Double (n-γ) and triple (n-n'γ) angular correlations in neutron inelastic scattering on ^{12}C

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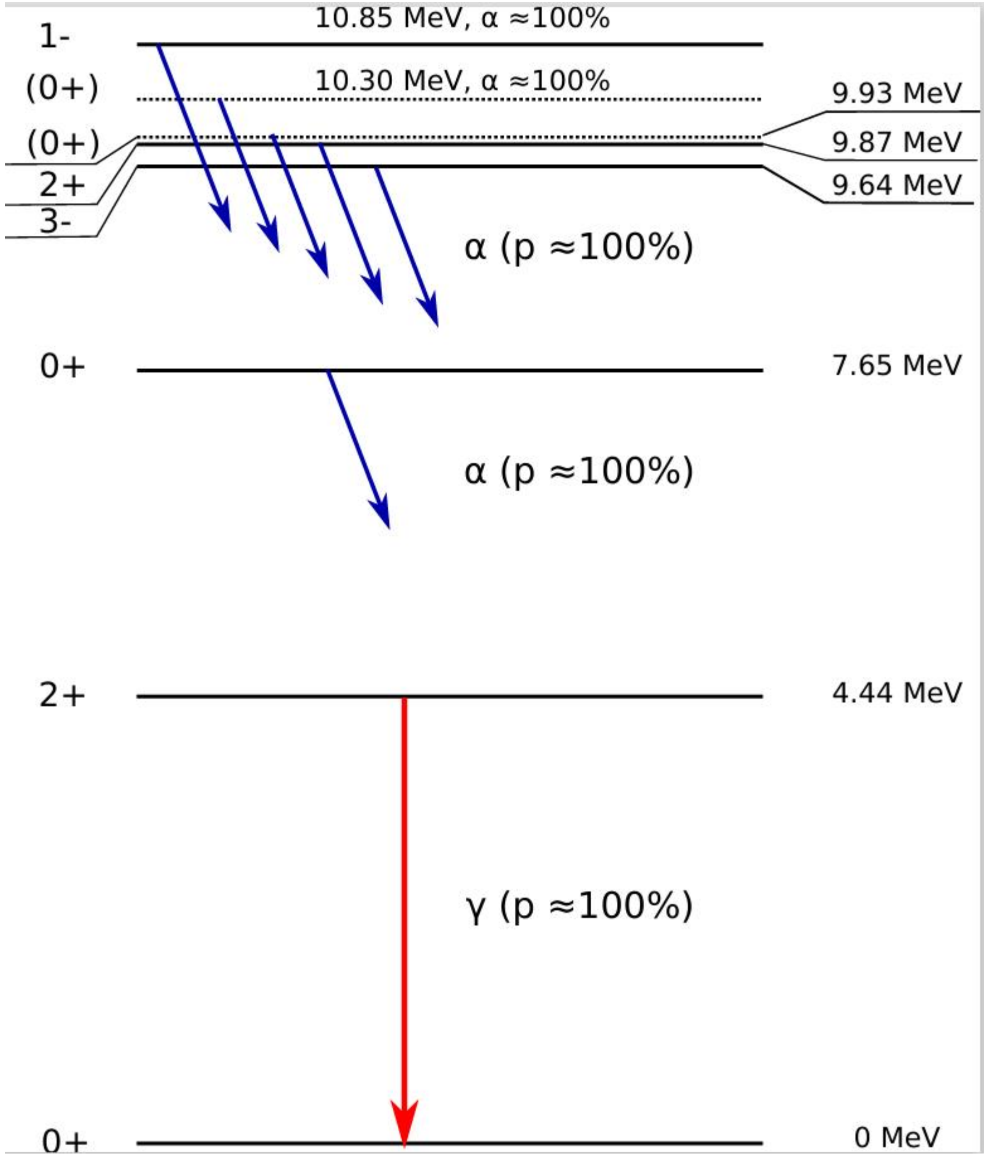
Outline

Considered reaction:
$^{12}C(n, n'\gamma)^{12}C$

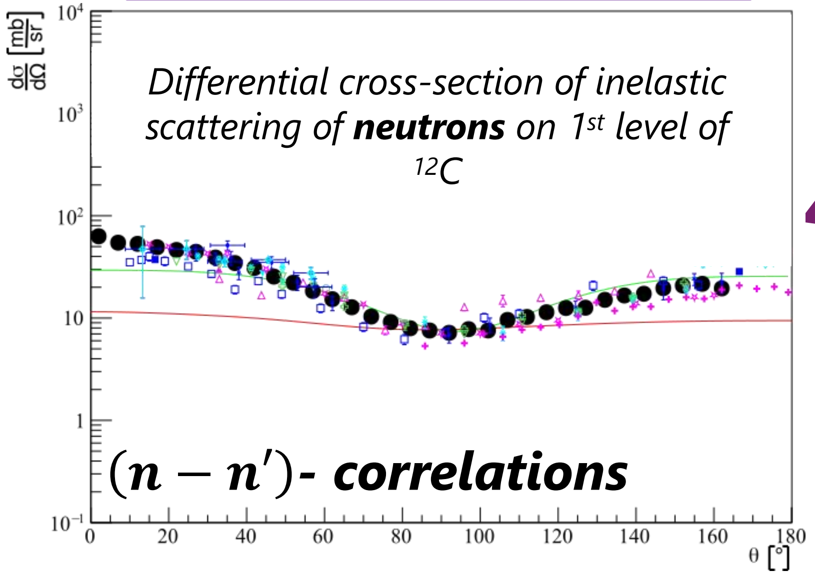
- Motivation
- Theory
- How to write 500 pages PDF file with formulas?
- S-matrix from TALYS

