Frank
Laboratory
of Neutron Physics

TO A NEUTRON DISPERSION LAW FOR MATTER MOVING WITH ACCELERATION

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Neutron Dispersion Law in Matter

Neutron Dispersion Law in Matter
Moving with Acceleration

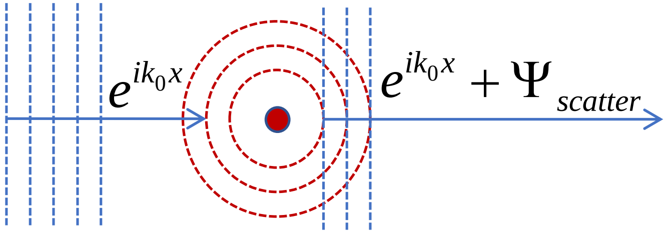
Phase distorbtion
Acceleration effect

Interaction of the Neutron Wave with Nuclei
Moving with Acceleration

Neutron Dispersion Law in Matter

Effective potential is a basis of UCN Optics

Interaction of a neutron wave with an atomic nucleus



Fermi's Pseudo Potential

$$U_{Fermi} = \frac{2\pi\hbar^2}{m} b\delta(\vec{r} - \vec{r}_0)$$

m – neutron mass
ρ – bulk density of nuclei
b – the length of neutron scattering on the nuclei

Effective Potential for Matter

$$U_{Effective} = \sum_i \frac{2\pi\hbar^2}{m} b_i(\vec{r} - \vec{r}_i) \to \frac{2\pi\hbar^2}{m}\rho b$$

Schrödinger Equation Solution

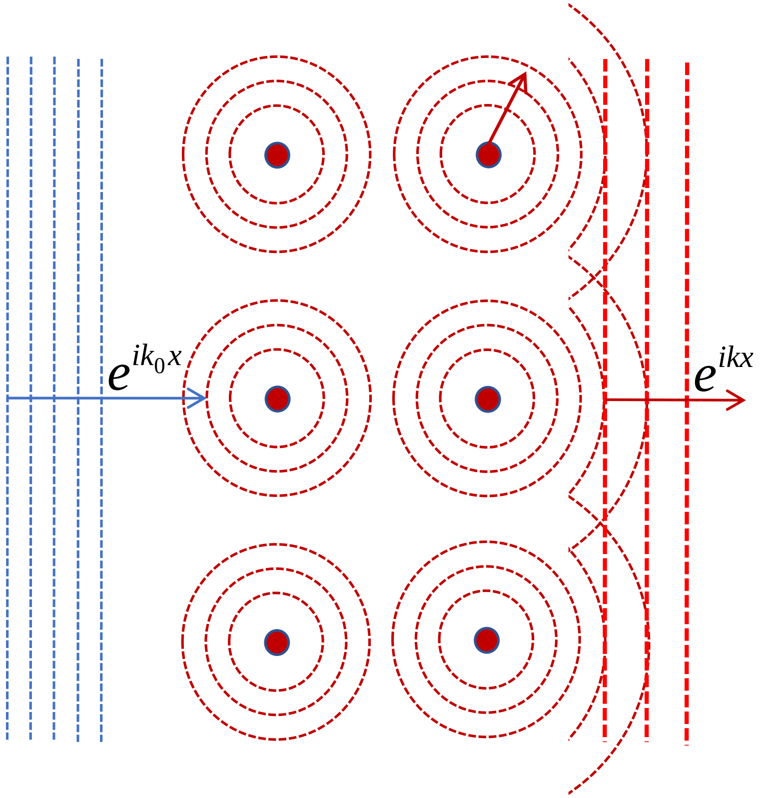
$$\Psi_{scatter} = -b\frac{e^{ik_0 r}}{r}$$

First Born Approximation!

Only first approximation for delta-function is available

Neutron dispersion law in matter

Effective potential is a basis of UCN Optics



The sum of scattered waves from all nuclei in a layer

$$\sum \Psi_{scatter} = -b\int 2\pi\rho d \frac{e^{ik_0 r}}{r} zdz \to -i\frac{2\pi\rho b}{k_0} d e^{ik_0 x}$$

m – neutron mass
ρ – bulk density of nuclei
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Interference of scattered waves with incident one.

$$\Psi_{initial} + \sum \Psi_{scatter} = \left(1 - i\frac{2\pi\rho b}{k_0} d\right) e^{ik_0 x} \underset{N\to\infty}{\overset{d\to 0}{\to}} e^{ik_0\left(1 - \frac{1}{2}\frac{4\pi\rho b}{k_0^2}\right)x} = e^{ik_0 n x}$$

