



Institute of Theoretical Physics
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Symmetry dictates Operators

From relativistic EFT to HQET

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The 7th National Heavy Flavor and QCD Workshop

04-19, 2025 @ Nanjing

Outline

Apologize for missing references in this talk

- Amplitude-operator correspondence by symmetry
- Relativistic EFT operators from Poincare symmetry
- Nonlinear Lorentz symmetry and spin symmetry
- Heavy quark effective theory operators from spin symmetry
- Summary and outlook

Symmetry dictates interaction

C.N. Yang: quantization, **symmetry**, and phase factor

Gauge symmetry dictates interactions



Weyl $U(1)$



Yang-Mills $SU(2)$



GSW $SU(2) \times U(1)$

Can spacetime symmetry dictate interactions?



Lorentz symmetry



Minkowski



Poincare symmetry

QFT: two equivalent descriptions

Lagrangian based on local field

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\mathcal{D} \leq 4} + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + \frac{\mathcal{L}_7}{\Lambda^3} + \frac{\mathcal{L}_8}{\Lambda^4} + \frac{\mathcal{L}_9}{\Lambda^5} + \dots$$

Expansion with mass dimension

Amplitudes based on on-shell particle

$$\mathcal{S} = \text{---} \bullet \text{---} + \text{---} \bullet \text{---} + \text{---} \bullet \text{---} + \text{---} \bullet \text{---} + \text{---} \bullet \text{---} + \dots$$

Classified by scattering particle numbers

$$\psi(x) = \sum_s \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ip \cdot x} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ip \cdot x} \right]$$

Field transform under Lorentz rep

$$A^\mu(x) \rightarrow U A^\mu(x) U^\dagger = (\Lambda^{-1})^\mu{}_\nu A^\nu(\Lambda x) + \Lambda^{-1} \partial^\nu \Omega(\Lambda x)$$

Impose EOM

$$\begin{aligned} \partial_\mu F^{\mu\nu} + m^2 A^\nu &= 0 \\ \partial_\mu A^\mu &= 0 \end{aligned}$$

Impose gauge condition

$$\epsilon^\mu(\vec{k}, \lambda) \rightarrow \epsilon^\mu(\vec{k}, \lambda) + \frac{k^\mu}{m}$$

Symmetry is manifest !

Particle transform under Little group

$$\epsilon_\mu^{IJ} = \frac{\langle p^{(I} | \sigma_\mu | p^{J)} \rangle}{\sqrt{2}m} \quad \epsilon_\mu^+ = \frac{\langle \zeta | \sigma_\mu | \lambda \rangle}{\sqrt{2} \langle \lambda \zeta \rangle}, \quad \epsilon_\mu^- = \frac{\langle \lambda | \sigma_\mu | \zeta \rangle}{\sqrt{2} [\lambda \zeta]}$$

Satisfy EOM and gauge inv automatically

$$\eta_\alpha \rightarrow \eta'_\alpha = a \eta_\alpha + b \lambda_\alpha$$

Poincare symmetry for scattering amplitudes

Scattering amplitudes transform under the Poincare symmetry

$$\mathcal{M}(p_a, \sigma_a) = \delta^D(p_{a_1}^\mu + \cdots p_{a_n}^\mu) M(p_a, \sigma_a)$$

$$p_{\alpha\dot{\alpha}} \equiv \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} \equiv |p\rangle_\alpha [p]_{\dot{\alpha}}$$

$$\mathbf{p}_{\alpha\dot{\beta}} = \lambda_\alpha^I \tilde{\lambda}_{\dot{\beta}I} = \epsilon_{IJ} |p^I\rangle_\alpha [p^J]_{\dot{\beta}}$$

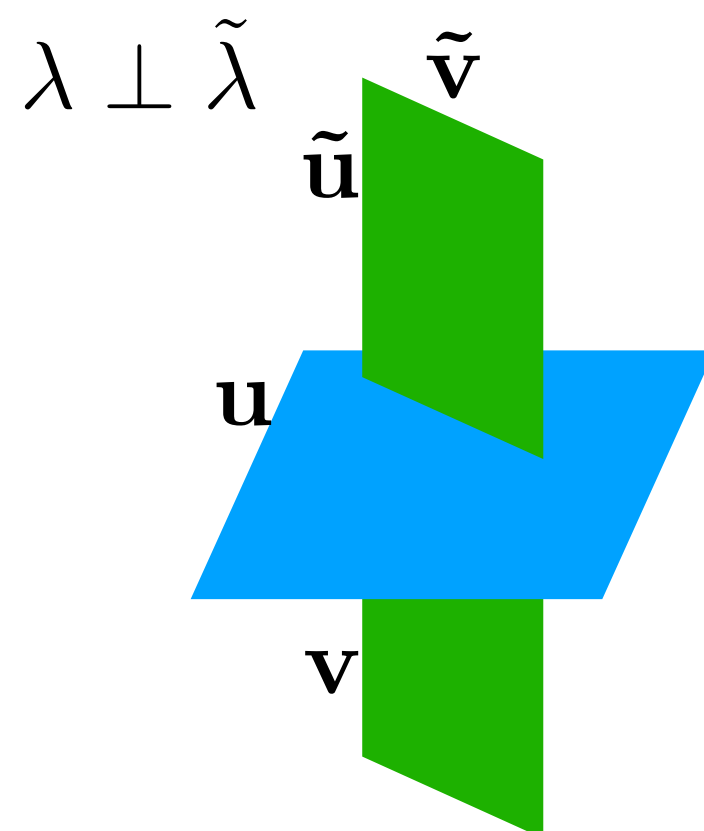
Translation symmetry

$$f(\{\lambda^i, \tilde{\lambda}_i\}) \delta^{(4)}\left(\sum_{i=1}^N \tilde{\lambda}_i^{\dot{a}} \lambda^{ia}\right)$$

Momentum conservation

$$P^{\dot{a}a} = \sum_{i=1}^N \tilde{\lambda}_i^{\dot{a}} \lambda^{ia} = 0$$

$$\begin{pmatrix} \lambda_1^1 & \lambda_1^2 & \cdots & \lambda_1^N \\ \lambda_2^1 & \lambda_2^2 & \cdots & \lambda_2^N \end{pmatrix} \xrightarrow{U(N)} \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}$$



