NNLO matching to parton shower with Vincia

山东大学物理学院
SCHOOL OF PHYSICS, SHANDONG UNIVERSITY

Haitao Li (李海涛)

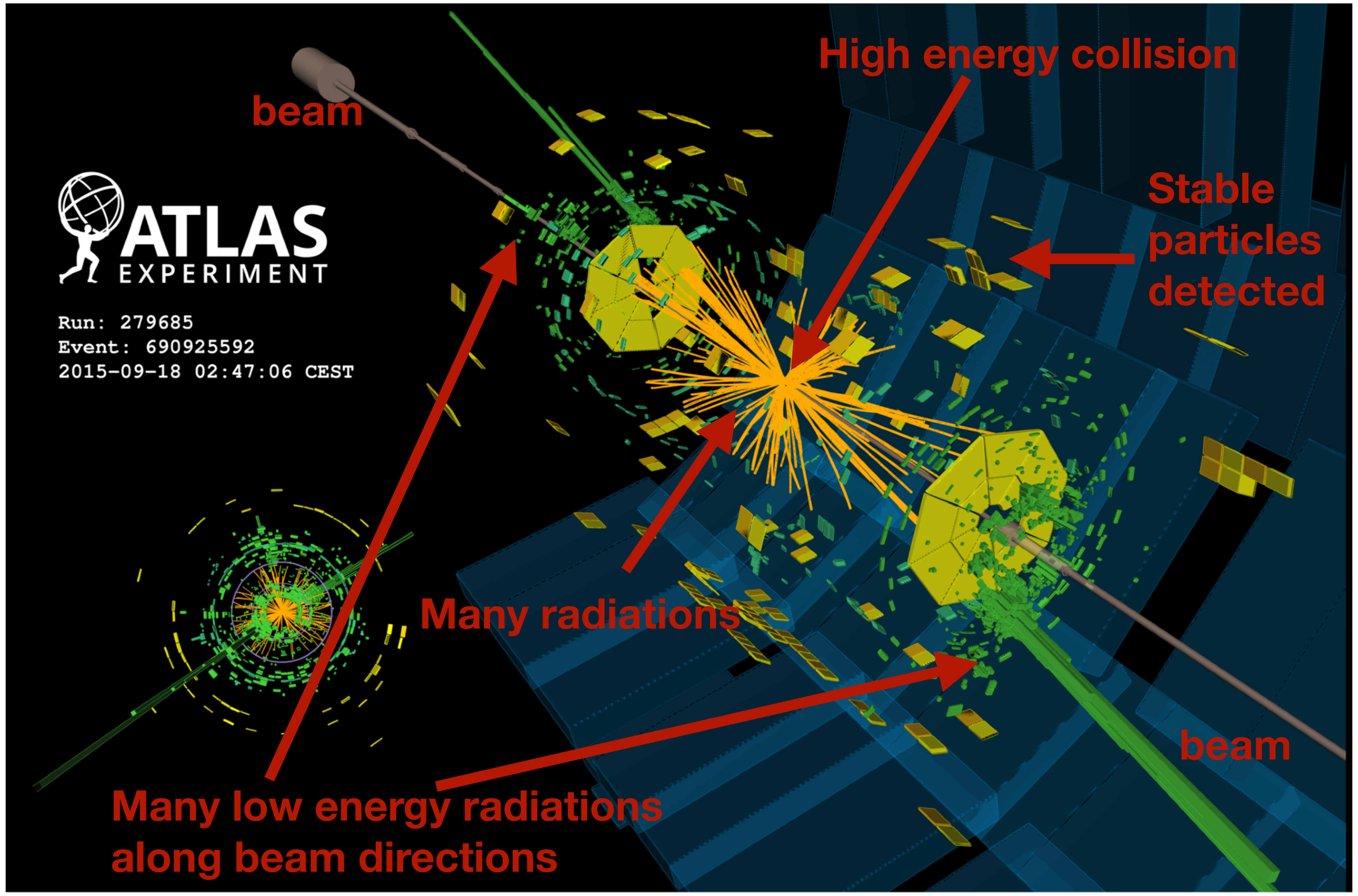
HTL, Skands, PLB 2017
Campbell, Hoche, HTL, Skands, Preuss, PLB 2023
& HTL, Skands, works in preparation

