Symmetry dictates Operators

From relativistic EFT to HQET

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Outline

Apologize for missing references in this talk

- Amplitude-operator correspondence by symmetry
- Relativistic EFT operators from Poincare symmetry
- Nonlinear Lorentz symmetry and spin symmetry
- Heavy quark effective theory operators from spin symmetry
- Summary and outlook

Symmetry dictates interaction

C.N. Yang: quantization, symmetry, and phase factor

Gauge symmetry dictates interactions

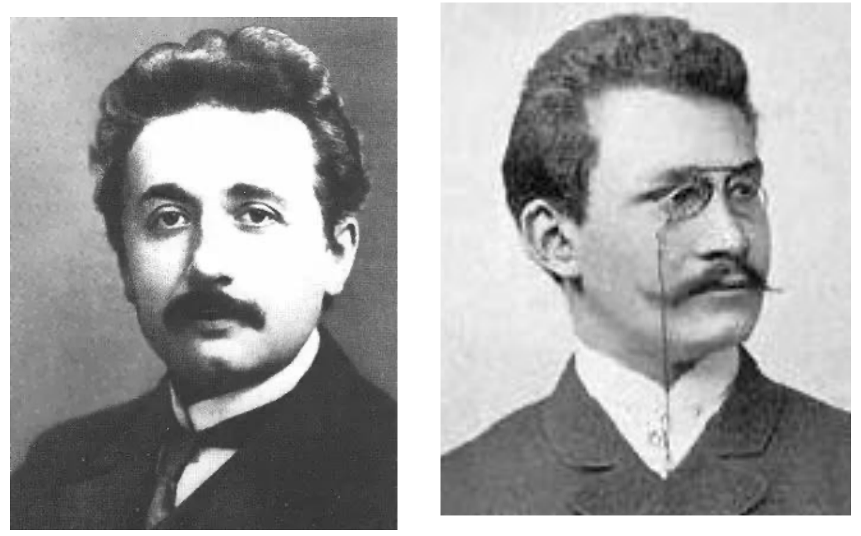
Weyl U(1)

Yang-Mills SU(2)

GSW SU(2) x U(1)

Can spacetime symmetry dictate interactions?

Lorentz symmetry



Minkowski

Poincare symmetry

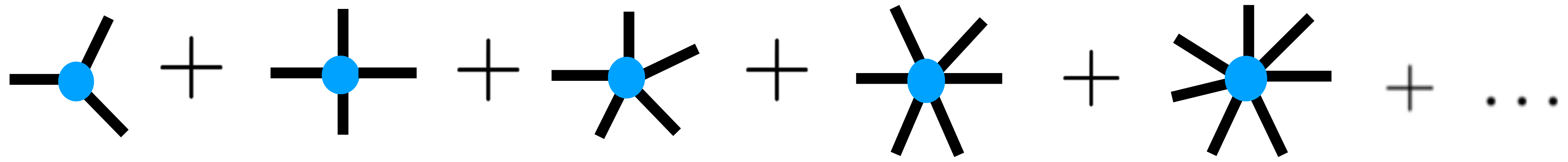
QFT: two equivalent descriptions

Lagrangian based on local field

$$\mathscr{L}_{\rm EFT} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \frac{\mathscr{L}_7}{\Lambda^3} + \frac{\mathscr{L}_8}{\Lambda^4} + \frac{\mathscr{L}_9}{\Lambda^5} + \ldots$$

Expansion with mass dimension

Amplitudes based on on-shell particle

$\mathcal{S} =$ 

Classified by scattering particle numbers

$$\psi(x) = \sum_s \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ip\cdot x} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ip\cdot x} \right]$$

Symmetry is manifest !

Field transform under Lorentz rep

$$A^\mu(x) \to U A^\mu(x) U^\dagger = \left(\Lambda^{-1}\right)^\mu{}_\nu\, A^\nu(\Lambda x) + \Lambda^{-1}\partial^\nu \Omega(\Lambda x)$$

Impose EOM

$$\partial_\mu F^{\mu\nu} + m^2 A^\nu = 0$$
$$\partial_\mu A^\mu = 0$$

Impose gauge condition

$$\epsilon^\mu(\vec{k},\lambda) \to \epsilon^\mu(\vec{k},\lambda) + \frac{k^\mu}{m}$$

Particle transform under Little group

$$\varepsilon_\mu^{IJ} = \frac{\langle p^{(I}|\sigma_\mu|p^{J)}]}{\sqrt{2}m} \qquad \epsilon_\mu^+ = \frac{\langle\zeta|\sigma_\mu|\lambda]}{\sqrt{2}\langle\lambda\zeta\rangle}, \quad \epsilon_\mu^- = \frac{\langle\lambda|\sigma_\mu|\zeta]}{\sqrt{2}[\lambda\zeta]}$$

Satisfy EOM and gauge inv automatically

$$\eta_\alpha \to \eta'_\alpha = a\eta_\alpha + b\lambda_\alpha$$

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Poincare symmetry for scattering amplitudes

Scattering amplitudes transform under the Poincare symmetry

$$\mathcal{M}(p_a, \sigma_a) = \delta^D(p^\mu_{a_1} + \cdots p^\mu_{a_n}) M(p_a, \sigma_a)$$

$$p_{\alpha\dot{\alpha}} \equiv \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} \equiv |p\rangle_\alpha [p|_{\dot{\alpha}}$$

$$\mathbf{p}_{\alpha\dot{\beta}} = \lambda^I_\alpha \tilde{\lambda}_{\dot{\beta}I} = \epsilon_{IJ} |p^I\rangle_\alpha [p^J|_{\dot{\beta}}$$

Translation symmetry

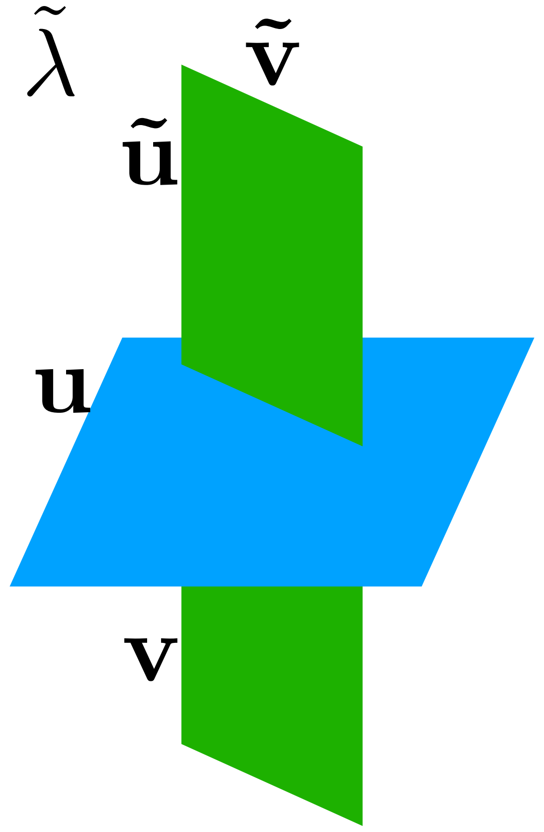
$$f(\{\lambda^i, \widetilde{\lambda}_i\})\, \delta^{(4)}\Big(\sum_{i=1}^{N} \widetilde{\lambda}^{\dot{a}}_i \lambda^{i\,a}\Big)$$

Momentum conservation

$$P^{\dot{a}a} = \sum_{i=1}^{N} \widetilde{\lambda}^{\dot{a}}_i \lambda^{i\,a} = 0$$

$$\begin{pmatrix} \lambda^1_1 & \lambda^2_1 & \cdots & \lambda^N_1 \\ \lambda^1_2 & \lambda^2_2 & \cdots & \lambda^N_2 \end{pmatrix} \xrightarrow{U(N)} \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}$$

$\lambda \perp \tilde{\lambda}$



Lorentz symmetry

$$M^\Lambda(p_a, \sigma_a) = \prod_a \left(D_{\sigma_a \sigma'_a}(W)\right) M((\Lambda p)_a, \sigma'_a)$$

Massless U(1) Little group

$$\lambda \to t\lambda, \quad \tilde{\lambda} \to t^{-1}\tilde{\lambda}$$

$$A(1^{h_1} \cdots n^{h_n}) \to \prod_i t_i^{-2h_i} A(1^{h_1} \cdots n^{h_n})$$

Little group scaling

Massive SU(2) Little group

$$\lambda^I \to W^I_J \lambda^J, \quad \tilde{\lambda}_I \to (W^{-1})^J_I \tilde{\lambda}_J$$

$$\mathcal{M}(p_a, \sigma_a) \to \mathcal{M}^\Lambda(p_a, \sigma_a) = \prod_a (D_{\sigma_a \sigma'_a}(W)) \mathcal{M}((\Lambda p)_a, \sigma'_a)$$

SU(2) transformation

$$\begin{cases} \lambda^I_\alpha & = -\lambda_\alpha \zeta^{-I} + \eta_\alpha \zeta^{+I}, \\ \tilde{\lambda}^I_{\dot{\alpha}} & = \tilde{\lambda}_{\dot{\alpha}} \zeta^{+I} + \tilde{\eta}_{\dot{\alpha}} \zeta^{-I}, \end{cases}$$

$$\mathbf{p}_{\alpha\dot{\alpha}} \equiv p_{\alpha\dot{\alpha}} + \eta_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} + \eta_\alpha \tilde{\eta}_{\dot{\alpha}}.$$

3-point amplitudes (massless) dictates interactions

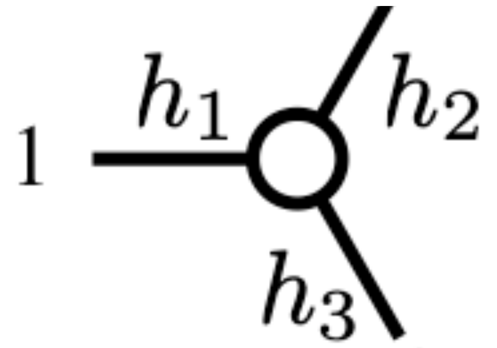
The building blocks of scattering amplitudes are 3-point amplitudes

Mom. Conserv. + Little group scaling

[Benincasa, Cachazo, 2007]