$$\sqrt{1-\chi} \to 1 - \frac{1}{2}\chi + ..$$

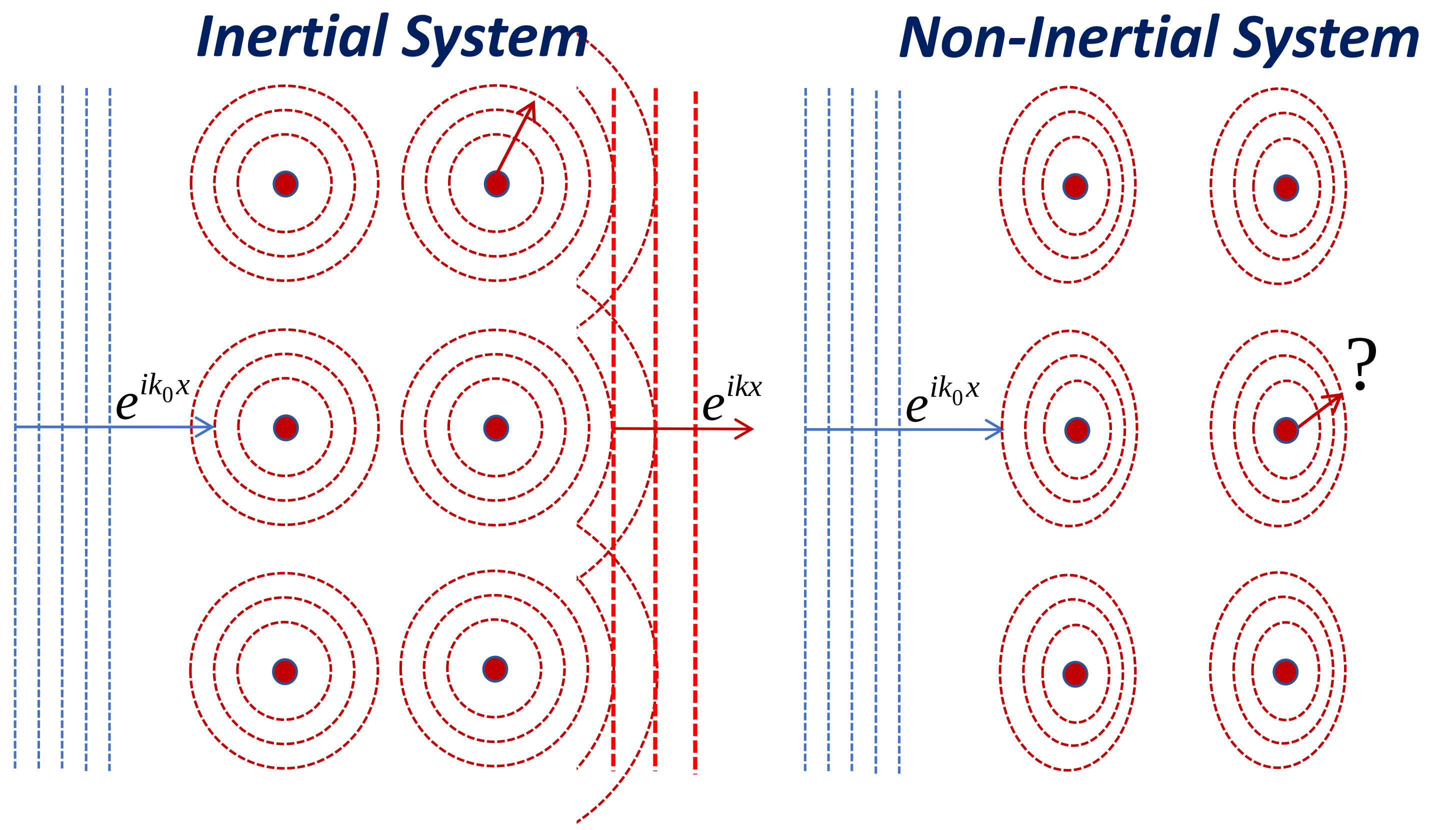
Neutron dispersion law

$$k^2 = k_0^2 - 4\pi\rho b \qquad n^2 = 1 - \frac{4\pi}{k_0^2}\rho b$$

(L.Foldy, 1945)

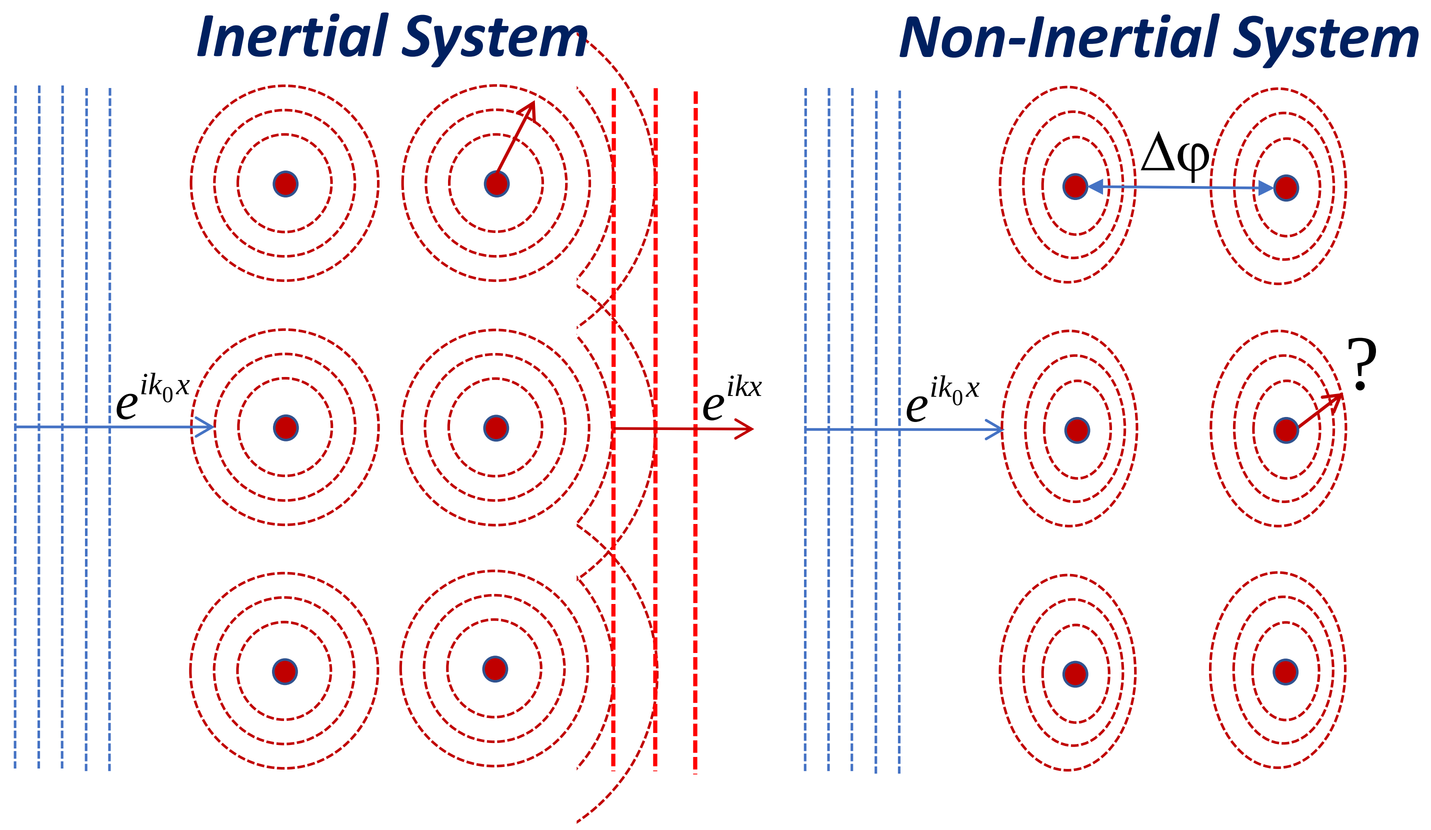
Neutron Dispersion Law in Matter Moving with Acceleration

Neutron dispersion law in matter moving with acceleration



What happens in non-inertia systems?

Neutron dispersion law in matter moving with acceleration



Phase distortion

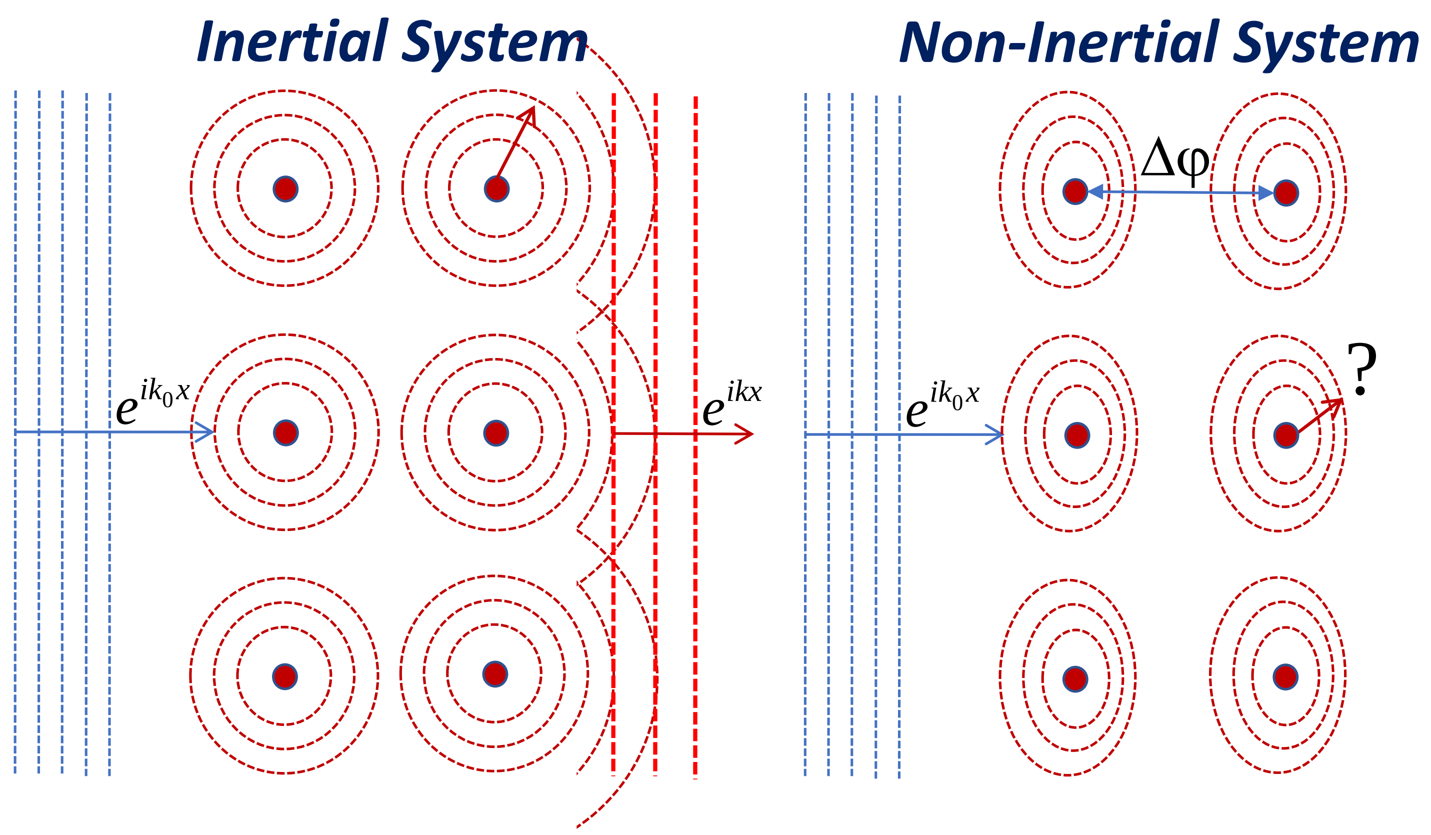
Additional phase shift due to non-inertia of the system