第三届高能物理理论与实验融合发展研讨

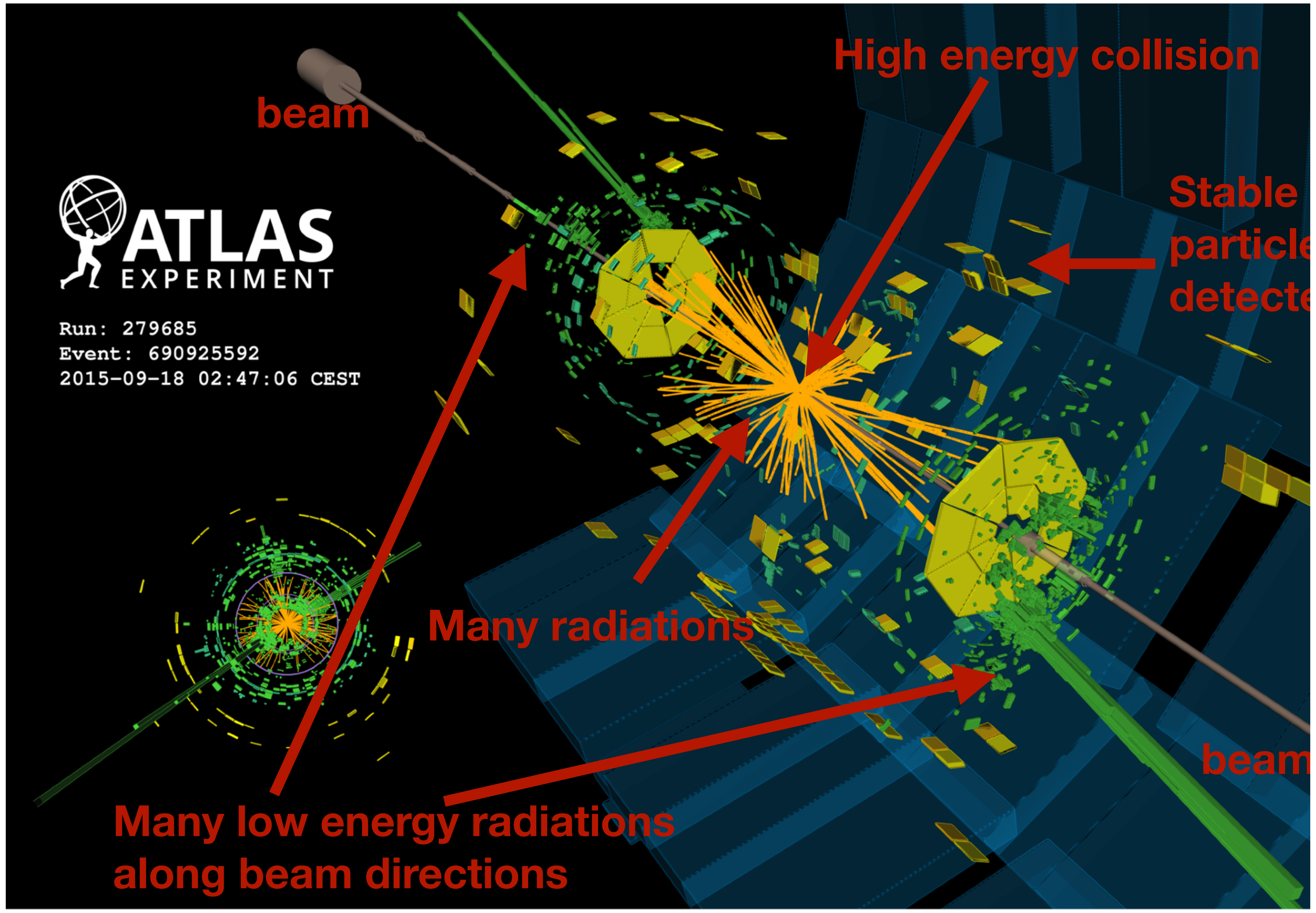
辽宁师范大学

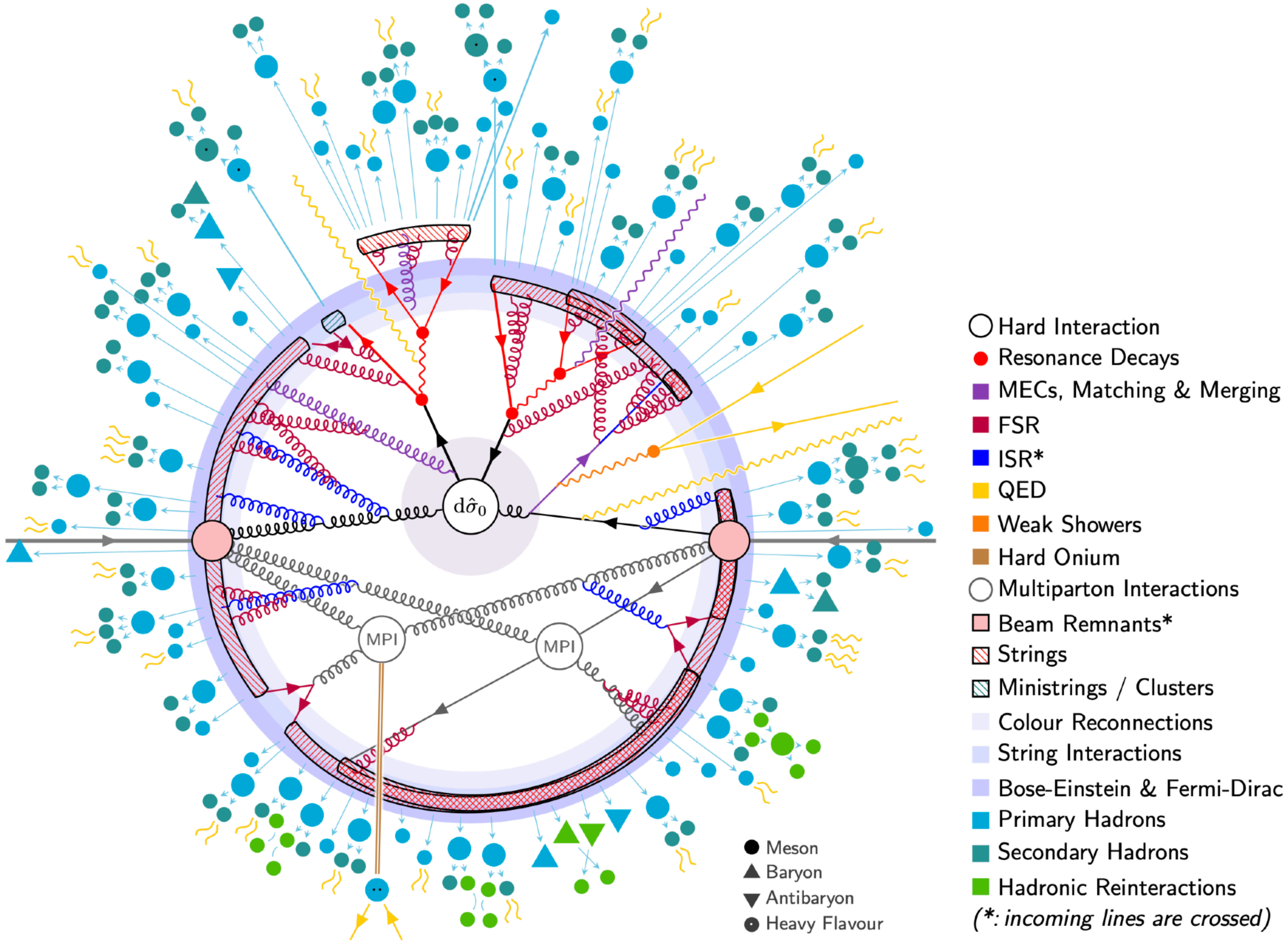
2024年11月2日

Monte Carlo Event generator



Monte Carlo Event generator





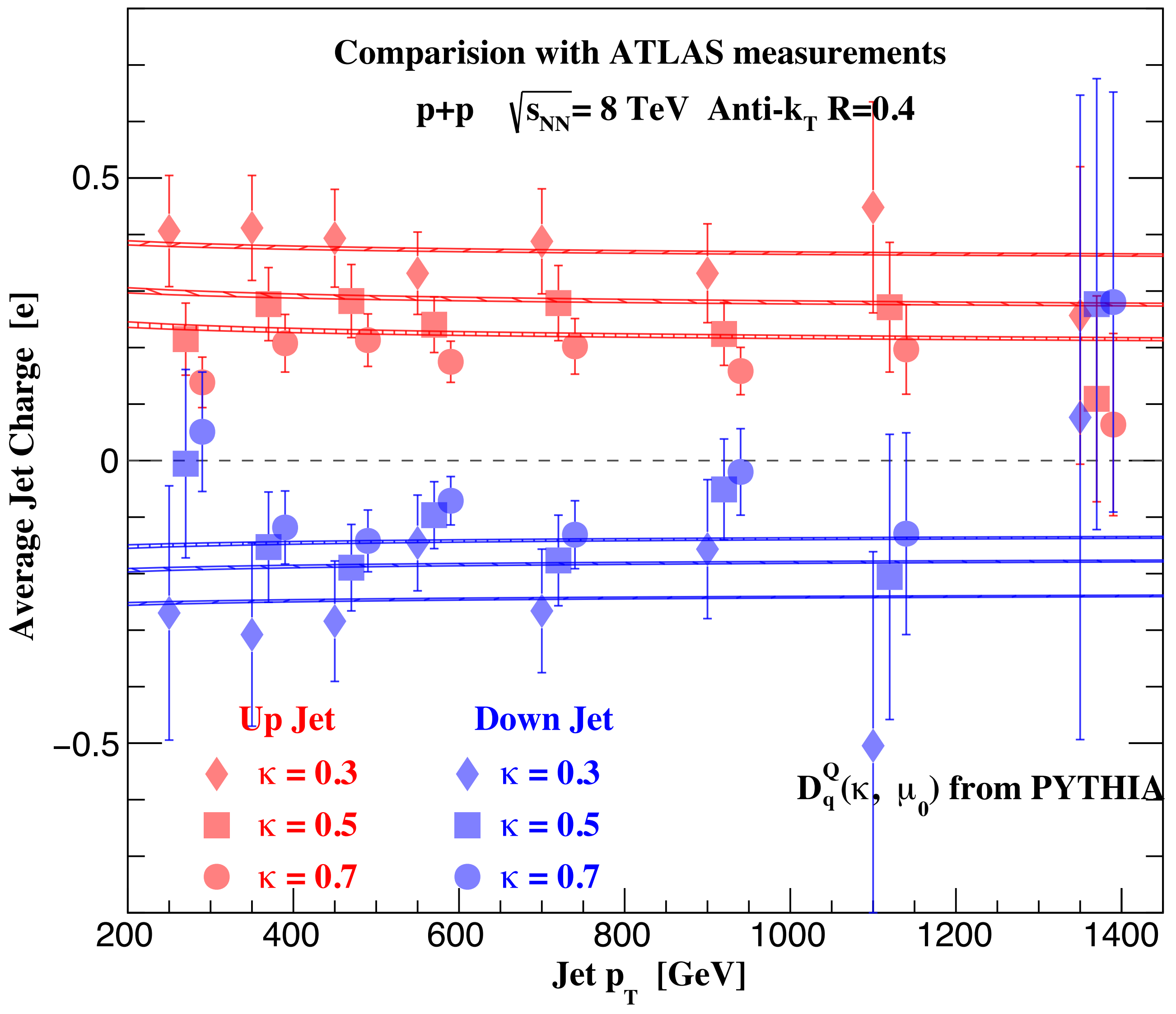
From PYTHIA 8.3

Monte Carlo Event generator

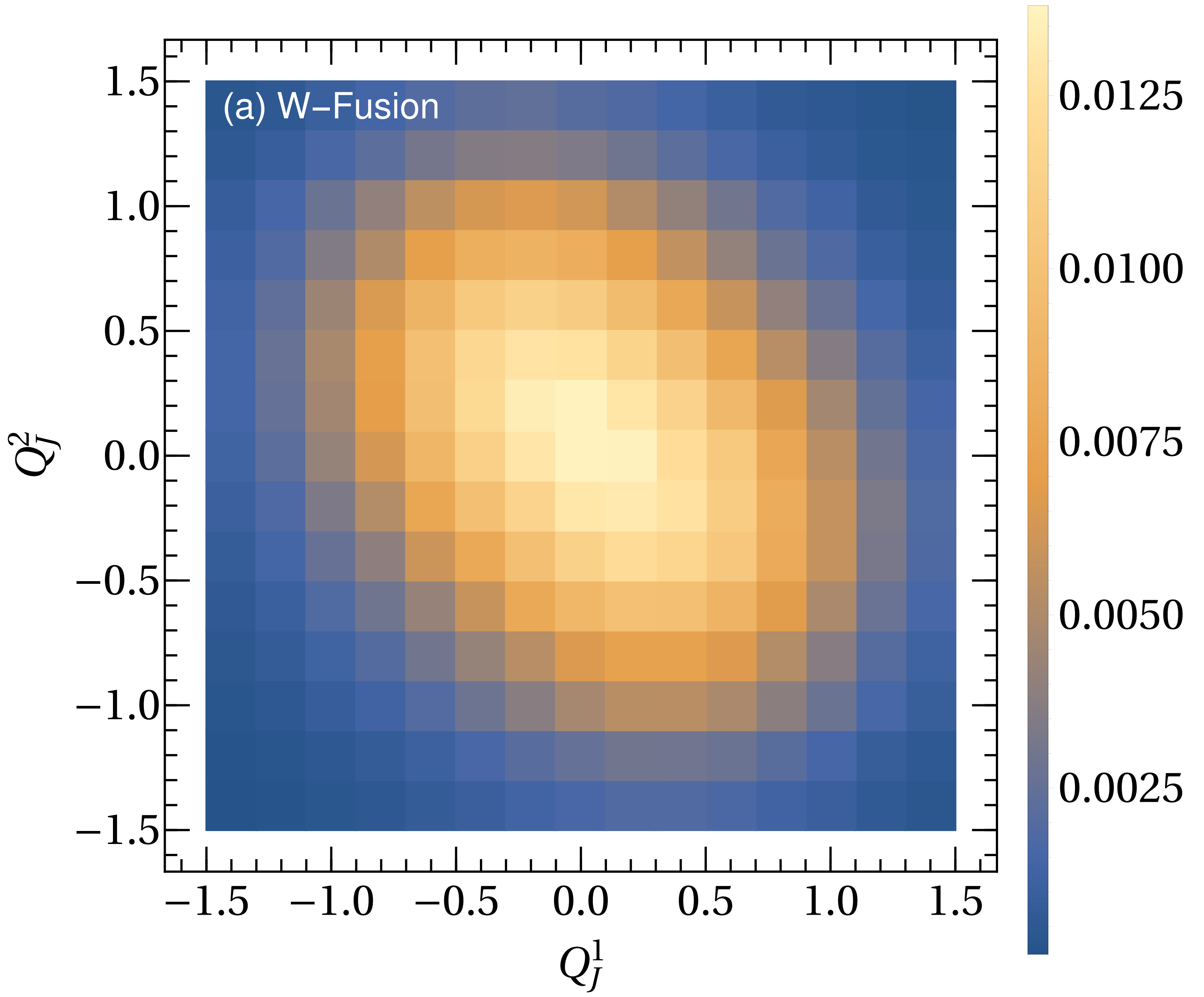
Parton Showers:

- generate a cascade of emissions.
- simulate the rich structure of the events or the jets

