# Modeling the Neutron Whispering Gallery to Search for New Short-Range Forces


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University of Virginia
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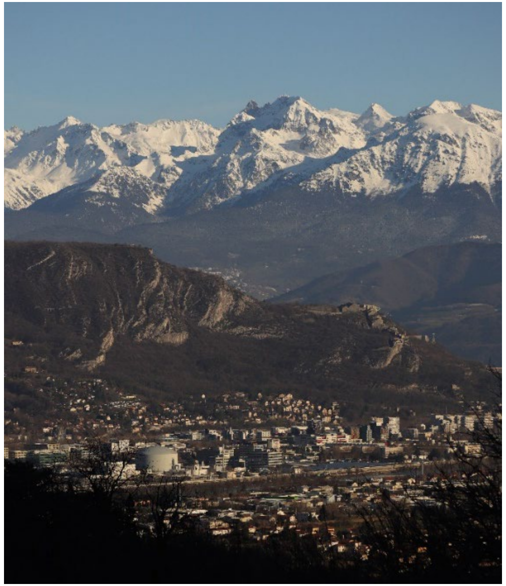

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NEUTRONS FOR SCIENCE®

# New Short-Range Forces

There are several reasons one could expect a new short-range interaction.
**Dark matter** could be explained by the existence of a new particle, which could be a new force mediator. Some theories with **extra spatial dimensions** also predict such a force.

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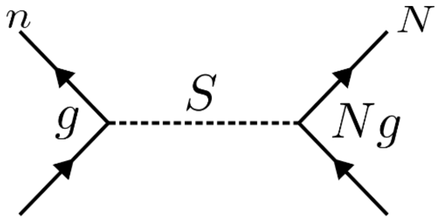


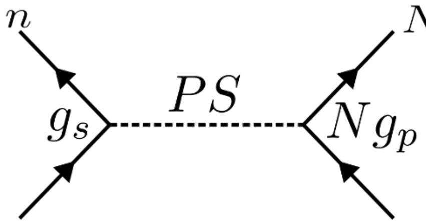


$$\lambda = \frac{\hbar}{m_{DM} c}$$

$$V_5^S(r) \sim N g^2 \frac{e^{-r/\lambda}}{r}$$

$$V_5^{PS}(r) \sim N g_s g_p \left(\frac{1}{r\lambda} + \frac{1}{r^2}\right) e^{-r/\lambda} \vec{\sigma} \cdot \hat{r}$$ [1]

[1] Moody, J. E. & Wilczek, F. New macroscopic forces?
*Phys. Rev. D* **30**, 130–138 (1984).

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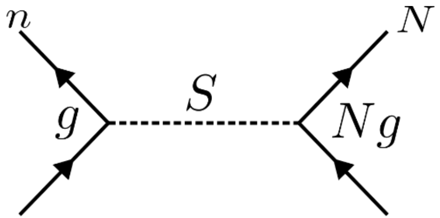


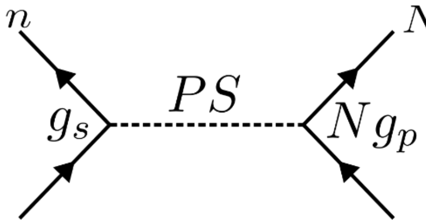


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The macroscopic effect of the new force from a material slab can be calculated by integrating the 5th force potential over the volume of the slab.

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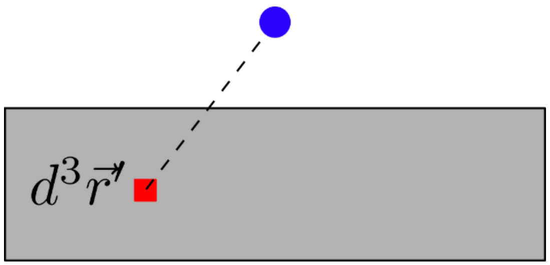
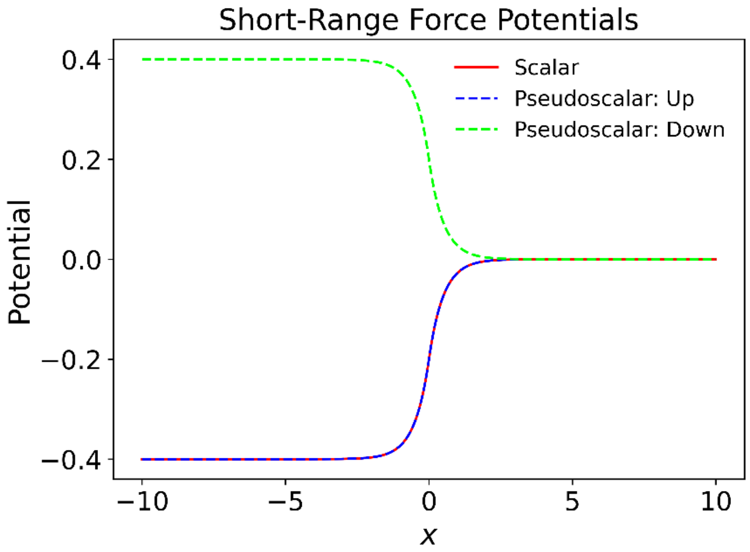
# New Short-Range Forces

$$V_5(\vec{r}) = \frac{\rho}{m}\int V_5^{nN}(\vec{r}-\vec{r}\,')d^3\vec{r}\,'$$



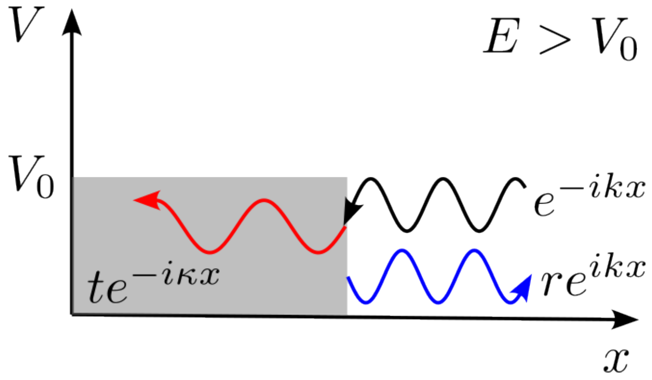




$$V_S(x) = \begin{cases} -W_S e^{-\frac{x}{\lambda}} & x \geq 0 \\ -W_S\left(2 - e^{-\frac{x}{\lambda}}\right) & x < 0 \end{cases}$$

$$V_{PS}^{\pm}(x) = \sigma_z \begin{cases} -W_{PS} e^{-\frac{x}{\lambda}} & x \geq 0 \\ -W_{PS}\left(2 - e^{-\frac{x}{\lambda}}\right) & x < 0 \end{cases}$$

# The Neutron Whispering Gallery

Neutrons can be reflected off material slabs (mirror) if $E_\perp \lesssim V_0$ where $V_0$ is the “optical potential”



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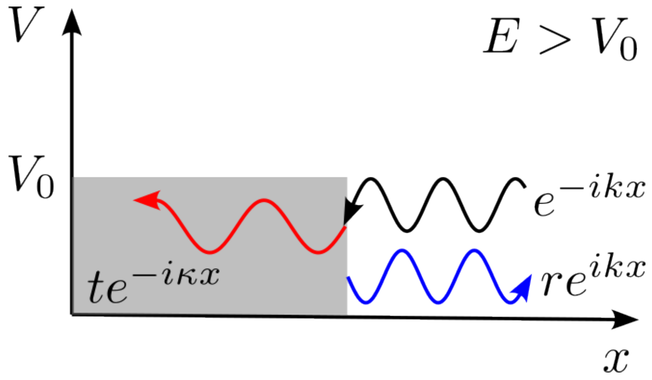
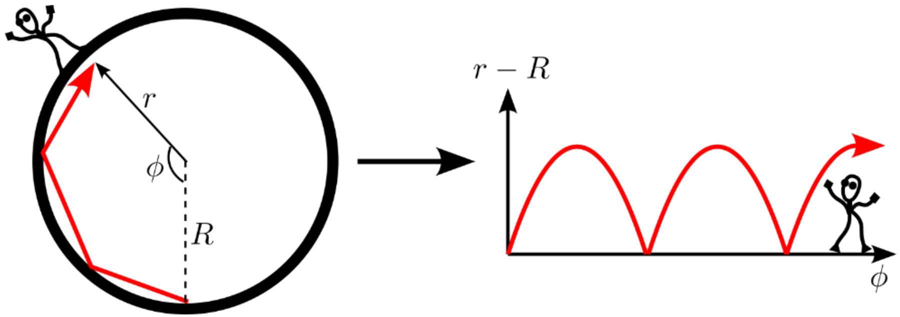



Neutrons incident onto a curved surface with a small grazing angle can be reflected many times.



