Estimation of Cosmic Muon-Induced Neutron Yields via Muon Spallation in Deep Underground Environments Using (e, e′xn) Cross-Section Measurements of ^{181}Ta

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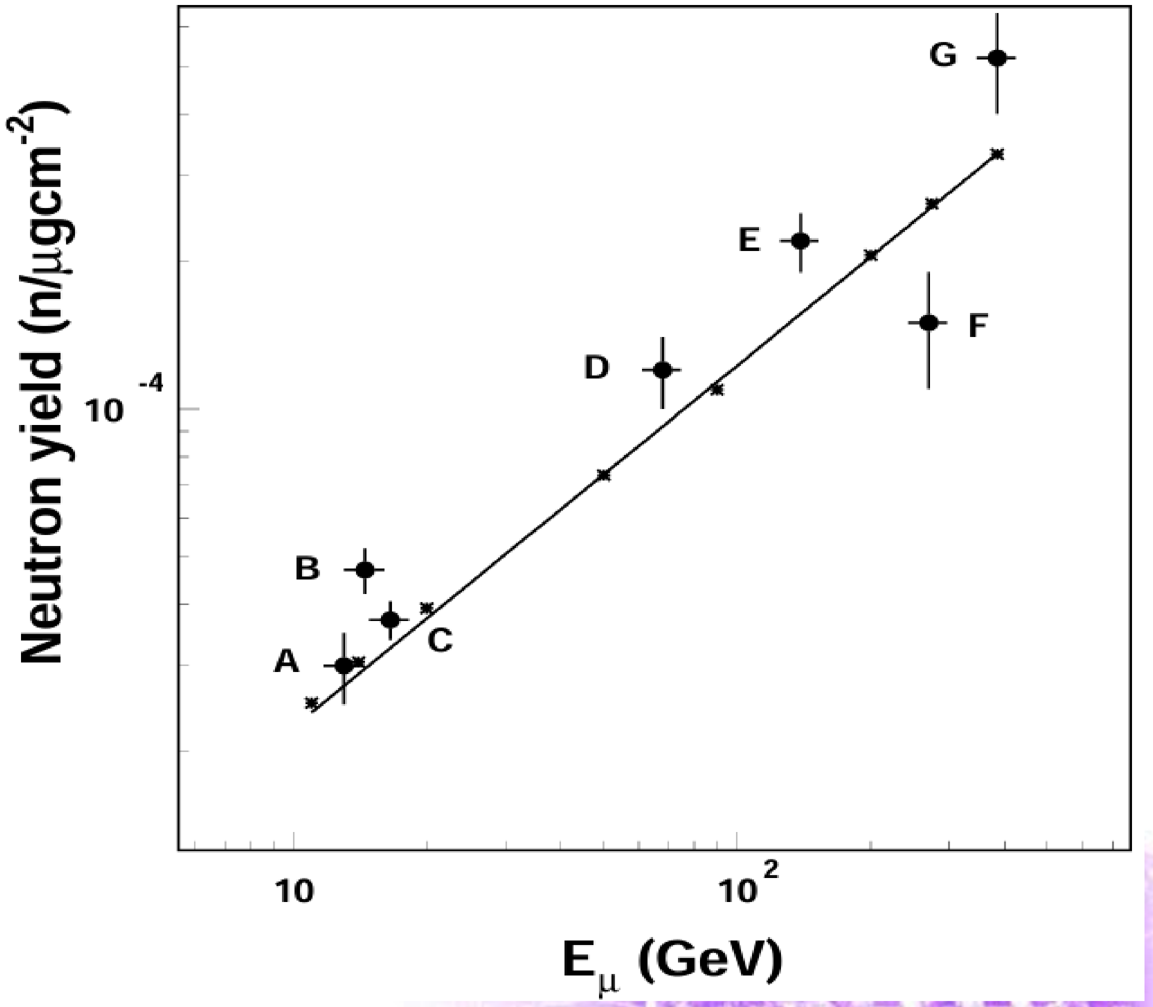
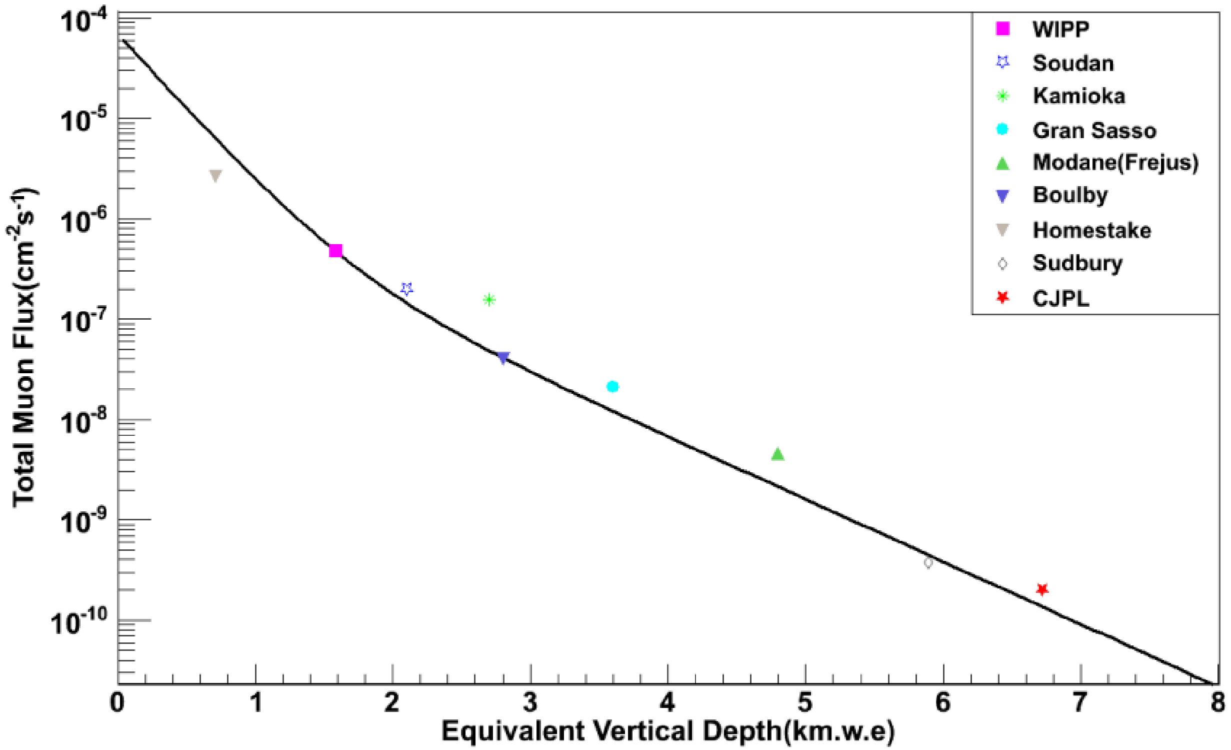
Cosmic-muon-induced neutrons – Conflicting Data & Models

- Fast neutrons from cosmic-ray muons represent an important background for many rare-event experiments such as searches for dark matter and neutrino oscillation experiments.
- In these rare-event experiments, neutrons can mimic the target signal.
- For example, the Palo Verde reactor neutrino oscillation experiment found such neutrons to be their dominant background.
- Low-energy solar neutrino experiments such as SNO and Borexino also have to estimate such backgrounds.
- The understanding of neutron backgrounds may be relevant in resolving the controversy between the CDMS and DAMA results on dark matter searches.

