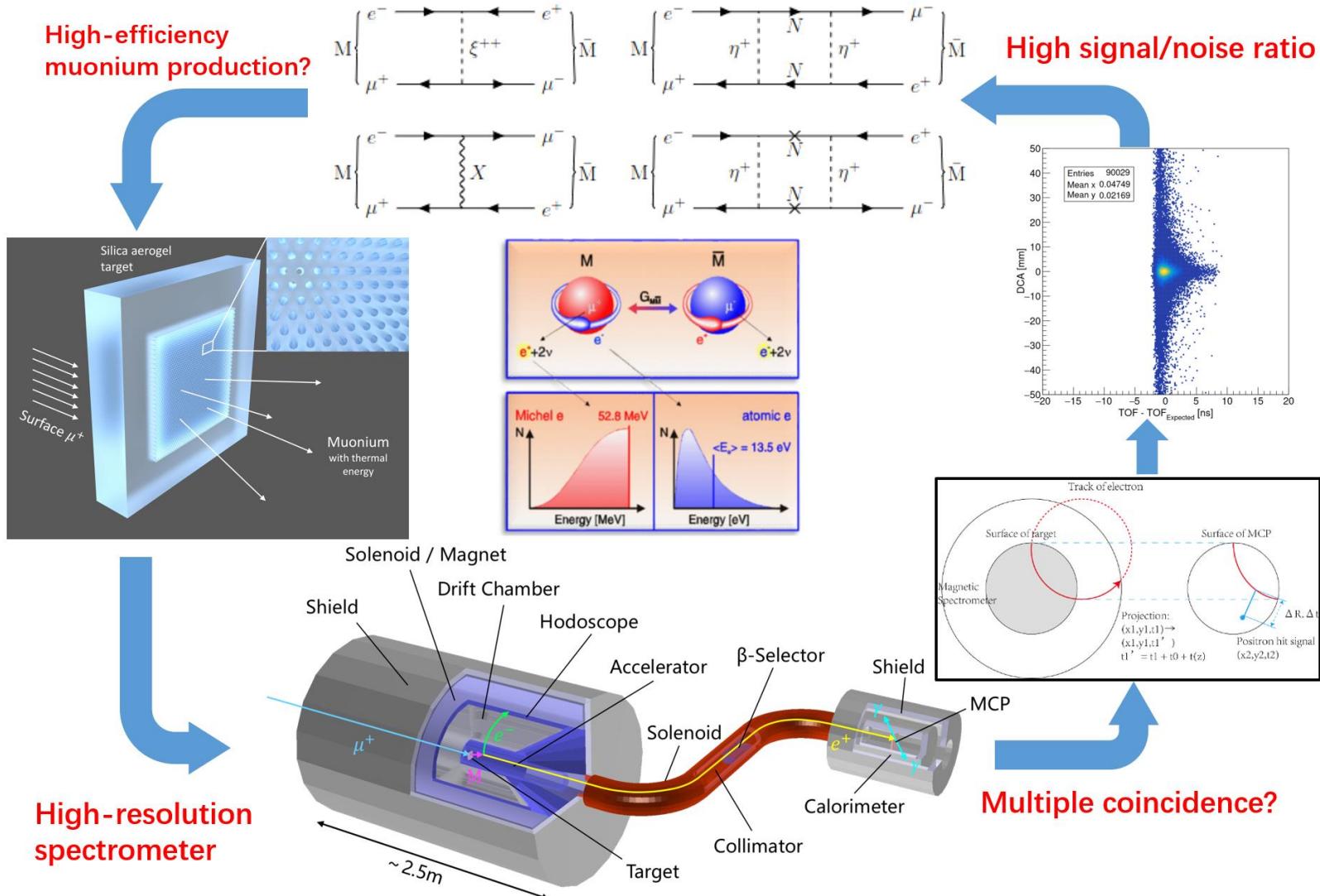


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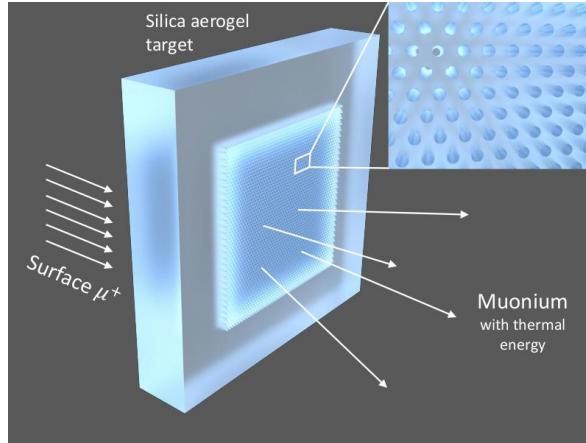
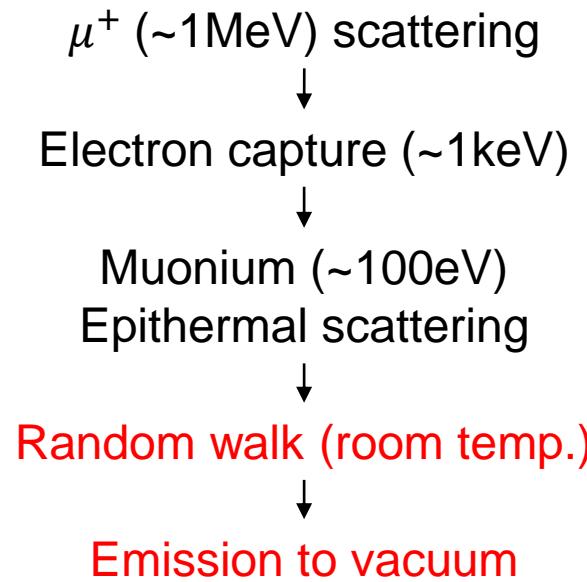
- Review of the past experience
- Motivations for muon physics
- Conceptual design of MACE
- Local laboratory: SMOOTH

MACE roadmap

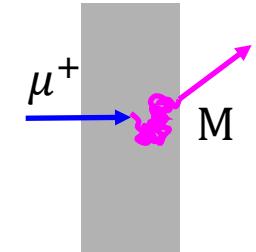
Muonium-to-Antimuonium Conversion Experiment



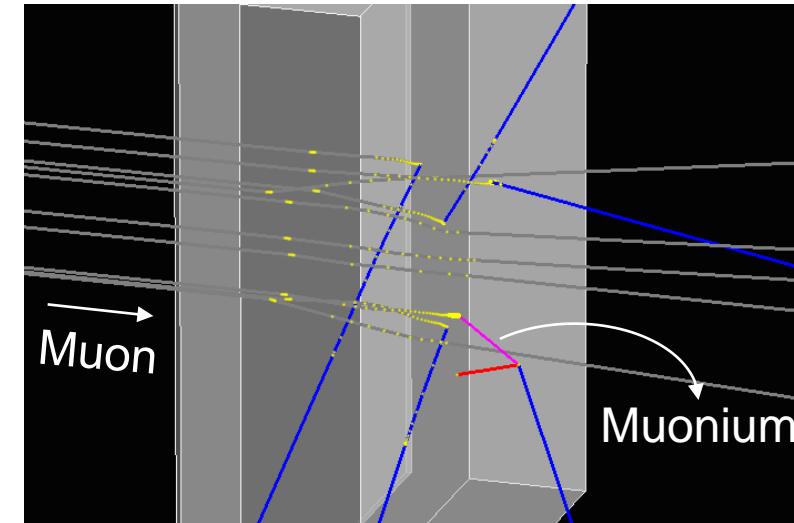
Muonium production in silica aerogel



MC simulation for muonium transport has been developed under the MACE offline software framework.

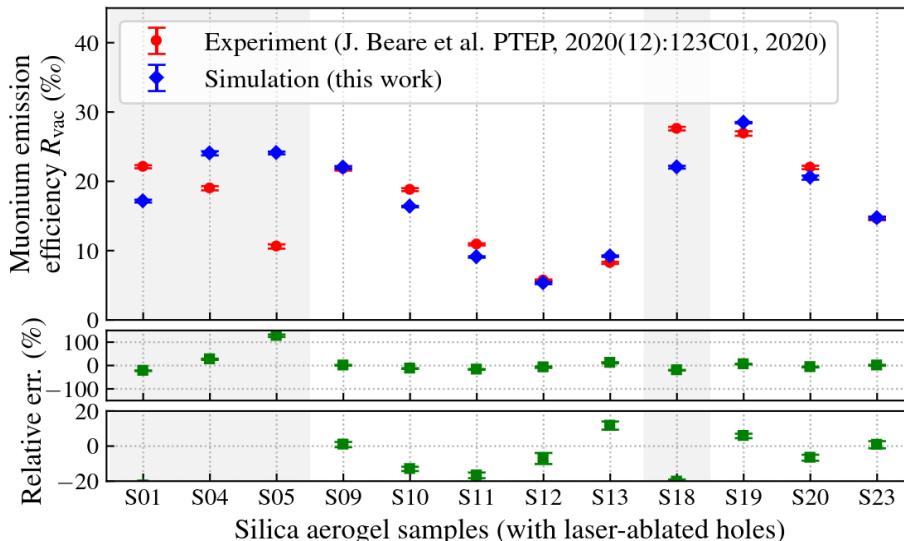


- ① Geant4 low-energy EM process.
- ② Geant4 AtRest process, modeled phenomenologically.
- ③ Random walk approach to thermal muonium tracking.

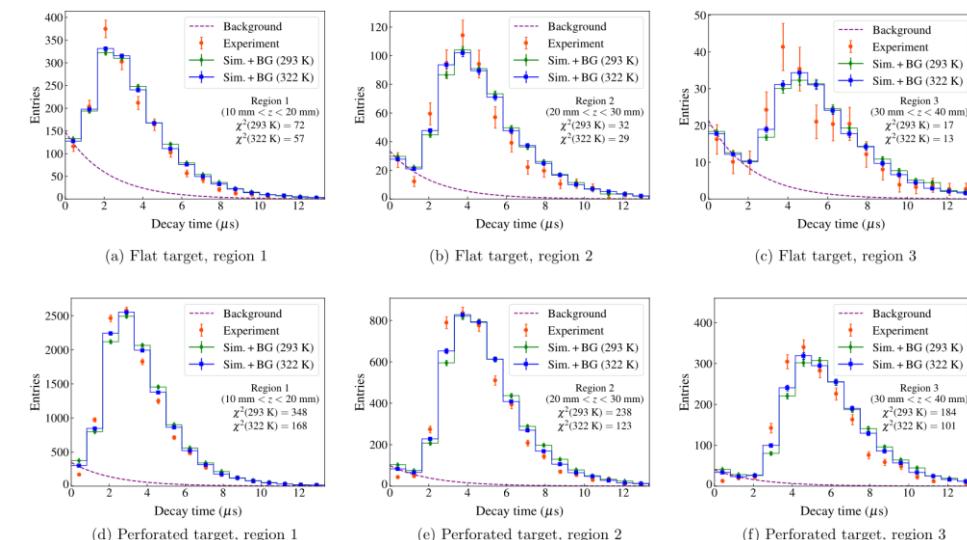
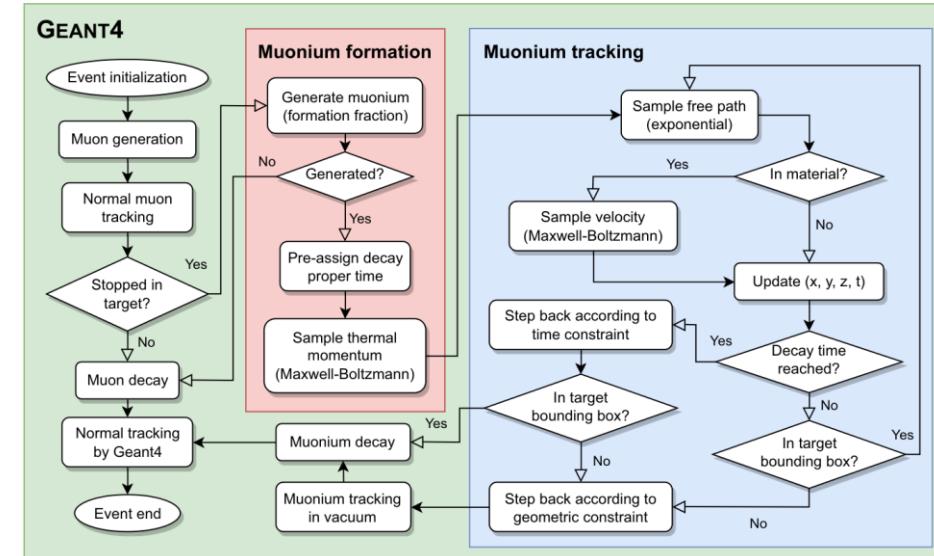


