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Sum Rules for Semi-leptonic $b \to c$ & $b \to u$ Decays

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2410.21384

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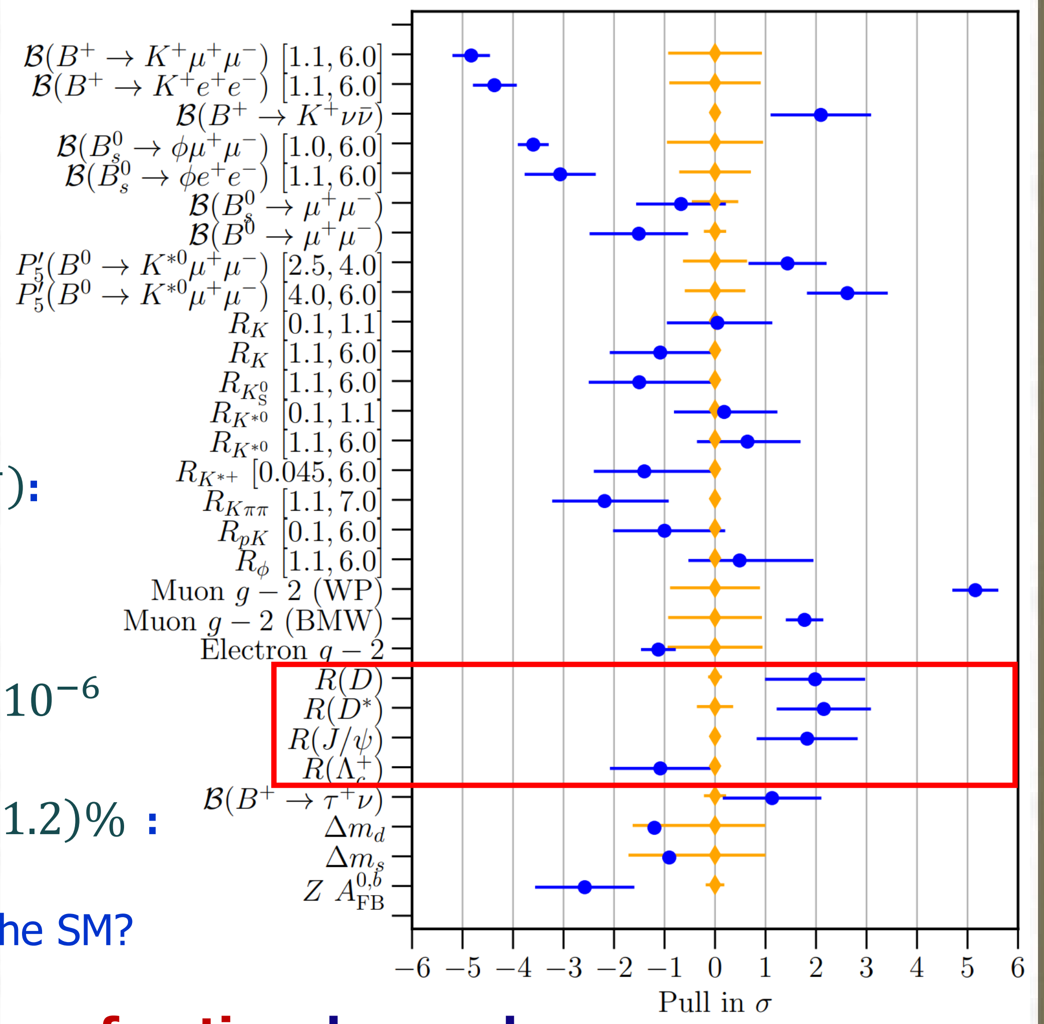
Outline

- Introduction
- History & current status of $R(D)$ & $R(D^*)$
- Global fits to $R(D^{(*)})$ within EFT framework
- Sum rules for $b \to cl\nu$ & $b \to ul\nu$ decays
- Summary

Flavor Anomalies

https://www.nikhef.nl/~pkoppenb/anomalies.html



- **Several interesting discrepancies between SM & experimental data observed in B physics:**

  - $R(D^{(*)}) = \frac{\mathcal{B}(B\to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B\to D^{(*)}l\nu_l)}$: Lepton flavor university violation?

  - $\mathcal{B}(B^+ \to K^{(*)+}\mu^+\mu^-), \mathcal{B}(B^+ \to K^+\nu\bar{\nu}), P_5'(B^0 \to K^{*0}\mu^+\mu^-)$:

    insufficient QCD estimates or NP in rare FCNC $b \to s$ decays?

  - $\mathcal{B}(B^0 \to \pi^0\pi^0) = (0.3-0.9)\times 10^{-6}$ vs $(1.55 \pm 0.16)\times 10^{-6}$

    $\Delta A_{CP}(B \to \pi K) = A_{CP}(\pi^0 K^-) - A_{CP}(\pi^+ K^-) = (11.0 \pm 1.2)\%$:

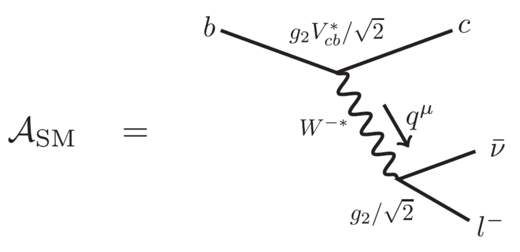
    pert. vs non-pert. QCD effects before discussing NP beyond the SM?

- **However, no clear evidence of NP from high-energy frontier observed**

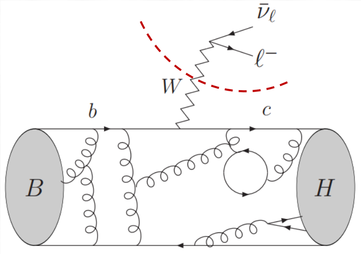
➡ **Even if NP existed, they must have specific flavor structures; new flavor- & CP-violating sources!**

Semi-leptonic b-hadron Decays

- **At quark-level, tree-level $b \to cl\nu$ transition mediated by $W^{\pm}$:**



quarks dressed into hadrons



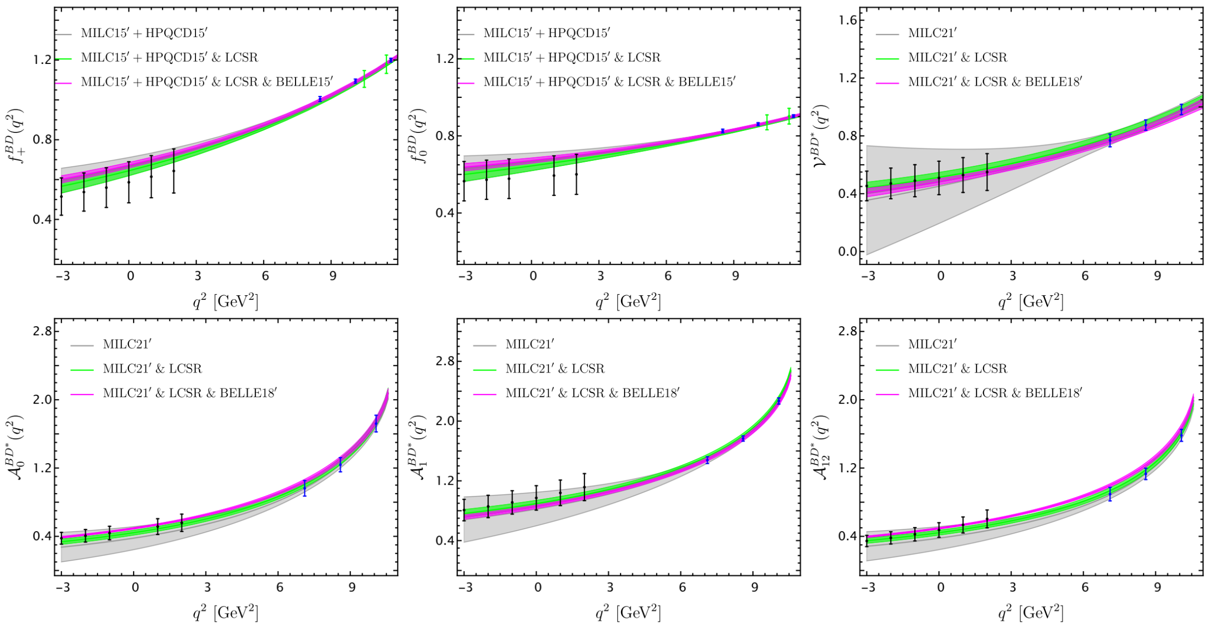
all QCD effects encoded by

***B* → *H* transition form factors**

$$\left\langle {}^{2s_c+1}\left(L^c\right)_{J_c} \middle| \bar{c}\Gamma b \middle| {}^{2s_b+1}\left(L^b\right)_{J_b} \right\rangle$$

$$\langle D(k)|\bar{c}\gamma^\mu b|\bar{B}(p)\rangle = \left((p+k)^\mu - \frac{m_B^2 - m_D^2}{q^2} q^\mu\right) f_+^{B\to D}(q^2) + \left(\frac{m_B^2 - m_D^2}{q^2} q^\mu\right) f_0^{B\to D}(q^2)$$

# of FFs	$B \to Pl\nu_l$	$B \to Vl\nu_l$
$l = e, \mu$	1	3
$l = \tau$	2	4



Bo-Yan Cui, Yong-Kang Huang, Yu-Ming Wang, Xue-Chen Zhao, 2301.12391

- **Form factors calculated by Lattice QCD, QCDSR, LCSR, LFQM, DS equations, …**

- **Mainly from Lattice QCD @ high q^2 (small recoil) & LCSR @ low $q^2 = (p_l + p_\nu)^2$ (large recoil)**

Determination of $|V_{cb}|$

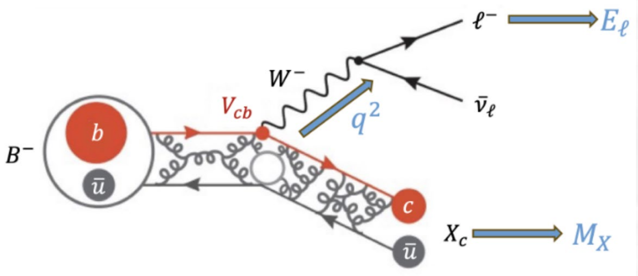
☐ Exclusive $B \to D^{(*)}\ell\nu$ decays: plagued by $B \to D^{(*)}$ form factors

$$\frac{d\Gamma(\overline{B} \to D\ell^-\overline{\nu}_\ell)}{dw} = \frac{G_F^2 m_D^3}{48\pi^3}(m_B + m_D)^2(w^2-1)^{3/2}\eta_{EW}^2\mathcal{G}^2(w)|V_{cb}|^2$$

$$\frac{d\Gamma(\overline{B} \to D^*\ell^-\overline{\nu}_\ell)}{dw} = \frac{G_F^2 m_{D^*}^3}{48\pi^3}(m_B - m_{D^*})^2\chi(w)\eta_{EW}^2\mathcal{F}^2(w)|V_{cb}|^2$$

$\Rightarrow$ $|V_{cb}|_{\text{excl}} = (40.25 \pm 0.71) \cdot 10^{-3}$

M. Bordone, A. Juttner, 2406.10074



☐ Inclusive $B \to X_c\ell\nu$ decays: heavy-quark expansion

$$\frac{d^3\Gamma}{dE_\ell dq^2 dm_X^2} = |V_{cb}|^2 G_F^2 \frac{m_b^5}{16\pi^3}\frac{d^3 f}{dE_\ell dq^2 dm_X^2}(m_b, m_c, \mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, \ldots)$$

Heavy Quark Expansion (HQE)

$$f(m_b, m_c, \ldots) = f^{\text{LP}} + f^{\text{NLP},\pi}\frac{\mu_\pi^2}{m_b^2} + f^{\text{NLP},G}\frac{\mu_G^2}{m_b^2} + f^{\text{NNLP},D}\frac{\rho_D^3}{m_b^3} + f^{\text{NNLP},LS}\frac{\rho_{LS}^3}{m_b^3} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^4}{m_b^4}\right)$$

$\Rightarrow$ $|V_{cb}|_{\text{incl}} = (41.97 \pm 0.48) \cdot 10^{-3}$

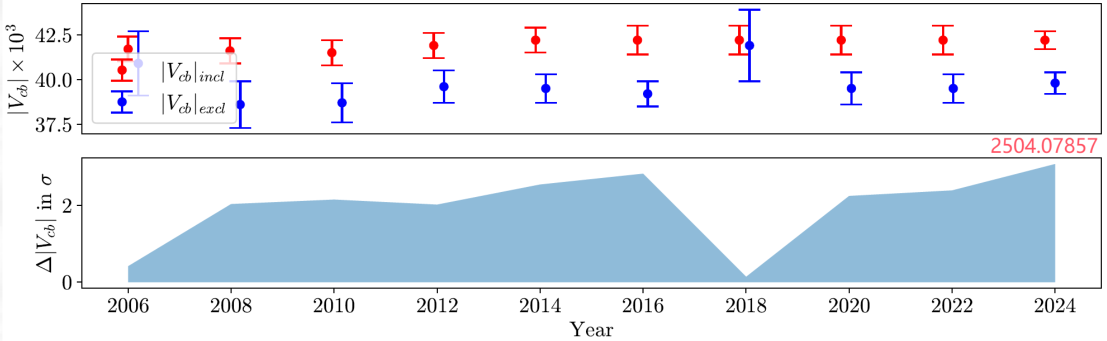
G. Finauri, P. Gambino, 2310.20324

☐ PDG 2024 review:

$|V_{cb}|_{\text{excl}} = (42.2 \pm 0.5) \cdot 10^{-3}$

$|V_{cb}|_{\text{incl}} = (39.8 \pm 0.6) \cdot 10^{-3}$

$2 - 3\sigma$



Lepton Flavor University (LFU)

- **Within the SM, three families of leptons couple with $W^{\pm}, Z^0$ & γ with equal strengths**

$$\mathcal{L}_{\rm cc}^{\ell} = g_W \sum_{i=1,2,3} \bar{\nu}^i{}_L \gamma_\mu \left(V_{\rm PMNS}^{ie} e_L + V_{\rm PMNS}^{i\mu} \mu_L + V_{\rm PMNS}^{i\tau} \tau_L \right) W^{+\mu} + \rm h.c. \quad \Rightarrow \quad \left|\mathcal{M}_j\right|^2 \propto \sum_{i=1,2,3} \left|V_{\rm PMNS}^{ij}\right|^2 = 1\ \forall j$$

the W-boson couples with different strengths to three lepton families. However, as the neutrino flavor is not observed

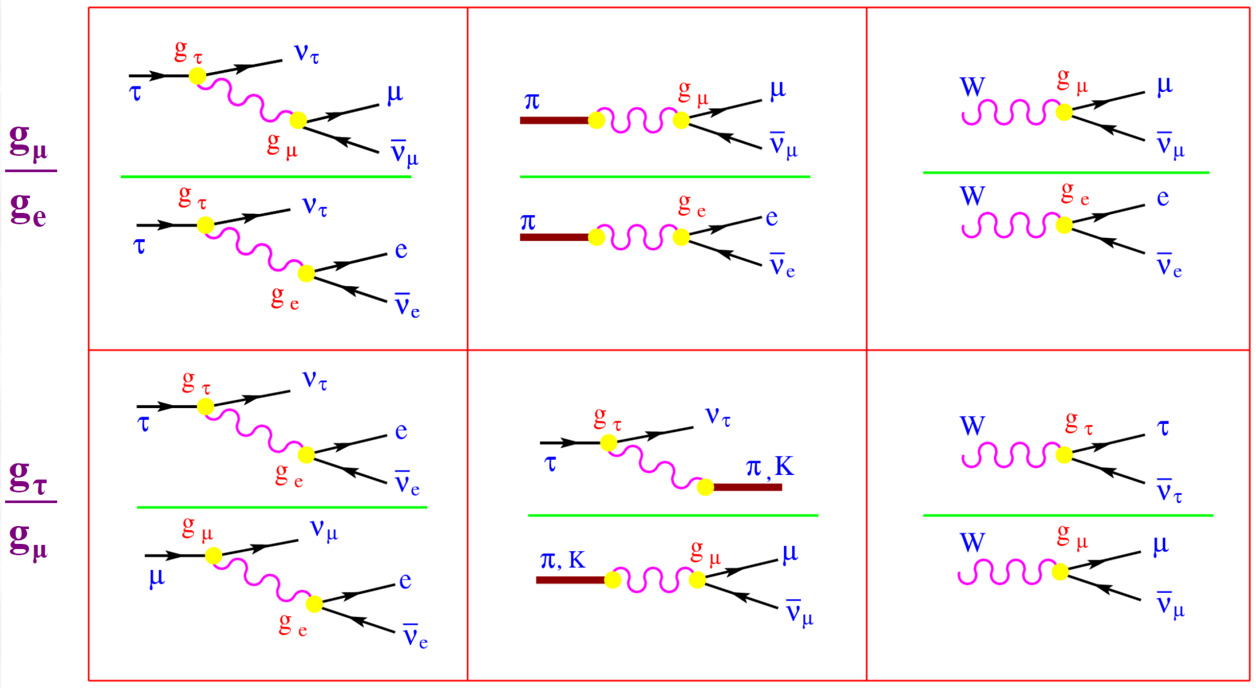
$$\mathcal{L}_{\rm nc}^{\ell} = \left(\bar{e}\gamma_\mu e + \bar{\mu}\gamma_\mu \mu + \bar{\tau}\gamma_\mu \tau\right)\left(g_\gamma A^\mu + g_Z Z^\mu\right)$$

both the γ- and the Z-boson couple with the same strengths to the three lepton families.

- **LFU now well established in W, Z, τ & π, K, D (semi-)leptonic decays:**

	$\Gamma_{\tau\to\mu}/\Gamma_{\tau\to e}$	$\Gamma_{\pi\to\mu}/\Gamma_{\pi\to e}$	$\Gamma_{K\to\mu}/\Gamma_{K\to e}$	$\Gamma_{K\to\pi\mu}/\Gamma_{K\to\pi e}$	$\Gamma_{W\to\mu}/\Gamma_{W\to e}$
$\lvert g_\mu/g_e\rvert$	1.0018 (14)	1.0021 (16)	0.9978 (20)	1.0010 (25)	0.996 (10)
	$\Gamma_{\tau\to e}/\Gamma_{\mu\to e}$	$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$	$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$	$\Gamma_{W\to\tau}/\Gamma_{W\to\mu}$	
$\lvert g_\tau/g_\mu\rvert$	1.0011 (15)	0.9962 (27)	0.9858 (70)	1.034 (13)	
	$\Gamma_{\tau\to\mu}/\Gamma_{\mu\to e}$	$\Gamma_{W\to\tau}/\Gamma_{W\to e}$			
$\lvert g_\tau/g_e\rvert$	1.0030 (15)	1.031 (13)	LEP		A. Pich, 1310.7922

0.992 ± 0.013 from ATLAS, 2007.14040

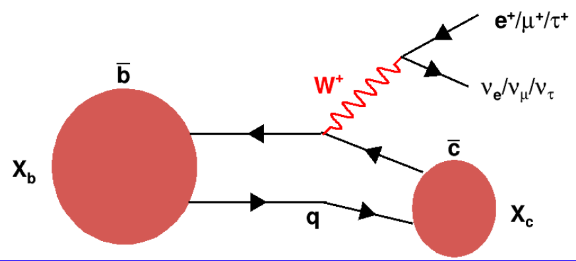


LFU in Semi-leptonic B-hadron Decays

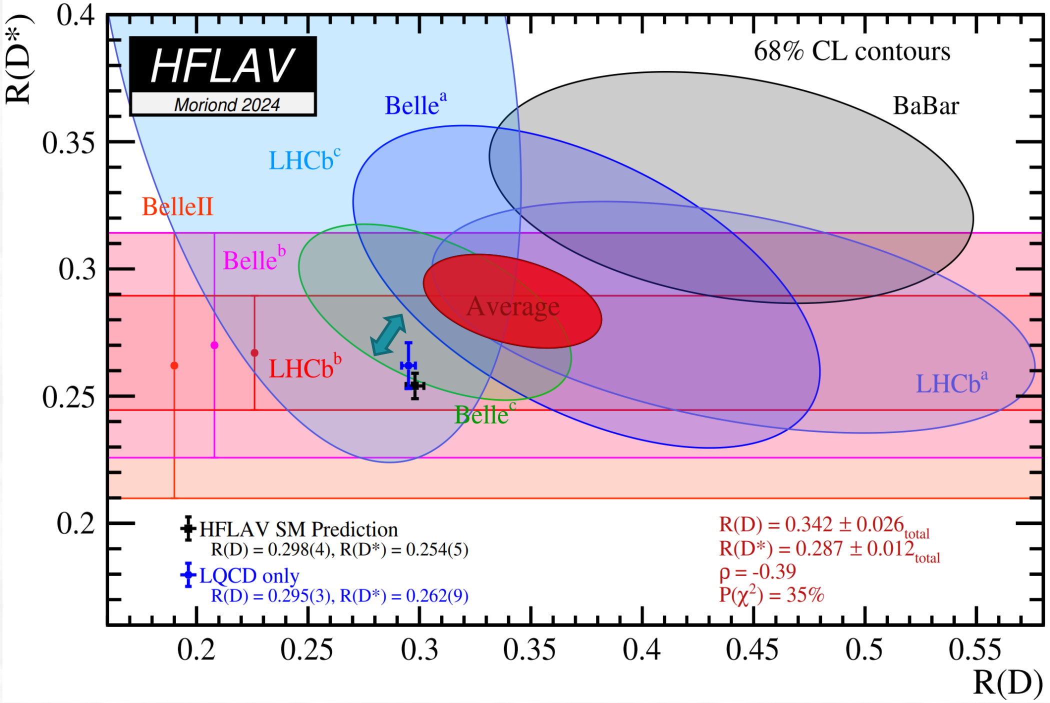
- **LFU can be further probed in tree-level $b \to c l \nu$ decays:**

$$R(X_c) = \frac{\mathcal{B}(X_b \to X_c \tau^+ \nu_\tau)}{\mathcal{B}(X_b \to X_c \ell^+ \nu_\ell)}$$

$$X_b = B^0, B^+_{(c)}, B^0_s, \Lambda_b, \ldots \quad X_c = D, D^*, D_s, \Lambda_c, \ldots$$



- **Ratios between $b \to c\tau\nu_\tau / b \to c\ell\nu_\ell$ featured by**
  - Dependence on $|V_{cb}|$ completely cancelled
  - Hadronic uncertainties from FFs mostly cancelled
  - More sensitive to possible enhanced coupling to 3rd generation fermions predicted by some BSM models



- **However, $\sim 3.31\sigma$ deviation still observed**

➡ ***need to understand dynamics behind it!***

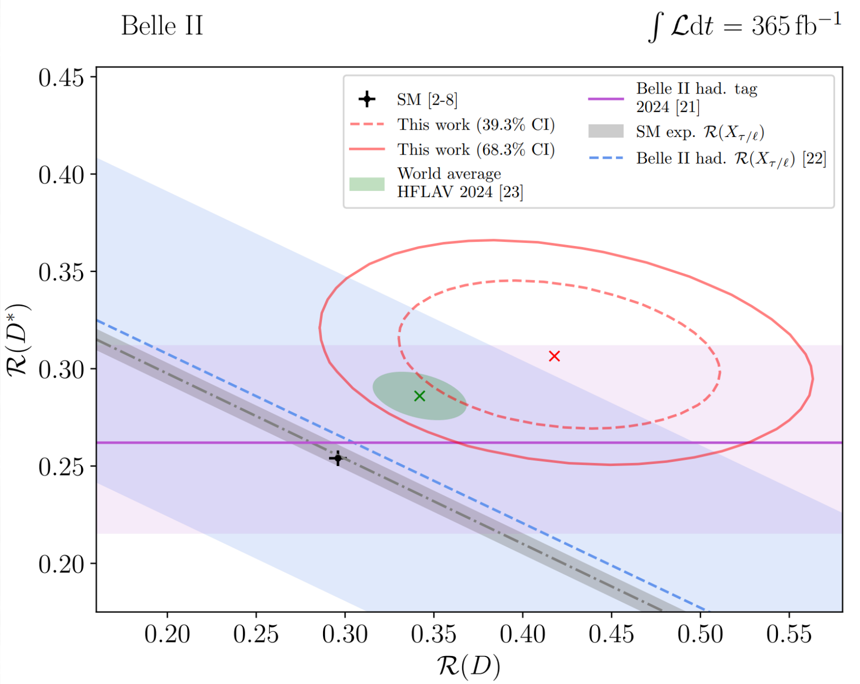
New Results from Belle II & LHCb 2025

□ New result from Belle II 2504.11220:

$$\mathcal{R}(D^+) = 0.418 \pm 0.074\,(\text{stat}) \;\pm 0.051(\text{syst})$$

$$\mathcal{R}(D^{*+}) = 0.306 \pm 0.034(\text{stat}) \pm 0.018(\text{syst})$$

- ➢ the first semi-leptonic-tagged results from Belle II
- ➢ compatible with the SM within 1.7σ, and agrees also with HFLAV 2024 average



□ Evidence for $B^- \to D^{**0}\tau^-\bar{\nu}_\tau$ from LHCb 2501.14943:

$$\mathcal{R}(D_{1,2}^{**0}) = 0.13 \pm 0.03\,(\text{stat}) \;\pm 0.01\,(\text{syst}) \;\pm 0.02\,(\text{ext})$$

- ➢ $D^{**0} \in (D_1'(2400)^0, D_1(2420)^0, D_2(2460)^0)$; one of largest syst. uncertainties from D^{**0} contamination
- ➢ agrees with SM prediction 0.09 ± 0.03 [Bernlochner, Ligeti, and Robinson, 1711.03110]
- ➢ This measurement is in line with the various $R(D^{**})$ assumptions used in experiments
- ➢ D^{**} feed-down is not the source of the present disagreement with SM for $R(D)$ & $R(D^*)$