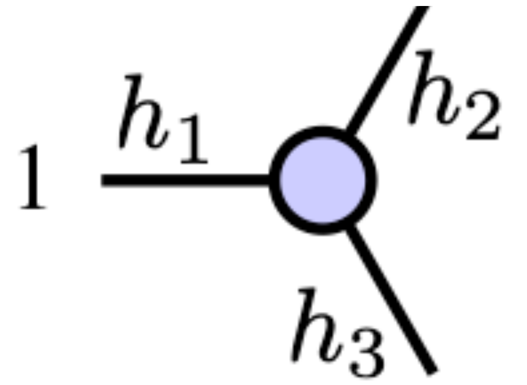
[Witten, 2003]

$\tilde{\lambda}_1 \propto \tilde{\lambda}_2 \propto \tilde{\lambda}_3$;



$\langle 12\rangle^{h_3-h_1-h_2}\langle 23\rangle^{h_1-h_2-h_3}\langle 31\rangle^{h_2-h_3-h_1}$

$\lambda_1 \propto \lambda_2 \propto \lambda_3$



$[12]^{h_1+h_2-h_3}[23]^{h_2+h_3-h_1}[31]^{h_3+h_1-h_2}$

QED Theory $(--+)$

Massless

$$A\left(1^{-\frac{1}{2}}2^{+\frac{1}{2}}3^{+}\right) = \frac{\langle 13\rangle^2}{\langle 12\rangle}$$

$$A\left(1^{+\frac{1}{2}}2^{-\frac{1}{2}}3^{-}\right) = \frac{[13]^2}{[12]}$$

$$\bar{\psi}\gamma^\mu\psi A_\mu$$

Yang-Mills Theory $(--+)$

$$A\left(1_a^- 2_b^- 3_c^+\right) = f_{abc}\frac{\langle 12\rangle^3}{\langle 13\rangle\langle 32\rangle}$$

$$A\left(1_a^+ 2_b^+ 3_c^-\right) = f_{abc}\frac{[12]^3}{[13][32]}$$

$$f_{abc}\left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a\right)A^{\mu b}A^{\nu c}$$

Einstein Theory $(--+)$

$$A(1^{--}2^{--}3^{++}) = \frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 32\rangle^2}$$

$$A(1^{++}2^{++}3^{--}) = \frac{[12]^6}{[13]^2[32]^2}$$

Yukawa Theory $(--0)$

Massless

$$A\left(1^{+\frac{1}{2}}2^{+\frac{1}{2}}3^{0}\right) = \langle 12\rangle$$

$$A\left(1^{-\frac{1}{2}}2^{-\frac{1}{2}}3^{0}\right) = [12]$$

$$\bar{\psi}\psi\phi$$

Amplitude-interaction correspondence

3-point amplitudes (massless) dictates Operators

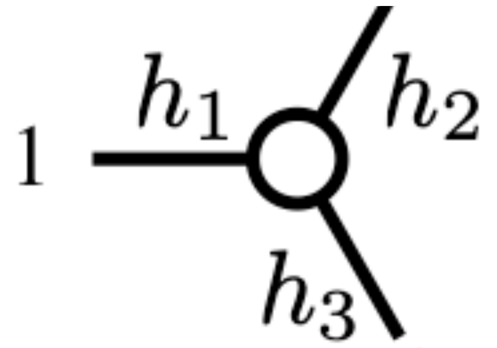
Also determines non-renormalizable interactions involving in 3-particles

Mom. Conserv. + Little group scaling

[Benincasa, Cachazo, 2007]

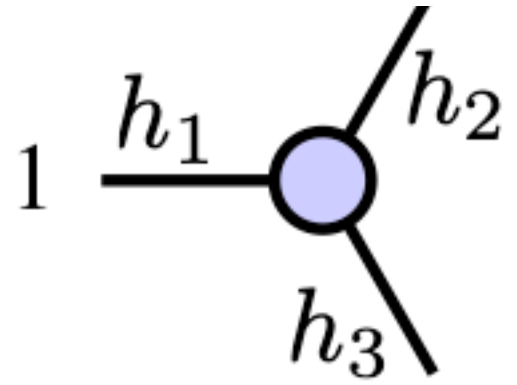
[Witten, 2003]

$\tilde\lambda_1 \propto \tilde\lambda_2 \propto \tilde\lambda_3$;



$\langle 12\rangle^{h_3-h_1-h_2}\langle 23\rangle^{h_1-h_2-h_3}\langle 31\rangle^{h_2-h_3-h_1}$

$\lambda_1 \propto \lambda_2 \propto \lambda_3$



$[12]^{h_1+h_2-h_3}[23]^{h_2+h_3-h_1}[31]^{h_3+h_1-h_2}$

$(- - -)$ **Magnetic dipole QED**

Massless

$$A\left(1^{+\frac{1}{2}}2^{+\frac{1}{2}}3^{+}\right) = \langle 13\rangle\langle 23\rangle$$
$$A\left(1^{-\frac{1}{2}}2^{-\frac{1}{2}}3^{-}\right) = [13][23]$$

$$\bar\psi\sigma^{\mu\nu}\psi F_{\mu\nu}$$

$(- - -)$ **Dim-6 Yang-Mills**

$$A(1_a^- 2_b^- 3_c^-) = f_{abc}\langle 12\rangle\langle 23\rangle\langle 31\rangle$$
$$A(1_a^+ 2_b^+ 3_c^+) = f_{abc}[12][23][31],$$

$$f_{abc}F_\mu^{a\nu}F_\nu^{b\rho}F_\rho^{c\mu}$$

$(- - -)$ **Modified Gravity**

$$A(1^{--}2^{--}3^{--}) = (\langle 12\rangle\langle 23\rangle\langle 31\rangle)^2$$
$$A\left(1^{++}2^{++}3^{++}\right) = ([12][23][31])^2$$

$$R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\alpha\beta}R_{\alpha\beta}{}^{\mu\nu}$$

$(- + 0)$ **x**

$$A\left(1^{+\frac{1}{2}}2^{-\frac{1}{2}}3^{0}\right) = g^{-}\frac{\langle 13\rangle}{\langle 23\rangle} + g^{+}\frac{[13]}{[23]}$$

inconsistent Locality

Amplitude-operator correspondence

Massive 3-point amplitudes

No simple little group scaling, momentum conservation complicated

[Arkani-Hamed, Huang, Huang, 2017]

$$M^{I_1\ldots I_{2s_1} J_1\ldots J_{2s_2}}_{K_1\ldots K_{2s_3}} = \left(\lambda^{I_1}_{1\alpha_1}\ldots\lambda^{I_{2s_1}}_{1\alpha_{2s_1}}\right)\left(\lambda^{J_1}_{2\alpha_1}\ldots\lambda^{I_{2s_2}}_{2\alpha_{2s_2}}\right)\left(\lambda^{\gamma_1}_{3K_1}\ldots\lambda^{\gamma_{2s_3}}_{3K_{2s_1}}\right)\bar{M}^{\alpha_1\ldots\alpha_{2s_1}\beta_1\ldots\beta_{2s_2}}_{\gamma_1\ldots\gamma_{2s_3}}$$

Find 2 linearly independent basis span the SL(2,C) space

$$\mathscr{I}^{(\alpha_1\cdots\alpha_{2s_1}),(\beta_1\cdots\beta_{2s_2}),(\gamma_1\cdots\gamma_{2s_3})} = \sum_{i=0}^{1}\sum_{\sigma_i} g_{\sigma_i}\left(\mathscr{O}^{s_1+s_2+s_3-i}\varepsilon^i\right)_{\sigma_i}^{(\alpha_1\cdots\alpha_{2s_1}),(\beta_1\cdots\beta_{2s_2}),(\gamma_1\cdots\gamma_{2s_3})}$$

symmetric tensor $\mathscr{O}$, antisymmetric tensor ε

QED Theory

$\langle\mathbf{13}\rangle[\mathbf{23}]$ $[\mathbf{13}]\langle\mathbf{23}\rangle$

$[\mathbf{13}][\mathbf{23}]$ $\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle$

$\bar{\psi}_1\gamma^\mu P_R\psi_2\varepsilon_{3,\mu}$

$\bar{\psi}_1\sigma^{\mu\nu}P_R\psi_2\varepsilon_{3,\mu}p_{3,\nu}$

Electroweak Theory

$\langle\mathbf{12}\rangle[\mathbf{13}][\mathbf{23}] + [\mathbf{12}]\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{12}][\mathbf{13}]\langle\mathbf{23}\rangle$

$[\mathbf{12}][\mathbf{13}][\mathbf{23}]$ $\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle$

$\langle\mathbf{12}\rangle\left(\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{13}]\langle\mathbf{23}\rangle\right)$

$[\eta_{\mu\nu}(p_1-p_2)_\rho + \eta_{\nu\rho}(p_2-p_3)_\mu + \eta_{\rho\mu}(p_3-p_1)_\nu]\,\varepsilon_1^\mu\varepsilon_2^\nu\varepsilon_3^\rho$

X

Just list Lorentz structures

Yukawa Theory

$[\mathbf{12}]$ $\langle\mathbf{12}\rangle$

Any spacetime symmetry to guarantee exhaust all possible structure?