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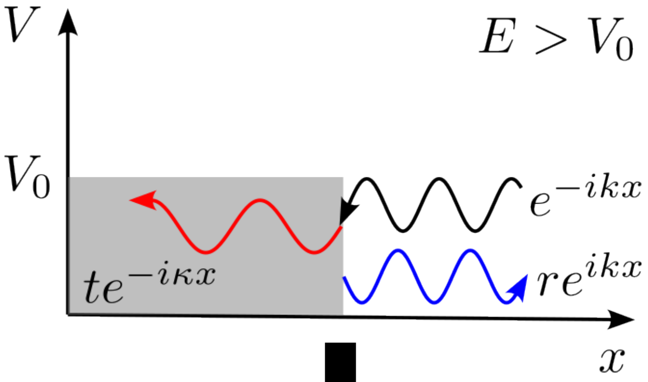
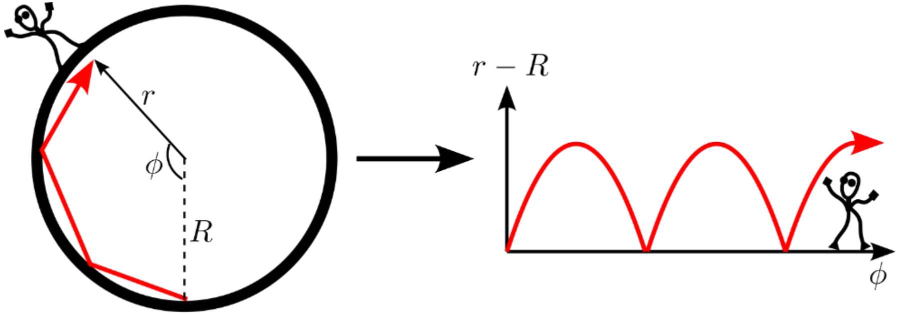
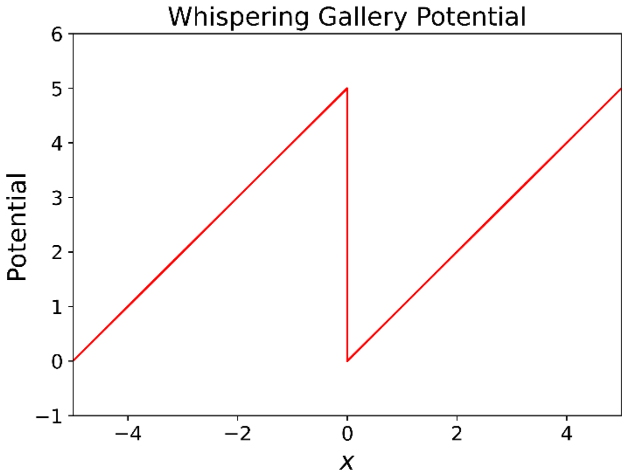
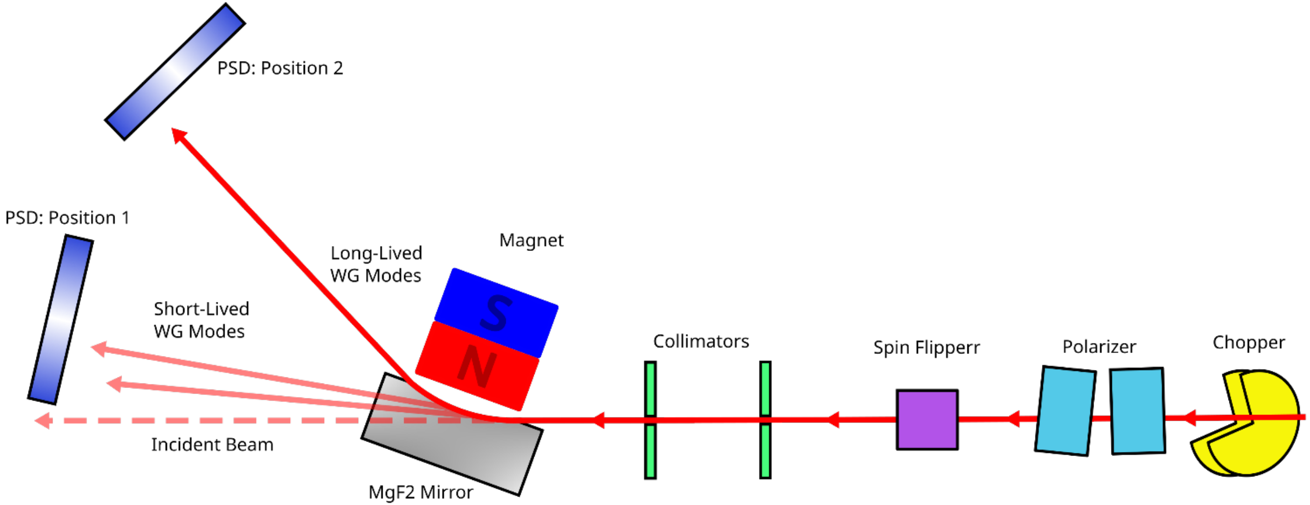
$$V(x) = V_0\Theta(-x) + \frac{mv^2}{R}x$$
$$x = r - R$$



# Experimental Realization



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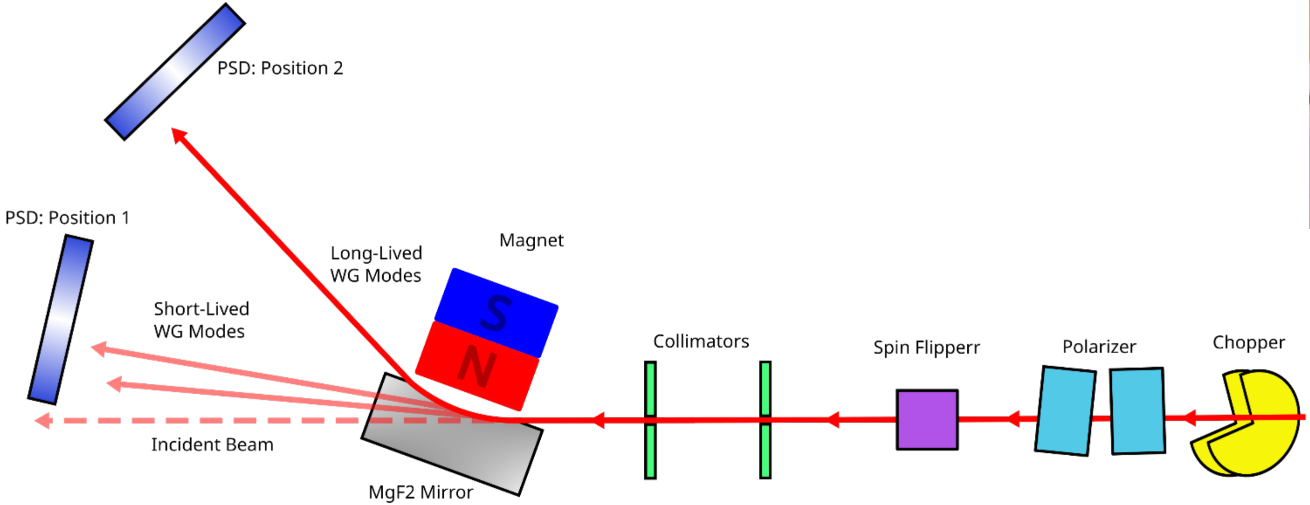
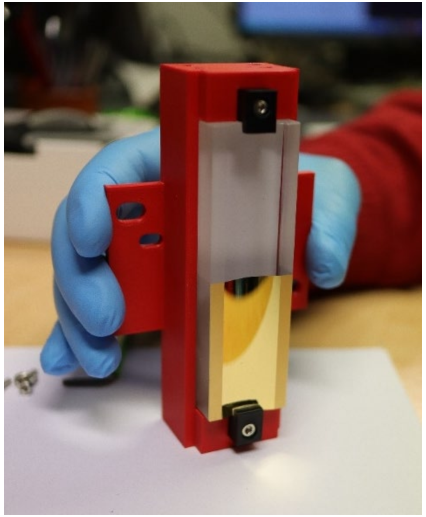


