Parity and Bell tests

第三届高能物理理论与
实验融合发展研讨会

arXiv:2409.15418

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- ## Research background

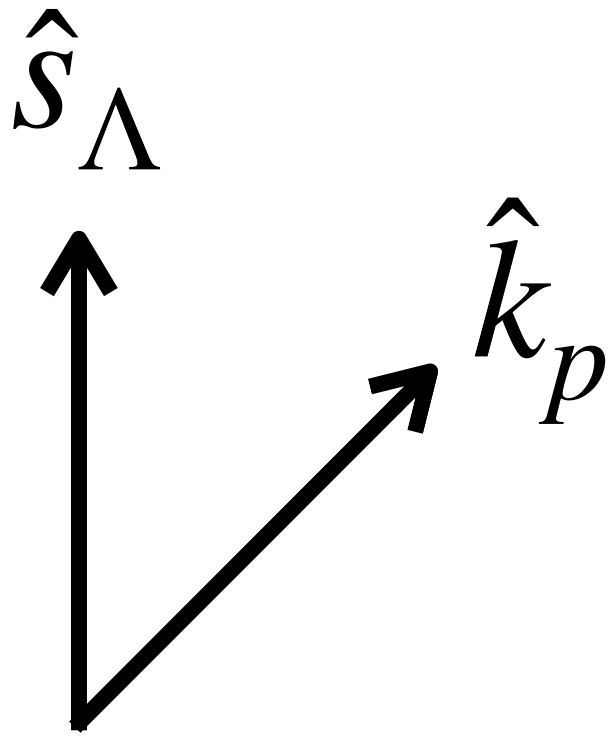
$$\Lambda \to p\pi^-$$

In **1956**, ***parity tests*** were proposed by Lee and Yang.

Tested in the same year by ***Chien-Shiung Wu***:

$$^{60}_{27}\mathrm{Co} \to {}^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}_e + 2\gamma$$

$$\langle \hat{s}_\Lambda \cdot \hat{k}_p \rangle \xrightarrow{P} -\langle \hat{s}_\Lambda \cdot \hat{k}_p \rangle$$



It is now an essential cornerstone of the ***standard model*** !

In **1935**, ***entanglements*** (spooky interactions) were discussed by Einstein, Podolsky and Rosen.

In **1964**, Bell test was proposed to rule out local variable theories.

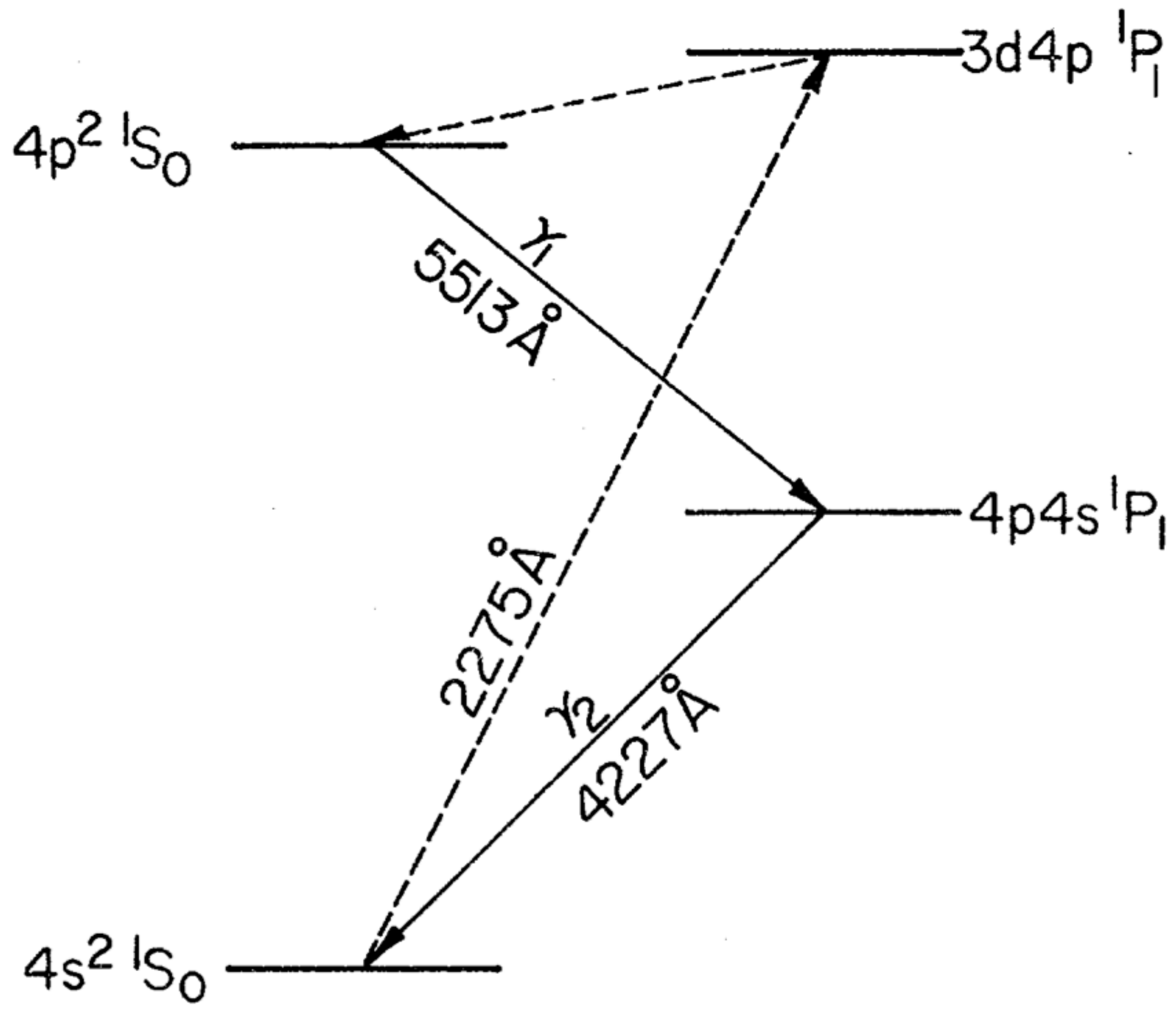
In **1970**, the first ***Bell test*** by Kasday, Ullman and ***Chien-Shiung Wu***.

$e^+e^- \to 2\gamma$, *no violations of Bell inequality was found.*

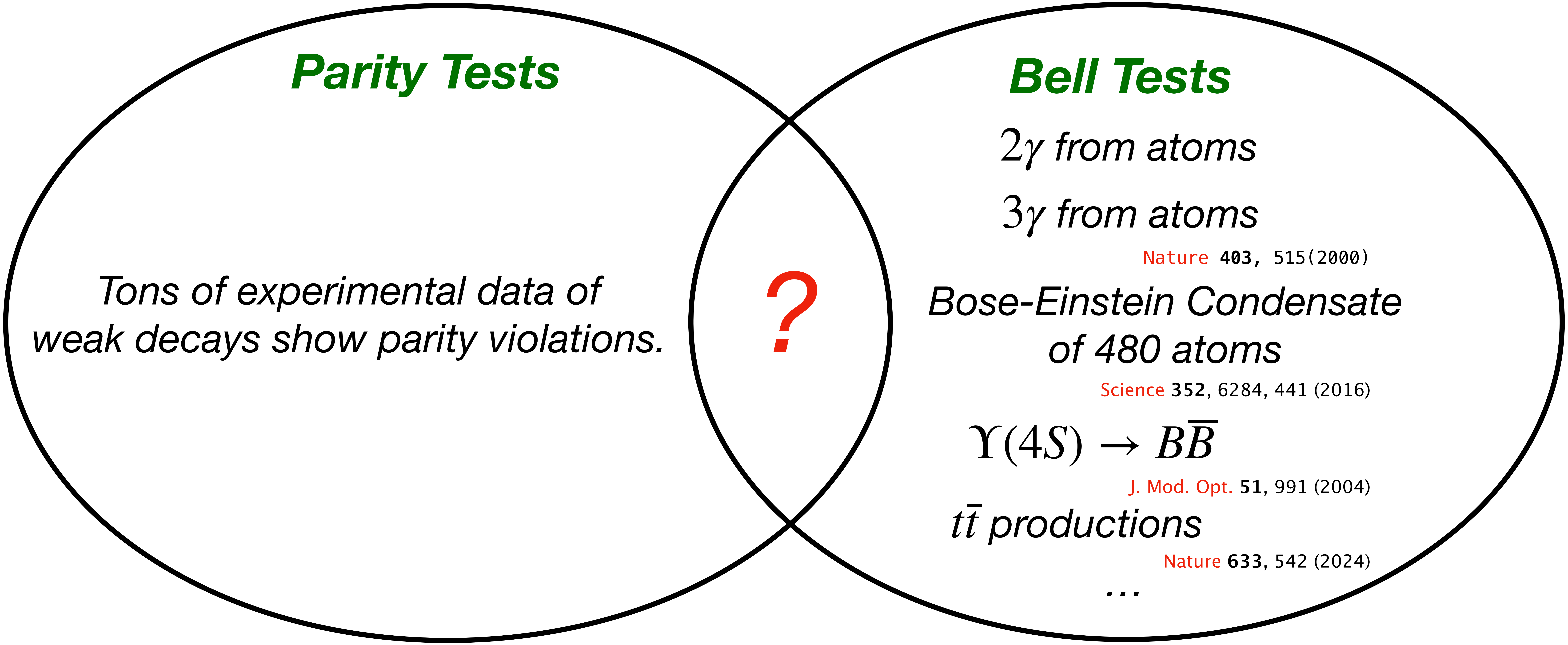
In **1972**, the first violation was observed by Freedman and Clauser.

Excited state of calcium to two photons.

It is now an essential prediction in ***quantum theories*** !



- **Research background**



Interesting to be tested!

Entanglements of two fermions produced by weak interactions

$i \to f_1 \bar{f}_2$ with the initial states spin-0 or spin-1

Remarks on measurements

Examples: $B^0 \to \Lambda_c^+ \overline{\Xi}_c^-$, $Z \to f\bar{f} \cdots$

Magnetic field in detectors; fictitious states

- **Formalism**

What is an entangled state?

$$|\psi\rangle_B = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

*It is **not** a disentangled state* $|\psi\rangle_D$ *.*

$$|\psi\rangle_D = \Big(A_0|\uparrow\rangle + A_1|\downarrow\rangle\Big) \otimes \Big(B_0|\uparrow\rangle + B_1|\downarrow\rangle\Big)$$

$$= |\uparrow\rangle_{\hat{n}_A} \otimes |\uparrow\rangle_{\hat{n}_B}$$

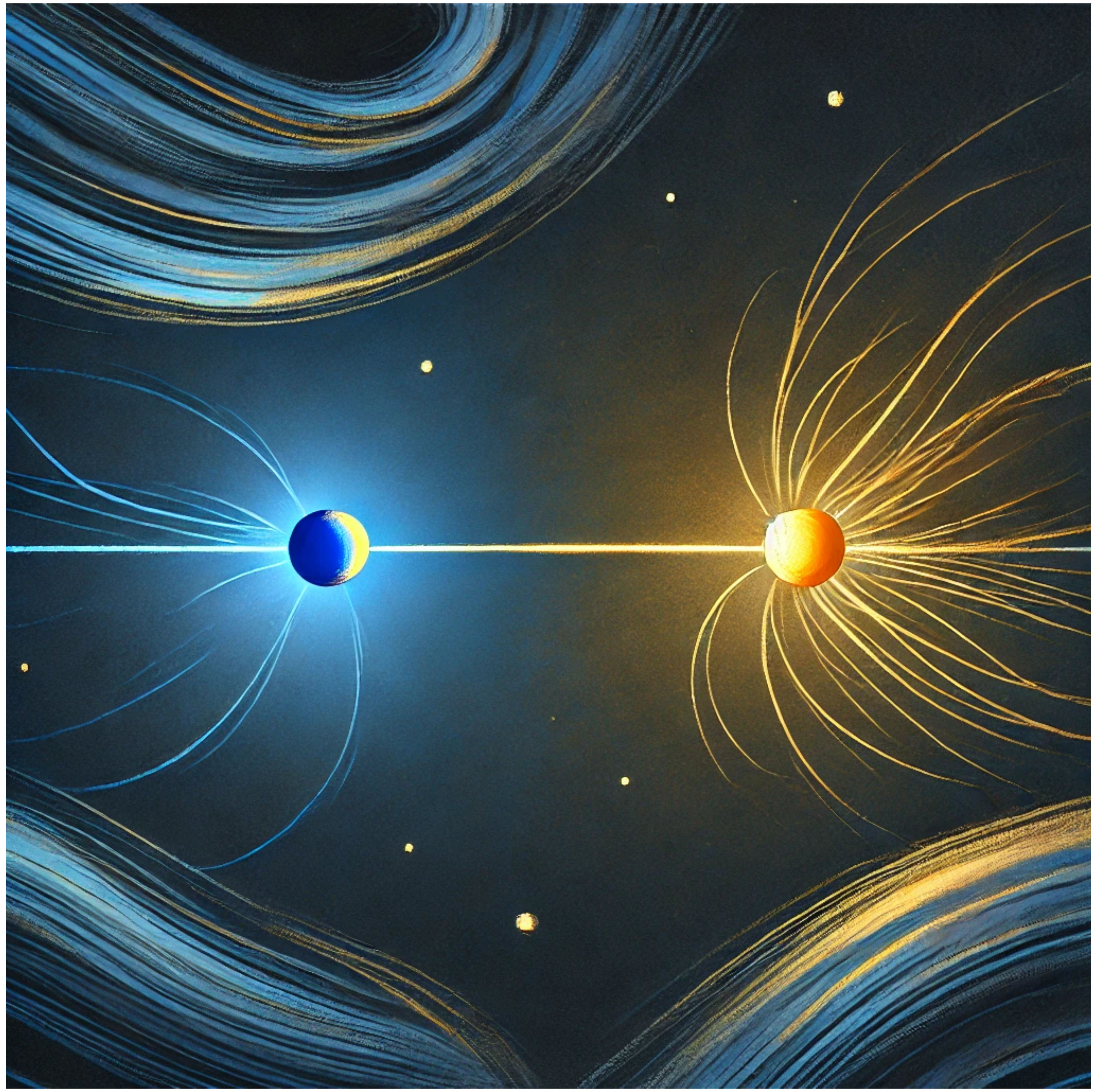
How to quantify it? Concurrence:

$$\mathscr{C} = \Big|\langle\psi|\tilde{\psi}\rangle\Big| \,.$$

$$|\psi\rangle \xrightarrow{TR} |\tilde{\psi}\rangle$$

$$|\uparrow\rangle_{\hat{n}} \xrightarrow{TR} |\downarrow\rangle_{\hat{n}}$$

$$\Big|{}_D\langle\psi|\tilde{\psi}\rangle_D\Big| = 0\,, \qquad \Big|{}_B\langle\psi|\tilde{\psi}\rangle_B\Big| = 1.$$