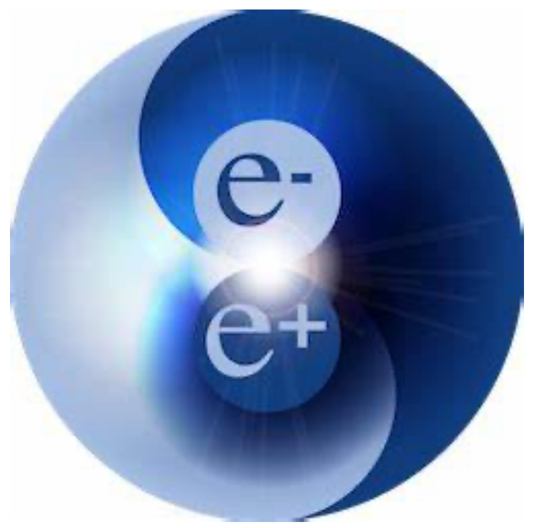
Spacetime symmetry extension

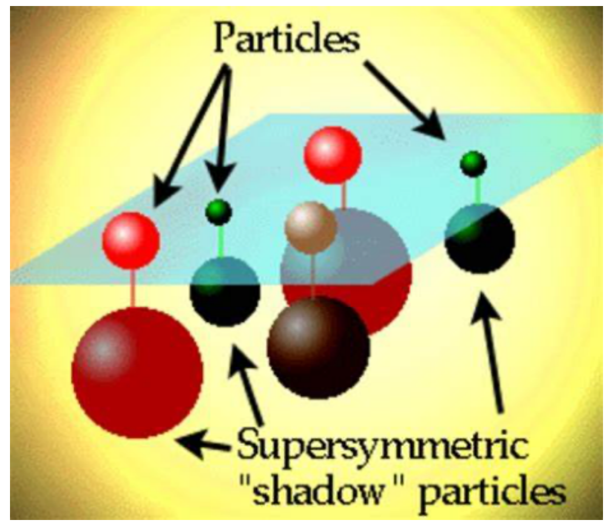
Spacetime and internal symmetry can't be mixed except supersymmetry, massless CFT, discrete, SSB theory

[Yu-Han Ni, Chao Wu, Yi-Ning Wang, **J.H.Yu**, 2412.03762]

Superspace (Grassmann)

$\phi \longleftrightarrow \psi$





$(\lambda, \tilde{\lambda}, \eta^A)$

$\{\eta^A, \partial_{\eta^B}\} = \delta^A_B$

$(\lambda, \tilde{\lambda}, \eta, \bar{\eta})$

Hyperspace (spinor)

$\lambda^I_\alpha \longleftrightarrow \tilde{\lambda}^I_{\dot{\alpha}}$

$(\lambda, \tilde{\lambda}, \eta, \tilde{\eta})$

p_μ , λ^I_α $\longrightarrow$ $P_M = (p_\mu, m, \tilde{m})$, $\lambda^I_A = \{\lambda^I_\alpha, \tilde{\lambda}^{\dot{\alpha} I}\}$

SO(5,1)/[SO(2) x SO(3,1)]

Internal space

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu \qquad \left\{ [p|^a \frac{\partial}{\partial \eta}, |p\rangle^{\dot{a}} \eta \right\} = |p\rangle^{\dot{a}} [p|^a$$

with R-charge

$$Q_\alpha \to e^{-i\lambda} Q_\alpha \quad \text{and} \quad \bar{Q}_{\dot{\alpha}} \to e^{i\lambda} \bar{Q}_{\dot{\alpha}} \qquad \eta \to e^{i\lambda} \eta$$

$$[R, Q_\alpha] = -Q_\alpha \quad \text{and} \quad [R, \bar{Q}_{\dot{\alpha}}] = +\bar{Q}_{\dot{\alpha}}$$

Internal space

$$[m, \tilde{m}] = 0. \qquad m \equiv \frac{1}{2}\lambda_{\alpha I}\lambda^{\alpha I}, \quad \tilde{m} \equiv \frac{1}{2}\tilde{\lambda}_{\dot{\alpha} I}\tilde{\lambda}^{\dot{\alpha} I}.$$

with transversality-charge

$$m \to e^{-i\phi} m, \quad \tilde{m} \to e^{i\phi} \tilde{m}.$$

$$[D_-, m] = -m, \quad [D_-, \tilde{m}] = +\tilde{m}$$

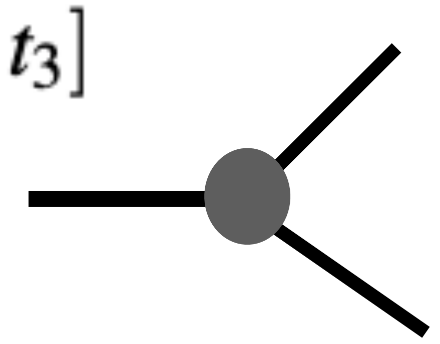
Relate to chirality

Massive 3-point amplitudes again

SO(5,1) isomorphic to conformal group: highest weight rep by scaling, then descendent ones

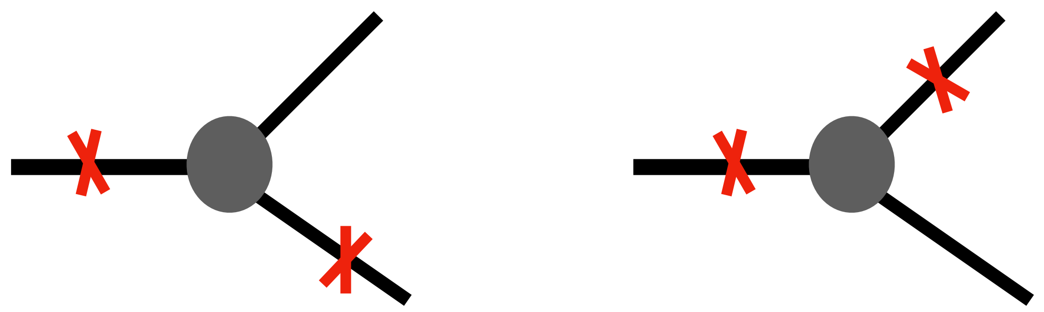
[Yu-Han Ni, Chao Wu, Yi-Ning Wang, **J.H.Yu**, 2412.03762]

$[t_1, t_2, t_3]$



$T_i^- \cdot T_j^-$

Chirality flip



$[s_1, s_2, s_3]: \quad [\mathbf{12}]^{\Delta s_3}[\mathbf{23}]^{\Delta s_1}[\mathbf{31}]^{\Delta s_2}$

$[s_1-1, s_2-1, s_3]: \quad \langle\mathbf{12}\rangle[\mathbf{12}]^{\Delta s_3-1}[\mathbf{23}]^{\Delta s_1}[\mathbf{31}]^{\Delta s_2}$

$[s_1, s_2-1, s_3-1]: \quad \langle\mathbf{23}\rangle[\mathbf{12}]^{\Delta s_3}[\mathbf{23}]^{\Delta s_1-1}[\mathbf{31}]^{\Delta s_2}$

$[s_1-1, s_2, s_3-1]: \quad \langle\mathbf{31}\rangle[\mathbf{12}]^{\Delta s_3}[\mathbf{23}]^{\Delta s_1}[\mathbf{31}]^{\Delta s_2-1}$

QED Theory | **Electroweak Theory** | **X** | **Yukawa Theory**

Massive

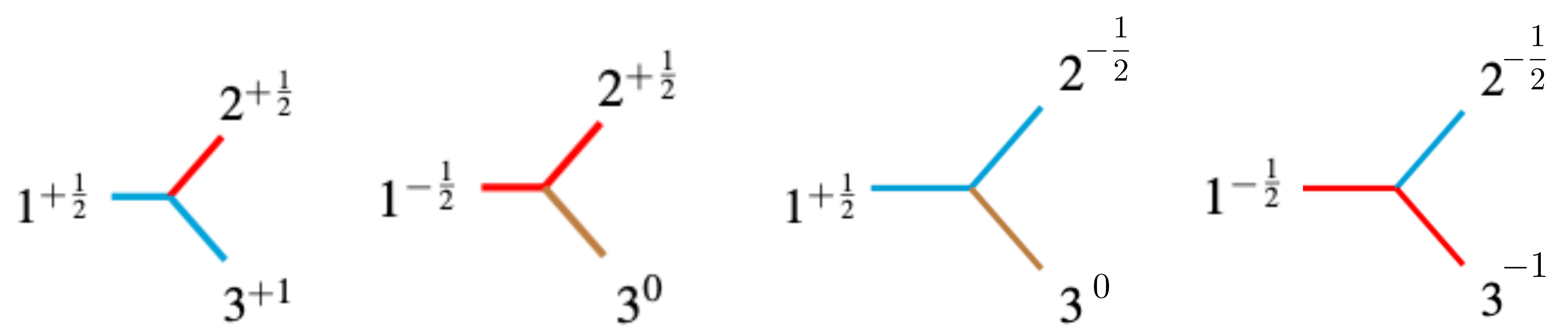
Massive

$\langle\mathbf{12}\rangle[\mathbf{13}][\mathbf{23}] + [\mathbf{12}]\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{12}][\mathbf{13}]\langle\mathbf{23}\rangle$

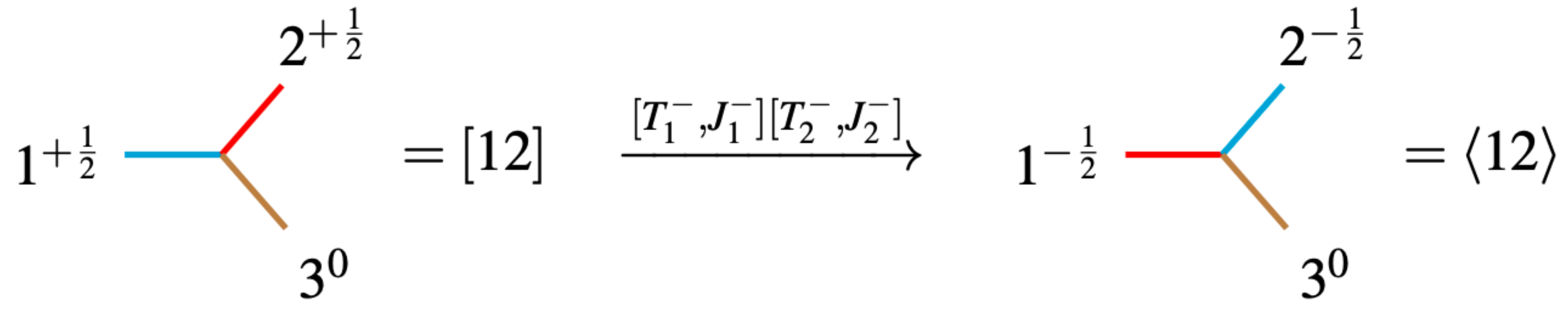
$[\mathbf{12}][\mathbf{13}][\mathbf{23}] \quad \langle\mathbf{12}\rangle\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle$

$\langle\mathbf{12}\rangle\left(\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{13}]\langle\mathbf{23}\rangle\right)$