- play a key role in the resummation of soft and collinear emissions
- help to simulate the hadronization process
- help in modeling jet production in hadronic collisions
- provide insights into the dynamics of QCD



HTL, Vitev, PRD, 2020; PRL, 2021



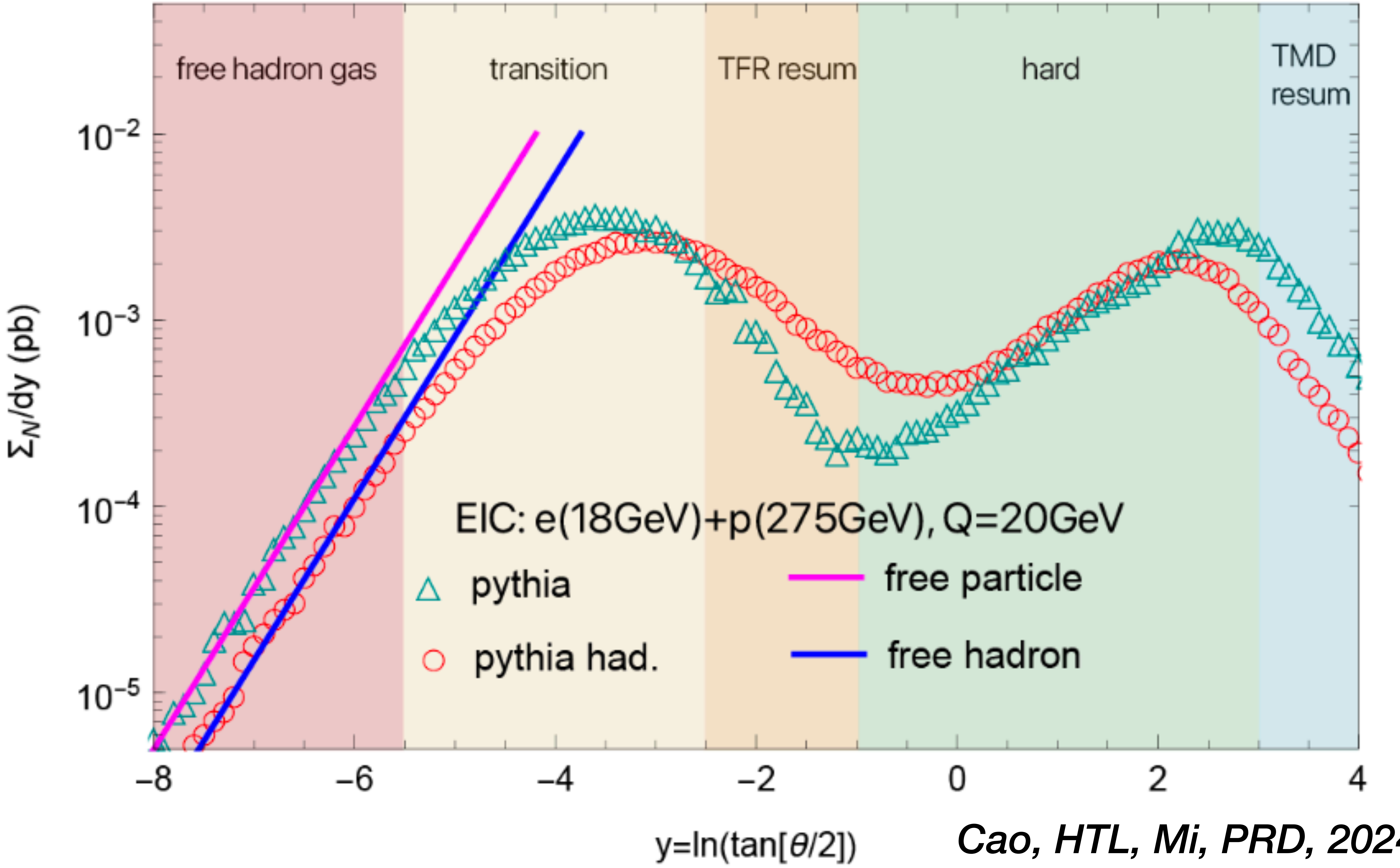
HTL, Yan, Yuan, PRL, 2023

Monte Carlo Event generator

Parton Showers:

- generate a cascade of emissions.
- simulate the rich structure of the events or the jets

- play a key role in the resummation of soft and collinear emissions
- help to simulate the hadronization process
- help in modeling jet production in hadronic collisions
- provide insights into the dynamics of QCD



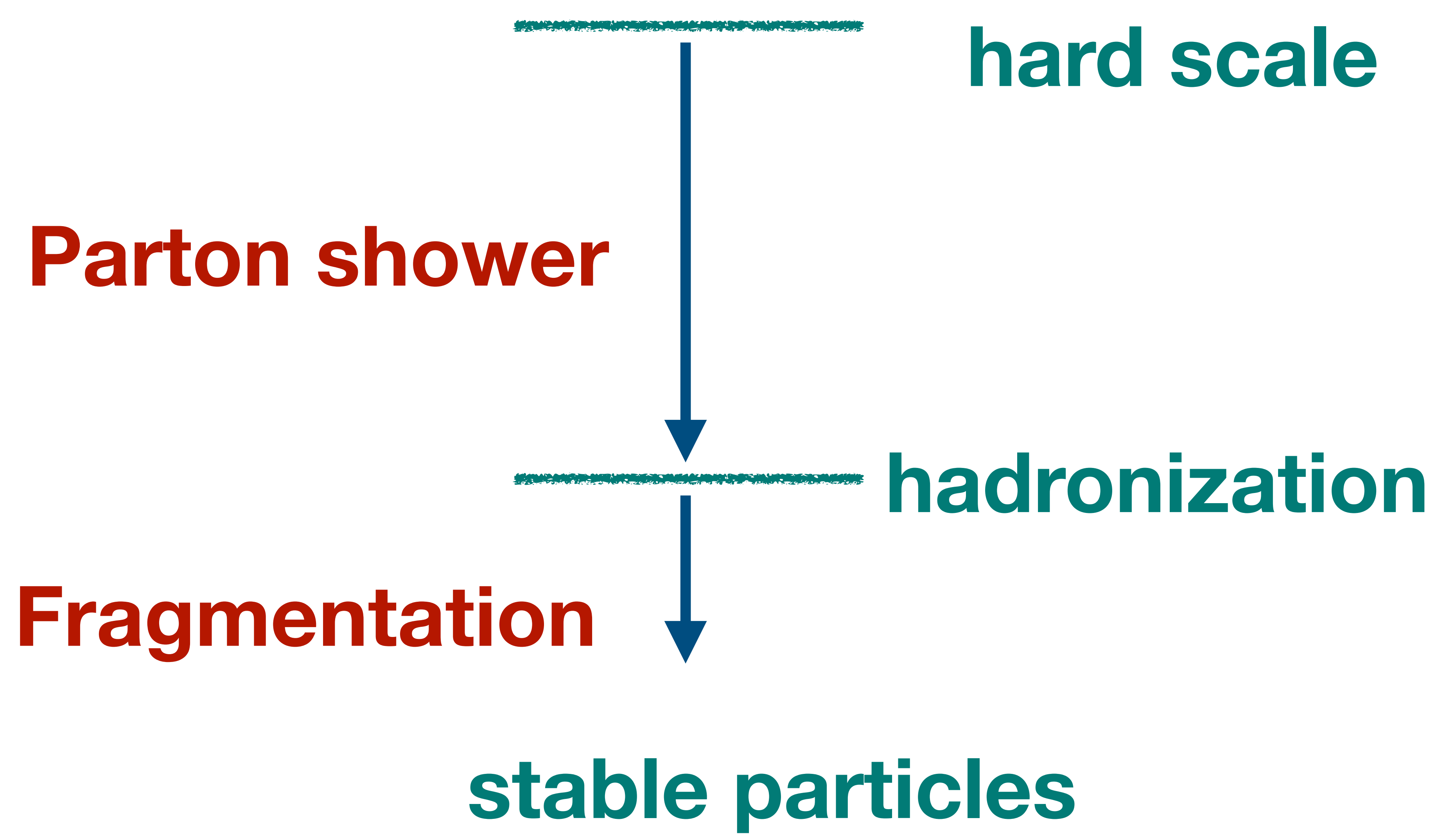
Cao, HTL, Mi, PRD, 2024

Monte Carlo Event generator

The purpose of Monte Carlo event generators is to generate events in as much details as nature (generate average and fluctuation right)

$$\mathscr{P}_{\text{event}} = \mathscr{P}_{\text{Hard}} \otimes \mathscr{P}_{\text{Decay}} \otimes \mathscr{P}_{\text{ISR}} \otimes \mathscr{P}_{\text{FSR}} \otimes \mathscr{P}_{\text{MPI}} \otimes \mathscr{P}_{\text{Had}} \cdots$$

- ❏ **Hard process in high energy**
- ❏ **Transition from high energy to low energy**
  —parton shower
- ❏ **Low energy soft regime**
  —fragmentation



Parton shower: a model for the evolution from high scale to hadronization scale

Parton Shower

In the collinear or soft limit, the matrix element can be factorized as

$$|M(\cdots, p_i, p_j, \cdots)|^2 \xrightarrow{i||j} g_s^2 \mathscr{C} \frac{P(z)}{s_{ij}} |M(\cdots, p_i + p_j, \cdots)|^2$$

$$|M(\cdots, p_i, q, p_j, \cdots)|^2 \xrightarrow{q \to 0} g_s^2 \mathscr{C} \frac{p_i \cdot p_j}{p_i \cdot q \; p_j \cdot q} |M(\cdots, p_i, p_j, \cdots)|^2$$

n+1 external legs (left-hand side) **n external legs** (right-hand side)