Lorentz symmetry

$$M^\Lambda(p_a, \sigma_a) = \prod_a (D_{\sigma_a \sigma'_a}(W)) M((\Lambda p)_a, \sigma'_a)$$

Massless U(1) Little group

$$\lambda \rightarrow t\lambda, \quad \tilde{\lambda} \rightarrow t^{-1}\tilde{\lambda}$$

$$A(1^{h_1} \cdots n^{h_n}) \rightarrow \prod_i t_i^{-2h_i} A(1^{h_1} \cdots n^{h_n})$$

Little group scaling

Massive SU(2) Little group

$$\lambda^I \rightarrow W_J^I \lambda^J, \quad \tilde{\lambda}_I \rightarrow (W^{-1})_I^J \tilde{\lambda}_J$$

$$\mathcal{M}(p_a, \sigma_a) \rightarrow \mathcal{M}^\Lambda(p_a, \sigma_a) = \prod_a (D_{\sigma_a \sigma'_a}(W)) \mathcal{M}((\Lambda p)_a, \sigma'_a)$$

SU(2) transformation

$$\begin{cases} \lambda_\alpha^I &= -\lambda_\alpha \zeta^{-I} + \eta_\alpha \zeta^{+I}, \\ \tilde{\lambda}_{\dot{\alpha}}^I &= \tilde{\lambda}_{\dot{\alpha}} \zeta^{+I} + \tilde{\eta}_{\dot{\alpha}} \zeta^{-I}, \end{cases}$$

$$\mathbf{p}_{\alpha\dot{\alpha}} \equiv p_{\alpha\dot{\alpha}} + \eta_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} + \eta_\alpha \tilde{\eta}_{\dot{\alpha}}.$$

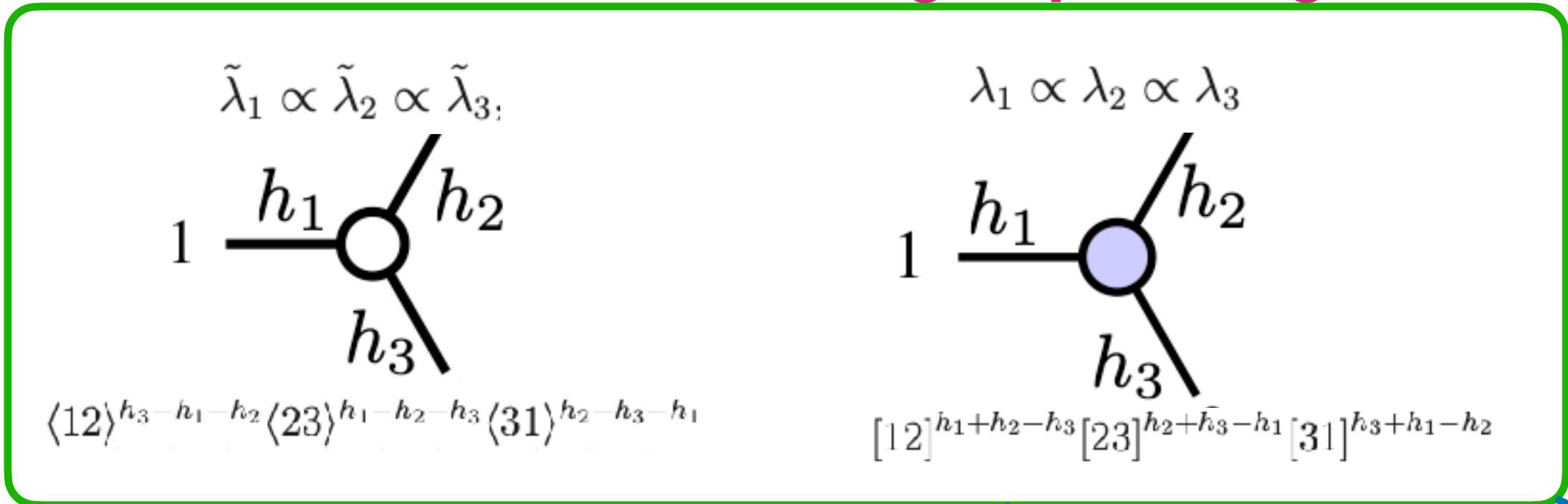
3-point amplitudes (massless) dictates interactions

The building blocks of scattering amplitudes are 3-point amplitudes

Mom. Conserv. + Little group scaling

[Benincasa, Cachazo, 2007]

[Witten, 2003]



(- - +)

(- - +)

(- - +)

(- - 0)

QED Theory

Yang-Mills Theory

Einstein Theory

Yukawa Theory

Massless

Massless

$$A(1^{-\frac{1}{2}} 2^{+\frac{1}{2}} 3^+) = \frac{\langle 13 \rangle^2}{\langle 12 \rangle}$$

$$A(1^{+\frac{1}{2}} 2^{-\frac{1}{2}} 3^-) = \frac{[13]^2}{[12]}$$

$$\bar{\psi} \gamma^\mu \psi A_\mu$$

$$A(1_a^- 2_b^- 3_c^+) = f_{abc} \frac{\langle 12 \rangle^3}{\langle 13 \rangle \langle 32 \rangle}$$

$$A(1_a^+ 2_b^+ 3_c^-) = f_{abc} \frac{[12]^3}{[13] [32]}$$

$$f_{abc} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) A^{\mu b} A^{\nu c}$$

$$A(1^{--} 2^{--} 3^{++}) = \frac{\langle 12 \rangle^6}{\langle 13 \rangle^2 \langle 32 \rangle^2}$$