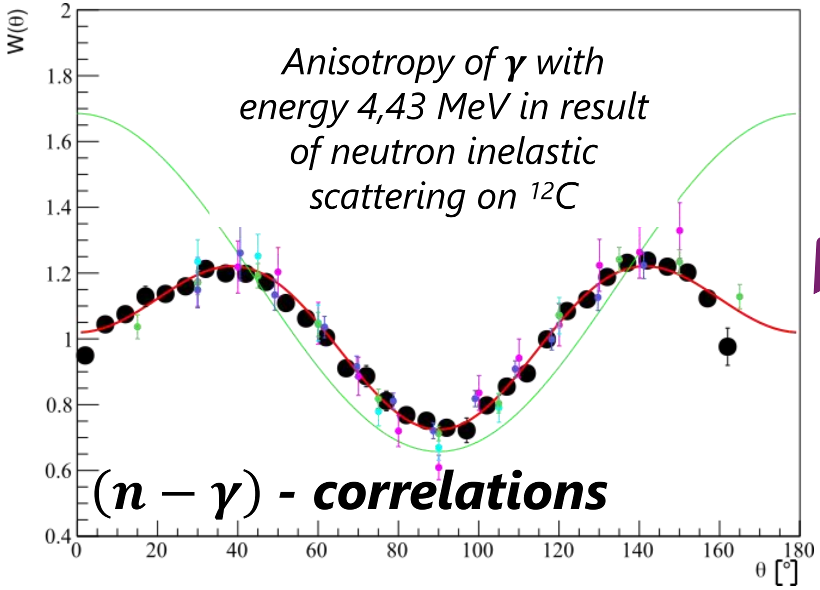


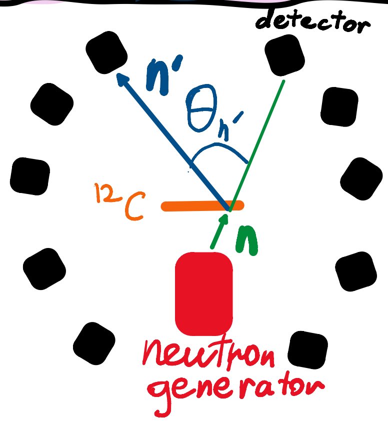
Levels of ^{12}C

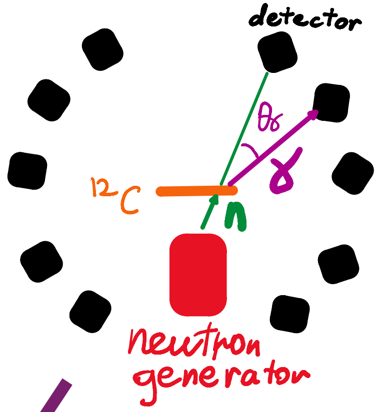
Types of angular correlations

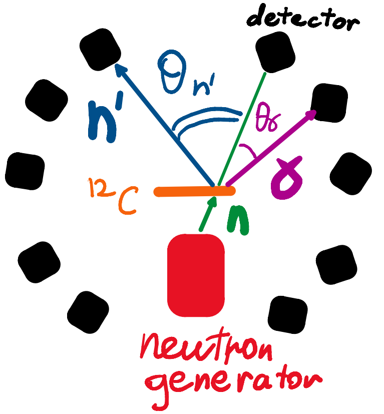
Double correlations



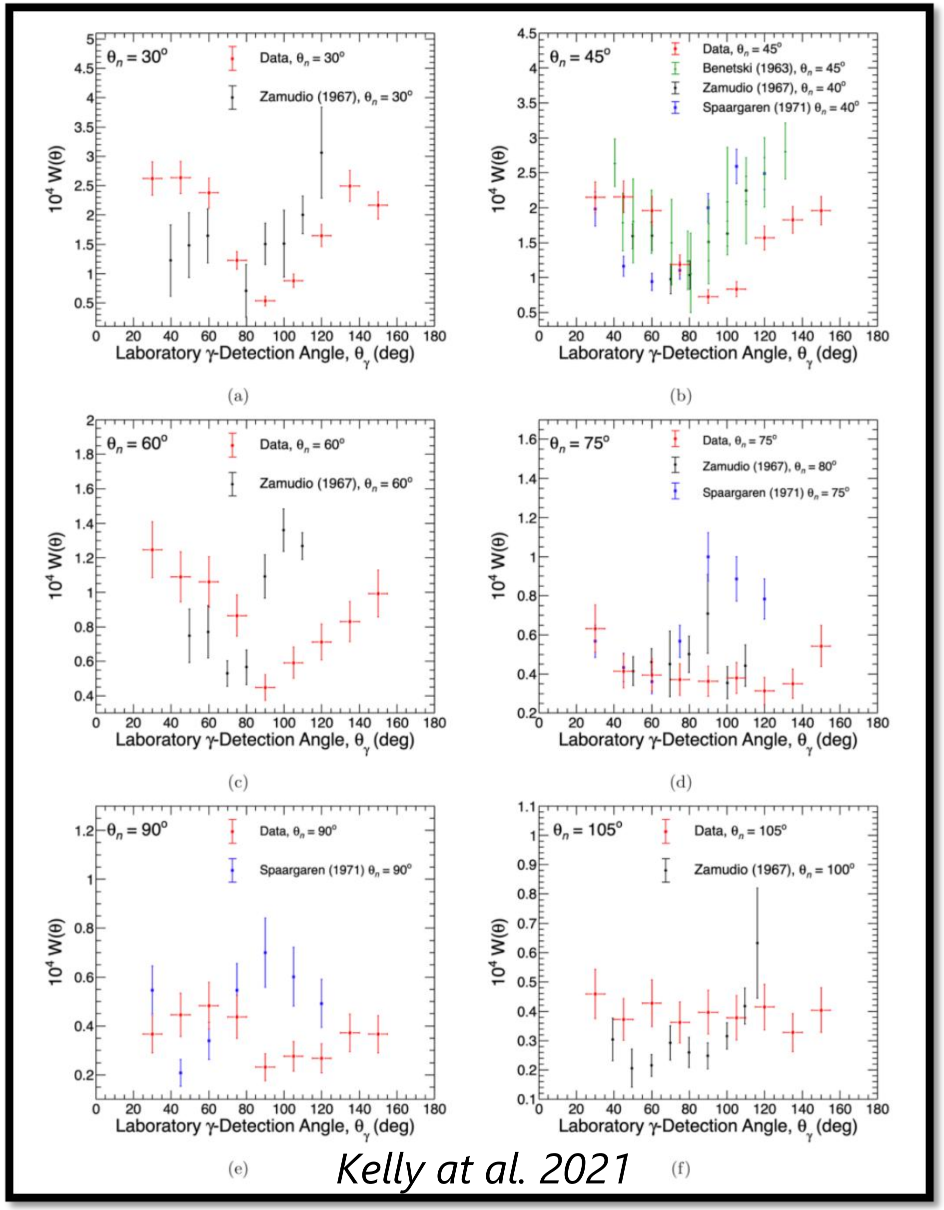








Triple correlations



$(n - n'\gamma)$ - correlations

Inelastic scattering of neutron on 12C

n $\vec{k}_\alpha$ ^{12}C $\quad$ n' $\vec{k}_\beta$ $\quad$ $^{12}C^*$ (2^+-state)

$$\Psi(\alpha \to \beta) = \sum \tilde{a}_{\mu_\beta \sigma_\beta} \chi^{\beta}_{I_\beta \mu_\beta} \chi^{\beta}_{s\sigma_\beta}$$

$$E_\alpha = 0\ MeV \to E_\beta = 4{,}44\ MeV$$

$$\Psi(\alpha \to \beta) = \left(\frac{m_\beta k_\beta}{m_\alpha k_\alpha}\right)^{1/2} \sum_{\lambda_\alpha} a(\lambda_\alpha) \sum_{\lambda_\beta} S_J(\lambda_\alpha \to \lambda_\beta)\, h^{(+)}_{l_\beta}(k_\beta r_\beta)\, \varphi^{\beta}_{\lambda_\beta}$$

Wave-function of binary system (nucleus+neutron)

S-matrix of transition in channel λ_β

Partial waves

Spin-angular functions

$$\to \quad a(\lambda_\alpha) \equiv a(l_\alpha j_\alpha JM) = 2\pi \sum_{\mu_\alpha \sigma_\alpha m_\alpha \nu_\alpha} C^{j_\alpha \nu_\alpha}_{l_\alpha m_\alpha s \sigma_\alpha} C^{JM}_{j_\alpha \nu_\alpha I_\alpha \mu_\alpha} a_{\mu_\alpha}(I_\alpha) a_{\sigma_\alpha}(s) Y^{*}_{l_\alpha m_\alpha}(\vec{n}_k)$$

neutron $\quad$ total

$$\to \quad \varphi^{\beta}_{\lambda_\beta} \equiv \varphi^{\beta}_{l_\beta j_\beta JM} = \sum_{\mu_\beta \sigma_\beta m_\beta \nu_\beta} C^{j_\beta \nu_\beta}_{l_\beta m_\beta s \sigma_\beta} C^{JM}_{j_\beta \nu_\beta I_\beta \mu_\beta} \chi^{\beta}_{I_\beta \mu_\beta} \chi^{\beta}_{s\sigma_\beta} i^{l_\beta} Y_{l_\beta m_\beta}(\vec{n}_\beta)$$