- ## Formalism

CHSH inequality (a type of Bell inequality)

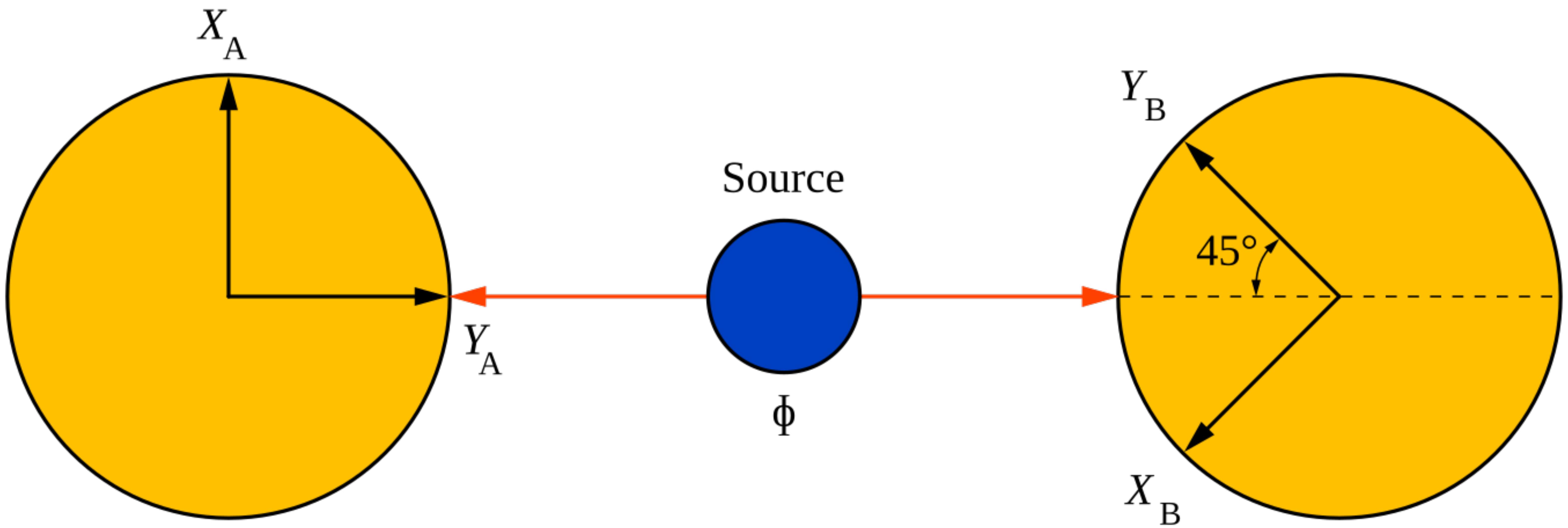
$$\mathscr{B} = \left| \langle X_A X_B \rangle + \langle X_A Y_B \rangle + \langle Y_A X_B \rangle - \langle Y_B Y_B \rangle \right|$$

$X_{A,B}$ and $Y_{A,B}$ take the values of ± 1.

Local Realism

If the measurements of **Alice** and **Bob** were independent, $\langle AB \rangle = \langle A \rangle \langle B \rangle$:

$\mathscr{B} = \left| \langle X_A + Y_A \rangle \langle X_B \rangle + \langle X_A - Y_A \rangle \langle Y_B \rangle \right| \leq 2\,.$ Either $\langle X_A + Y_A \rangle$ or $\langle X_A - Y_A \rangle$ vanishes.



Alice **Bob**

• Formalism

What we obtained in the experiments are **mix states**:

$$\rho = \sum_n P_n |\psi_n\rangle\langle\psi_n| = \frac{1}{4}\left(1 + \overrightarrow{B}^+ \cdot \vec{s}_1 + \overrightarrow{B}^- \cdot \vec{s}_2 + \vec{s}_1 \cdot \overleftrightarrow{C} \cdot \vec{s}_2\right).$$

$\overrightarrow{B}^\pm$ are the **polarizations** of fermions, and $\overleftrightarrow{C}$ their **correlation**.

$\max\left(\mathscr{B}(\rho)\right) = 2\sqrt{\mu_1^2 + \mu_2^2} \leq 2\sqrt{2}$, where $\mu_{1,2}$ are eigenvalues of $\overleftrightarrow{C}$.

$\mathscr{C}(\rho) = \max\left(0,2\lambda_{\max} - \mathrm{Tr}(\mathscr{R})\right)$, where $\lambda_{\max}$ is the largest eigenvalue of

$\mathscr{R} = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$. Note $\rho \xrightarrow{TR} \tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$.

• Formalism— spin-0 to two fermions ($h_i \to f_1 \bar{f}_2$)

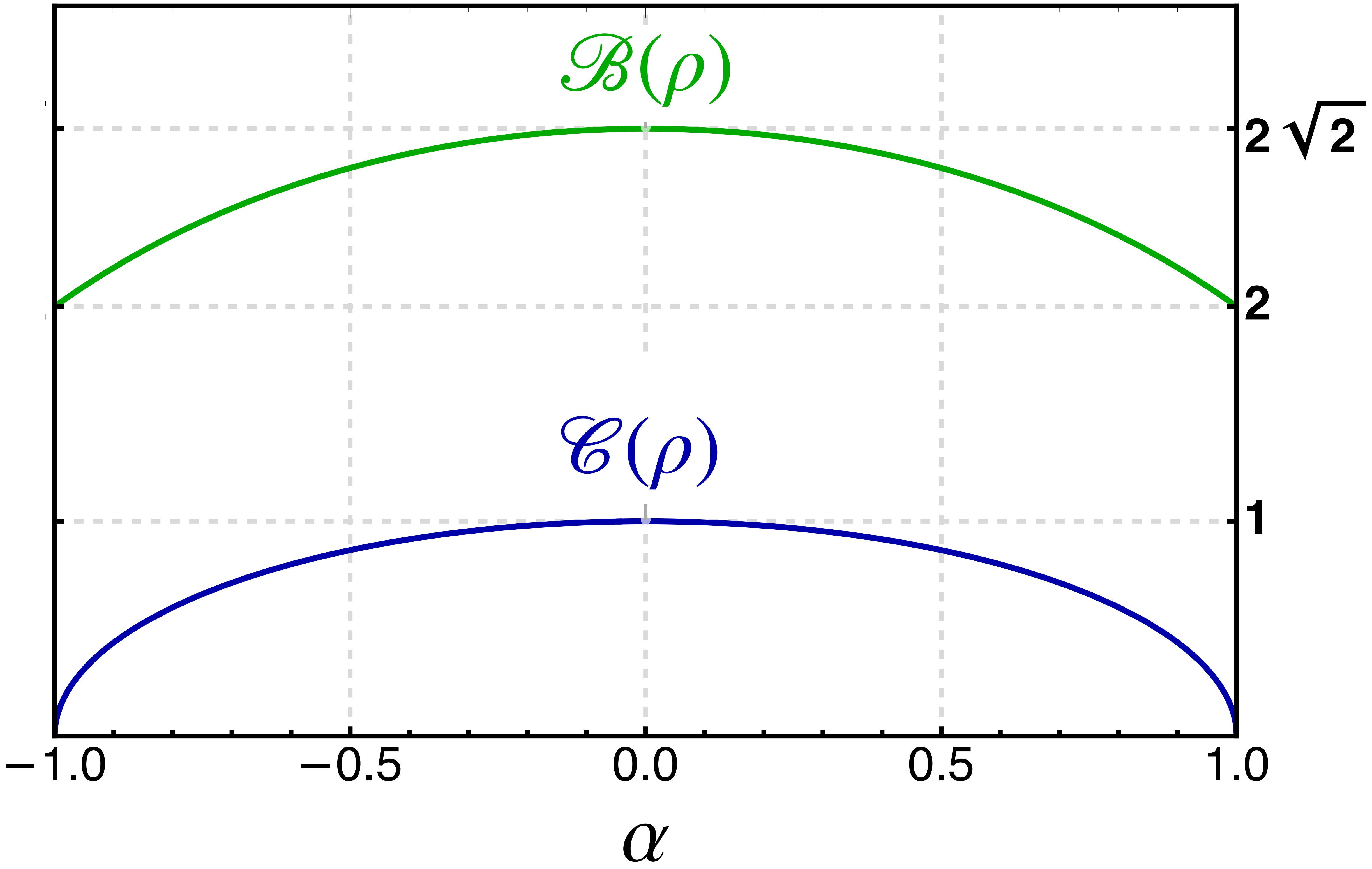
$$M_{\text{scalar}} = \bar{f}_1 \left(g_S - g_P \gamma_5\right) f_2 , \quad S = g_S \sqrt{m_i^2 - (m_1 + m_2)^2} ,$$

$$\alpha = \frac{2\mathrm{Re}(S^* P)}{|S|^2 + |P|^2} , \qquad P = g_P \sqrt{m_i^2 - (m_1 - m_2)^2} .$$

1. The α quantifies the size of **parity violation**.

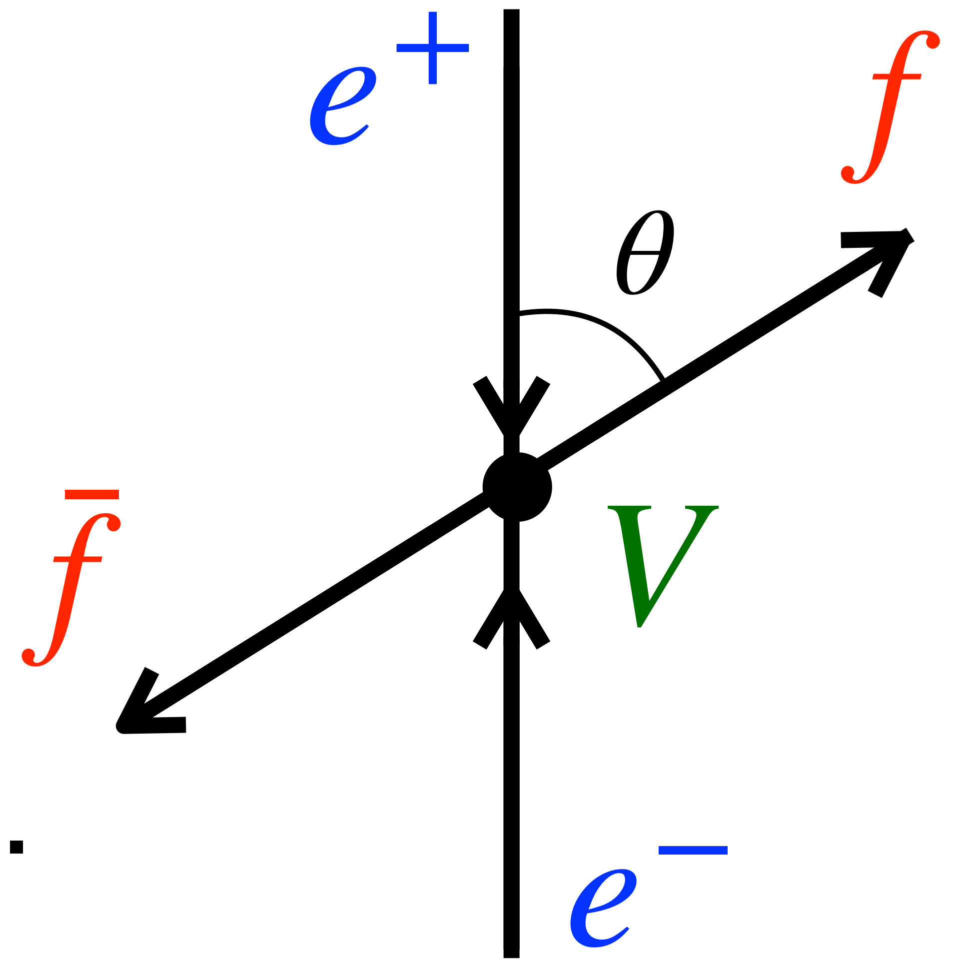
2. In extreme case, $\mathscr{C}$ and $\mathscr{B}$ overlap with **classical** (local realistic) boundaries.

3. They are **independent** of other Lee-Yang parameters: $\beta = \frac{2\mathrm{Im}(S^* P)}{|S|^2 + |P|^2}, \gamma = \frac{|S|^2 - |P|^2}{|S|^2 + |P|^2}$.



