Next-to-Leading-Order Weak-Annihilation Correction to Rare B $\to$ $(K,\pi)\ell^+\ell^-$ Decays

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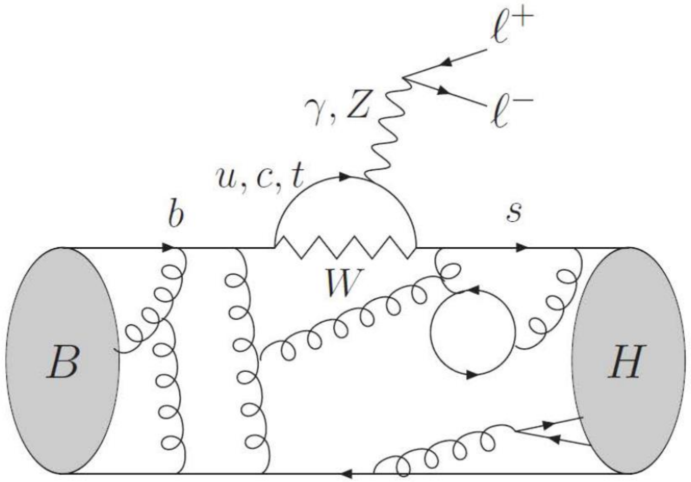
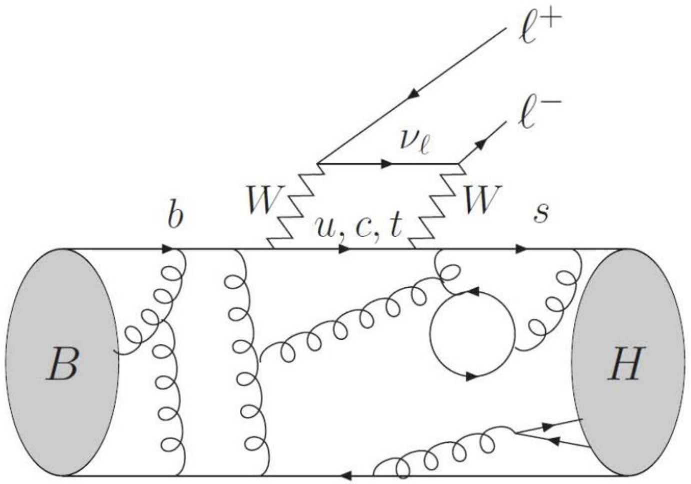
Based on： Y. K. Huang(黄勇康), YLS, C. Wang(王超) and Y. M. Wang(王玉明), *Phys.Rev.Lett.* 134 (2025) 9, 091901

Motivation

◆Flavor changing neutral current(FCNC) processes: prominent tools in searching for new physics signals





$$b \to (s,d)\ell^+\ell^-$$

Motivation

◆Observables in $B \to P\ell^+\ell^-(P = K, \pi)$

Angular distribution

$$\frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} = a_\ell(q^2) + b_\ell(q^2)\cos\theta + c_\ell(q^2)\cos^2\theta$$

Forward-backward asymmetry and flat term

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_\ell} = \frac{3}{4}(1 - F_H^\ell)(1 - \cos^2\theta) + \frac{1}{2}F_H^\ell + A_{FB}\cos\theta$$

Differential decay width

$$\frac{d\Gamma}{dq^2} = 2[a_\ell(q^2) + \frac{1}{3}c_\ell(q^2)]$$

CP asymmetry

$$A_{CP} = \frac{d\Gamma[\overline{B} \to \overline{K}\ell^+\ell^-]/dq^2 - d\Gamma[B \to K\ell^+\ell^-]/dq^2}{d\Gamma[\overline{B} \to \overline{K}\ell^+\ell^-]/dq^2 + d\Gamma[B \to K\ell^+\ell^-]/dq^2}$$

Isospin asymmetry asymmetry

$$A_I = \frac{\Gamma\left[\overline{B^0} \to \overline{K^0}\ell^+\ell^-\right] - \Gamma[B^- \to K^-\ell^+\ell^-]}{\Gamma\left[\overline{B^0} \to \overline{K^0}\ell^+\ell^-\right] + \Gamma[B^- \to K^-\ell^+\ell^-]}$$

Motivation

◆Branching ratios of $B \to K\ell^+\ell^-$

$$\mathcal{B}^{[1.1,2.0],SM}_{B^+\to K^+\mu^+\mu^-} = (0.33 \pm 0.03) \times 10^{-7}$$

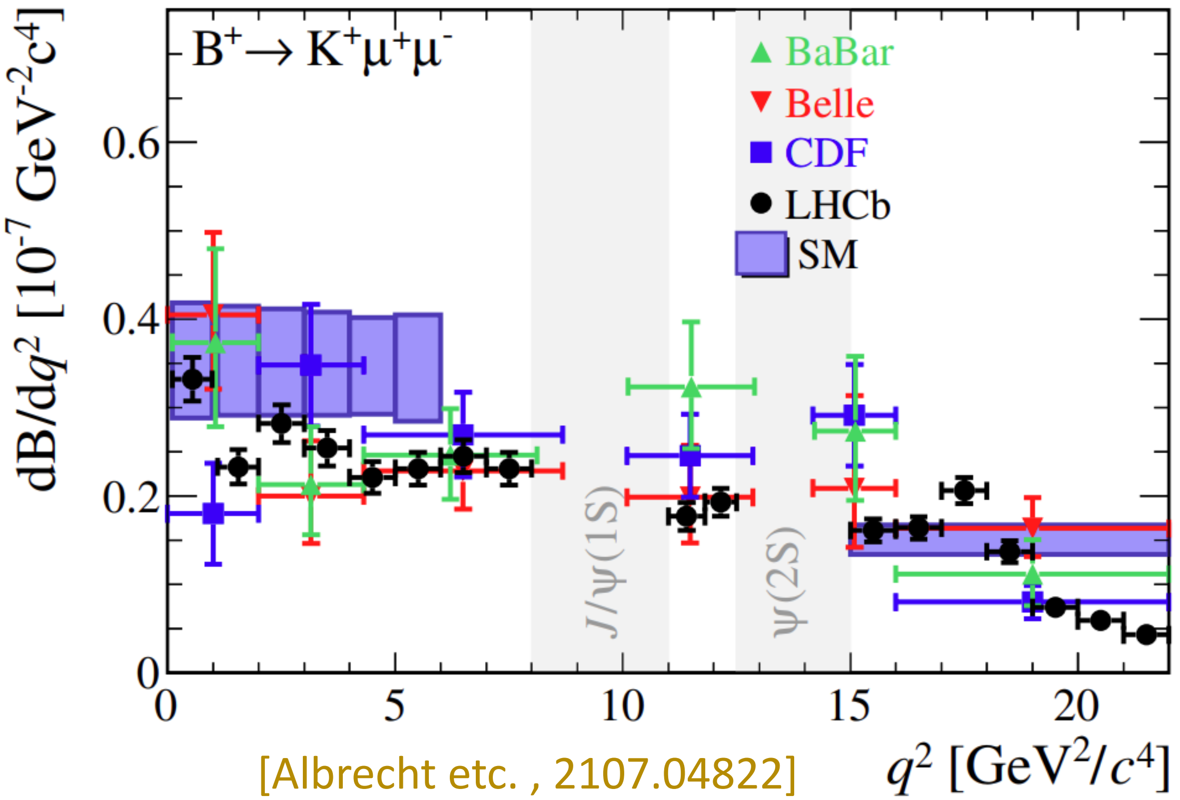
$$\mathcal{B}^{[4.0,5.0],SM}_{B^+\to K^+\mu^+\mu^-} = (0.37 \pm 0.03) \times 10^{-7}$$