$$\varphi = kx\left(1 - \frac{max}{2E}\right) \qquad \Delta\varphi = k\frac{max^2}{2E}$$

The hypothesis is that the usual dispersion law holds if the phase distortion due to acceleration at the interatomic distance is much smaller than the phase shift ***kb*** caused by scattering at the nuclei

A.I.Frank. JETP Letters., 100, 613 (2014)

Neutron dispersion law in matter moving with acceleration



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Phase distortion

Additional phase shift due to non-inertia of the system

$$\varphi = kx\left(1 - \frac{max}{2E}\right) \qquad \Delta\varphi = k\frac{max^2}{2E}$$

Validity condition for dispersion theory

$$\Delta\varphi << kb \qquad a << \frac{4Eb}{md^2}$$

Conditions for the visibility of the effect

$x \rightarrow$ interatomic distance d

For UCN

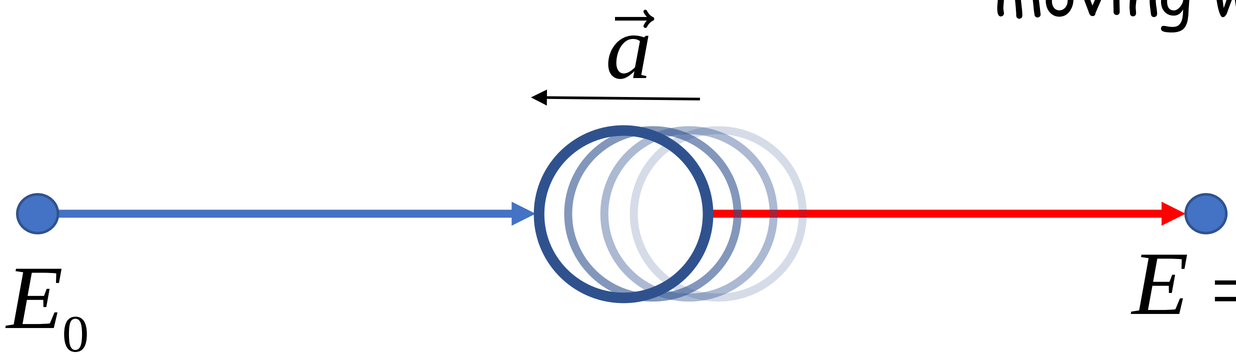
E ≈ 100 neV, b ≈ 5 10^{-15} m, a ≈ 8 10^{5} m/s^2

A.I.Frank. JETP Letters., 100, 613 (2014)

Neutron dispersion law in matter moving with acceleration

Acceleration effect

Acceleration effect is a general non-stationary phenomenon, when a particle interacts with object, moving with acceleration, and…



$$E = E_0 + \Delta E$$

$$\Delta E = \hbar k a \tau$$

…and changes it's energy after interaction.

k – wave numeric,
a – object's acceleration,
τ – time of interaction

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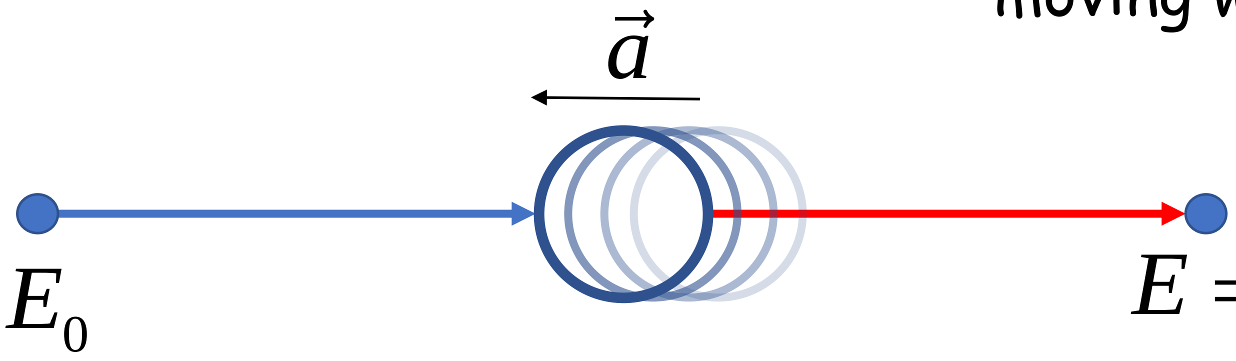

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Talk by Alexander Frank

Investigation the acceleration effect

Interaction with refractive sample moving with acceleration

Light

$$\Delta\omega \cong \frac{\omega a d}{c^2}(n-1)$$

Tanaka formulae

$$\tau = \frac{nd}{c} - \frac{d}{c} = \frac{d}{c}(n-1) \quad \Delta\omega = \omega\frac{\Delta v}{c} = \omega\frac{a\tau}{c}$$

Tanaka K., Phys. Rev. A, **25,** 385 (1982)

Neutrons

$$\Delta E \cong mad\left(\frac{1}{n} - 1\right)$$

Kowalski-Nosov-Frank formulae

$$\tau = \frac{d}{v}\left(\frac{1-n}{n}\right) \quad \Delta v = a\tau \quad \Delta E = mv\cdot\Delta v$$

Kowalski F. V., Phys. Lett. A, **182,** 335 (1993)

Nosov V. G., Frank A.I., Phys. of Atomic Nuclei, **61,** 613 (1998)

Frank A.I. et al., Nuclear Physics, **71,** 1686 (2008)

Frank A.I. et al., JETP Letters, **93,** 403 (20011)