Optimization of muonium yield in perforated silica aerogel

- Intensity of in-vacuum muonium source: $I_M^{\text{vac}} = I_{\text{beam}} Y_{\mu \rightarrow M}$
- $Y_{\mu \rightarrow M}$ can be improved by utilizing porous materials, ideally perforated silica aerogel.
- An simulation method is developed to accurately simulate muonium production and diffusion.
- The simulation is validated by muonium yield data measured in TRIUMF and J-PARC.



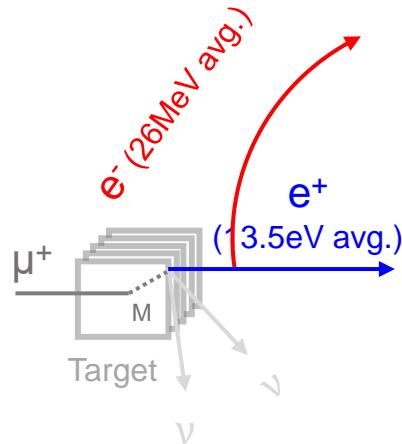
Shihuan Zhao and Jian Tang, Optimization of muonium yield in perforated silica aerogel, *Phys. Rev. D* 109, 072012. arXiv 2401.00222



MACE signal and background

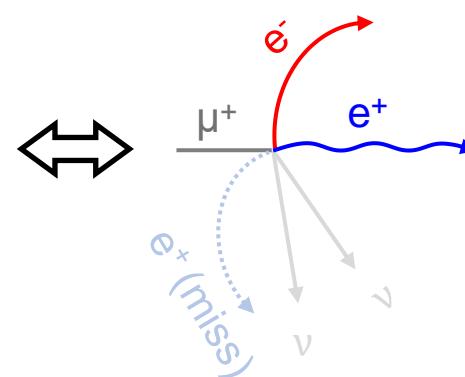
Signal:

fast e^- + slow e^+



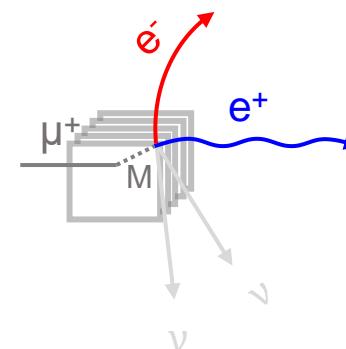
1. Internal conv. (IC) decay

$$\square \mu^+ \rightarrow e^+ e^- e^+ \bar{\nu}_\mu \nu_e$$



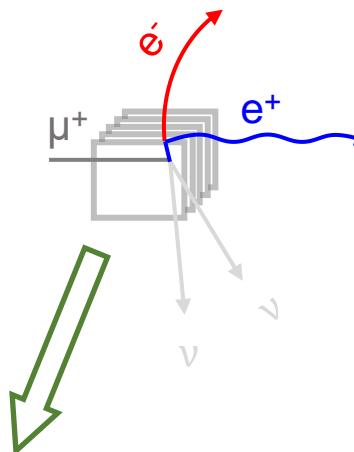
2. Final state scattering

$$\square M \rightarrow e^+ \bar{\nu}_\mu \nu_e e^-$$

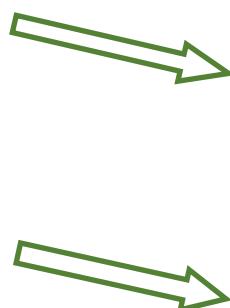


3. Accidental bkg.

- Scattering/conv. e^-
- Misreconstruction
- Cosmic ray, etc.

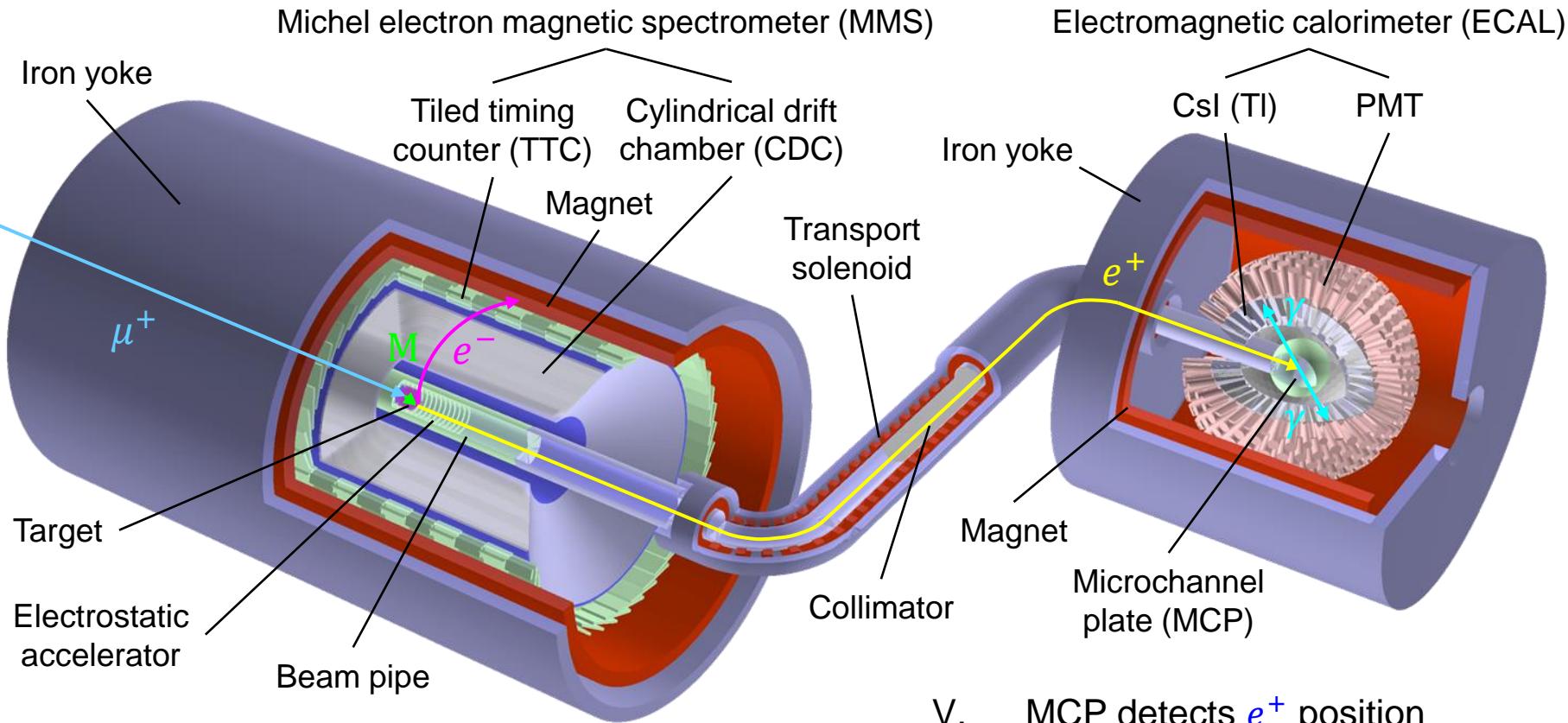


- **Coincidence of a fast e^- and a slow e^+**
- **Common vertex (by selecting e^+/e^- track DCA)**
 - ✓ Select p_{xy} of e^+
 - ✓ Reject accidental e^-
- **Time coincidence (by selecting e^+ TOF)**
 - ✓ Select p_z of e^+
 - ✓ Reject e^+ from IC decay or Bhabha scattering
- **Charge identification (by e^- track & e^+ annihilation)**



- A "clean" data taking duration
 - Pulsed muon beam
- Excellent vertex resolution
 - e^+/e^- spatial resolution
 - Precise e^+ transport in EM field
- Excellent time resolution
 - e^+/e^- time resolution

MACE baseline design v1



- I. Surface muon stop in target → muonium
- II. M diffuse into vacuum & convert to \bar{M}
- III. Decay in a vacuum: $\bar{M} \rightarrow e^+ e^- \nu_\mu \bar{\nu}_e$
- IV. CDC detects **Michel e^-** track
- V. Transport **atomic e^+** to MCP (conserving transverse position)

- V. MCP detects **e^+** position
- VI. **e^+** annihilates on MCP
- VII. ECal detects 2 back-to-back annihilation γ

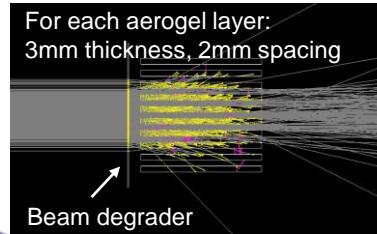
Triple coincidence:
 ➤ **MMS + MCP + ECal**

Michel e^- Atomic e^+

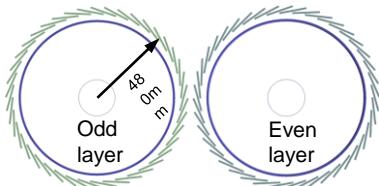
MACE baseline design v1

Muonium target:

- Silica aerogel with perforation surface.
- Multilayer design, 4% muonium yield in a vacuum.