Parton Shower

In the collinear or soft limit, the matrix element can be factorized as

$$|M(\cdots, p_i, p_j, \cdots)|^2 \xrightarrow{i||j} g_s^2 \mathscr{C} \frac{P(z)}{s_{ij}} |M(\cdots, p_i + p_j, \cdots)|^2$$

$$|M(\cdots, p_i, q, p_j, \cdots)|^2 \xrightarrow{q \to 0} g_s^2 \mathscr{C} \frac{p_i \cdot p_j}{p_i \cdot q \; p_j \cdot q} |M(\cdots, p_i, p_j, \cdots)|^2$$

n+1 external legs **n external legs**

Together with phase space integration, the cross section is

$$d\sigma_{n+1} = \frac{1}{2s} \int d\phi_{n+1} |M_{n+1}|^2 = d\sigma_n \otimes d\phi_{n \to n+1} \times \frac{|M_{n+1}|^2}{|M_n|^2}$$

Parton Shower

In the collinear or soft limit, the matrix element can be factorized as

$$|M(\cdots,p_i,p_j,\cdots)|^2 \xrightarrow{i||j} g_s^2 \mathscr{C} \frac{P(z)}{s_{ij}} |M(\cdots,p_i+p_j,\cdots)|^2$$

$$|M(\cdots,p_i,q,p_j,\cdots)|^2 \xrightarrow{q\to 0} g_s^2 \mathscr{C} \frac{p_i\cdot p_j}{p_i\cdot q\; p_j\cdot q} |M(\cdots,p_i,p_j,\cdots)|^2$$

n+1 external legs **n external legs**

Together with phase space integration, the cross section is

$$d\sigma_{n+1} = \frac{1}{2s}\int d\phi_{n+1}|M_{n+1}|^2 = d\sigma_n \otimes d\phi_{n\to n+1} \times \frac{|M_{n+1}|^2}{|M_n|^2}$$

If we want to get the single unresolved limit correct, $\frac{|M_{n+1}|^2}{|M_n|^2}$ can be written as universal functions.

higher multiplicities can be obtained recursively

Parton Shower

Non-branching effects

$$d\sigma_{n+1} = \frac{1}{2s}\int d\phi_{n+1}\,|M_{n+1}|^2 = d\sigma_n \otimes d\phi_{n\to n+1} \times \frac{|M_{n+1}|^2}{|M_n|^2}$$

In the exact single-unresolved limit

$s_{ij} = 0$ or $E_q = 0$

Parton Shower

Non-branching effects

$$d\sigma_{n+1} = \frac{1}{2s}\int d\phi_{n+1} |M_{n+1}|^2 = d\sigma_n \otimes d\phi_{n\to n+1} \times \frac{|M_{n+1}|^2}{|M_n|^2}$$

$$d\sigma_{n+2} = \frac{1}{2s}\int d\phi_{n+1} |M_{n+1}|^2 = d\sigma_n \times \frac{1}{2}\left(\int d\phi_{n\to n+1} \times \frac{|M_{n+1}|^2}{|M_n|^2}\right)^2$$

In the exact single-unresolved limit

$s_{ij} = 0$ or $E_q = 0$

$$d\sigma_{n+m} = \frac{1}{2s}\int d\phi_{n+1} |M_{n+1}|^2 = d\sigma_n \times \frac{1}{m!}\left(\int d\phi_{n\to n+1} \frac{|M_{n+1}|^2}{|M_n|^2}\right)^m$$

Parton Shower

Non-branching effects

$$d\sigma_{n+1} = \frac{1}{2s}\int d\phi_{n+1}|M_{n+1}|^2 = d\sigma_n \otimes d\phi_{n\to n+1} \times \frac{|M_{n+1}|^2}{|M_n|^2}$$

$$d\sigma_{n+2} = \frac{1}{2s}\int d\phi_{n+1}|M_{n+1}|^2 = d\sigma_n \times \frac{1}{2}\left(\int d\phi_{n\to n+1} \times \frac{|M_{n+1}|^2}{|M_n|^2}\right)^2$$

In the exact single-unresolved limit

$s_{ij} = 0$ or $E_q = 0$

$$d\sigma_{n+m} = \frac{1}{2s}\int d\phi_{n+1}|M_{n+1}|^2 = d\sigma_n \times \frac{1}{m!}\left(\int d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right)^m$$

take $m \to \infty$

add together from $m = 0$ to $m = \infty$

$$d\sigma_n \times \exp\left[\int d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right]$$

no additional radiation observed

with the probability function $\Delta = \exp\left[\int d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2}\right]$

Parton Shower

Phase space mapping $\int d\phi_{n\to n+1} \frac{|M_{n+1}|^2}{|M_n|^2} = \int_{q^2}^{Q^2} \frac{\mathrm{d}k^2}{k^2} \frac{\alpha_s}{2\pi} \int_{Q_0^2/k^2}^{1-Q_0^2/k^2} \mathrm{d}z P_{ji}(z)$

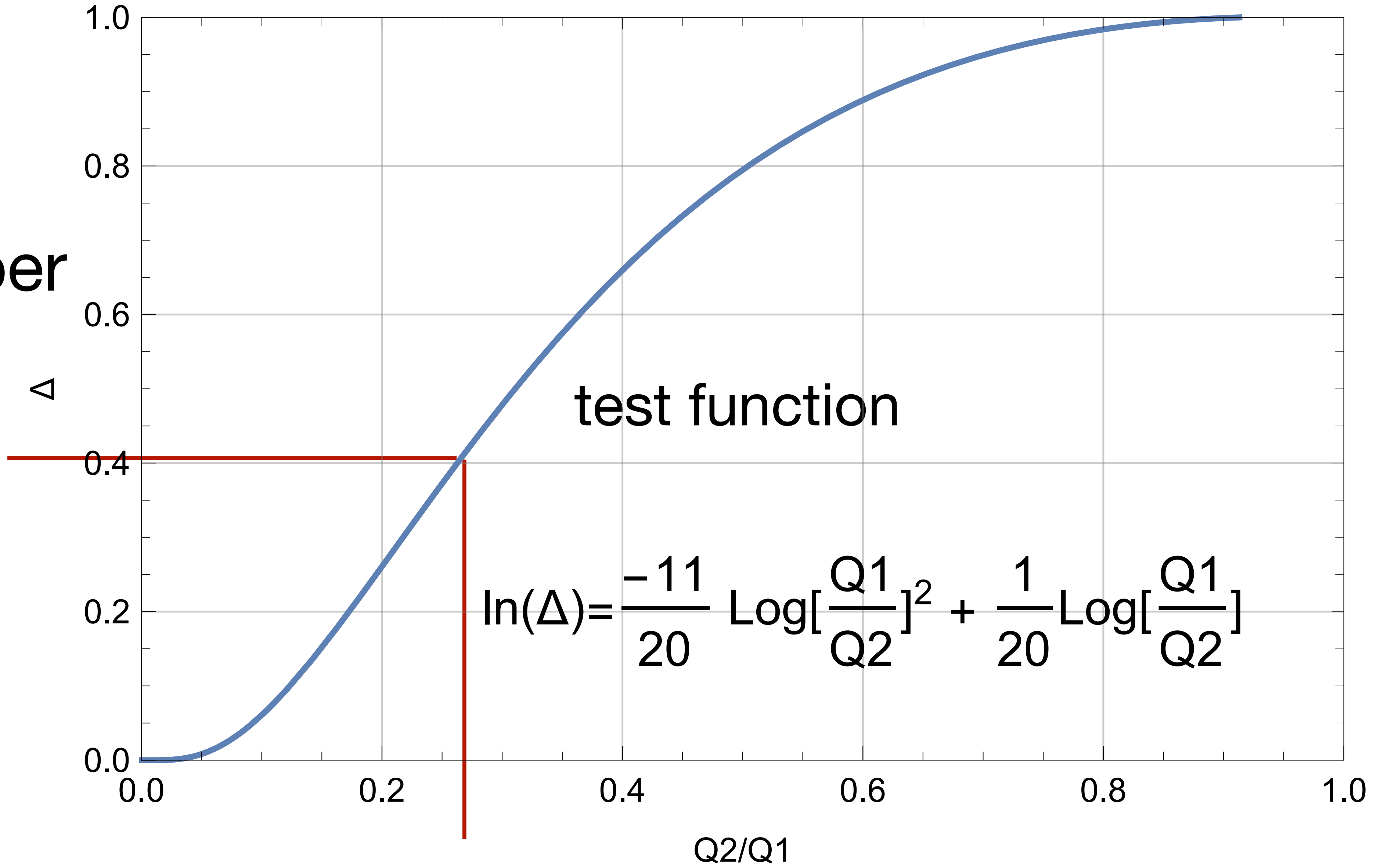
$$\frac{d\theta^2}{\theta^2} = \frac{dq^2}{q^2} = \frac{dk_\perp^2}{k_\perp^2}$$