$$A(1^{++} 2^{++} 3^{--}) = \frac{[12]^6}{[13]^2 [32]^2}$$

Amplitude-interaction correspondence

$$A(1^{+\frac{1}{2}} 2^{+\frac{1}{2}} 3^0) = \langle 12 \rangle$$

$$A(1^{-\frac{1}{2}} 2^{-\frac{1}{2}} 3^0) = [12]$$

$$\bar{\psi} \psi \phi$$

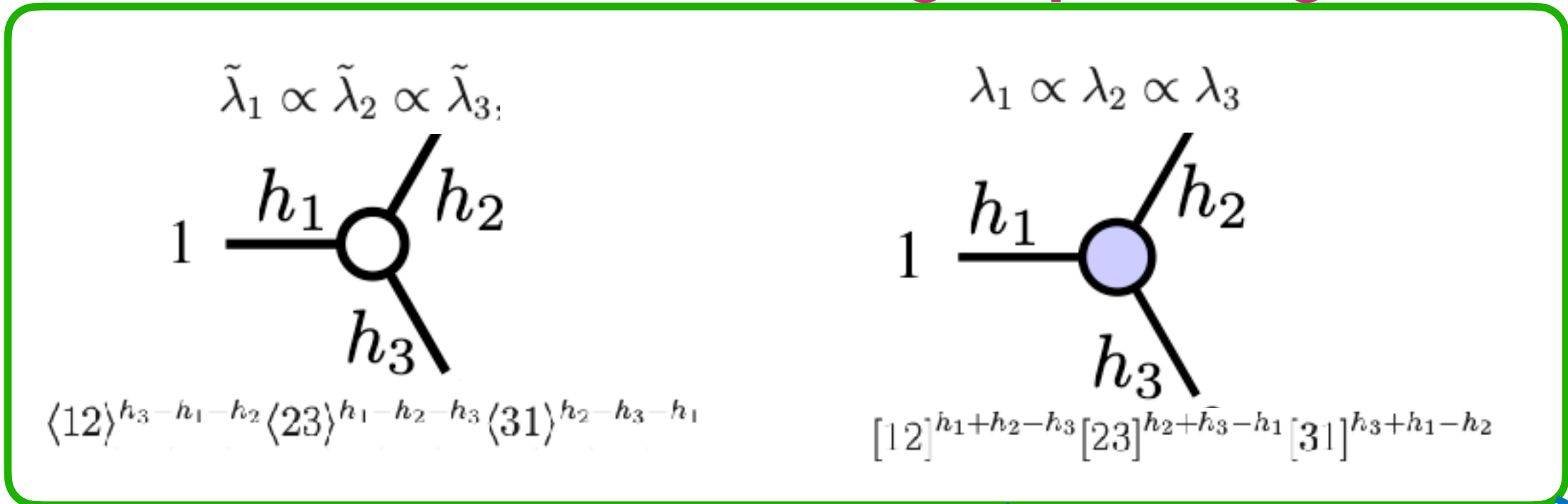
3-point amplitudes (massless) dictates Operators

[Benincasa, Cachazo, 2007]

Also determines non-renormalizable interactions involving in 3-particles

[Witten, 2003]

Mom. Conserv. + Little group scaling



(- - -)

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(- + 0)

Magnetic dipole QED

Massless

Dim-6 Yang-Mills

Modified Gravity

x

$$A(1^{+\frac{1}{2}} 2^{+\frac{1}{2}} 3^+) = \langle 13 \rangle \langle 23 \rangle$$

$$A(1^{-\frac{1}{2}} 2^{-\frac{1}{2}} 3^-) = [13][23]$$

$$\bar{\psi} \sigma^{\mu\nu} \psi F_{\mu\nu}$$

$$A(1_a^- 2_b^- 3_c^-) = f_{abc} \langle 12 \rangle \langle 23 \rangle \langle 31 \rangle$$

$$A(1_a^+ 2_b^+ 3_c^+) = f_{abc} [12][23][31],$$

$$f_{abc} F_{\mu}^{a\nu} F_{\nu}^{b\rho} F_{\rho}^{c\mu}$$

$$A(1^- 2^- 3^-) = (\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle)^2$$

$$A(1^{++} 2^{++} 3^{++}) = ([12][23][31])^2$$

$$R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\alpha\beta} R_{\alpha\beta}{}^{\mu\nu}$$

$$A(1^{+\frac{1}{2}} 2^{-\frac{1}{2}} 3^0) = g^- \frac{\langle 13 \rangle}{\langle 23 \rangle} + g^+ \frac{[13]}{[23]}$$

inconsistent
Locality

Amplitude-operator correspondence

Massive 3-point amplitudes

No simple little group scaling, momentum conservation complicated

[Arkani-Hamed, Huang, Huang, 2017]

$$M_{K_1 \dots K_{2s_3}}^{I_1 \dots I_{2s_1} J_1 \dots J_{2s_2}} = \left(\lambda_{1\alpha_1}^{I_1} \dots \lambda_{1\alpha_{2s_1}}^{I_{2s_1}} \right) \left(\lambda_{2\alpha_1}^{J_1} \dots \lambda_{2\alpha_{2s_2}}^{J_{2s_2}} \right) \left(\lambda_{3K_1}^{\gamma_1} \dots \lambda_{3K_{2s_3}}^{\gamma_{2s_3}} \right) \tilde{M}_{\gamma_1 \dots \gamma_{2s_3}}^{\alpha_1 \dots \alpha_{2s_1} \beta_1 \dots \beta_{2s_2}}$$

Find 2 linearly independent basis span the SL(2,C) space

$$\mathcal{J}^{(\alpha_1 \cdots \alpha_{2s_1}), (\beta_1 \cdots \beta_{2s_2}), (\gamma_1 \cdots \gamma_{2s_3})} = \sum_{i=0}^1 \sum_{\sigma_i} g_{\sigma_i} (\mathcal{O}^{s_1+s_2+s_3-i} \epsilon^i)_{\sigma_i}^{(\alpha_1 \cdots \alpha_{2s_1}), (\beta_1 \cdots \beta_{2s_2}), (\gamma_1 \cdots \gamma_{2s_3})}$$

symmetric tensor $\mathcal{O}$, antisymmetric tensor ϵ



$$\langle 13 \rangle [23] \quad [13] \langle 23 \rangle$$

$$[13] [23] \quad \langle 13 \rangle \langle 23 \rangle$$

$$\bar{\psi}_1 \gamma^\mu P_R \psi_2 \epsilon_{3,\mu}$$

$$\bar{\psi}_1 \sigma^{\mu\nu} P_R \psi_2 \epsilon_{3,\mu} p_{3,\nu}$$

$$\langle 12 \rangle [13] [23] + [12] \langle 13 \rangle [23] + [12] [13] \langle 23 \rangle$$

$$[12] [13] [23] \quad \langle 12 \rangle \langle 13 \rangle \langle 23 \rangle$$

$$\langle 12 \rangle (\langle 13 \rangle [23] + [13] \langle 23 \rangle)$$

$$[\eta_{\mu\nu} (p_1 - p_2)_\rho + \eta_{\nu\rho} (p_2 - p_3)_\mu + \eta_{\rho\mu} (p_3 - p_1)_\nu] \epsilon_1^\mu \epsilon_2^\nu \epsilon_3^\rho$$

Just list Lorentz structures

Any spacetime symmetry to guarantee exhaust all possible structure?