$R(D)$ & $R(D^*)$ Anomalies

- $R(D)$ & $R(D^*)$ measured by BaBar, Belle (II), & LHCb, with leptonic or hadronic τ decays

Experiments

Preliminary reports are removed

2012: first BaBar measurement
($\tau\to l\nu\nu$, had. tag)

2015: first Belle ($\tau\to l\nu\nu$, had. tag)
first LHCb ($\tau\to\mu\nu\nu$) D* only
first HFLAV average

2016: new two Belle D* only
($\tau\to l\nu\nu$, semi-lept. tag) ($\tau\to\pi\nu$, had. tag)

2018: new LHCb ($\tau\to 3\pi\nu$) D* only

2019: Belle update 2016 with D&D*
($\tau\to l\nu\nu$, semi-lept. tag)

2023: LHCb ($\tau\to\mu\nu\nu$) update 2015 with D&D*
LHCb ($\tau\to 3\pi\nu$) update 2018, D* only

2024: first Belle II D* only

Theory

2008: first robust RD calc.
CLN with 2008 combined data

2012: first RD* calc.
CLN with 2010 Belle data

charged Higgs disfavored
inconsistent with BaBar

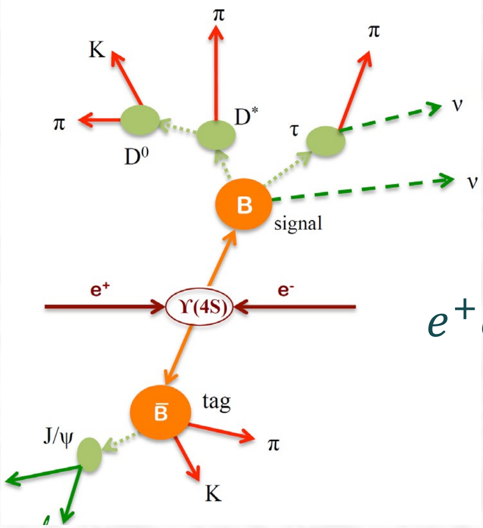
2013: leptoquark studies
possible solutions to "anomaly"

2016: first Lattice for B→D
BGL available for RD calc.

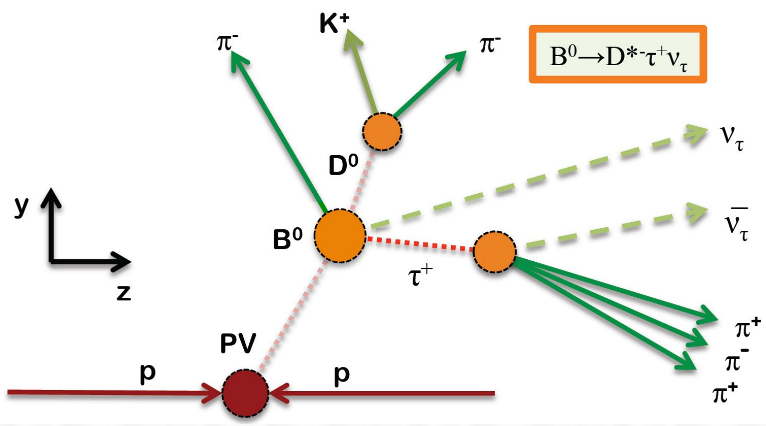
2017: first Lattice for D* at 0-recoil
BGL & general HQET studied

2018: first LCSR large recoil fit

2021: Lattice for D* at non 0-recoil
2021: FNAL-MILC
2023: JLQCD, HPQCD



e^+e^- collision



pp collision

BGL vs CLN parametrizations of $B \to D^{(*)}$ FFs

- **Mostly adopted parametrizations:** FFs expressed as a power series in conformal mapping parameter z

$$\langle D^*(\varepsilon,p')|\bar{c}\gamma^\mu b|\bar{B}(p)\rangle = ig\epsilon^{\mu\nu\alpha\beta}\varepsilon^*_\nu p_\alpha p'_\beta,$$
$$\langle D^*(\varepsilon,p')|\bar{c}\gamma^\mu\gamma^5 b|\bar{B}(p)\rangle = f\varepsilon^{*\mu} + (\varepsilon^*\cdot p)[a_+(p+p')^\mu + a_-(p-p')^\mu],$$

$$g(z) = \frac{1}{P_g(z)\phi_g(z)}\sum_{n=0}^{N} a_n z^n, \quad f(z) = \frac{1}{P_f(z)\phi_f(z)}\sum_{n=0}^{N} b_n z^n, \quad \mathcal{F}_1(z) = \frac{1}{P_{\mathcal{F}_1}(z)\phi_{\mathcal{F}_1}(z)}\sum_{n=0}^{N} c_n z^n,$$

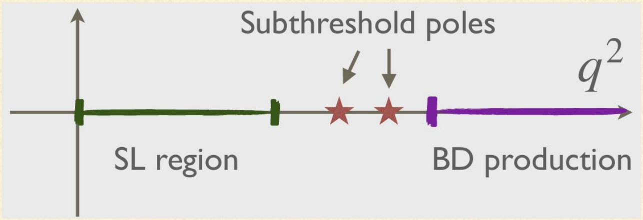
Combination of f and a_+

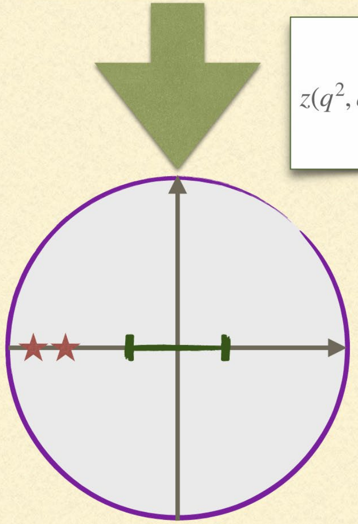
Conformal variable z:
$$z = \frac{\sqrt{w+1}-\sqrt{2}a}{\sqrt{w+1}+\sqrt{2}a}$$

QCD encoded in coefficients:
$$\{a_n, b_n, c_n\}$$

$c_0 =$ **constants** $\times\, b_0$

C.G. Boyd, B. Grinstein, R.F. Lebed, hep-ph/9412324





$$z(q^2,q_0^2) = \frac{\sqrt{q_+^2-q^2}-\sqrt{q_+^2-q_0^2}}{\sqrt{q_+^2-q^2}+\sqrt{q_+^2-q_0^2}}$$

conformal mapping

$$1 \geq \frac{1}{2\pi i}\oint \frac{dz}{z}\left|B(z)\Phi(z)f(z)\right|^2$$

$$f(z) = \frac{1}{\Phi(z)B(z)}\sum_{i=0}^{\infty} a_i z^i \qquad 1 \geq \sum_{i=0}^{\infty}|a_i|^2$$

$$\mathcal{G}(z)^2 = \frac{4r}{(1+r)^2} f_+(z)^2$$

$$f_+(z) = \frac{1}{P_+(z)\phi_+(z)}\sum_{n=0}^{N} a_{+,n} z^n$$

$$\phi_+(z) = 1.1213(1+z)^2(1-z)^{1/2}[(1+r)(1-z)+2\sqrt{r}(1+z)]^{-5}$$

- ✓ based on QCD dispersion relations
- ✓ the a_n coefficients must satisfy the unitarity bound: $\sum_{n=0}^{\infty}|a_n|^2 \leq 1$

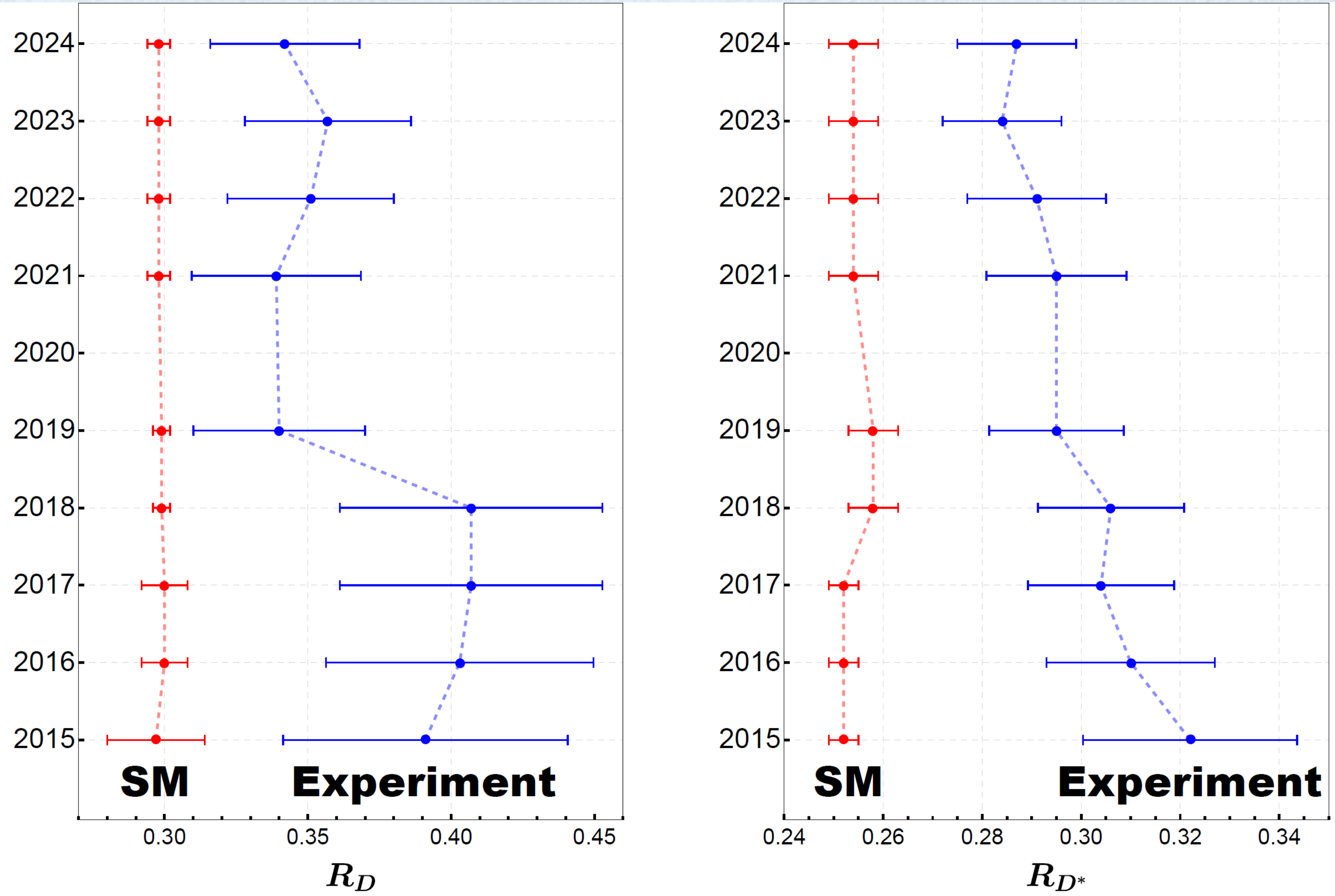
$$\mathcal{G}(z) = \mathcal{G}(1)\left[1-\rho^2 z + (51\rho^2-10)z^2 - (252\rho^2-84)z^3\right]$$

I. Caprini, L. Lellouch, M. Neubert, hep-ph/9712417

- ✓ # of free parameters reduced with dispersive constraints & symmetries
- ✓ this parameterization is, however, model-dependent

HFLAV Average

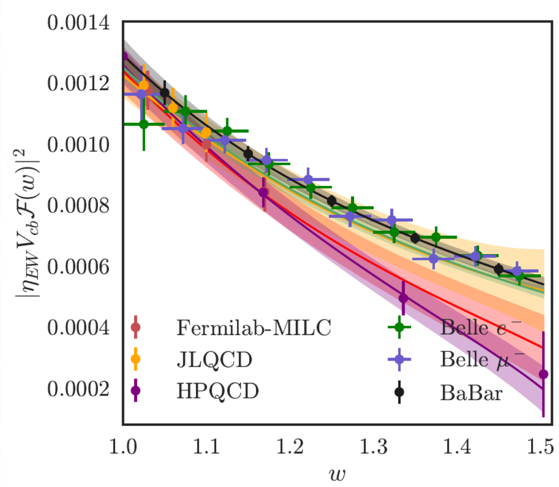
- Since 2015 first $R(D)$ & $R(D^*)$ world average, up to now