many choices for the evolution variables

Monte-Carlo Technique and resummation

Sudakov factor $\Delta\left(Q_1^2, Q_2^2\right)$

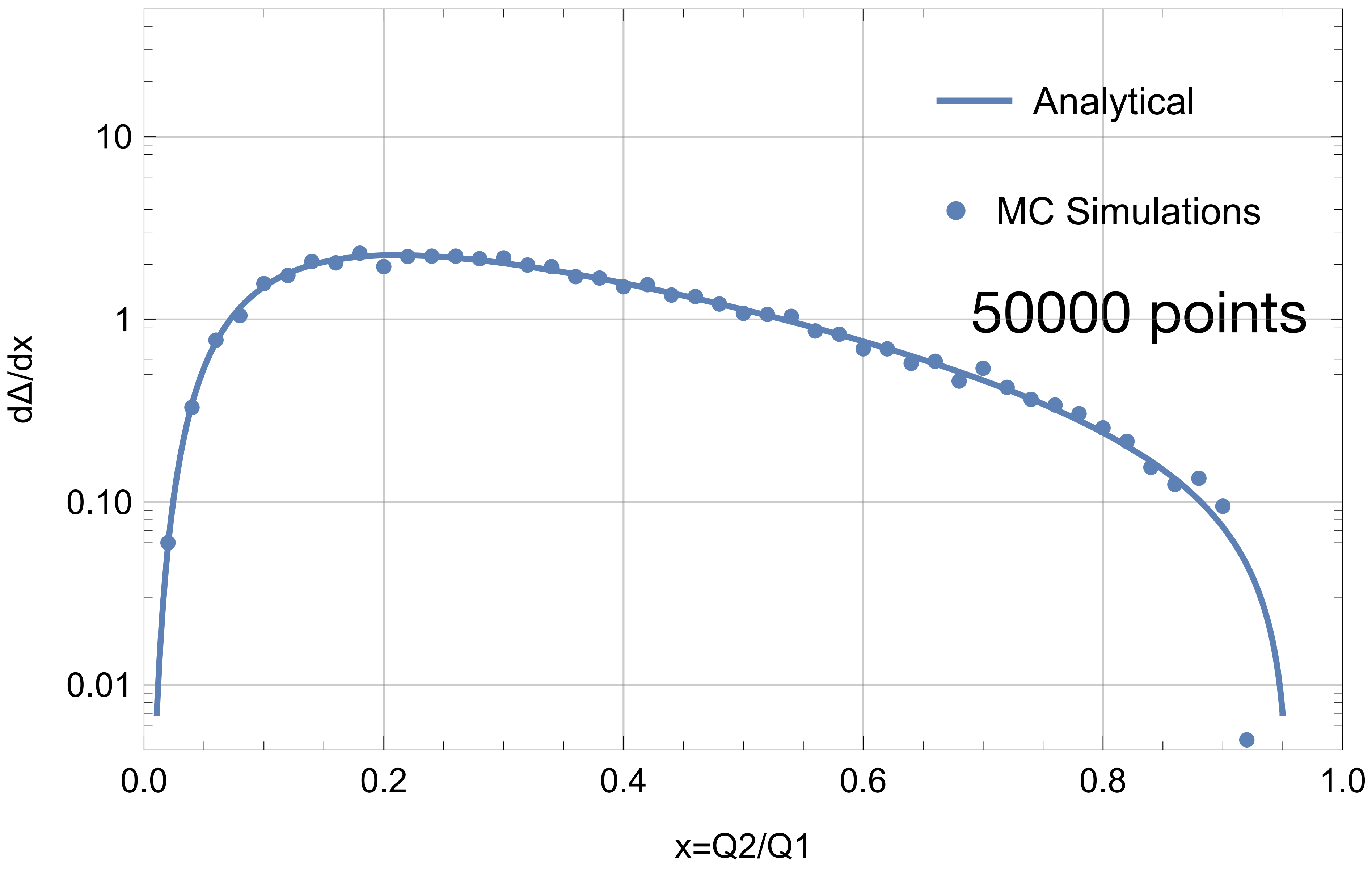
Generate random number $R \in (0,1)$



Solve $R = \Delta$ for Q1/Q2

Q_2/Q_1 **distribution generated by**

$$\frac{d}{dq^2}\Delta\left(Q^2, q^2\right) = \Delta\left(Q^2, q^2\right) \times d\phi_{n\to n+1} \frac{|M_{n+1}|^2}{|M_n|^2}$$



new phase space point generated according to the new scales

Parton Shower

- start from Sudakov form factor: Non-branching probability

  for example $\Delta(Q^2, q^2) = \exp\left[\int_{Q^2}^{q^2} \mathbf{d}\phi_{\mathbf{n}\to\mathbf{n+1}} \frac{|M_{n+1}|^2}{|M_n|^2}\right]$

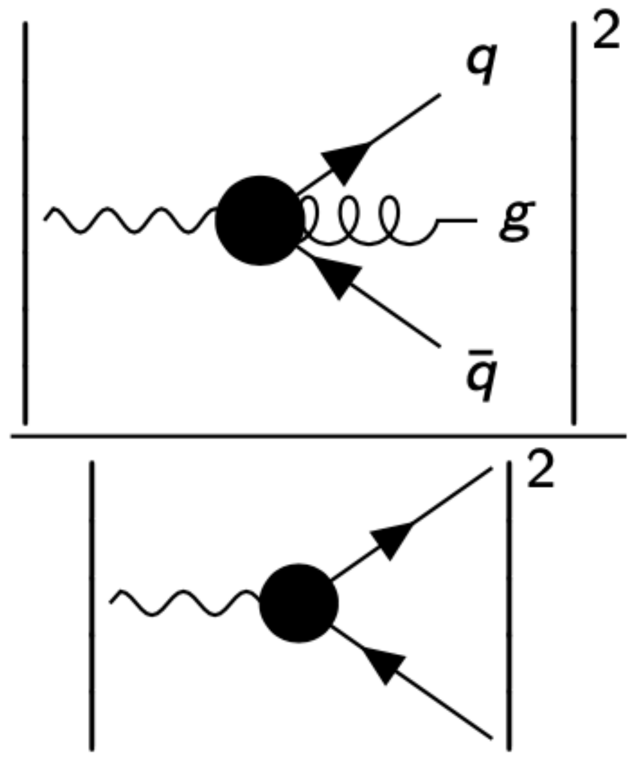
- use Monte Carlo Method to generate kinematics variables such as $k_\perp, z, \phi$

- use these variables to construct $\phi_n \to \phi_{n+1}$

- repeat the above algorithm recursively

Parton Shower

Infrared structure for single unresolved limit is well known



$$\rightarrow \begin{cases} \underbrace{\frac{P_{q\mapsto qg}(z_q)}{s_{qg}}}_{q\text{-collinear}} + \underbrace{\frac{P_{\bar{q}\mapsto \bar{q}g}(z_{\bar{q}})}{s_{g\bar{q}}}}_{\bar{q}\text{-collinear}} & \text{DGLAP} \\ \underbrace{\frac{K_{q\mapsto qg;\bar{q}}(z_q)}{s_{qg}}}_{q\text{-(soft-)collinear}} + \underbrace{\frac{K_{\bar{q}\mapsto \bar{q}g;q}(z_{\bar{q}})}{s_{g\bar{q}}}}_{\bar{q}\text{-(soft-)collinear}} & \text{CS Dipoles} \\ \underbrace{\frac{2s_{q\bar{q}}}{s_{qg}s_{g\bar{q}}}}_{\text{soft}} + \underbrace{\frac{1}{s}\left(\frac{s_{g\bar{q}}}{s_{qg}} + \frac{s_{qg}}{s_{g\bar{q}}}\right)}_{\text{collinear}} & \text{Antennae} \end{cases}$$

DGLAP splitting functions used

applied widely used CS dipole subtraction terms

antenna function obtained directly from matrix element square

Phase space mapping $\int d\phi_{n\to n+1}\frac{|M_{n+1}|^2}{|M_n|^2} = \int_{q^2}^{Q^2}\frac{\mathrm{d}k^2}{k^2}\frac{\alpha_s}{2\pi}\int_{Q_0^2/k^2}^{1-Q_0^2/k^2} \mathrm{d}z P_{ji}(z)$

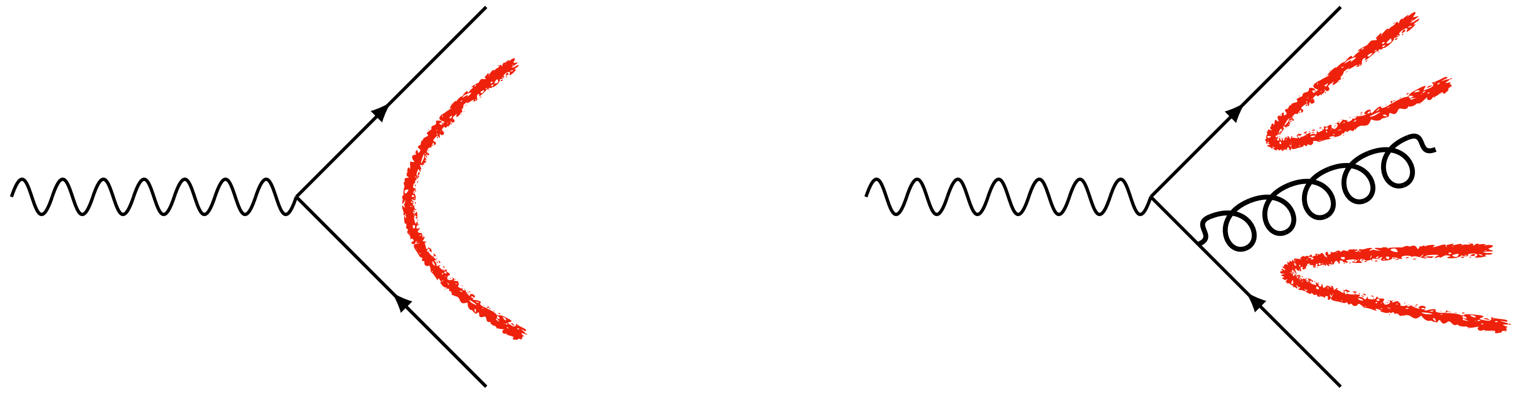
$$\frac{d\theta^2}{\theta^2} = \frac{dq^2}{q^2} = \frac{dk_\perp^2}{k_\perp^2}$$

many choices for the evolution variables

Parton Shower

LO parton shower

From parton shower



$1 - \Delta_i(Q^2, Q_0^2)$

$$\sigma_{\rm NLO}^{\rm PS} = \sigma_0 \Pi_i \left(\Delta_i(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \Delta_i(Q^2, q^2) \int dz P_{ji}(z) \right)$$

0-radiation **1-radiation (Sudakov suppressed)**

Parton Shower

LO parton shower

From parton shower

$1 - \Delta_i(Q^2, Q_0^2)$

$$\sigma_{\mathrm{NLO}}^{\mathrm{PS}} = \sigma_0 \Pi_i \left(\Delta_i(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \Delta_i(Q^2, q^2) \int dz P_{ji}(z) \right)$$