Massive



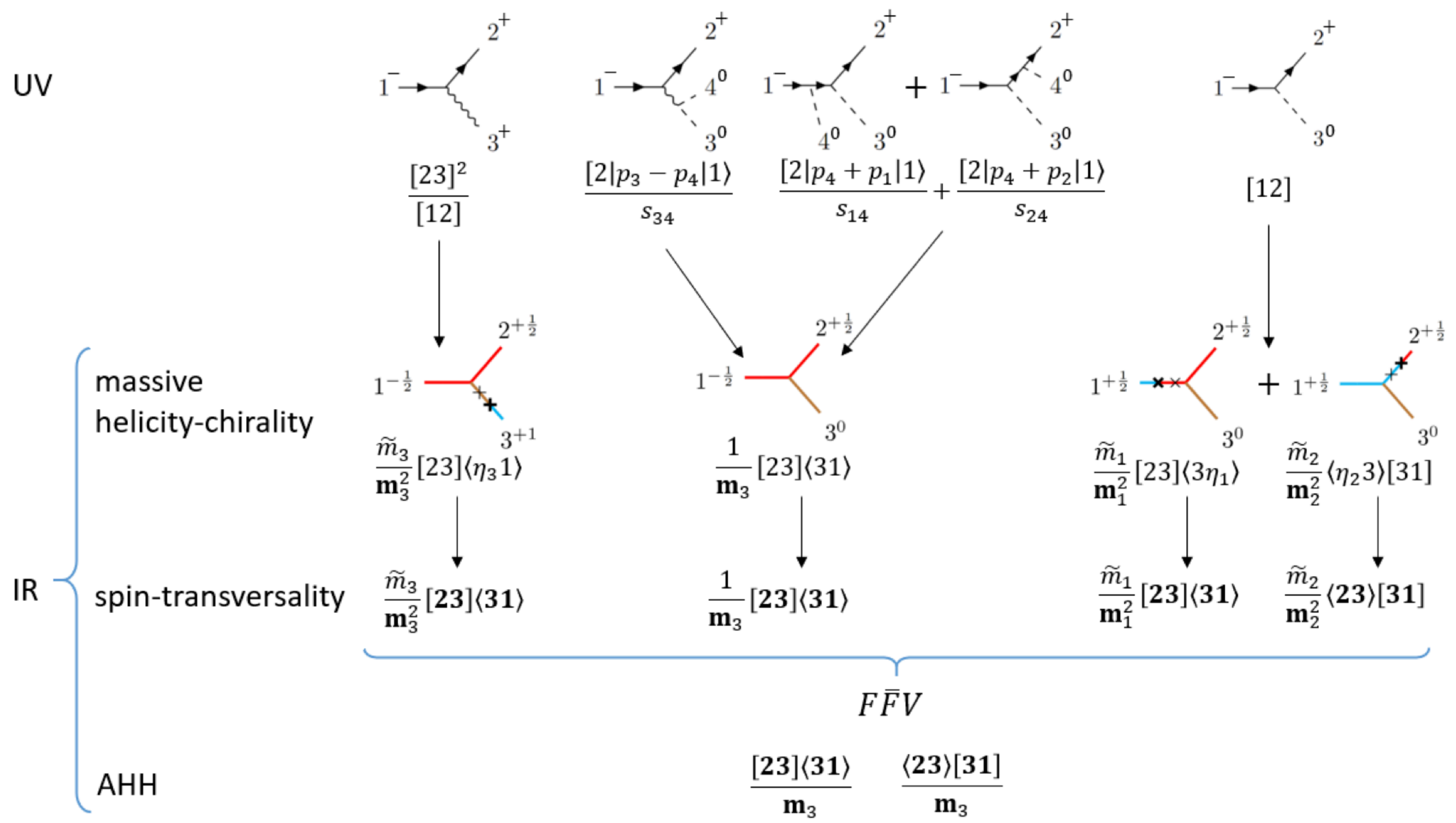
$[23][31] \quad [23]\langle 31\rangle \quad \langle 23\rangle[31] \quad \langle 23\rangle\langle 31\rangle$



Massless-massive correspondence

On-shell Higgs mechanism: massless Goldstone bosons are eaten by longitudinal gauge boson

[Yu-Han Ni, Chao Wu, Yi-Ning Wang, **J.H.Yu**, 2501.09062]



U(N) symmetry extension

In above, 3-pt amplitude dictates operators with 3-particles, how about operators with n-particles?

SUSY with U(N) R symmetry

$$(\lambda, \tilde{\lambda}, \eta, \bar{\eta})$$

$$(\lambda, \tilde{\lambda}, \eta^A) \qquad \{\eta^A, \partial_{\eta^B}\} = \delta^A_B$$

Chiral superspace

$$\{Q_{\alpha A}, Q^{\dagger B}_{\dot{\beta}}\} = -2\delta^B_A(\sigma^\mu_{\alpha\dot{\beta}})P_\mu$$

$$\{Q_{\alpha A}, Q_{\beta B}\} = Z_{AB}\epsilon_{\alpha\beta} \qquad Z_{AB} = 0$$

SU(N) R symmetry (no central charge)

Spinor with U(N) symmetry

$$(\lambda, \tilde{\lambda}, \eta, \tilde{\eta})$$

$$\begin{pmatrix} \lambda_1^1 & \lambda_1^2 & \cdots & \lambda_1^N \\ \lambda_2^1 & \lambda_2^2 & \cdots & \lambda_2^N \end{pmatrix} \xrightarrow{U(N)} \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}$$

$$\downarrow SL(2,\mathbb{C})$$

$$\begin{pmatrix} \lambda_a^1 & \lambda_a^2 & \cdots & \lambda_a^N \end{pmatrix}$$

U(1)^n little group → **SL(2,C) x SU(N)**

$$\lambda \to t\lambda, \quad \tilde{\lambda} \to t^{-1}\tilde{\lambda}$$

$$A(1^{h_1} \cdots n^{h_n}) \to \prod_i t_i^{-2h_i} A(1^{h_1} \cdots n^{h_n})$$

$$\lambda_i \to \sum_j U_i^j \lambda_j, \quad \tilde{\lambda}^i \to \sum_k U^{\dagger i}{}_k \tilde{\lambda}^k,$$

$$K_{\dot{a}a} = -\sum_{i=1}^N \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{a}}} \frac{\partial}{\partial \lambda^{ia}}$$

$$f(\{\lambda^i, \tilde{\lambda}_i\})\,\delta^{(4)}\Big(\sum_{i=1}^N \tilde{\lambda}_i^{\dot{a}} \lambda^{ia}\Big)$$

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Operator-amplitude correspondence

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2201.04639]

Standard model effective field theory

SMEFT provides systematic parametrization on all possible Lorentz-invariant new physics

$$\mathscr{L}_{\rm EFT} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \frac{\mathscr{L}_7}{\Lambda^3} + \frac{\mathscr{L}_8}{\Lambda^4} + \frac{\mathscr{L}_9}{\Lambda^5} + \cdots$$

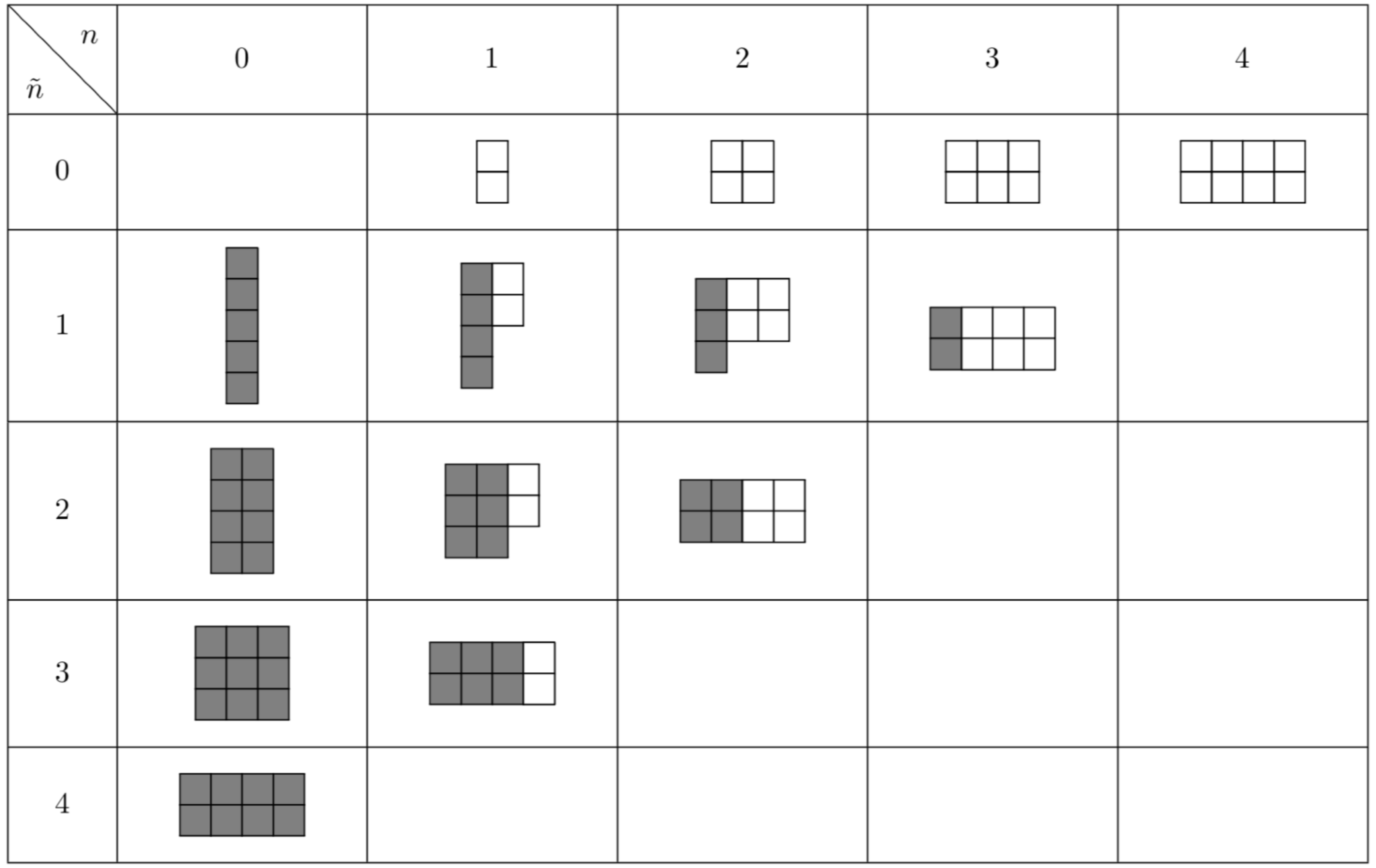
Standard Model	Weinberg Operator	84	30	993	560