γ-emission from the 1st excited state of ^{12}C

В формуле (3) вычислим сумму квадрат модуля проекциям (чего?):

$$\sum_{\mu=\pm1}\left|\sum_{\nu} D^{L*}_{\nu\mu}(\vec{n}_\gamma)\,\mu\, a_{L\nu}(e)\right|^2 = \sum_{\mu=\pm1}\left|\sum_{\nu} D^{L*}_{\nu\mu}(\vec{n}_\gamma)\sum_{\mu_i} a^*_{\mu_i}(I_i) C^{I_i\mu_i}_{I_f\mu_f L\nu} a_L(e)\right|^2 =$$

$$\sum_{\mu=\pm1}\sum_{LL'}\sum_{\nu\nu'} D^{L*}_{\nu\mu}(\vec{n}_\gamma)\, D^{L'*}_{\nu'\mu}(\vec{n}_\gamma)\mu^2 \sum_{\mu_i\mu_i'} a^*_{\mu_i}(I_i) a_{\mu_i'}(I_i) C^{I_i\mu_i}_{I_f\mu_f L\nu} C^{I_i\mu_i'}_{I_f\mu_f L'\nu'}\,|a_L(e)|^2$$

Согласно *B.114* из [17]:

$$D^{L*}_{\nu\mu}(\vec{n}_\gamma)\cdot D^{L'}_{\nu'\mu}(\vec{n}_\gamma) = \sum_{Qq}\frac{\sqrt{4\pi(2Q+1)}}{2L+1} C^{L\mu}_{L'\mu Q0} C^{L'\nu'}_{L\nu Qq} Y_{Qq}(\vec{n}_\gamma)$$

Тогда при подстановке (45) в (44) проведем суммирование произведения трё
эффициентов Клебша-Гордана (ККГ) согласно *A.156* и поменяв местами инде
ККГ (*A.66* и *A.68*):

$$\sum_{\nu\nu'\mu_f} C^{L'\nu'}_{L\nu \underline{Qq}}\cdot C^{I_i\mu_i}_{I_f\mu_f L\nu}\cdot C^{I_i\mu_i'}_{I_f\mu_f L'\nu'} = (-1)^q\cdot(-1)^{L+L'}\sqrt{\frac{2L'+1}{2L+1}}\, C^{I_i\mu_i}_{I_i\mu_i' Q-q}\cdot U(I_f I_i L'Q; LI_i$$

В результате формула (44) согласно определению спин-тензора (7) преобразуется к виду:

$$\sum_{\mu=\pm1}\sum_{LL'}\sum_{\mu_i\mu_i'}\sum_{Qq}(-1)^{-q}(-1)^{L+L'}\sqrt{\frac{2L'+1}{2L+1}}\frac{\sqrt{4\pi(2Q+1)}}{2L+1}\cdot\mu^2\cdot|a_L(e)|^2 C^{L'\mu}_{L\mu Q0}\cdot$$

$$C^{I_i\mu_i}_{I_i\mu_i' Q-q}\, a^*_{\mu_i}(I_i)\, a_{\mu_i'}(I_i)\cdot U(I_f I_i L'Q; LI_i)\cdot Y_{Qq} = \sum_{\mu=\pm1}\sum_{LL'}\sum_{Qq}(-1)^{-q}(-1)^{L+L'} \qquad (47)$$

$$\sqrt{\frac{2L'+1}{2L+1}}\frac{\sqrt{4\pi(2Q+1)}}{2L+1}\cdot\mu^2\cdot|a_L(e)|^2\cdot C^{L'\mu}_{L\mu Q0}\,\tau_{Q-q}(I_i)\, U(I_f I_i L'Q; LI_i)\cdot Y_{Qq}$$

Суммирование по поляризациям приводит к тому, что остаются только чётные Q:

$$\sum_{\mu=\pm1} C^{L'\mu}_{L\mu Q0}\,\mu^2 = \sum_{\mu=\pm1}(C^{L'\mu}_{L\mu Q0} + C^{L'-\mu}_{L-\mu Q0}) = (1+(-1)^Q)\to Q = 0,2,4,\ldots \qquad (48)$$

Проведя данные вычисления приходим к формуле (6).

$$dw(\vec{n}_\gamma) = \frac{2\omega a_L^2(e)}{\hbar c^3}\sum_{Q=0,2,4,\ldots}\sqrt{4\pi(2Q+1)}C^{L1}_{L1Q0}U(I_f I_i LQ, LI_i)\sum_q Y_{Qq}(\vec{n}_\gamma)\tilde{\tau}^*_{Qq}(I_i)\,d\Omega_\gamma$$

Spin-tensor

Differential probability of γ-emission per time unit

$$dw(\vec{n}_\gamma) = \frac{2\omega a_L^2(e)}{\hbar c^3} \sum_{Q=0,2,4,\ldots} \sqrt{4\pi(2Q+1)} C_{L1Q0}^{L1} U(I_f I_i L Q, L I_i) \sum_q Y_{Qq}(\vec{n}_\gamma) \tilde{\tau}_{Qq}^*(I_i) d\Omega_\gamma$$

Unnormalized spin-tensor

$$\tilde{\tau}_{Qq}(I_i) = \sum_{\mu_i \mu_i'} C_{I_i \mu_i Q q}^{I_i \mu_i'} \sum_{\sigma_\beta} \tilde{a}_{\mu_i \sigma_\beta} \tilde{a}_{\mu_i' \sigma_\beta}^*$$

$$\tau_{Qq}^*(I_i) = (-1)^{-q} \tau_{Q-q}(I_i)$$

Wave function of binary system (nucleus+neutron)

$$\Psi(\alpha \to \beta) = \sum_{\mu_\beta \sigma_\beta} \tilde{a}_{\mu_\beta \sigma_\beta} \chi_{I_\beta \mu_\beta}^{\beta} \chi_{s \sigma_\beta}^{\beta}$$

Normalized spin-tensor

$$\tilde{\tau}_{00}(I_i) = \sum_{\mu_i \sigma_\beta} |\tilde{a}_{\mu_i \sigma_\beta}|^2$$

$$\tau_{Qq}(I_i) = \frac{\tilde{\tau}_{Qq}(I_i)}{\tilde{\tau}_{00}(I_i)}$$

A.2 Вычисление спин-тензора

Спин-тензор представлен формулой (15), тогда согласно (17) проведём вычисления:

$$
\begin{aligned}
\tilde{\tau}_{Qq}(I_i) &= \sum_{\mu_i\mu_i'} C^{I_i\mu_i'}_{I_i\mu_i Qq} \sum_{\sigma_\beta} \langle \tilde{a}_{\mu_i}(I_i)\, \tilde{a}^*_{\mu_i'}(I_i)\rangle \\
&= \sum_{\mu_i\mu_i'} \sum_{JMJ'M'} \sum_{l_\alpha l'_\alpha l_\beta l'_\beta} \sum_{j_\alpha j'_\alpha j_\beta j'_\beta} \sum_{\mu_\alpha\sigma_\alpha\sigma_\beta} \sum_{\xi\xi'\nu_\alpha\nu'_\alpha} \sum_{mm'\nu_\beta\nu'_\beta} C^{I_i\mu_i'}_{I_i\mu_i Qq} C^{j_\alpha\nu_\alpha}_{l_\alpha\xi s\sigma_\alpha} C^{JM}_{j_\alpha\nu_\alpha I_\alpha\mu_\alpha} \cdot \\
& C^{j'_\alpha\nu'_\alpha}_{l'_\alpha\xi' s\sigma_\alpha} C^{J'M'}_{j'_\alpha\nu'_\alpha I_\alpha\mu_\alpha} C^{j_\beta\nu_\beta}_{l_\beta m s\sigma_\beta} C^{JM}_{j_\beta\nu_\beta I_i\mu_i} C^{j'_\beta\nu'_\beta}_{l'_\beta m' s\sigma_\beta} C^{J'M'}_{j'_\beta\nu'_\beta I_i\mu_i'} \cdot \\
& \frac{1}{2s+1}\,\frac{1}{2I_\alpha+1}\,\frac{1}{(k_\beta r)^2}\left(\frac{m_\beta k_\beta}{m_\alpha k_\alpha}\right) Y^*_{l_\alpha\xi}(\vec{n}_k) Y_{l'_\alpha\xi'}(\vec{n}_k) Y_{l_\beta m}(\vec{n}_\beta) Y^*_{l'_\beta m'}(\vec{n}_\beta) \cdot \\
& S_J(l_\alpha j_\alpha \to l_\beta j_\beta)\, S^*_{J'}(l'_\alpha j'_\alpha \to l'_\beta j'_\beta)
\end{aligned}
\tag{49}
$$

Согласно *A.176* из [17] сферические функции можно представить в виде:

$$
\begin{aligned}
Y_{l_\beta m}(\vec{n}_\beta) \cdot Y^*_{l'_\beta m'}(\vec{n}_\beta) &= \sum_{\Lambda\lambda} \sqrt{\frac{(2\Lambda+1)(2l'_\beta+1)}{4\pi(2l_\beta+1)}} \cdot C^{l_\beta 0}_{l'_\beta 0\Lambda 0}\, C^{l_\beta m}_{l'_\beta m'\Lambda\lambda}\, Y_{\Lambda\lambda}(\vec{n}_\beta) \\
Y^*_{l_\alpha\xi}(\vec{n}_k) \cdot Y_{l'_\alpha\xi'}(\vec{n}_k) &= \sum_{\Lambda'\lambda'} \sqrt{\frac{(2\Lambda'+1)(2l'_\alpha+1)}{4\pi(2l_\alpha+1)}} \cdot C^{l_\alpha 0}_{l'_\alpha 0\Lambda' 0}\, C^{l_\alpha\xi}_{l'_\alpha\xi'\Lambda'\lambda'}\, Y_{\Lambda'\lambda'}(\vec{n}_k)
\end{aligned}
\tag{50}
$$

Далее согласно *A.156* можно просуммировать три коэффициента Клебша-Гордана:

$$
\sum_{\xi\xi'\sigma_\alpha} C^{j_\alpha\nu_\alpha}_{l_\alpha\xi s\sigma_\alpha} C^{j'_\alpha\nu'_\alpha}_{l'_\alpha\xi' s\sigma_\alpha} C^{l_\alpha\xi}_{l'_\alpha\xi'\Lambda'\lambda'} = \sqrt{\frac{(2l_\alpha+1)(2j'_\alpha+1)}{(2l'_\alpha+1)(2j_\alpha+1)}}\, C^{j_\alpha\nu_\alpha}_{j'_\alpha\nu'_\alpha\Lambda'\lambda'}\, U(sj'_\alpha l_\alpha\Lambda'; l'_\alpha j_\alpha) \tag{51}
$$

Полученный в (51) коэффициент Клеббша-Гордана $C^{j_\alpha\nu_\alpha}_{j'_\alpha\nu'_\alpha\Lambda'\lambda'}$ аналогично просуммируем по $\nu'_\alpha, \nu_\alpha, \mu_\alpha$ с двумя другими коэффициентами $C^{JM}_{j_\alpha\nu_\alpha I_\alpha\mu_\alpha}$ и $C^{J'M'}_{j'_\alpha\nu'_\alpha I_\alpha\mu_\alpha}$, получим:

$$
\sum_{\nu'_\alpha\nu_\alpha\mu_\alpha} C^{j_\alpha\nu_\alpha}_{j'_\alpha\nu'_\alpha\Lambda'\lambda'} C^{JM}_{j_\alpha\nu_\alpha I_\alpha\mu_\alpha} C^{J'M'}_{j'_\alpha\nu'_\alpha I_\alpha\mu_\alpha} = \sqrt{\frac{(2j_\alpha+1)(2J'+1)}{(2l'_\alpha+1)(2J+1)}} C^{JM}_{J'M'\Lambda'\lambda'} U(I_\alpha J' j_\alpha\Lambda'; j'_\alpha J) \tag{52}
$$

Запишем ещё одну сумму:

$$
\sum_{mm'\sigma_\beta} C^{j_\beta\nu_\beta}_{l_\beta m s\sigma_\beta} C^{j'_\beta\nu'_\beta}_{l'_\beta m' s\sigma_\beta} C^{l_\beta m}_{l'_\beta m'\Lambda\lambda} = \sqrt{\frac{(2l_\beta+1)(2j'_\beta+1)}{(2l'_\beta+1)(2j_\beta+1)}} C^{j_\beta\nu_\beta}_{j'_\beta\nu'_\beta\Lambda\lambda} U(sj'_\beta l_\beta\Lambda; l'_\beta j_\beta) \tag{53}
$$

После этого воспользуемся формулой *A.68*:

$$
C^{I_i\mu_i'}_{I_i\mu_i Qq} = (-1)^q\, C^{I_i\mu_i}_{I_i\mu_i' Q-q} \tag{54}
$$

После преобразования (54) мы можем применить формулу A.165:

$$
\begin{aligned}
&\sum_{\mu\mu'\nu\nu'MM'} (-1)^q\, C^{JM}_{J'M'\Lambda'\lambda'} C^{J'M'}_{j'_\beta\nu'_\beta I_i\mu_i'} C^{JM}_{j_\beta\nu_\beta I_i\mu_i} C^{j_\beta\nu_\beta}_{j'_\beta\nu'_\beta\Lambda\lambda} C^{I_i\mu_i}_{I_i\mu_i' Q-q} = \\
&(-1)^q\,(2J+1)\sqrt{\frac{(2J'+1)(2j_\beta+1)(2I_i+1)}{(2\Lambda'+1)}} C^{\Lambda'\lambda'}_{\Lambda\lambda Q-q} \begin{Bmatrix} J & j_\beta & I_i \\ J' & j'_\beta & I_i \\ \Lambda & \Lambda' & Q \end{Bmatrix}
\end{aligned}
\tag{55}
$$

Собрав все результаты промежуточных вычислений, чтобы получить формулу для спин-тензора (18), необходимо применить формулу *A.68*:

$$
C^{\Lambda'\lambda'}_{\Lambda\lambda Q-q} = \sqrt{\frac{2\Lambda'+1}{2\Lambda+1}} (-1)^{\Lambda-\Lambda'} C^{\Lambda\lambda}_{\Lambda'\lambda' Qq} \tag{56}
$$

и поменять Λ и Λ' местами во всей формуле.