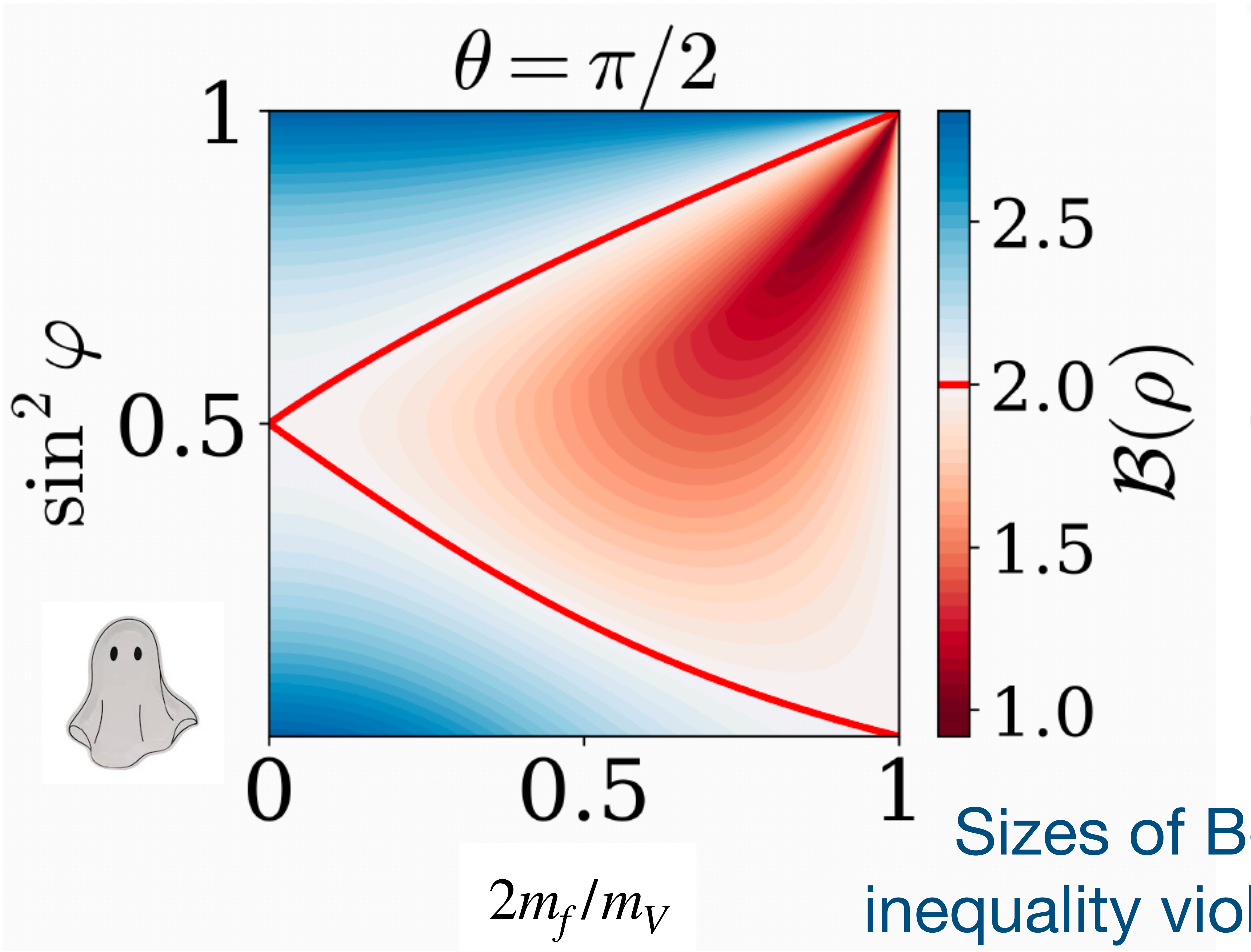
$$\mathscr{C}(\rho) = \sqrt{1 - \alpha^2} , \quad \mathscr{B}(\rho) = 2\sqrt{2 - \alpha^2} .$$

• Formalism— spin-1 to two fermions

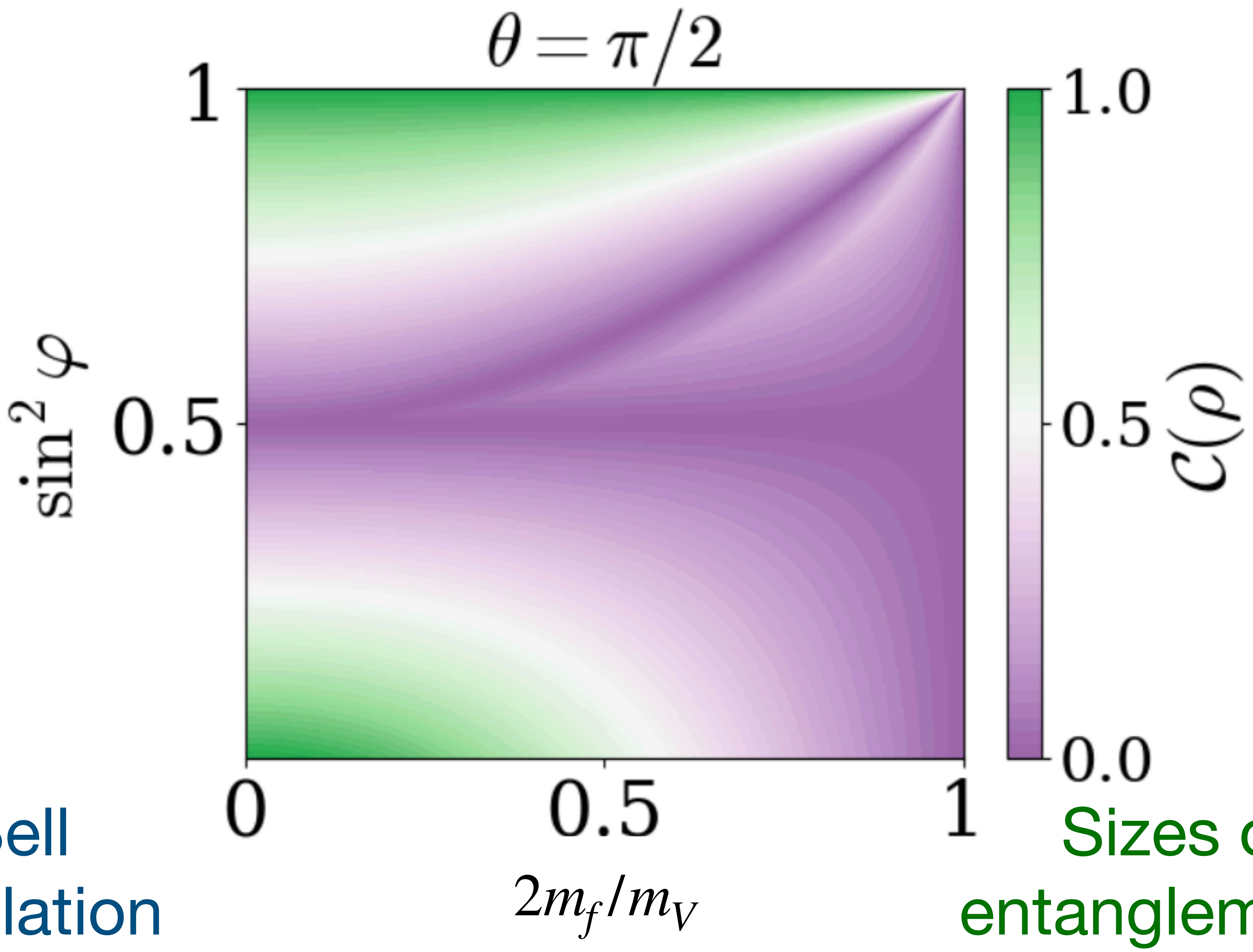


$$M_{\text{vector}} = g_f \epsilon_\mu \bar{u} \gamma^\mu (\cos\varphi + \sin\varphi\gamma_5) v\,,$$ where g_f is some constant.

$\mathscr{C}$ and $\mathscr{B}$ reach **maximums** at $\theta = \pi/2$, and are invariant under $\theta \to \pi - \theta$.