Weinberg Operator: $\frac{c_{ij}}{\Lambda}(L_iH)(L_jH) + \text{h.c.}$

Dim-8: 993 operators from 16 Young diagrams

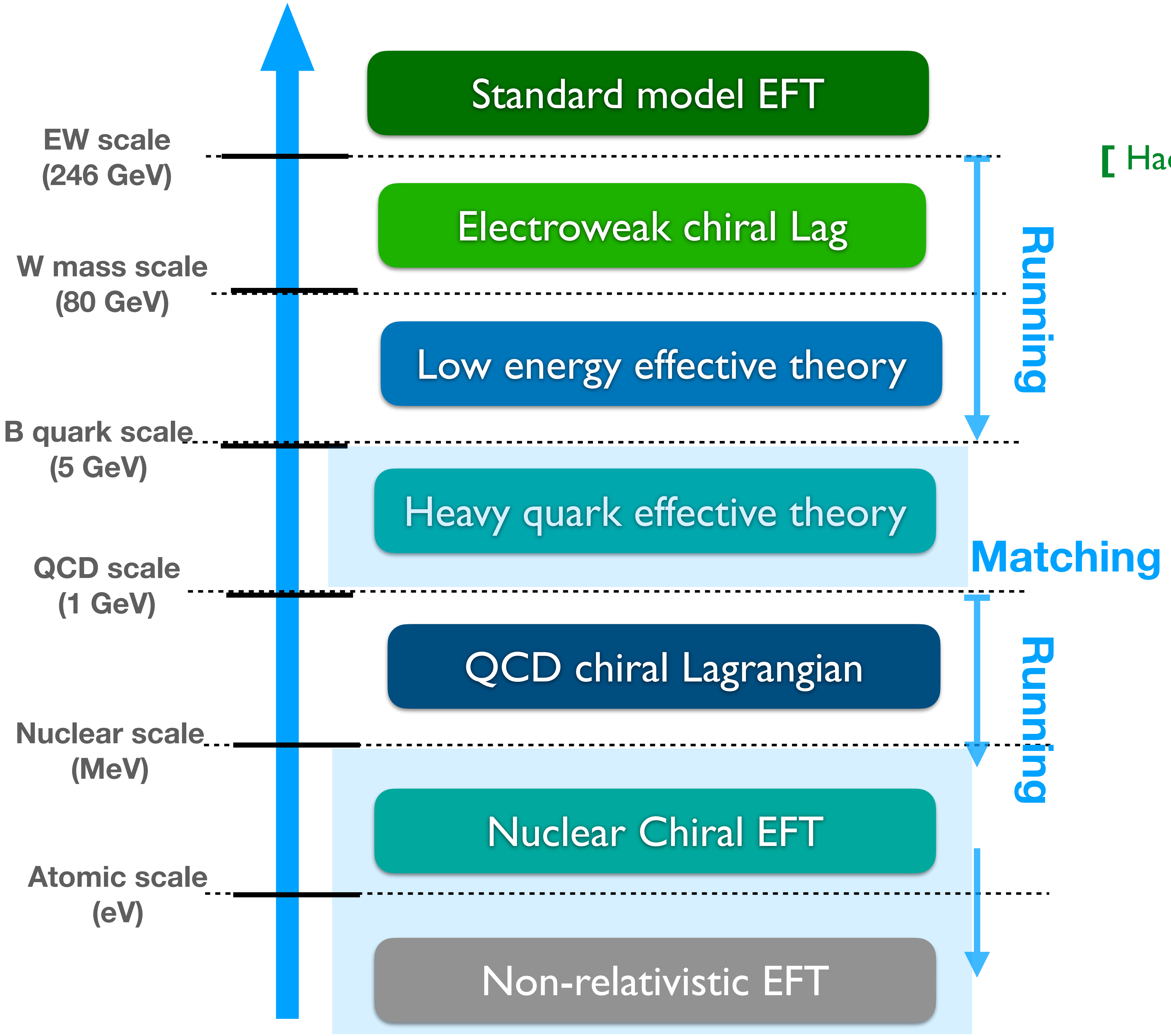
[Hao-Lin Li, Jing Shu, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2005.00008]

$\bar{n}$ \ n	0	1	2	3	4
0	ϕ^8	$\psi^2\phi^5$	$\psi^4\phi^2$, $F_{\rm L}\psi^2\phi^3$, $F_{\rm L}^2\phi^4$	$F_{\rm L}\psi^4$, $F_{\rm L}^2\psi^2\phi$, $F_{\rm L}^3\phi^2$	$F_{\rm L}^4$
1	$\psi^{\dagger 2}\phi^5$	$\psi^{\dagger 2}\psi^2\phi^2$, $\psi^\dagger\psi\phi^4 D$, $\phi^6 D^2$	$F_{\rm L}\psi^{\dagger 2}\psi^2$, $F_{\rm L}^2\psi^{\dagger 2}\phi$, $\psi^\dagger\psi^3\phi D$, $F_{\rm L}\psi^\dagger\psi\phi^2 D$, $\psi^2\phi^3 D^2$, $F_{\rm L}\phi^4 D^2$	$F_{\rm L}^2\psi^\dagger\psi D$, $\psi^4 D^2$, $F_{\rm L}\psi^2\phi D^2$, $F_{\rm L}^2\phi^2 D^2$	
2	$\psi^{\dagger 4}\phi^2$, $F_{\rm R}\psi^{\dagger 2}\phi^3$, $F_{\rm R}^2\phi^4$	$F_{\rm R}\psi^{\dagger 2}\psi^2$, $F_{\rm R}^2\psi^2\phi$, $\psi^{\dagger 3}\psi\phi D$, $F_{\rm R}\psi^\dagger\psi\phi^2 D$, $\psi^{\dagger 2}\phi^3 D^2$, $F_{\rm R}\phi^4 D^2$	$F_{\rm R}^2 F_{\rm L}^2$, $F_{\rm R}F_{\rm L}\psi^\dagger\psi D$, $\psi^{\dagger 2}\psi^2 D^2$, $F_{\rm R}\psi^2\phi D^2$, $F_{\rm L}\psi^{\dagger 2}\phi D^2$, $F_{\rm R}F_{\rm L}\phi^2 D^2$, $\phi^4 D^4$, $\psi^\dagger\psi\phi^2 D^3$		
3	$F_{\rm R}\psi^{\dagger 4}$, $F_{\rm R}^2\psi^{\dagger 2}\phi$, $F_{\rm R}^3\phi^2$	$F_{\rm R}^2\psi^\dagger\psi D$, $\psi^{\dagger 4} D^2$, $F_{\rm R}\psi^{\dagger 2}\phi D^2$, $F_{\rm R}^2\phi^2 D^2$			
4	$F_{\rm R}^4$				



Tower of EFTs from EW to low scales

Describe collider physics, flavor physics, hadronic physics, nuclear physics and atomic physics



[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2201.04639]

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[Chuan-Qiang Song, Hao Sun, **J.H.Yu**, 2404.15047]

[Xuan-He Li, Hao Sun, Feng-Jie Tang, **J.H.Yu**, 2404.14152]

[Chuan-Qiang Song, Hao Sun, **J.H.Yu**, 2501.09787]

[Hao Sun, Yi-Ning Wang, **J.H.Yu**, 2501.14018]

[Yong-Kang Li, Yi-Ning Wang, **J.H.Yu**, in préparation]

Scattering amplitudes at low energy

The spin symmetry (little group) plays essential roles to describe physical particles

$$\psi(x) = \sum_s \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ip\cdot x} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ip\cdot x}\right]$$

Field transform under SU(2)A x SU(2)B Lorentz rep

non-unitary rep

Particle transform under SU(2)j Little group

unitary irreps

Weinberg: Wave function is just CG Coefficients of two groups

$$u_{ab}(0,\sigma) = (2m)^{-1/2} C_{AB}(j\sigma; ab)$$

[Yi-Ning Wang, **J.H.Yu**, in préparation]

3-particle amplitudes can be determined by CG Coefficients from spin-orbital couple and then boost back

$$\langle \vec{p_1}, \sigma_1 | \langle \vec{p_2}, \sigma_2 | \vec{P}, J, \sigma \rangle = C^{J,\sigma}_{l,\sigma_l; s,\sigma_s} C^{s,\sigma_s}_{s_1,\sigma_1; s_2,\sigma_2} \; Y_{L\sigma_l}(\Omega) W(L_P^{-1}, p_1)^{\sigma_1}_{\sigma_1'} W(L_P^{-1}, p_2)^{\sigma_2}_{\sigma_2'}$$

$$\mathcal{A}^{\sigma_2\sigma_3}_{\sigma_1}(\mathbf{k}_1, \mathbf{p}_2^*, \mathbf{p}_3^*) = \underbrace{\Gamma^{\alpha_2\alpha_3}_{\alpha_1}(\mathbf{k}_1, \mathbf{p}_2^*, \mathbf{p}_3^*)}_{\text{pure-orbital part}} \times \underbrace{\bar{u}^{\alpha_1}_{\sigma_1}(s_1)\, u^{\sigma_2}_{\alpha_2}(\mathbf{p}_2^*, s_2)\, u^{\sigma_3}_{\alpha_3}(\mathbf{p}_3^*, s_3)}_{\text{pure-spin part}} = \underbrace{\Gamma^{\beta_2\beta_3}_{\alpha_1}(\mathbf{k}_1, \mathbf{p}_2^*, \mathbf{p}_3^*)\, D_{\beta_2}^{\ \alpha_2}(\Lambda_{2^*})\, D_{\beta_3}^{\ \alpha_3}(\Lambda_{3^*})}_{\text{pure-orbital part}} \times \underbrace{\bar{u}^{\alpha_1}_{\sigma_1}(s_1)\, u^{\sigma_2}_{\alpha_2}(s_2)\, u^{\sigma_3}_{\alpha_3}(s_3)}_{\text{pure-spin part}}$$

B.S.Zou and D.V.Bugg, Eur.Phys.J.A,16,537-547 (2003)

3-point covariant spin-orbital amplitudes

The spin symmetry in spinor-helicity provides spin-orbital covariant decomposition without the need of boost

Spin space projection + covariant orbital

[Yi-Ning Wang, **J.H.Yu**, in préparation]

$$M^{I_1^{(1)}\dots I_{2s_1}^{(1)} I_1^{(2)}\dots I_{2s_2}^{(2)}}_{I_1^{(3)}\dots I_{2s_3}^{(3)}}$$

$$(\tau_1^{2s_1})^{\{I^{(1)}\}}_{\{J\}}(\tau_2^{2s_2})^{\{I^{(2)}\}}_{\{K\}}[\mathbf{3}|1|\mathbf{3}\rangle^L_{\{N\}} C^{s_1,\{J\};s_2,\{K\}}_{S,\{M\}} C^{S,\{M\};L,\{N\}}_{s_3,\{I^{(3)}\}}.$$

Two body decay | **Subsequent decay** | **Fermion bilinear** | **Dark matter scattering**

$s_1 = s_2 = \frac{1}{2}$.