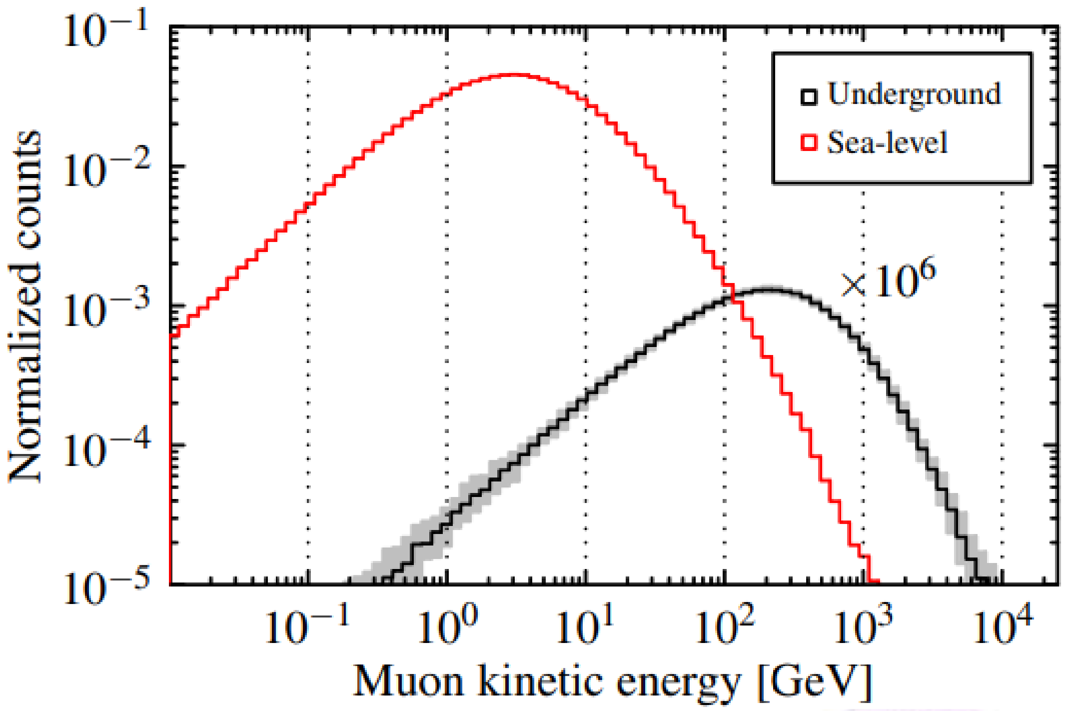


[1]Wang, Y-F., et al. Physical Review D 64.1 (2001): 013012.

Challenge on Measurements



Wu, Yu-Cheng, et al. Chinese physics C 37.8 (2013): 086001.



Guo, Zi-yi, et al. Chinese Physics C 45.2 (2021): 025001.

- CJPL (2400 m.w.e.) recorded just 343 muons in 820 days, yielding only ~8 detected neutrons.
- Black: Muon spectrum in CJPL

- Due to the high energy and low flux of cosmic muons, it is challenging to directly measure the neutron yield.

Unavoidable Neutrons Produced in the Detector

Fast neutrons from cosmic-ray muons are produced in the following processes:

a) Muon interactions with nuclei via a virtual photon producing a nuclear disintegration. This process is usually referred to as "muon spallation" and is the main source of theoretical uncertainty.

b) Muon elastic scattering with neutrons bound in nuclei.

c) Photo-nuclear reactions associated with electromagnetic showers generated by muons.

d) Secondary neutron production following any of the above processes.

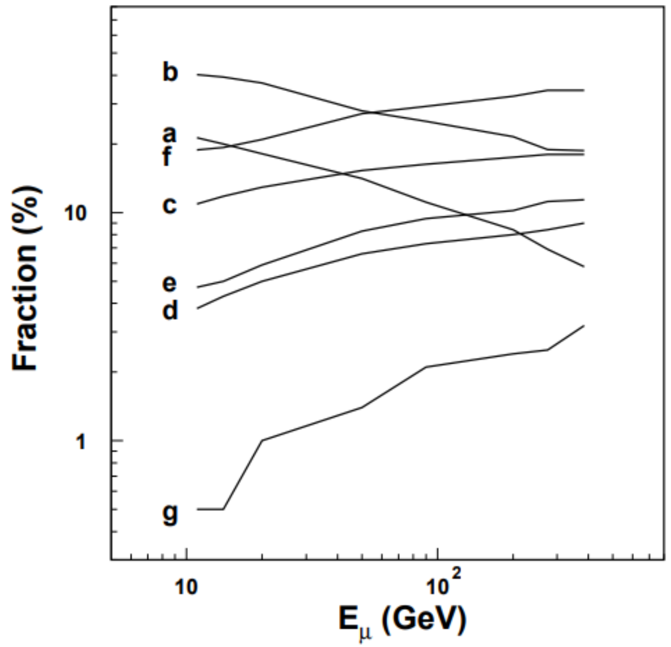


FIG. 3. Origin of neutrons: a) direct muon spallation, b) real photo-nuclear disintegration, c) neutron spallation, d) proton spallation, e) π^+ spallation, f) π^- spallation and capture, g) others.

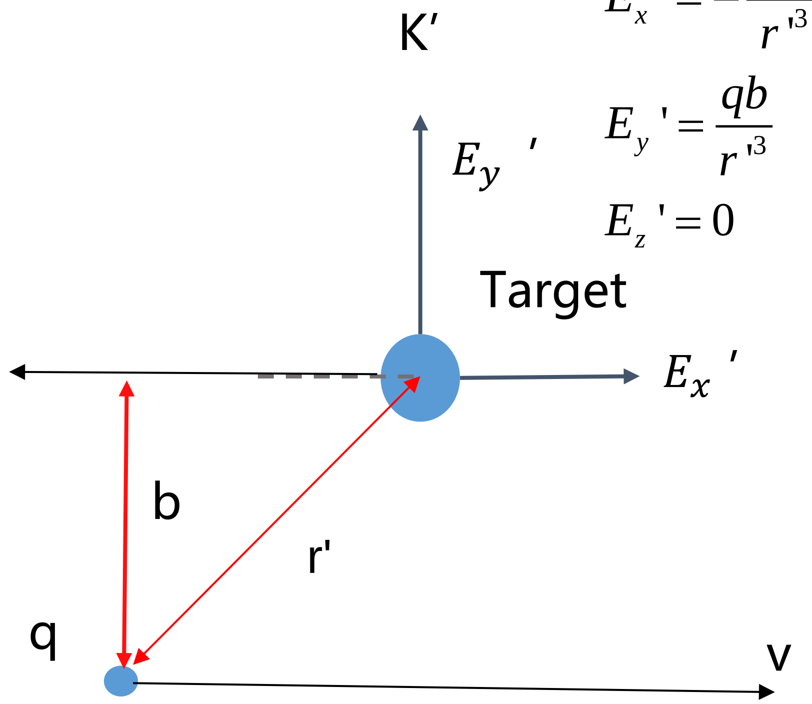
- Even if we can shield as much as possible against fast neutrons generated outside the detector, fast neutrons produced in the detector by muon spallation will always be present.

- According to the equivalent photon approximation method, the Coulomb excitation process of leptons with the same relativistic factor is similar.

(e,e'xn) ➡ (μ,μ'xn)

[1]Wang, Y-F., et al. Physical Review D 64.1 (2001): 013012.