General expression

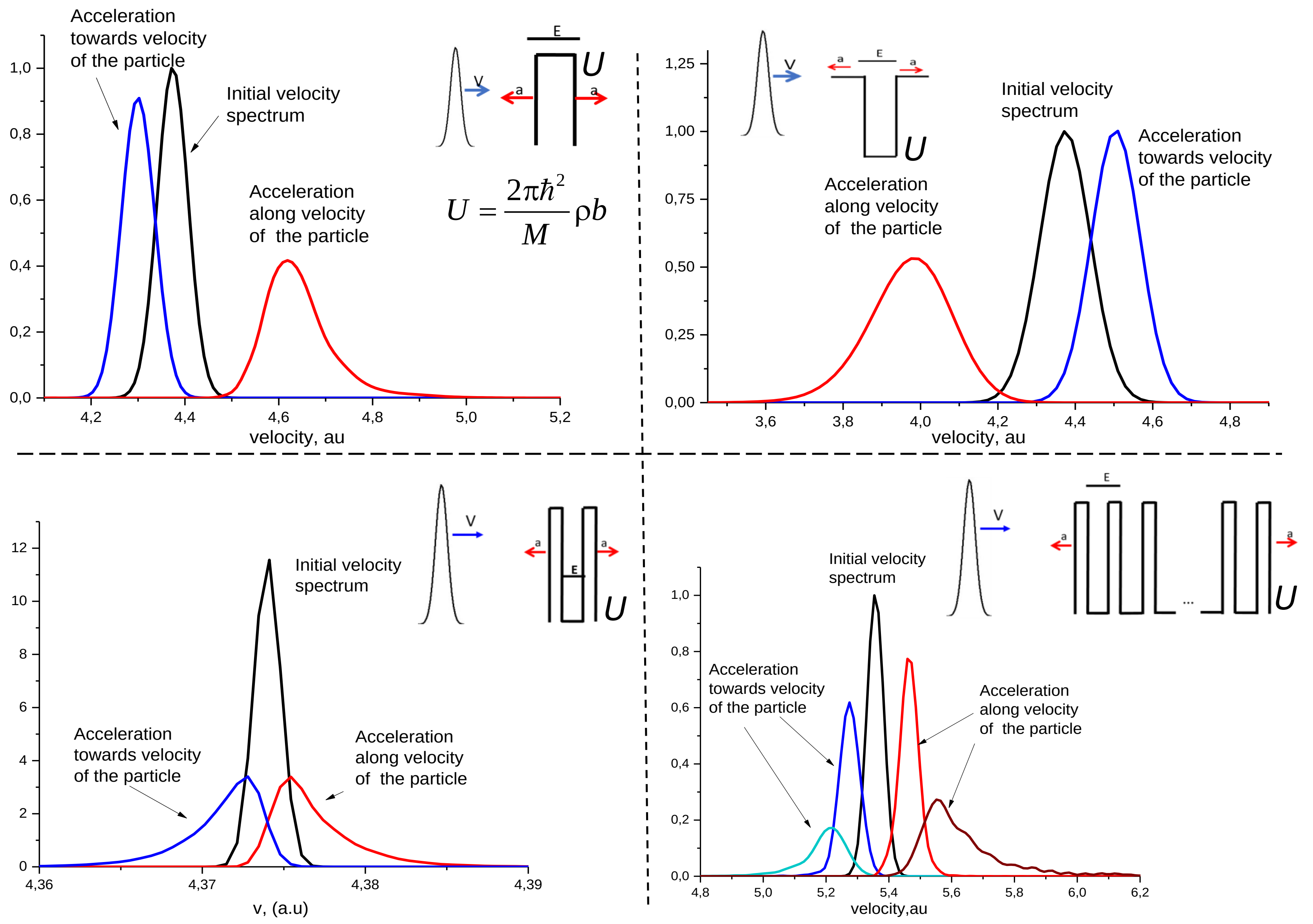
$$\Delta\omega = ka\tau$$

It doesn't matter the nature of the time τ

Frank A.I. *et al*. *Phys. Atom. Nuclei* **71,** 1656 (2008).
Frank A.I., Phys. Usp., **63**, 500 (2020).

Numerical Theoretical Study of UCN Interaction with Accelerated Quantum Objects

The Acceleration effect should take place in quantum mechanics.



Change in:

$$\Delta\omega = ka\tau$$ ***frequency***

$$\Delta E = \hbar ka\tau$$ ***energy***

$$\Delta v = a\tau$$ ***velocity***

$$\tau = \hbar \frac{\partial\varphi}{\partial E}$$

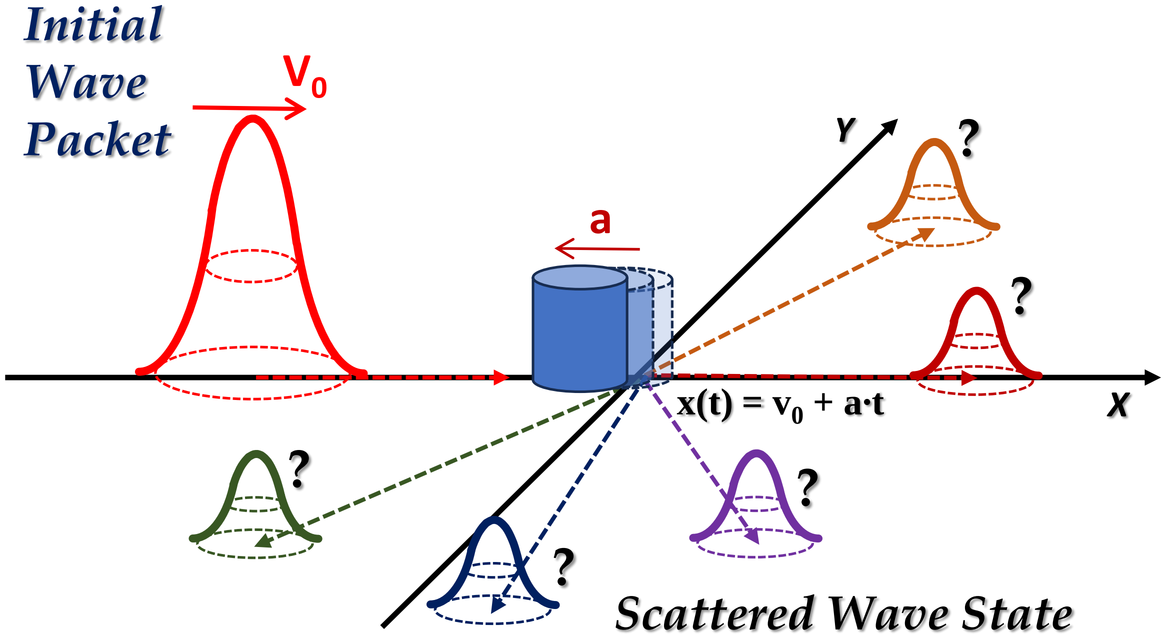
Group delay time
(Bohm, Wigner, 1952-55)

k – wave numeric,
a – object acceleration,
φ – phase of the interaction amplitude,
E – particle energy