$(L,S,J) = (1,0,1)$

$([\mathbf{12}] - \langle\mathbf{12}\rangle)[\mathbf{3}|p_1|\mathbf{3}\rangle$

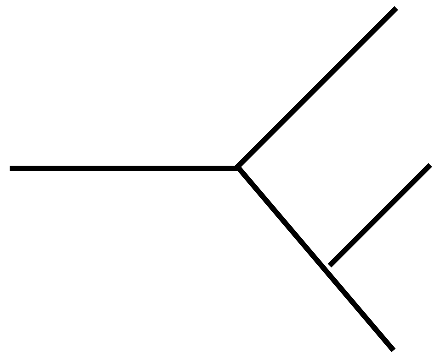
$(0,1,1),$

$([\mathbf{31}] - \langle\mathbf{31}\rangle)([\mathbf{32}] - \langle\mathbf{32}\rangle)$

$(1,1,1)$

$([\mathbf{32}][\mathbf{31}] - \langle\mathbf{32}\rangle\langle\mathbf{31}\rangle) + m_3(\langle\mathbf{32}\rangle[\mathbf{31}] - \langle\mathbf{31}\rangle[\mathbf{32}])$.

N-body amplitudes without boost



bilinear	r=(L,S,J)	C	P
$\bar\psi\psi$	(1,1,0))	+	+
$i\bar\psi\gamma_5\psi$	(0,0,0)	+	-
$\bar\psi\gamma_0\psi$	none	-	+
$\bar\psi\gamma_i\psi$	(0,1,1)+(2,1,1)	-	-
$\bar\psi\gamma_5\gamma_0\psi$	(0,0,0)	+	-
$\bar\psi\gamma_5\gamma_i\psi$	(1,1,1)	+	+
$\bar\psi\sigma^{0i}\psi$	(0,1,1)+(2,1,1)	-	-
$\bar\psi\sigma^{ij}\psi$	(1,0,1)	-	+

NR partial wave analysis

Relic abundance: s-wave, p-wave

DM bilinear	*SM fermion bilinear*			
fermion DM	$\bar f f$	$\bar f\gamma^5 f$	$\bar f\gamma^\mu f$	$\bar f\gamma^\mu\gamma^5 f$
$\bar\chi\chi$	$\sigma v \sim v^2,\ \sigma_{\rm SI}\sim 1$	$\sigma v\sim v^2,\ \sigma_{\rm SD}\sim q^2$	–	–
$\bar\chi\gamma^5\chi$	$\boldsymbol{\sigma v\sim 1,\ \sigma_{\rm SI}\sim q^2}$	$\boldsymbol{\sigma v\sim 1,\ \sigma_{\rm SD}\sim q^4}$	–	–
$\bar\chi\gamma^\mu\chi$ (Dirac only)	–	–	$\sigma v\sim 1,\ \sigma_{\rm SI}\sim 1$	$\boldsymbol{\sigma v\sim 1,\ \sigma_{\rm SD}\sim v_\perp^2}$
$\bar\chi\gamma^\mu\gamma^5\chi$	–	–	$\sigma v\sim v^2,\ \sigma_{\rm SI}\sim v_\perp^2$	$\boldsymbol{\sigma v\sim 1,\ \sigma_{\rm SD}\sim 1}$

Explicit spin symmetry

[Yong-Kang Li, Yi-Ning Wang, **J.H.Yu**, in préparation]

Since the spin symmetry could determines 3-pt amplitudes, suggest the Lorentz symmetry can be hidden

Lorentz symmetry

$$\begin{cases} \hat{P}^\mu, & \text{Translations,} \\ \hat{J}^i, & \text{Rotations,} \\ \hat{K}^i, & \text{Boosts,} \end{cases}$$

$$R^4 \rtimes SO(3,1)$$

$$\langle \Omega | \hat{P}^\mu | \Omega \rangle \equiv m v^\mu$$

$$v_0^\mu = (1,0,0,0)$$

Spin symmetry

$$\begin{cases} \hat{H}_v = v \cdot \hat{P} & , \text{Time Translation,} \\ (\hat{P}_v)^i = v_\perp^i \cdot \hat{P} & , \text{Spatial Translations,} \\ (\hat{J}_v)^i = (v_\perp^i)^\mu \epsilon_{\mu\nu\rho\sigma} \hat{J}^{\nu\rho} v^\sigma & , \text{Rotations,} \end{cases}$$

$$R^{1,3} \rtimes SO(3)$$

CCWZ Coset description

$$v^\mu = L(\vec{\eta}(v)) v_0^\mu \quad L(\vec{\eta}) = e^{i\vec{\eta}\cdot\vec{K}}$$

Missing Goldstone

$$|v, 0, \sigma\rangle + |v, \vec{k}\rangle = |v, \vec{k}, \sigma\rangle$$

Rest state **Goldstone** **k state**

$$|v, \vec{k}, \sigma\rangle = U(L_v(k))|v, 0, \sigma\rangle$$

Boost relates different k

Shift symmetry

$$\text{rotation}: \vec{\eta} \to \quad \vec{\eta}\,' = R\vec{\eta},$$

$$\text{boost}: \vec{\eta} \to \quad \vec{\eta}\,' = \vec{\eta} + \frac{\vec{q}}{m} + \mathcal{O}(q^2).$$

$$v^\mu \to v^\mu + \frac{q^\mu}{m}$$

Reparametrization invariance (RPI)

Heavy quark effective theory

Two equivalent descriptions on heavy quark effective theory (HQET) Lagrangian with NR fields

Top down integrate out

$$\Psi(x) = e^{-imv\cdot x}(Q_v(x) + B_v(x))$$

$$\mathcal{L} = \bar{\Psi}(i\not{D} - m)\Psi + \sum_n c_n \bar{\Psi}\frac{\mathcal{O}^{(n)}_{eff}}{m^n}\Psi,$$

$$Q_v(x) = e^{i\mathcal{P}x}P_+\Psi(x) = e^{imvx}\frac{1+\not{v}}{2}\Psi(x),$$

$$B_v(x) = e^{i\mathcal{P}x}P_-\Psi(x) = e^{imvx}\frac{1-\not{v}}{2}\Psi(x),$$

$$\mathcal{L} = \bar{Q}_v(iv\cdot D + \sum_n c_n \frac{\mathcal{O}^{(n)}_{eff}}{m^n})Q_v + \bar{Q}_v(i\not{D}_\perp + \sum_n c_n \frac{\mathcal{O}^{(n)}_{eff}}{m^n}) \times (iv\cdot D + 2m - \sum_x c_x \frac{\mathcal{O}^{(x)}_{eff}}{m^x})^{-1}(i\not{D}_\perp + \sum_m c_m \frac{\mathcal{O}^{(m)}_{eff}}{m^m})Q_v.$$

Expand to a non-relativistic basis

Bottom up nonlinear trans.

$$U(L(\vec{q}))|v,\vec{k},\sigma\rangle = \sum_{\sigma'} D_{\sigma'\sigma}(W(L(\vec{q}),\vec{k}))|v, L(\vec{q})\vec{k},\sigma'\rangle.$$

$$U(\Lambda)Q_{v,l}U(\Lambda)^\dagger = D[W(\Lambda, i\partial)]_{l\bar{l}}Q_{v,\bar{l}}$$

$$\mathcal{L} = N^\dagger\{iD_t + c_2\frac{\vec{D}^2}{2m} + c_F g\frac{\vec{\sigma}\cdot\vec{B}}{2m} + c_D g\frac{[D_i,E^i]}{8m^2} + ic_S g\frac{\epsilon^{ijk}\sigma^i\{D_j,E^k\}}{8m^2} + c_4\frac{\vec{D}^4}{8m^3}\}N$$

$$Q_v(x) = e^{imt}P_+\Psi(x) \equiv \begin{bmatrix} N(x) \\ 0 \end{bmatrix}$$

$$N'(x) \to e^{i\vec{q}\cdot\vec{x}}[1 - i\frac{\vec{q}\cdot\vec{D}}{2m^2} + \frac{\vec{\sigma}\cdot\vec{q}\times\vec{D}}{4m^2} + \mathcal{O}(g, \frac{1}{m^4})]N'(\Lambda^{-1}x).$$

$$c_2 = 1,$$
$$c_4 = 1,$$
$$c_S = 2c_F - 1,$$
$$2c_M = c_D - c_F,$$

FW trans.

Nonlinear little group trans. = RPI

3-point amplitudes for HQET and black hole EFT

Third description: spinor-helicity formalism with projection to determines the HQET operators

[Yong-Kang Li, Yi-Ning Wang, **J.H.Yu**, in préparation]

3-pt scaling for highest weight

$$|k^+\rfloor \equiv (\psi^+)^s_I \qquad |k^-\rfloor \equiv (\psi^-)^s_I$$

$$\lceil 1^\pm 2^\pm \rfloor^{S_1+S_2-S_3} \lceil 1^\pm 3^\pm \rfloor^{S_1+S_3-S_2} \lceil 2^\pm 3^\pm \rfloor^{S_2+S_3-S_1}$$

$$|k^-\rfloor = \frac{1}{2m + v\cdot k}|k^+\rfloor.$$

Direct construction

dim	Operators
5	$N^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{B}N \quad N^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{B}\chi \quad \chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{B}\chi$
6	$N^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{B}\partial_t N \quad N^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{B}\partial_t \chi \quad \chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{B}\partial_t \chi$ $N^\dagger \boldsymbol{\sigma}\cdot\partial_t\boldsymbol{B}N \quad N^\dagger \boldsymbol{\sigma}\cdot\partial_t\boldsymbol{B}\chi \quad \chi^\dagger \boldsymbol{\sigma}\cdot\partial_t\boldsymbol{B}\chi$