Equivalent Photon Approximation

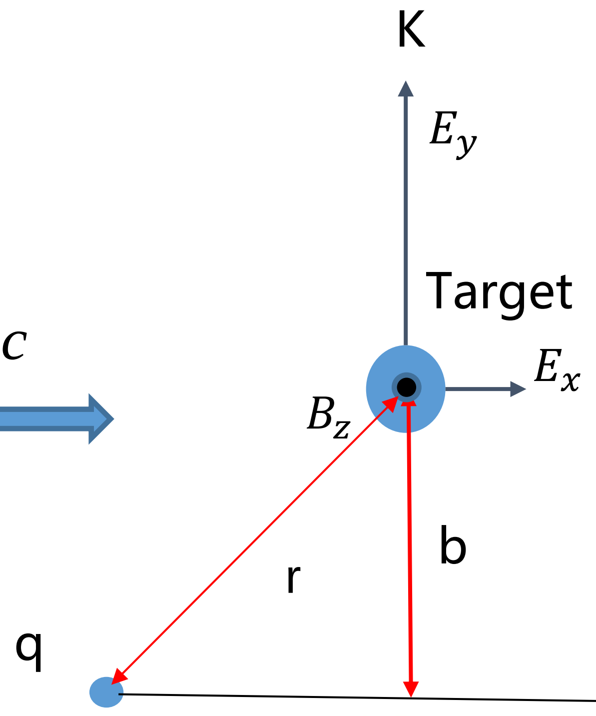


$$E_x' = -\frac{qvt'}{r'^3} \qquad B_x' = 0$$

$$E_y' = \frac{qb}{r'^3} \qquad B_y' = 0$$

$$E_z' = 0 \qquad B_z' = 0$$

$$v = \beta c$$



$$E_x = E_x' = -\frac{q\gamma vt}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

$$E_y = \gamma E_y' = \frac{q\gamma b}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

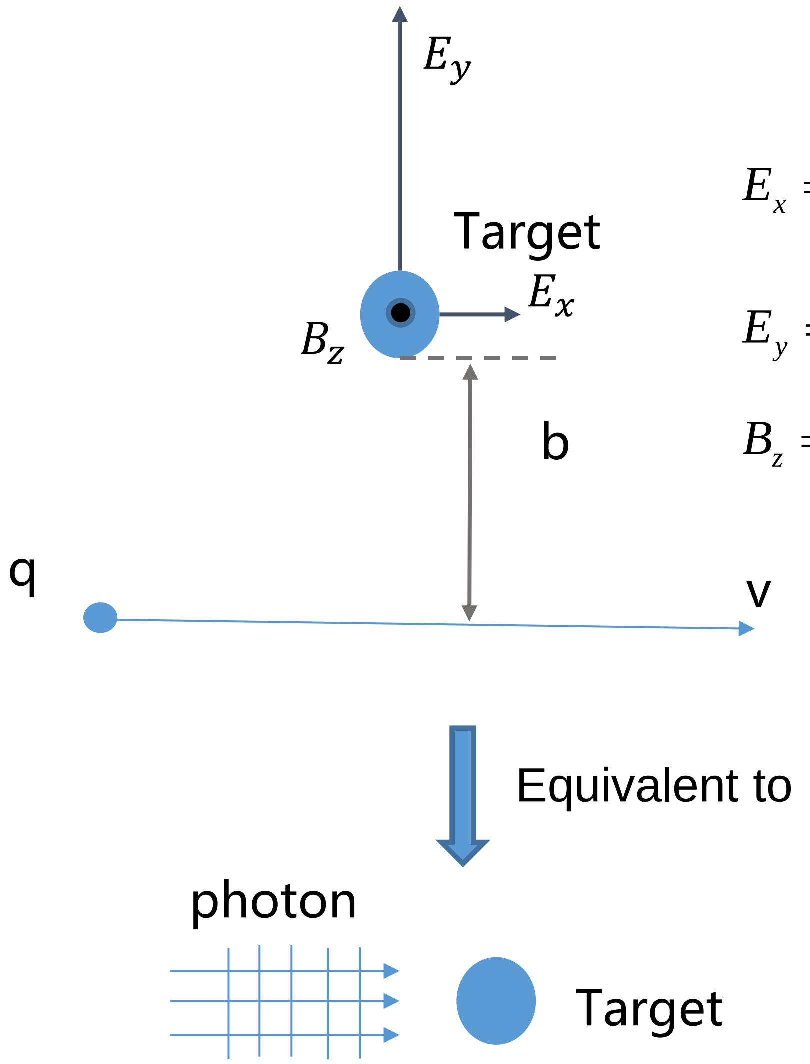
$$B_z = \gamma\beta E_y' = \beta E_y$$

$$E_x = E_x' \qquad B_x = B_x'$$

$$E_y = \gamma(E_y' + \beta B_z') \qquad B_y = \gamma(B_y' - \beta E_z')$$

$$E_z = \gamma(E_z' - \beta B_y') \qquad B_z = \gamma(B_z' + \beta E_y')$$

The equivalent photon approximation



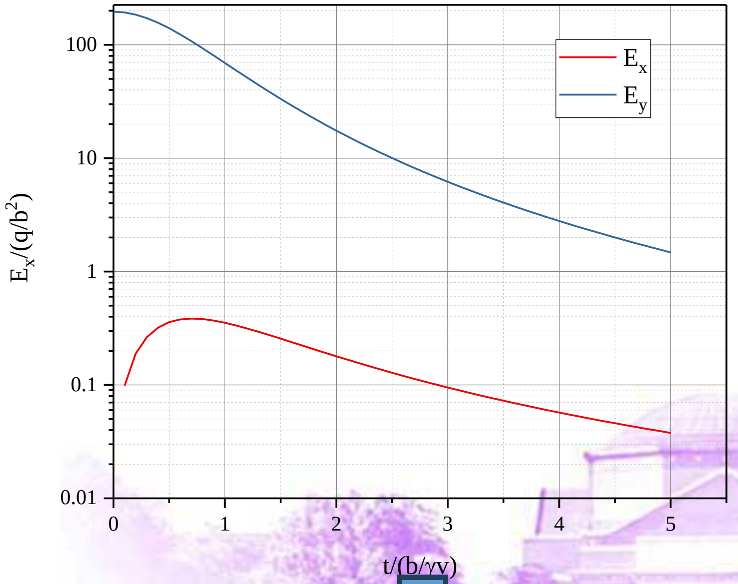
$$E_x = E_x' = -\frac{q\gamma vt}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

$$E_y = \gamma E_y' = \frac{q\gamma b}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

$$B_z = \gamma\beta E_y' = \beta E_y$$

$$\beta = \frac{v}{c} \approx 1 \Rightarrow E_y \approx B_z$$

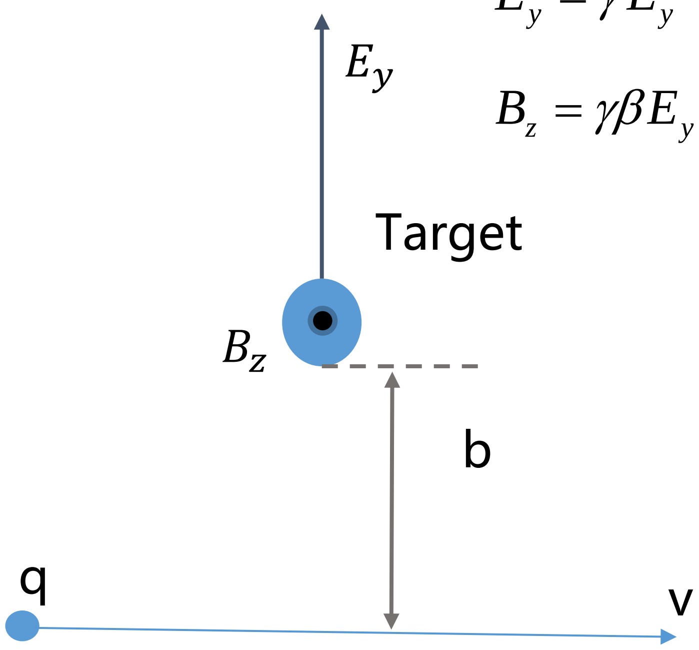
E_x and E_y when γ=200



E_x can be ignored

The equivalent photon approximation

Weizsäcker-Williams method of virtual quanta:



$$E_y = \gamma E_y' = \frac{q\gamma b}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

$$B_z = \gamma\beta E_y' = \beta E_y$$

Fourier transform $\Rightarrow E(\omega) \xrightarrow{\frac{d^2 I}{d\omega dA}(\omega) = c|E(\omega)|^2} \frac{d^2 I}{d\omega dA}(\omega)$

$$\frac{dI}{d\omega}(\omega) = \int \frac{d^2 I}{d\omega dA}(\omega) 2\pi b db$$

$$\frac{dI}{d\omega}(\omega)$$

$$\frac{dI}{d\omega}(\omega) d\omega = \hbar\omega N(\hbar\omega) d(\hbar\omega)$$

$$N(\hbar\omega) = \frac{2}{\pi}\frac{q^2}{\hbar c}\left(\frac{c}{v}\right)^2 \frac{1}{\hbar\omega}\{xK_0(x)K_1(x) - \frac{v^2}{2c^2}x^2 \times [K_1^2(x) - K_0^2(x)]\}$$

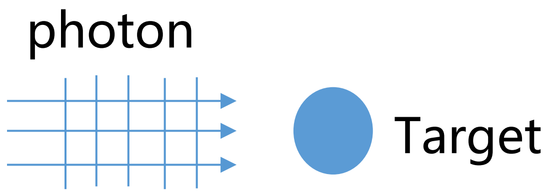
$$x = \frac{\omega b_{min}}{\gamma v}$$

minimum impact parameter b_{min}:

$$b_{min} = \max\left(\frac{\hbar}{p}, R\right)$$

$K_n(x)$: Modified Bessel function of the second kind

Equivalent to



Same impact parameter

Same Lorentz factor

Low energy transfer

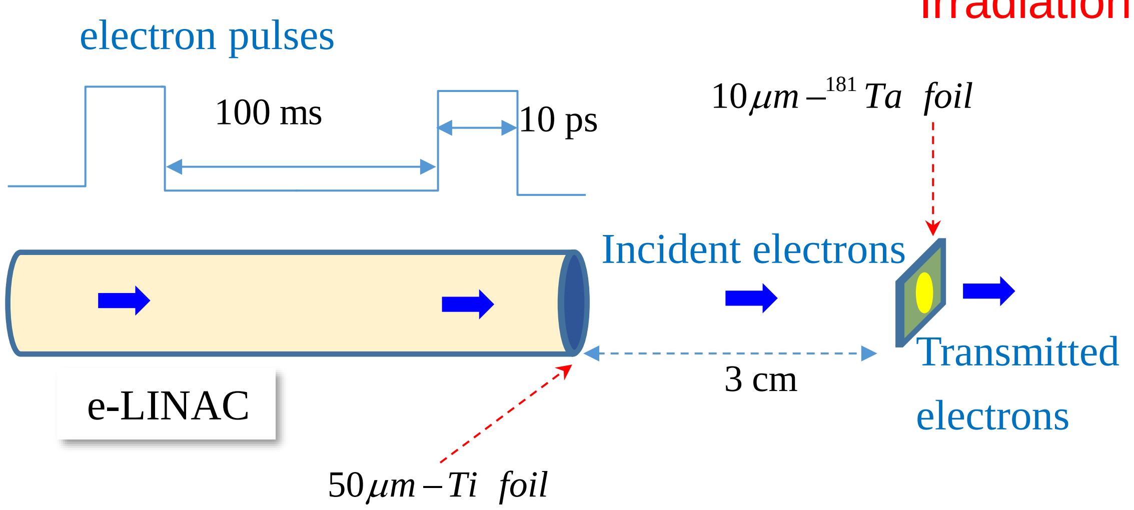
$$\Rightarrow N(E_{e^-}) = N(E_{\mu^-} = \frac{m_{\mu^-}}{m_{e^-}} E_{e^-} \approx 207 E_{e^-})$$

$$E_{e^-} = 100 MeV \Leftrightarrow E_{\mu^-} = 20.7 GeV\ (example)$$

Experimental process

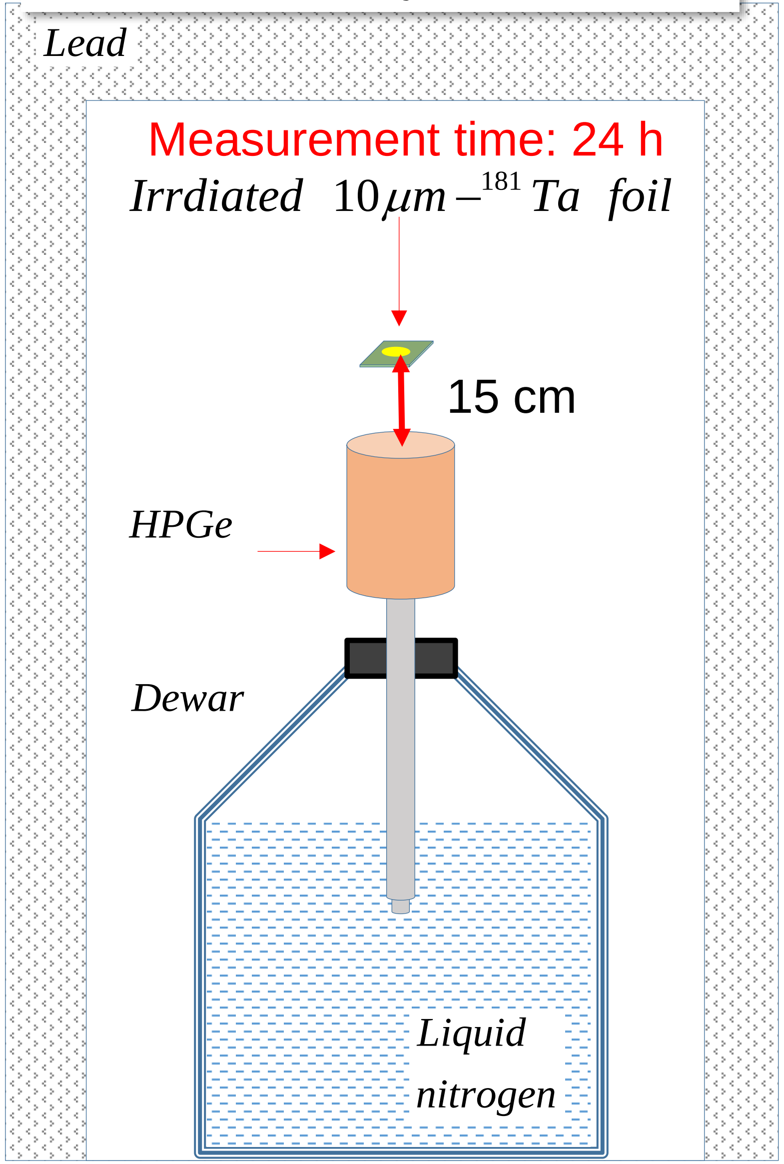
(1) 10μm-^{181}Ta foil irradiated by electrons