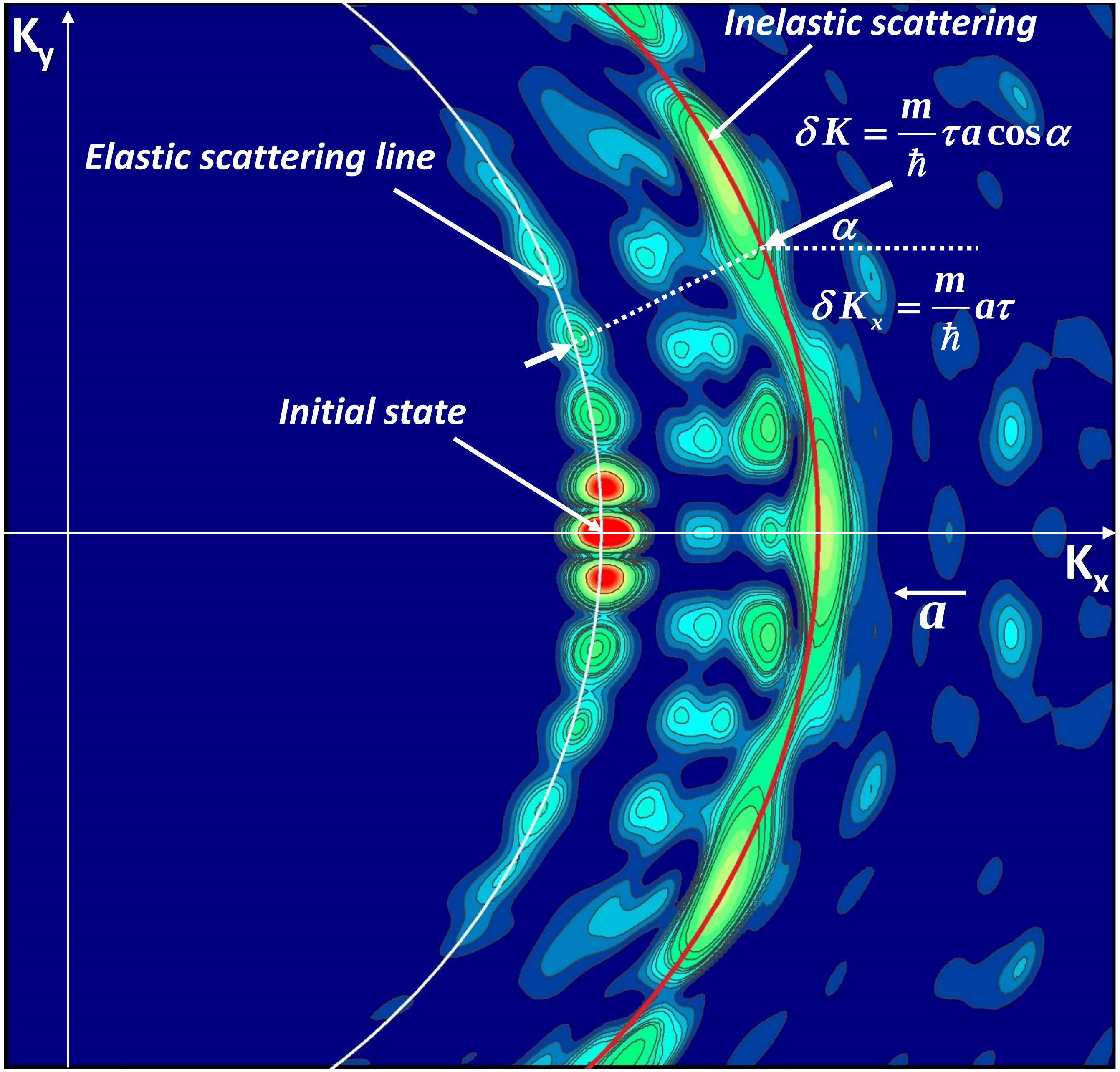
M.A. Zakharov et al. Eur. Phys. *J. D* 75, 47 (2021).

Numerical Theoretical Study of UCN Interaction with

2D quantum well moving with acceleration



Final State Spectrum



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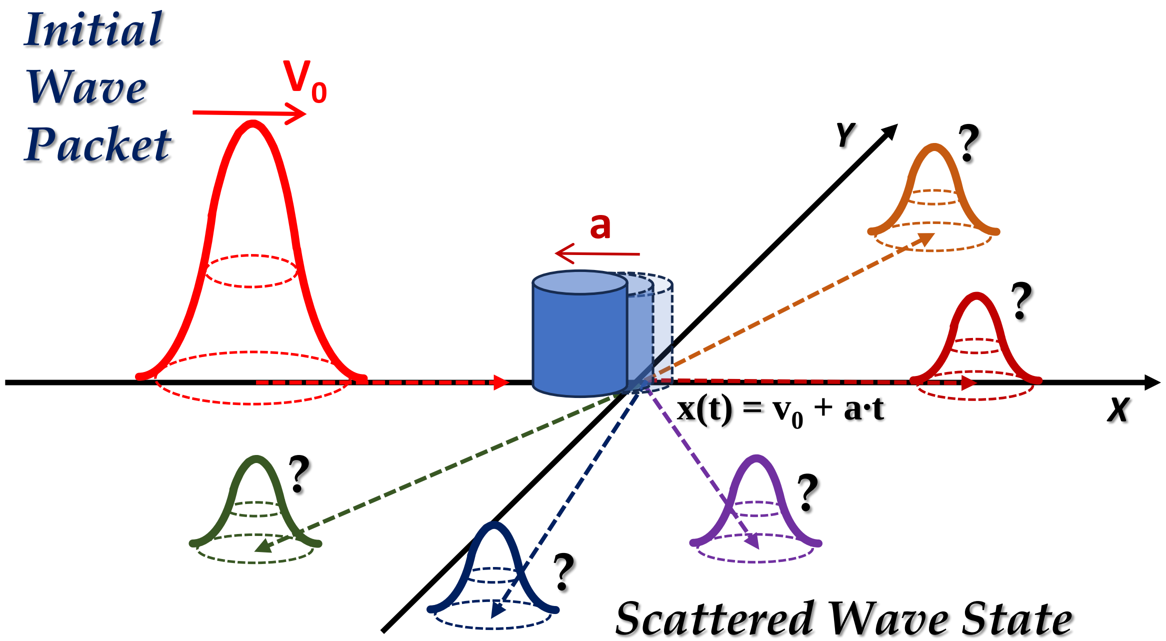
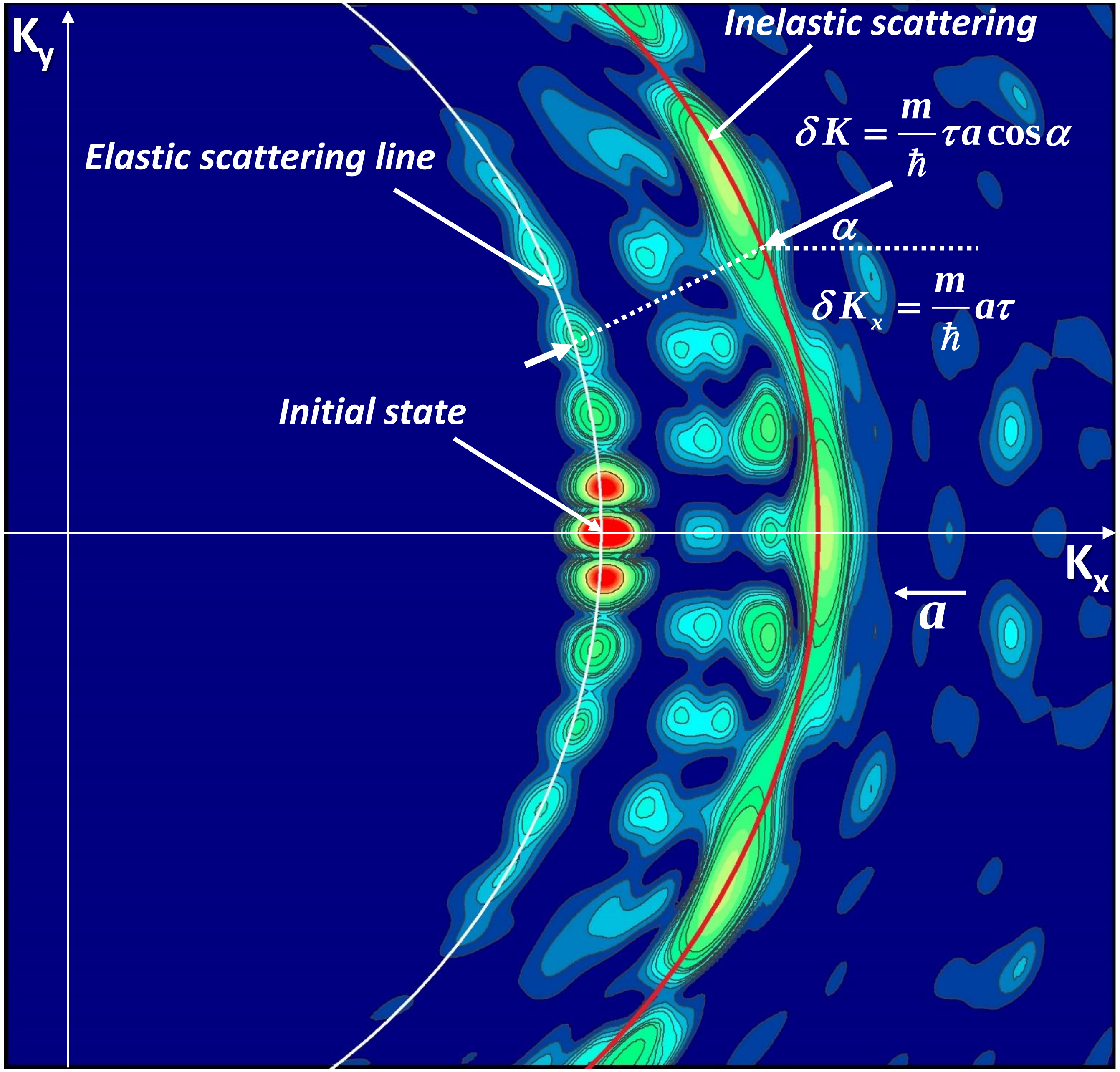
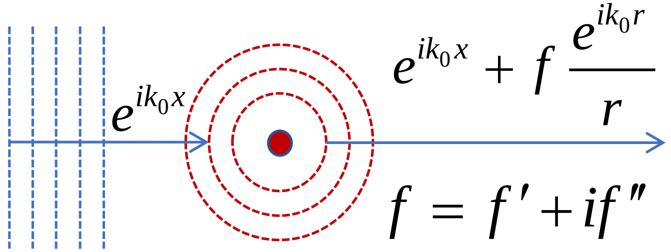