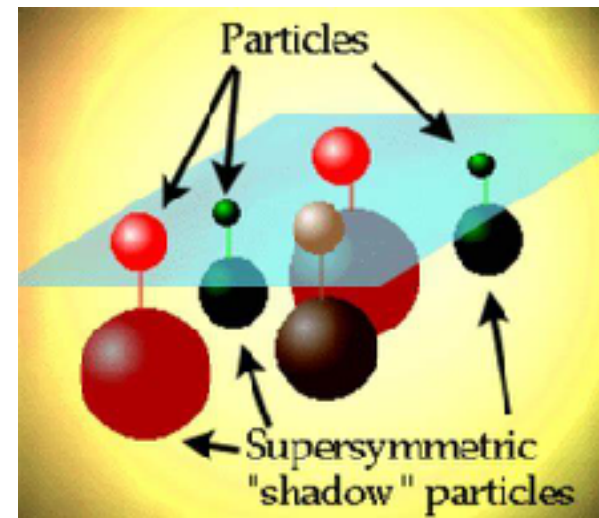
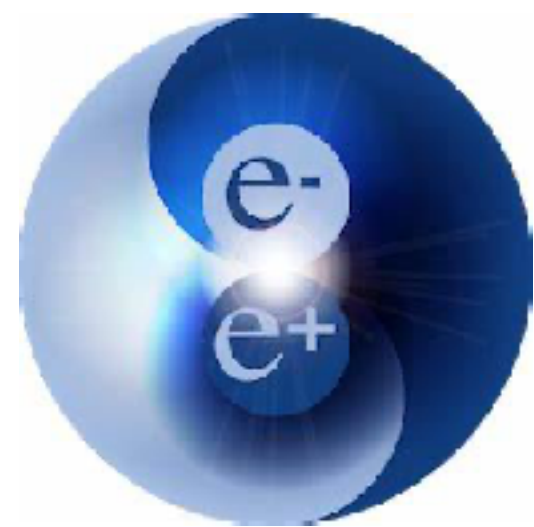
Spacetime symmetry extension

Spacetime and internal symmetry can't be mixed except supersymmetry, massless CFT, discrete, **SSB** theory

[Yu-Han Ni, Chao Wu, Yi-Ning Wang, **J.H.Yu**, 2412.03762]

Superspace (Grassmann)

$$\phi \longleftrightarrow \psi$$



$$(\lambda, \tilde{\lambda}, \eta^A)$$

$$\{\eta^A, \partial_{\eta^B}\} = \delta_B^A$$

$$(\lambda, \tilde{\lambda}, \eta, \bar{\eta})$$

Hyperspace (spinor)

$$\lambda_\alpha^I \longleftrightarrow \tilde{\lambda}_{\dot{\alpha}}^I$$

$$(\lambda, \tilde{\lambda}, \eta, \bar{\eta})$$

$$\begin{matrix} p_\mu \\ \lambda_\alpha^I \end{matrix} \longrightarrow \begin{matrix} P_M = (p_\mu, m, \tilde{m}) \\ \lambda_A^I = \{\lambda_\alpha^I, \tilde{\lambda}^{\dot{\alpha}I}\} \end{matrix}$$

$$\text{SO}(5,1)/[\text{SO}(2) \times \text{SO}(3,1)]$$

Internal space

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma_{\alpha\dot{\alpha}}^\mu P_\mu \quad \left\{ [p]^a \frac{\partial}{\partial \eta}, |p\rangle^{\dot{a}} \eta \right\} = |p\rangle^{\dot{a}} [p]^a$$

with R-charge

$$Q_\alpha \rightarrow e^{-i\lambda} Q_\alpha \quad \text{and} \quad \bar{Q}_{\dot{\alpha}} \rightarrow e^{i\lambda} \bar{Q}_{\dot{\alpha}} \quad \eta \rightarrow e^{i\lambda} \eta$$

$$[R, Q_\alpha] = -Q_\alpha \quad \text{and} \quad [R, \bar{Q}_{\dot{\alpha}}] = +\bar{Q}_{\dot{\alpha}}$$

Internal space

$$[m, \tilde{m}] = 0. \quad m \equiv \frac{1}{2} \lambda_{\alpha I} \lambda^{\alpha I}, \quad \tilde{m} \equiv \frac{1}{2} \tilde{\lambda}_{\dot{\alpha} I} \tilde{\lambda}^{\dot{\alpha} I}.$$