$$\mathcal{B}^{[5.0,6.0],SM}_{B^+\to K^+\mu^+\mu^-} = (0.37 \pm 0.03) \times 10^{-7}$$

$$\mathcal{B}^{[1.1,2.0],LHCb}_{B^+\to K^+\mu^+\mu^-} = (0.21 \pm 0.02) \times 10^{-7} (4.0\sigma)$$

$$\mathcal{B}^{[4.0,5.0],LHCb}_{B^+\to K^+\mu^+\mu^-} = (0.22 \pm 0.02) \times 10^{-7} (4.4\sigma)$$

$$\mathcal{B}^{[5.0,6.0],LHCb}_{B^+\to K^+\mu^+\mu^-} = (0.23 \pm 0.02) \times 10^{-7} (4.0\sigma)$$



[Albrecht etc. , 2107.04822]

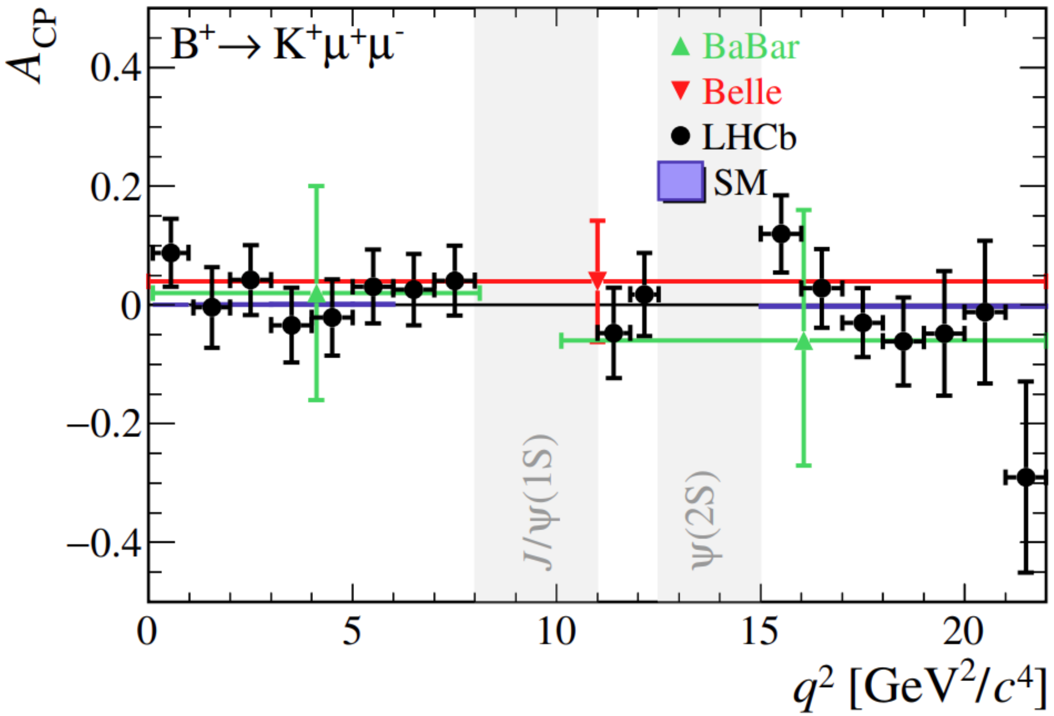
◆$R_{K^{(*)}}$: lepton flavor universality

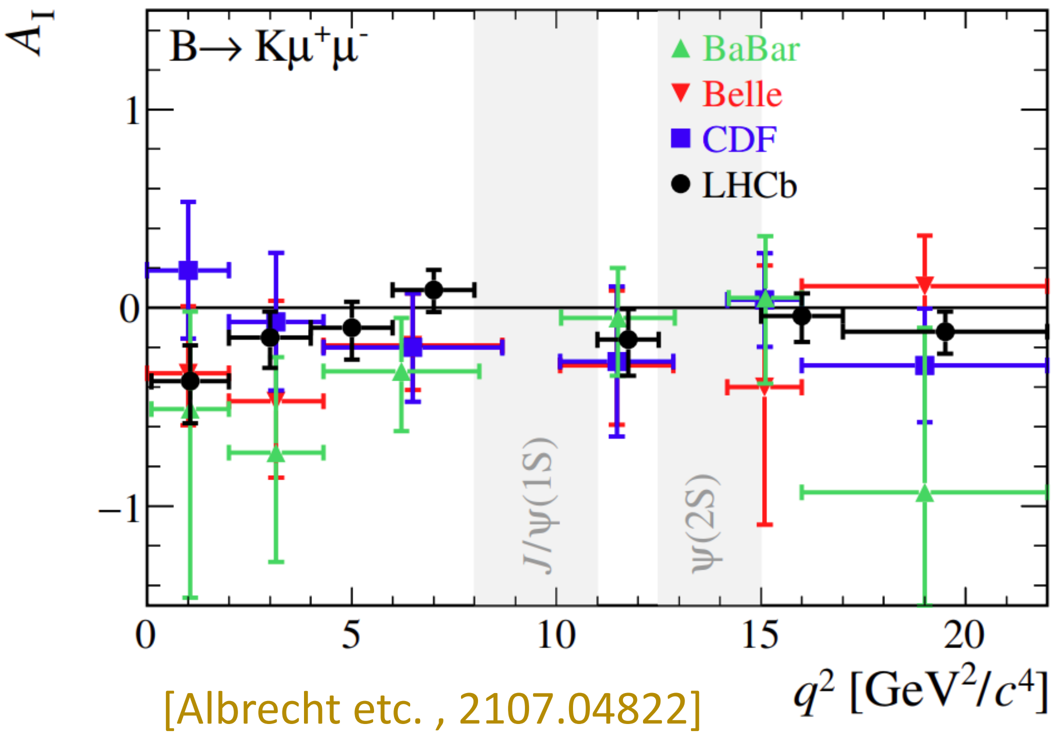
$$R_{K^{(*)}} = \mathcal{B}_{B^+\to K^+\mu^+\mu^-}/\mathcal{B}_{B^+\to K^+e^+e^-} = 1 + O(\alpha_{em}, m_\mu^2)$$