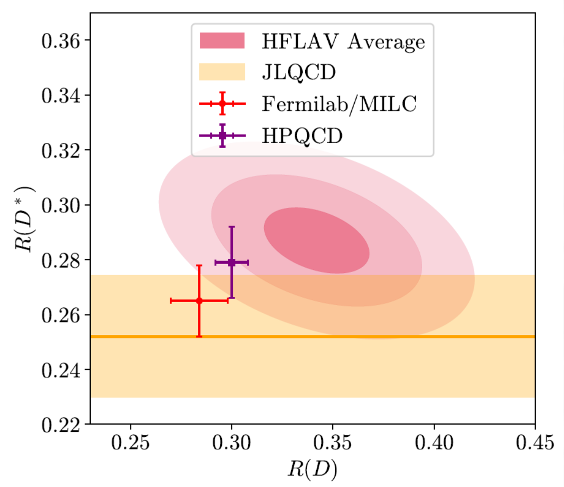


- For $R(D)_{\rm SM}$: SM prediction stable for a long time
- For $R(D)_{\rm exp}$: big changes since 2019 mainly due to Belle update; getting closer to, but still deviates from SM
- For $R(D^*)_{\rm SM}$: new lattice QCD results of FF available in recent years; their q^2 parameterization improved → rising central value
- For $R(D^*)_{\rm exp}$: every update lowers central value, but still indicates a large deviation from the SM value

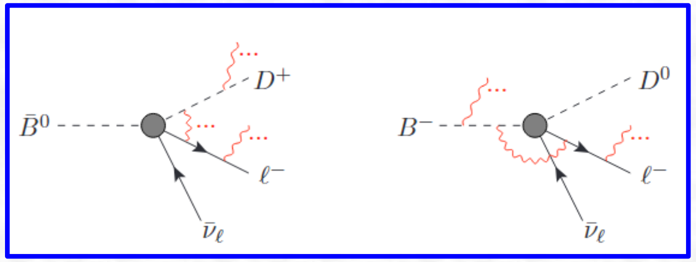
New Physics Interpretations

- For $R(D^{(*)})$ **anomalies**, further improvements of $B \to D^{(*)}$ **form factors** urgently needed



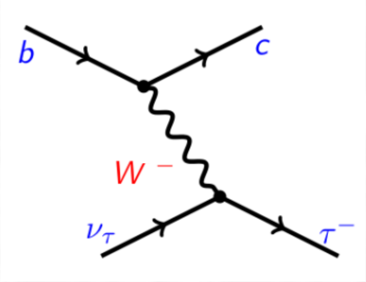


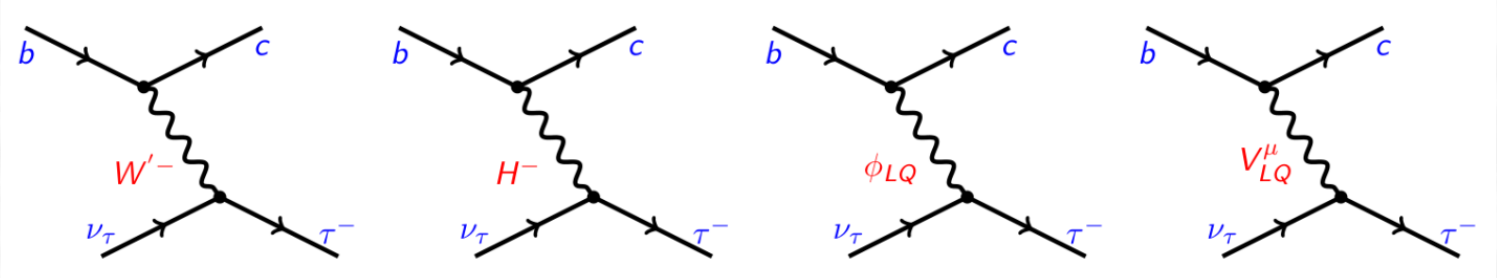
Exp. data + Lattice/LCSR + unitarity bounds + HQE to even higher orders to obtain more precise $B \to D^{(*)}$ form factors



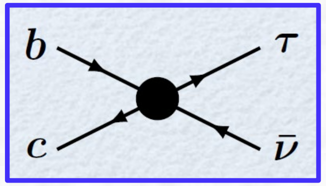
- **EM corrections** to $R(D^+)$ & $R(D^0)$ by 5% & 3%, for **soft-photon** with energy cut at 20-40MeV Boer, Kitahara, Nisandzic, 1803.05881

- $b \to c\tau\nu_\tau$: tree-level $W^\pm$-mediated; sensitive to new mediators coupled to 3rd generations

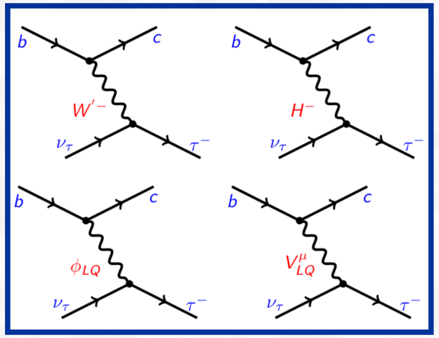




Model-independent Analysis in LEFT



- **Most general low-energy effective field theory (LEFT) for $b \to c\tau\nu_\tau$**



$m_R \gg m_b$

$$\mathcal{L}_{\rm SM}^{(6)} = -\frac{4G_F}{\sqrt{2}} V_{cb}\,\mathcal{O}_{V_L} + \text{H.c.}, \quad (V-A)\otimes(V-A)$$

$$\mathcal{L}_{\rm NP}^{(6)} = -\frac{4G_F}{\sqrt{2}} V_{cb}\left(C_{V_L}\mathcal{O}_{V_L} + C_{V_R}\mathcal{O}_{V_R} + C_{S_L}\mathcal{O}_{S_L} + C_{S_R}\mathcal{O}_{S_R} + C_T\mathcal{O}_T\right) + \text{H.c.}$$

$$\mathcal{O}_{V_{L(R)}} = (\bar{c}\gamma^\mu P_{L(R)} b)(\bar{\tau}\gamma_\mu P_L \nu_\tau),$$
$$\mathcal{O}_{S_{L(R)}} = (\bar{c} P_{L(R)} b)(\bar{\tau} P_L \nu_\tau),$$
$$\mathcal{O}_T = (\bar{c}\sigma^{\mu\nu} P_L b)(\bar{\tau}\sigma_{\mu\nu} P_L \nu_\tau).$$

FF inputs from

Lattice (2014+2016+2017) + LCSR + Belle (2017 + 2018)

FF parameterization

general HQET up to 1/mc^2 (See 2004.10208)

HFLAV average: RD = 0.342(26) RD* = 0.287(12) corr. = -0.39

Private average: FLD* = 0.49(5) Belle 2019 + LHCb 2023

- **Latest global fits with a single operator:**

✓ $\mathcal{O}^\tau_{V_L}$ gives the best Pull

✓ $\mathcal{O}^\tau_{S_R}$ gives the worst Pull

✓ $\mathcal{O}^\tau_{V_R}$ & $\mathcal{O}^\tau_{S_L}$ need very large WCs

$\to$ disfavored by collider bounds

✓ $\mathcal{O}^\tau_T$ also a viable solution

$(\bar{c}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu)$ ✔	$C^\tau_{V_L} \approx 0.08$	$\text{Pull} \equiv \sqrt{\chi^2_{\rm SM} - \chi^{2,\rm best}_{\rm NP}} \approx 4.8$ $\left(\sqrt{\chi^2_{\rm SM}} \approx 4.8\right)$
$(\bar{c}\gamma^\mu P_R b)(\bar{\ell}\gamma_\mu P_L \nu)$	$C^\tau_{V_R} \approx 0.01 \pm i0.41$	$\text{Pull} \approx 4.4$
$(\bar{c} P_L b)(\bar{\ell} P_L \nu)$	$C^\tau_{S_L} \approx -0.79 \pm i0.86$	$\text{Pull} \approx 4.3$
$(\bar{c} P_R b)(\bar{\ell} P_L \nu)$	$C^\tau_{S_R} \approx 0.18$	$\text{Pull} \approx 3.9$
$(\bar{c}\sigma^{\mu\nu} P_L b)(\bar{\ell}\sigma_{\mu\nu} P_L \nu)$ ✔	$C^\tau_T \approx 0.02 \pm i0.13$	$\text{Pull} \approx 4.3$

S. Iguro, T. Kitahara, R. Watanabe, 2405.06062

Model-independent Analysis in SMEFT

☐ model-indep. LEFT analysis can be extended to SMEFT:

Buchmuller, Wyler ‘86; Grzadkowski etal, 1008.4884

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{SM} + \frac{1}{\Lambda} C^{(5)} O^{(5)} + \frac{1}{\Lambda^2} \sum_i C_i^{(6)} O_i^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right)$$

- Includes SM fields only.
- Follows $SU(3)_C \times SU(2)_L \times U(1)_Y$.
- Electroweak (EW) symmetry linearly realized.

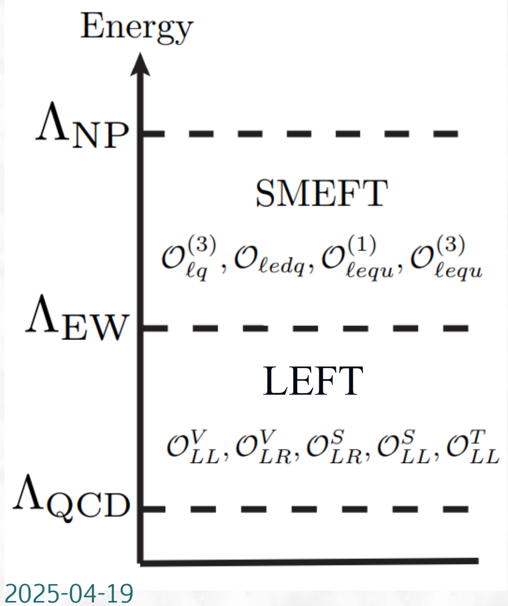
well-motivated framework by data

☐ Operators contributing to $b \to c\tau\nu$:

$$Q_{lq}^{(3)} = (\bar{l}\gamma_\mu \tau^I l)(\bar{q}\gamma^\mu \tau^I q), \qquad Q_{ledq} = (\bar{l}^j e)(\bar{d} q^j),$$
$$Q_{lequ}^{(1)} = (\bar{l}^j e)\varepsilon_{jk}(\bar{q}^k u), \qquad Q_{lequ}^{(3)} = (\bar{l}^j \sigma_{\mu\nu} e)\varepsilon_{jk}(\bar{q}^k \sigma^{\mu\nu} u),$$

☐ Basic procedure:

- ➢ evolution from $\Lambda_{\rm NP}$ to $\Lambda_{\rm EW}$ for SMEFT
- ➢ matching SMEFT onto LEET at $\Lambda_{\rm EW}$
- ➢ evolution from $\Lambda_{\rm EW}$ down to μ_b for LEFT



LEFT ↓ SMEFT ↓

$$C_{V_L} = -\frac{\sqrt{2}}{2G_F\Lambda^2} \sum_n \left[C_{lq}^{(3)}\right]_{332n} \frac{V_{nb}}{V_{cb}},$$
$$C_{S_R} = -\frac{\sqrt{2}}{4G_F\Lambda^2} \frac{1}{V_{cb}} \left[C_{ledq}\right]_{3332}^*,$$
$$C_{S_L} = -\frac{\sqrt{2}}{4G_F\Lambda^2} \sum_n \left[C_{lequ}^{(1)}\right]_{33n2}^* \frac{V_{nb}}{V_{cb}},$$
$$C_T = -\frac{\sqrt{2}}{4G_F\Lambda^2} \sum_n \left[C_{lequ}^{(3)}\right]_{33n2}^* \frac{V_{nb}}{V_{cb}}.$$

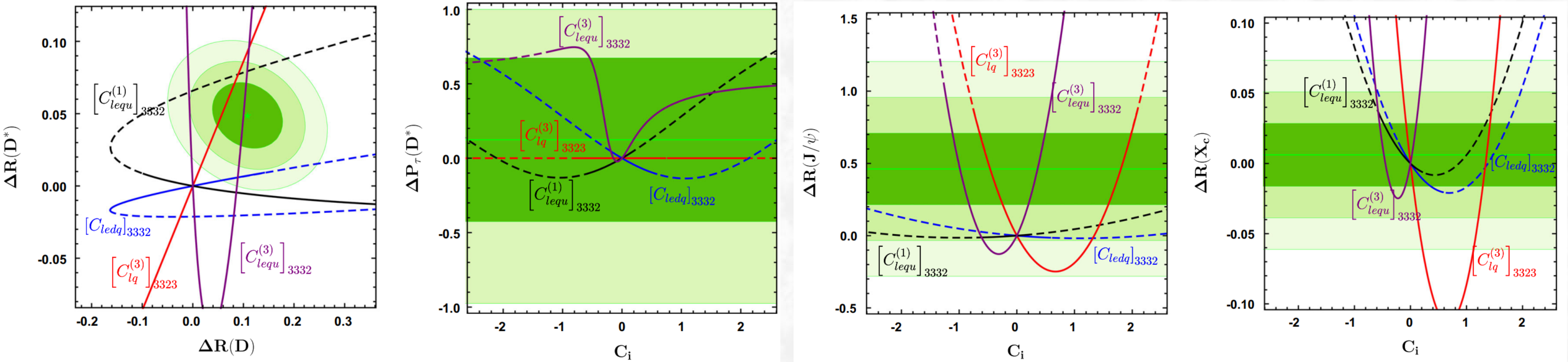
Evolution & matching →

$$C_{V_L}(\mu_b) = -1.503 \left[C_{lq}^{(3)}\right]_{3323}(\Lambda),$$
$$C_{V_R}(\mu_b) = 0,$$
$$C_{S_L}(\mu_b) = -1.257 \left[C_{lequ}^{(1)}\right]_{3332}(\Lambda) + 0.2076 \left[C_{lequ}^{(3)}\right]_{3332}(\Lambda),$$
$$C_{S_R}(\mu_b) = -1.254 \left[C_{ledq}\right]_{3332}(\Lambda),$$
$$C_T(\mu_b) = 0.002725 \left[C_{lequ}^{(1)}\right]_{3332}(\Lambda) - 0.6059 \left[C_{lequ}^{(3)}\right]_{3332}(\Lambda),$$