with transversality-charge

$$m \rightarrow e^{-i\phi} m, \quad \tilde{m} \rightarrow e^{i\phi} \tilde{m}.$$

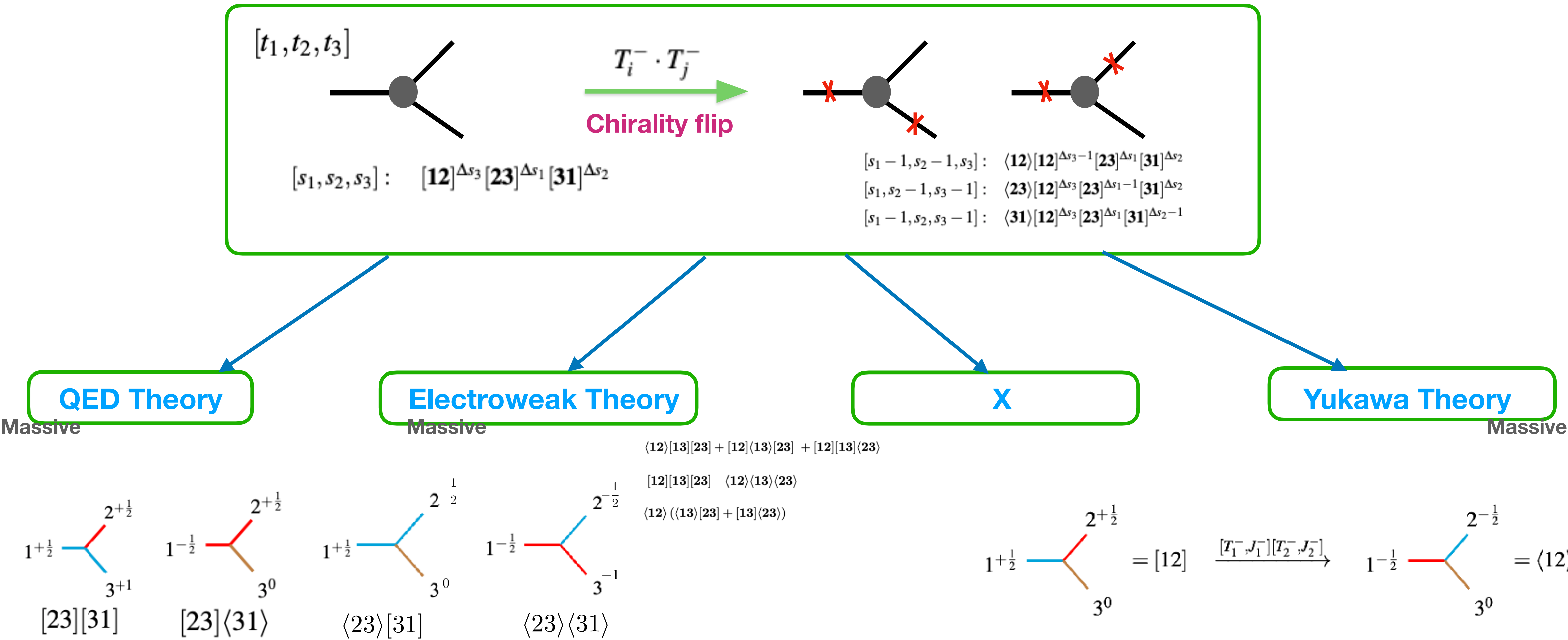
$$[D_-, m] = -m, \quad [D_-, \tilde{m}] = +\tilde{m},$$

Relate to chirality

Massive 3-point amplitudes again

SO(5,1) isomorphic to conformal group: highest weight rep by scaling, then descendent ones

[Yu-Han Ni, Chao Wu, Yi-Ning Wang, **J.H.Yu**, 2412.03762]



$U(N)$ symmetry extension

In above, 3-pt amplitude dictates operators with 3-particles, how about operators with n-particles?

SUSY with $U(N)$ R symmetry

$$(\lambda, \tilde{\lambda}, \eta, \bar{\eta})$$

$$(\lambda, \tilde{\lambda}, \eta^A) \quad \{\eta^A, \partial_{\eta^B}\} = \delta_B^A$$

Chiral superspace

$$\{Q_{\alpha A}, Q_{\dot{\beta}}^{\dagger B}\} = -2\delta_A^B (\sigma^\mu_{\alpha\dot{\beta}}) P_\mu$$

$$\{Q_{\alpha A}, Q_{\beta B}\} = Z_{AB} \epsilon_{\alpha\beta} \quad Z_{AB} = 0$$

$SU(N)$ R symmetry (no central charge)

Spinor with $U(N)$ symmetry

$$(\lambda, \tilde{\lambda}, \eta, \tilde{\eta})$$

$$\begin{pmatrix} \lambda_1^1 & \lambda_1^2 & \cdots & \lambda_1^N \\ \lambda_2^1 & \lambda_2^2 & \cdots & \lambda_2^N \end{pmatrix} \xrightarrow{U(N)} \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}$$

$$\downarrow SL(2, \mathbb{C})$$

$$\begin{pmatrix} \lambda_a^1 & \lambda_a^2 & \cdots & \lambda_a^N \end{pmatrix}$$

$U(1)^n$ little group

$$\lambda \rightarrow t\lambda, \quad \tilde{\lambda} \rightarrow t^{-1}\tilde{\lambda}$$

$$A(1^{h_1} \cdots n^{h_n}) \rightarrow \prod_i t_i^{-2h_i} A(1^{h_1} \cdots n^{h_n})$$

$SL(2, \mathbb{C}) \times SU(N)$

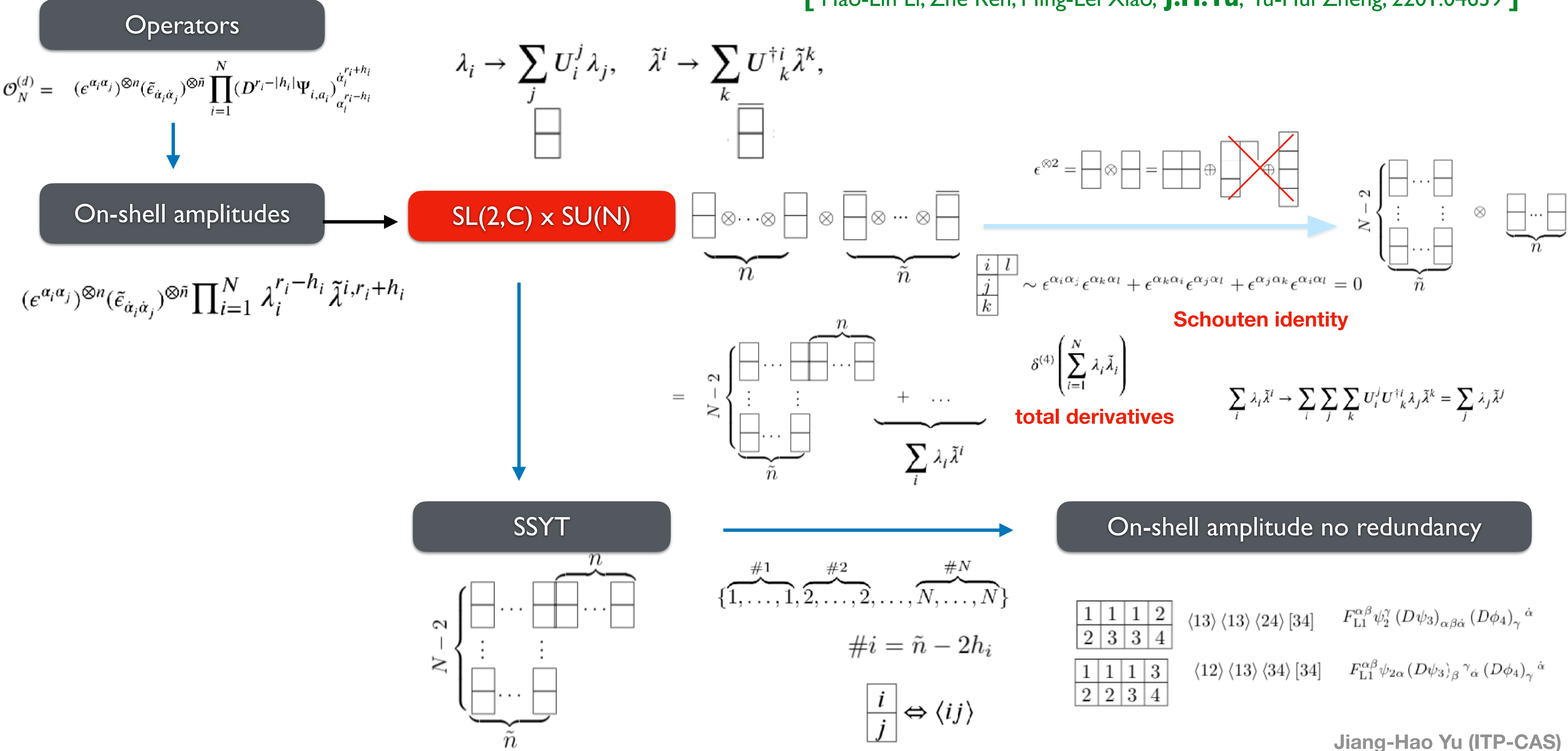
$$\lambda_i \rightarrow \sum_j U_i^j \lambda_j, \quad \tilde{\lambda}^i \rightarrow \sum_k U^{\dagger i}_k \tilde{\lambda}^k,$$