<u>Mirror Parameters</u>

$R \approx 3$ cm

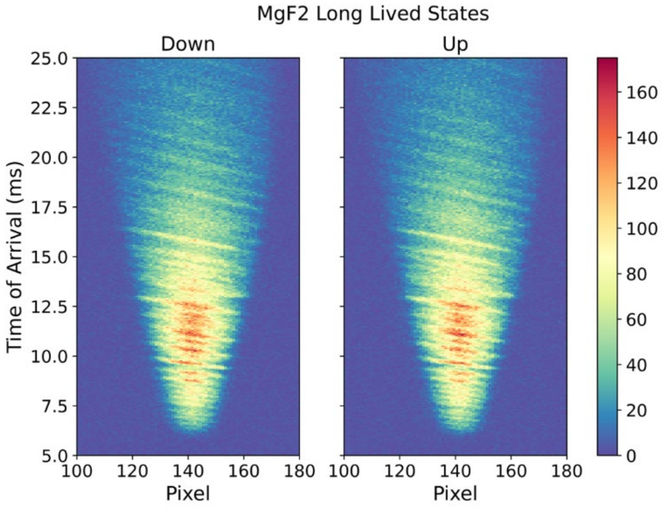
$\Theta \approx 40^{\circ}$

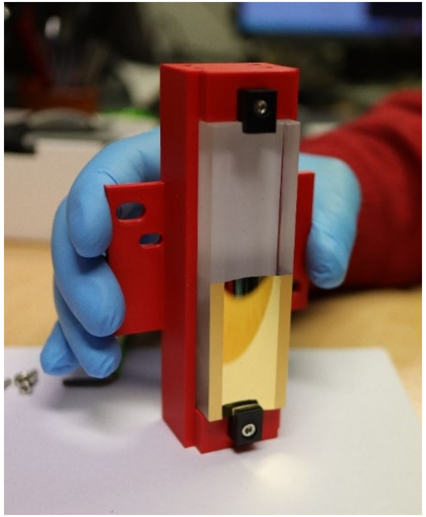
# Experimental Realization

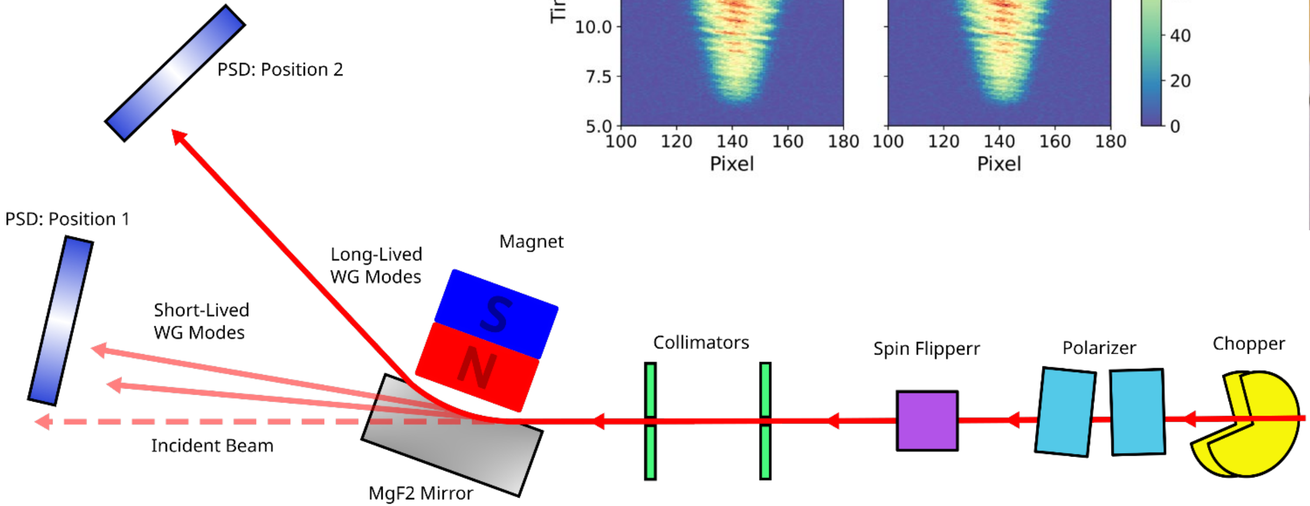
## Characteristic Numbers

$$160 < v < 1320\frac{m}{s}$$
$$3Å < \lambda < 25Å$$

$$32.5 \text{ nm} < l_0 < 133 \text{ nm}$$
$$1 \text{ neV} < \epsilon_0 < 20 \text{ neV}$$
$$33\ ns < \tau_0 < 565 \text{ ns}$$



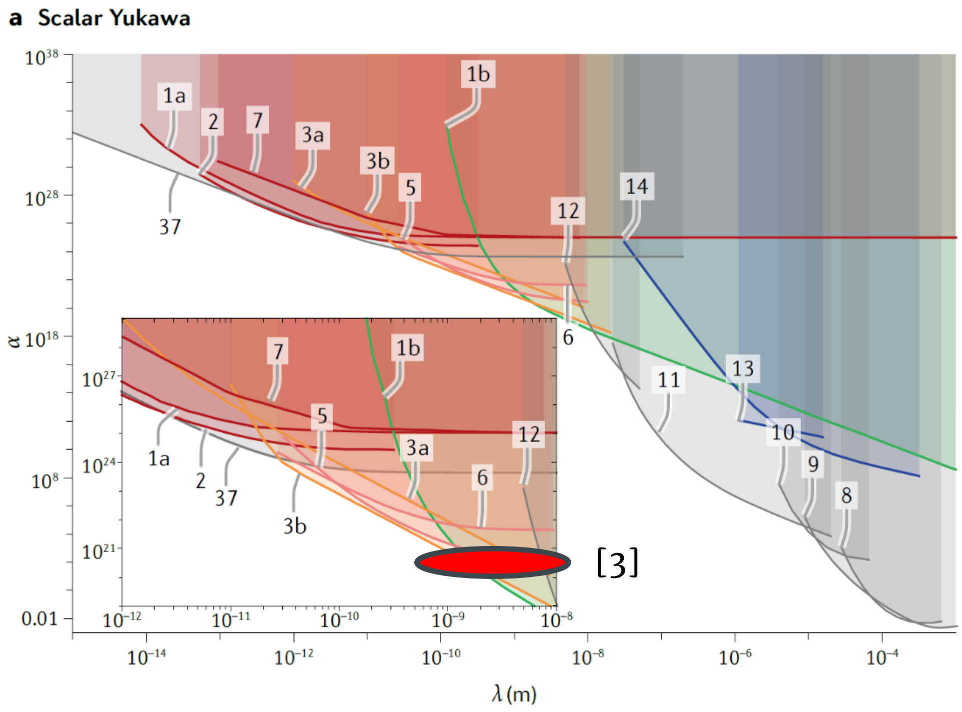



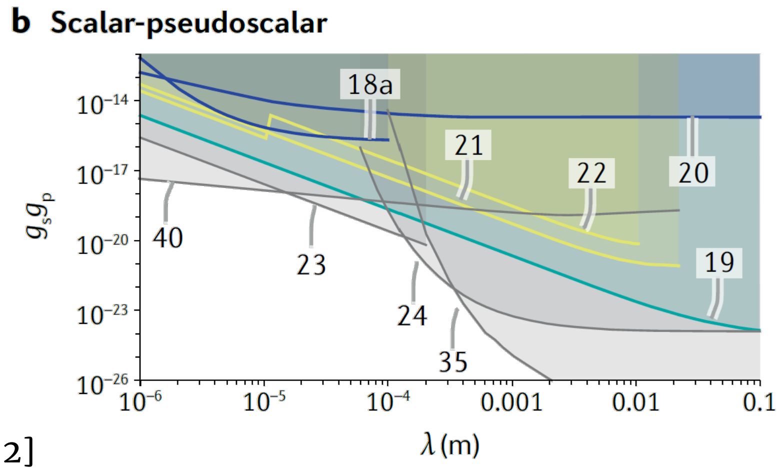



## Mirror Parameters

$$R \approx 3 \text{ cm}$$
$$\Theta \approx 40°$$

# Constraints on Short-Range Forces

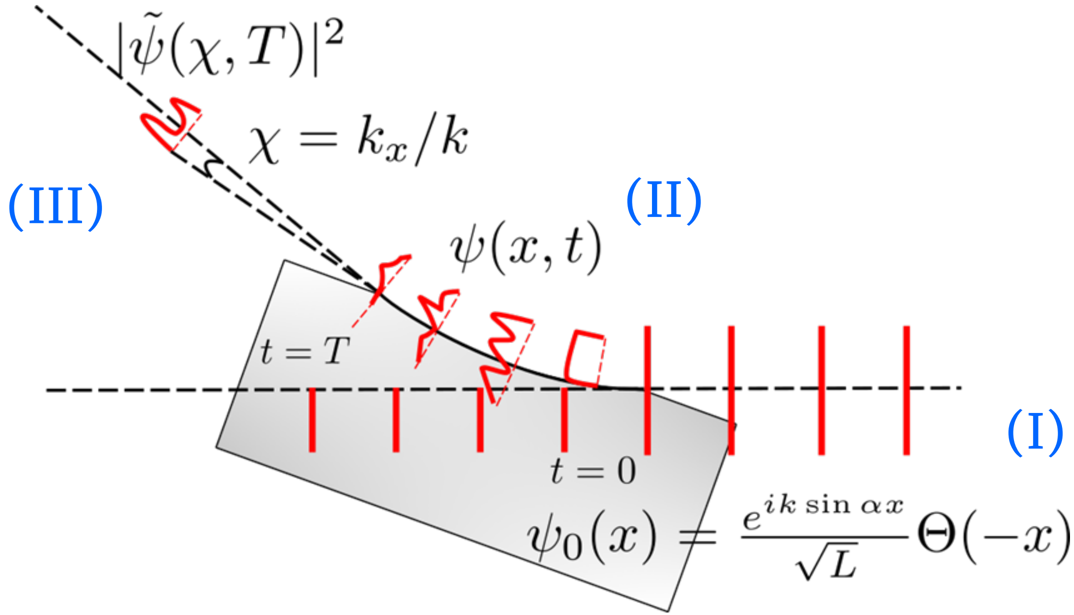



[2]


[2] Sponar, S et. al. (2021). Tests of fundamental quantum mechanics and dark interactions with low-energy neutrons. Nature Reviews Physics, 3(5), 309–327
[3] Emily Perry Master's Report

# Theoretical Description




Region (I) – Beam has small divergence and large spatial size $(100\mu\text{m} \gg l_0 \sim 10\text{ nm})$