0-radiation **1-radiation (Sudakov suppressed)**

From the definition of Sudakov factor, we have

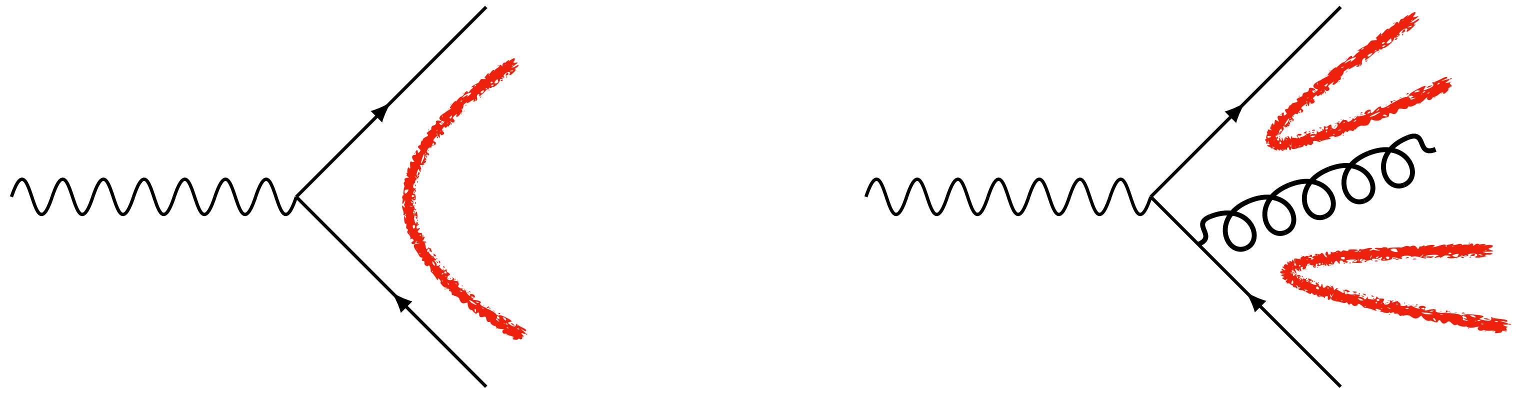
$$\mathscr{P}(\text{unresolved}) + \mathscr{P}(\text{resolved}) = 1$$

probability conservation from the definition of Δ

Parton Shower

LO parton shower

From parton shower



$$1 - \Delta_i(Q^2, Q_0^2)$$

$$\sigma_{\text{NLO}}^{\text{PS}} = \sigma_0 \Pi_i \left(\Delta_i(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \Delta_i(Q^2, q^2) \int dz P_{ji}(z) \right)$$

0-radiation **1-radiation (Sudakov suppressed)**

From the definition of Sudakov factor, we have

$$\mathscr{P}(\text{unresolved}) + \mathscr{P}(\text{resolved}) = 1$$

probability conservation from the definition of Δ

From NLO calculations

$$\sigma_{\text{NLO}} = \sigma_0 + \left(\int d\Phi_n V + \int d\Phi_{n+1} S \right) \mathcal{O}_n + \int d\Phi_{n+1} (R \mathcal{O}_{n+1} - S \mathcal{O}_n)$$

virtual **integrated subtraction** **subtracted real**

$$\sigma_{\text{NLO}} = \sigma_0^n + \int_0^{t_n} d\sigma_{(1)}^n + \int_{t_n} d\sigma_{(1)}^{n+1}$$

t_n as the resolution scale for 1-radiation

LO parton showers reproduce the NLO singular behavior of the underlying hard process with unitarity assumption $V + \int R = 0.$

Parton Shower

NNLO QCD corrections with subtraction method

$$\delta\sigma_{\text{NNLO}} = \underbrace{\left(\int d\Phi_n V_2 + \int d\Phi_{n+2} S^{(1)}_{n+2} + \int d\Phi_{n+2} S_{n+2} \right) \mathcal{O}_n}_{double\ virtual} + \underbrace{\left(\int d\Phi_{n+1} VR\mathcal{O}_{n+1} - \int d\Phi_{n+1} S^{(1)}_{n} \mathcal{O}_n + \int d\Phi_{n+2} S_{n+1} \mathcal{O}_{n+1} \right)}_{real\ virtual (single\ unresolved\ and\ resolved)}$$

$$+ \quad \underbrace{\int d\Phi_{n+2}(R\mathcal{O}_{n+2} - S_{n+1}\mathcal{O}_{n+1} - S_{n+2}\mathcal{O}_n)}_{double\ real\ (double/single\ unresolved, and\ double\ resolved)}$$

Parton Shower

NNLO QCD corrections with subtraction method

$$\delta\sigma_{\text{NNLO}} = \underbrace{\left(\int d\Phi_n V_2 + \int d\Phi_{n+2} S^{(1)}_{n+2} + \int d\Phi_{n+2} S_{n+2} \right) \mathcal{O}_n}_{\textit{double virtual}} + \underbrace{\left(\int d\Phi_{n+1} VR\mathcal{O}_{n+1} - \int d\Phi_{n+1} S^{(1)}_n \mathcal{O}_n + \int d\Phi_{n+2} S_{n+1} \mathcal{O}_{n+1} \right)}_{\textit{real virtual(single unresolved and resolved)}}$$

$$+ \underbrace{\int d\Phi_{n+2} (R\mathcal{O}_{n+2} - S_{n+1}\mathcal{O}_{n+1} - S_{n+2}\mathcal{O}_n)}_{\textit{double real (double/single unresolved,and double resolved)}}$$

In kinematics, there are $n+2$, $n+1$, and n particle final state

$$\delta\sigma_{\text{NNLO}} = \int_0^{t_0} d\sigma^n_{(2)} + \int_{t_n}^{t_{n+1}} d\sigma^{n+1}_{(2)} + \int_{t_{n+1}} d\sigma^{n+2}_{(2)}$$

Parton Shower

NNLO QCD corrections with subtraction method

$$\delta\sigma_{\text{NNLO}} = \underbrace{\left(\int d\Phi_n V_2 + \int d\Phi_{n+2} S^{(1)}_{n+2} + \int d\Phi_{n+2} S_{n+2}\right)\mathcal{O}_n}_{double\ virtual} + \underbrace{\left(\int d\Phi_{n+1} VR\mathcal{O}_{n+1} - \int d\Phi_{n+1} S^{(1)}_n \mathcal{O}_n + \int d\Phi_{n+2} S_{n+1}\mathcal{O}_{n+1}\right)}_{real\ virtual(single\ unresolved\ and\ resolved)}$$

$$+ \underbrace{\int d\Phi_{n+2}(R\mathcal{O}_{n+2} - S_{n+1}\mathcal{O}_{n+1} - S_{n+2}\mathcal{O}_n)}_{double\ real\ (double/single\ unresolved,and\ double\ resolved)}$$

In kinematics, there are $n+2$, $n+1$, and n particle final state

$$\delta\sigma_{\text{NNLO}} = \int_0^{t_0} d\sigma^n_{(2)} + \int_{t_n}^{t_{n+1}} d\sigma^{n+1}_{(2)} + \int_{t_{n+1}} d\sigma^{n+2}_{(2)}$$

How to defined a NLO parton shower?

- expected to have a similar structure as NNLO does
- expected to resummed the singular terms in NNLO corrections

Parton Shower

In kinematics, there are $n+2$, $n+1$, and n particle final state

$$\delta\sigma_{\text{NNLO}} = \int_0^{t_0} d\sigma_{(2)}^{n} + \int_{t_n}^{t_{n+1}} d\sigma_{(2)}^{n+1} + \int_{t_{n+1}} d\sigma_{(2)}^{n+2}$$

Parton shower algorithm requires $\mathscr{P}(\text{unresolved}) + \mathscr{P}(\text{resolved}) = 1$

we want to get $\int_{t_n}^{t_{n+1}} d\sigma_{(2)}^{n+1} + \int_{t_{n+1}} d\sigma_{(2)}^{n+2}$ correctly

$$\sigma_{\text{NLO}}^{\text{PS}} = \sigma_0 \Pi_i \left(\boxed{\Delta_i(Q^2, Q_0^2)} + \boxed{\int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \Delta_i(Q^2, q^2) \int dz P_{ji}(z)} \right)$$

non-radiation

with radiation

$\int_0^{t_0} d\sigma_{(2)}^{n}$ is obtained from unitarity

Parton Shower

In kinematics, there are $n+2$, $n+1$, and n particle final state

$$\delta\sigma_{\rm NNLO} = \int_0^{t_0} d\sigma_{(2)}^{n} + \int_{t_n}^{t_{n+1}} d\sigma_{(2)}^{n+1} + \int_{t_{n+1}} d\sigma_{(2)}^{n+2}$$

Parton shower algorithm requires $\mathscr{P}(\text{unresolved}) + \mathscr{P}(\text{resolved}) = 1$

we want to get $\int_{t_n}^{t_{n+1}} d\sigma_{(2)}^{n+1} + \int_{t_{n+1}} d\sigma_{(2)}^{n+2}$ correctly

$$\sigma_{\rm NLO}^{\rm PS} = \sigma_0 \Pi_i \left(\Delta_i(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \Delta_i(Q^2, q^2) \int dz P_{ji}(z) \right)$$

non-radiation **with radiation**

$\int_0^{t_0} d\sigma_{(2)}^{n}$ is obtained from unitarity

Parton Shower

$$\frac{d}{dQ^2}\left(1 - \Delta(Q_0^2, Q^2)\right) = -\int \frac{d\Phi_3}{d\Phi_2}\, \delta(Q^2 - Q^2(\Phi_3))\, a_3^0 \Delta(Q_0^2, Q^2)$$