Motivation

◆CP Asymmetries in $B \to K\ell^+\ell^-$

◆Isospin Asymmetries in $B \to K\ell^+\ell^-$





[Albrecht etc. , 2107.04822]

$$A \sim V_{tb}V_{ts}^*P + V_{ub}V_{ud}^*T$$

Sensitive to weak annihilation contribution

- $B \to \pi\ell^+\ell^-$: larger CP and isospin asymmetry are expected

- Measured branching ratio [LHCb, 1505.00414]

$$BR(B^+ \to \pi^+\ell^+\ell^-) = (1.83 \pm 0.24 \pm 0.05) \times 10^{-8}$$

- Direct CP asymmetry [LHCb, 1505.00414]

$$A_{CP}(B^+ \to \pi^+\ell^+\ell^-) = -0.11 \pm 0.12 \pm 0.01$$

Motivation

◆Angular observables in $B \to K^* \ell^+ \ell^-$

$$\frac{d^2\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_K d\phi}$$

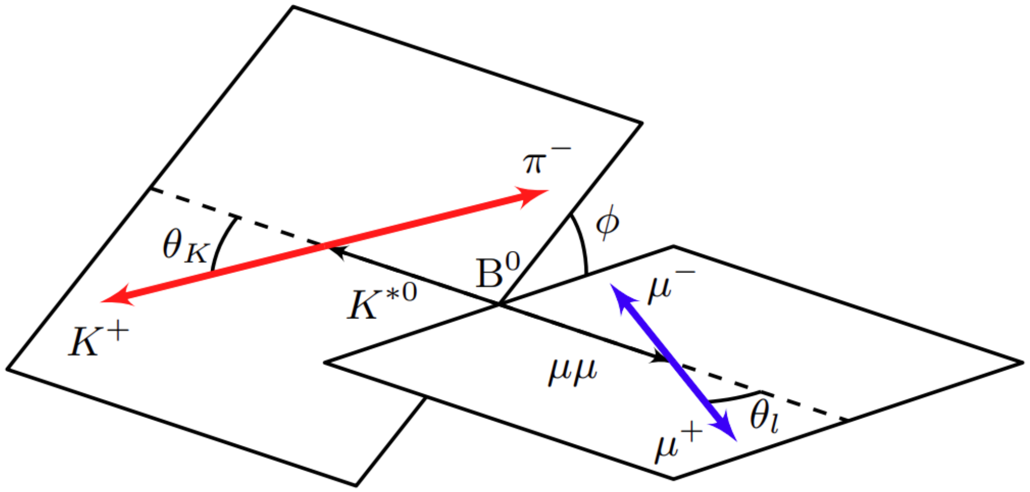
$$\propto \frac{9}{32\pi}[\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta$$

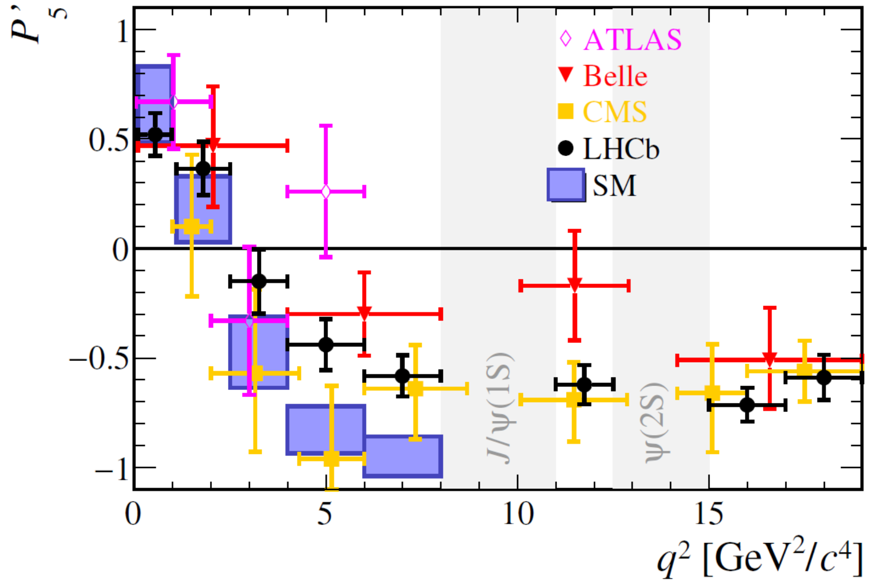
$$+\frac{1}{4}(1-F_L)\sin^2\theta_K\cos 2\theta_\ell - F_L\cos^2\theta_K\cos 2\theta_\ell$$

$$+S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi$$

$$+S_5\sin 2\theta_K\sin\theta_\ell\cos\phi + S_6\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\phi$$

$$+S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi]$$

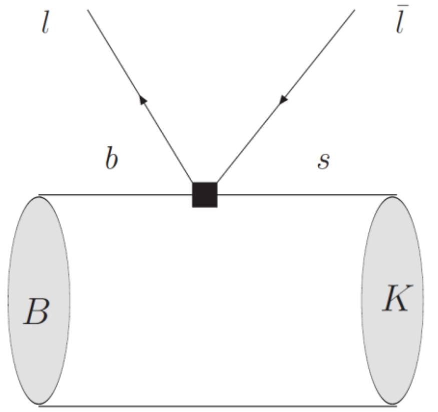




Typical Angular observable $P_i' = \frac{S_i}{\sqrt{F_L(1-F_L)}}$; $i = 4,5,8$

◆The local contribution

$$\mathcal{A}_{SM}^{local}(\bar{B} \to P\ell^+\ell^-) = \frac{G_F \alpha_{em} V_{tb} V_{tD}^*}{\sqrt{2}\pi}\left[C_9 L_V^\mu + C_{10} L_A^\mu\right]\mathcal{F}_\mu^{B\to P}$$



$$O_{9V} = \frac{\alpha_e}{4\pi}(\bar{s}_L\gamma_\mu b_L)(\bar{\ell}\gamma^\mu\ell)$$

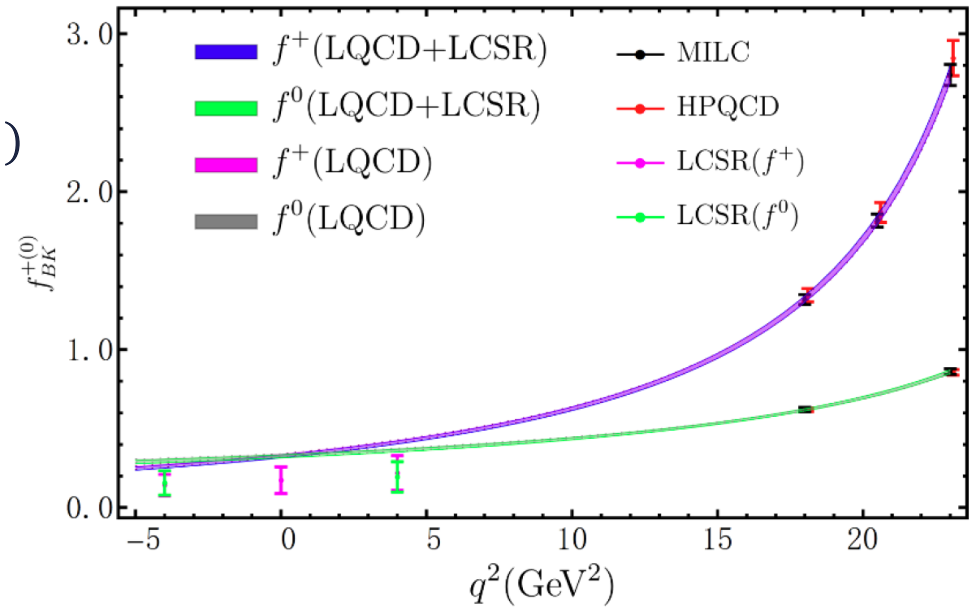
$$O_{10A} = \frac{\alpha_e}{4\pi}(\bar{s}_L\gamma_\mu b_L)(\bar{\ell}\gamma^\mu\gamma_5\ell)$$