$$K_{aa} = - \sum_{i=1}^N \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{a}}} \frac{\partial}{\partial \lambda^{ia}}$$

$$f(\{\lambda^i, \tilde{\lambda}_i\}) \delta^{(4)} \left(\sum_{i=1}^N \tilde{\lambda}_i^{\dot{a}} \lambda^{ia} \right)$$

Operator-amplitude correspondence

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2201.04639]



Standard model effective field theory

SMEFT provides systematic parametrization on all possible Lorentz-invariant new physics

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\mathcal{D} \leq 4} + \frac{\mathcal{L}_5}{\Lambda} + \frac{\mathcal{L}_6}{\Lambda^2} + \frac{\mathcal{L}_7}{\Lambda^3} + \frac{\mathcal{L}_8}{\Lambda^4} + \frac{\mathcal{L}_9}{\Lambda^5} + \dots$$

Standard Model

Weinberg Operator

84

30

993

560

$$\frac{c_{ij}}{\Lambda}(L_i H)(L_j H) + \text{h.c.}$$

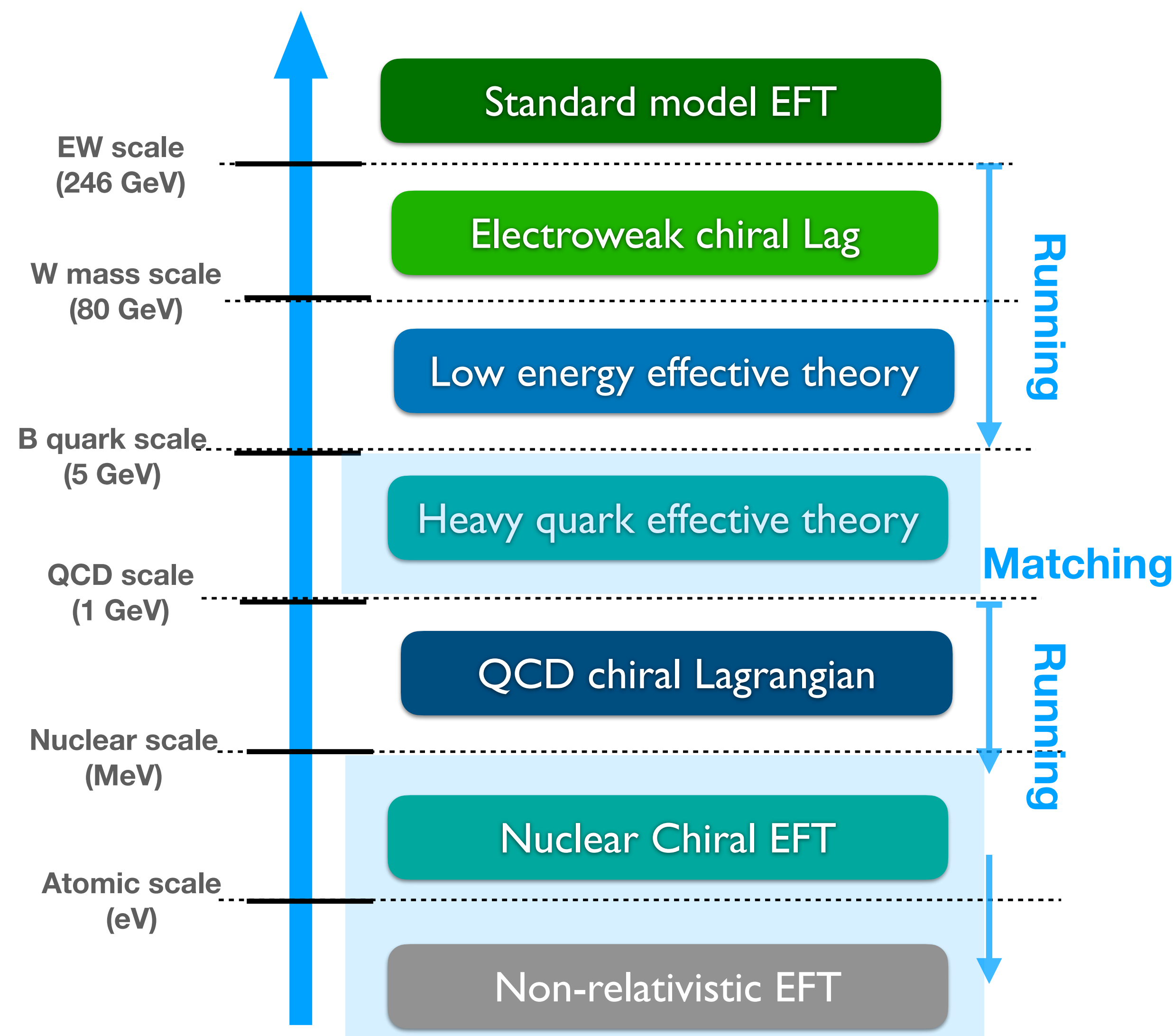
Dim-8: 993 operators from 16 Young diagrams [Hao-Lin Li, Jing Shu, Zhe Ren, Ming-Lei Xiao, J.H.Yu, Yu-Hui Zheng, 2005.00008]