- ✓ with 3-loop QCD & 1-loop EW/QED evolutions
- ✓ here $\mu_b = 4.18$GeV and $\Lambda = 1$TeV

Analysis within the SMEFT

- **Fit with a single SMEFT operator:** Q. Y. Hu, X. Q. Li, Y. D. Yang, 1810.04939 **dashed: $\mathrm{Br}(B_c \to \tau\,\nu_\tau) \leq 10\%$**



$$C_{V_L}(\mu_b) = -1.503\,[C_{lq}^{(3)}]_{3323}\,(\Lambda),$$
$$C_{V_R}(\mu_b) = 0,$$
$$C_{S_L}(\mu_b) = -1.257\,[C_{lequ}^{(1)}]_{3332}\,(\Lambda) + 0.2076\,[C_{lequ}^{(3)}]_{3332}\,(\Lambda),$$
$$C_{S_R}(\mu_b) = -1.254\,[C_{ledq}]_{3332}\,(\Lambda),$$
$$C_T(\mu_b) = 0.002725\,[C_{lequ}^{(1)}]_{3332}\,(\Lambda) - 0.6059\,[C_{lequ}^{(3)}]_{3332}\,(\Lambda),$$

- ✓ $[Q_{lequ}^{(1)}]_{3332}$ already excluded by $R(D^{(*)})$ @ 3σ
- ✓ $[Q_{ledq}]_{3332}$ explains $R(D^{(*)})$ only marginally @~2σ
- ✓ $[Q_{lq}^{(3)}]_{3323}$ or $[Q_{lequ}^{(3)}]_{3323}$ can explain $R(D^{(*)})$ @~1σ

- ✓ $\Delta P_\tau(D^*)$ helpful to discriminate the solutions $[Q_{lq}^{(3)}]_{3323}$ & $[Q_{lequ}^{(3)}]_{3323}$
- ✓ $\Delta R(J/\psi)$ & $\Delta R(X_c)$ further exclude some allowed intervals of $[C_{lq}^{(3)}]_{3323}$ & $[C_{lequ}^{(3)}]_{3323}$

- **Due to $SU(2)_L$ invariance, interesting correlations among $b \to c\tau\nu$ & $b \to s\tau\tau, s\nu\bar{\nu}$ observables**

$\Lambda_b \to \Lambda_c l\nu$ Decays Mediated by $b \to cl\nu$

- **$R(\Lambda_c) = \mathrm{Br}(\Lambda_b \to \Lambda_c \tau \nu_\tau)/\mathrm{Br}(\Lambda_b \to \Lambda_c \ell \nu_\ell)$:**
  - At the quark level, also mediated by $b \to cl\nu$ transitions
  - First result reported by LHCb with large error [arXiv:2201.03497]

$$R_{\Lambda_c}^{\mathrm{LHCb}} = 0.242 \pm 0.026 \pm 0.04 \pm 0.059$$

$$R_{\Lambda_c}^{\mathrm{SM}} = 0.324 \pm 0.004$$

exp. central value lower than SM

- **General expressions for the ratios in LEFT:**

$$\mathcal{H}_{\mathrm{eff}} = 2\sqrt{2} G_F V_{qb} \left[\left(1 + C_{V_L}^{ql}\right) \mathcal{O}_{V_L}^{ql} + C_{V_R}^{ql} \mathcal{O}_{V_R}^{ql} + C_{S_L}^{ql} \mathcal{O}_{S_L}^{ql} + C_{S_R}^{ql} \mathcal{O}_{S_R}^{ql} + C_T^{ql} \mathcal{O}_T^{ql} \right]$$

$$\mathcal{O}_{V_L}^{ql} = (\bar{q}\gamma^\mu P_L b)\left(\bar{l}\gamma_\mu P_L \nu_l\right), \qquad \mathcal{O}_{V_R}^{ql} = (\bar{q}\gamma^\mu P_R b)\left(\bar{l}\gamma_\mu P_L \nu_l\right),$$

$$\mathcal{O}_{S_L}^{ql} = (\bar{q} P_L b)\left(\bar{l} P_L \nu_l\right), \qquad \mathcal{O}_{S_R}^{ql} = (\bar{q} P_R b)\left(\bar{l} P_L \nu_l\right),$$

$$\mathcal{O}_T^{ql} = (\bar{q}\sigma^{\mu\nu} P_L b)\left(\bar{l}\sigma_{\mu\nu} P_L \nu_l\right)$$

LEFT

- ✓ assuming only τ modes affected by NP
- ✓ light neutrinos always left-handed
- ✓ a_X^{ij} calculable with FF inputs together with uncertainties inherited from them

$$\frac{R_P}{R_P^{\mathrm{SM}}} = |1 + C_{V_L}^{q\tau} + C_{V_R}^{q\tau}|^2 + a_P^{SS}|C_{S_L}^{q\tau} + C_{S_R}^{q\tau}|^2 + a_P^{TT}|C_T^{q\tau}|^2 + a_P^{VS}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau} + C_{V_R}^{q\tau}\right)\left(C_{S_L}^{q\tau*} + C_{S_R}^{q\tau*}\right)\right] + a_P^{VT}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau} + C_{V_R}^{q\tau}\right) C_T^{q\tau*}\right],$$

$$\frac{R_V}{R_V^{\mathrm{SM}}} = |1 + C_{V_L}^{q\tau}|^2 + |C_{V_R}^{q\tau}|^2 + a_V^{SS}|C_{S_L}^{q\tau} - C_{S_R}^{q\tau}|^2 + a_V^{TT}|C_T^{q\tau}|^2 + a_V^{V_L V_R}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau}\right) C_{V_R}^{q\tau*}\right] + a_V^{VS}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau} - C_{V_R}^{q\tau}\right)\left(C_{S_L}^{q\tau*} - C_{S_R}^{q\tau*}\right)\right] + a_V^{V_L T}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau}\right) C_T^{q\tau*}\right] + a_V^{V_R T}\mathrm{Re}\left[C_{V_R}^{q\tau} C_T^{q\tau*}\right],$$

$$\frac{R_H}{R_H^{\mathrm{SM}}} = |1 + C_{V_L}^{q\tau}|^2 + |C_{V_R}^{q\tau}|^2 + a_H^{SS}\left[|C_{S_L}^{q\tau}|^2 + |C_{S_R}^{q\tau}|^2\right] + a_H^{TT}|C_T^{q\tau}|^2 + a_H^{V_L V_R}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau}\right) C_{V_R}^{q\tau*}\right] + a_H^{VS_1}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau}\right) C_{S_L}^{q\tau*} + C_{V_R}^{q\tau} C_{S_R}^{q\tau*}\right] + a_H^{VS_2}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau}\right) C_{S_R}^{q\tau*} + C_{V_R}^{q\tau} C_{S_L}^{q\tau*}\right] + a_H^{S_L S_R}\mathrm{Re}\left[C_{S_L}^{q\tau} C_{S_R}^{q\tau*}\right] + a_H^{V_L T}\mathrm{Re}\left[\left(1 + C_{V_L}^{q\tau}\right) C_T^{q\tau*}\right] + a_H^{V_R T}\mathrm{Re}\left[C_{V_R}^{q\tau} C_T^{q\tau*}\right],$$

Sum Rule for $b \to c$ Sector

$|1 + C_{V_L}^{q\tau}|^2$ (crossed out)

$\mathrm{Re}[(1 + C_{V_L}^{q\tau}) C_{S_L}^{q\tau *}]$ (crossed out)

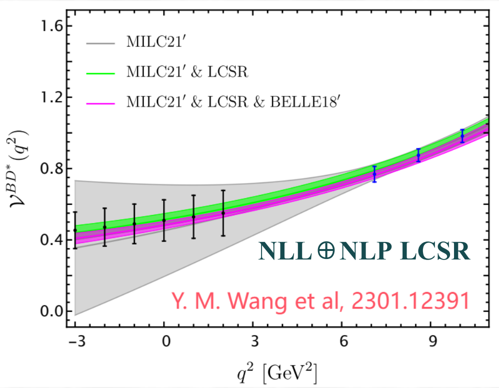
Sum rule for $R(D)$, $R(D^*)$ & $R(\Lambda_c)$: M. Blanke et al., 1811.09603, 1905.08253

$$\frac{R_H}{R_H^{\rm SM}} = b\frac{R_P}{R_P^{\rm SM}} + c\frac{R_V}{R_V^{\rm SMM}} + \delta_H(C_i)$$

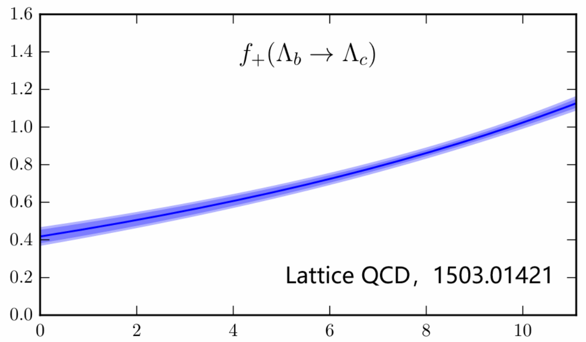
→ $b + c = 1$ & $a_P^{VS} b + a_V^{VS} c = a_H^{VS_1}$, so that $\delta_H(C_i)$ small

→ **model-indep. & holds for any tau-philic NP**

State-of-the-art prediction: W. F. Duan, S. Iguro, X. Q. Li, R. Watanabe, Y. D. Yang, 2410.21384



	Lattice		LCSR		Lattice + LCSR
	SM	Tensor	SM	Tensor	SM
$B \to D$	Refs. [85, 86]	no data	Ref. [90, 91]	Ref. [90]	Ref. [91] (**)
$B \to D^*$	Refs. [87–89]	no data (*)	Ref. [90, 91]	Ref. [90]	Ref. [91] (**)
$\Lambda_b \to \Lambda_c$	Ref. [80]	Ref. [92]	no data	no data	–



$$\frac{R_{\Lambda_c}}{R_{\Lambda_c}^{\rm SM}} = (0.272 \pm 0.015)\frac{R_D}{R_D^{\rm SM}} + (0.728 \mp 0.015)\frac{R_{D^*}}{R_{D^*}^{\rm SM}} + \delta_{\Lambda_c}$$

$$\begin{aligned}\delta_{\Lambda_c} = &(-0.001 \pm 0.005)\left(|C_{S_L}^{c\tau}|^2 + |C_{S_R}^{c\tau}|^2\right) + (-0.007 \pm 0.005)\,\mathrm{Re}\left(C_{S_L}^{c\tau} C_{S_R}^{c\tau *}\right)\\ &+ (-2.681 \pm 6.907)\,|C_T^{c\tau}|^2 + (-0.561 \pm 1.439)\,\mathrm{Re}\left(C_{V_R}^{c\tau} C_T^{c\tau *}\right)\\ &+ \mathrm{Re}\left[\left(1 + C_{V_L}^{c\tau}\right)\left\{(0.041 \pm 0.034) C_{V_R}^{c\tau *} + (0.594 \pm 1.274) C_T^{c\tau *}\right\}\right]\\ &+ (-0.002 \pm 0.009)\,\mathrm{Re}\left[\left(1 + C_{V_L}^{c\tau}\right) C_{S_R}^{c\tau *} + C_{S_L}^{c\tau} C_{V_R}^{c\tau *}\right]\end{aligned}$$

- important to properly consider correlations among FF para.s
- δ_{Λ_c} mostly sensitive to the tensor operator $\mathcal{O}_T = (\bar{c}\sigma^{\mu\nu} P_L b)(\bar{\tau}\sigma_{\mu\nu} P_L \nu_\tau)$

Sum rule for $b \to c$ sector

- **Sum rule implications: shift factor $|\delta_{\Lambda_c}(C_X)| \ll 1$ for $C_X < 1$; almost independent of NP**

$(\bar{c}\sigma^{\mu\nu}P_L b)(\bar{\ell}\sigma_{\mu\nu}P_L\nu)$ $\quad C_T^{\tau} \approx 0.02 \pm i0.13$ $\quad\Rightarrow\quad \delta_{\Lambda_c}(C_T = 0.02 \pm i0.13) \simeq -0.035 \pm 0.096$

tensor FFs

- **Sum-rule prediction for $R(\Lambda_c)$:**

$$\frac{R_{\Lambda_c}}{R_{\Lambda_c}^{\rm SM}} = (0.272 \pm 0.015)\frac{R_D}{R_D^{\rm SM}} + (0.728 \mp 0.015)\frac{R_{D^*}}{R_{D^*}^{\rm SM}} + \delta_{\Lambda_c}$$