What about a nuclei?

Final State Spectrum



Group delay time at neutrons scattering by atomic nucleus



Estimates of scattering time

$$\tau = \hbar \frac{\partial \varphi}{\partial E}$$

Group delay time
(Bohm, Wigner, 1952-55)

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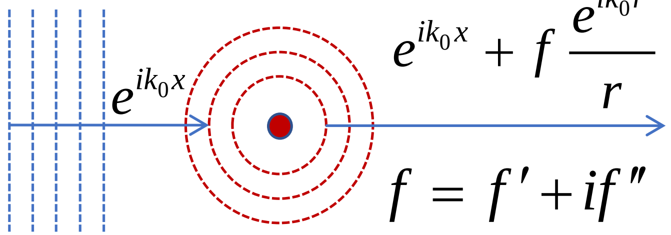

$$\varphi = \frac{f''}{f'} = \frac{k\sigma_t\sqrt{4\pi}}{4\pi\sqrt{\sigma_s}} = \frac{k(\sigma_s+\sigma_a)}{\sqrt{4\pi\sigma_s}}$$

$$\sigma_t = \sigma_s + \sigma_a = \frac{4\pi}{k} f''$$

$$\sigma_s = 4\pi(f')^2; \quad \sigma_a \sim \frac{1}{k}$$

σ_t - total cross-section

σ_s - scattering cross-section

σ_a - capture cross-section

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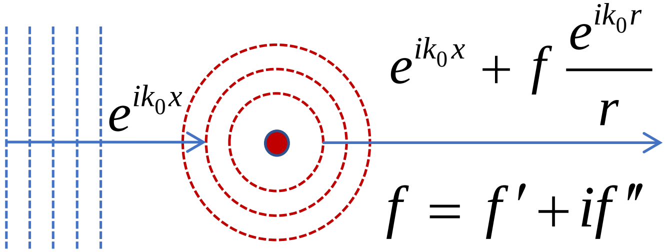

$$f = f' + if''$$

$$\varphi = \frac{f''}{f'} = \frac{k\sigma_t\sqrt{4\pi}}{4\pi\sqrt{\sigma_s}} = \frac{k(\sigma_s+\sigma_a)}{\sqrt{4\pi\sigma_s}}$$

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Estimates of scattering time

$$\tau = \hbar\frac{\partial\varphi}{\partial E}$$

Group delay time
(Bohm, Wigner, 1952-55)

$$\tau = \hbar\frac{d\varphi}{dE} = \frac{1}{v}\sqrt{\frac{\sigma_s}{4\pi}}$$

$$\tau = \frac{|b|}{v}$$

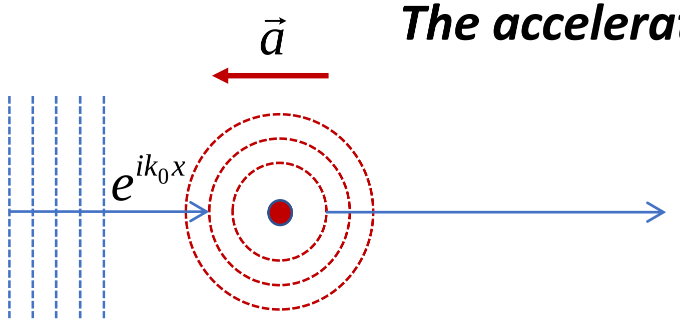
b – the length of neutron scattering on the nuclei

For thermal neutrons $\tau \approx 10^{-18}\,s$

For UCN $\tau \approx 10^{-15}\,s$

Group delay time at neutrons scattering by atomic nucleus

The acceleration effect during the interaction between UCN and an atomic nucleus.



$$\Delta E = \hbar k a \tau$$

k – wave numeric,
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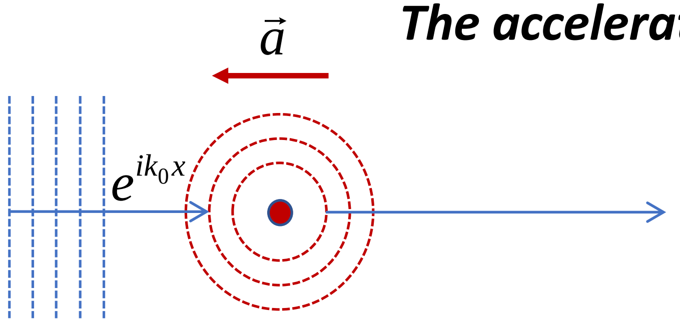
Quasi-classical consideration

$$\tau = \frac{|b|}{v} \approx 10^{-15} s$$

Group delay time at neutrons scattering by atomic nucleus



The acceleration effect during the interaction between UCN and an atomic nucleus.