Energy: 20~110 MeV
Irradiation time: 1 h

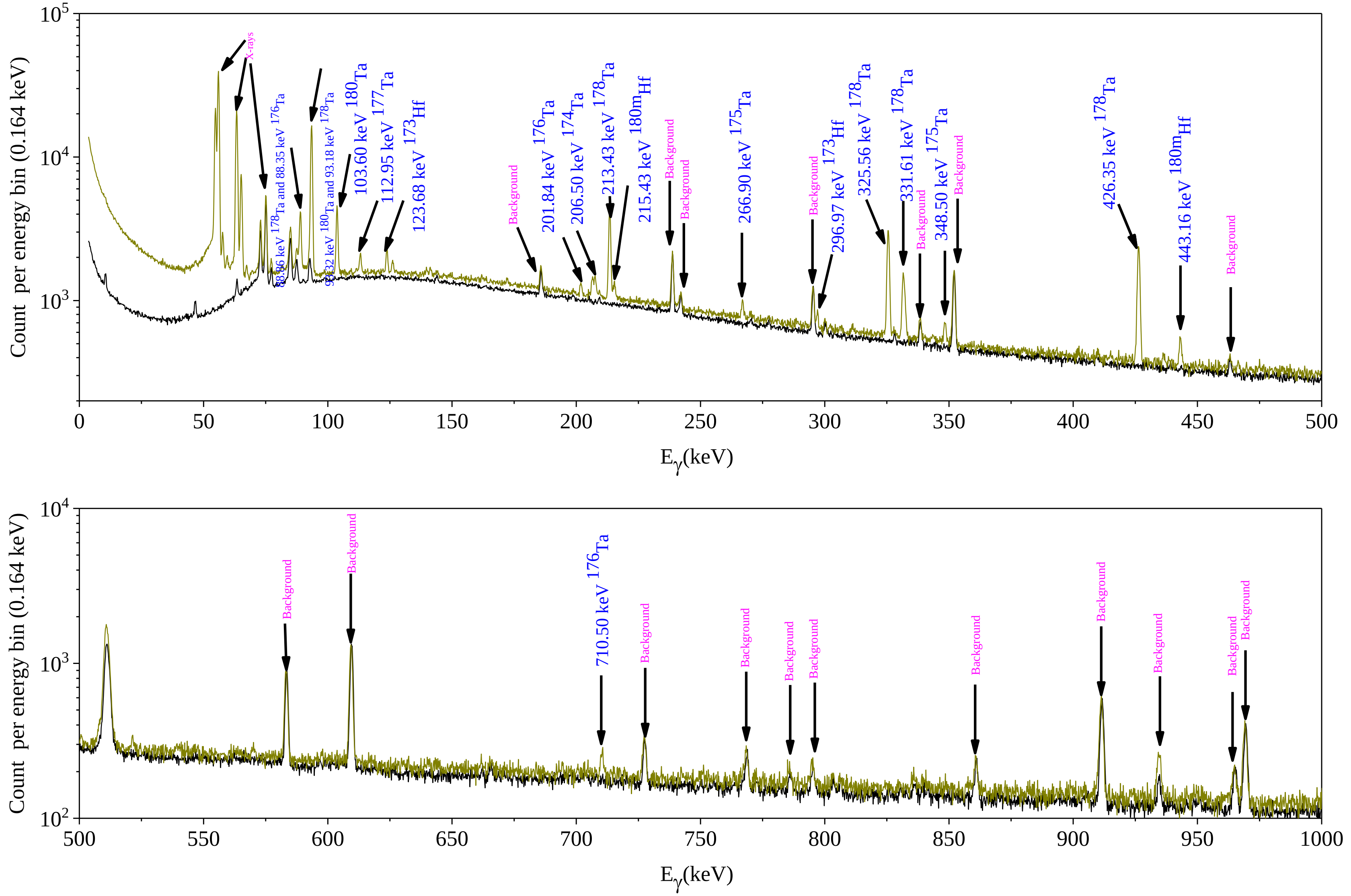


The 120 MeV Electron Linac at the State Key Laboratory of Intense Pulsed Radiation Simulation and Effect

(2) : residual activity of the 10μm-^{181}Ta foil measured by a HPGe detector



Spectrum



Yellow line: residual γ-activity spectrum

Black line: background

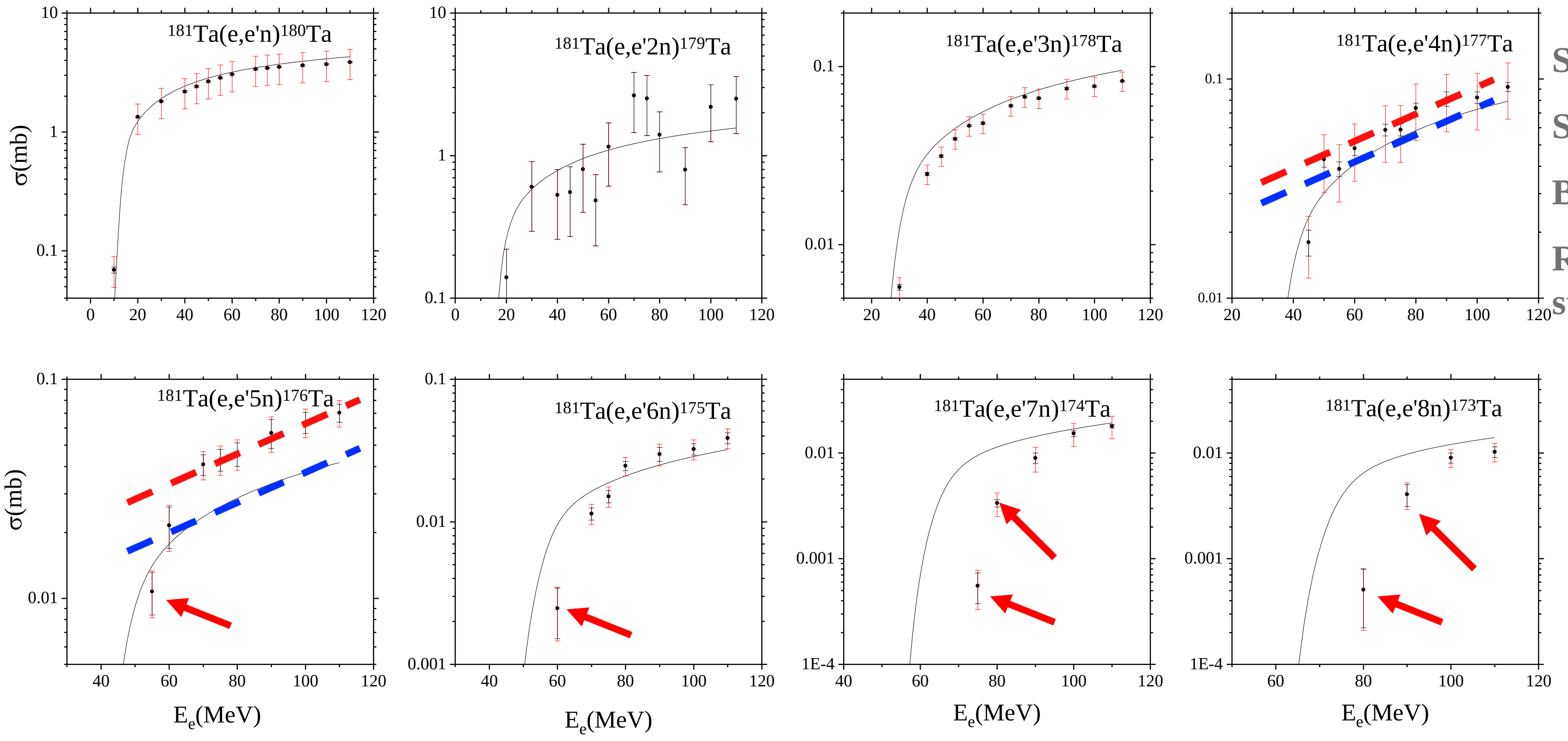
Calculation of measured cross sections

$$\sigma_{(e,e'xn)}^{\exp} = -\frac{1}{ND}\ln\left[1 - \frac{N_{meas}}{N_e I_\gamma \varepsilon (1-e^{-\lambda t_{irr}}) e^{-\lambda t_{cool}} (1-e^{-\lambda t_{meas}})}\right]$$