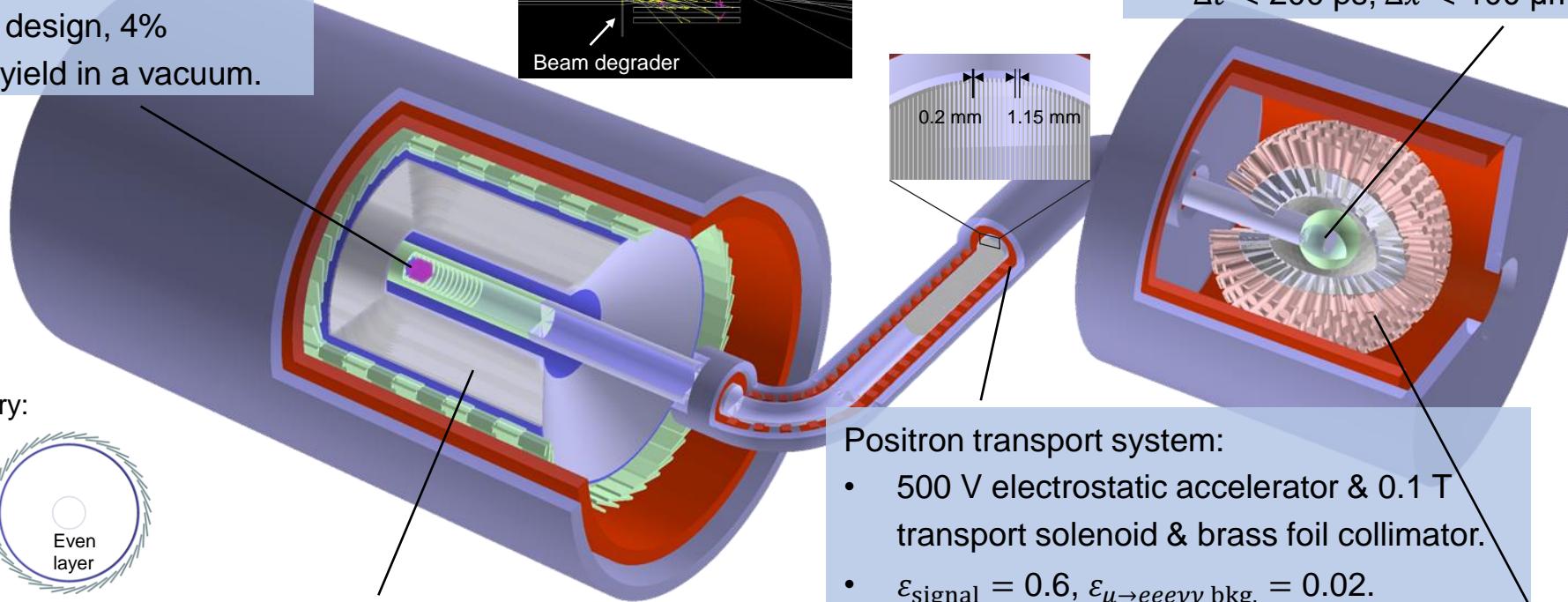


TTC geometry:



Magnetic spectrometer:

- 0.1 T axial magnetic field.
- CDC: $\text{He}(\text{C}_4\text{H}_{10})$ gas, 21 layers, 3540 cells. 89% geometry acceptance, $\Delta p \approx 500$ keV.
- TTC: 756 fast scintillators with SiPM readout, slant ± 15 deg, $\Delta t < 100$ ps.



Microchannel plate (MCP) specifications:

- Signal (e^+ 500 eV) efficiency > 0.7
- $\Delta t < 200$ ps, $\Delta x < 100$ μm .

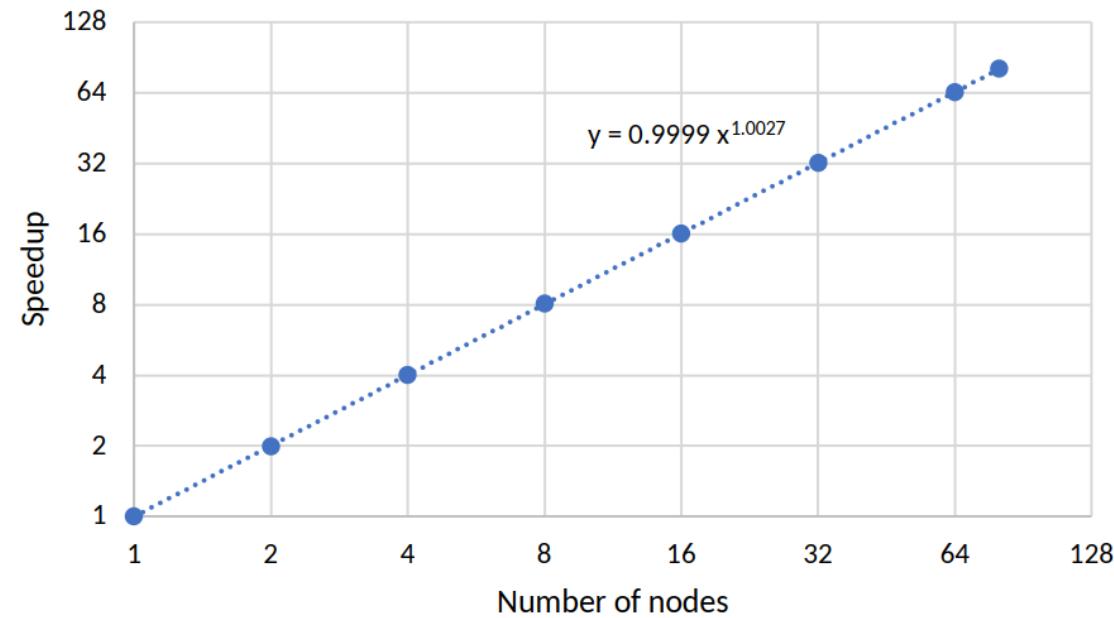
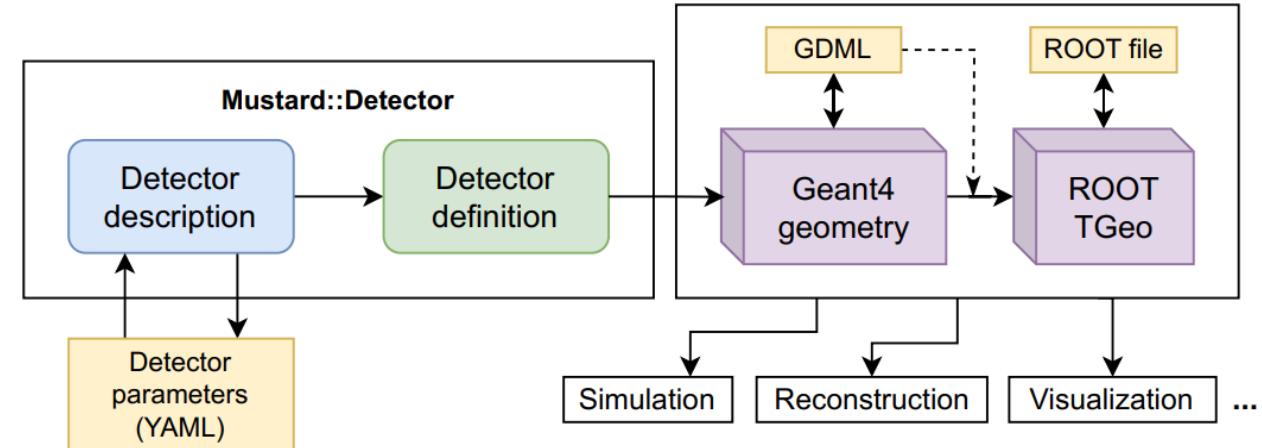
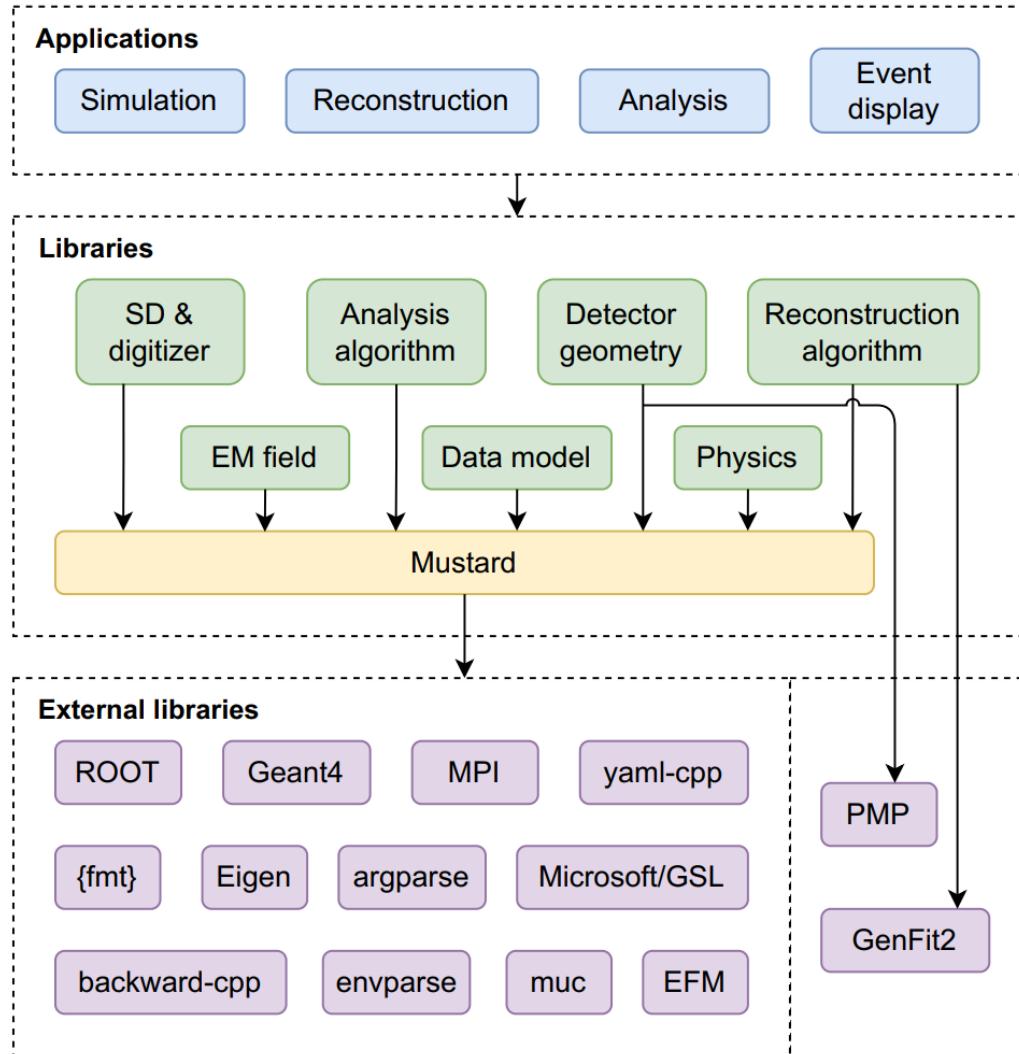
Positron transport system:

- 500 V electrostatic accelerator & 0.1 T transport solenoid & brass foil collimator.
- $\varepsilon_{\text{signal}} = 0.6$, $\varepsilon_{\mu \rightarrow eeevv \text{ bkg.}} = 0.02$.
- Signal e^+ position error 100 μm .

Electromagnetic calorimeter:

- Geometry: Class-I GP(4,0) Goldberg polyhedron.
- 622 CsI(Tl) crystals with 10 cm length, PMT readout.
- 97% geoemtry acceptance, $\Delta E/E = 7.5\%$ (signal 2γ event), 67.5% signal efficiency.

MACE offline software





MACE sensitivity

- Summary of current full simulation results:

Background		count / (10 ⁸ μ/s × 365 d)
	$\mu^+ \rightarrow e^+ e^- e^+ \bar{\nu}_\mu \nu_e$	0.287 ± 0.020
Accidental	Beam positron	< 0.07
	Cosmic ray (w/ veto)	< 0.1
Total		< 1

✓ O(10⁻¹⁴) single event sensitivity is expected:

$$\text{SES} = \frac{1}{\varepsilon_{\text{Geom}} \varepsilon_{\text{MMS}} \varepsilon_{\text{MCP}} \varepsilon_{\text{ECal}} \varepsilon_{\text{cut}} y_M N_{\mu^+}} = 1.3 \times 10^{-13}$$

- More background simulations and refined data analyses to be updated!