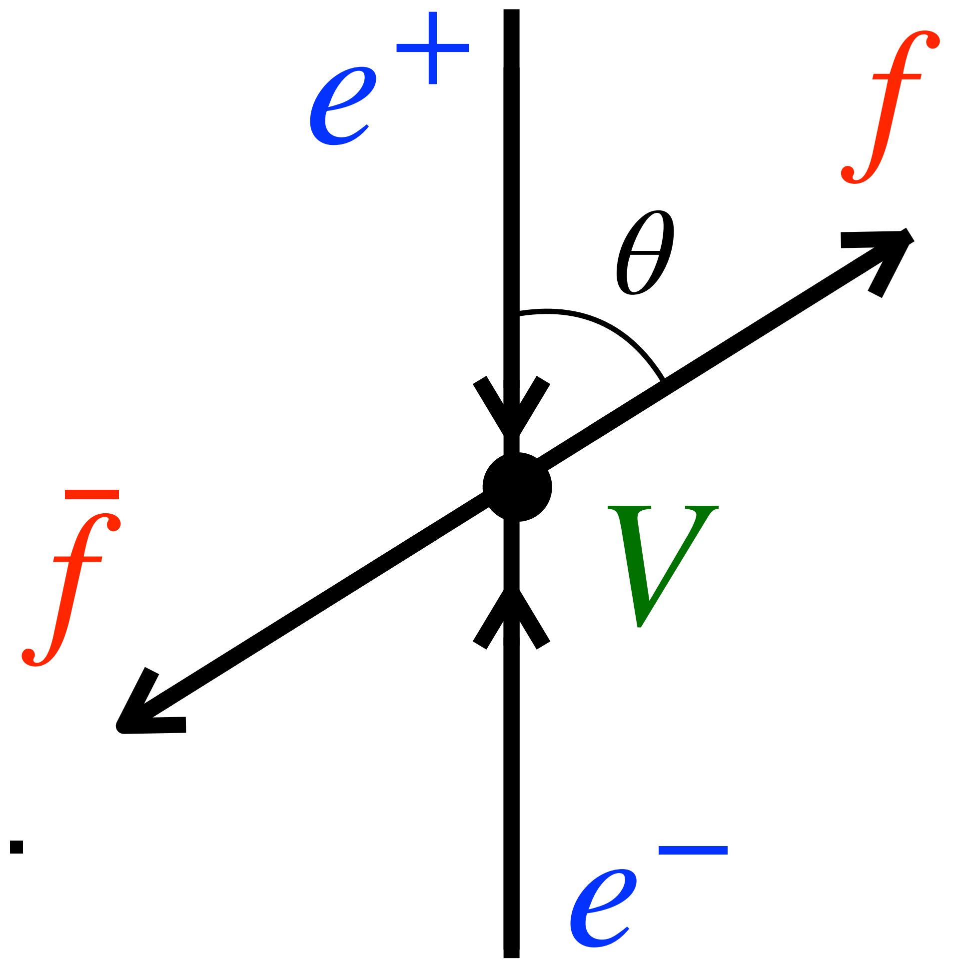


Sizes of Bell inequality violation



Sizes of entanglements

• Formalism— spin-1 to two fermions



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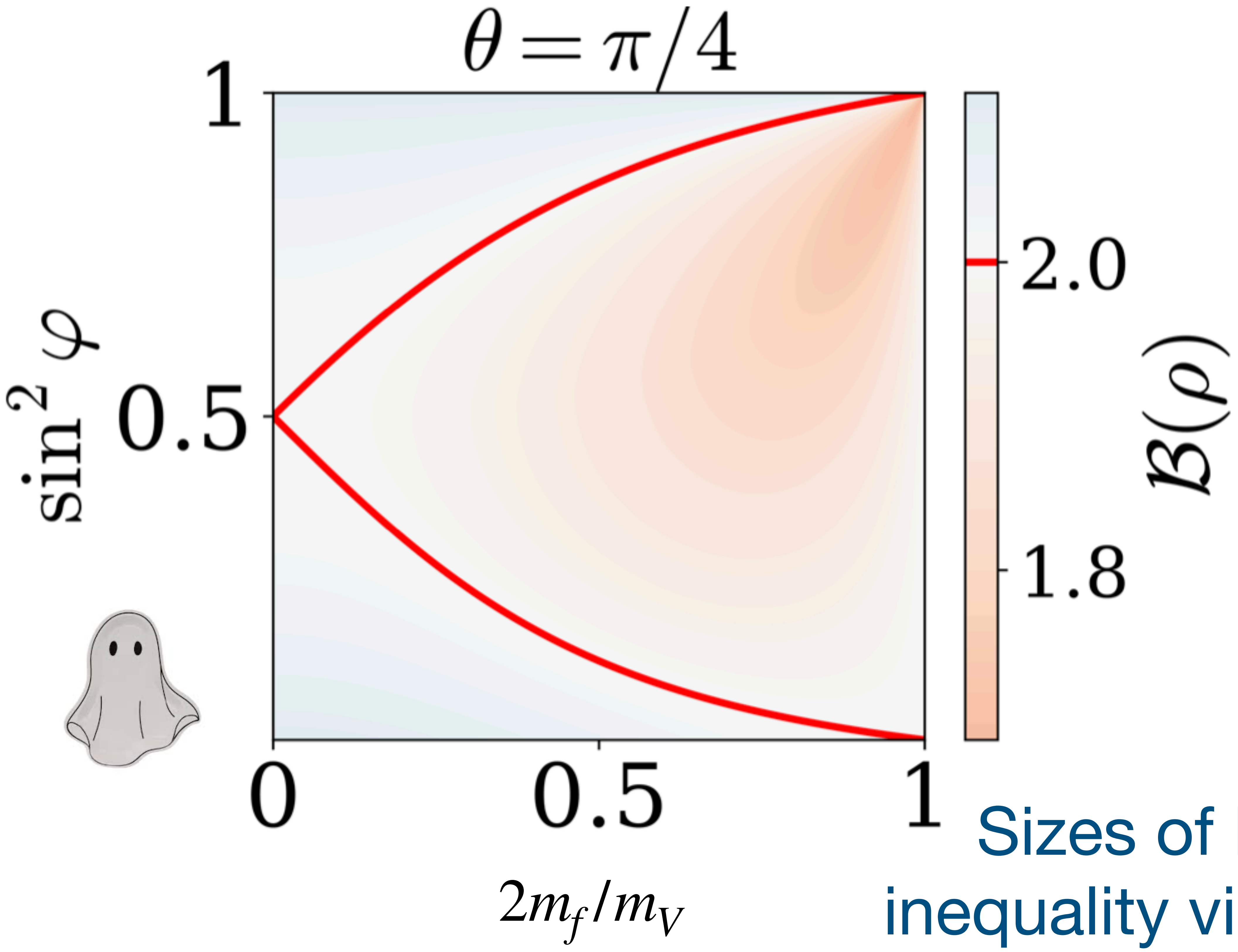
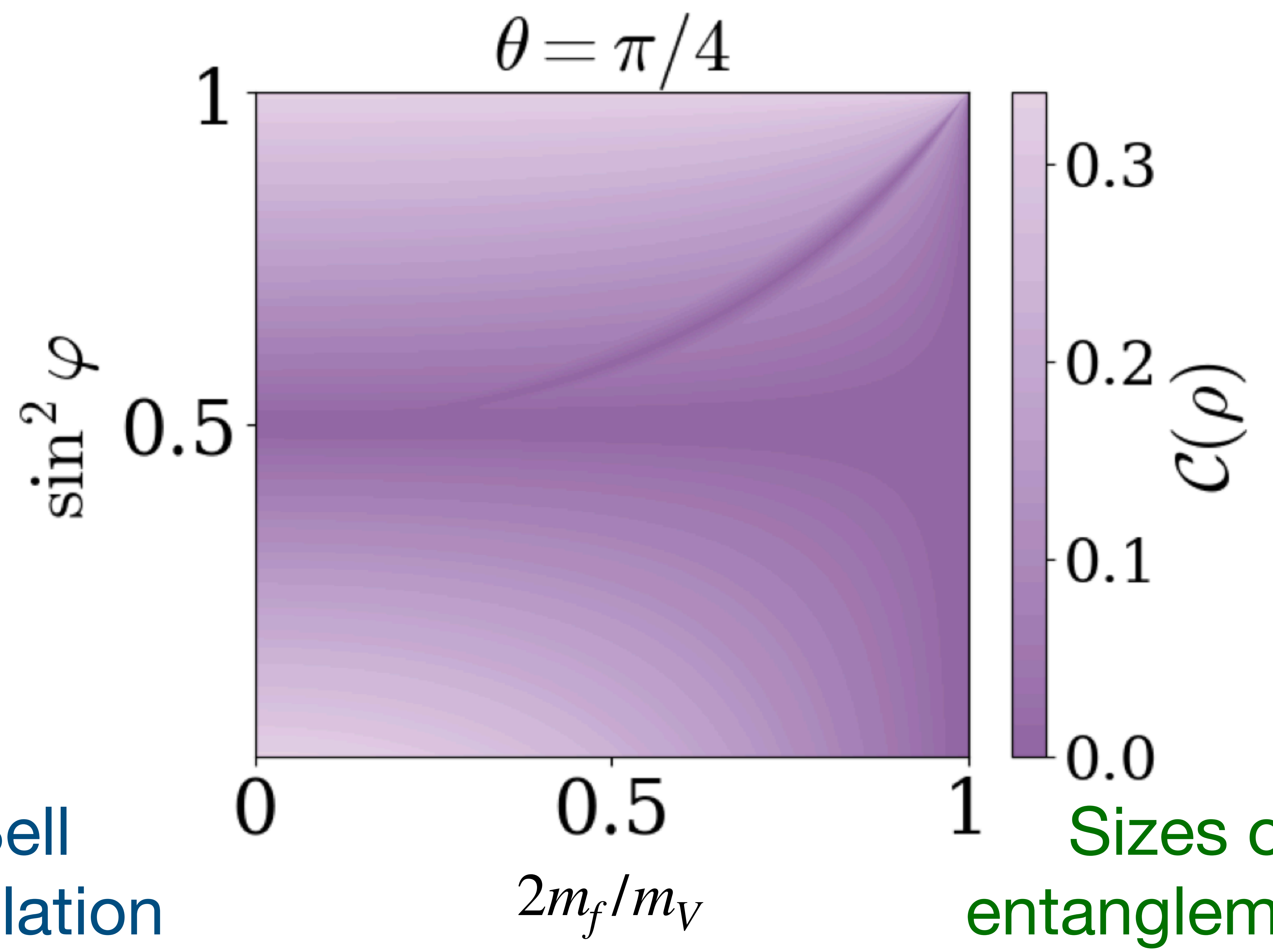


Sizes of Bell inequality violation



Sizes of entanglements

Entanglements of two fermions produced by weak interactions

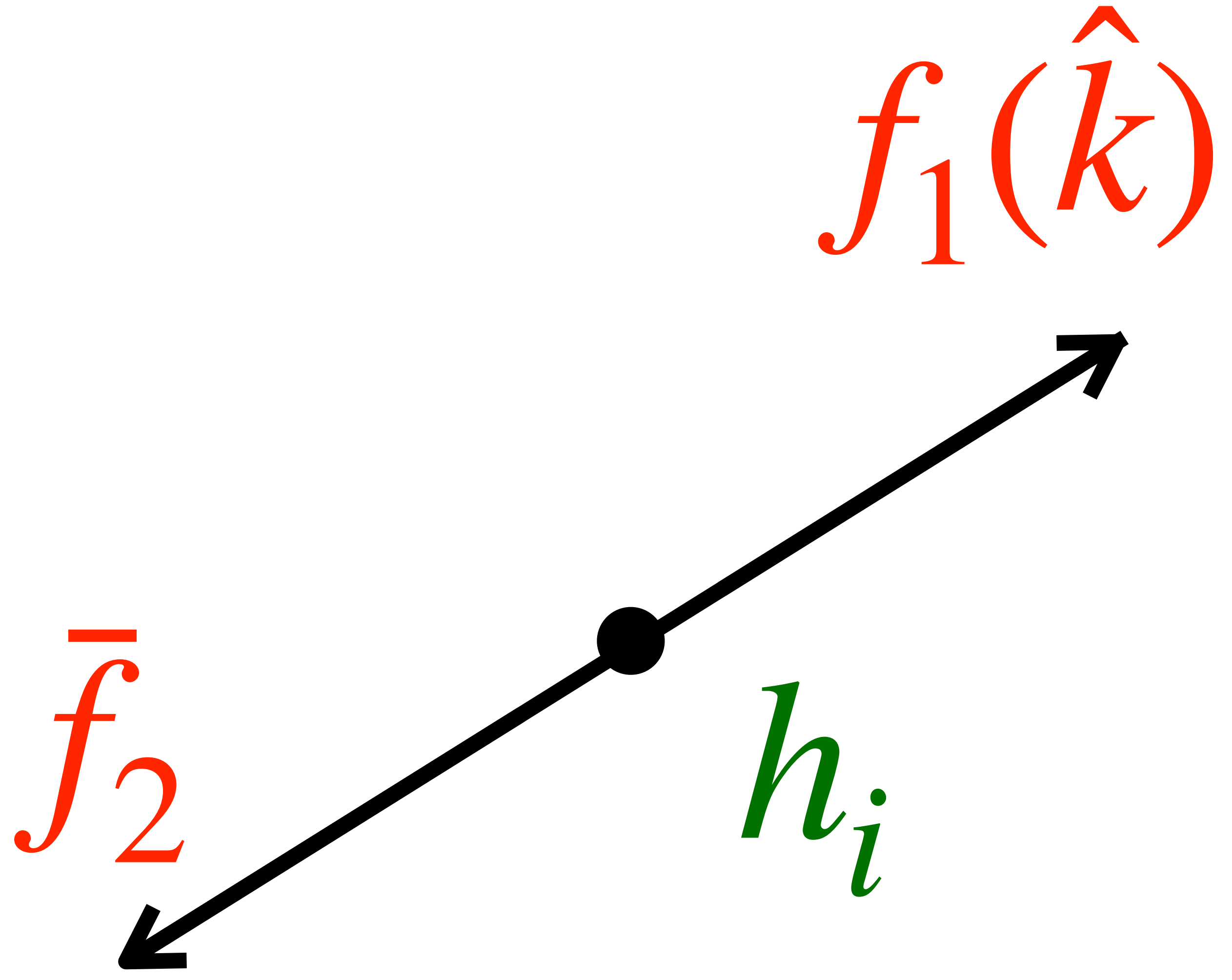
$i \to f_1\bar{f}_2$ **with the initial states spin-0 or spin-1**

Remarks on measurements

Examples: $B^0 \to \Lambda_c^+\overline{\Xi}_c^-,\ Z \to f\bar{f}\cdots$

Magnetic field in detectors; fictitious states

- **Measurements in spin-0 to two fermions ($h_i \to f_1 \bar{f}_2$)**



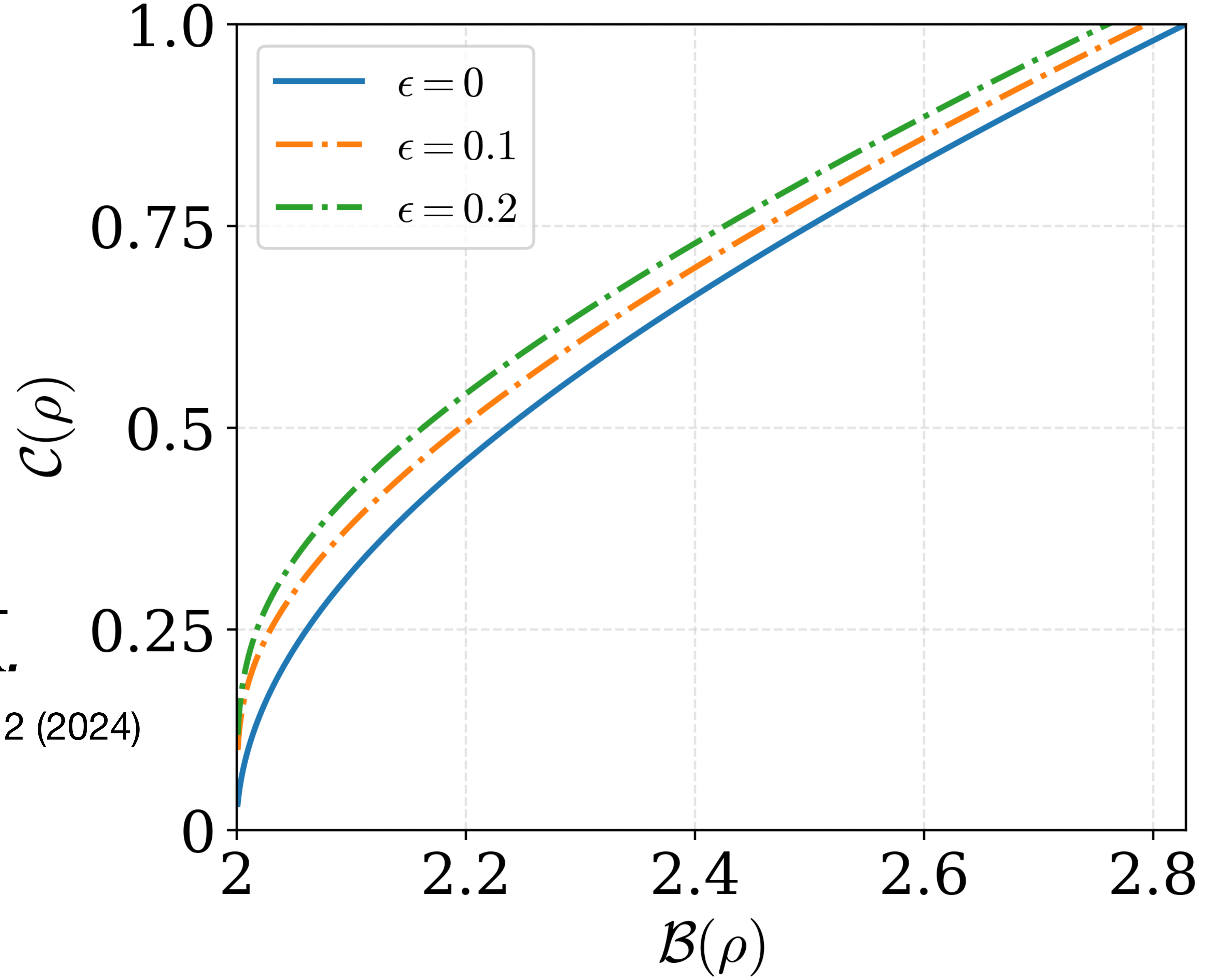
$$\rho = \frac{1}{4}\left(1 + b_{1k}\hat{k}\cdot\vec{s}_1 - b_{1k}\hat{k}\cdot\vec{s}_2 + \vec{s}_1 \cdot \overleftrightarrow{C} \cdot \vec{s}_2\right).$$

$$C_{ij} = (-1 - 2c_5/3)\delta_{ij} + c_2\epsilon_{ijl}\hat{k}_l + c_5(\hat{k}_i\hat{k}_j - \delta_{ij}/3).$$

Angular momentum conservation: $\rho(\vec{s}_1 = \vec{s}_2 = \pm \hat{k}) = 0.$

One-to-one correspondence of $\mathscr{C}$ and $\mathscr{B}$ is broken by ϵ.

$$\epsilon = 1 - b_{1k}^2 - c_2^2 - (1 + c_5)^2 \geq 0.$$



Note that $(b_{1k}, c_2, c_5) = (\alpha, \beta, -1-\gamma)$ and $\epsilon = 0$ in QFT.