Local Form factors

$$\mathcal{F}_\mu^{B\to K} = \langle K(p)|\bar{s}\gamma_\mu(1-\gamma_5)|\bar{B}(p+q)\rangle$$
$$= \left[(2p+q)_\mu - \frac{m_B^2 - m_K^2}{q^2} q_\mu\right] f_+(q^2) + \frac{m_B^2 - m_K^2}{q^2} q_\mu f_0(q^2)$$

Lattice: calculation at large q^2+extrapolation
[Bailey etc., 1509.06325; HPQCD, 2207.12468]

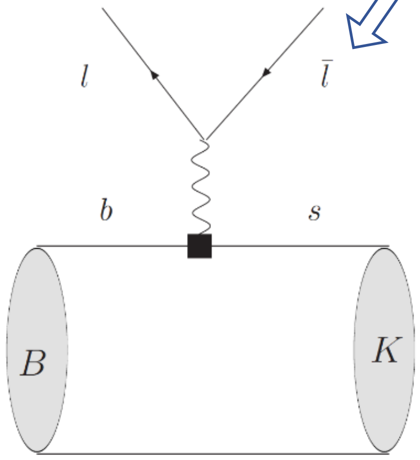
LCSR: calculation at small q^2
[Cui etc., 2212.11624]

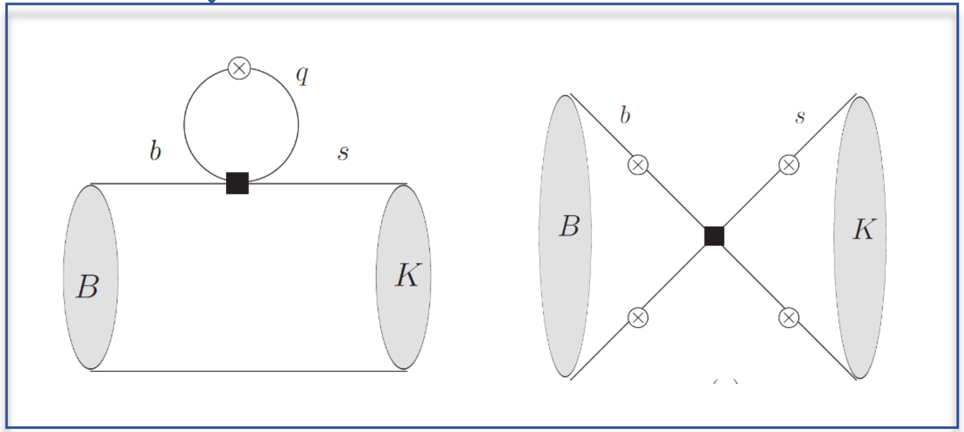


QCD factorization for $B \to P\ell^+\ell^-(P = K,\pi)$

◆The nonlocal contribution

$$\mathcal{A}_{SM}^{\text{nonlocal}}(\bar{B} \to P\ell^+\ell^-) = \frac{G_F\alpha_{em}}{\sqrt{2}\pi}\frac{L_V^{\mu}}{q^2}\{V_{tb}V_{tD}^{*}\left[2im_bC_7\mathcal{F}_{T\mu}^{B\to P} + 16\pi^2\mathcal{H}_{\mu}^{(t)}\right] + V_{ub}V_{uD}^{*}\,16\pi^2\mathcal{H}_{\mu}^{(u)}\}$$





$$H_{eff}^{u} = C_1(Q_1^c - Q_1^u) + C_2(Q_2^c - Q_2^u)$$

$$H_{eff}^{t} = C_1Q_1^c + C_2Q_2^c + \sum C_iQ_i + C_{8g}O_{8g}$$

$$\mathcal{H}_{\mu}^{(t)} = iQ_p\int d^4x e^{iq\cdot x}\langle P(p)|T[\bar{p}\gamma_{\mu}p(x), H_{eff}^{t}]|\bar{B}(p+q)\rangle$$

$$\mathcal{H}_{\mu}^{(u)} = iQ_p\int d^4x e^{iq\cdot x}\langle P(p)|T[\bar{p}\gamma_{\mu}p(x), H_{eff}^{u}]|\bar{B}(p+q)\rangle$$

◆The nonlocal form factor

$$\mathcal{H}_\mu^{B\to P} = iQ_p \int d^4x e^{iq\cdot x} \langle P(p)|T[\bar{p}\gamma_\mu p(x), C_1 Q_1^p + C_2 Q_2^p + \sum C_i Q_i + C_{8g} O_{8g}]|\bar{B}(p+q)\rangle$$

- Large recoil region $4m_c^2 - q^2 \gg \Lambda m_b$: light-cone OPE

  A. Khodjamirian etc., 1006.4945 N. Gubernari etc., 2011.09813

- Small recoil region $|q^2| = \mathcal{O}(m_b^2)$: local OPE (assuming quark hadron duality)

  B. Grinstein etc., hep-ph/0404250 M Beylich etc., 1101.5118

- Resonance region: violation of quark hadron duality

QCD factorization for $B \to P\ell^+\ell^-(P = K, \pi)$

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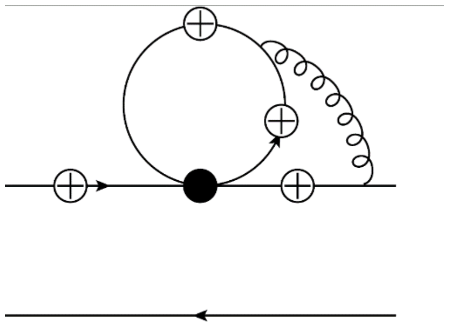
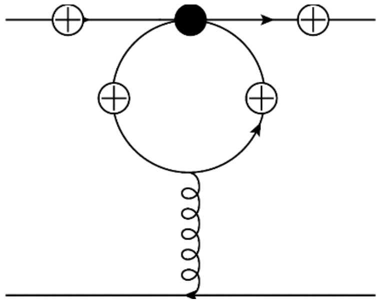
◆The nonlocal form factor: leading power contribution at Large recoil region