Detector, component or analysis	Efficiency type	Efficiency value
Magnetic spectrometer (MMS)	Geometric efficiency ($\varepsilon_{\text{MMS}}^{\text{geom}}$)	84.6%
	Reconstruction efficiency ($\varepsilon_{\text{MMS}}^{\text{recon}}$)	$\sim 80\%$
Positron transport system (PTS)	Transmission efficiency (ε_{PTS})	65.8%
	Microchannel plate (MCP)	Detection efficiency (ε_{MCP})
Electromagnetic calorimeter (ECAL)	Incident efficiency $\varepsilon_{\text{ECAL}}^{\text{In}}$	63.4%
	Geometric efficiency $\varepsilon_{\text{ECAL}}^{\text{Geom}}$	95.3%
	Reconstruction efficiency $\varepsilon_{\text{ECAL}}^{\text{Recon}}$	94.0%
Total detection efficiency		8.25%
Analysis		Signal efficiency (ε_{Cut})
Total signal efficiency		6.6%

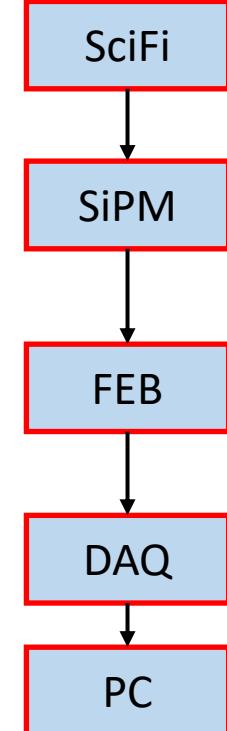
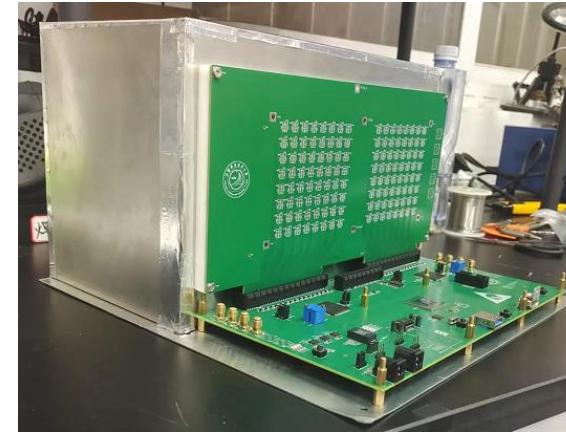
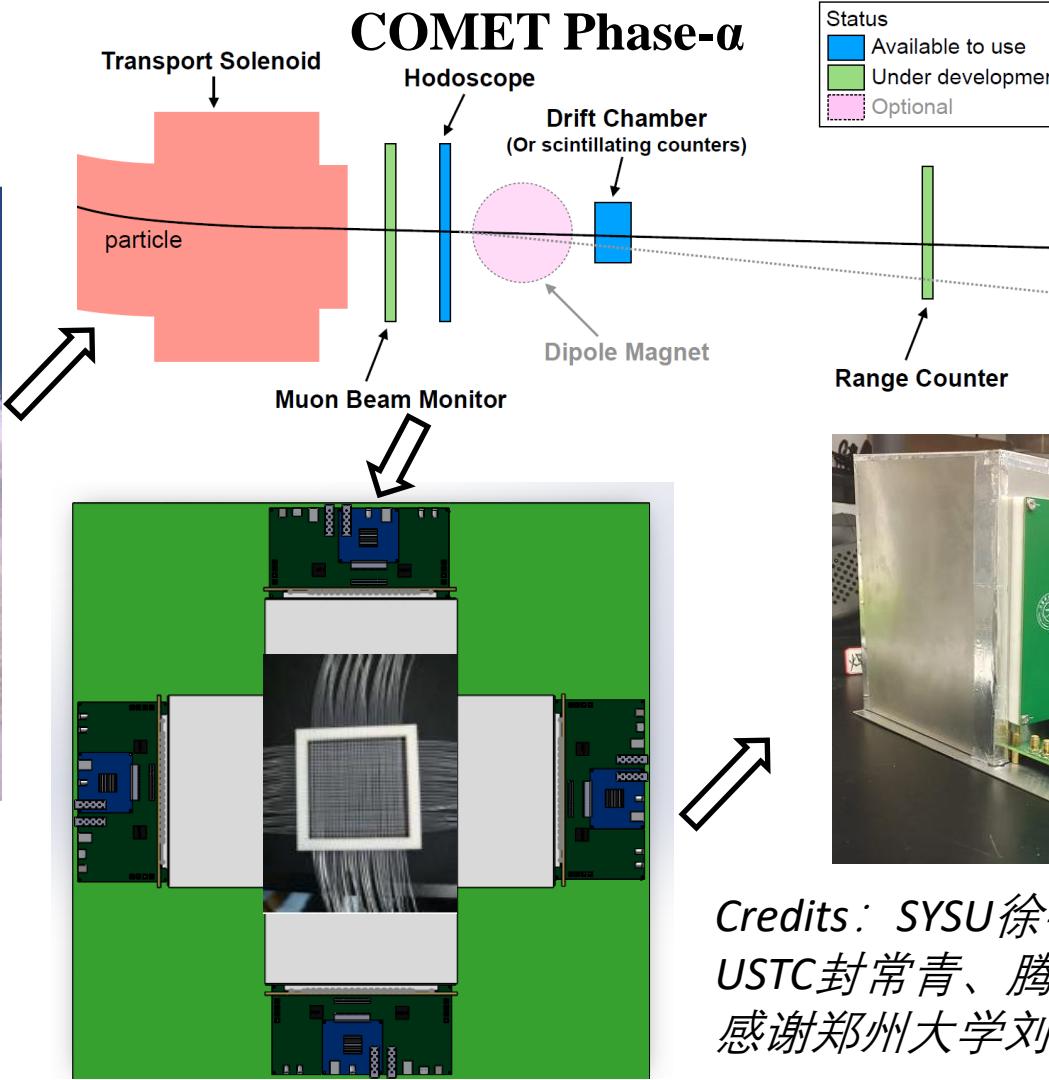
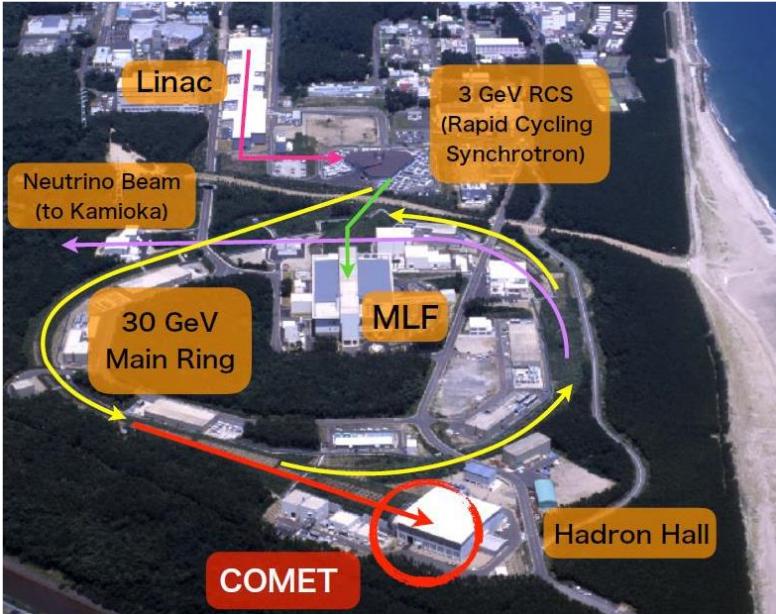


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- Motivations for muon physics
- Conceptual design of MACE
- Local laboratory: SMOOTH



Muon beam monitor for COMET



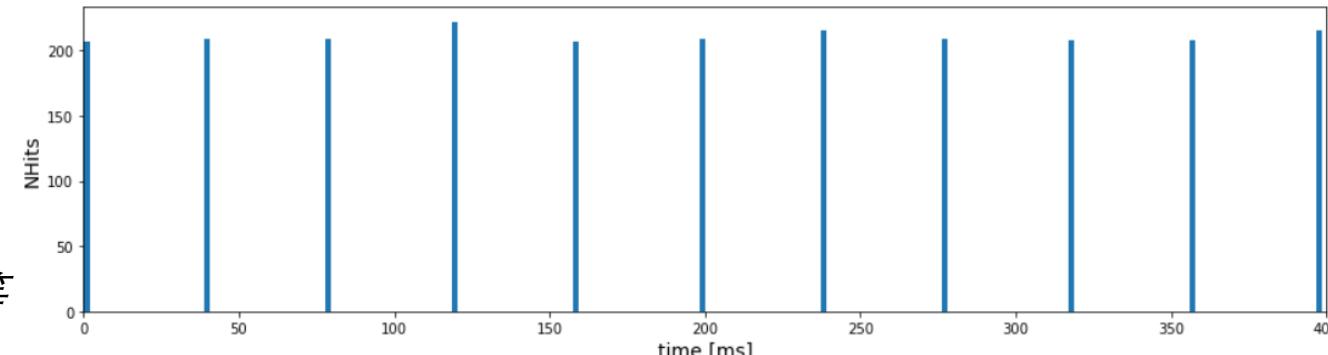
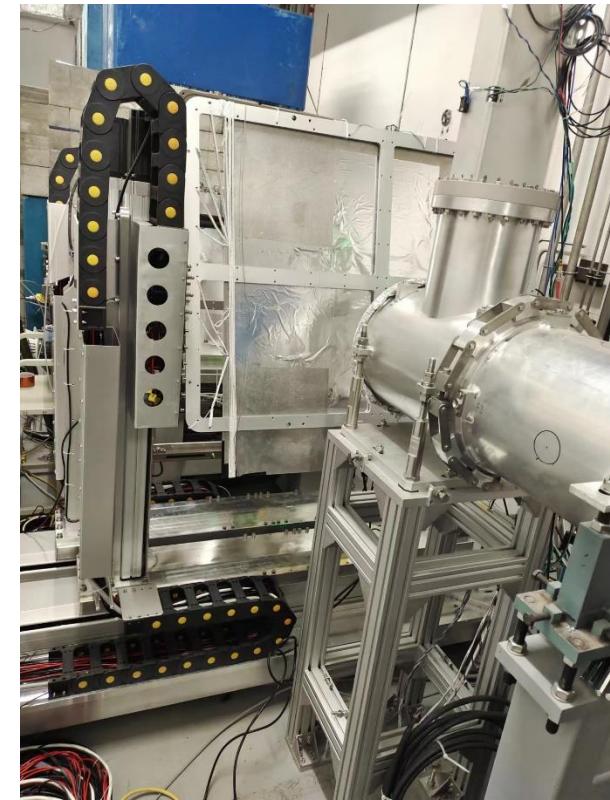
e-Print: [2308.15253](https://arxiv.org/abs/2308.15253) [physics.ins-det]

Nuclear Science and Techniques 35 (2024) 4, 79

Credits: SYSU徐宇、宁云松、孙铭辰、余涛等
USTC封常青、腾尧、秦治臻等
感谢郑州大学刘义的帮助!