  - provide a **unique** prediction of $R(\Lambda_c)$ model-independently

HFLAG 2024: $R_D^{\rm exp} = 0.342 \pm 0.026, \quad R_{D^*}^{\rm exp} = 0.287 \pm 0.012$

$$R_{\Lambda_c}^{\rm SR} = 0.372 \pm 0.017 \quad \text{vs} \quad R_{\Lambda_c}^{\rm LHCb} = 0.242 \pm 0.076$$

$$\Rightarrow \delta_{\rm SR/\,exp} = \frac{R_{\Lambda_c}^{\rm exp}}{R_{\Lambda_c}^{\rm SM}} - \left(0.272\frac{R_D^{\rm exp}}{R_D^{\rm SM}} + 0.728\frac{R_{D^*}^{\rm exp}}{R_{D^*}^{\rm SM}}\right) = -0.39 \pm 0.23$$

  - sum rule relation without the shift factor $\delta_{\Lambda_c}(C_X)$ seems to be violated
  - with $\delta_{\Lambda_c}(C_T = 0.02 \pm i0.13) \simeq -0.035 \pm 0.096$ included, still explain the sum rule deviation

- **This exercise encourages a more precise measurement of $\Lambda_b \to \Lambda_c\tau\nu$ decays from LHCb**

Other processes mediated by $b \to cl\nu$ decays

- **Sum rule for $R(D)$, $R(D^*)$ & $R(X_c)$ $= \mathrm{Br}(B \to X_c\tau\nu_\tau)/\mathrm{Br}(B \to X_c\ell\nu_\ell)$:**

$$\frac{R_{X_c}}{R_{X_c}^{\rm SM}} \simeq 0.288\,\frac{R_D}{R_D^{\rm SM}} + 0.712\,\frac{R_{D^*}}{R_{D^*}^{\rm SM}} + \delta_{X_c}$$

$$\begin{aligned}\delta_{X_c} \simeq\, & 0.015\,\left(|C_{S_L}^{c\tau}|^2 + |C_{S_R}^{c\tau}|^2\right) - 0.003\,\mathrm{Re}\left(C_{S_L}^{c\tau} C_{S_R}^{c\tau *}\right) - 1.655\,|C_T^{c\tau}|^2 \\ & + \mathrm{Re}\left[\left(1 + C_{V_L}^{c\tau}\right)\left\{0.192 C_{V_R}^{c\tau *} + 0.896 C_T^{c\tau *}\right\}\right] - 3.405\,\mathrm{Re}\left(C_{V_R}^{c\tau} C_T^{c\tau *}\right) \\ & + 0.043\mathrm{Re}\left[\left(1 + C_{V_L}^{c\tau}\right) C_{S_R}^{c\tau *} + C_{S_L}^{c\tau} C_{V_R}^{c\tau *}\right]\end{aligned}$$

➡ $R_{X_c}^{\rm SR} \simeq 0.247 \pm 0.008\,|_{R_X^{\rm SM,exp}}$ vs $R_{X_c}^{\rm exp} = 0.228 \pm 0.039$ [Belle II, 2311.07248]

- $\Gamma(B \to X_c\ell\nu_\ell) = \sum \Gamma(B \to D\ell\nu_\ell) + \Gamma(B \to D^*\ell\nu_\ell) + \Gamma(B \to D^{**}\ell\nu_\ell)$, saturated already by excl. rate?
- the sum rule relation provides another complementary test of the dynamics behind the decays

- **Sum rule for $R(D^*)$ & $R(J/\psi)$ $= \mathrm{Br}(B \to J/\psi\tau\nu_\tau)/\mathrm{Br}(B \to J/\psi\ell\nu_\ell)$:**

$$\frac{R_{J/\psi}}{R_{J/\psi}^{SM}} \simeq \frac{R_{D^*}}{R_{D^*}^{SM}} \quad \Rightarrow \quad \frac{R_{J/\psi}}{R_{J/\psi}^{SM}} - \frac{R_{D^*}}{R_{D^*}^{SM}} = 1.2 \pm 0.7$$

- satisfied within the 2σ error bars; would be significant once $R_{J/\psi}$ measurement improved

Sum rule for $b \to u$ sector

Sum rule for $R(\pi)$, $R(\rho)$ & $R(p)$:

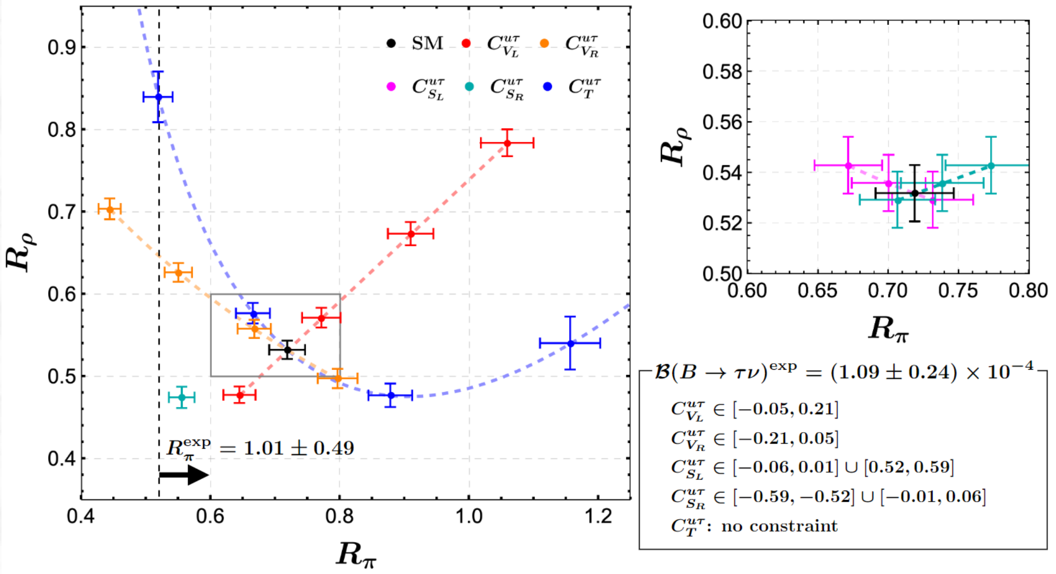
$$\frac{R_p}{R_p^{\rm SM}} = (0.284 \pm 0.037)\,\frac{R_\pi}{R_\pi^{\rm SM}} + (0.716 \mp 0.037)\,\frac{R_\rho}{R_\rho^{\rm SM}} + \delta_p$$

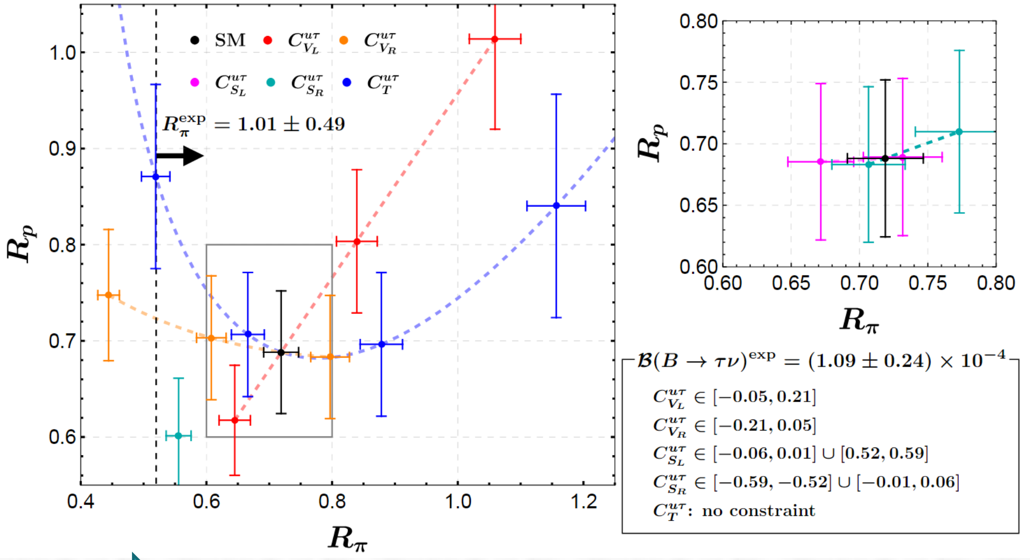
$$\begin{aligned}\delta_p = {} & (-0.090 \pm 0.059)\,\left(|C_{S_L}^{u\tau}|^2 + |C_{S_R}^{u\tau}|^2\right) + (-0.185 \pm 0.038)\,{\rm Re}\left(C_{S_L}^{u\tau} C_{S_R}^{u\tau *}\right) \\ & + (-0.913 \pm 2.403)\,|C_T^{u\tau}|^2 + (-0.203 \pm 0.538)\,{\rm Re}\left(C_{V_R}^{u\tau} C_T^{u\tau *}\right) \\ & + {\rm Re}\left[\left(1 + C_{V_L}^{u\tau}\right)\left\{(0.169 \pm 0.158) C_{V_R}^{u\tau *} + (0.370 \pm 0.632) C_T^{u\tau *}\right\}\right] \\ & + (-0.079 \pm 0.056)\,{\rm Re}\left[\left(1 + C_{V_L}^{u\tau}\right) C_{S_R}^{u\tau *} + C_{S_L}^{u\tau} C_{V_R}^{u\tau *}\right]\,.\end{aligned}$$

➡ sum rule for $b \to u$ more (less) sensitive to scalar (tensor) NP compared to $b \to c$

	Lattice		LCSR		Lattice + LCSR
	SM	Tensor	SM	Tensor	SM + Tensor
$B \to \pi$	Refs. [98–100]	Ref. [101]	Refs. [90, 103–105]		Ref. [106]
$B \to \rho$	no data	no data	Refs. [77, 90, 107]		B.-Y. Cui et al., 2212.11624
$\Lambda_b \to p$	Ref. [80]	no data	Ref. [108]	no data	–

Correlation among $R(\pi)$, $R(\rho)$ & $R(p)$:





➡ NP contributions to R_π & R_ρ well distinctive from SM

➡ R_p less predictive due to large FF uncertainty

Summary

- $R(D)$ & $R(D^*)$ **anomalies: even much progress achieved since 2012, still** $\sim 3.31\sigma$ **deviation**
- **In LEFT & SMEFT,** $\mathcal{O}^{\tau}_{V_L} = (\bar{c}\gamma^{\mu}P_L b) \otimes (\bar{\tau}\gamma_{\mu}P_L\nu_{\tau})$ **&** $\mathcal{O}^{\tau}_{T} = (\bar{c}\sigma^{\mu\nu}P_L b) \otimes (\bar{\tau}\sigma_{\mu\nu}P_L\nu_{\tau})$ **provide two good solutions, and also indistinguishable by other observables or other processes**

$$(\bar{c}\gamma^{\mu}P_L b)(\bar{\ell}\gamma_{\mu}P_L\nu) \quad C_{V_L} \approx 0.08 \quad / \quad (\bar{c}\sigma^{\mu\nu}P_L b)(\bar{\ell}\sigma_{\mu\nu}P_L\nu) \quad C_T \approx 0.02 \pm i0.13$$

- **Sum rules for** $b \to cl\nu$ **&** $b \to ul\nu$**: model-independent & complementary information on** $R(D)$ **&** $R(D^*)$ **anomalies**

$$\frac{R_H}{R_H^{\rm SM}} = b\frac{R_P}{R_P^{\rm SM}} + c\frac{R_V}{R_V^{\rm SMM}} + \delta_H(C_i)$$

➡ sum-rule based prediction for $R_{\Lambda_c}^{\rm SR}$ higher than $R_{\Lambda_c}^{\rm LHCb}$, but can be explained by tensor NP

- **More precise results for** $B \to P$, $B \to V$, **&** $\Lambda_b \to \Lambda_c(p)$ **FFs expected from LQCD & LCSR**
- **Bright future from LHC, Belle-II, FCC, CEPC: more data, more process, more observables**

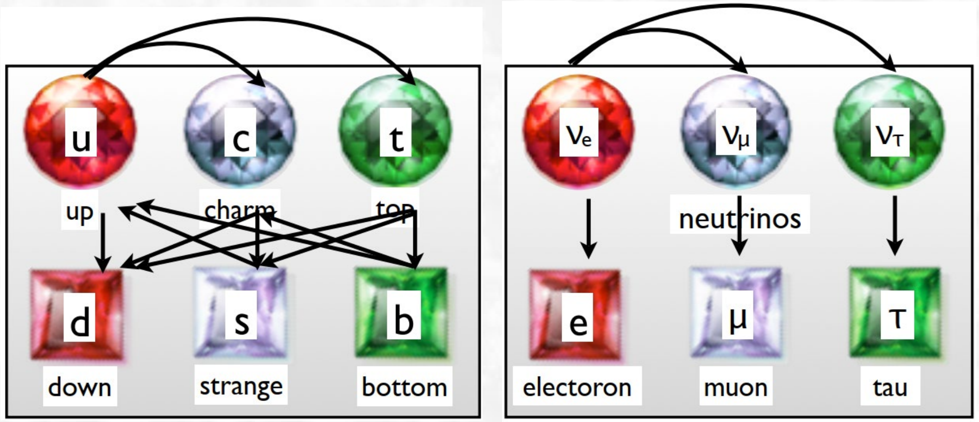
➡ understand the true dynamics behind the $R(D)$ & $R(D^*)$ anomalies