Spin-tensor

$$\tilde{\tau}_{Qq}(I_i) = \frac{\pi}{k_\alpha k_\beta r_\beta^2} \frac{m_\beta}{m_\alpha} \sum_{JJ'} g_{J'} \sum_{l_\alpha l'_\alpha j_\alpha j'_\alpha l_\beta l'_\beta j_\beta j'_\beta} S_J(l_\alpha j_\alpha \to l_\beta j_\beta)\, S^*_{J'}(l'_\alpha j'_\alpha \to l'_\beta j'_\beta) \times$$
$$\sum_{\Lambda\Lambda'} (-1)^{\Lambda-\Lambda'} \sqrt{(2I_i+1)(2\Lambda+1)(2J+1)(2j'_\beta+1)}\, C^{l_\alpha 0}_{l'_\alpha 0 \Lambda 0} C^{l_\beta 0}_{l'_\beta 0 \Lambda' 0} \times$$
$$U(sj'_\alpha l_\alpha \Lambda, l'_\alpha j_\alpha)\, U(I_\alpha J' j_\alpha \Lambda, j'_\alpha J)\, U(sj'_\beta l_\beta \Lambda', l'_\beta j_\beta) \begin{Bmatrix} J & j_\beta & I_i \\ J' & j'_\beta & I_i \\ \Lambda & \Lambda' & Q \end{Bmatrix} \times$$
$$\sum_{\lambda\lambda'} C^{\Lambda'\lambda'}_{\Lambda\lambda Qq} Y^*_{\Lambda\lambda}(\vec{n}_k)\, Y_{\Lambda'\lambda'}(\vec{n}_\beta),$$

$$\tilde{\tau}_{00}(I_i) = \frac{m_\beta}{4k_\alpha k_\beta r_\beta^2 m_\alpha} \sum_Q (2Q+1) C_Q P_Q(\vec{n}_k \vec{n}_\beta)$$

Differential probability of γ-emission

$$d\widetilde{w}(\vec{n}_\gamma) = A \sum_{Q=0,2,4} \sum_{\Lambda\Lambda'} D^{\Lambda}_{\Lambda' Q} \, \phi^{\Lambda}_{\Lambda' Q}(\vec{n}_k, \vec{n}_\beta, \vec{n}_\gamma) \, d\Omega_\gamma$$

$$g_J = \frac{2J+1}{(2s+1)(2I_\alpha+1)}$$

Invariant convolution of 3 spherical harmonics

$$\phi^{\Lambda}_{\Lambda' Q}(\vec{n}_k, \vec{n}_\beta, \vec{n}_\gamma) = (4\pi)^{3/2} \sum_{\mu\nu q} C^{\Lambda\mu}_{\Lambda'\nu Qq} Y^*_{\Lambda\mu}(\vec{n}_k) \cdot Y_{\Lambda'\nu}(\vec{n}_\beta) \cdot Y_{Qq}(\vec{n}_\gamma)$$

$$A = \frac{\omega \, a_L(e)^2}{2\hbar c^3 k_\alpha k_\beta r_\beta^2} \frac{m_\beta}{m_\alpha}$$

$$D^{\Lambda}_{\Lambda' Q} = \sqrt{2Q+1} \, C^{L1}_{L1Q0} \, U(I_f I_i L Q, L I_i) \sqrt{(2I_i+1)(2\Lambda'+1)} \times$$

$$\sum_{JJ'} g_{J'} \sum_{l_\alpha l'_\alpha j_\alpha j'_\alpha l_\beta l'_\beta j_\beta j'_\beta} S_J(l_\alpha j_\alpha \to l_\beta j_\beta) \, S^*_{J'}(l'_\alpha j'_\alpha \to l'_\beta j'_\beta) \sqrt{(2J+1)(2j'_\beta+1)} \times$$

$$C^{l_\alpha 0}_{l'_\alpha 0 \Lambda 0} C^{l_\beta 0}_{l'_\beta 0 \Lambda' 0} \, U(s j'_\alpha l_\alpha \Lambda, l'_\alpha j_\alpha) \, U(I_\alpha J' j_\alpha \Lambda, j'_\alpha J) \, U(s j'_\beta l_\beta \Lambda', l'_\beta j_\beta) \begin{Bmatrix} J & j_\beta & I_i \\ J' & j'_\beta & I_i \\ \Lambda & \Lambda' & Q \end{Bmatrix}$$

Differential cross-section of neutron inelastic scattering

$$\frac{d\sigma_{\alpha\beta}(\vec{n}_\beta)}{d\Omega_\beta} = \frac{1}{4k_\alpha^2}\sum_Q (2Q+1)C_Q P_Q(\vec{n}_k\vec{n}_\beta)$$

$$d\sigma_{\alpha\beta}(\vec{n}_\beta) = \frac{m_\alpha k_\beta r_\beta^2}{m_\beta k_\alpha}\tilde{\tau}_{00}(I_i)d\Omega_\beta$$

$$\tilde{\tau}_{00}(I_i) = \frac{m_\beta}{4k_\alpha k_\beta r_\beta^2 m_\alpha}\sum_Q (2Q+1)C_Q P_Q(\vec{n}_k\vec{n}_\beta)$$

$$\tilde{\tau}_{00}(I_i) = \frac{m_\beta k_\alpha}{m_\alpha k_\beta r_\beta^2}\frac{d\sigma_{\alpha\beta}(\vec{n}_\beta)}{d\Omega_\beta}$$

$$C_Q = \sum_{JJ'} g_J \sum_{l_\alpha l'_\alpha j_\alpha j'_\alpha l_\beta l'_\beta j_\beta j'_\beta} S_J(l_\alpha j_\alpha \to l_\beta j_\beta)\, S^*_{J'}(l'_\alpha j'_\alpha \to l'_\beta j'_\beta)\, C^{l_\alpha 0}_{l'_\alpha 0Q0}\, C^{l_\beta 0}_{l'_\beta 0Q0} \times$$
$$U(sj'_\alpha l_\alpha Q, l'_\alpha j_\alpha)\, U(I_\alpha J' j_\alpha Q, j'_\alpha J)\, U(sj'_\beta l_\beta Q, l'_\beta j_\beta)\, U(I_i j_\beta J' Q, J j'_\beta).$$

Double differential cross-section of neutron inelastic scattering

$$d^2\sigma(\vec{n}_\beta, \vec{n}_\gamma) = d\sigma_{\alpha\beta}(\vec{n}_\beta)\, dW(\vec{n}_\gamma) = d\sigma_{\alpha\beta}(\vec{n}_\beta)\, \frac{dw(\vec{n}_\gamma)}{w}$$

Unnormalized differential probability of γ-emission per time unit

$$d\widetilde{w}(\vec{n}_\gamma) = A \sum_{Q=0,2,4,\ldots} \sum_{\Lambda\Lambda'} D^{\Lambda}_{\Lambda' Q}\, \phi^{\Lambda}_{\Lambda' Q}(\vec{n}_k, \vec{n}_\beta, \vec{n}_\gamma)\, d\Omega_\gamma$$

Normalized

$$dw(\vec{n}_\gamma) = \frac{d\tilde{w}(n_\gamma)}{\tilde{\tau}_{00}(I_i)}$$

$$d\sigma_{\alpha\beta}(\vec{n}_\beta) = \frac{m_\alpha k_\beta r_\beta^2}{m_\beta k_\alpha}\, \tilde{\tau}_{00}(I_i) d\Omega_\beta$$