LO parton shower

2 to 3 phase space mapping

LO antenna function

Parton Shower

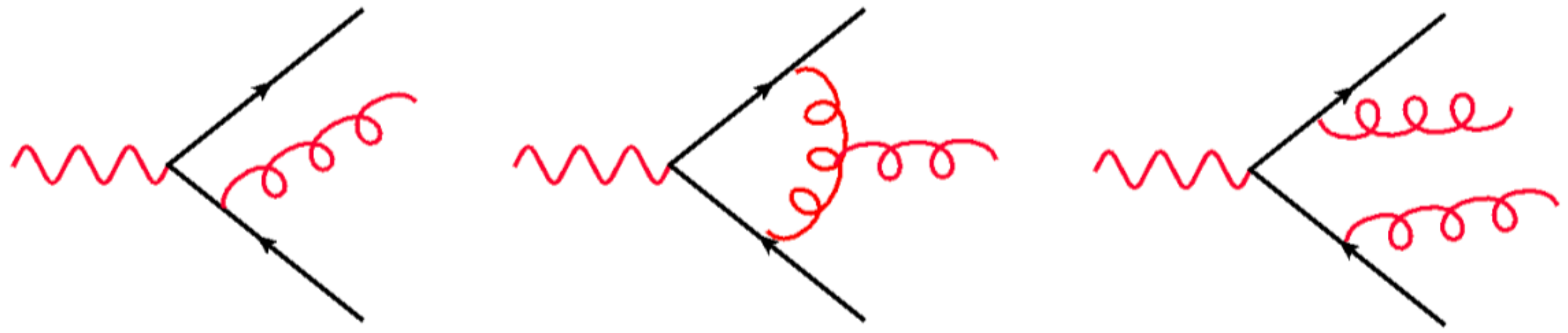
LO parton shower

$$\frac{d}{dQ^2}\left(1-\Delta(Q_0^2,Q^2)\right) = -\int \frac{d\Phi_3}{d\Phi_2}\,\delta(Q^2-Q^2(\Phi_3))\;a_3^0\Delta(Q_0^2,Q^2)$$

2 to 3 phase space mapping

LO antenna function

What we expect for NLO showers



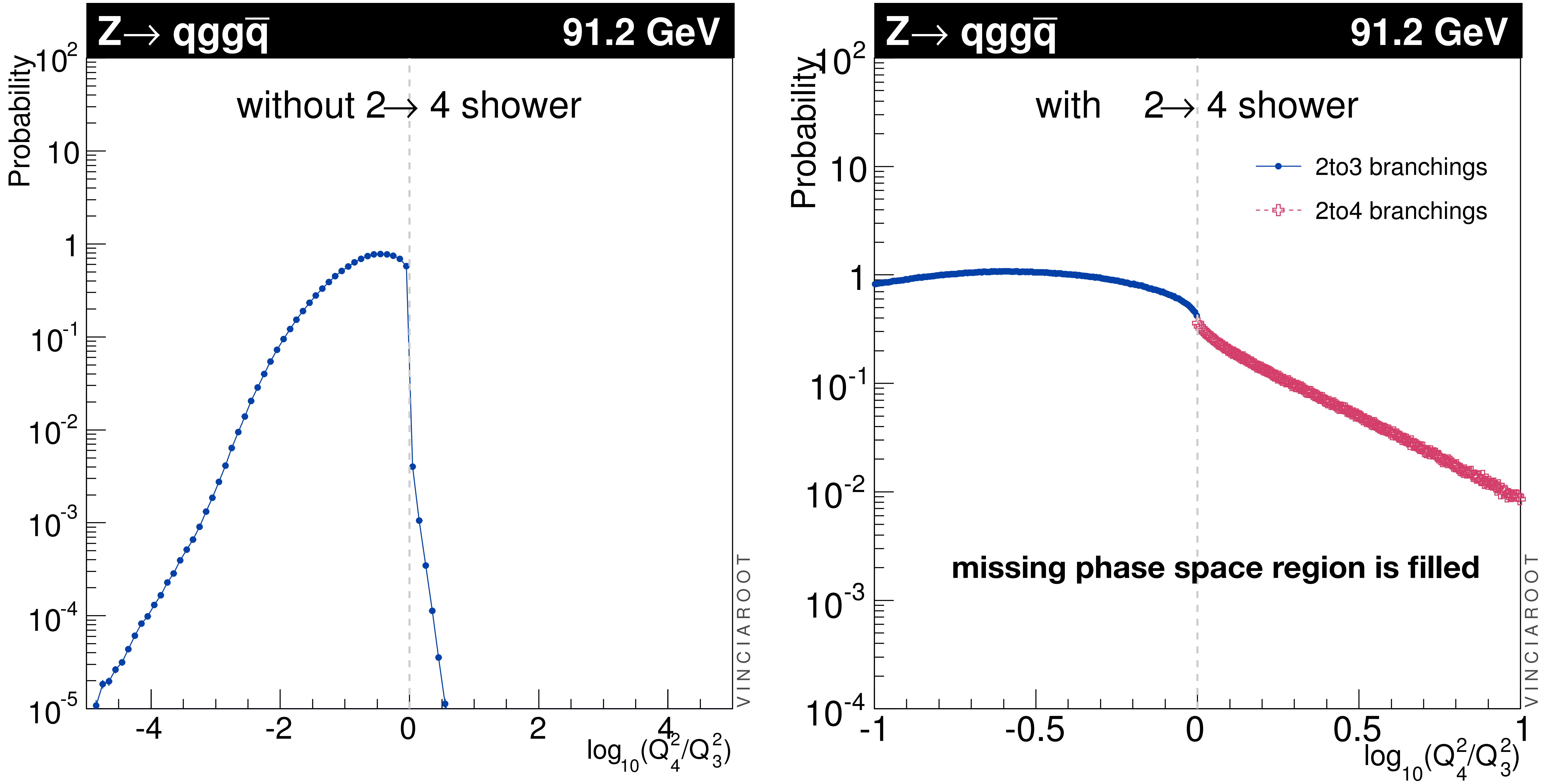
NLO parton shower

$$\frac{d}{dQ^2}\underbrace{\left(1-\Delta\left(Q_0^2,Q^2\right)\right)}_{\text{branching probability}} = \underbrace{-\int\frac{d\Phi_3}{d\Phi_2}\delta\left(Q^2-Q^2\left(\Phi_3\right)\right)\left(a_3^0+a_3^1\right)}_{\text{born and virtual correction}}\Delta\left(Q_0^2,Q^2\right)$$

$$\underbrace{-\int\frac{d\Phi_4}{d\Phi_2}\delta\left(Q^2-Q^2\left(\Phi_4\right)\right)a_4^0}_{\text{real correction}}\Delta\left(Q_0^2,Q^2\right)$$

Parton Shower

single unresolved



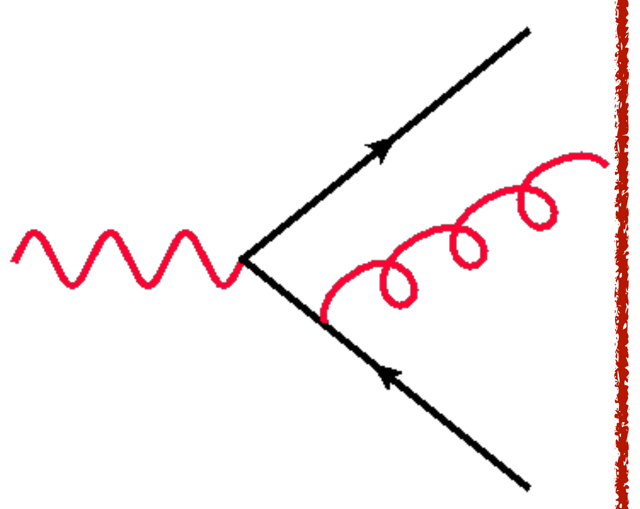
double unresolved

$2 \to 4$ shower fills in the unordered phase space, and, in the limit $Q_4 \sim Q_3$, consistently matches onto the $2 \to 3$ result.

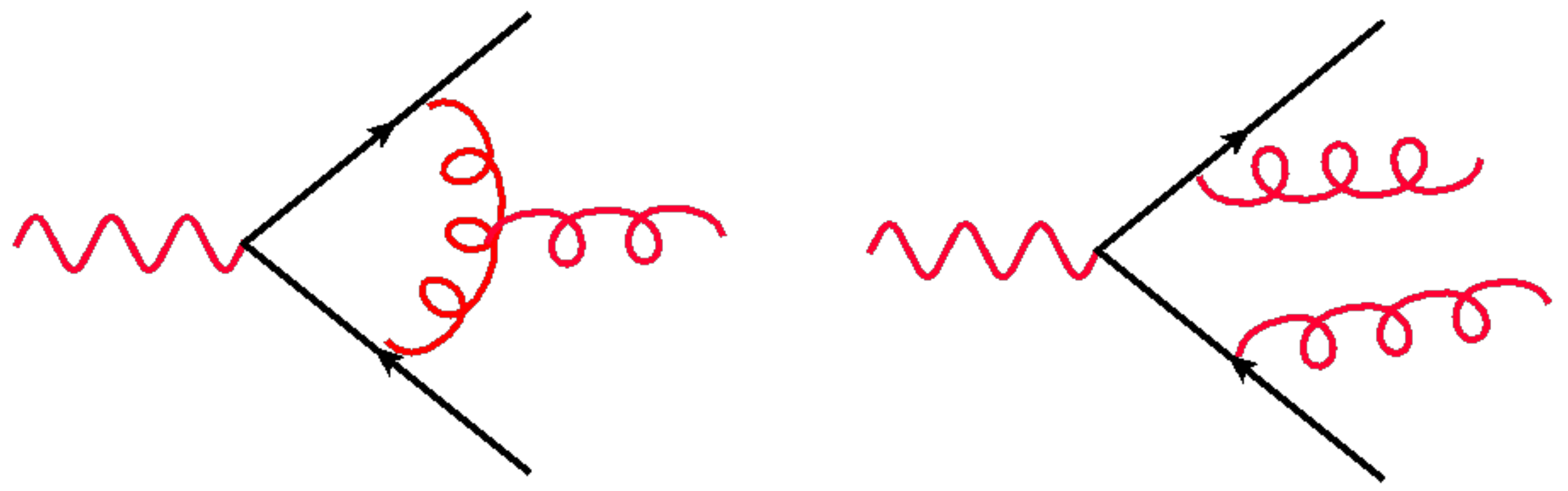
Parton Shower

Resummation of NLO parton showers

Evolution kernel reproduces the singular of the matrix element at NNLO



LO matrix element



NLO corrections

3 parton final state **3+4 parton final state**

Two-loop anomalous dimensions are included correctly at leading color

Many efforts in this direction
- **resummation beyond NLL**
- **NNLL if three loop cusp included**