Symbol	Meaning	Error
N_{meas}	Counts of full-energy peaks	1% ~ 10%
N_e	Number of bombarding electrons	2.5%
ε	Absolute detection efficiency for full-energy peaks of gamma-rays	2%
I_γ	Intensity of emitted gamma-ray for each decay of $^{181-x}$Ta	1 ~ 20%
λ	Decay constant of $^{181-x}$Ta	0.1% ~ 7%
$t_{irr}, t_{cool}, t_{meas}$	Irradiation, cooling and measurement times	/
N	^{181}Ta nuclei's number density	/
D	Thickness of tantalum foil	/

IAEA

Measurement result



Solid line: simulated cross-section

Solid circles: measured cross-section

Black error bars: statistical errors

Red error bars: total errors (both statistical and systematic errors).

- **When the number of escaping neutrons reaches 5–8, a significant difference arises between the experimental and theoretical cross sections, especially when electron energies are close to threshold.**
- **These differences not only cast our doubt on the accuracy of the TALYS results but also highlight the potential of our approach to correct the simulation outcomes.**

Energy Dependence of Neutron Yield

Weizsäcker-Williams method of virtual quanta

$$N(\hbar\omega)=\frac{2}{\pi}\frac{q^2}{\hbar c}\left(\frac{c}{v}\right)^2\frac{1}{\hbar\omega}\{xK_0(x)K_1(x)-\frac{v^2}{2c^2}x^2\times[K_1^2(x)-K_0^2(x)]\}$$

$$\xrightarrow{\hbar\omega\ll E_e} N(\hbar\omega)=\frac{2e^2}{\pi c\hbar^2\omega}[\ln(\frac{1.123\gamma c}{\omega b_{\min}})-\frac{1}{2}]$$

$$\sigma_{(e,e'xn)}(E_e)=\int_{E_{th}}^{E_e}\sigma_{(\gamma,xn)}(E_\gamma)N_{virtual}(E_\gamma,E_e)dE_\gamma$$

$$=\int_{E_{th}}^{E_e}\sigma_{(\gamma,xn)}(E_\gamma)\frac{2e^2}{\pi c\hbar^2\omega}[\ln(E_e)+\ln(\frac{1.123c}{0.511\omega b_{\min}})-\frac{1}{2}]dE_\gamma$$

$$=\int_{E_{th}}^{E_e}\sigma_{(\gamma,xn)}(E_\gamma)\frac{2e^2}{\pi c\hbar^2\omega}dE_\gamma\ln(E_e)+\int_{E_{th}}^{E_e}\sigma_{(\gamma,xn)}(E_\gamma)\frac{2e^2}{\pi c\hbar^2\omega}[\ln(\frac{1.123c}{0.511\omega b_{\min}})-\frac{1}{2}]dE_\gamma$$

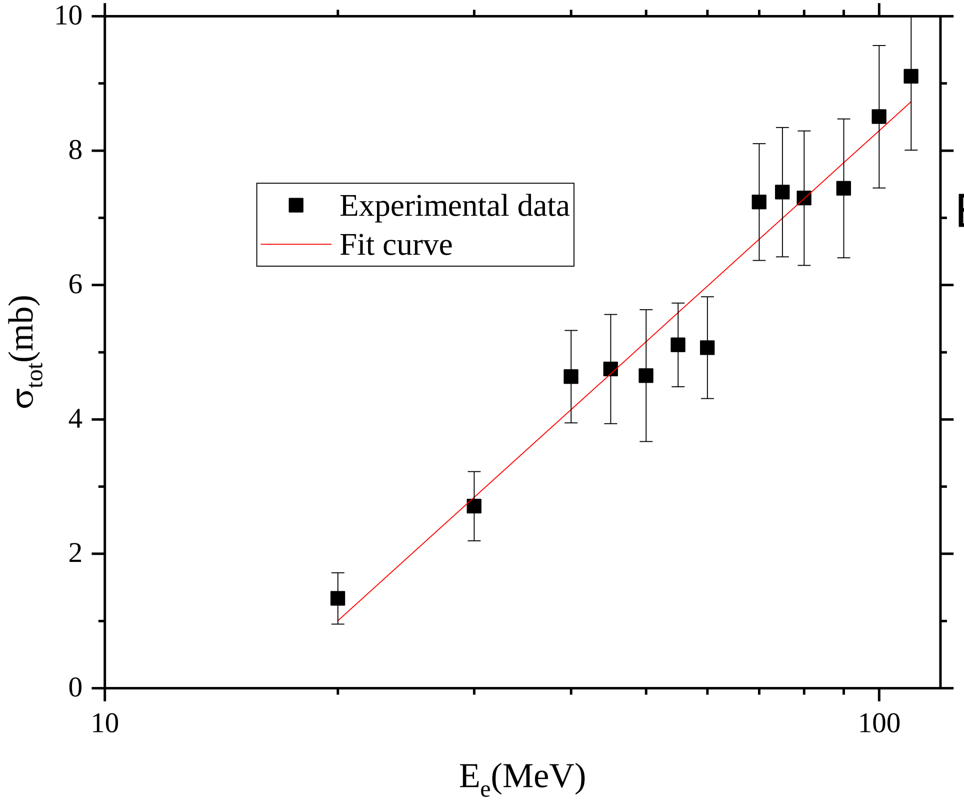
$$=A\bullet\ln(E_e)+B$$

$\Downarrow$

For electrons with energies far above the energy at the peak of the cross-section, the trend of the Coulomb excitation cross-section should be a logarithmic increase as the electron energy increases.