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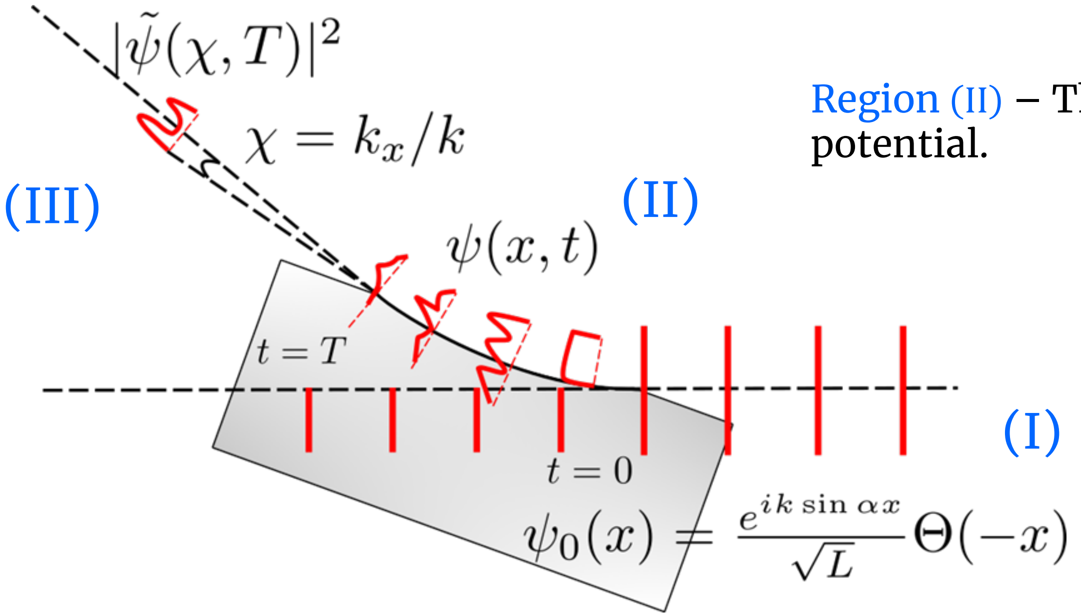
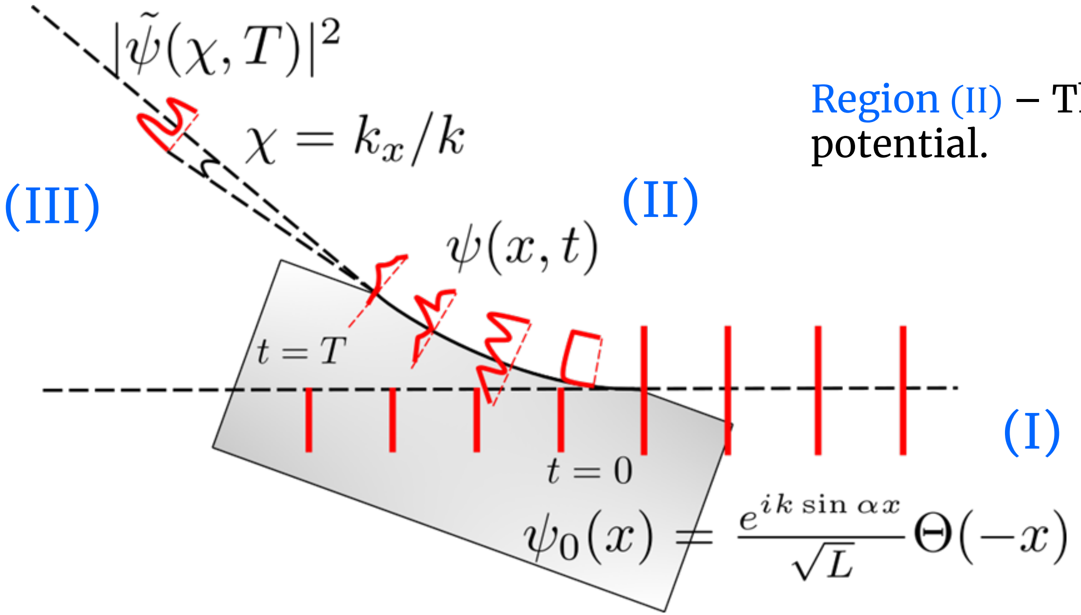



Region (II) – The wave packet evolves in WG potential.

Region (I) – Beam has small divergence and large spatial size ($100\mu m \gg l_0 \sim 10$ nm)

# Theoretical Description

Region (III) – Wave packet evolves in free space. In the far-field, we measure its Fourier Transform/transverse velocity distribution




Region (II) – The wave packet evolves in WG potential.

Region (I) - Beam has small divergence and large spatial size (100μm ≫ $l_0$~10 nm)

# WG Solution: Continuum Expansion

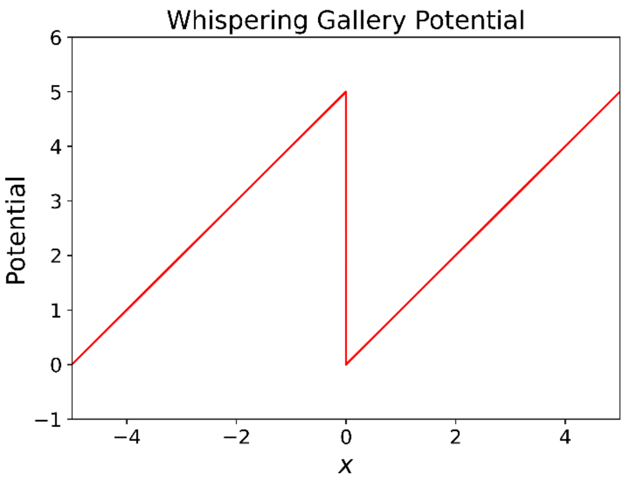
There are no bound states permissible in this potential.

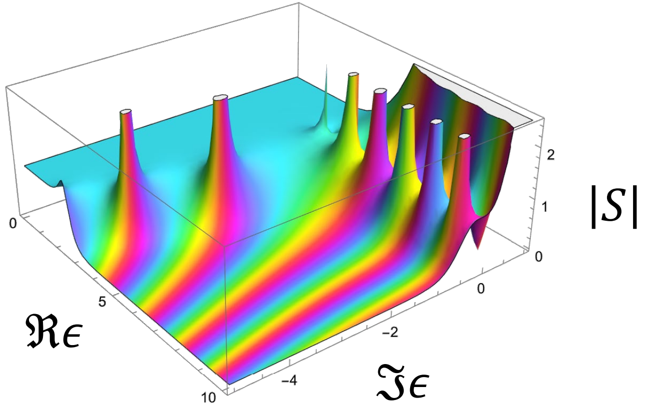
$$\psi(x,t) = \int c_\epsilon \psi_\epsilon(x) e^{-i\epsilon\tau} d\epsilon$$

$$c_\epsilon = \int \psi_0(x') \psi_\epsilon^*(x') dx'$$

The solution to the Schrödinger equation

$$\psi_\epsilon(x) = \frac{1}{2}\left[\chi_\epsilon^+(x) - S_\epsilon \chi_\epsilon^-(x)\right]$$



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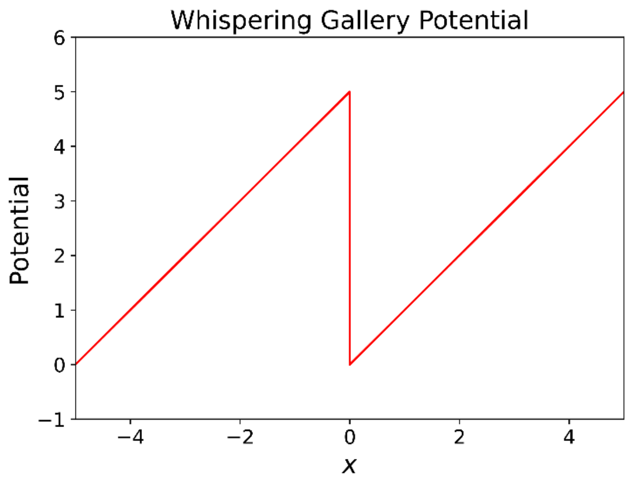


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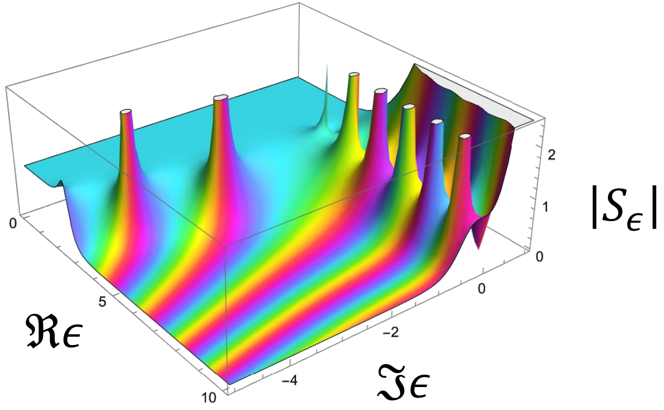
$$\psi_\epsilon(x) = \frac{1}{2}[\chi_\epsilon^+(x) - S_\epsilon \chi_\epsilon^-(x)]$$

where

$$\psi_\epsilon(x) = \begin{cases} \mathrm{Ai}(x-\epsilon) & x \geq 0 \\ \mathrm{Ci}^+(x+u-\epsilon) - S_\epsilon \mathrm{Ci}^-(x+u-\epsilon) & x \leq 0 \end{cases}$$

and $\mathrm{Ci}^\pm(z) = \mathrm{Ai}(z) \pm i\mathrm{Bi}(z)$



# Quasi-Stationary States

Looking at the wave functions at the poles of $S_\epsilon$, we find **Quasi-Stationary States** which have discrete and complex eigen energies