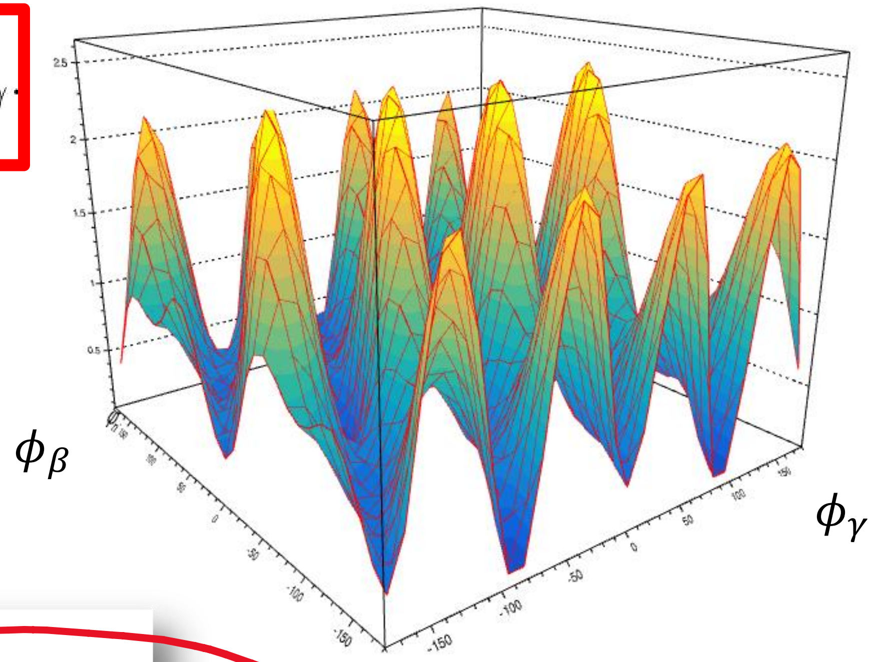
$$d^2\sigma(\vec{n}_\beta, \vec{n}_\gamma) = \frac{1}{16\pi k_\alpha^2} \sum_{Q=0,2,4,\ldots} \sum_{\Lambda\Lambda'} D^{\Lambda}_{\Lambda' Q}\, \phi^{\Lambda}_{\Lambda' Q}(\vec{n}_k, \vec{n}_\beta, \vec{n}_\gamma)\, d\Omega_\beta d\Omega_\gamma.$$

(n-n’γ)-angular correlations

Double differential cross-section of neutron inelastic scattering with γ-emission

$$d^2\sigma(\vec{n}_\beta,\vec{n}_\gamma) = \frac{1}{16\pi k_\alpha^2}\sum_{Q=0,2,4,\dots}\sum_{\Lambda\Lambda'} D^{\Lambda}_{\Lambda' Q}\,\phi^{\Lambda}_{\Lambda' Q}(\vec{n}_k,\vec{n}_\beta,\vec{n}_\gamma)\,d\Omega_\beta d\Omega_\gamma.$$

$$A = \frac{\omega\, a_L(e)^2}{2\hbar c^3 k_\alpha k_\beta r_\beta^2}\,\frac{m_\beta}{m_\alpha}\,,$$



$$D^{\Lambda}_{\Lambda' Q} = \sqrt{2Q+1}\,C^{L1}_{L1Q0}\,U(I_f I_i L Q, L I_i)\sqrt{(2I_i+1)(2\Lambda'+1)}\,\times$$
$$\sum_{JJ'} g_{J'} \sum_{l_\alpha l'_\alpha j_\alpha j'_\alpha l_\beta l'_\beta j_\beta j'_\beta} S_J(l_\alpha j_\alpha \to l_\beta j_\beta)\, S^*_{J'}(l'_\alpha j'_\alpha \to l'_\beta j'_\beta)\sqrt{(2J+1)(2j'_\beta+1)}\,\times$$
$$C^{l_\alpha 0}_{l'_\alpha 0 \Lambda 0} C^{l_\beta 0}_{l'_\beta 0 \Lambda' 0}\, U(s j'_\alpha l_\alpha \Lambda, l'_\alpha j_\alpha)\, U(I_\alpha J' j_\alpha \Lambda, j'_\alpha J)\, U(s j'_\beta l_\beta \Lambda', l'_\beta j_\beta) \begin{Bmatrix} J & j_\beta & I_i \\ J' & j'_\beta & I_i \\ \Lambda & \Lambda' & Q \end{Bmatrix}.$$

S-matrix coefficients from TALYS

SymPy - Python library for symbolic mathematics

SymPy

Quantum Mechanics

Abstract

Contains Docstrings of Physics-Quantum module

Quantum Functions

- Anticommutator
- Clebsch-Gordan Coefficients
- Commutator
- Constants
- Dagger
- Inner Product
- Tensor Product

States and Operators

- Cartesian Operators and States
- Hilbert Space
- Operator
- Operator/State Helper Functions
- Qapply
- Represent
- Spin
- State

Quantum Computation

- Circuit Plot
- Gates
- Grover's Algorithm
- QFT
- Qubit
- Shor's Algorithm

Clebsch-Gordan Coefficients

Clebsch-Gordon Coefficients.

`class sympy.physics.quantum.cg.CG(j1, m1, j2, m2, j3, m3)`

Class for Clebsch-Gordan coefficient.

Parameters:

j1, m1, j2, m2 : Number, Symbol

Angular momenta of states 1 and 2.

j3, m3: Number, Symbol

Total angular momentum of the coupled system.

```
In [12]:  CG(3,0,6,0,3,0)
```

Out[12]: C^{30}_{3060}

```
In [13]:  CG(3,0,6,0,3,0).doit()
```

Out[13]: $-\frac{10\sqrt{429}}{429}$

```
In [14]:  latex(CG(3,0,6,0,3,0))
Out[14]: 'C^{30}_{3060}'
```

```
In [15]:  latex(CG(3,0,6,0,3,0).doit())
Out[15]: '- \\frac{10 \\sqrt{429}}{429}'
```

1	D^{0}_{00}	3
2	D^{1}_{10}	7
3	D^{2}_{20}	25
4	D^{3}_{30}	57
5	D^{4}_{40}	81
6	D^{5}_{50}	96
7	D^{6}_{60}	99
8	D^{2}_{02}	101
9	D^{1}_{12}	111
10	D^{3}_{12}	129
11	D^{0}_{22}	149
12	D^{2}_{22}	159
13	D^{4}_{22}	191
14	D^{1}_{32}	213
15	D^{3}_{32}	233
16	D^{5}_{32}	257
17	D^{2}_{42}	266
18	D^{4}_{42}	288
19	D^{6}_{42}	304
20	D^{3}_{52}	309
21	D^{5}_{52}	318
22	D^{4}_{62}	321
23	D^{6}_{62}	326
24	D^{4}_{04}	328