$\bar{n} \backslash n$	0	1	2	3	4
0	ϕ^8	$\psi^2 \phi^5$	$\psi^4 \phi^2, F_L \psi^2 \phi^3, F_L^2 \phi^4$	$F_L \psi^4, F_L^2 \psi^2 \phi, F_L^3 \phi^2$	F_L^4
1	$\psi^{\dagger 2} \phi^5$	$\psi^{\dagger 2} \psi^2 \phi^2, \psi^{\dagger} \psi \phi^4 D, \phi^6 D^2$	$F_L \psi^{\dagger 2} \psi^2, F_L^2 \psi^{\dagger 2} \phi, \psi^{\dagger} \psi^3 \phi D, F_L \psi^{\dagger} \psi \phi^2 D, \psi^2 \phi^3 D^2, F_L \phi^4 D^2$	$F_L^2 \psi^{\dagger} \psi D, \psi^4 D^2, F_L \psi^2 \phi D^2, F_L^2 \phi^2 D^2$	
2	$\psi^{\dagger 4} \phi^2, F_R \psi^{\dagger 2} \phi^3, F_R^2 \phi^4$	$F_R \psi^{\dagger 2} \psi^2, F_R^2 \psi^2 \phi, \psi^{\dagger 3} \psi \phi D, F_R \psi^{\dagger} \psi \phi^2 D, \psi^{\dagger 2} \phi^3 D^2, F_R \phi^4 D^2$	$F_R^2 F_L^2, F_R F_L \psi^{\dagger} \psi D, \psi^{\dagger 2} \psi^2 D^2, F_R \psi^2 \phi D^2, F_L \psi^{\dagger 2} \phi D^2, F_R F_L \phi^2 D^2, \phi^4 D^4, \psi^{\dagger} \psi \phi^2 D^3$		
3	$F_R \psi^{\dagger 4}, F_R^2 \psi^{\dagger 2} \phi, F_R^3 \phi^2$	$F_R^2 \psi^{\dagger} \psi D, \psi^{\dagger 4} D^2, F_R \psi^{\dagger 2} \phi D^2, F_R^2 \phi^2 D^2$			
4	F_R^4				

$\bar{n} \backslash n$	0	1	2	3	4
0					
1					
2					
3					
4					

Tower of EFTs from EW to low scales

Describe collider physics, flavor physics, hadronic physics, nuclear physics and atomic physics



[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2201.04639]

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2007.07899]

[Hao-Lin Li, Jing Shu, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2005.00008]

[Zhe Ren, **J.H.Yu**, 2211.01420]

[Hao Sun, Ming-Lei Xiao, **J.H.Yu**, 2206.07722]

[Hao Sun, Ming-Lei Xiao, **J.H.Yu**, 2210.14939]

[Hao Sun, Yi-Ning Wang, **J.H.Yu**, 2211.11598]

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2012.09188]

[Hao-Lin Li, Zhe Ren, Ming-Lei Xiao, **J.H.Yu**, Yu-Hui Zheng, 2105.09329]

[Huayang Song, Hao Sun, **J.H.Yu**, 2305.16770]

[Huayang Song, Hao Sun, **J.H.Yu**, 2306.05999]

[Chuan-Qiang Song, Hao Sun, **J.H.Yu**, 2404.15047]

[Xuan-He Li, Hao Sun, Feng-Jie Tang, **J.H.Yu**, 2404.14152]

[Chuan-Qiang Song, Hao Sun, **J.H.Yu**, 2501.09787]

[Hao Sun, Yi-Ning Wang, **J.H.Yu**, 2501.14018]

[Yong-Kang Li, Yi-Ning Wang, **J.H.Yu**, in preparation]

Scattering amplitudes at low energy

The spin symmetry (little group) plays essential roles to describe physical particles

$$\psi(x) = \sum_s \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left[a_s(\vec{p}) u_s(\vec{p}) e^{-ip \cdot x} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{ip \cdot x} \right]$$

Field transform under $SU(2)_A \times SU(2)_B$ Lorentz rep
non-unitary rep

Particle transform under $SU(2)_j$ Little group
unitary irreps

Weinberg: Wave function is just CG Coefficients of two groups

$$u_{ab}(0, \sigma) = (2m)^{-1/2} C_{AB}(j\sigma; ab)$$

[Yi-Ning Wang, J.H.Yu, in préparation]

3-particle amplitudes can be determined by CG Coefficients from spin-orbital couple and then boost back

$$\langle \vec{p}_1, \sigma_1 | \langle \vec{p}_2, \sigma_2 | \vec{P}, J, \sigma \rangle = C_{l, \sigma_l; s, \sigma_s}^{J, \sigma} C_{s_1, \sigma_1; s_2, \sigma_2}^{s, \sigma_s} Y_{L\sigma_l}(\Omega)$$

$$\mathcal{A}_{\sigma_1}^{\sigma_2 \sigma_3}(\mathbf{k}_1, \mathbf{p}_2^*, \mathbf{p}_3^*) = \underbrace{\Gamma_{\alpha_1}^{\alpha_2 \alpha_3}(\mathbf{k}_1, \mathbf{p}_2^*, \mathbf{p}_3^*)}_{\text{pure-orbital part}} \times \underbrace{\bar{u}_{\sigma_1}^{\alpha_1}(s_1) u_{\alpha_2}^{\sigma_2}(\mathbf{p}_2^*, s_2) u_{\alpha_3}^{\sigma_3}(\mathbf{p}_3^*, s_3)}_{\text{pure-spin part}} = \underbrace{\Gamma_{\alpha_1}^{\beta_2 \beta_3}(\mathbf{k}_1, \mathbf{p}_2^*, \mathbf{p}_3^*) D_{\beta_2}^{\alpha_2}(\Lambda_{2^*}) D_{\beta_3}^{\alpha_3}(\Lambda_{3^*})}_{\text{pure-orbital part}} \times \underbrace{\bar{u}_{\sigma_1}^{\alpha_1}(s_1) u_{\alpha_2}^{\sigma_2}(s_2) u_{\alpha_3}^{\sigma_3}(s_3)}_{\text{pure-spin part}}$$