Muon beam monitor for COMET

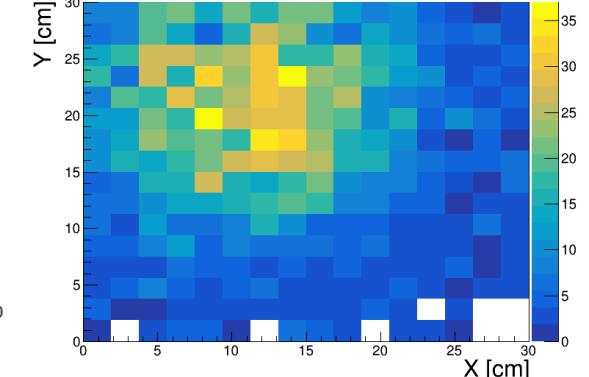
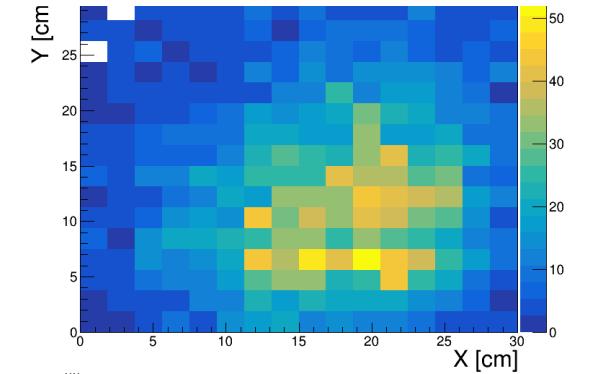
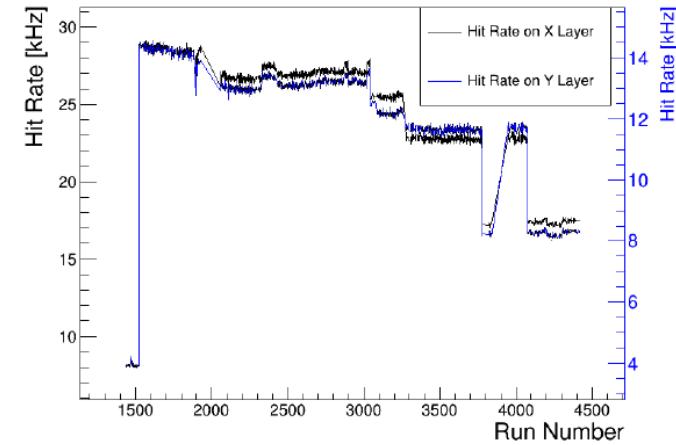
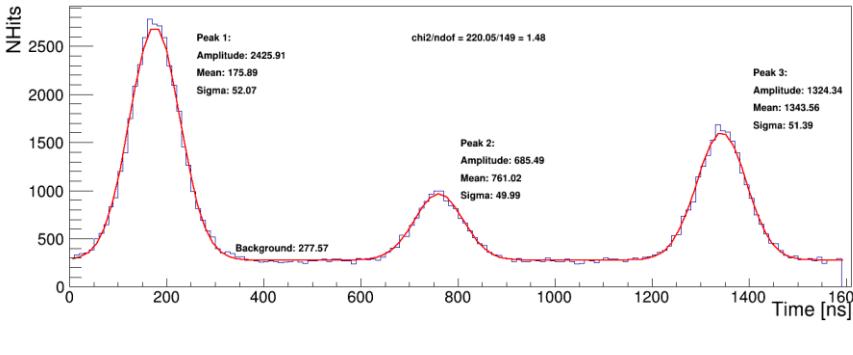
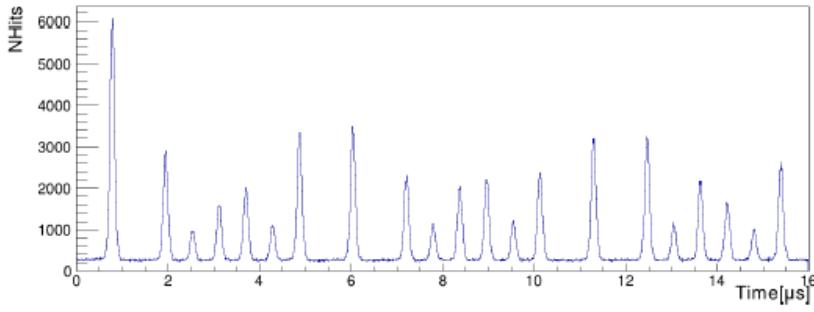
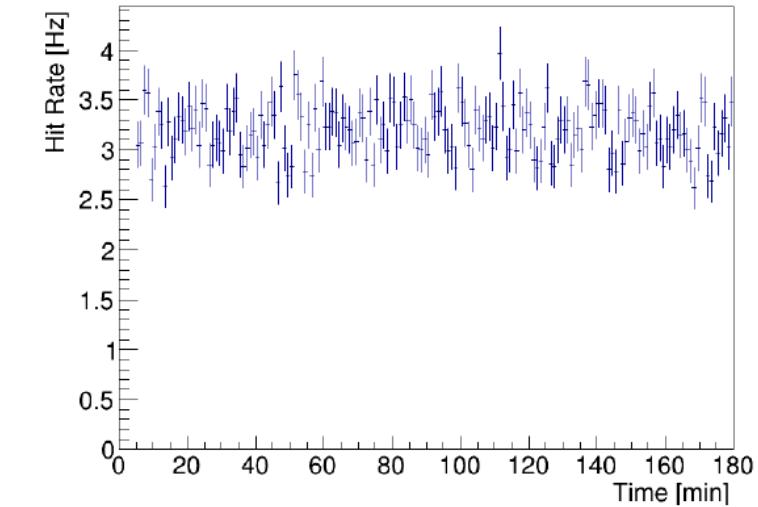
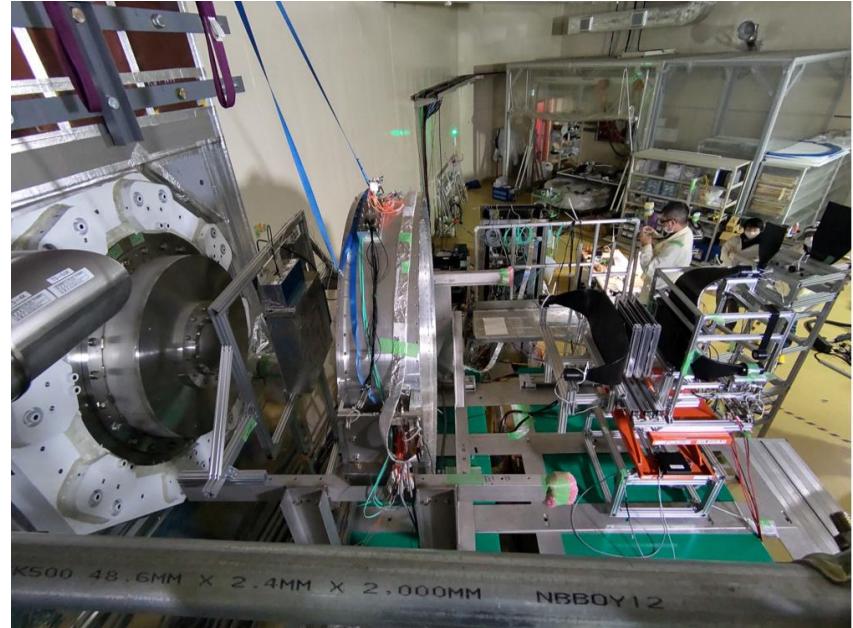
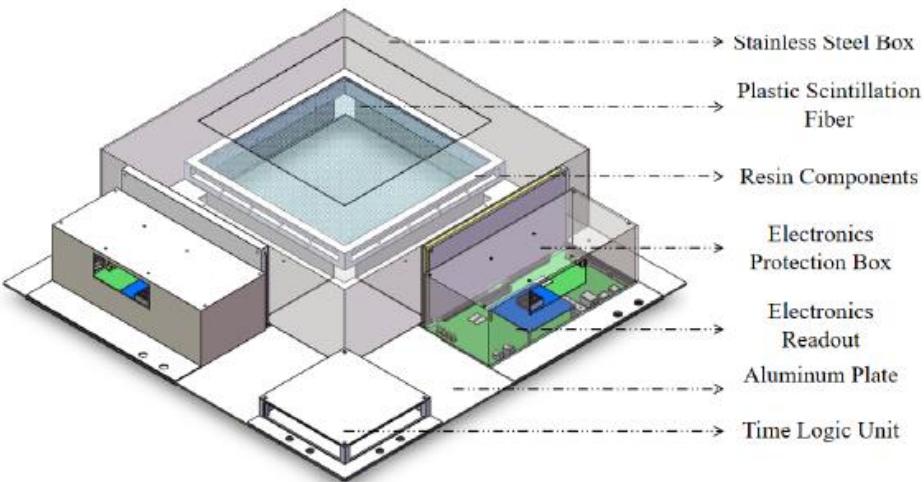
- CSNS proton beam time: 2022/7/20
- Beam window:
 - $1\text{cm} \times 1\text{cm}$
 - Energy: 30 MeV, 35 MeV, 40 MeV, 45 MeV, 50 MeV, 55 MeV, 60 MeV
 - Time: 90s per point
 - Beam rate: 1.7×10^7 protons/s/cm²