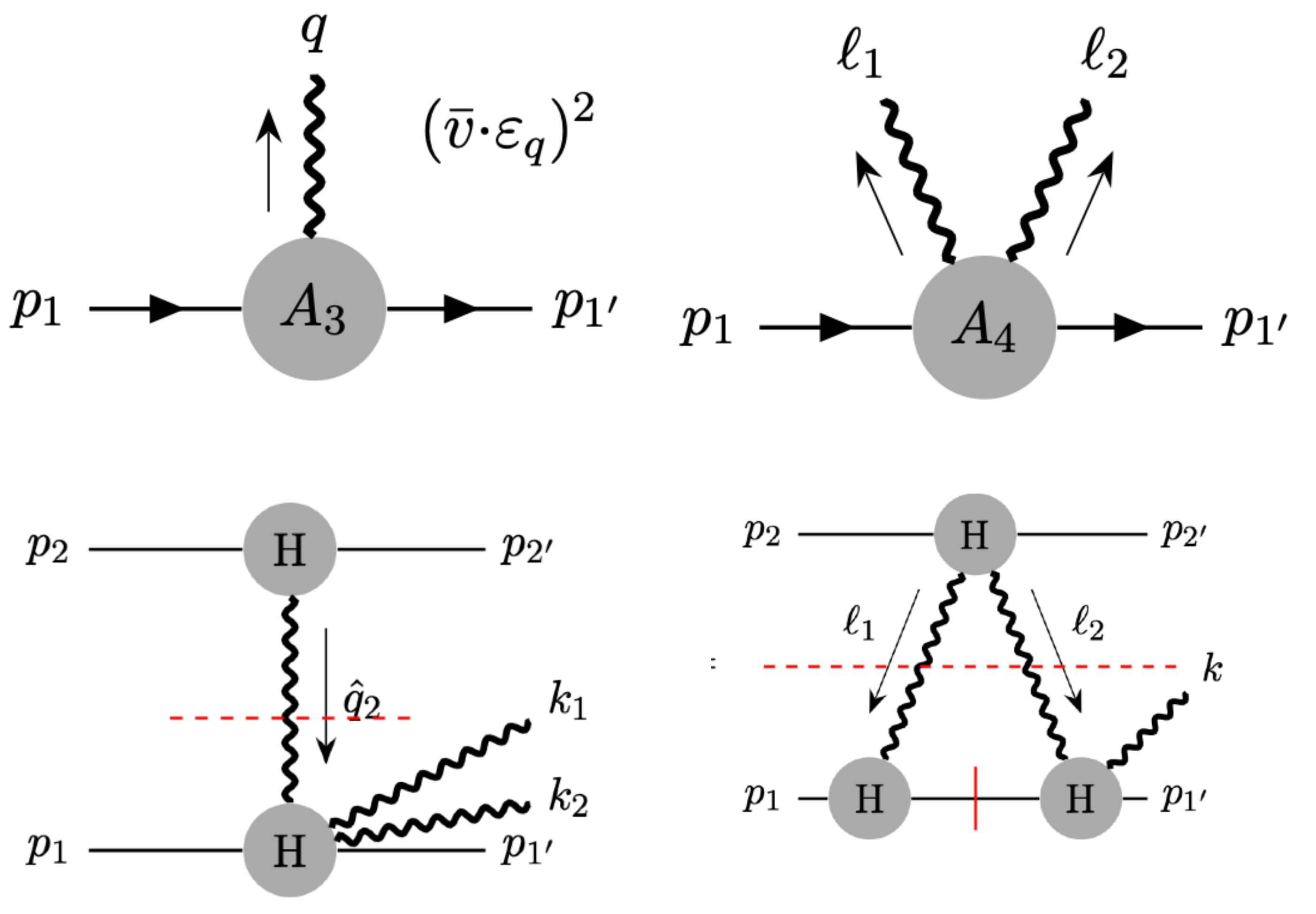
	Amplitudes		
	$\lceil 12^+\rfloor\lceil 13^+\rfloor$	$\lceil 12^-\rfloor\lceil 13^+\rfloor$	$\lceil 12^-\rfloor\lceil 13^-\rfloor$
$(v\cdot k_2)\times$	$\lceil 12^+\rfloor\lceil 13^+\rfloor$	$\lceil 12^-\rfloor\lceil 13^+\rfloor$	$\lceil 12^-\rfloor\lceil 13^-\rfloor$
$(v\cdot q_1)\times$	$\lceil 12^+\rfloor\lceil 13^+\rfloor$	$\lceil 12^-\rfloor\lceil 13^+\rfloor$	$\lceil 12^-\rfloor\lceil 13^-\rfloor$

Linear RPI

Heavy quark effective theory

dim	Amplitudes	Operators
5	$\lceil 12^+\rfloor\lceil 13^+\rfloor$	$N^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{B}N$
6	$\lceil 12^+\rfloor\lceil 1\|q_{1\perp}\|3^+\rfloor$	$N^\dagger\epsilon^{ijk}\sigma^i[\nabla^j, B^k]N$
	$\lceil 12^+\rfloor\lceil 1\|k_{2\perp}\|3^+\rfloor$	$N^\dagger\boldsymbol{B}\cdot\boldsymbol{D}N$
	$\lceil 13^+\rfloor\lceil 1\|k_{2\perp}\|2^+\rfloor$	$N^\dagger\epsilon^{ijk}\sigma^i B^j D^k N$
7	$\lceil 1\|q_{1\perp}\|2^+\rfloor\lceil 1\|k_{2\perp}\|3^+\rfloor$	$N^\dagger\epsilon^{ijk}\{D^i, [\partial^j B^k]\}N$
	$\lceil 1\|q_{1\perp}\|2^+\rfloor\lceil 1\|q_{1\perp}\|3^+\rfloor$	$N^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{D}\boldsymbol{B}\cdot\boldsymbol{D}N$
	$\lceil 1\|k_{2\perp}\|2^+\rfloor\lceil 1\|k_{2\perp}\|3^+\rfloor$	$N^\dagger\boldsymbol{D}\cdot\boldsymbol{B}\boldsymbol{\sigma}\cdot\boldsymbol{D}N$
	$\mathrm{tr}[q_{1\perp}q_{1\perp}]\lceil 12^+\rfloor\lceil 13^+\rfloor$	$N^\dagger[\partial^2, \boldsymbol{\sigma}\cdot\boldsymbol{B}]N$
	$\mathrm{tr}[k_{2\perp}k_{2\perp}]\lceil 12^+\rfloor\lceil 13^+\rfloor$	$N^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{B}D^2N$
	$\mathrm{tr}[q_{1\perp}k_{2\perp}]\lceil 12^+\rfloor\lceil 13^+\rfloor$	$N^\dagger\epsilon_{ijk}D^i\boldsymbol{\sigma}\cdot\boldsymbol{B}D^iN$

Heavy black hole effective theory

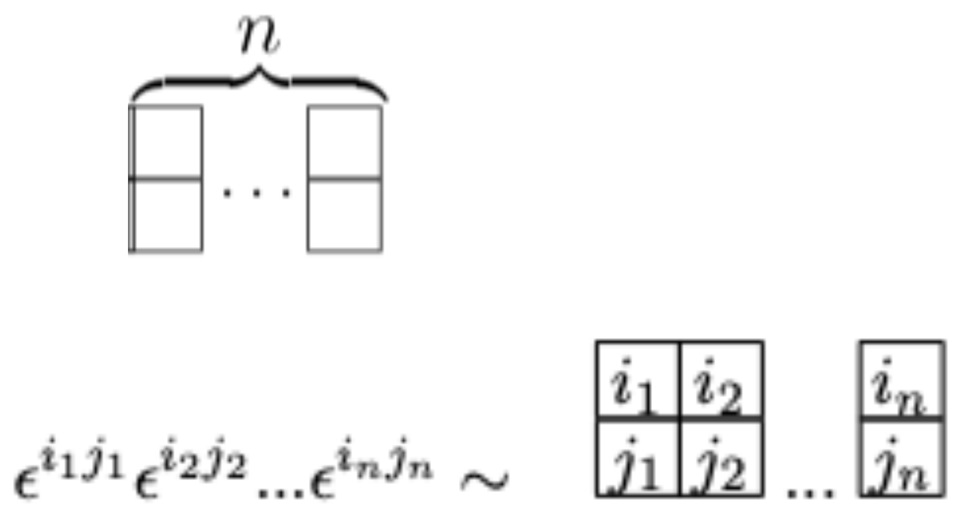


Heavy quark effective theory

[Yong-Kang Li, Yi-Ning Wang, **J.H.Yu**, in préparation]

Young diagram construction for n-pt amplitudes

Dual space operator → Spin space operator → SU(2) x SU(N)



Fields	$SO(3)\times SU(2)$
∇^I	$(1,0)$
B^I	$(1,0)$
$(S^I)_{ij}$	$(1,1)$
$\xi_i^\dagger$	$(0,\frac{1}{2})$
ξ_i	$(0,\frac{1}{2})$

$$\begin{aligned} &N_i, \\ N_j^\dagger &\equiv \epsilon_{ji} N^{\dagger i}, \\ \nabla_{kj} &\equiv \epsilon_{ki}\nabla_I(\sigma^I)^i_j, \\ E_{kj} &\equiv \epsilon_{ki}E^I(\sigma^I)^i_j, \end{aligned}$$

$$O = \epsilon^n \prod_{n_k}^{n_E}\prod_{n_l}^{n_B} N^\dagger(\nabla^{\omega_k}E_{n_k})(\nabla^{\omega_l}B_{n_l})\nabla^{\omega_j}N$$