$$\Delta E = \hbar k a \tau$$

Quasi-classical consideration

k – wave numeric,
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$$\tau = \frac{|b|}{v} \approx 10^{-15} s$$

Uncertainty relation

$$\delta E \cdot \delta t > \hbar$$

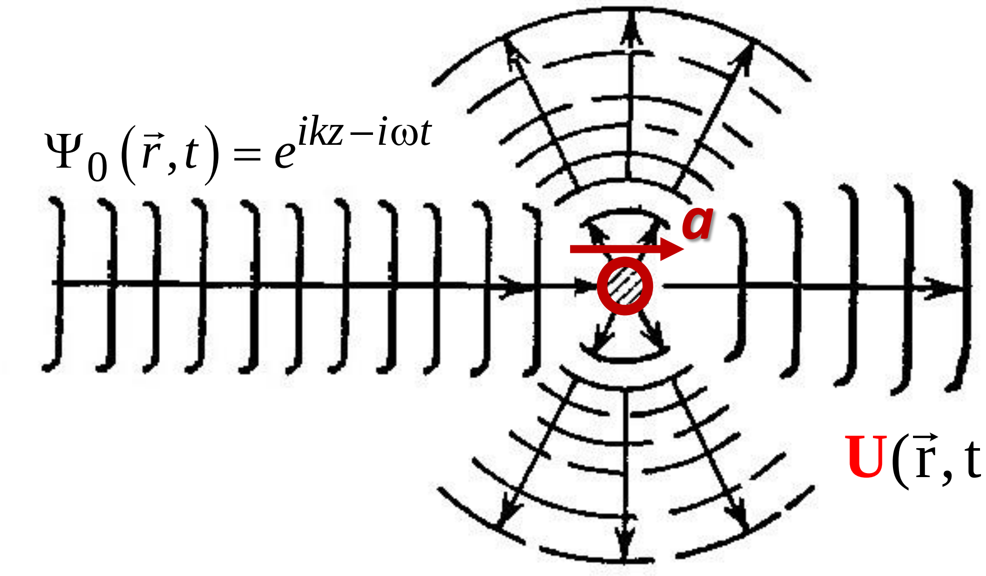
$$\delta E > \hbar / \delta t \sim 0.66 \ eV >> \Delta E$$

Quantum effects are large!

A rigorous quantum description is necessary

Interaction of the Neutron Wave With Nuclei Moving with Acceleration

Neutron wave scattering on an accelerating atomic nucleus



The problem of interaction of a neutron wave with a potential in the form of a delta function moving with acceleration

$$\mathbf{U}(\vec{r},t) = b\frac{2\pi\hbar^2}{m}\delta\left(\vec{r} - \vec{r}_0 - \vec{\mathbf{a}}t^2/2\right)$$

Time-dependent Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = -\frac{\hbar^2}{2m}\Delta\Psi(\vec{r},t) + \mathbf{U}(\vec{r},t)\Psi(\vec{r},t)$$

Accelerated Coordinate System

$$\tilde{x} = x;\ \tilde{y} = y;\ \tilde{z} = z + \mathbf{a}t^2/2$$

x, y, z – *Lab. Coord. System*

$\tilde{x}, \tilde{y}, \tilde{z}$ – *Accel. Coord. System*

Modified Time-dependent Schrödinger equation

$$i\hbar\left(\frac{\partial}{\partial\tilde{t}} + \mathbf{a}\tilde{t}\frac{\partial}{\partial\tilde{z}}\right)\Psi(\tilde{\vec{r}},t) = -\frac{\hbar^2}{2m}\Delta\Psi(\tilde{\vec{r}},t) + \mathbf{U}(\tilde{\vec{r}})\Psi(\tilde{\vec{r}},t)$$

Neutron wave scattering on an accelerating atomic nucleus

Scattered Wave — Nuclear potential — Initial Wave (In Borhn approx.) — Green's function

Lippmann-Schwinger Integral Equation

$$\boldsymbol{\Psi}\left(\tilde{\vec{r}}\right)=\int_{-\infty}^{t}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mathbf{U}\left(\vec{r}\,'\right)\cdot\boldsymbol{\Psi}_{\mathbf{0}}\left(\vec{r}\,',t\,'\right)\cdot\mathbf{G}\left(\tilde{\vec{r}},t;\vec{r}\,',t\,'\right)d\vec{r}\,'dt\,'$$

Nuclear potential – Fermi pseudo potential

$$\mathbf{U}\left(x',y',z'\right)=b\frac{2\pi\hbar^2}{m}\delta\left(x'-x_0\right)\delta\left(y'-y_0\right)\delta\left(z'-z_0\right)$$

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Initial Wave in non-inertial reference frame

$$\Psi_0\left(\vec{r},t\right)=e^{ikz-i\omega t}$$

Transition operator to...

$$\Psi_0\left(\tilde{\vec{r}},\tilde{t}\right)=\hat{\Theta}_a\Psi_0\left(\vec{r},t\right)=e^{-\frac{i}{\hbar}m\mathbf{a}\tilde{t}\tilde{z}-\frac{i}{\hbar}\frac{m\mathbf{a}^2\tilde{t}^3}{6}}e^{ik\tilde{z}+ik\frac{\mathbf{a}\tilde{t}^2}{2}-i\omega\tilde{t}}$$

...a non-inertial system

Neutron wave scattering on an accelerating atomic nucleus

Lippmann-Schwinger Integral Equation

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Green's function

$$\mathbf{G}\left(\vec{r},t;\vec{r}\,',t'\right)=-\frac{1}{\sqrt{i}\left(2\pi\right)^{3/2}}\frac{m^{3/2}}{\hbar^{5/2}}\frac{\theta\left(t-t'\right)}{\left(t-t'\right)^{3/2}}e^{i\frac{m}{2\hbar}\frac{\left(x-x'\right)^2+\left(y-y'\right)^2+\left(z-z'-\frac{\mathbf{a}}{2}\left(t^2-t'^2\right)\right)^2}{t-t'}}$$

The function of point-like momentary source in accelerating frame

Neutron wave scattering on an accelerating atomic nucleus

Lippmann-Schwinger Integral Equation

Scattered Wave — Nuclear potential — Initial Wave (In Borhn approx.) — Green's function

$$\Psi\left(\tilde{\vec{r}}\right) = \int_{-\infty}^{t}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathbf{U}\left(\vec{r}\,'\right)\cdot\Psi_0\left(\vec{r}\,',t'\right)\cdot\mathbf{G}\left(\tilde{\vec{r}},t;\vec{r}\,',t'\right)d\vec{r}\,'dt'$$

Applying some reasonable restrictions

1. let's consider the wave function that is far from the scatterer point,

2. …but not so far that the fictitious forces of the non-inertial system would significantly change the energy of the incident wave,

3. one will assume that the acceleration is small,

4. one will also consider such times that the neutron speed is always much greater than the speed of the potential in the laboratory system.

$$\begin{cases} z_0 \ll z \\ maz \ll \hbar\omega \\ a \to small \\ v_{neutron} \gg v_{nuclear} \end{cases}$$