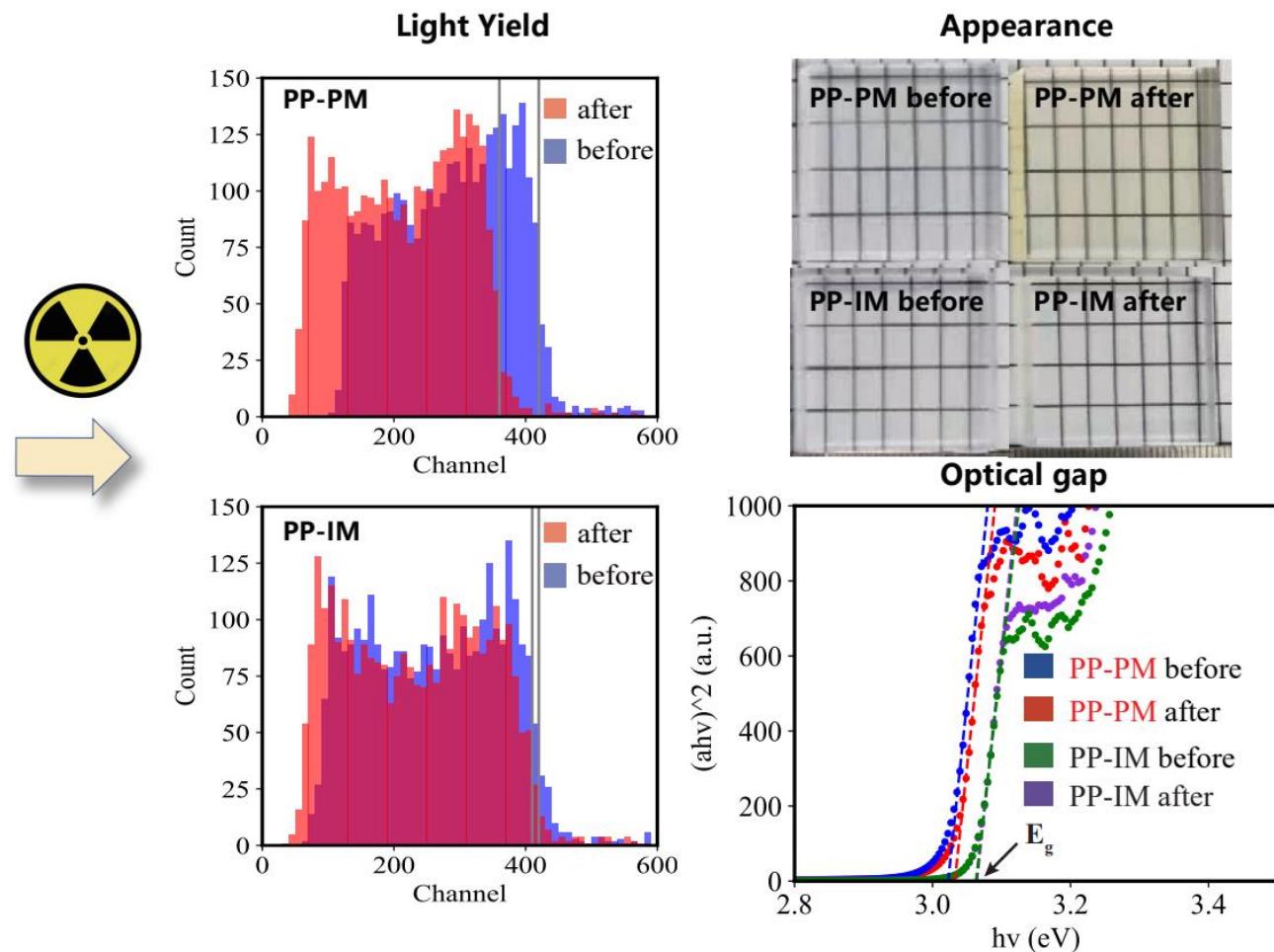
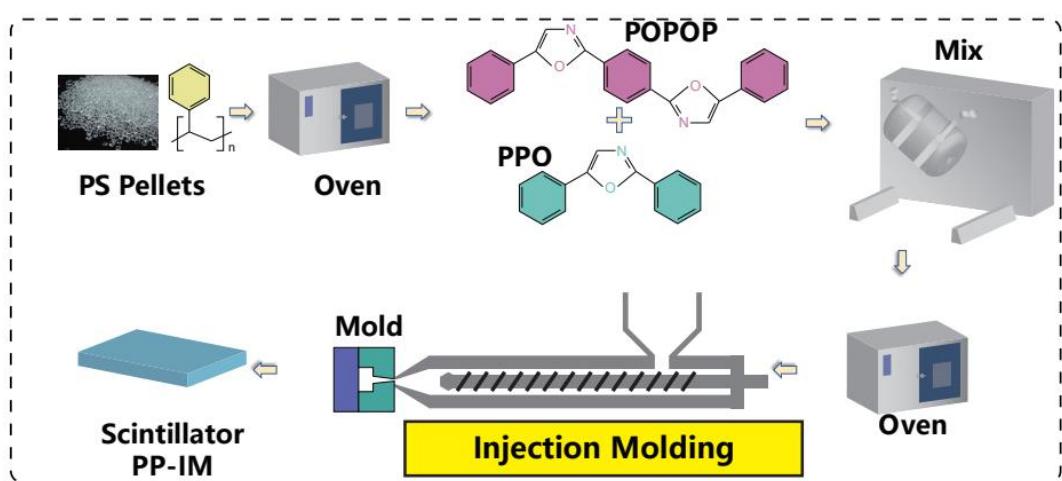
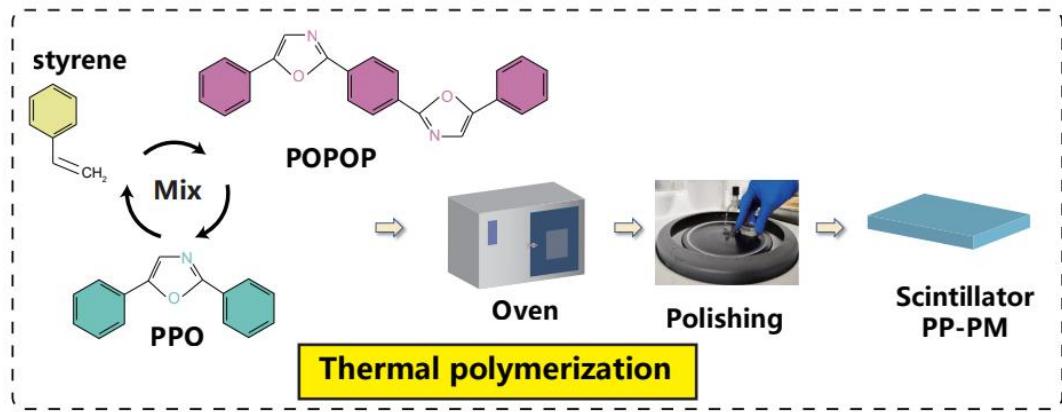
致谢: CSNS质子束流平台敬汉涛、谭志新等



Muon beam monitor for COMET



R&D of new plastic scintillators for muon detections



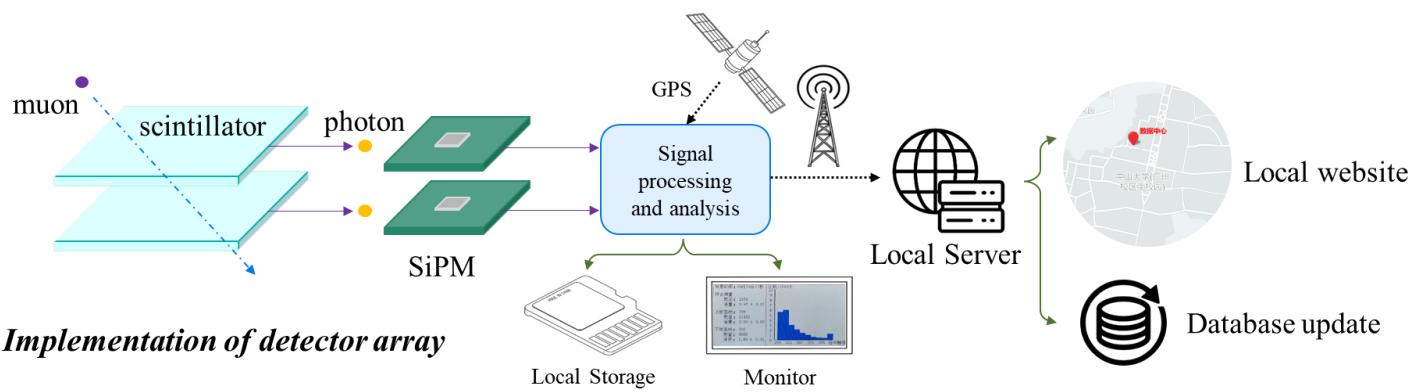
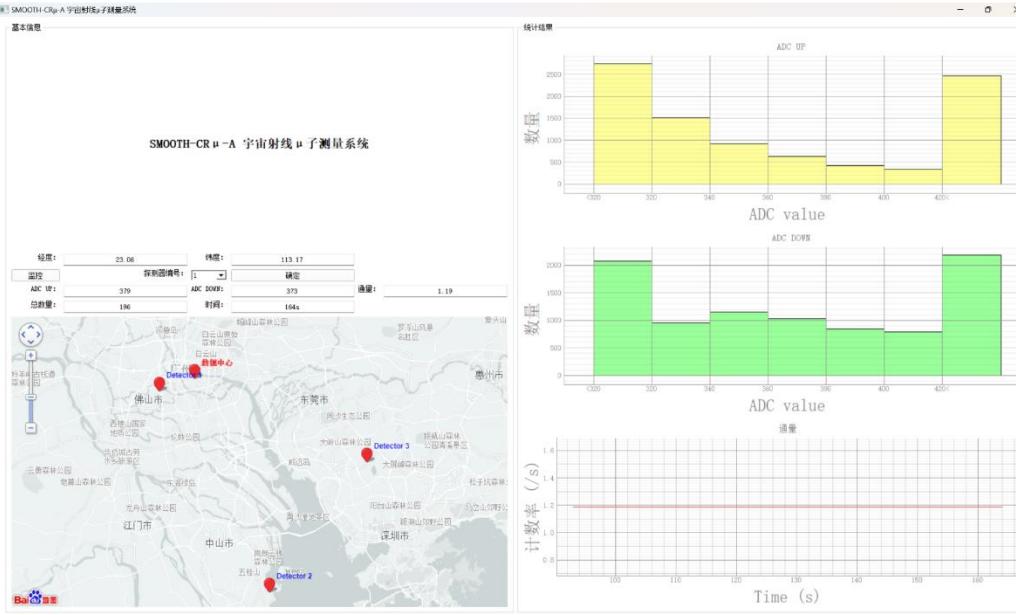
Enhancing plastic scintillator performance through advanced injection molding techniques