Young tableau	Amplitudes	Operators
$\begin{smallmatrix} i_1 & i_2 \\ i_2 & i_3 \end{smallmatrix}$	$\lceil 1^+2\rfloor\lceil 23^+\rfloor + \lceil 1^+\tilde{2}\rfloor\lceil \tilde{2}3^+\rfloor$	$N_1^\dagger \boldsymbol{\sigma}\cdot \boldsymbol{B}_2 N_3$
$\begin{smallmatrix} i_1 & i_2 & j_3 \\ i_3 & j_3 & i_2 \end{smallmatrix}$	$-\left(\lceil 1^+3^+\rfloor\lceil 2\vert k_{3\perp}\vert 2\rfloor + \lceil 1^+3^+\rfloor\lceil \tilde{2}\vert k_{3\perp}\vert \tilde{2}\rfloor\right)$	$-2N_1^\dagger \boldsymbol{B}_2\cdot \boldsymbol{D}N_3$
$\begin{smallmatrix} i_1 & i_2 & j_3 \\ i_2 & j_3 & i_3 \end{smallmatrix} - \begin{smallmatrix} i_1 & j_3 & i_2 \\ j_3 & i_2 & i_3 \end{smallmatrix}$	$-(\lceil 1^+2\rfloor\lceil 2\vert k_{3\perp}\vert 3^+\rfloor + \lceil 1^+\tilde{2}\rfloor\lceil \tilde{2}\vert k_{3\perp}\vert 3^+\rfloor)$ $+(\lceil 1^+\vert k_{3\perp}\vert 2\rfloor\lceil 23^+\rfloor + \lceil 1^+\vert k_{3\perp}\vert \tilde{2}\rfloor\lceil \tilde{2}3^+\rfloor)$	$2iN_1^\dagger \boldsymbol{\sigma}\cdot \boldsymbol{B}_2\times \boldsymbol{D}N_3$
$\begin{smallmatrix} i_1 & i_{\tilde{2}} \\ i_{\tilde{2}} & i_3 \end{smallmatrix}$	$i\lceil 1^+2\rfloor\lceil 23^+\rfloor - i\lceil 1^+\tilde{2}\rfloor\lceil \tilde{2}3^+\rfloor$	$N_1^\dagger \boldsymbol{\sigma}\cdot \boldsymbol{E}_2 N_3$
$\begin{smallmatrix} i_1 & i_{\tilde{2}} & j_3 \\ i_3 & j_3 & i_{\tilde{2}} \end{smallmatrix}$	$-i\left(\lceil 1^+3^+\rfloor\lceil 2\vert k_{3\perp}\vert 2\rfloor - \lceil 1^+3^+\rfloor\lceil \tilde{2}\vert k_{3\perp}\vert \tilde{2}\rfloor\right)$	$-2N_1^\dagger \boldsymbol{E}_2\cdot \boldsymbol{D}N_3$
$\begin{smallmatrix} i_1 & i_{\tilde{2}} & j_3 \\ i_{\tilde{2}} & j_3 & i_3 \end{smallmatrix} - \begin{smallmatrix} i_1 & j_3 & i_{\tilde{2}} \\ j_3 & i_{\tilde{2}} & i_3 \end{smallmatrix}$	$-i(\lceil 1^+2\rfloor\lceil 2\vert k_{3\perp}\vert 3^+\rfloor - \lceil 1^+\tilde{2}\rfloor\lceil \tilde{2}\vert k_{3\perp}\vert 3^+\rfloor)$ $+i(\lceil 1^+\vert k_{3\perp}\vert 2\rfloor\lceil 23^+\rfloor - \lceil 1^+\vert k_{3\perp}\vert \tilde{2}\rfloor\lceil \tilde{2}3^+\rfloor)$	$2iN_1^\dagger \boldsymbol{\sigma}\cdot \boldsymbol{E}_2\times \boldsymbol{D}N_3$

Validated by Hilbert series

Similarly heavy baryon chiral EFT

Light dark matter detection

phonon, plasmon, superfluid, etc

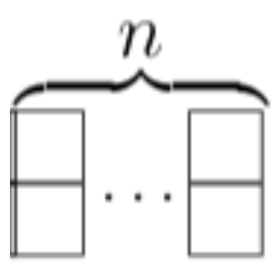
Obtain the complete/non-redundant operator basis for HQET

Non-relativistic effective theory

The NR operator basis for **any spins** can be obtained, and perform spin-orbital partial wave decomposition

[Yong-Kang Li, Yi-Ning Wang, **J.H.Yu**, in préparation]

SU(2) x SU(N) Young diagram



NR 4 fermion operators

Pion-less nuclear EFT

Cold atom EFT

Spin-s dark matter operators

Spinning binary black hole

NR Fermion bilinear

NR partial wave analysis

Integrate-out amplitudes		Operators
$\lceil 1^+2^+\rfloor\lceil 3^+4^+\rfloor$	$\lceil 1^+3^+\rfloor\lceil 2^+4^+\rfloor$	$(N^\dagger N)(N^\dagger N)$ $(N^\dagger \boldsymbol{\sigma} N)\cdot(N^\dagger \boldsymbol{\sigma} N)$
$\lceil 1^+\vert k_{2\perp}\vert 2^+\rfloor\lceil 3^+\vert k_{2\perp}\vert 4^+\rfloor$	$\lceil 1^+\vert k_{2\perp}\vert 3^+\rfloor\lceil 2^+\vert k_{2\perp}\vert 4^+\rfloor$	$: (N^\dagger\vec{\sigma}\cdot\overrightarrow{\nabla}\overrightarrow{\nabla}^i N^\dagger)(N\sigma^i N)+h.c.$
$\lceil 1^+\vert k_{2\perp}k_{2\perp}\vert 2^+\rfloor\lceil 3^+4^+\rfloor$	$\lceil 1^+3^+\rfloor\lceil 2^+\vert k_{3\perp}k_{2\perp}\vert 4^+\rfloor$	$: -(N^\dagger\overrightarrow{\nabla}^2 N^\dagger)(NN)+h.c.$
$\lceil 1^+\vert k_{2\perp}k_{3\perp}\vert 2^+\rfloor\lceil 3^+4^+\rfloor$	$\lceil 1^+\vert k_{2\perp}k_{3\perp}\vert 4^+\rfloor\lceil 2^+3^+\rfloor$	$: -(N^\dagger\vec{\sigma}\cdot\overrightarrow{\nabla}\sigma^i N^\dagger)(N\overleftarrow{\nabla}^i N)+h.c.$
$\lceil 1^+\vert k_{2\perp}\vert 2^+\rfloor\lceil 3^+\vert k_{3\perp}\vert 4^+\rfloor$	$\lceil 1^+2^+\rfloor\lceil 3^+4^+\rfloor tr[k_{2\perp}k_{3\perp}]$	$: -(N^\dagger\vec{\sigma}\cdot\overrightarrow{\nabla}N^\dagger)(N\vec{\sigma}\cdot\overleftarrow{\nabla}N)+h.c.$
$\lceil 1^+3^+\rfloor\lceil 2^+4^+\rfloor tr[k_{2\perp}k_{4\perp}]$	$\lceil 1^+\vert k_{2\perp}k_{4\perp}\vert 3^+\rfloor\lceil 2^+4^+\rfloor$	$: -2(N^\dagger N)(N^\dagger\overleftarrow{\nabla}\cdot\overrightarrow{\nabla}N)+h.c.$
$\lceil 1^+\vert k_{2\perp}k_{4\perp}\vert 4^+\rfloor\lceil 2^+3^+\rfloor$	$\lceil 1^+\vert k_{2\perp}\vert 2^+\rfloor\lceil 3^+\vert k_{4\perp}\vert 4^+\rfloor$	$: (N^\dagger\sigma^i\sigma^j N)(N^\dagger\overleftarrow{\nabla}^i\overrightarrow{\nabla}^j N)+h.c.$

Young tableau	Amplitudes	Operators
$\begin{smallmatrix} i_1 & i_3 \\ i_2 & i_4 \end{smallmatrix}$	$\lceil 1^+2^+\rfloor\lceil 3^+4^+\rfloor$	$(N_1^\dagger N_2)(\xi_3^\dagger\xi_4)$
$\begin{smallmatrix} i_1 & i_2 \\ i_3 & i_4 \end{smallmatrix}$	$\lceil 1^+3^+\rfloor\lceil 2^+4^+\rfloor$	$(N_1^\dagger\xi_3^\dagger)(N_2\xi_4)$
$\begin{smallmatrix} i_1 & i_2 & j_2 \\ j_2 & i_3 & i_4 \end{smallmatrix}$	$-\lceil 1^+\vert k_{2\perp}\vert 4^+\rfloor\lceil 2^+3^+\rfloor$	$-(N_1^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}_2\xi_4)(N_2\xi_3^\dagger)$
$\begin{smallmatrix} i_1 & i_2 & i_3 \\ j_2 & j_2 & i_4 \end{smallmatrix}$	$\lceil 1^+\vert k_{2\perp}\vert 2^+\rfloor\lceil 3^+4^+\rfloor$	$(N_1^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}_2 N_2)(\xi_3^\dagger\xi_4)$
$\begin{smallmatrix} i_1 & j_2 & j_2 \\ i_2 & i_3 & i_4 \end{smallmatrix}$	$\lceil 1^+2^+\rfloor\lceil 3^+\vert k_{2\perp}\vert 4^+\rfloor$	$(N_1^\dagger N_2)(\xi_3^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}_2\xi_4)$
$\begin{smallmatrix} i_1 & i_2 & i_3 \\ j_3 & j_3 & i_4 \end{smallmatrix}$	$\lceil 1^+\vert k_{3\perp}\vert 2^+\rfloor\lceil 3^+4^+\rfloor$	$(N_1^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}_3 N_2)(\xi_3^\dagger\xi_4)$

	Amp	$^{2S+1}L_J$	C	P
NN	$\lfloor 1^+2^+\rfloor$	$^1S_0$	\	+
$N\sigma^i N$	$\vert 1^+\rfloor^{(I}\vert 2^+\rfloor^{J)}$	$^3S_1$	\	+
$N\overleftrightarrow{\nabla}^i N$	$q^{IJ}\lfloor 1^+2^+\rfloor$	$^1P_0$	\	-
$N(\overleftrightarrow{\nabla}\cdot\sigma)N$	$\lfloor 1^+\vert \mathrm{q}\vert 2^+\rfloor$	$^3P_0$	\	-
$N\overleftrightarrow{\nabla}\times\vec{\sigma}N$	$q_I^K\vert 1^+\rfloor^{(I}\vert 2^+\rfloor^{J)}$	$^3P_1$	\	-
$N(\overleftrightarrow{\nabla}^i\vec{\sigma}^j-\frac{\overleftrightarrow{\nabla}\cdot\sigma}{3}\delta^{ij})N$	$q^{(KL}\vert 1^+\rfloor^I\vert 2^+\rfloor^{J)}$	$^3P_2$	\	-

Summary and outlook

Explore the internal symmetry
Field 1927 - 1980
[Weyl, Yang-Mills, Weinberg-Salam, Gell-mann….]

Explore the spacetime symmetry
Particle 2000 -
[Witten, Cachazo, Arkani-Hamed, ….]

The spacetime symmetry dictates interactions (operator basis)

Poincare symmetry — Relativistic EFT

- Massless EFT: SMEFT, Gravity EFT
- Massive EFT: Weak EFT, Low energy EFT
- EFT without anti-particle: HQET, Heavy baryon ChPT

Goldstone

Spin symmetry — NR EFT

- NR EFT: Pion-less nuclear EFT, Spinning black hole, Cold atom
- NR with more modes: NRQCD, Phonon, plasmon, Superfluid, …

Thanks for your attention!