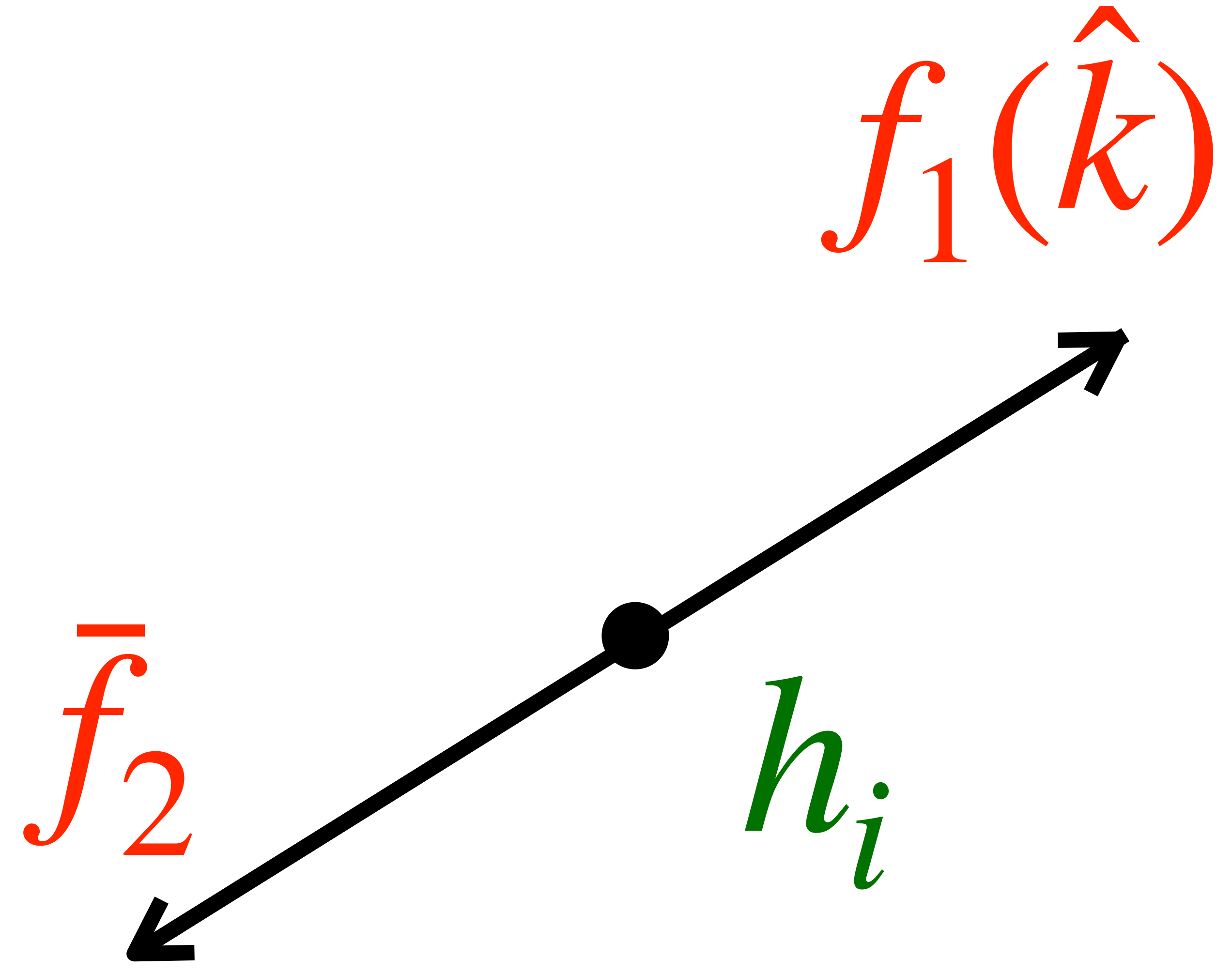
Purity tests can also be done in hyperons decays, $\chi_c^0 \to \Lambda\bar{\Lambda}$.

PRD **110**, 054012 (2024)

$\epsilon(\Lambda \to p\pi^-) = -0.025 \pm 0.154$ in 1962

Precision can hit 10^{-3} at BESIII *!*

- **Measurements in spin-0 to two fermions ($h_i \to f_1 \bar{f}_2$)**



$$\rho = \frac{1}{4}\left(1 + b_{1k}\hat{k}\cdot\vec{s}_1 - b_{1k}\hat{k}\cdot\vec{s}_2 + \vec{s}_1\cdot\overleftrightarrow{C}\cdot\vec{s}_2\right).$$

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Angular momentum conservation: $\rho(\vec{s}_1 = \vec{s}_2 = \pm\hat{k}) = 0.$

From CP symmetry	b_{1k}	c_2	c_5
$B^0_{sL} \to \Lambda_c^+\overline{\Lambda}_c^-$:	0	0	-2
$B^0_{sH} \to \Lambda_c^+\overline{\Lambda}_c^-$:	0	0	0

Uncertainties from B and $\mathbf{B}_c$ LCDAs, the scale dependence, and the Sudakov resummation, in pQCD

Mode	b_{1k}	c_2	$-1-c_5$
$B^- \to \Xi_c^0\bar{\Lambda}_c^-$	$-0.01^{+0.10+0.12+0.05+0.01}_{-0.10-0.29-0.14-0.01}$	$-0.99^{+0.01+0.09+0.00+0.00}_{-0.00-0.01-0.00-0.00}$	$-0.07^{+0.07+0.38+0.04+0.07}_{-0.06-0.13-0.05-0.08}$
$\bar{B}_s^0 \to \Lambda_c^+\bar{\Lambda}_c^-$	$-0.03^{+0.05+0.03+0.05+0.01}_{-0.04-0.04-0.03-0.00}$	$-0.57^{+0.02+0.02+0.00+0.05}_{-0.03-0.02-0.02-0.05}$	$-0.82^{+0.03+0.02+0.01+0.04}_{-0.01-0.01-0.00-0.03}$
$\bar{B}^0 \to \Lambda_c^+\bar{\Lambda}_c^-$	$0.17^{+0.08+0.08+0.03+0.02}_{-0.08-0.05-0.18-0.01}$	$-0.97^{+0.04+0.06+0.02+0.02}_{-0.03-0.00-0.02-0.01}$	$-0.15^{+0.17+0.54+0.14+0.09}_{-0.14-0.16-0.11-0.11}$

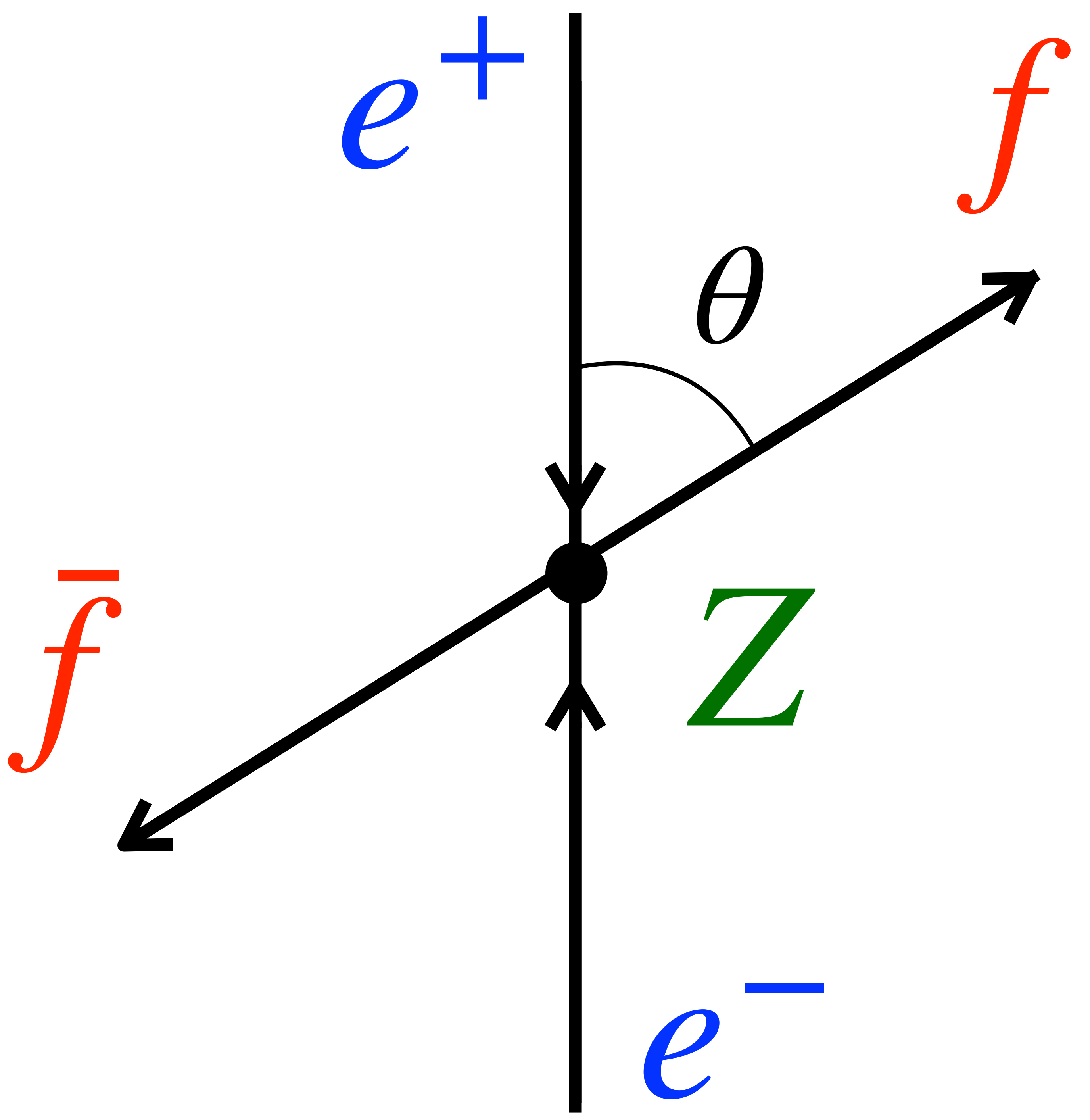
Zhou, Zou and Li, arXiv:2409.16113

• Measurements in spin-1 to two fermions ($Z \to f\bar{f}$)

Fictitious states come from the needs of accumulating enough data over θ:

$$\bar{\rho} = \frac{1}{\mathcal{N}} \int_{\theta_1}^{\theta_2} \rho(\theta) \frac{d\sigma(\theta)}{d\cos\theta} d\cos\theta.$$

σ is the scattering rate.



It induces **basis-dependence**. For example, using (x, y, z) would **differ** from (r, θ, ϕ).

$$\bar{\rho}' = \frac{1}{\mathcal{N}} \int_{\theta_1}^{\theta_2} U_1(\theta)\rho(\theta)U_2^\dagger(\theta) \frac{d\sigma(\theta)}{d\cos\theta} d\cos\theta\,.$$

Here $U_{1,2}(\theta)$ are $SU(2) \otimes SU(2)$ rotations.

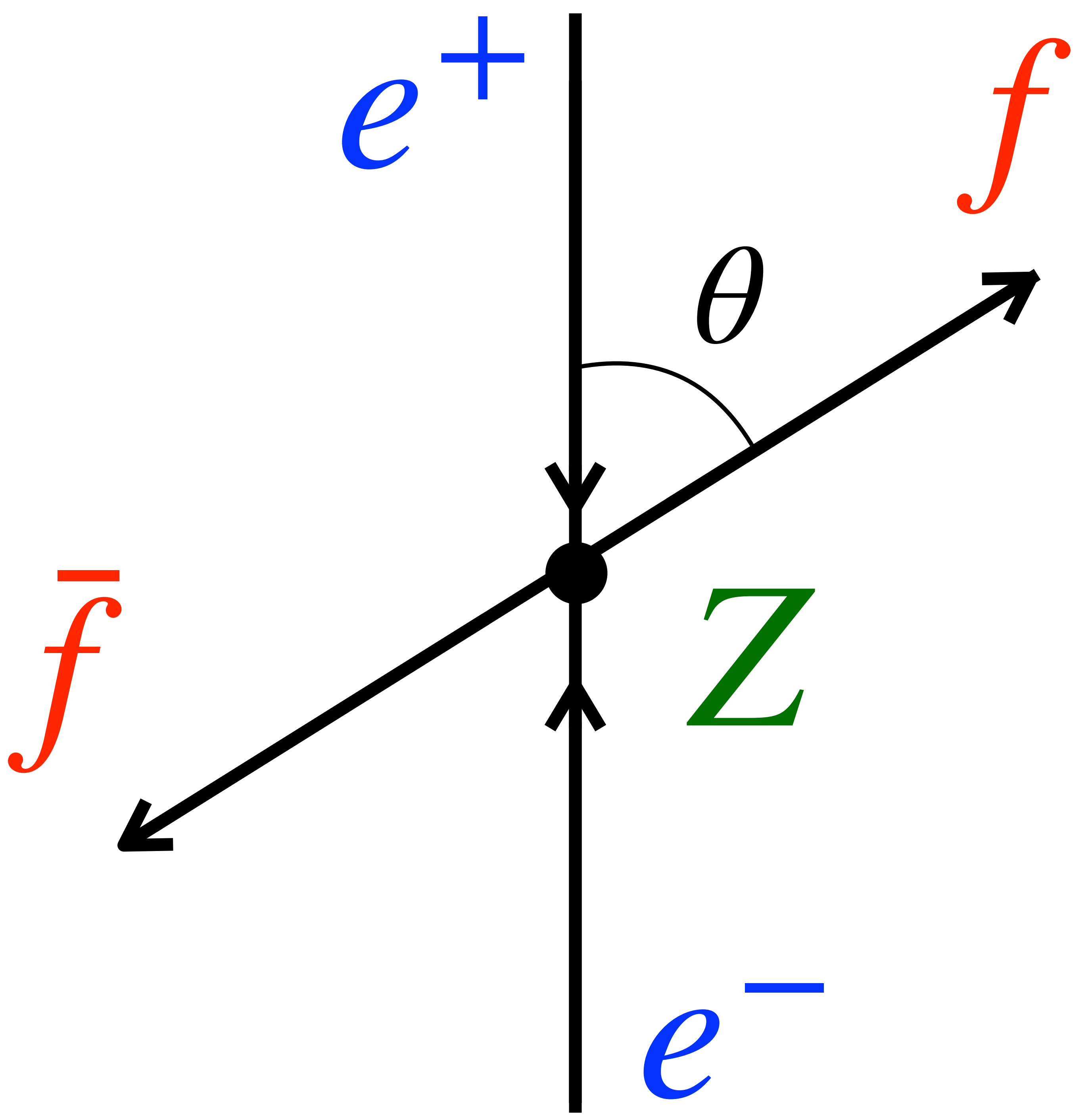
- **Measurements in spin-1 to two fermions ($Z \to f\bar{f}$)**

$$\rho = \frac{1}{4}\left(1 + \overrightarrow{B}^{+} \cdot \vec{s}_1 + \overrightarrow{B}^{-} \cdot \vec{s}_2 + \vec{s}_1 \cdot \overleftrightarrow{C} \cdot \vec{s}_2\right).$$