Credits: 钟嘉豪、阮天龙、Nouman、周剑等
Radiation Physics and Chemistry 226 (2025) 112193

Detector R&D with cosmic muons: CR μ SR



CR μ edu. kit array

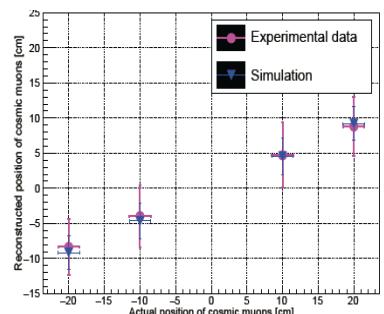
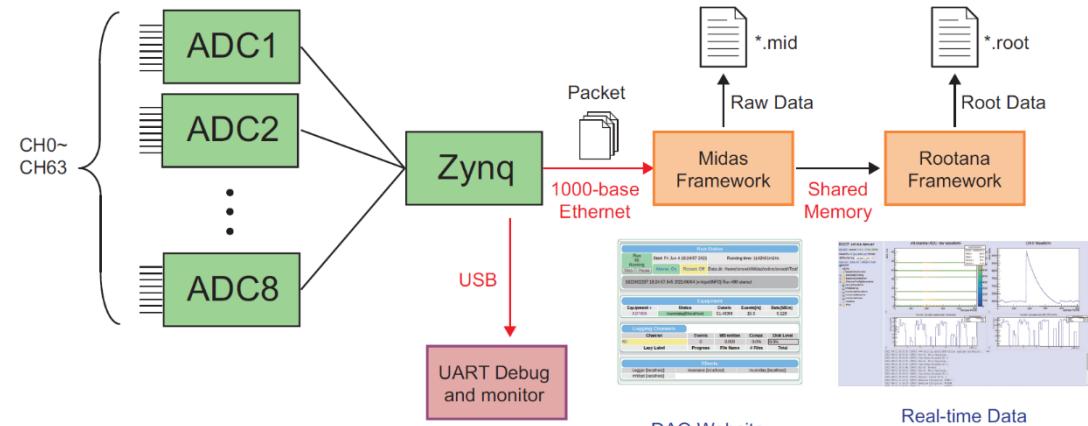
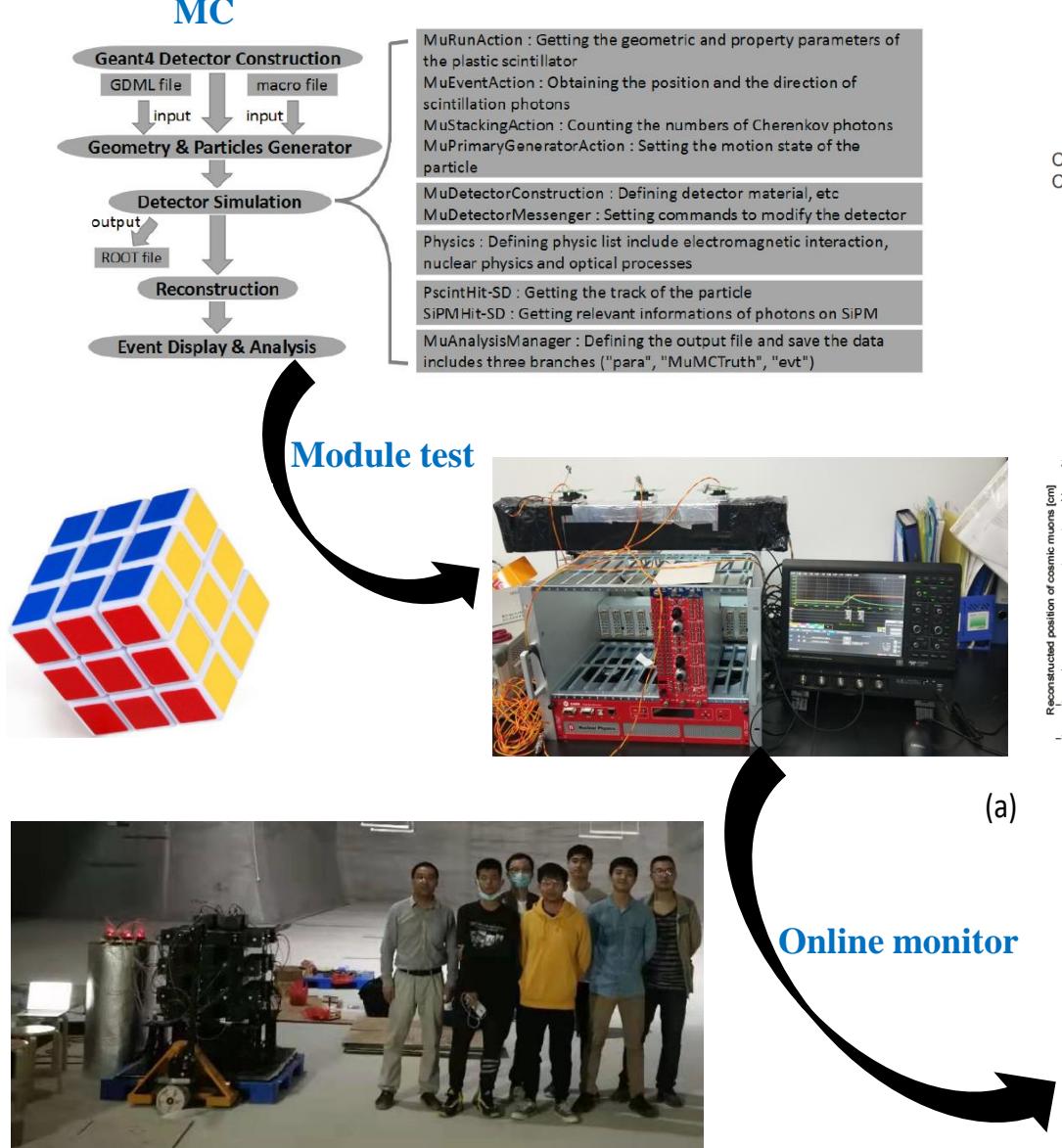


CR μ SR prototype

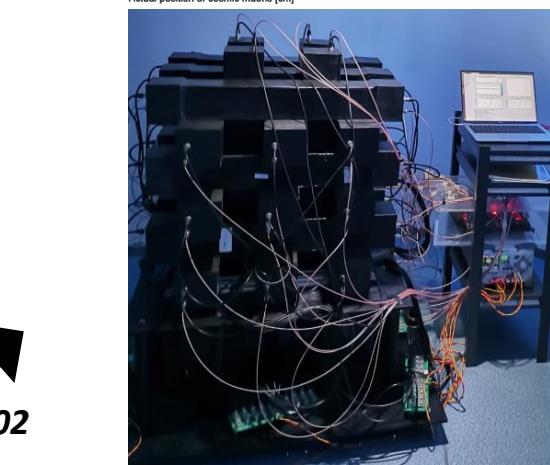


Taking data!

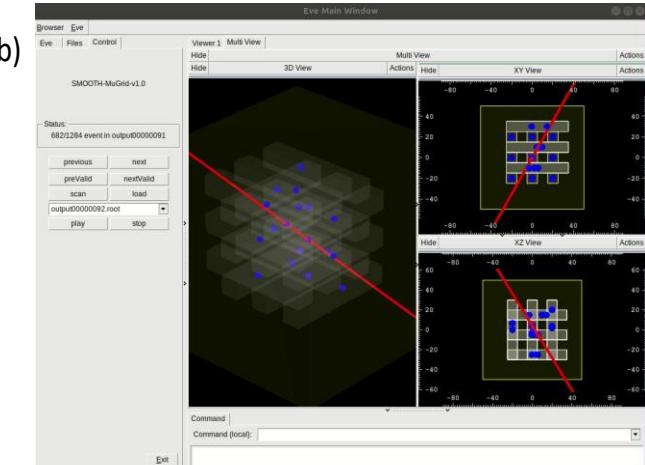
Detector R&D with cosmic muons: MuGrid



(a)



(b)



Customized electronics

Track reconstruction



Real-time Data Monitoring Website



Summary

- Muon physics is in the ascendant, enabling precise tests of QED theory and search for new physics beyond SM.
- MACE experiment will achieve a breakthrough in muon physics.
- Significant progress has been made in experiment design, muonium target design, and offline software development.
- Ongoing development of sub-detectors (MBM, EMCAL, etc.) and reconstruction algorithms.
- Local muon lab SMOOTH: cosmic muon detector, muon beam monitoring detector, μ SR prototype
- MACE Conceptual Design Report completed; Cutting-edge science will drive technological applications; looking forward to multidisciplinary applications after a development of SMOOTH- μ SR prototype.
- Great potential in muon physics — small sparks can ignite a prairie fire!
- Welcome collaborations and fruitful results!

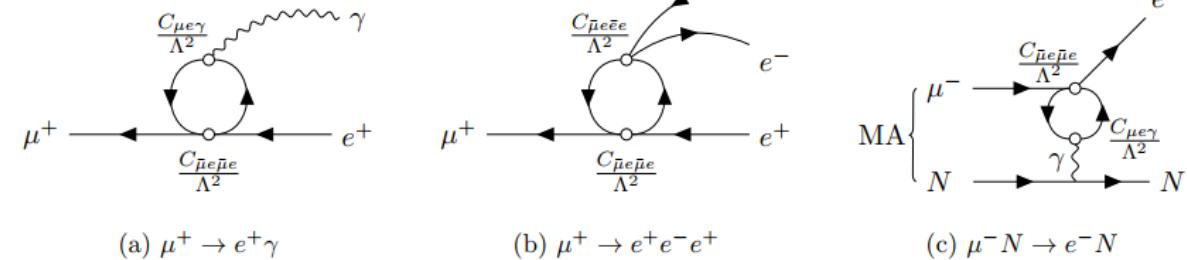
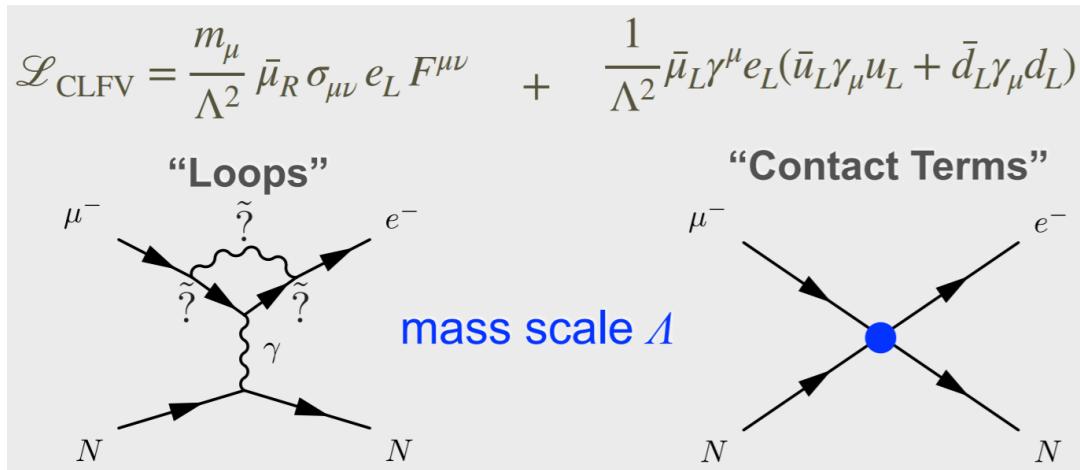
- *Thanks for the invitation from the local organization crew.*
- *In collaboration with Prof. Changqin Feng at electronics readout in detector R&D.*
- *Appreciate fabrication of Silica aerogel at school of material science and engineer by Prof. Jian Zhou.*
- *Supported by NSFC no. 12075326, Guangdong province and Guangzhou natural science foundation.*
- *Special thanks to SYSU and excellent bachelor students!*

Thanks!



SMEFT

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_{d>4} \sum_n \frac{c_n^{(d)}}{\Lambda^{d-4}} \mathcal{O}_n^{(d)}$$



REF: By A. DeGouvea and P. Vogel, arXiv:1303.4097. EFT treatment by S. Davidson and B. Echenard. arXiv: 2010.00317



SMEFT

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} \sum_i C_i^{(5)} Q_i^{(5)} + \frac{1}{\Lambda^2} \sum_i C_i^{(6)} Q_i^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right).$$

- The first-order effective operator beyond the Standard Model has a dimension of 5, corresponding to Λ^{-1} ; this order generates the Majorana mass term for neutrinos.
- Subsequent effective Lagrangian corresponds to Λ^{-2} , where operators of this order can produce cLFV at tree-level.
- Different processes typically exhibit sensitivity to certain classes of operators while being insensitive to others.
 - For example, muonium conversion is sensitive to the $\bar{\mu}e\bar{\mu}e$ coupling but not to $\bar{\mu}e\bar{e}e$ or $\bar{\mu}e\gamma$; so conversely do $\mu \rightarrow eee$ and $\mu \rightarrow e\gamma$.
- Muon conversion is directly generated by the $\bar{\mu}e\bar{\mu}e$ coupling, with $M^2 \propto \frac{1}{\Lambda^4}$;
- In contrast, $\mu \rightarrow e\gamma$ at the EFT tree-level does not involve the $\mu\bar{e}\bar{\mu}e$ coupling; if one insists to involve $\mu\bar{e}\bar{\mu}e$, it would require two EFT vertices, resulting in $M^2 \propto \frac{1}{\Lambda^8}$ suppression.



SMEFT

Ref: Julian Heeck and Mikheil Sokhashvili. Lepton flavor violation by two units. *Phys. Lett. B*, 852:138621, 2024.