Energy Dependence of Neutron Yield

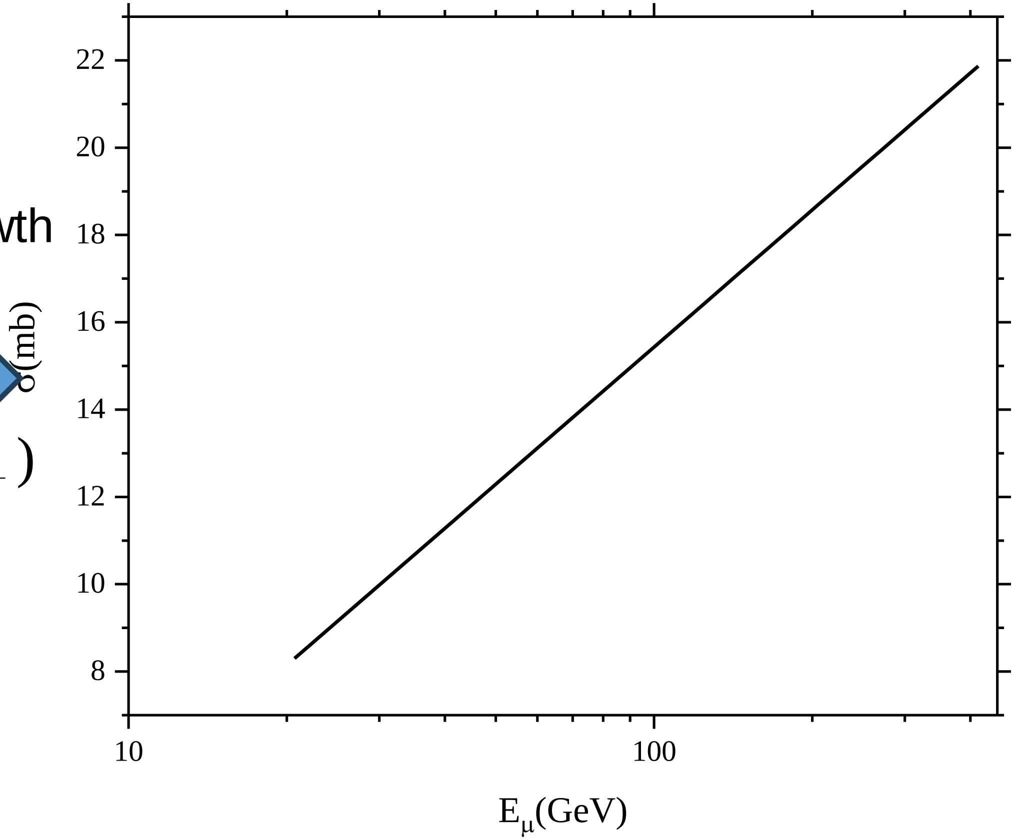
Electron-induced-neutron–production cross section vs Electron energy



Extrapolate logarithmic growth trend to high energies

$$N(E_{e^-}) = N(E_{\mu^-} = 207E_{e^-})$$

Muon-induced-neutron–production cross section vs Muon energy



Nuclear Dependence of Neutron Yield

- Assume the neutron–production cross section is dominated by the GDR contribution
- GDR sum rule:：

$$\int \sigma_\gamma(E_\gamma)dE_\gamma = \frac{2\pi^2 e^2\hbar}{Mc}\cdot\frac{NZ}{A}$$

- GDR resonance energy vs. mass number A：

$$E_r = 31.2A^{-1/3} + 20.6A^{-1/6}$$

- Virtual photon spectrum vs. photon energy：

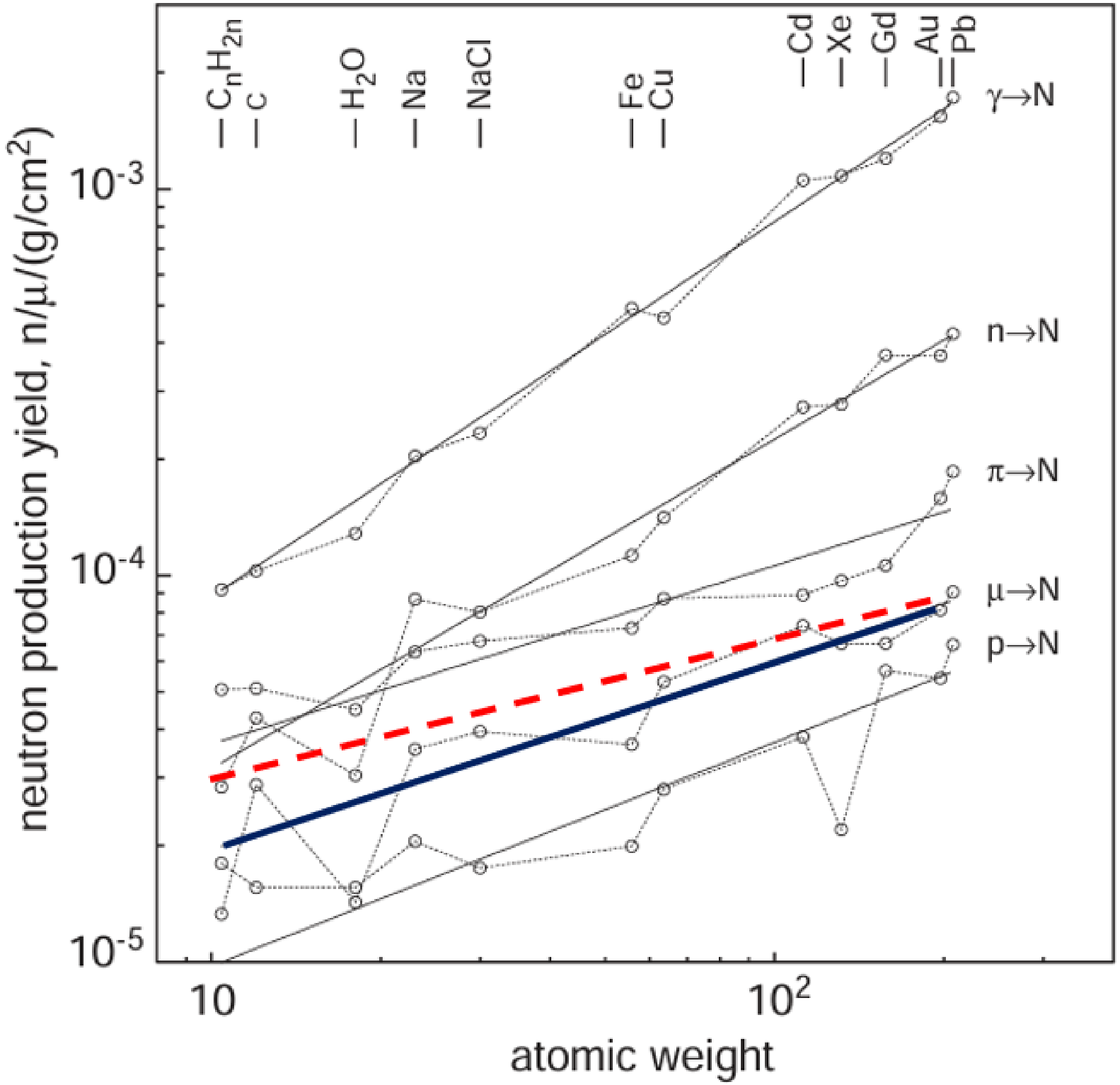
$$N(\hbar\omega) = \frac{2e^2}{\pi c\hbar^2\omega}[\ln(\frac{1.123\gamma c}{\omega b_{\min}}) - \frac{1}{2}]$$

- Assume Virtual photon spectrum varies slowly across the GDR region, then

$$\sigma_\mu(E_\mu) = \int N_{virtual}(E_\gamma, E_\mu)\sigma_\gamma(E_\gamma)dE_\gamma$$

$$= N_{virtual}(E_r, E_\mu)\int \sigma_\gamma(E_\gamma)dE_\gamma \propto \frac{NZ}{A}\cdot\frac{A - B\ln E_r}{E_r}$$

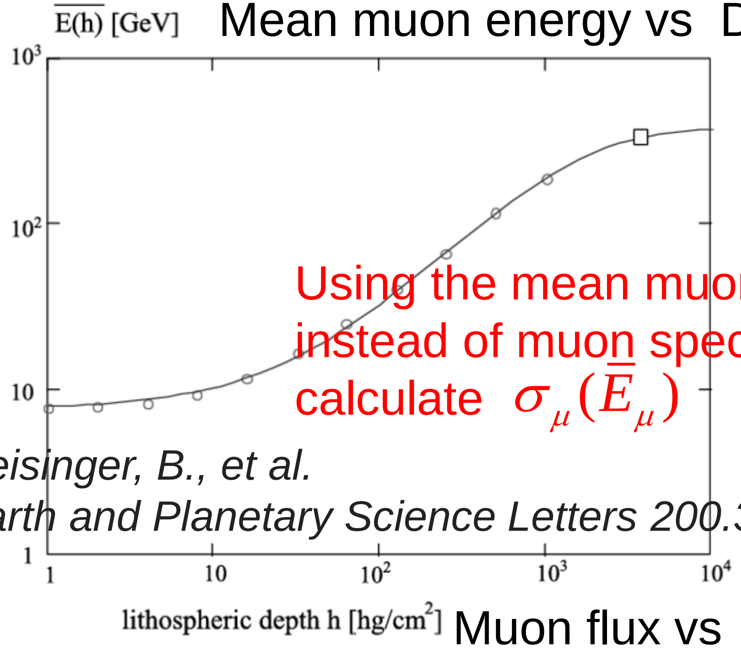
$F(A,Z)$



[1] Araujo, H. M., et al. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 545.1-2 (2005): 398-411.