42 D^{6}_{64}

$$D^{6}_{64} = -\tfrac{2\sqrt{910}}{7} \times ($$

$$4 \; S_{\frac{5}{2}}(3\tfrac{5}{2} \to 3\tfrac{5}{2}) S^{*}_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{7}{2}) \, 4\sqrt{3} \times$$

$$C^{30}_{3060} \, C^{30}_{3060} \; U\left(\tfrac{1}{2}\tfrac{7}{2}36; 3\tfrac{5}{2}\right) U\left(\tfrac{1}{2}\tfrac{7}{2}36; 3\tfrac{5}{2}\right) U\left(0\tfrac{7}{2}\tfrac{5}{2}6; \tfrac{7}{2}\tfrac{5}{2}\right) \left\{ \begin{array}{ccc} \frac{5}{2} & \frac{5}{2} & 2 \\ \frac{7}{2} & \frac{7}{2} & 2 \\ 6 & 6 & 4 \end{array} \right\} +$$

$$4 \; S_{\frac{5}{2}}(3\tfrac{5}{2} \to 3\tfrac{7}{2}) S^{*}_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{5}{2}) \, 6 \times$$

$$C^{30}_{3060} \, C^{30}_{3060} \; U\left(\tfrac{1}{2}\tfrac{7}{2}36; 3\tfrac{5}{2}\right) U\left(\tfrac{1}{2}\tfrac{5}{2}36; 3\tfrac{7}{2}\right) U\left(0\tfrac{7}{2}\tfrac{5}{2}6; \tfrac{7}{2}\tfrac{5}{2}\right) \left\{ \begin{array}{ccc} \frac{5}{2} & \frac{7}{2} & 2 \\ \frac{7}{2} & \frac{5}{2} & 2 \\ 6 & 6 & 4 \end{array} \right\} +$$

$$4 \; S_{\frac{5}{2}}(3\tfrac{5}{2} \to 3\tfrac{7}{2}) S^{*}_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{7}{2}) \, 4\sqrt{3} \times$$

$$C^{30}_{3060} \, C^{30}_{3060} \; U\left(\tfrac{1}{2}\tfrac{7}{2}36; 3\tfrac{5}{2}\right) U\left(\tfrac{1}{2}\tfrac{7}{2}36; 3\tfrac{7}{2}\right) U\left(0\tfrac{7}{2}\tfrac{5}{2}6; \tfrac{7}{2}\tfrac{5}{2}\right) \left\{ \begin{array}{ccc} \frac{5}{2} & \frac{7}{2} & 2 \\ \frac{7}{2} & \frac{7}{2} & 2 \\ 6 & 6 & 4 \end{array} \right\} +$$

$$3 \; S_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{5}{2}) S^{*}_{\frac{5}{2}}(3\tfrac{5}{2} \to 3\tfrac{7}{2}) \, 8 \times$$

$$C^{30}_{3060} \, C^{30}_{3060} \; U\left(\tfrac{1}{2}\tfrac{5}{2}36; 3\tfrac{7}{2}\right) U\left(\tfrac{1}{2}\tfrac{7}{2}36; 3\tfrac{5}{2}\right) U\left(0\tfrac{5}{2}\tfrac{7}{2}6; \tfrac{5}{2}\tfrac{7}{2}\right) \left\{ \begin{array}{ccc} \frac{7}{2} & \frac{5}{2} & 2 \\ \frac{5}{2} & \frac{7}{2} & 2 \\ 6 & 6 & 4 \end{array} \right\} +$$

$$4 \; S_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{5}{2}) S^{*}_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{7}{2}) \, 8 \times$$

$$C^{30}_{3060} \, C^{30}_{3060} \; U\left(\tfrac{1}{2}\tfrac{7}{2}36; 3\tfrac{7}{2}\right) U\left(\tfrac{1}{2}\tfrac{7}{2}36; 3\tfrac{5}{2}\right) U\left(0\tfrac{7}{2}\tfrac{7}{2}6; \tfrac{7}{2}\tfrac{7}{2}\right) \left\{ \begin{array}{ccc} \frac{7}{2} & \frac{5}{2} & 2 \\ \frac{7}{2} & \frac{7}{2} & 2 \\ 6 & 6 & 4 \end{array} \right\} +$$

$$3 \; S_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{7}{2}) S^{*}_{\frac{5}{2}}(3\tfrac{5}{2} \to 3\tfrac{5}{2}) \, 4\sqrt{3} \times$$

$$C^{30}_{3060} \, C^{30}_{3060} \; U\left(\tfrac{1}{2}\tfrac{5}{2}36; 3\tfrac{7}{2}\right) U\left(\tfrac{1}{2}\tfrac{5}{2}36; 3\tfrac{7}{2}\right) U\left(0\tfrac{5}{2}\tfrac{7}{2}6; \tfrac{5}{2}\tfrac{7}{2}\right) \left\{ \begin{array}{ccc} \frac{7}{2} & \frac{7}{2} & 2 \\ \frac{5}{2} & \frac{5}{2} & 2 \\ 6 & 6 & 4 \end{array} \right\} +$$

$$3 \; S_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{7}{2}) S^{*}_{\frac{5}{2}}(3\tfrac{5}{2} \to 3\tfrac{7}{2}) \, 8 \times$$

$$C^{30}_{3060} \, C^{30}_{3060} \; U\left(\tfrac{1}{2}\tfrac{5}{2}36; 3\tfrac{7}{2}\right) U\left(\tfrac{1}{2}\tfrac{7}{2}36; 3\tfrac{7}{2}\right) U\left(0\tfrac{5}{2}\tfrac{7}{2}6; \tfrac{5}{2}\tfrac{7}{2}\right) \left\{ \begin{array}{ccc} \frac{7}{2} & \frac{7}{2} & 2 \\ \frac{5}{2} & \frac{7}{2} & 2 \\ 6 & 6 & 4 \end{array} \right\} +$$

$$4 \; S_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{7}{2}) S^{*}_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{5}{2}) \, 4\sqrt{3} \times$$

$$C^{30}_{3060} \, C^{30}_{3060} \; U\left(\tfrac{1}{2}\tfrac{7}{2}36; 3\tfrac{7}{2}\right) U\left(\tfrac{1}{2}\tfrac{5}{2}36; 3\tfrac{7}{2}\right) U\left(0\tfrac{7}{2}\tfrac{7}{2}6; \tfrac{7}{2}\tfrac{7}{2}\right) \left\{ \begin{array}{ccc} \frac{7}{2} & \frac{7}{2} & 2 \\ \frac{7}{2} & \frac{5}{2} & 2 \\ 6 & 6 & 4 \end{array} \right\} +$$

$$4 \; S_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{7}{2}) S^{*}_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{7}{2}) \, 8 \times$$

$$C^{30}_{3060} \, C^{30}_{3060} \; U\left(\tfrac{1}{2}\tfrac{7}{2}36; 3\tfrac{7}{2}\right) U\left(\tfrac{1}{2}\tfrac{7}{2}36; 3\tfrac{7}{2}\right) U\left(0\tfrac{7}{2}\tfrac{7}{2}6; \tfrac{7}{2}\tfrac{7}{2}\right) \left\{ \begin{array}{ccc} \frac{7}{2} & \frac{7}{2} & 2 \\ \frac{7}{2} & \frac{7}{2} & 2 \\ 6 & 6 & 4 \end{array} \right\})$$

$$= -\tfrac{400\sqrt{7854}}{99099} \; S_{\frac{5}{2}}(3\tfrac{5}{2} \to 3\tfrac{5}{2}) S^{*}_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{7}{2}) +$$

$$\tfrac{4000\sqrt{3927}}{891891} \; S_{\frac{5}{2}}(3\tfrac{5}{2} \to 3\tfrac{7}{2}) S^{*}_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{5}{2}) +$$

$$\tfrac{400\sqrt{1309}}{297297} \; S_{\frac{5}{2}}(3\tfrac{5}{2} \to 3\tfrac{7}{2}) S^{*}_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{7}{2}) +$$

$$\tfrac{4000\sqrt{3927}}{891891} \; S_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{5}{2}) S^{*}_{\frac{5}{2}}(3\tfrac{5}{2} \to 3\tfrac{7}{2}) +$$

$$-\tfrac{800\sqrt{3927}}{891891} \; S_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{5}{2}) S^{*}_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{7}{2}) +$$

$$-\tfrac{400\sqrt{7854}}{99099} \; S_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{7}{2}) S^{*}_{\frac{5}{2}}(3\tfrac{5}{2} \to 3\tfrac{5}{2}) +$$

$$\tfrac{400\sqrt{1309}}{297297} \; S_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{7}{2}) S^{*}_{\frac{5}{2}}(3\tfrac{5}{2} \to 3\tfrac{7}{2}) +$$

$$-\tfrac{800\sqrt{3927}}{891891} \; S_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{7}{2}) S^{*}_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{5}{2}) +$$

$$-\tfrac{800\sqrt{1309}}{297297} \; S_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{7}{2}) S^{*}_{\frac{7}{2}}(3\tfrac{7}{2} \to 3\tfrac{7}{2})$$