<u>***Q:***</u> How to choose $U_{1,2}(\theta)$

$$\bar{\rho}' = \frac{1}{\mathcal{N}} \int_{\cos\theta_1}^{\cos\theta_2} U_1(\theta)\rho(\theta)U_2^\dagger(\theta) \frac{d\sigma(\theta)}{d\cos\theta} d\cos\theta\,.$$

to **maximize** $\mathscr{B}(\overline{\rho}')$?



<u>***A:***</u> $\overleftrightarrow{C}'(\theta)$ in $\rho'(\theta) = U_1(\theta)\rho(\theta)U_2^\dagger(\theta)$ is diagonal and

$$\mathscr{B}(\overline{\rho}') = \frac{2}{\mathcal{N}}\sqrt{\sum_{i=1,2}\left[\int_{\theta_1}^{\theta_2} \mu_i \left(\frac{d\sigma}{d\cos\theta}\right) d\cos\theta\right]^2}$$

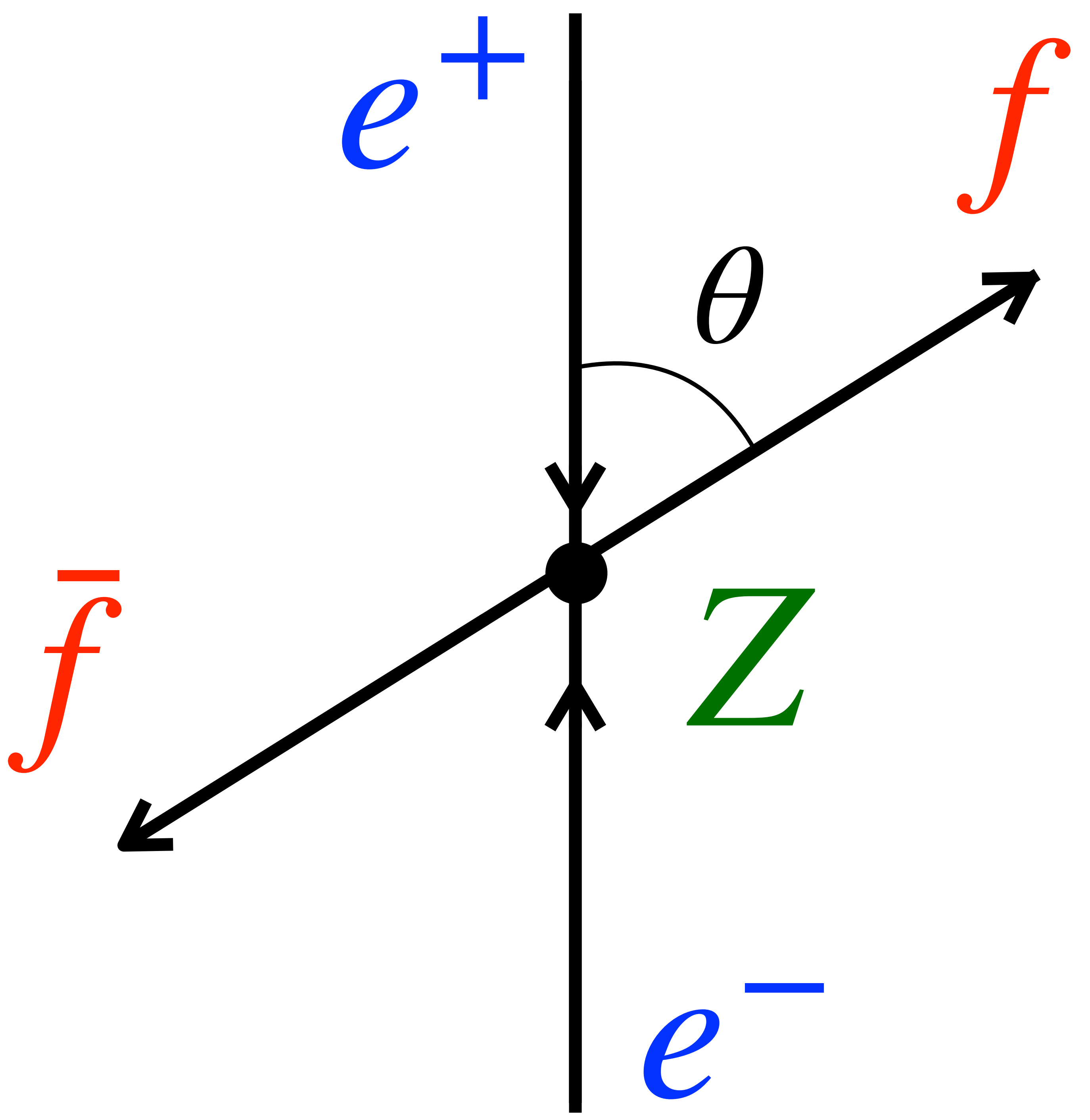
Here $\mu_{1,2}$ are the eigenvalues of $\overleftrightarrow{C}'(\theta)$.

arXiv:2407.01672, 2409.15418.

• Measurements in spin-1 to two fermions ($Z \to f\bar{f}$)

The upper and lower scripts represent $\cos\theta_2$ and $\cos\theta_1$: $\bar{\rho}' = \frac{1}{\mathcal{N}} \int_{\cos\theta_1}^{\cos\theta_2} U_1(\theta)\rho(\theta)U_2^\dagger(\theta)\frac{d\sigma(\theta)}{d\cos\theta} d\cos\theta$.

Processes	$-\alpha_f$	$\mathcal{B}_{-1.0}^{-0.5}$	$\mathcal{B}_{-0.5}^{-0.3}$	$\mathcal{B}_{-0.3}^{-0.1}$	$\mathcal{B}_{-0.1}^{0.1}$	$\mathcal{B}_{0.1}^{0.3}$	$\mathcal{B}_{0.3}^{0.5}$	$\mathcal{B}_{0.5}^{1.0}$
$Z \to \Lambda_b^0 \bar{\Lambda}_b^0$	0.94	2.01	2.04	2.08	2.10	2.10	2.06	2.01
$Z \to \Lambda_c^+ \bar{\Lambda}_c^-$	0.70	2.03	2.24	2.40	2.49	2.46	2.31	2.05
$Z \to \tau^- \tau^+$	0.21	2.06	2.45	2.69	2.81	2.71	2.47	2.07



- **Heavy quark symmetry** has been used in Λ_Q.

  It relates the heavy quark spins to Λ_Q.
- The parity violation on the production side has been considered. Causing **asymmetries** between θ and $\pi - \theta$.
- $|\alpha_f|$ denotes the sizes of parity violation. A larger $|\alpha_f|$ implies a lower violation of Bell inequality.

Magnetic fields in general

$$\rho = \frac{1}{4}\left(1 + \vec{B}^{+} \cdot \vec{s}_1 + \vec{B}^{-} \cdot \vec{s}_2 + \vec{s}_1 \cdot \overleftrightarrow{C} \cdot \vec{s}_2\right).$$

- Impact of magnetic fields on CHSH parameters are around 10^{-3}.

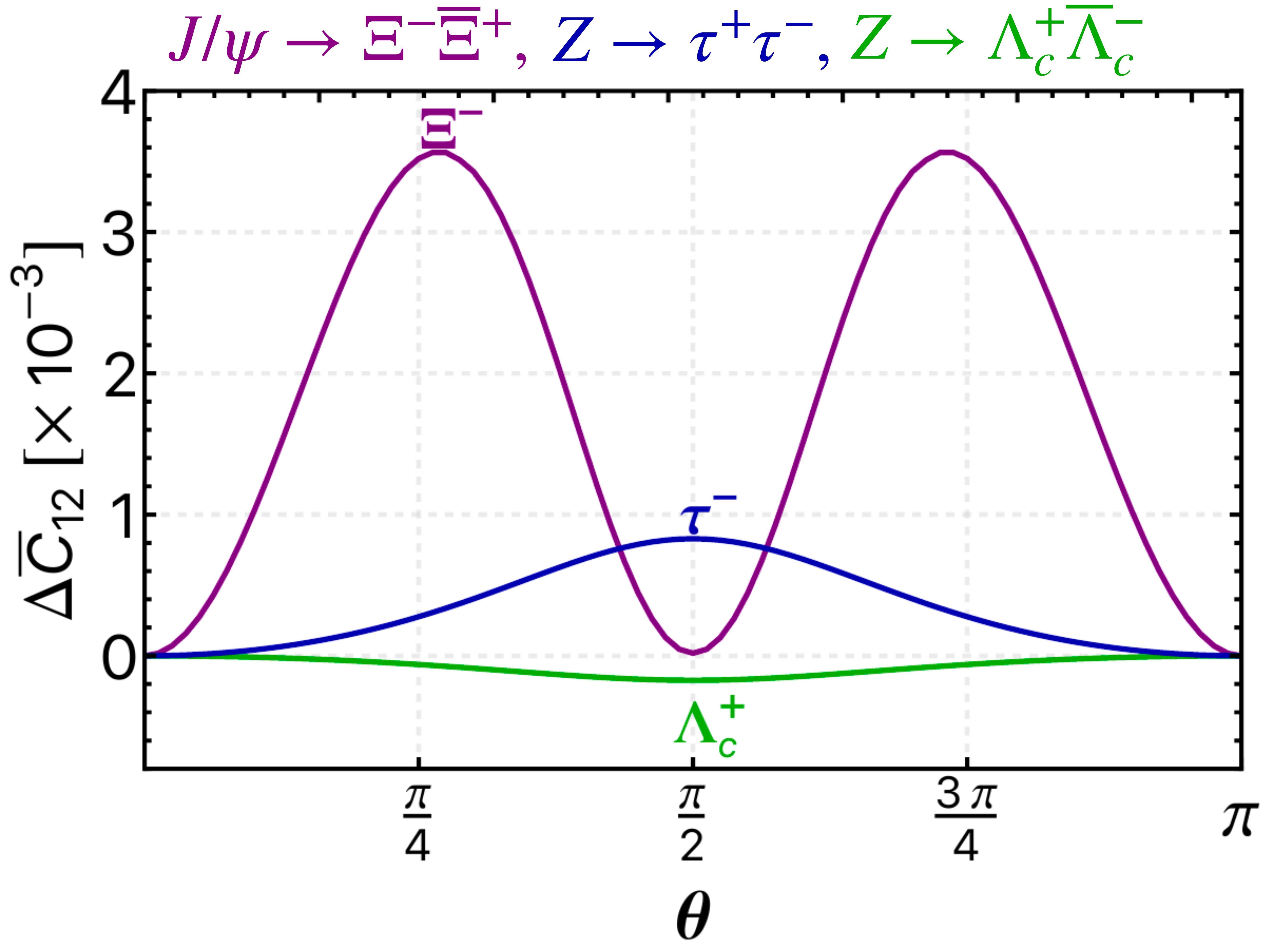
- $\Delta \overleftrightarrow{C}_{12} = \overleftrightarrow{C}_{12} - \overleftrightarrow{C}_{21} \neq 0$ indicates **CP violation** in the absence of magnetic fields.