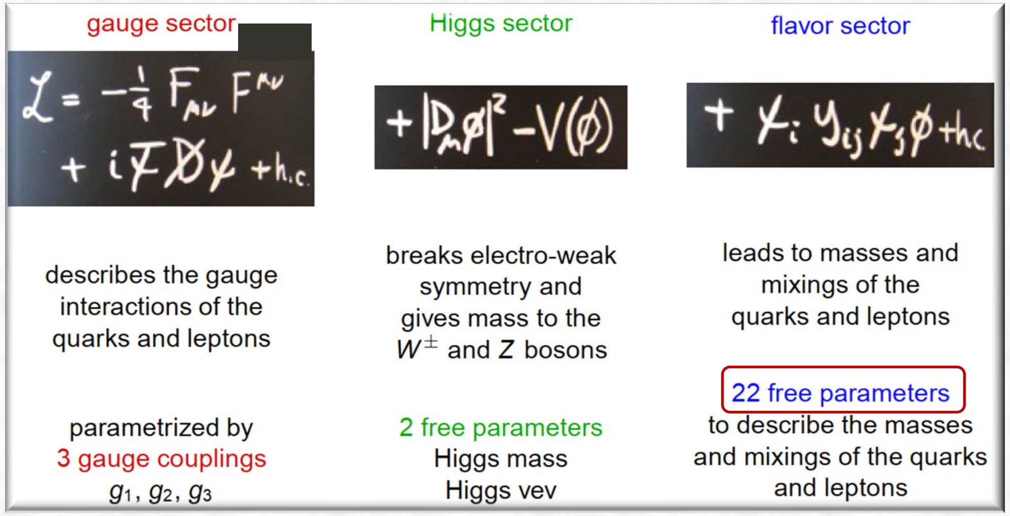
Thank You for Your Attention!

backup

Heavy Flavor Physics

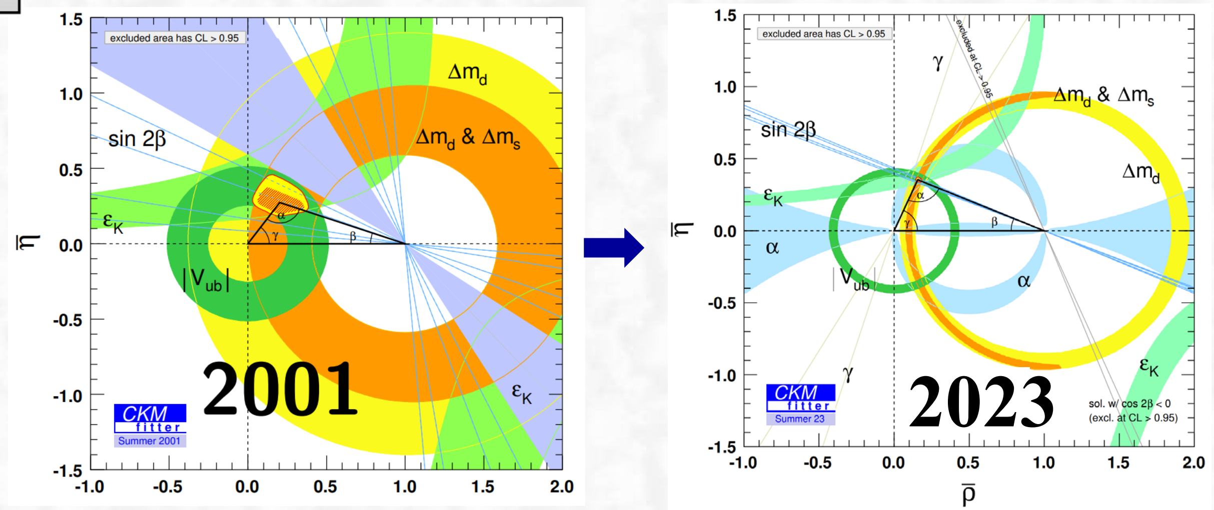
- **An important branch of particle physics: most free parameters from flavor sector**





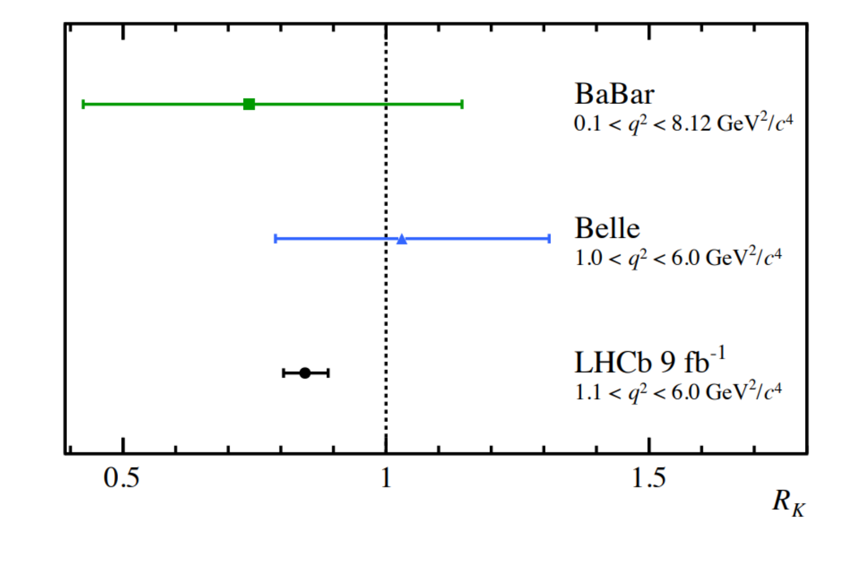
- **Much theo. & exp. progress achieved, now entering a *precision flavor era!***

- **With present LHCb, Belle-II & future STCF, CEPC, FCC, …: bright prospect**

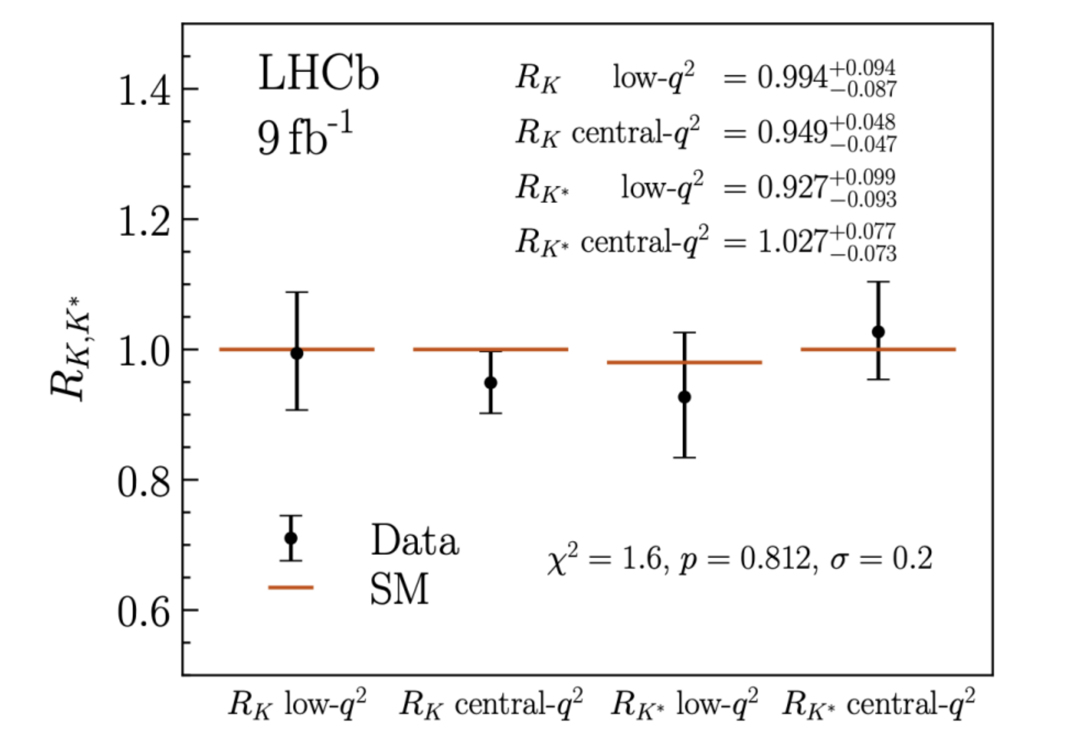


LFUV in $b \to s\ell^+\ell^-$ decays

- LFUV can also be probed in $b \to s\ell^+\ell^-$ FCNC decays through $R(K^{(*)}) = \frac{\mathcal{B}(B \to K^{(*)}\mu^+\mu^-)}{\mathcal{B}(B \to K^{(*)}e^+e^-)}$



LHCb Nature Phys. 18 (2022) 277



arXiv:2212.09153 arXiv:2212.09152

- The new LHCb results are now consistent with the SM predictions.

- We do learn quite a lot from $R(K^{(*)})$, and much progress achieved in theory!

Specific NP Scenarios with Single Mediator

☐ Applied to some specific single-mediator NP scenarios:

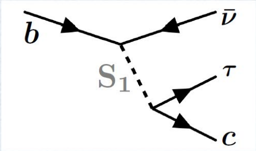
S. Iguro, T. Kitahara, R. Watanabe, 2405.06062

	Spin	Charge	Operators	R_D	R_{D^*}	LHC	Flavor
$H^\pm$	0	$(\mathbf{1}, \mathbf{2}, 1/2)$	O_{S_L}	✓	✓	$b\tau\nu$	$B_c \to \tau\nu$, $F_L^{D^*}$, $P_\tau^{D^*}$, M_W
S_1	0	$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	O_{V_L}, O_{S_L}, O_T	✓	✓	$\tau\tau$	ΔM_s, P_τ^D, $B \to K^{(*)}\nu\nu$
$R_2^{(2/3)}$	0	$(\mathbf{3}, \mathbf{2}, 7/6)$	O_{S_L}, O_T, (O_{V_R})	✓	✓	$b\tau\nu$, $\tau\tau$	$P_\tau^{D^*}$, M_W, $Z \to \tau\tau$, d_N
U_1	1	$(\mathbf{3}, \mathbf{1}, 2/3)$	O_{V_L}, O_{S_R}	✓	✓	$b\tau\nu$, $\tau\tau$	ΔM_s, $R_{K^{(*)}}$, $B_s \to \tau\tau$, d_N
$V_2^{(1/3)}$	1	$(\bar{\mathbf{3}}, \mathbf{2}, 5/6)$	O_{S_R}	✓	2σ	$\tau\tau$	$B_s \to \tau\tau$, $B_u \to \tau\nu$, M_W

Three LQ bosons are capable of the RD(*) solution: S1, R2, U1 [arXiv:1309.0301]

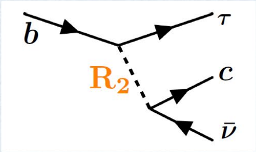
LQs have independent and specific WCs

Leptoquarks (LQs)



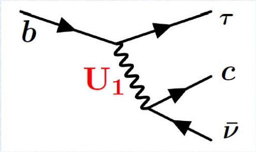
S_1 $(\bar{3}, 1, 1/3)$ scalar: C_{V_L}, C_{S_L}, C_T

$$\mathcal{L}_{S_1} = \left(h_S^{ij}\bar{q}_L^{c,i}(i\sigma_2)\ell_L^j + h_S'^{ij}\bar{u}_R^{c,i}\ell_R^j\right) S_1 + \text{h.c.}$$



R_2 $(3, 2, 7/6)$ scalar: C_{V_R}, C_{S_L}, C_T

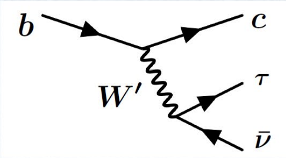
$$\mathcal{L}_{R_2} = \left(h_R^{ij}\bar{d}_L^i\ell_R^j + h_R'^{ij}\bar{u}_R^i\nu_R^j\right) R_2 + \text{h.c.}$$



U_1 $(3, 1, 2/3)$ vector: C_{V_L}, C_{S_R}

$$\mathcal{L}_{U_1} = \left(h_U^{ij}\bar{q}_L^i\gamma_\mu\ell_L^j + h_U'^{ij}\bar{d}_R^i\gamma_\mu\ell_R^j\right) U_1^\mu + \text{h.c.}$$

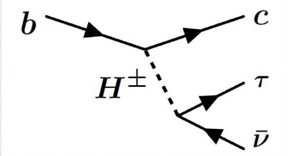
Vector boson (W'): $C_{V_L}^\tau \approx 0.08$, or $C_{V_R}^\tau \approx 0.01 \pm i0.41$



The V_L best pull implies: $M_{W'}/g_{W'} \approx 3$ TeV ✗

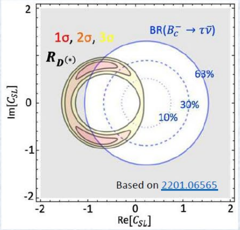
SU(2)' model inevitably includes Z' that is very constrained due to tree-level FCNC

Charged Higgs ($H^\pm$): $C_{S_L}^\tau \approx -0.79 \pm i0.86$, or $C_{S_R}^\tau \approx 0.18$



Typical 2HDM cannot achieve the solution

ex) $C_{S_R}^{\text{Type II}} = -\tan^2\beta \frac{m_b m_\tau}{m_{H^\pm}^2} \Big/ \left(2\sqrt{2}V_{cb}G_F\right)$ ✗



General 2HDM can be a viable model but need to concern $B_c \to \tau\nu$ △

Present bound: Br(Bc → τν) < 60% [arXiv:2201.06565]

Future prospect: CEPC can observe this decay!