- $\mu^+ e^- \rightarrow \mu^- e^+$ SMEFT Lagrangian with vector $\bar{\mu}e\bar{\mu}e$ couplings only:

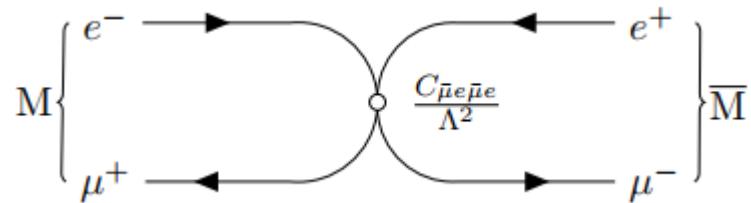
$$\begin{aligned}\mathcal{L}_{\text{SMEFT}}^{\Delta L_\mu=2} \supset & \frac{1}{\Lambda^2} \left(C_{\mu e \mu e}^{LL} (\bar{\mu}_L \gamma^\alpha e_L) (\bar{\mu}_L \gamma_\alpha e_L) \right. \\ & + C_{\mu e \mu e}^{LR} (\bar{\mu}_L \gamma^\alpha e_L) (\bar{\mu}_R \gamma_\alpha e_R) \\ & \left. + C_{\mu e \mu e}^{RR} (\bar{\mu}_R \gamma^\alpha e_R) (\bar{\mu}_R \gamma_\alpha e_R) \right) + \text{h.c.}\end{aligned}$$

- 3 Wilson coefficients, time-independent conversion probability writes

$$\begin{aligned}P \approx & \frac{1}{\Lambda^4} \left(\frac{7.58 \times 10^{-7}}{G_F^2} |C_{\mu e \mu e}^{LL} + C_{\mu e \mu e}^{RR} - 1.68 C_{\mu e \mu e}^{LR}|^2 \right. \\ & \left. + \frac{4.27 \times 10^{-7}}{G_F^2} |C_{\mu e \mu e}^{LL} + C_{\mu e \mu e}^{RR} + 0.68 C_{\mu e \mu e}^{LR}|^2 \right).\end{aligned}$$



SMEFT



Complete $\mu^+ e^- \rightarrow \mu^- e^+$ SMEFT Lagrangian, with vector and scalar couplings:

$$\begin{aligned}\mathcal{L}_{\text{SMEFT}} &= \mathcal{L}_{\text{SM}} + \sum_{n>4} \frac{1}{\Lambda^{n-4}} \sum_i C_i^{(n)} Q_i^{(n)} \\ &= \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \sum_i C_i^{(5)} Q_i^{(5)} + \frac{1}{\Lambda^2} \sum_i C_i^{(6)} Q_i^{(6)} + \dots\end{aligned}$$

$$Q_V^{LL} = (\bar{\mu}_L \gamma_\alpha e_L) (\bar{\mu}_L \gamma^\alpha e_L), \quad Q_V^{RR} = (\bar{\mu}_R \gamma_\alpha e_R) (\bar{\mu}_R \gamma^\alpha e_R),$$

$$Q_V^{LR} = (\bar{\mu}_L \gamma_\alpha e_L) (\bar{\mu}_R \gamma^\alpha e_R),$$

$$Q_S^{LR} = (\bar{\mu}_L e_R) (\bar{\mu}_L e_R), \quad Q_S^{RL} = (\bar{\mu}_R e_L) (\bar{\mu}_R e_L).$$

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} (C_V^{LL} Q_V^{LL} + C_V^{RR} Q_V^{RR} + C_V^{LR} Q_V^{LR} + C_S^{LR} Q_S^{LR} + C_S^{RL} Q_S^{RL} + C_V^{L\nu} Q_V^{L\nu} + C_V^{R\nu} Q_V^{R\nu})$$

Follows the same steps as that for the $B\bar{B}$ or $K\bar{K}$ mixing

$$P(M \rightarrow \bar{M}) = S_B(B_0, f_P) (f_P P(M_P \rightarrow \bar{M}_P) + (1 - f_P) P(M_V \rightarrow \bar{M}_V))$$

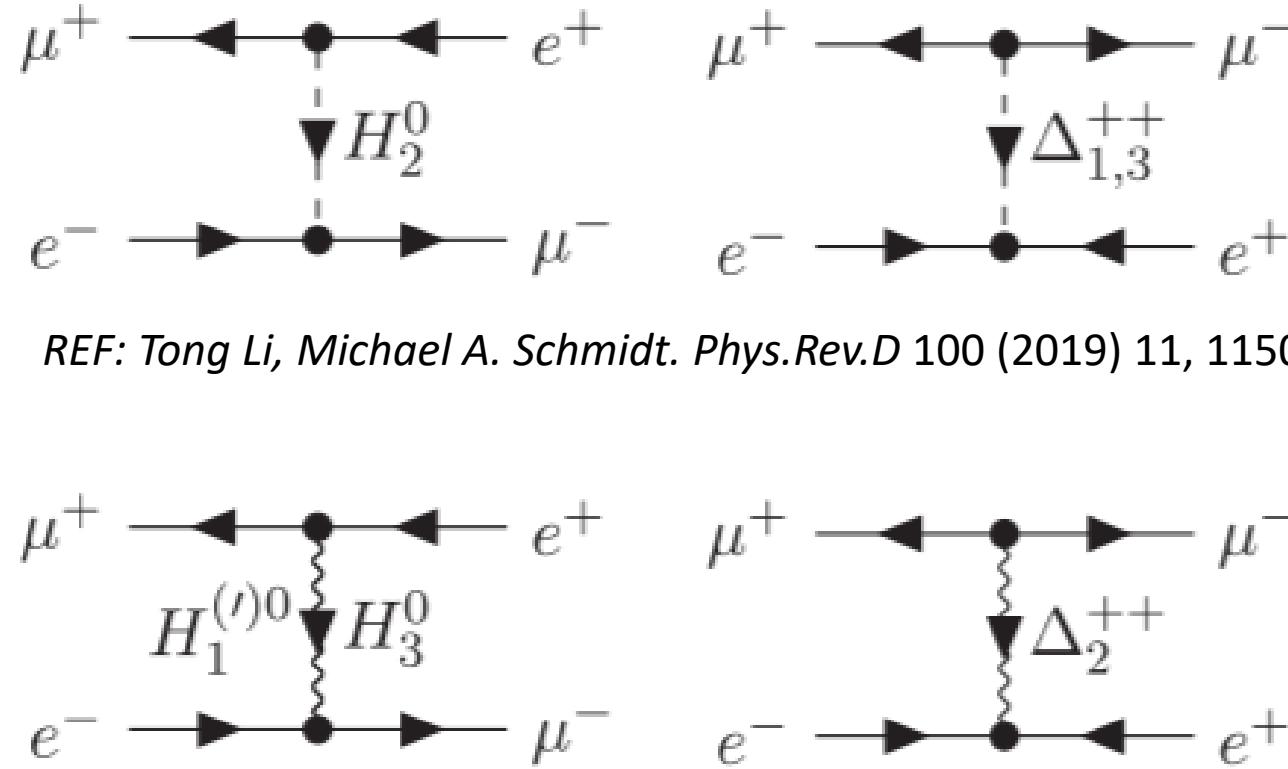
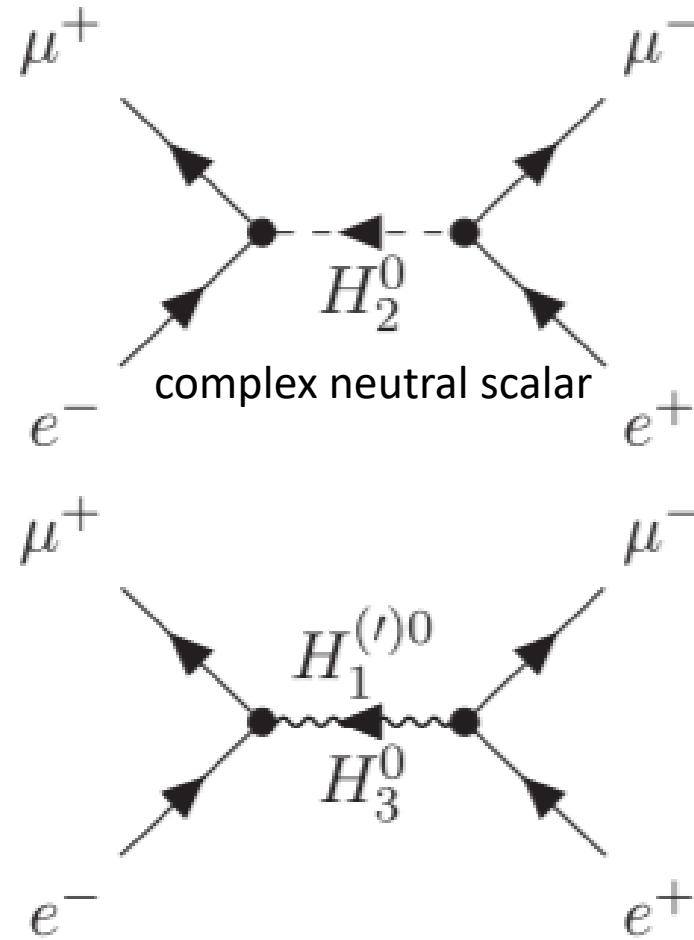
$$P(M \rightarrow \bar{M}) = \left(\frac{f_P}{2} (x_P^2 + y_P^2) + \frac{1 - f_P}{2} (x_V^2 + y_V^2) \right) S_B(B_0, f_P)$$

$$\begin{aligned}x_P &= \frac{4(\alpha\mu)^3}{\pi\Gamma\Lambda^2} \left(C_V^{LL} + C_V^{RR} - \frac{3}{2} C_V^{LR} - \frac{1}{4} (C_S^{LR} + C_S^{RL}) \right), & x_V &= -\frac{12(\alpha\mu)^3}{\pi\Gamma\Lambda^2} \left(C_V^{LL} + C_V^{RR} + \frac{1}{2} C_V^{LR} + \frac{1}{4} (C_S^{LR} + C_S^{RL}) \right), \\ y_P &= \frac{G_F}{\sqrt{2}\Lambda^2} \frac{m^2(\alpha\mu)^3}{\pi^2\Gamma} (C_V^{L\nu} - C_V^{R\nu}), & y_V &= -\frac{G_F}{\sqrt{2}\Lambda^2} \frac{m^2(\alpha\mu)^3}{\pi^2\Gamma} (5C_V^{L\nu} + C_V^{R\nu}).\end{aligned}$$

$$\begin{aligned}P(M \rightarrow \bar{M}) &= \frac{8(\alpha\mu)^6}{\pi^2\Gamma^2\Lambda^4} \left(f_P \left(C_V^{LL} + C_V^{RR} - \frac{3}{2} C_V^{LR} - \frac{1}{4} (C_S^{LR} + C_S^{RL}) \right)^2 \right. \\ &\quad \left. + 9(1 - f_P) \left(C_V^{LL} + C_V^{RR} + \frac{1}{2} C_V^{LR} + \frac{1}{4} (C_S^{LR} + C_S^{RL}) \right)^2 \right) S_B(B_0, f_P)\end{aligned}$$

Process	Type	Experiment	Current bound
$M \rightarrow \bar{M}$	$M - \bar{M}$ mixing	MACS [10], MACE	$P < 8.3 \times 10^{-11}/S_B(0.1 \text{ T})$ [10]
$\mu^+ e^- \rightarrow \mu^- e^+$			
$\mu^+ \mu^+ \rightarrow e^+ e^+$	Scattering	μ TRISTAN [39]	None
$\mu^+ \mu^+ \rightarrow \tau^+ \tau^+$			
$\mu^+ \rightarrow e^+ \bar{\nu}_e \nu_\mu$		τ_μ measurement	$\Delta\tau_\mu/\tau_\mu = 1 \times 10^{-6}$ [23]
$Z \rightarrow \ell'^\pm \ell'^\pm \ell^\mp \ell^\mp$	Decay	CEPC [40], FCC-ee [41]	None

Model dependent muonium conversion



REF: Tong Li, Michael A. Schmidt. Phys.Rev.D 100 (2019) 11, 115007