$$\alpha_{\Xi^-} = -0.376 \pm 0.007 \pm 0.003$$

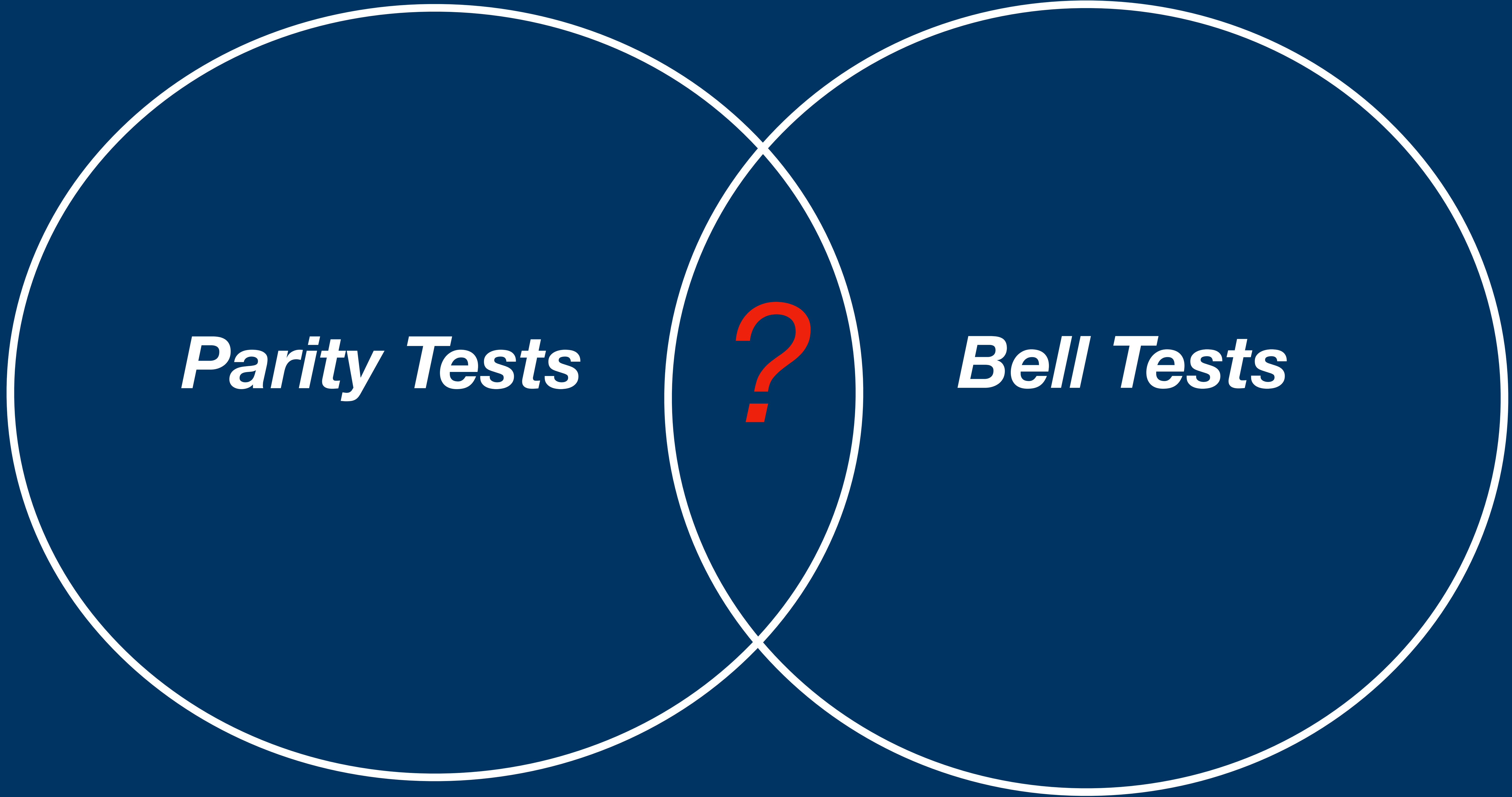
$$\alpha_{\overline{\Xi}^+} = 0.371 \pm 0.007 \pm 0.002$$

Nature **606**, 64–69 (2022)

- Precisions in STCF (Super Tau Charm Facility) are around 10^{-4}.



Outlook



物理学第一夫人

吴健雄 （*1912—1997*）

Backup slides

Q: Are Bell tests useful in searching for new physics?

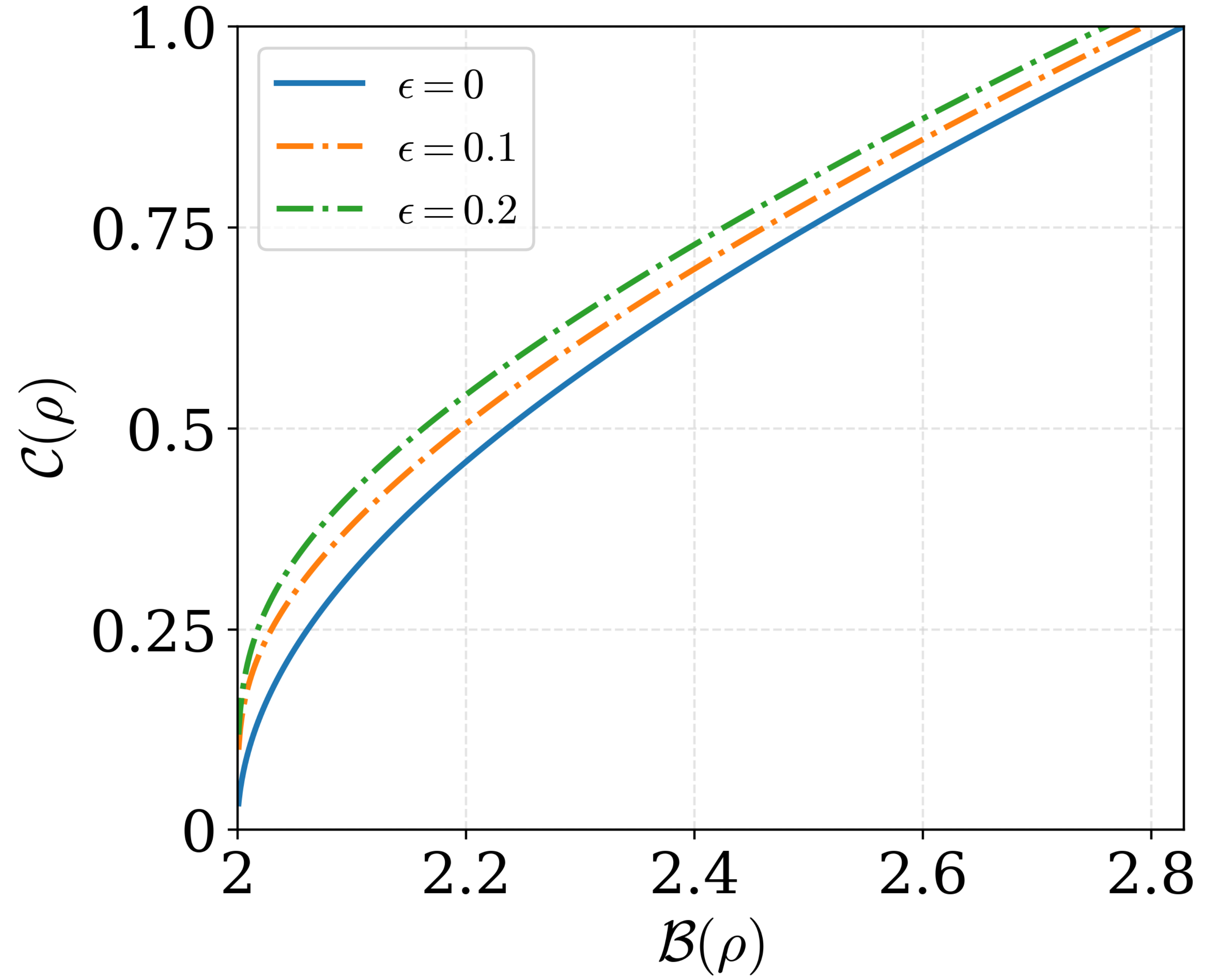
A: Not so useful to the NP model defined in QFT, with the feature:

CP violation, flavor changing, e.t.c..

Useful to nonstandard NP model that

violates Lorentz (CPT) symmetry and ***cannot*** defined within QFT…

We can so we should conduct Bell tests!



$$\mathrm{CEPC} \sim 7 \times 10^{11}$$

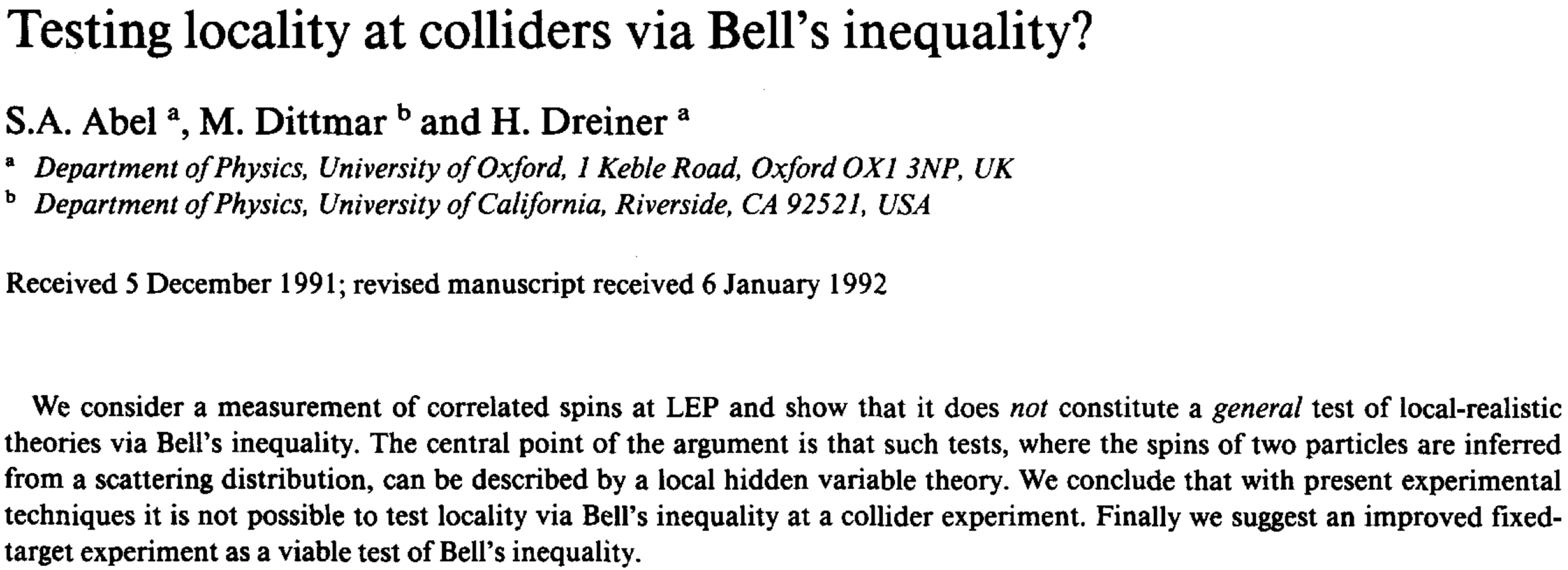
$$\mathrm{FCC-ee} \sim 3 \times 10^{12}$$

$$\mathbf{BR}(Z \to \tau^+\tau^-) = (3.33696 \pm 0.0066)\,\%$$

$$\mathbf{BR}(Z \to \Lambda_c^+ X) = (1.54 \pm 0.33)\,\%$$

$$\mathbf{BR}(Z \to b\ \mathrm{baryon}X) = (1.38 \pm 0.22)\,\%$$

- **Loopholes?**
  - **It requires the observables to be non-commutative, which are not satisfied in the collider particle pairs.**


Testing locality at colliders via Bell's inequality?

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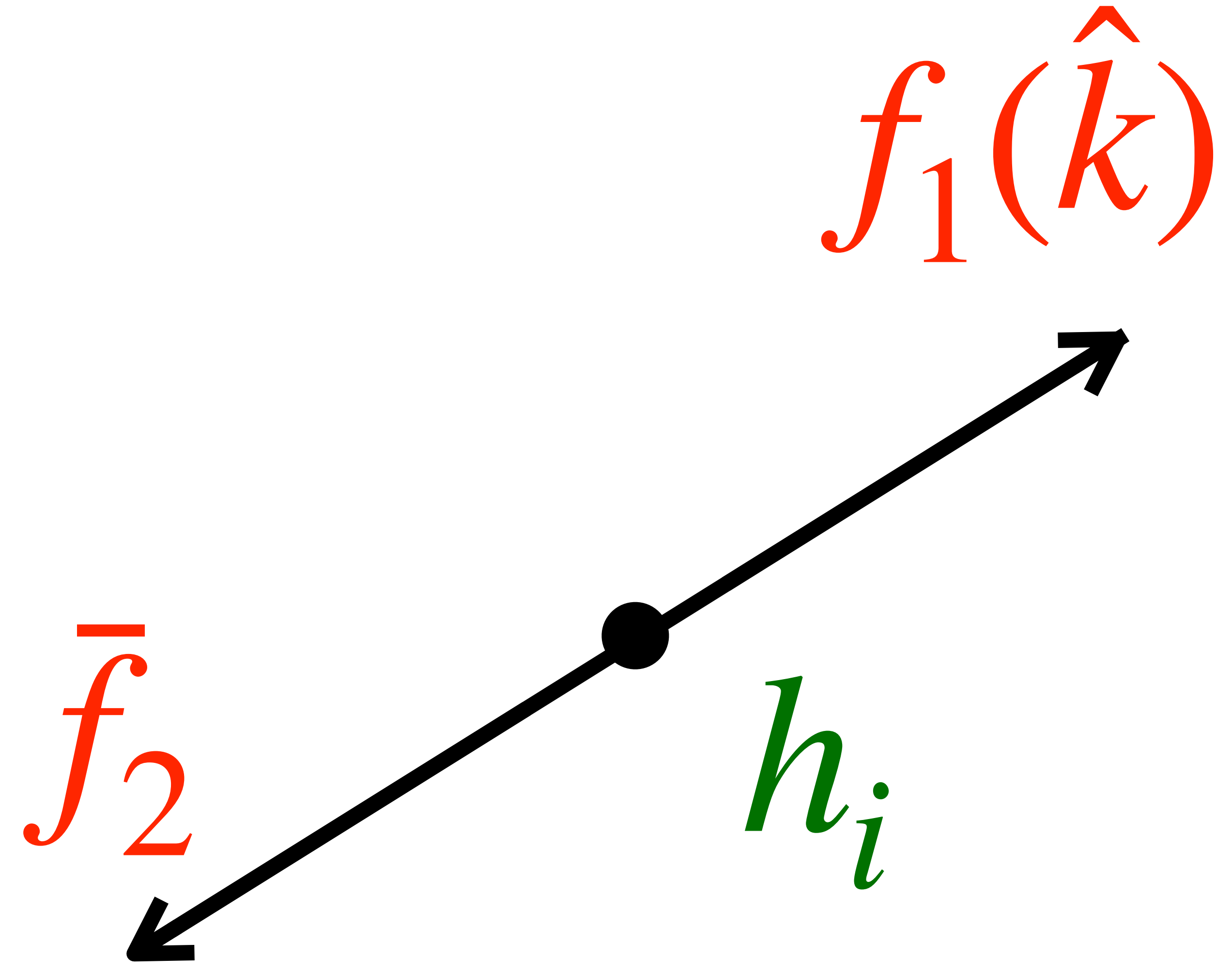
We consider a measurement of correlated spins at LEP and show that it does *not* constitute a *general* test of local-realistic theories via Bell's inequality. The central point of the argument is that such tests, where the spins of two particles are inferred from a scattering distribution, can be described by a local hidden variable theory. We conclude that with present experimental techniques it is not possible to test locality via Bell's inequality at a collider experiment. Finally we suggest an improved fixed-target experiment as a viable test of Bell's inequality.