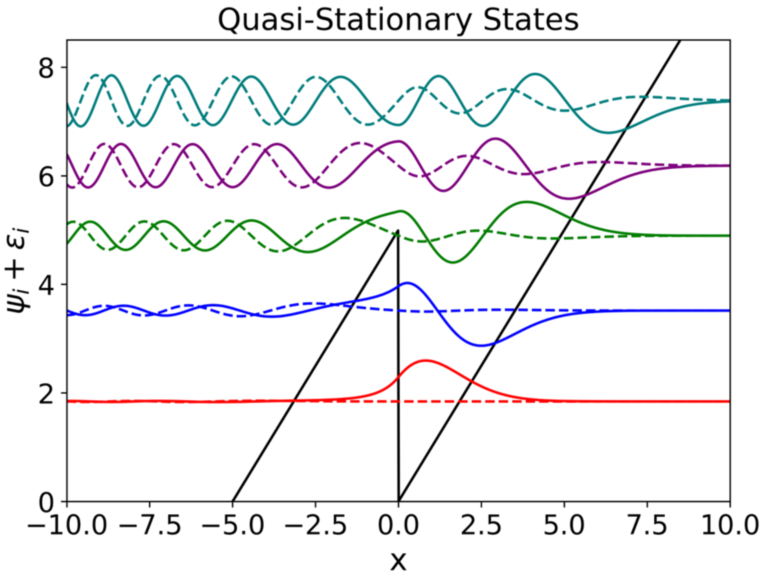
$$E = \epsilon - \frac{i\Gamma}{2}$$

It would be appealing to have

$$\psi(x,t) \approx \sum_n c_n(t)\psi_n(x)e^{-\frac{i\epsilon_n t}{\hbar}}$$

$$c_n(t) = c_n(0)e^{-\frac{\Gamma_n}{2\hbar}t}$$

This is both more physically intuitive to understand and significantly more efficient computationally

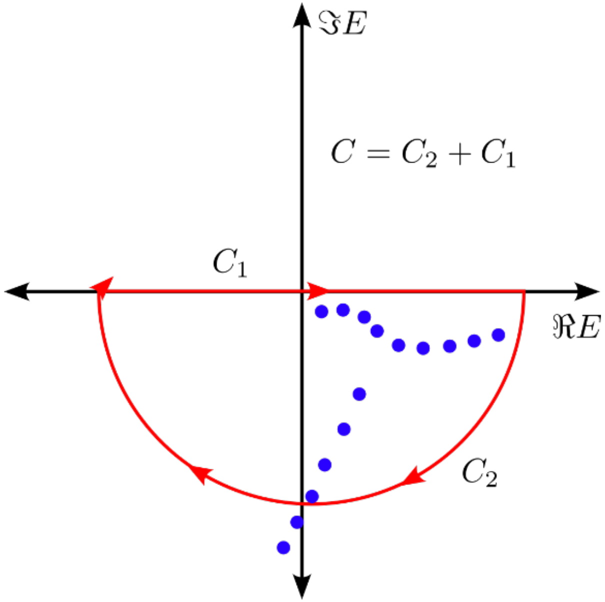


# WG Solution: Resonance Expansion

To utilize the quasi-stationary states in the lower-half plane, we integrate the continuum expansion over a contour $C$ which enclose those poles.

This yields

$$\psi(x,t) = \int c_\epsilon \psi_\epsilon(x) e^{-i\epsilon\tau} d\epsilon = \sum_i C_{\epsilon_i} N_{\epsilon_i} \psi_i e^{-i\epsilon_i\tau} + E(x,t)$$

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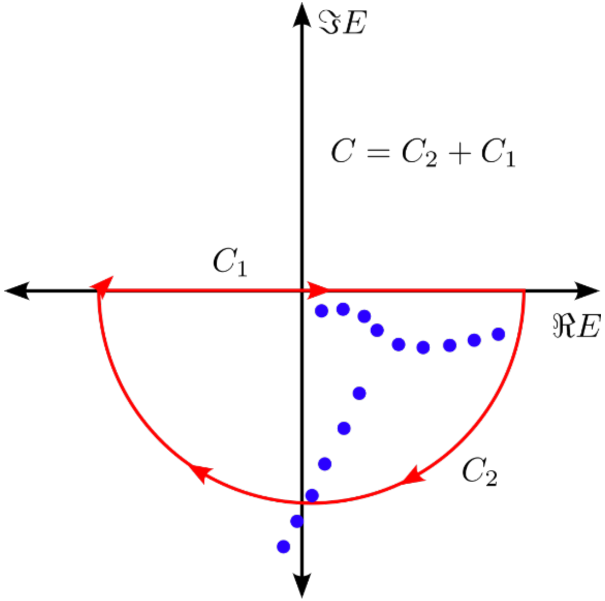
with

$$C_{\epsilon_i} = \int \psi_0(x') N_{\epsilon_i} \psi_i dx' \quad \& \quad N_{\epsilon_i}^2 = i\frac{\pi}{2}\mathrm{Res}(S_\epsilon, \epsilon_i)$$

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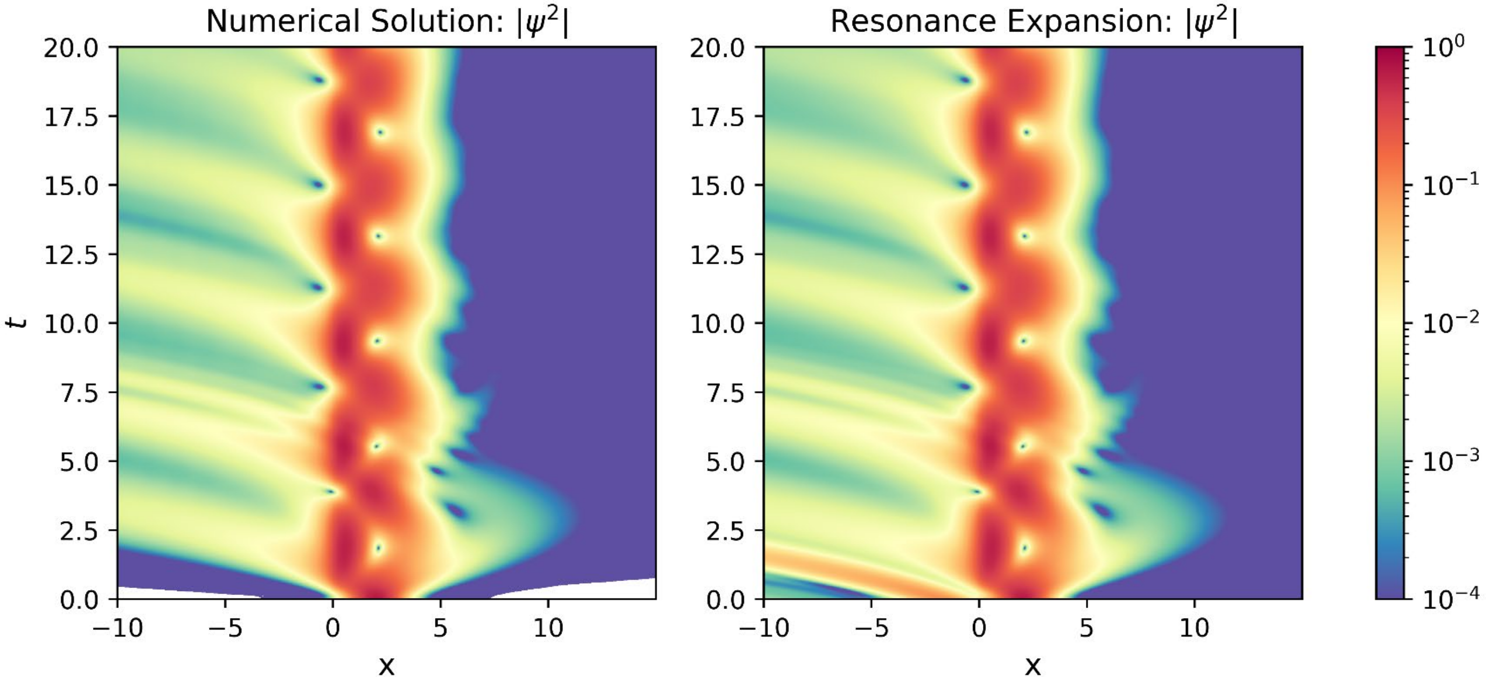
# Comparison to Numerical Simulation





[4] C. A. Moyer, Am. J. Phys. **72**, 351 (2004).



Credit to Anran Zhao for writing the numerical simulations.

# Perturbations

Our goal is to constrain theoretical models of new fundamental forces.

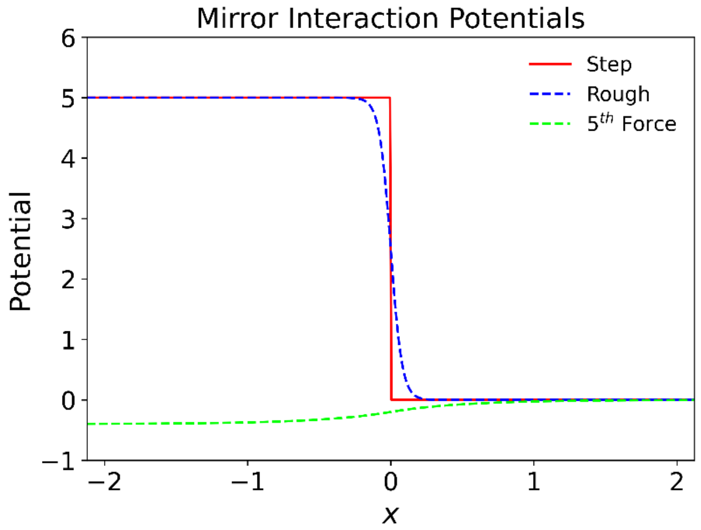
To do this accurately, we must consider two perturbations to our step potential model.