Depth Dependence of Neutron Yield

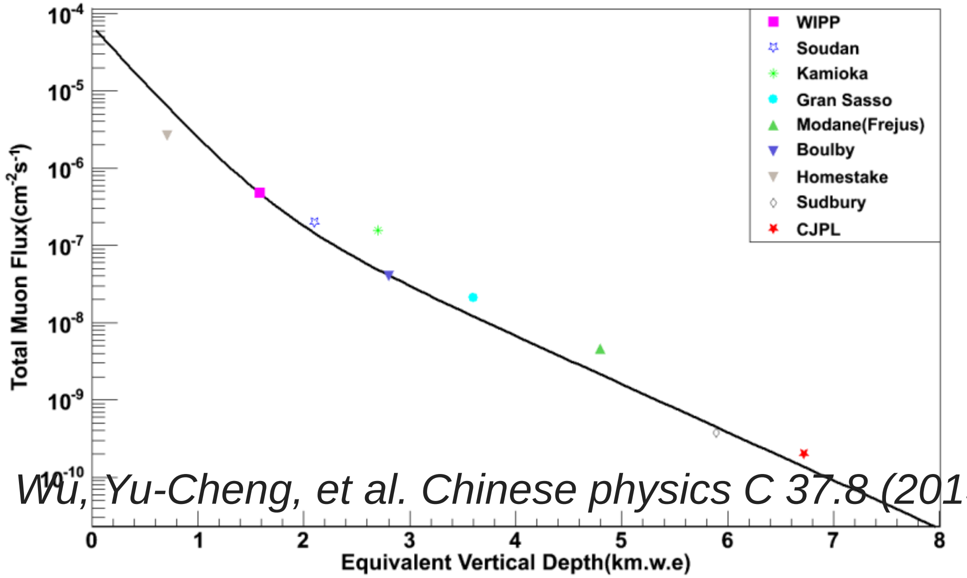
Mean muon energy vs Depth



Using the mean muon energy instead of muon spectrum to calculate $\sigma_\mu(\bar{E}_\mu)$

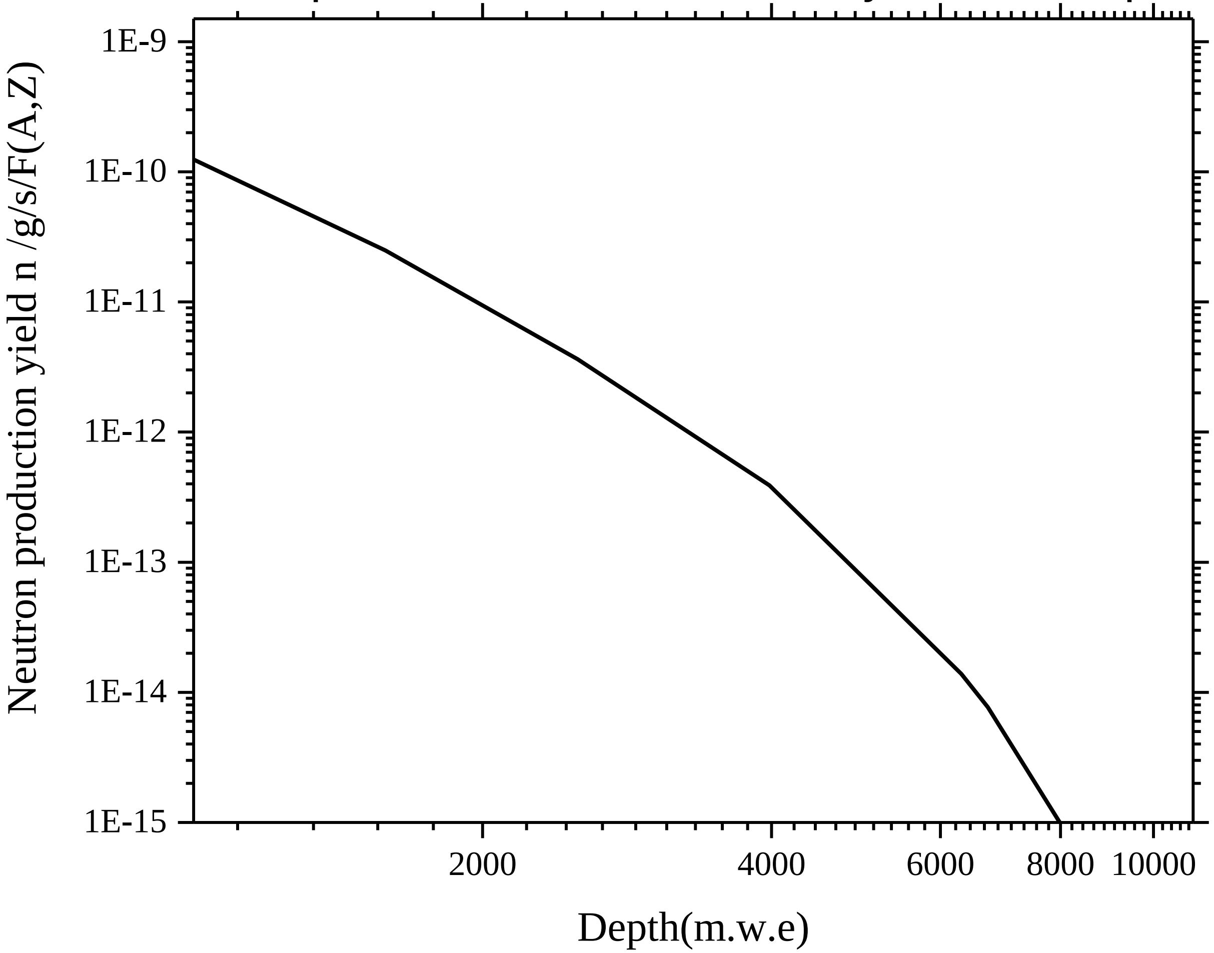
Heisinger, B., et al. Earth and Planetary Science Letters 200.3-4 (2002): 345-355.

Muon flux vs Depth



Wu, Yu-Cheng, et al. Chinese physics C 37.8 (2013): 086001.

Muon-Spallation-induced neutron yield vs Depth



Serving as a lower limit for the neutron background in deep-underground measurements

Conclusion

- According to the Weizsäcker-Williams virtual photon method, the Coulomb excitation process of leptons with the same relativistic factor is similar.

- Based on this, we propose a method to estimate the neutron yield induced by cosmic-ray muons via Coulomb excitation in detectors by measuring the neutron yield from electron Coulomb excitation, thereby providing a lower-bound estimate of the neutron background in deep-underground measurements.

- More measurements of electron-induced neutron yields for each target material are needed for more accurate evaluation of nuclear dependence of the neutron yield induced by cosmic-ray muons via Coulomb excitation.

THANK YOU FOR LISTENING!