Dulat, Prestel, Hoche, 2018; HTL, Skands, 2017

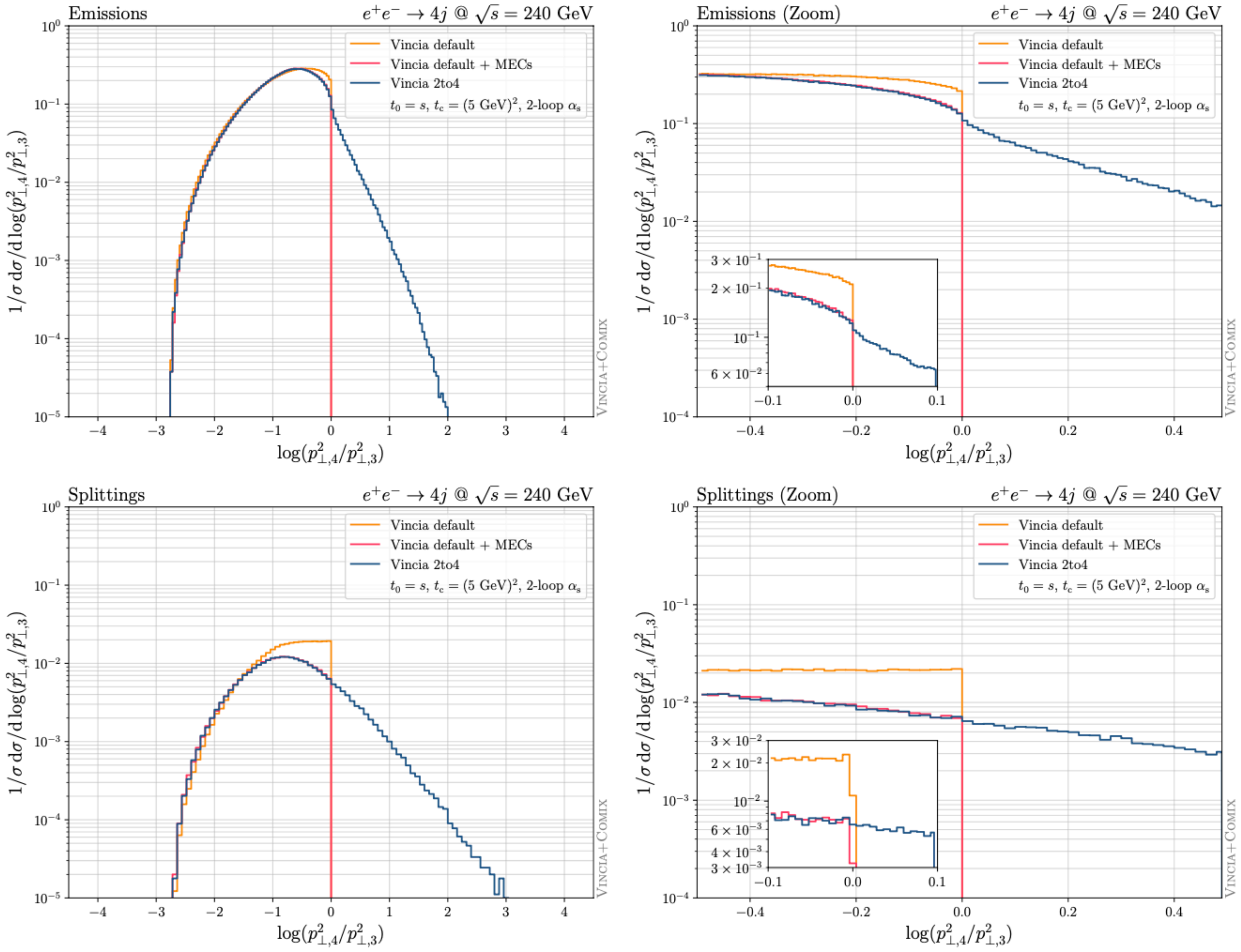
And also parton showers beyond Leading color,
Nagy, Soper, 2019; DeAngels, Forshw, Platzer, 2020; Hamilton etal 2021

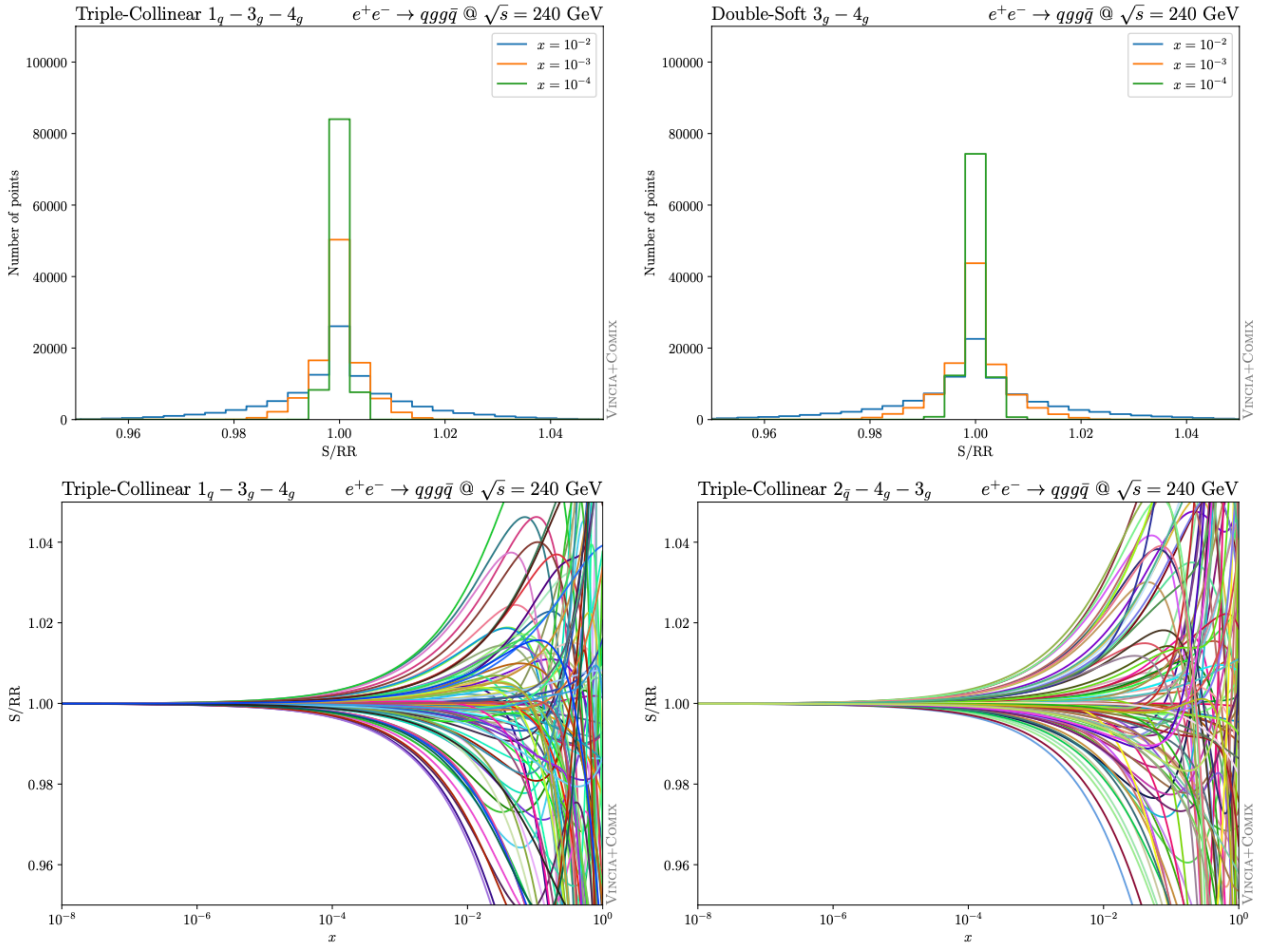
NLO Matching

Matching using NLO antenna shower

$$\Delta_2^{\mathrm{NLO}}\left(t_0, t\right) = \exp\left\{ -\int_t^{t_0} \mathrm{d}\Phi_{+1}\ \mathrm{A}^{(0)}_{2\mapsto 3}\left(\Phi_{+1}\right) w^{\mathrm{NLO}}_{2\mapsto 3}\left(\Phi_2, \Phi_{+1}\right) \right\} \times \exp\left\{ -\int_t^{t_0} \mathrm{d}\Phi^{>}_{+2}\mathrm{A}^{(0)}_{2\mapsto 4}\left(\Phi_{+2}\right) w^{\mathrm{LO}}_{2\mapsto 4}\left(\Phi_2, \Phi_{+2}\right) \right\}$$

Expanding the Sudakov factor to NNLO and compare it with full NNLO corrections **First fully differentially matching**





Summary

- Parton showers are built on soft and collinear approximations to the full cross sections
  - conserve flavor and four momentum, and
  - constructed with the assumption unitarity,
- Showers generate singular parts of higher-order matrix elements and evolve events from high scale to hadronization scale.
- Briefly discussed antenna shower and its NLO corrections
- The NLO shower can be matched to NNLO QCD corrections fully differentially

Indispensable tools for particle physics phenomenology at hadron colliders.

Summary

- Parton showers are built on soft and collinear approximations to the full cross sections
  - conserve flavor and four momentum, and
  - constructed with the assumption unitarity,
- Showers generate singular parts of higher-order matrix elements and evolve events from high scale to hadronization scale.
- Briefly discussed antenna shower and its NLO corrections
- The NLO shower can be matched to NNLO QCD corrections fully differentially

Indispensable tools for particle physics phenomenology at hadron colliders.

Thank you!