Roughness: $0.005 < a < 0.02$

$$V_R(x) = \frac{u_0}{1 + e^{\frac{x}{a}}}$$

Short-Range Force:

$$V_5(x) = \begin{cases} -W_S e^{-\frac{x}{\lambda}} & x \geq 0 \\ -W_S\left(2 - e^{-\frac{x}{\lambda}}\right) & x < 0 \end{cases}$$




$$V(x) = u_0\Theta(-x) + x + V_R(x) + V_5(x)$$

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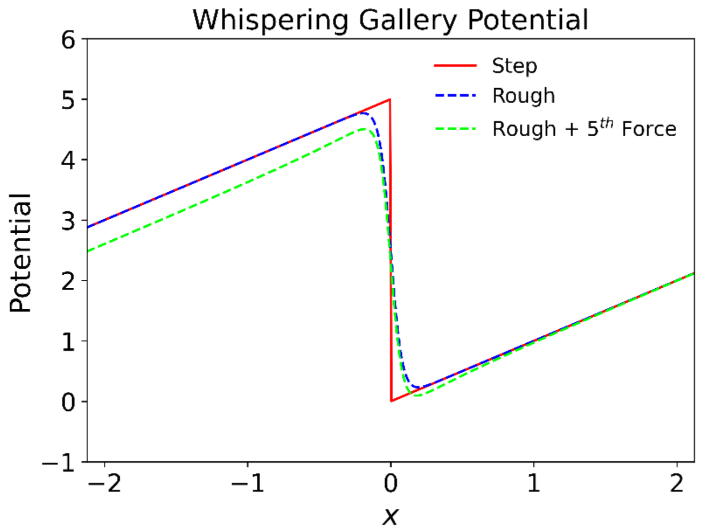


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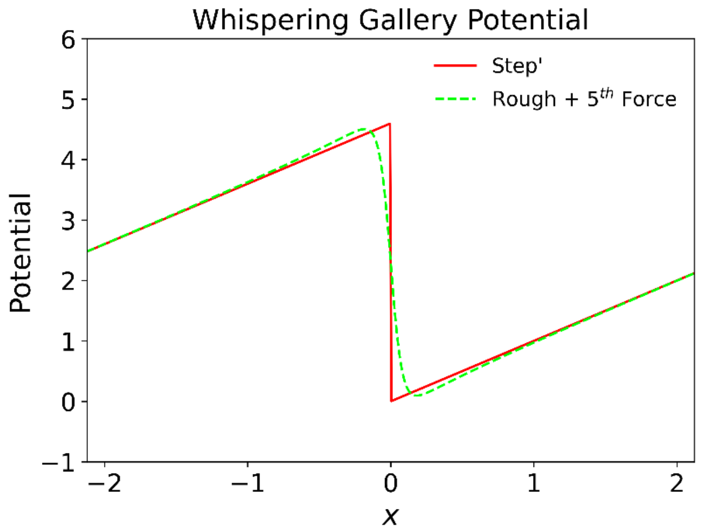


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# Logarithmic Perturbation Theory

**Advantage:** This does not depend on having a complete set of eigenstates and only requires only the eigenstate being perturbed.

Suppose $\psi = e^G$

Then

$$-\psi'' + (V_0 + \lambda V_1)\psi = E\psi$$

$$\rightarrow G'' + G'^2 = V_0 + \lambda V_1 - E$$

[5] Leung et. al formalize the theory for resonance states and considers potentials with tails [P T Leung *et al* *J. Phys. A: Math. Gen.* **31** 3271] (1998)

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If we express

$$G = G_0 + \lambda G_1 + \lambda^2 G_2 + \cdots$$

$$E = E_0 + \lambda E_1 + \lambda^2 E_2 + \cdots$$

$$\psi = \psi_0 + \lambda \psi_1 + \lambda^2 \psi_2 + \cdots$$

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Then we can find the corrections to $G$ order by order.

$$G_0'' + G'^2 = V_0 - E_0$$

$$G_1'' + 2G_0'G_1' = V_1 - E_1$$

$$G_2'' + 2G_2'G_0' = -E_2 - G_1'^2$$

$$\vdots$$

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# Logarithmic Perturbation Theory: Solutions

The solutions of which yield the energy corrections

$$E_1 = \frac{\int_a^b V_1(x)\psi_0^2\,dx}{\int_a^b \psi_0^2\,dx + \dot{G}_0'\psi_0^2\big|_a^b} \quad \& \quad E_2 = -\frac{\int_a^b G_1^2\psi_0^2\,dx + \frac{1}{2}E_1^2\,\ddot{G}_0{}'\,\psi_0^2\big|_a^b}{\int_a^b \psi_0^2\,dx + \dot{G}_0'\psi_0^2\big|_a^b}$$

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And wave function corrections

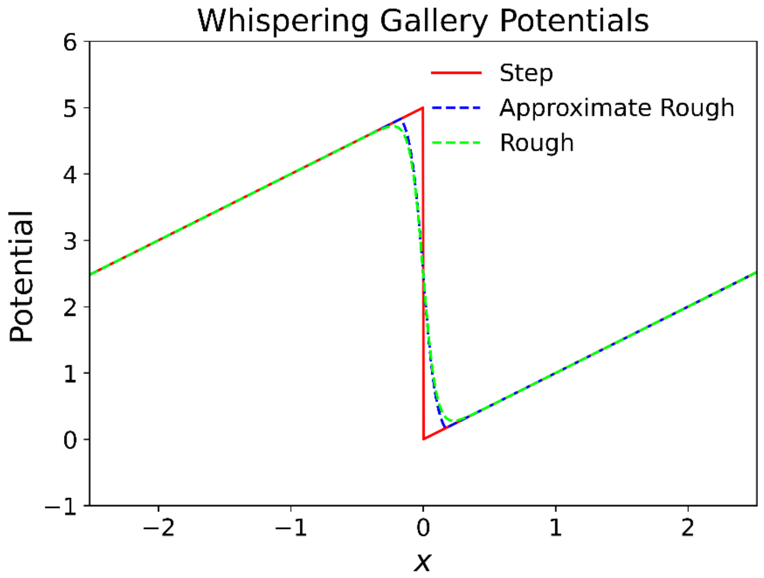
$$G_1(x) = \int_x^\infty \frac{1}{\psi_0^2(x')}\int_{x'}^\infty (V_1(x'') - E_1)\psi_0^2 dx''\,dx' \quad \& \quad G_2(x) = -\int_x^\infty \frac{1}{\psi_0^2(x')}\int_{x'}^\infty \left(G_1'^2(x'') + E_2\right)\psi_0^2 dx''\,dx'$$

$$\psi = \psi_0\left(1 + \lambda G_1 + \lambda^2\left(\frac{1}{2}G_1^2 + G_2\right) + \cdots\right) = \psi_0 + \lambda\psi_1 + \lambda^2\psi_2 + \cdots$$

# Logarithmic Perturbation Theory: Validation

To validate the perturbation corrections, we calculate the solutions to an exact potential model which approximates the mirror roughness.

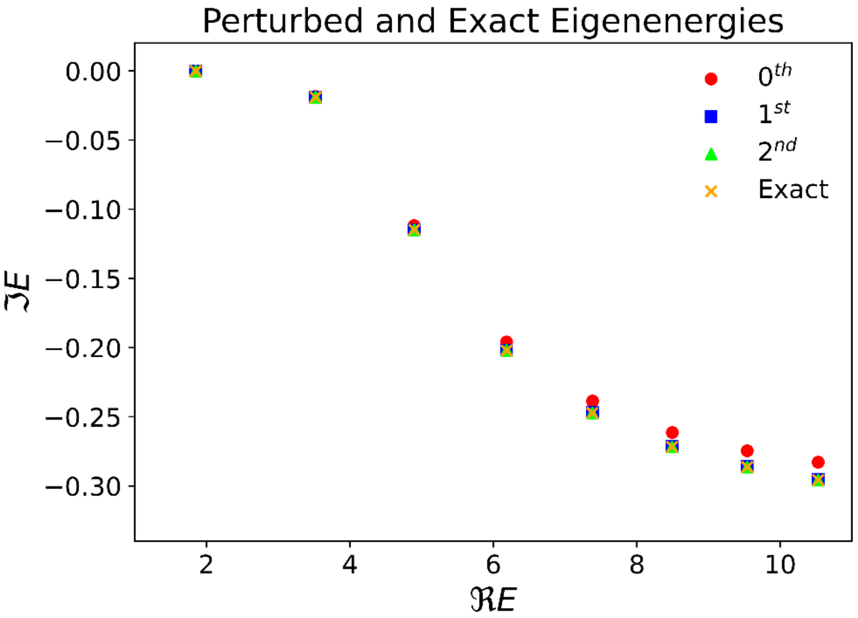
$$V_R \approx \begin{cases} u_0 + x & x < -l \\ \dfrac{u_0}{1 + e^{\frac{x}{a}}} & -l < x < l \\ x & l < x \end{cases}$$

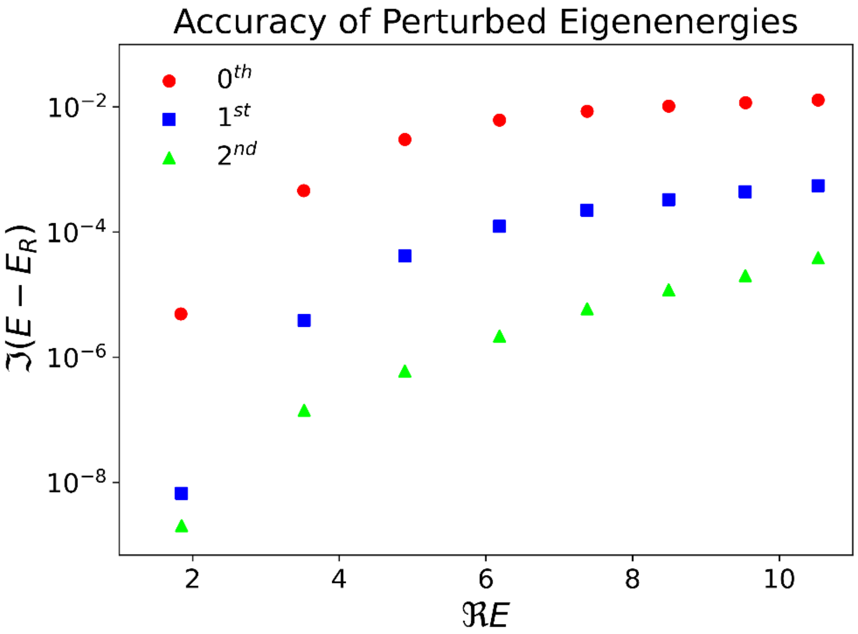



For small $a$, this potential is nearly identical to the “real” roughness model.