Neutron wave scattering on an accelerating atomic nucleus

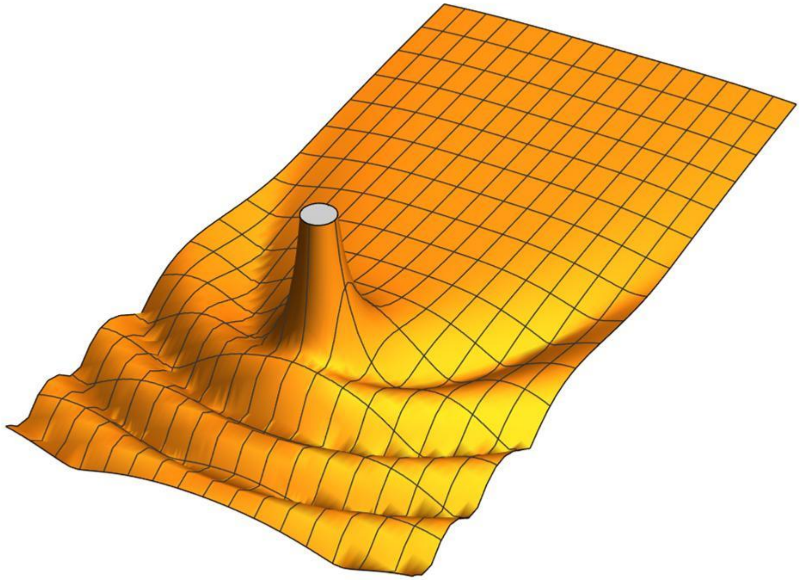
Convert to inertial laboratory coordinate system

$$\Psi(\vec{r},t) = -b e^{ik\left(z_0 + \frac{\mathbf{a}t^2}{2}\right) - i\omega t} \sqrt{\frac{1}{1 + \frac{5}{4}\frac{m\mathbf{a}\tilde{R}}{\hbar^2 k^2/2m}}} \frac{e^{ik\tilde{R} + i\frac{1}{4}\frac{m\mathbf{a}\tilde{R}}{\hbar^2 k^2/2m}k\tilde{R}}}{\tilde{R}}$$

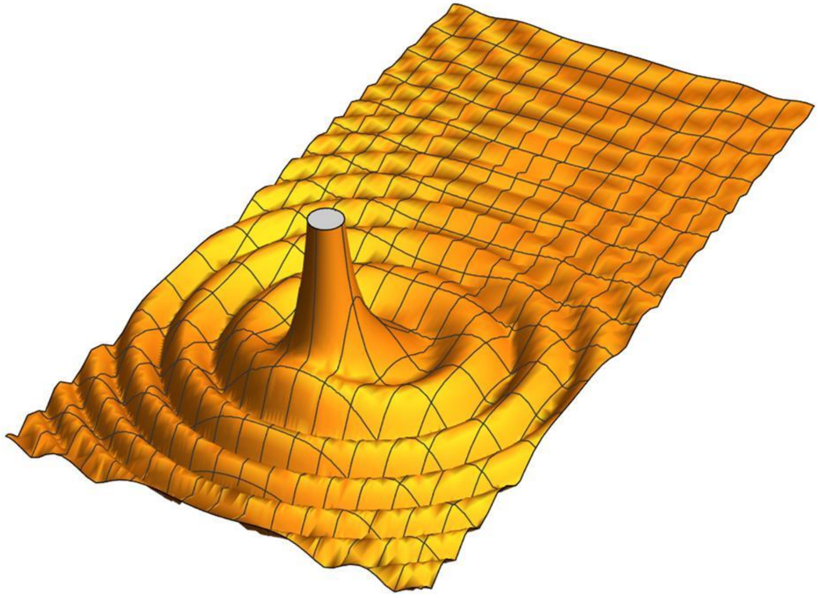
$$\tilde{R} = \sqrt{(x-x_0)^2 + (y-y_0)^2 + \left(z - z_0 - \mathbf{a}t^2/2\right)^2}$$

Interference of incident and scattered waves

without quantum effects



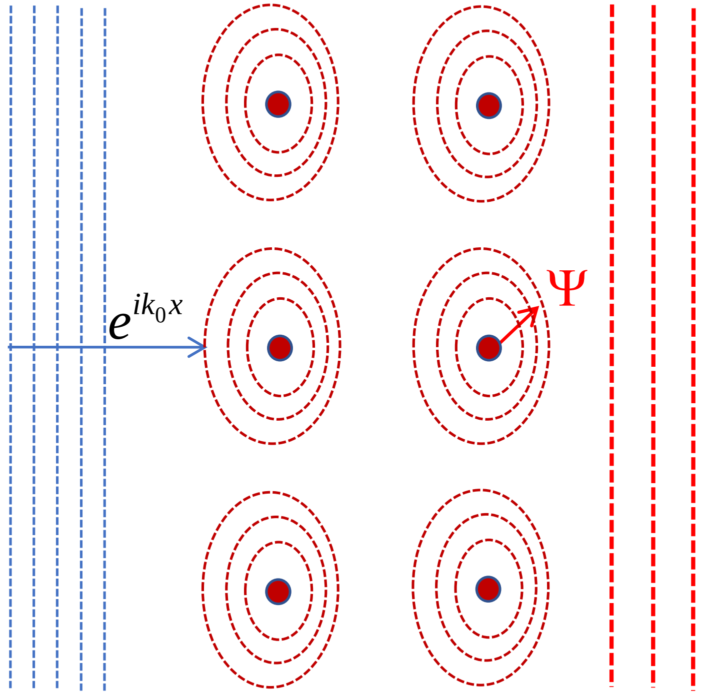
with quantum effects

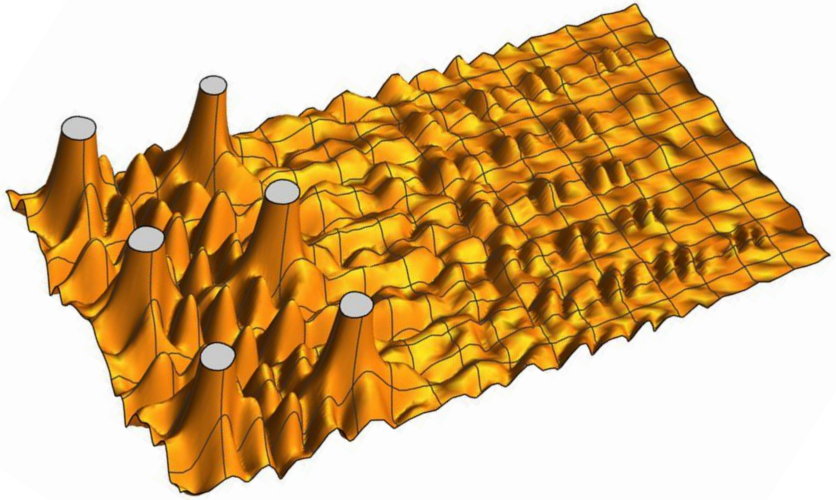


Perspectives

The way for studying the neutron wave in the layer of matter moving with acceleration IS OPEN

$$\sum \Psi_{scatter} = \int 2\pi\rho d\Psi(\vec{r},t) z dz$$





$$\Psi(\vec{r},t) = -be^{ik\left(z_0+\frac{\mathbf{a}t^2}{2}\right)-i\omega t}\sqrt{\frac{1}{1+\frac{5}{4}\frac{m\mathbf{a}\tilde{R}}{\hbar^2k^2/2m}}}\frac{e^{ik\tilde{R}+i\frac{1}{4}\frac{m\mathbf{a}\tilde{R}}{\hbar^2k^2/2m}k\tilde{R}}}{\tilde{R}}$$

$$\tilde{R}=\sqrt{(x-x_0)^2+(y-y_0)^2+\left(z-z_0-\mathbf{a}t^2/2\right)^2}$$

Thank you for your attention!

Many thanks to Alexander Frank
for fruitful discussions