$q^2 \in [1\text{–}7]\mathrm{GeV}^2$

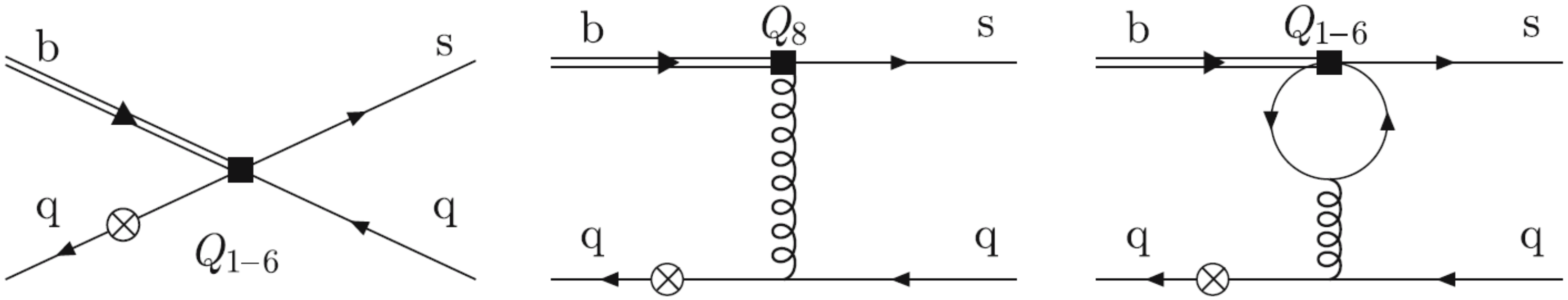
- Soft form factor



- Hard spectator scattering

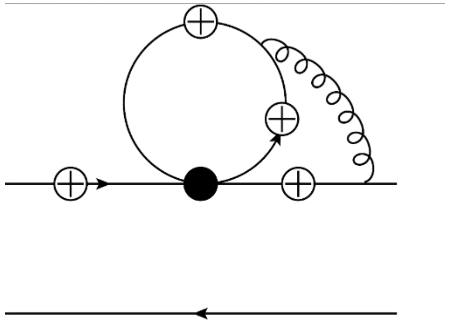


- Annihilation

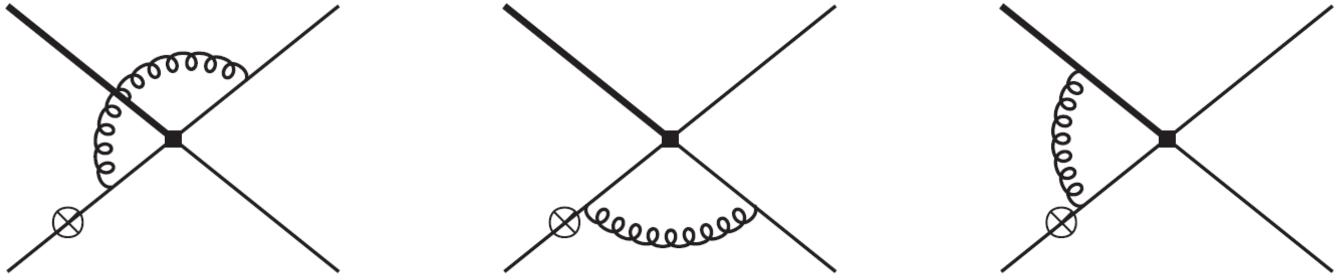


[Beneke etc., hep-ph/0106067; Ali etc. hep-ph/0601034]

◆Arriving at a complete leading power study at $O(\alpha_s)$



Penguin operator insertion



- NLO correction at leading power
- Soft-collinear factorization
- More pronounced for $B \to \pi\ell^+\ell^-$

NLO annihilation contribution

Factorization formula

$$\mathcal{H}_\mu^{(t,u)} = \frac{m_b}{4\pi^2 m_B} \mathcal{T}_P^{(t,u)}(q^2)[q^2(2p+q)_\mu - (m_B^2 - m_K^2)q_\mu]$$

$$\mathcal{T}_{P,\mathrm{anni}}^{(u,t)}(q^2) = -\frac{\pi^2}{N_c}\frac{\tilde{F}_B f_P}{m_B}\sum_{m=\pm}\int_0^\infty \frac{d\omega}{\omega}\int_0^1 du\, T_{P,m}^{t,u}(\omega,u,\mu)\phi_{B,m}(\omega,\mu)\phi_P(u,\mu)$$

$$\Downarrow$$

$$J_m \otimes \sum C_i H_i$$

strategy of the calculation: two step matching

$$\mathrm{QCD} \rightarrow \mathrm{SCET_I} \rightarrow \mathrm{SCET_{II}}$$

$$\Downarrow \qquad \Downarrow$$

$$\sum C_i H_i \qquad J_m$$

NLO annihilation contribution

- hard function

$$Q_i = H_i^I * O^{(A0)} + H_i^{II} * O^{(B1)}$$

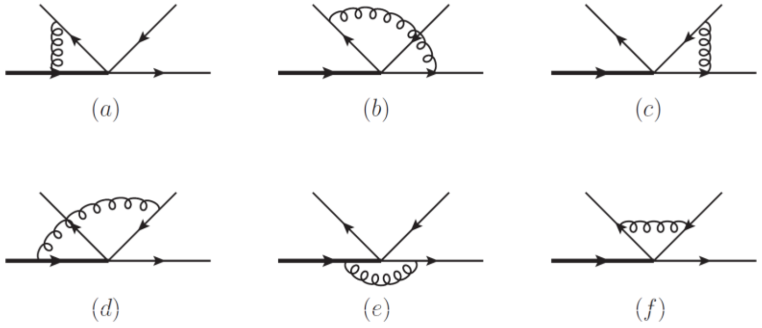
$$\begin{aligned}
\mathcal{O}^{(\mathrm{A0})} &= \left[(\bar{\chi}W_{\bar{c}})\,(t\bar{n})\,\frac{\not{\bar{n}}}{2}\,(1-\gamma_5)\,\left(W_{\bar{c}}^{\dagger}\chi\right)(0)\right] \\
&\quad \left[(\bar{\xi}W_c)\,(0)\;\not{n}\,(1-\gamma_5)\,h_v(0)\right], \\
\mathcal{O}^{(\mathrm{B1})} &= \frac{1}{m_b}\left[(\bar{\chi}W_{\bar{c}})\,(t\bar{n})\,\frac{\not{\bar{n}}}{2}\,(1-\gamma_5)\,\left(W_{\bar{c}}^{\dagger}\chi\right)(0)\right] \\
&\quad \left[(\bar{\xi}W_c)\,(0)\,\frac{\not{n}}{2}\left[W_c^{\dagger}i\,\not{D}_{\perp c}W_c\right](sn)\,(1+\gamma_5)\,h_v(0)\right]
\end{aligned}$$

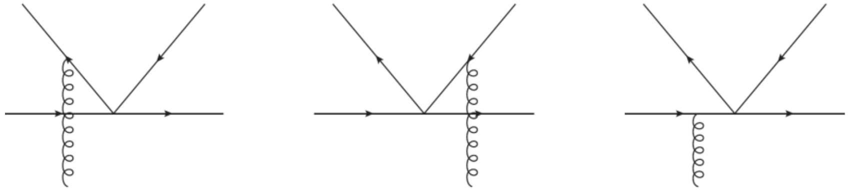
- the method [0911.3665,2002.03262]

$$\begin{aligned}
H_i^{\mathrm{I}(0)} &= A_{i1}^{(0)}\,, \\
H_i^{\mathrm{I}(1)} &= A_{i1}^{(1)} + Z_{\mathrm{ext}}^{(1)}A_{i1}^{(0)} + Z_{ij}^{(1)}A_{j1}^{(0)} - A_{i1}^{(0)}\,Y_{11}^{(1)}
\end{aligned}$$

$$\langle u(l')\bar{u}(\bar{u}p)s(up)g(l)|T\{\int d^4x\mathcal{L}_{\mathrm{QCD}}(x), P_i\}|b(p_b)\rangle^{(0)}$$

$$= H_{ia}^{\mathrm{II}(0)}\langle u(l')\bar{u}(\bar{u}p)s(up)g(l)|O_a^{\mathrm{II}}|b(p_b)\rangle_{\mathrm{FT}}^{(0)}$$