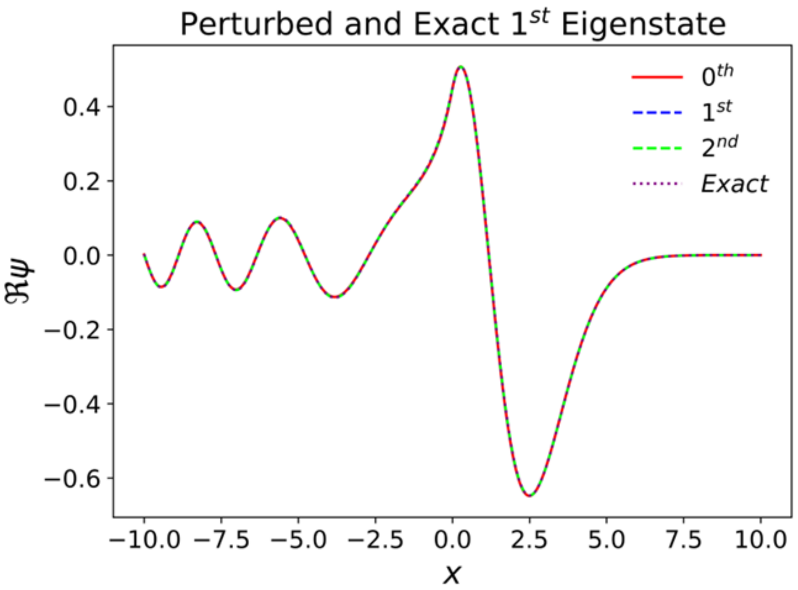
Credit to Serge Reynaud for this model.