☐ These single-mediator NP scenarios can be further tested by considering the other observables & LHC direct searches

$B \to X_c l\nu$ decays

- Include **NLO QCD** & **power corrections** for **all kinds of four-quark operators in LEFT**

$$\frac{d^3\Gamma}{dq^2 dE_\tau dE_{\nu_\tau}} = \frac{1}{4}\sum_{X_c}\sum_{\substack{\text{lepton}\\ \text{spins}}} \frac{|\langle X_c\tau^-\bar{\nu}_\tau|\mathcal{H}_{\rm eff}|\bar{B}\rangle|^2}{2m_B}\,\delta^{(4)}\left[p_B-(p_\tau+p_{\nu_\tau})-p_{X_c}\right]$$

$$= \frac{G_F^2|V_{cb}|^2}{4\pi^3}\sum_{i,j} C_i^* C_j\, (W^{ij})_{MN}\,(L^{ij})^{MN},$$

$$(L^{ij})^{MN} = \sum_{\substack{\text{lepton}\\ \text{spins}}} \langle 0|L^{(i)\dagger M}|\tau^-\bar{\nu}_\tau\rangle\,\langle\tau^-\bar{\nu}_\tau|L^{(j)N}|0\rangle,$$

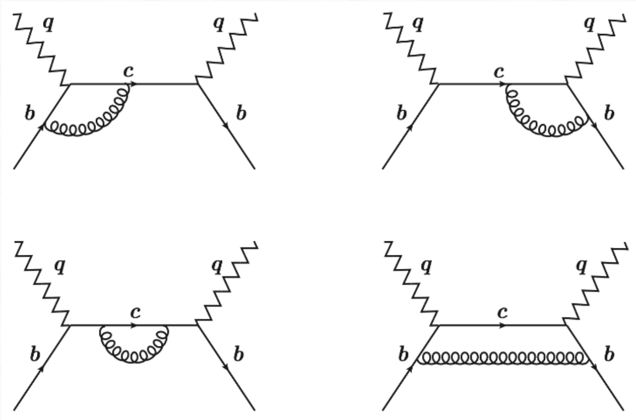
$$(W^{ij})_{MN} = \frac{1}{2m_B}\sum_{X_c}\langle\bar{B}|J_M^{(i)\dagger}|X_c\rangle\,\langle X_c|J_N^{(j)}|\bar{B}\rangle\,(2\pi)^3\delta^{(4)}\left[p_B-(p_\tau+p_{\nu_\tau})-p_{X_c}\right].$$

$$\left(W^{ij}\right)_{MN} = -\frac{1}{\pi}\mathrm{Im}\left(T^{ij}\right)_{MN}$$

$$\left(T^{ij}\right)_{MN} = -\frac{i}{2m_B}\int d^4x\, e^{-iq\cdot x}\langle\bar{B}|T\left[J_M^{(i)\dagger}(x)J_N^{(j)}(0)\right]|\bar{B}\rangle$$

$$\frac{d^3\Gamma}{dq^2 dE_\tau dE_{\nu_\tau}} = \frac{d^3\Gamma^{(0,0)}}{dq^2 dE_\tau dE_{\nu_\tau}} + \frac{\alpha_s}{\pi}\frac{d^3\Gamma^{(1,0)}}{dq^2 dE_\tau dE_{\nu_\tau}} + \frac{1}{m_b^2}\frac{d^3\Gamma^{(0,2)}}{dq^2 dE_\tau dE_{\nu_\tau}} + \ldots .$$

- **One-loop diagrams:**



- **Final results: depending on the quark mass scheme**

$$R(X_c)^{1\mathrm{S}} = 0.220\left[|C_1|^2+|C_2|^2+0.354(|C_3|^2+|C_4|^2)+11.194|C_5|^2-0.511\mathrm{Re}[C_1C_2^*]\right.$$
$$+0.360\mathrm{Re}[C_1C_3^*+C_2C_4^*]+0.564\mathrm{Re}[C_1C_4^*+C_2C_3^*]-2.705\mathrm{Re}[C_1C_5^*]$$
$$\left.+1.939\mathrm{Re}[C_2C_5^*]+0.553\mathrm{Re}[C_3C_4^*]+0\,\mathrm{Re}[C_3C_5^*+C_4C_5^*]\right].$$

$$R(X_c)^{\mathrm{kin}} = 0.211\left[|C_1|^2+|C_2|^2+0.353(|C_3|^2+|C_4|^2)+11.215|C_5|^2-0.544\mathrm{Re}[C_1C_2^*]\right.$$
$$+0.369\mathrm{Re}[C_1C_3^*+C_2C_4^*]+0.563\mathrm{Re}[C_1C_4^*+C_2C_3^*]-2.854\mathrm{Re}[C_1C_5^*]$$
$$\left.+2.343\mathrm{Re}[C_2C_5^*]+0.560\mathrm{Re}[C_3C_4^*]+0\,\mathrm{Re}[C_3C_5^*+C_4C_5^*]\right].$$

L. F. Lai, X. Q. Li, Y. Y. Li, Y. D. Yang, to appear soon

Analysis in SMEFT with flavor symmetry

- **SMEFT dim-6 operators contributing to $b \to ul\nu$ & $b \to cl\nu$ decays**

$$\sum_i c_i^{(6)} Q_i^{(6)}\Big|_{b\to ql\nu} = c_{H\ell}^{ij}\left(H^\dagger i\overleftrightarrow{D}^I_\mu H\right)\left(\bar{L}^i\gamma^\mu\tau^I L^j\right) + c_{Hq}^{mn}\left(H^\dagger i\overleftrightarrow{D}^I_\mu H\right)\left(\bar{Q}^m\gamma^\mu\tau^I Q^n\right)$$
$$+ c_V^{mnij}\left(\bar{Q}^m\gamma^\mu\tau^I Q^n\right)\left(\bar{L}^i\gamma_\mu\tau^I L^j\right) + \Big\{c_{H\bar{q}}^{mn}\left(\tilde{H}^\dagger i D_\mu H\right)\left(\bar{U}^m\gamma^\mu D^n\right)$$
$$+ c_{S_d}^{mnij}\left(\bar{L}^i E^j\right)\left(\bar{D}^m Q^n\right) + c_{S_u}^{mnij}\left(\bar{L}^{a,i} E^j\right)\epsilon_{ab}\left(\bar{Q}^{b,m} U^n\right)$$
$$+ c_T^{mnij}\left(\bar{L}^{a,i}\sigma_{\mu\nu}E^j\right)\epsilon_{ab}\left(\bar{Q}^{b,m}\sigma^{\mu\nu}U^n\right) + \text{h.c.}\Big\}$$

- $c_{H\ell}^{ij}$, c_{Hq}^{mn}, $c_{H\bar{q}}^{mn}$ affect $b \to ql\nu$ by modifying W & Z couplings to fermions → strong constrained by EWPO
- focus only on dim-6 four-fermion operators, with their WCs being generically flavor-dependent

- **To establish correlations between $b \to ul\nu$ & $b \to cl\nu$ decays, we must resort to specific flavor assumption** → **3rd generation-philic interaction & $U(2)^5$ symmetry**

$$U(2)^5 = U(2)_Q \otimes U(2)_U \otimes U(2)_D \otimes U(2)_L \otimes U(2)_E$$

$$C_{V_L}^{q\tau} = -\frac{v^2}{\Lambda^2}c_V'^0\left[1+\lambda_Q^s\left(\frac{V_{qs}}{V_{qb}} + \frac{V_{qd}}{V_{qb}}\frac{V_{td}^*}{V_{ts}^*}\right)\right] = -\frac{v^2}{\Lambda^2}c_V'^0\left(1-\lambda_Q^s\frac{V_{tb}^*}{V_{ts}^*}\right),$$

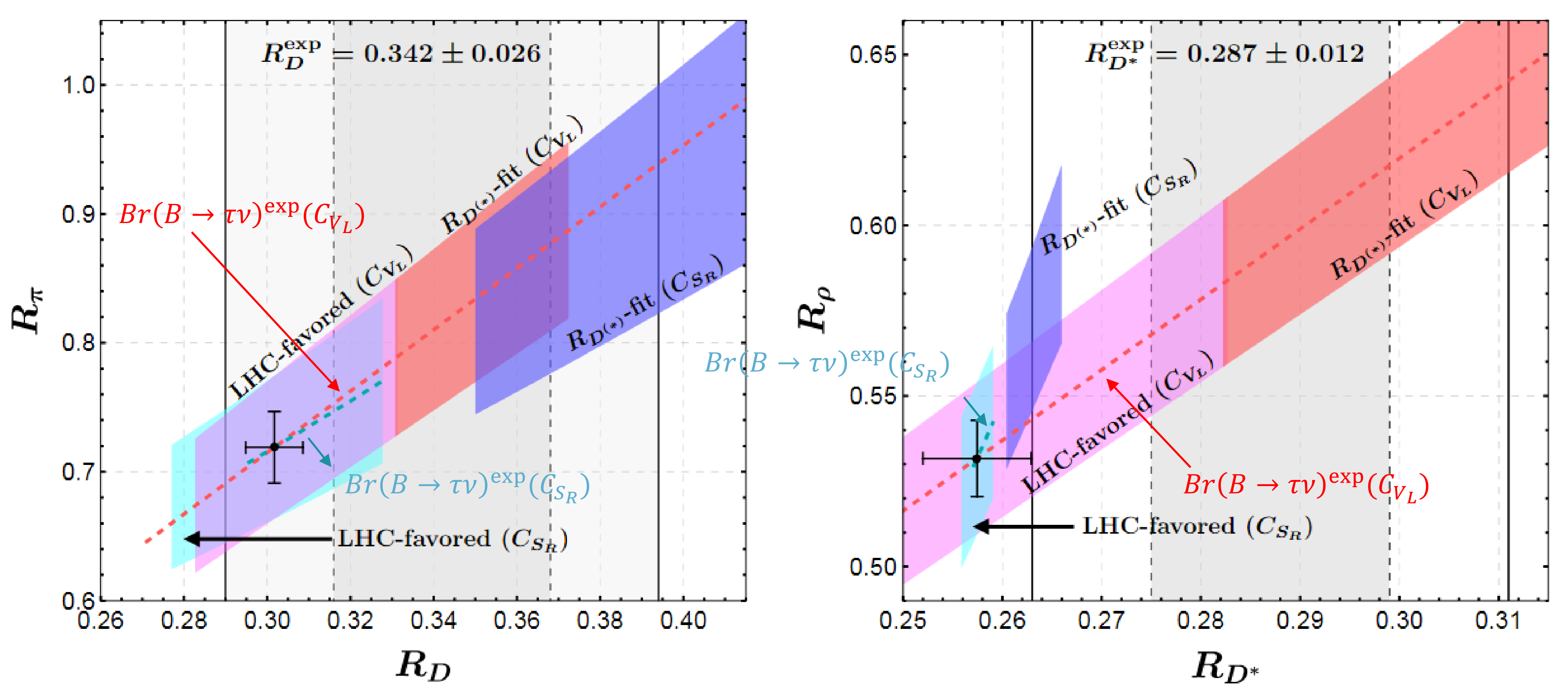
$$C_{S_R}^{q\tau} = -\frac{v^2}{2\Lambda^2}c_{S_u}'^0\left(1-\lambda_Q^s\frac{V_{tb}^*}{V_{ts}^*}\right),$$

$$C_{S_L}^{u\tau} = C_T^{u\tau} \simeq 0, \qquad C_{S_L}^{c\tau} = C_T^{c\tau} \propto m_c/m_t,$$

- $c_V'^0$ & $c_{S_u}'^0$: flavor-universal couplings → **correlation between these two sectors**
- left-handed vector & right-handed scalar NP have same sizes in $b \to u$ & $b \to c$ sectors

Analysis in SMEFT with flavor symmetry

- **Projections in $R(D)-R(\pi)$ & $R(D^*)-R(\rho)$ planes in SMEFT with $U(2)^5$ flavor symmetry:**



$R(D)-R(D^*)$ fit (not) consistent with $\text{Br}(B \to \tau\nu)^{exp}$ constraint in the $V_L(S_R)$-type NP scenario