S-matrix from ECIS (TALYS subprogram)

Changes in source file: directecis.f, dwbaecis.f, incidentecis.f → ecist.f

```
!
! ********************* Set ECIS input parameters *********************
!
! Specific ECIS flags:
!
open (unit = 9, file = 'ecisdisc.inp', status = 'replace')
rotational = .false.
vibrational = .true.
jlmloc = .false.
legendre = .true.
title = 'Direct discrete cross sections by DWBA          '
ecis1 = 'FFFFFTFFFFFFFFFFFFFFFFFFFFTFTFFFFFFFFFFFFFFFFFFFFF'
ecis2 = 'FFFFFTFFTTFFFTTFTTTFTTTFTFFFFFFFFFFFFFFFFFFFTFFFFF'
if (flagrel) ecis1(8:8) = 'T'
ncoll = 2
iterm = 1
if (flagstate) then
  npp = ncoll
```

lo **– array with logical flags in *ecis1* and *ecis2***

Print S-matrix in file fort.60:
lo(60) = T – 10th place in variable ***ecis2***

Print S-matrix in ecis output files (ex. ecisdisc.out):
lo(56) = T - 6th place in variable ***ecis2***

input

Файл Изменить Просмотр

```
projectile n
element    C
mass       12
energy   14.1
partable y
widthmode 0
fullhf y
outinverse y
outecis y
eciscompound y
ecissave y
optmodfileN 6 6N.loc
deformfile 6 C.def
```

S-matrix – file fort.60

fort.60

```
<s-matrix>      1.01      1.41000000D+01      12.00    0    40
      0.5     +     3     3
  1  1  1  0   0.5   2.3754952D-01  -3.3084799D-01 i     0.40729617
  1  2  2  2   1.5   6.1735684D-02  -1.0046878D-01 i     0.11792061
  1  3  2  2   2.5   1.1635605D-01  -6.0871448D-02 i     0.13131665
      0.5     -     3     3
  1  1  1  1   0.5  -3.0405097D-01  -4.4808248D-01 i     0.54150245
  1  2  2  1   1.5   1.5504123D-01   5.6294140D-03 i     0.15514339
  1  3  2  3   2.5  -2.1685912D-02  -3.6444128D-02 i     0.04240817
      1.5     -     5     5
  1  1  1  1   1.5  -2.7195675D-02  -5.0980312D-01 i     0.51052799
  1  2  2  1   0.5  -9.5765627D-02  -5.3598509D-03 i     0.09591550
  1  3  2  1   1.5  -1.0296216D-01  -3.8048495D-02 i     0.10976746
  1  4  2  3   2.5  -8.1479007D-03  -2.1379974D-02 i     0.02287994
  1  5  2  3   3.5  -1.4396438D-02  -5.3910644D-02 i     0.05579978
      1.5     +     5     5
  1  1  1  2   1.5  -1.7075794D-01  -8.1152197D-02 i     0.18906071
  1  2  2  0   0.5  -6.2495487D-02   5.7685787D-02 i     0.08504902
  1  3  2  2   1.5   6.7080697D-02   8.8331524D-02 i     0.11091563
  1  4  2  2   2.5  -5.0394620D-03  -6.9148897D-02 i     0.06933229
  1  5  2  4   3.5   1.2554986D-02  -1.0645735D-02 i     0.01646084
      2.5     +     6     6
  1  1  1  2   2.5  -1.2508429D-01  -4.2539450D-01 i     0.44340338
  1  2  2  0   0.5   5.7036078D-02  -1.5825555D-02 i     0.05919090
  1  3  2  2   1.5   4.7713827D-03   4.6588656D-02 i     0.04683235
```

ecisdisc.out

```
target               mass =   12.00000            product of charges =     0.           at**1/3 =    2.289428D+00
incident particle    mass =    1.00866      spin = 0.5    ap**1/3 =    1.002878D+00
              energy(lab) =    1.410000D+01 mev           energy(c. m.) =    1.299975D+01 mev.

n =   1 -    spin = 0.0+ excitation energy =   0.0000 mev          potential    1
n =   2 -    spin = 2.0+ excitation energy =   4.4398 mev          potential    1        phonon state with phonon  1
              phonon  1      1    2      k =   0      beta = 0.39636 0.39636 0.39636 0.39636 0.39636 0.39636 0.39636 0.39636

                                                                                                        4 4 4 4 4 5
channel spin and parity =     0.5+                                                                      5 6 7 8 9 0
                                                                                                        F F F F F F
  ic  icp  n     l    j               s matrix                 |s|       phase /with coul.
   1   1   1     0    0.5    2.3754952D-01 -3.3084799D-01 i    0.40729617-0.94809037-0.94809037     4 4 4 4 4 5
   1   2   2     2    1.5    6.1735684D-02 -1.0046878D-01 i    0.11792061-1.01980042-1.01980042     5 6 7 8 9 0
   1   3   2     2    2.5    1.1635605D-01 -6.0871448D-02 i    0.13131665-0.48199415-0.48199415     T F F F F F

channel spin and parity =     0.5-

  ic  icp  n     l    j               s matrix                 |s|       phase /with coul.
   1   1   1     1    0.5   -3.0405097D-01 -4.4808248D-01 i    0.54150245-2.16698783-2.16698783
   1   2   2     1    1.5    1.5504123D-01  5.6294140D-03 i    0.15514339 0.03629320 0.03629320
   1   3   2     3    2.5   -2.1685912D-02 -3.6444128D-02 i    0.04240817-2.10756480-2.10756480

channel spin and parity =     1.5-

  ic  icp  n     l    j               s matrix                 |s|       phase /with coul.
   1   1   1     1    1.5   -2.7195675D-02 -5.0980312D-01 i    0.51052799-1.62409126-1.62409126
   1   2   2     1    0.5   -9.5765627D-02 -5.3598509D-03 i    0.09591550-3.08568256-3.08568256
   1   3   2     1    1.5   -1.0296216D-01 -3.8048495D-02 i    0.10976746-2.78761863-2.78761863
   1   4   2     3    2.5   -8.1479007D-03 -2.1379974D-02 i    0.02287994-1.93490389-1.93490389
   1   5   2     3    3.5   -1.4396438D-02 -5.3910644D-02 i    0.05579978-1.83174966-1.83174966
```

Thank you for your attention!
感谢您的关注