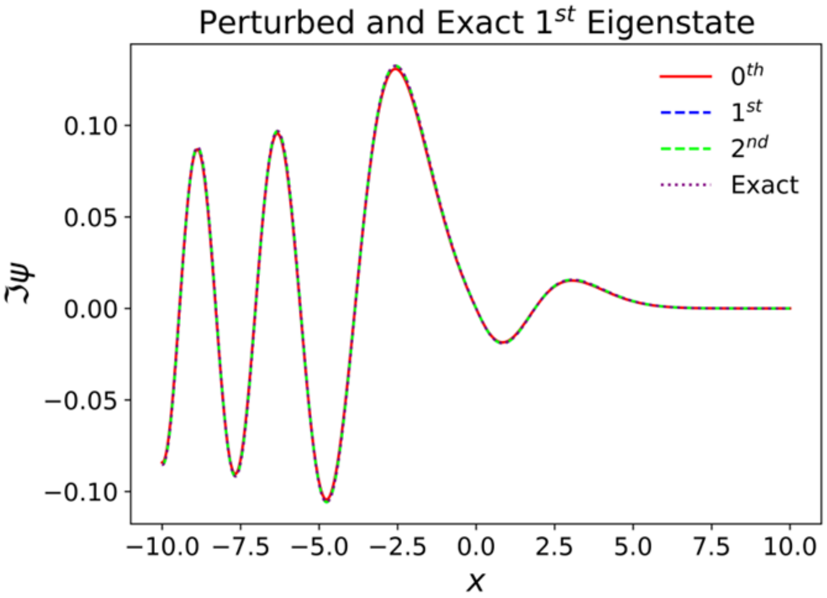
# Resonance Energies



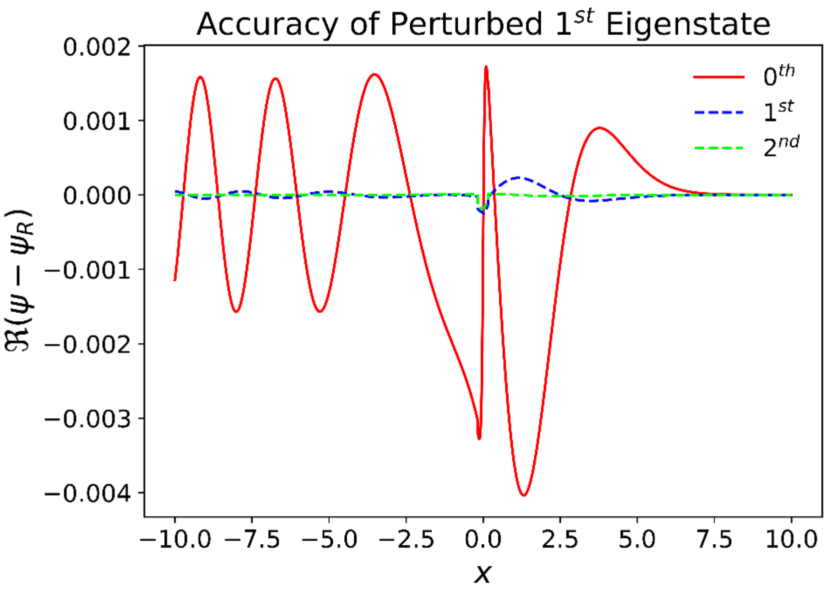



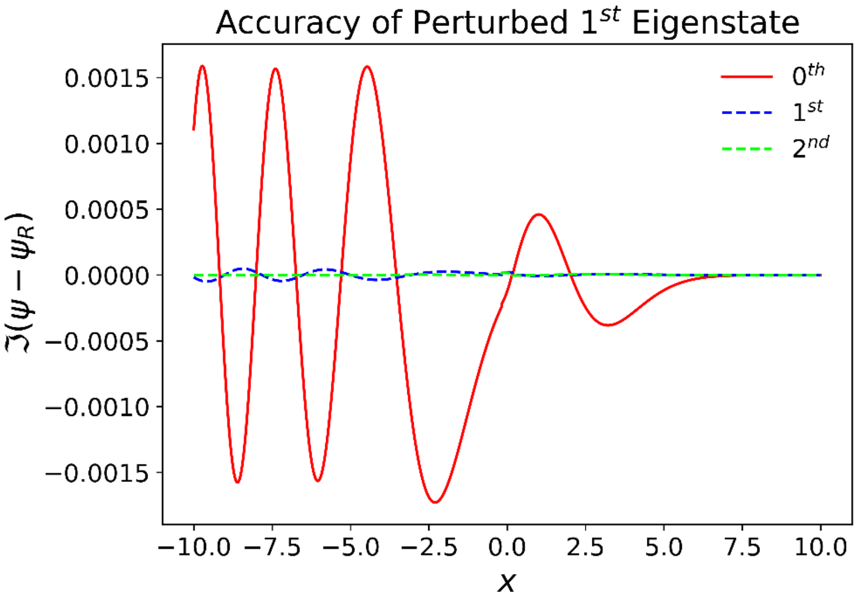
# Quasi-Stationary States: Comparison



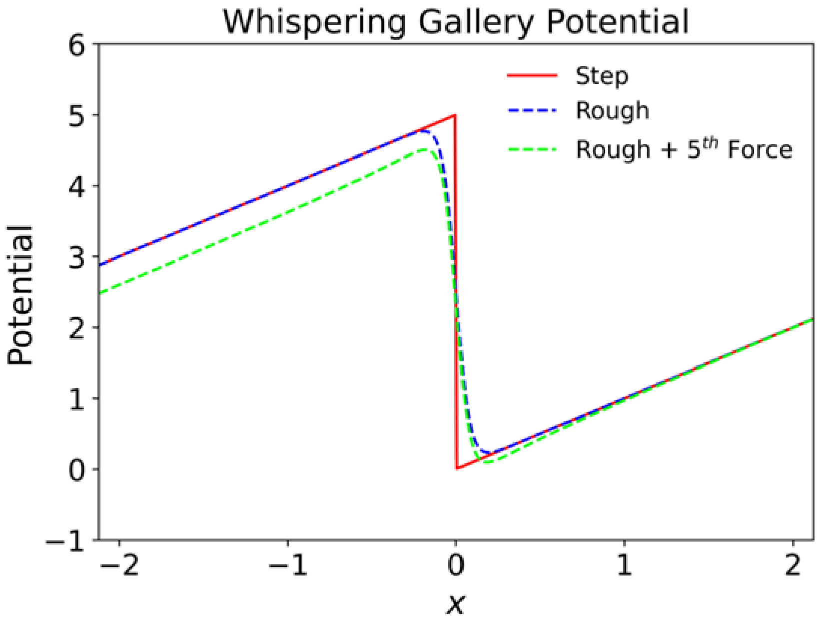



# Quasi-Stationary States: Comparison



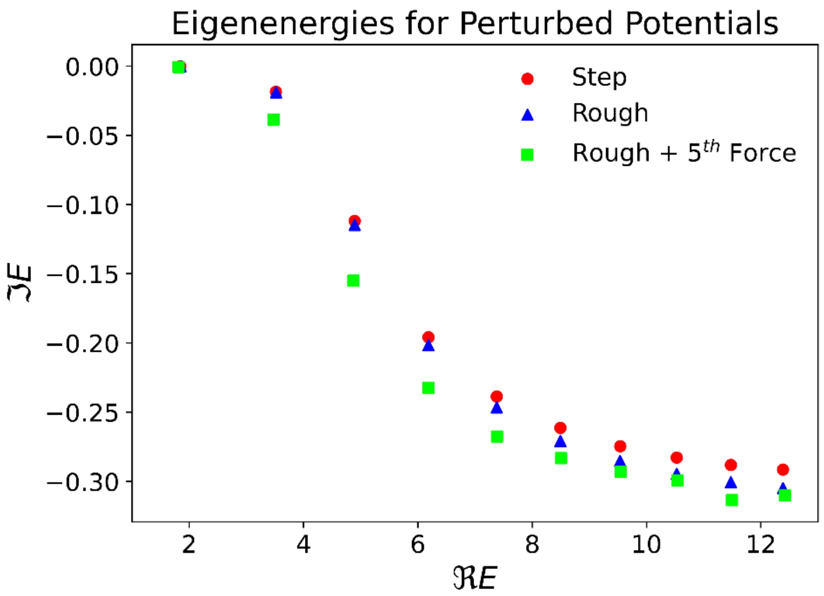



# Roughness + $5^{th}$ Force

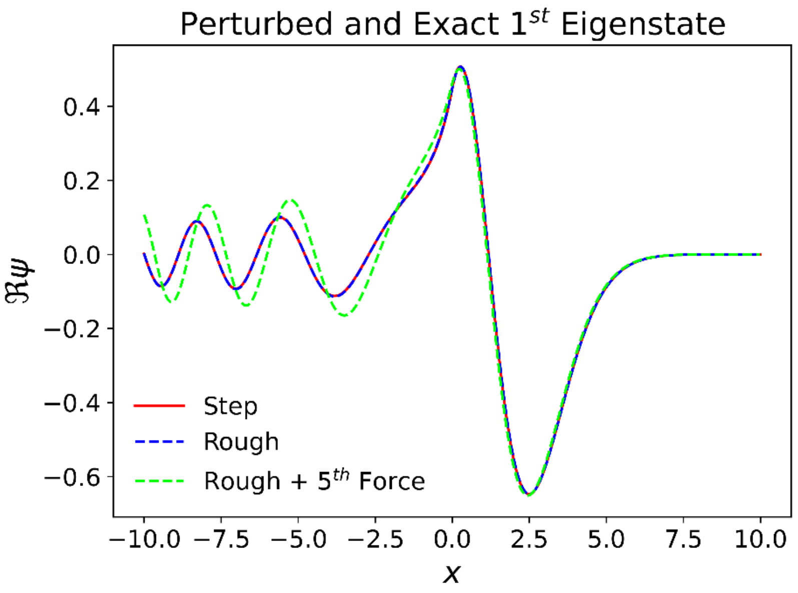


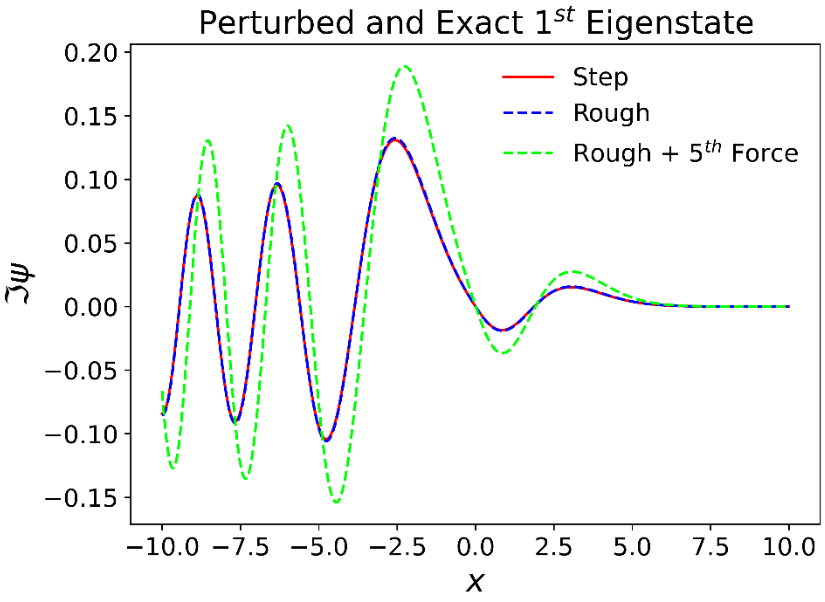

$$V(x) = u_0\Theta(-x) + x + V_R(x) + V_5(x)$$

# Roughness + $5^{th}$ Force: Energies



# Roughness + $5^{th}$ Force: States



# Likelihood Analysis of Data

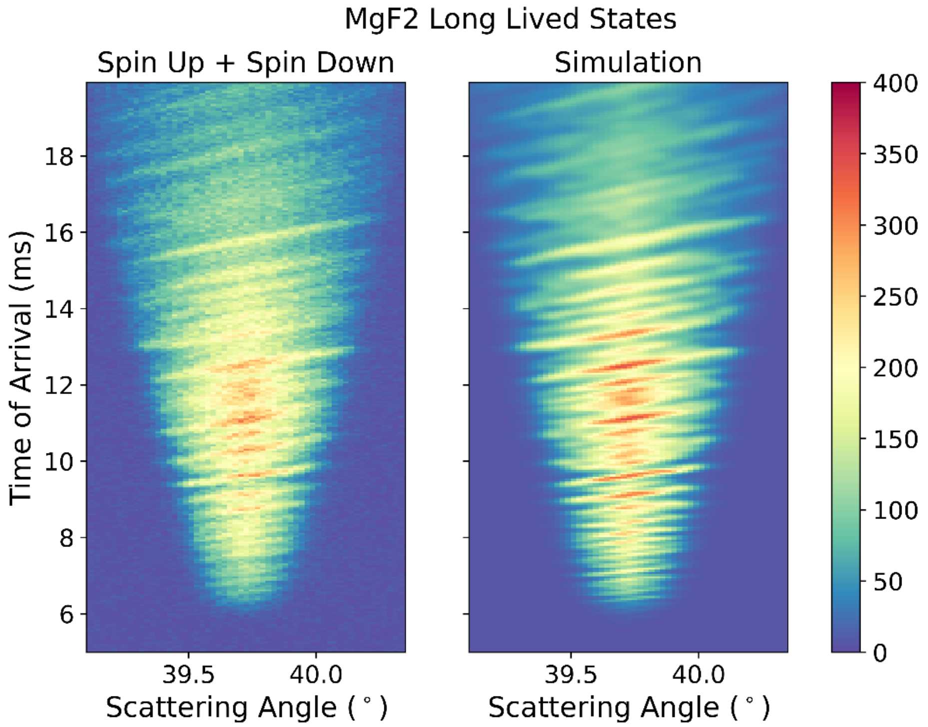
$$\mathcal{L}\left(\vec{N},\vec{\theta}\right)=\prod_i P\left(\vec{x}_i,N_i,\vec{\theta}\right)=\prod_i \frac{e^{-n\left(\vec{x}_i,\vec{\theta}\right)}n\left(\vec{x}_i,\vec{\theta}\right)^{N_i}}{N_i!}$$

$N_i$ – counts in bin $i$

$n_i=n\left(\vec{x}_i,\vec{\theta}\right)$ – fit function

$\vec{\theta}$ – fit parameters:

- $x_0$ – pixel offset
- $t_0$ – timing offset
- $D_0$ – distance from exiting edge of mirror to detector
- $D_{tof}$ – time of flight distance
- $N_0$ – count scale
- $B$ – Background
- $r$ – Roughness parameter
- $\alpha$ – Incidence Angle
- $\Theta$ – Angular Size of the mirror
- $R_0$ – Radius of Mirror



# Conclusions

The **neutron whispering gallery effect** is an interesting tool to investigate **new fundamental short-range interactions**.

A model has been developed to simulate this whispering gallery effect in a computationally efficient way with a resonance expansion.

Logarithmic perturbation theory enables us to calculate the effects of the mirror roughness and short-range forces.

Our simulation qualitatively reproduces the measured interference pattern, but more parameters need to be fit, and constraints must be made.

# People

There are many people involved in this work:

- Stefan Baessler, Anran Zhao (UVA)
- Valery Nesvizhevsky (ILL)
- Katharina Schreiner (LKB,SMI,ILL)
- Serge Reynaud (LKB), Pierre Cladé (LKB)
- Alexei Voronin (LPI),
- Mingyu Shi (ETH)

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- Ugne Miniotaite (KTH), Julien Vivier (UGA), Kenny Campbell (St. Andrews)

KTH VETENSKAP OCH KONST

UGA Université Grenoble Alpes

University of St Andrews

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