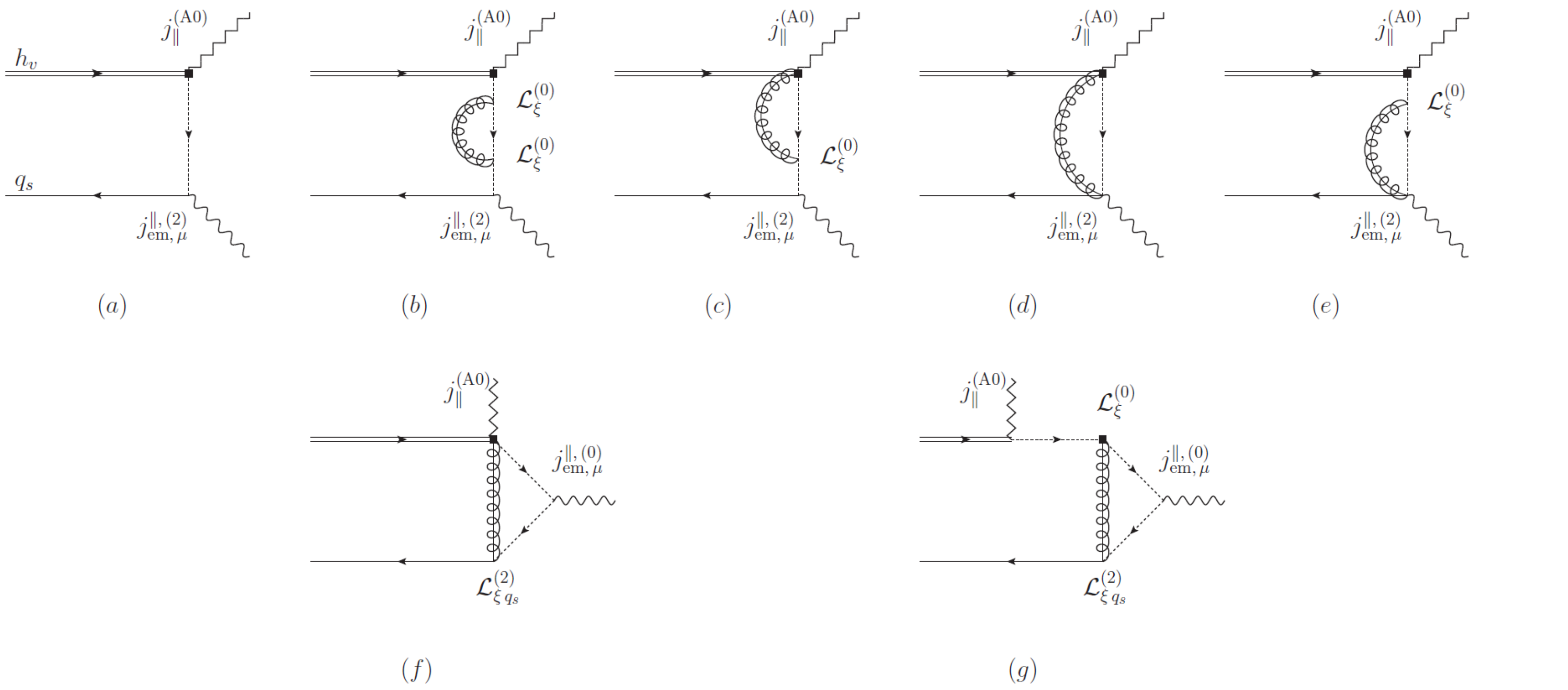




NLO annihilation contribution

- Jet function from A-type operators

$$\left\langle 0\left|T\left\{j_{em}^{(2)}, O^{(A0)}\right\}\right|\bar{B}\right\rangle_{FT} + \left\langle 0\left|T\left\{j_{em}^{(0)}, \int d^4y\, i\mathcal{L}_{\xi q}^{(2)}(y), O^{(A0)}\right\}\right|\bar{B}\right\rangle_{FT} = \frac{1}{2}\tilde{f}_B m_B \frac{J_{-}^{(A0)}}{\bar{n}\cdot q-\omega} \otimes \phi_B^{-}$$



NLO annihilation contribution

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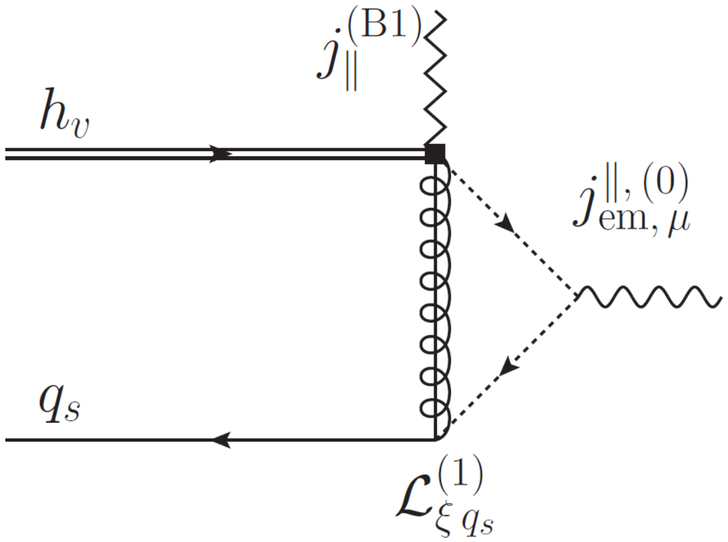
- Jet function from B-type operators

$$\left\langle 0 \left| T\left\{ j_{em}^{(0)}, \int d^4y\, i\mathcal{L}_{\xi q}^{(1)}(y), O^{(B1)} \right\} \right| \bar{B} \right\rangle_{FT} = \frac{1}{2}\tilde{f}_B m_B \sum J_+^{(B1)} \otimes \phi_B^+$$



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Resummation of large logarithms

Hard function

$$\frac{d}{d\ln\mu} C_i H_i(u,\mu) = \left[-\Gamma_{cusp}(\alpha_s)\ln\frac{\mu}{m_b} + \gamma_h(\alpha_s)\right] C_i H_i(u,\mu) - \int_0^1 dv\Gamma_{ERBL}(v,u,\alpha_s)\, C_i H_i(v,\mu)$$

$$\Rightarrow C_i\; H_i(u,\mu) = e^{S_h(\mu_h,\mu)}\left(\frac{\mu_h}{m_b}\right)^{-a(\mu_h,\mu)} \int_0^1 U_{\mathrm{ERBL}}(v,u,\mu_h,\mu)\, C_i H_i(u,\mu_h)$$