• Measurements in spin-0 to two fermions ($h_i \to f_1 \bar{f}_2$)

$$\rho = \frac{1}{4}\left(1 + b_{1k}\hat{k}\cdot\vec{s}_1 - b_{1k}\hat{k}\cdot\vec{s}_2 + \vec{s}_1\cdot\overleftrightarrow{C}\cdot\vec{s}_2\right).$$

$$C_{ij} = (-1 - 2c_5/3)\delta_{ij} + c_2\epsilon_{ijl}\hat{k}_l + c_5(\hat{k}_i\hat{k}_j - \delta_{ij}/3).$$



Angular momentum conservation: $\rho(\vec{s}_1 = \vec{s}_2 = \pm\hat{k}) = 0.$

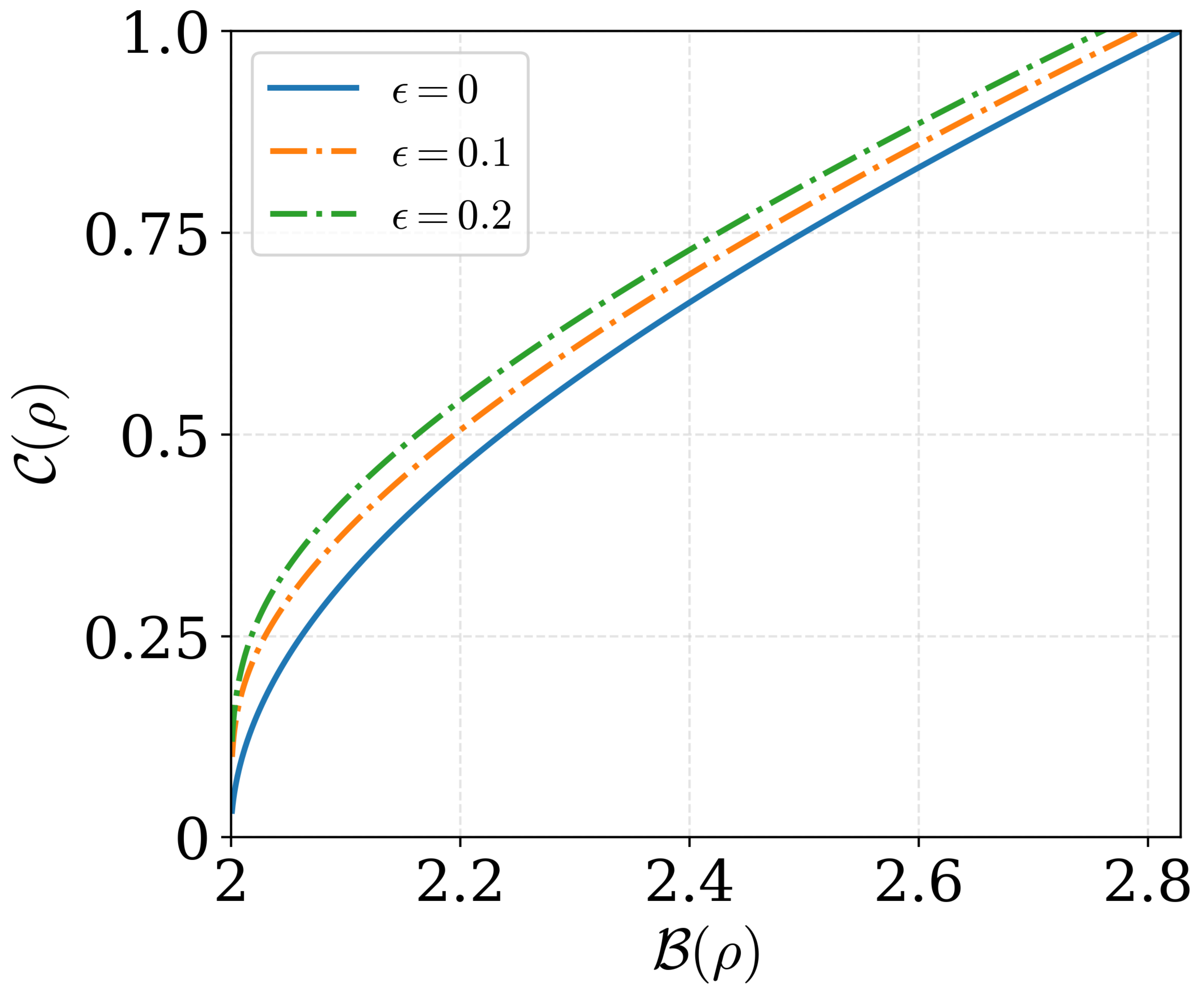
One-to-one correspondence of $\mathscr{C}$ and $\mathscr{B}$ is broken by ϵ.

$$\epsilon = 1 - b_{1k}^2 - c_2^2 - (1 + c_5)^2 \geq 0.$$

Note that $(b_{1k}, c_2, c_5) = (\alpha, \beta, -1-\gamma)$ and $\epsilon = 0$ in QFT.

$$\mathscr{B} = 2\sqrt{2 - b_{1k}^2 - \epsilon},$$

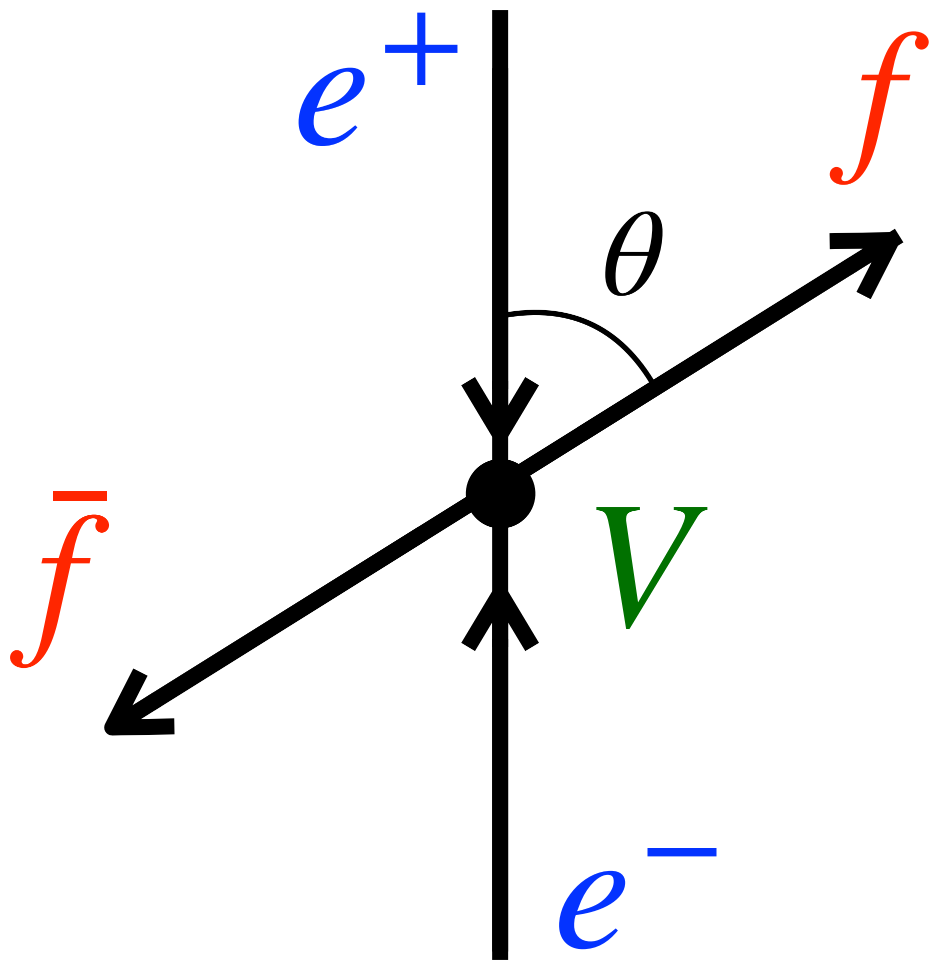
$$\mathscr{C}(\rho) = \frac{1}{2}[(\mathscr{B}(\rho)^2 - 4)(\mathscr{B}(\rho)^2 - 4 + 4\epsilon)]^{\frac{1}{4}}.$$

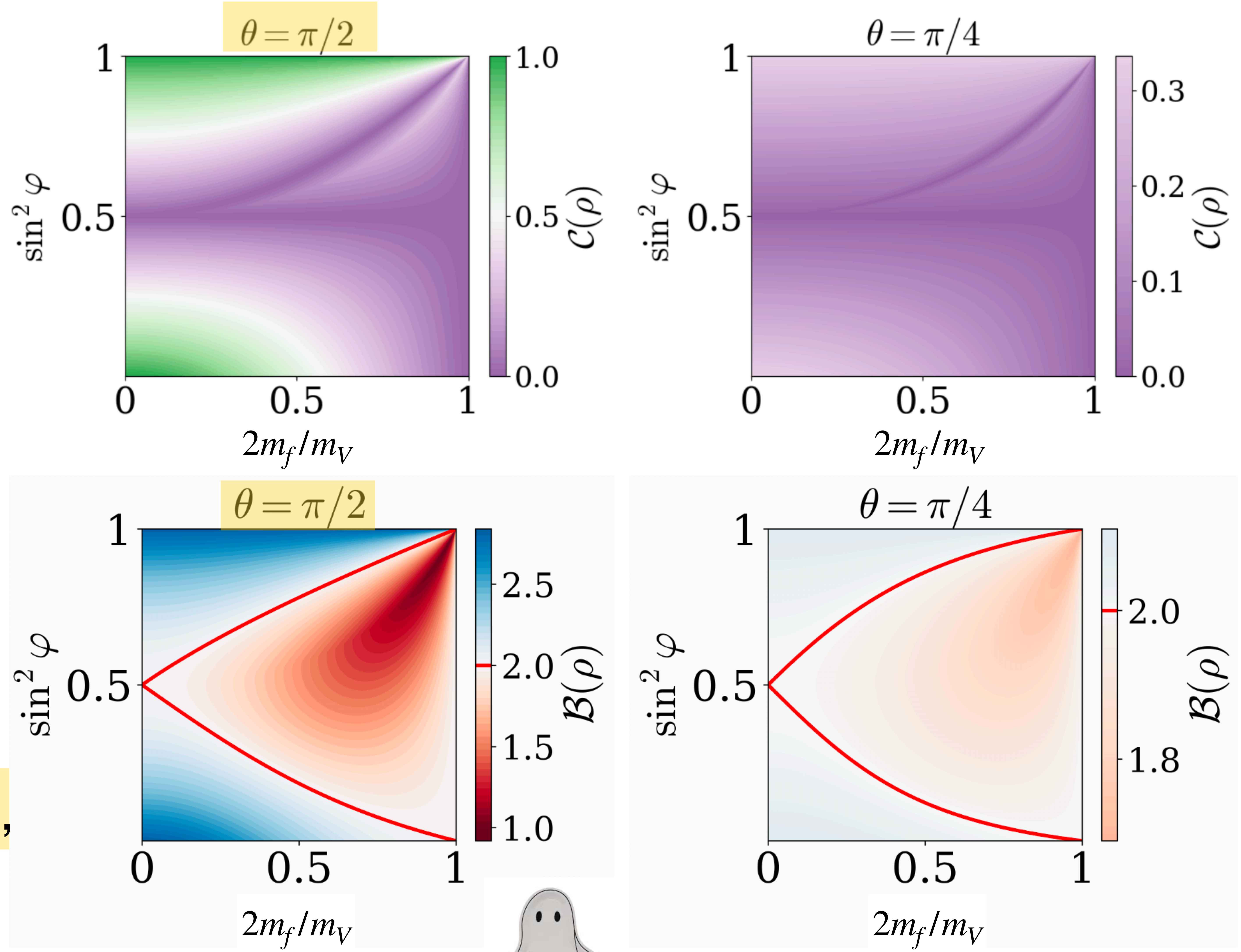


• Formalism— spin-1 to two fermions

$$M_{\text{vector}} = g_f \epsilon_\mu \bar{u} \gamma^\mu (\cos\varphi + \sin\varphi\gamma_5) v\,,$$ where g_f is some constant.

1. θ is defined as :





2. $\sin^2\varphi = 0.5$: maximal parity violations and the states **disentangle**.

3. $\mathscr{C}$ and $\mathscr{B}$ reach **maximums** at $\theta = \pi/2$, and are invariant under $\theta \to \pi - \theta$.