Jet function

$$\frac{d}{d\ln\mu}\frac{J_-(n\cdot q,\omega,\mu)}{\bar{n}\cdot q-1} = \left[\Gamma_{cusp}(\alpha_s)\ln\frac{\hat{\mu}^2}{\omega} + \gamma_{hc}(\alpha_s)\right] J_-(n\cdot q,\omega,\mu) - \int d\omega'\omega\Gamma_{LN,-}(\omega',\omega,\alpha_s)\frac{J_-(n\cdot q,\omega,\mu)}{\bar{n}\cdot q-1}$$

$$\Rightarrow \frac{J_-(n\cdot q,\omega,\mu)}{\bar{n}\cdot q-1} e^{S_h(\mu_{hc},\mu)} \int_0^\infty \frac{d\omega'}{\omega'}\left(\frac{\hat{\mu}_{hc}}{\omega'}\right)^{a(\mu_h c,\mu)} U_{LN,-}(\omega',\omega,\mu_{hc},\mu)\frac{J_-(n\cdot q,\omega,\mu c)}{\bar{n}\cdot q-1}$$

NLO annihilation contribution

- Convergence of the convolution

$$\int_0^\infty \frac{J_-(n\cdot q, \bar{n}\cdot q, \omega, \mu)}{\bar{n}\cdot q - \omega}\phi_B^-(\omega) \quad \text{and} \quad \int_0^\infty J_+(n\cdot q, \bar{n}\cdot q, \omega, \mu)\phi_B^+(\omega)$$

- $\omega \to 0$

$$\phi_B^-(\omega) \sim 1, \quad J_-(n\cdot q, \omega, \mu) \sim 1 \qquad \phi_B^+(\omega) \sim \omega, \quad J_+(n\cdot q, \omega, \mu) \sim 1$$

- $\omega \to \infty$

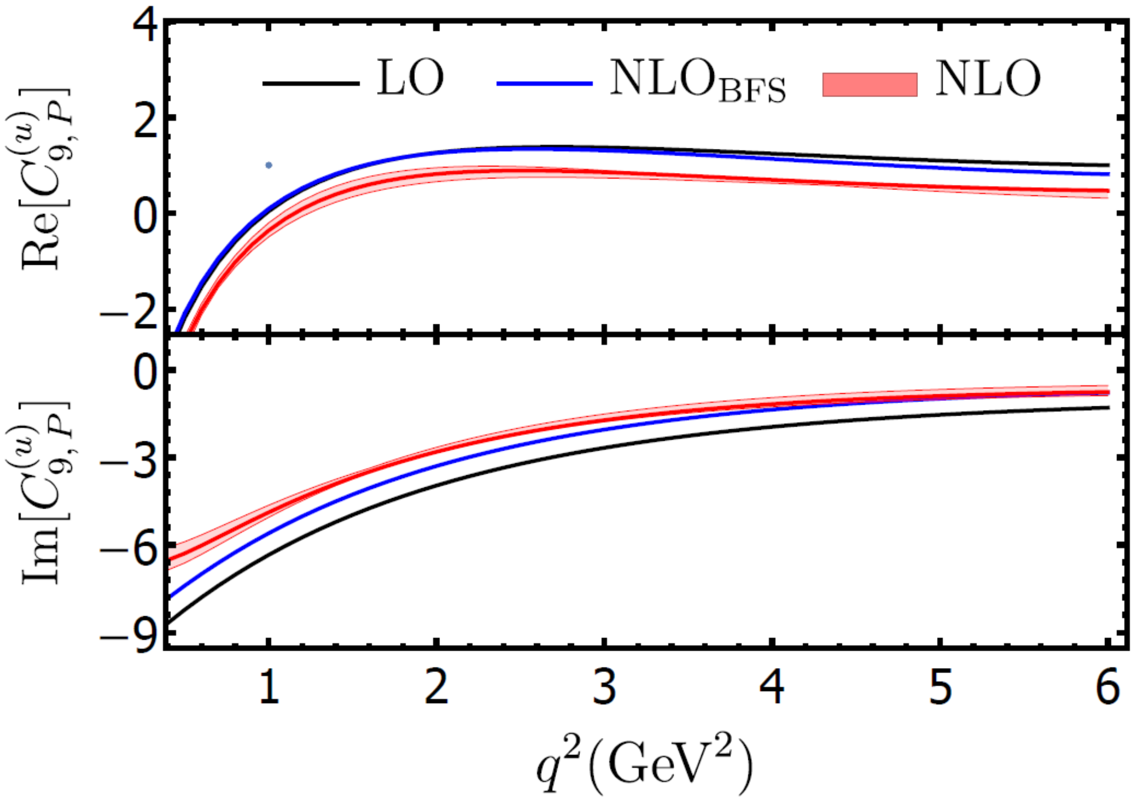
$$\phi_B^\pm(\omega) \sim \frac{1}{\omega}\ln\frac{\mu}{\omega} \qquad J_-(n\cdot q, \bar{n}\cdot q, \omega, \mu) \sim \ln^2\frac{\omega}{\bar{n}\cdot q} \qquad J_+(n\cdot q, \bar{n}\cdot q, \omega, \mu) \sim \frac{1}{\omega}\ln\frac{\omega}{\bar{n}\cdot q}$$

Phenomenology

- effective Wilson coefficient $$C_9^i = C_9\delta^{it} + \frac{2m_b}{m_B}\frac{\mathcal{T}_P^{(i)}}{m_B}, i = u, t$$

- C_9^u in $B \to \pi\ell^+\ell^-$:

NLO WA contribution brings about 35% reduction for real part, and 15% reduction for imaginary part



- C_9^t in $B \to \pi\ell^+\ell^-$:

Only 3% corrections to $\mathcal{T}_P^{(t)}$

Phenomenology

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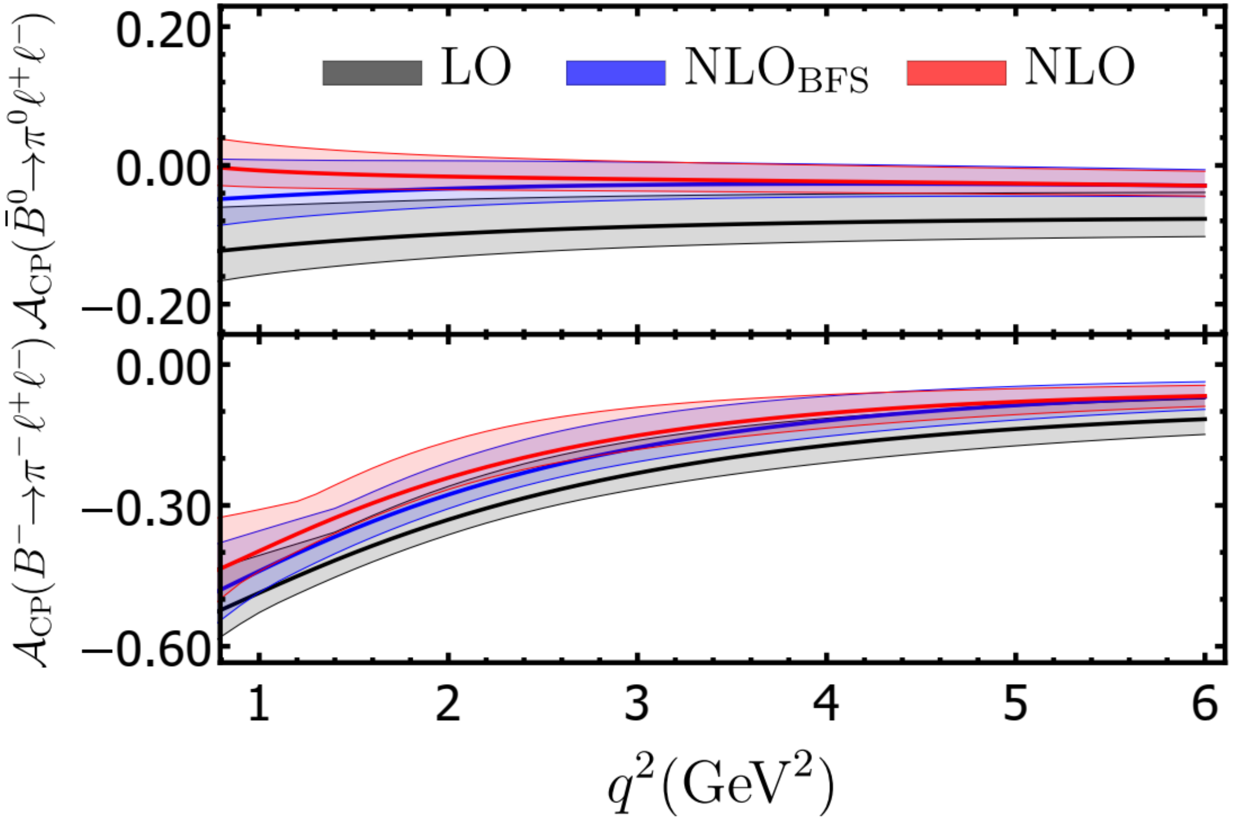
- Branching ratios：

  WA brings negligible corrections

- CP asymmetries in $B \to \pi \ell^+ \ell^-$



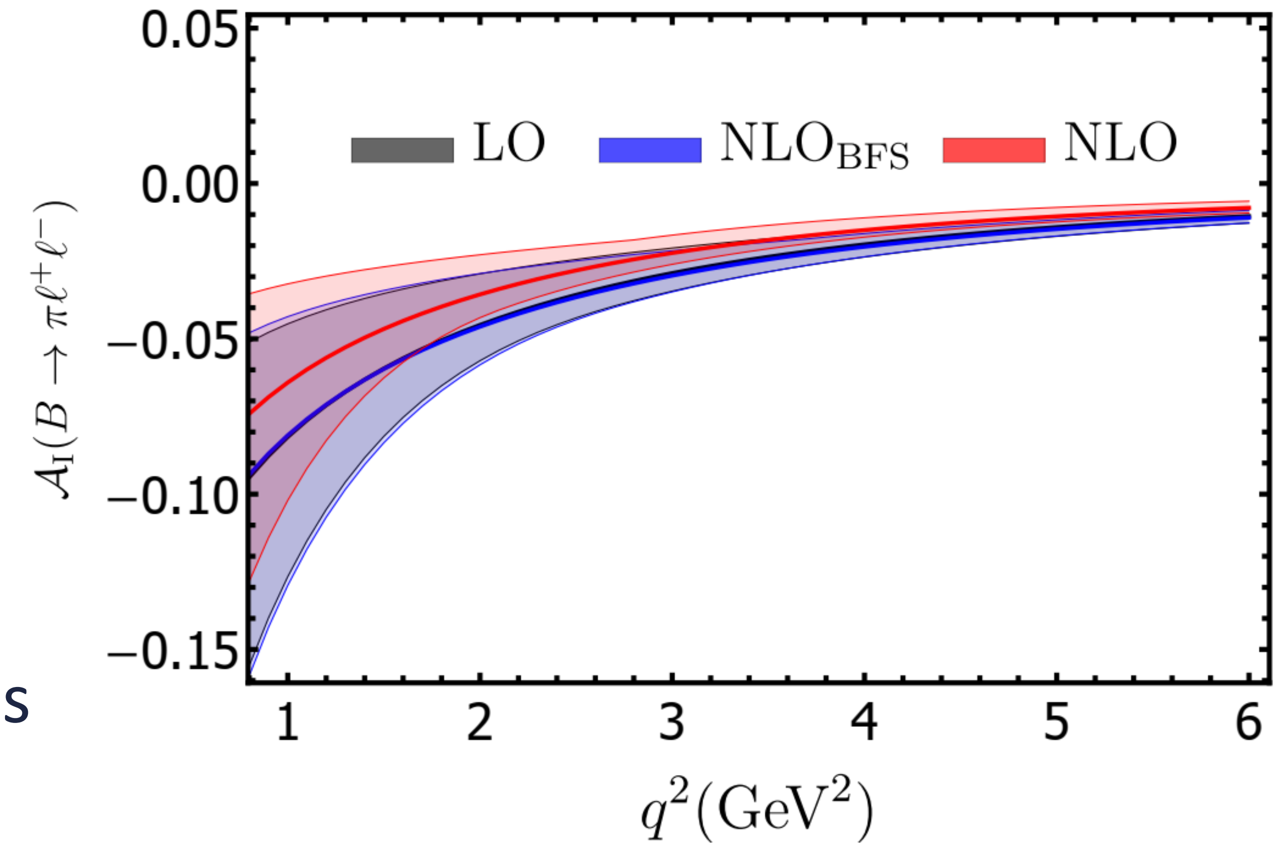
- CP asymmetries in $B \to K \ell^+ \ell^-$

q^2	[1.1,2.0]	[2.0,3.0]	[2.0,3.0]	[4.0,5.0]	[5.0,6.0]
$B^- \to K^- \ell^+ \ell^-$	$1.08^{+0.09}_{-0.26}$	$0.67^{+0.09}_{-0.26}$	$0.44^{+0.12}_{-0.19}$	$0.33^{+0.12}_{-0.14}$	$0.27^{+0.10}_{-0.11}$
$\bar{B}^0 \to \bar{K}^0 \ell^+ \ell^-$	$0.01^{+0.08}_{-0.12}$	$0.04^{+0.07}_{-0.11}$	$0.06^{+0.07}_{-0.10}$	$0.08^{+0.07}_{-0.10}$	$0.09^{+0.07}_{-0.09}$

- Isospin asymmetry

- NLO WA brings about 25% shift of A_I in $B \to \pi\ell^+\ell^-$

- A_I in $B \to \pi\ell^+\ell^-$ is 3-6 times larger than $B \to K\ell^+\ell^-$



Summary and outlook

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- We established the factorization formula for the weak annihilation contribution in $B \to P\ell^+\ell^-$ at leading power, and calculated NLO perturbation functions

- NLO weak annihilation contribution can bring sizable correction to the CP asymmetry and isospin asymmetry.

- The power suppressed contributions, such as long distance quark loop diagram, may have large impact on many observables