B.S.Zou and D.V.Bugg, Eur.Phys.J.A,16,537-547 (2003)

Explicit spin symmetry

[Yong-Kang Li, Yi-Ning Wang, **J.H.Yu**, in préparation]

Since the spin symmetry could determines 3-pt amplitudes, suggest the Lorentz symmetry can be hidden

Lorentz symmetry

$$\begin{cases} \hat{P}^\mu, & \text{Translations,} \\ \hat{J}^i, & \text{Rotations,} \\ \hat{K}^i, & \text{Boosts,} \end{cases}$$

$$R^4 \rtimes SO(3, 1)$$

$$\langle \Omega | \hat{P}^\mu | \Omega \rangle \equiv m v^\mu$$

$$v_0^\mu = (1, 0, 0, 0)$$

Spin symmetry

$$\begin{cases} \hat{H}_v &= v \cdot \hat{P} & , & \text{Time Translation,} \\ (\hat{P}_v)^i &= v_\perp^i \cdot \hat{P} & , & \text{Spatial Translations,} \\ (\hat{J}_v)^i &= (v_\perp^i)^\mu \epsilon_{\mu\nu\rho\sigma} \hat{J}^{\nu\rho} v^\sigma & , & \text{Rotations,} \end{cases}$$

$$R^{1,3} \rtimes SO(3)$$

CCWZ Coset description

$$v^\mu = L(\vec{\eta}(v)) v_0^\mu \quad L(\vec{\eta}) = e^{i\vec{\eta} \cdot \vec{K}}$$

Missing Goldstone

$$|v, 0, \sigma\rangle + |v, \vec{k}\rangle = |v, \vec{k}, \sigma\rangle$$

Rest state **Goldstone** **k state**

$$|v, \vec{k}, \sigma\rangle = U(L_v(k)) |v, 0, \sigma\rangle$$

Boost relates different k

Shift symmetry

$$\text{rotation : } \vec{\eta} \rightarrow \vec{\eta}' = R\vec{\eta},$$

$$\text{boost : } \vec{\eta} \rightarrow \vec{\eta}' = \vec{\eta} + \frac{\vec{q}}{m} + \mathcal{O}(q^2).$$

$$v^\mu \rightarrow v^\mu + \frac{q^\mu}{m}$$

Reparametrization invariance (RPI)

Heavy quark effective theory

Two equivalent descriptions on heavy quark effective theory (HQET) Lagrangian with NR fields

Top down integrate out

$$\Psi(x) = e^{-imv \cdot x} (Q_v(x) + B_v(x))$$

$$\mathcal{L} = \bar{\Psi} (i\not{D} - m) \Psi + \sum_n c_n \bar{\Psi} \frac{\mathcal{O}_{eff}^{(n)}}{m^n} \Psi,$$

$$Q_v(x) = e^{iP_x} P_+ \Psi(x) = e^{imvx} \frac{1 + \not{v}}{2} \Psi(x),$$

$$B_v(x) = e^{iP_x} P_- \Psi(x) = e^{imvx} \frac{1 - \not{v}}{2} \Psi(x),$$

$$\begin{aligned} \mathcal{L} = & \bar{Q}_v (iv \cdot D + \sum_n c_n \frac{\mathcal{O}_{eff}^{(n)}}{m^n}) Q_v + \bar{Q}_v (i\not{D}_\perp + \sum_n c_n \frac{\mathcal{O}_{eff}^{(n)}}{m^n}) \\ & \times (iv \cdot D + 2m - \sum_n c_n \frac{\mathcal{O}_{eff}^{(n)}}{m^n})^{-1} (i\not{D}_\perp + \sum_n c_n \frac{\mathcal{O}_{eff}^{(n)}}{m^n}) Q_v. \end{aligned}$$

Expand to a non-relativistic basis

Bottom up nonlinear trans.

$$U(L(\vec{q})) |v, \vec{k}, \sigma\rangle = \sum_{\sigma'} D_{\sigma'\sigma}(W(L(\vec{q}), \vec{k})) |v, L(\vec{q})\vec{k}, \sigma'\rangle.$$

$$U(\Lambda) Q_{v,l} U(\Lambda)^\dagger = D[W(\Lambda, i\partial)]_{ll'} Q_{v,l'}$$

$$\mathcal{L} = N^\dagger \{ iD_t + c_2 \frac{\vec{D}^2}{2m} + c_F g \frac{\vec{\sigma} \cdot \vec{B}}{2m} + c_D g \frac{[D_i, E^i]}{8m^2} + i c_S g \frac{\epsilon^{ijk} \sigma^i \{D_j, E^k\}}{8m^2} + c_4 \frac{\vec{D}^4}{8m^3} \} N$$

$$Q_v(x) = e^{imt} P_+ \Psi(x) \equiv \begin{bmatrix} N(x) \\ 0 \end{bmatrix}$$

$$N'(x) \rightarrow e^{i\vec{q} \cdot \vec{x}} [1 - i \frac{\vec{q} \cdot \vec{D}}{2m^2} + \frac{\vec{\sigma} \cdot \vec{q} \times \vec{D}}{4m^2} + \mathcal{O}(g, \frac{1}{m^4})] N'(\Lambda^{-1}x).$$

$$c_2 = 1,$$

$$c_4 = 1,$$

$$c_S = 2c_F - 1,$$

$$2c_M = c_D - c_F,$$

FW trans.

Nonlinear little group trans. = RPI

3-point amplitudes for HQET and black hole EFT

Third description: spinor-helicity formalism with projection to determines the HQET operators

3-pt scaling for highest weight

[Yong-Kang Li, Yi-Ning Wang, **J.H.Yu**, in préparation]

$$|k^+\rangle \equiv (\psi^+)_I^s \quad |k^-\rangle \equiv (\psi^-)_I^s$$

$$\left[1^{\pm}2^{\pm}\right]_{S_1+S_2-S_3}\left[1^{\pm}3^{\pm}\right]_{S_1+S_3-S_2}\left[2^{\pm}3^{\pm}\right]_{S_2+S_3-S_1}$$

$$|k^-] = \frac{1}{2m + v \cdot k} |k^+].$$

Direct construction

Heavy quark effective theory

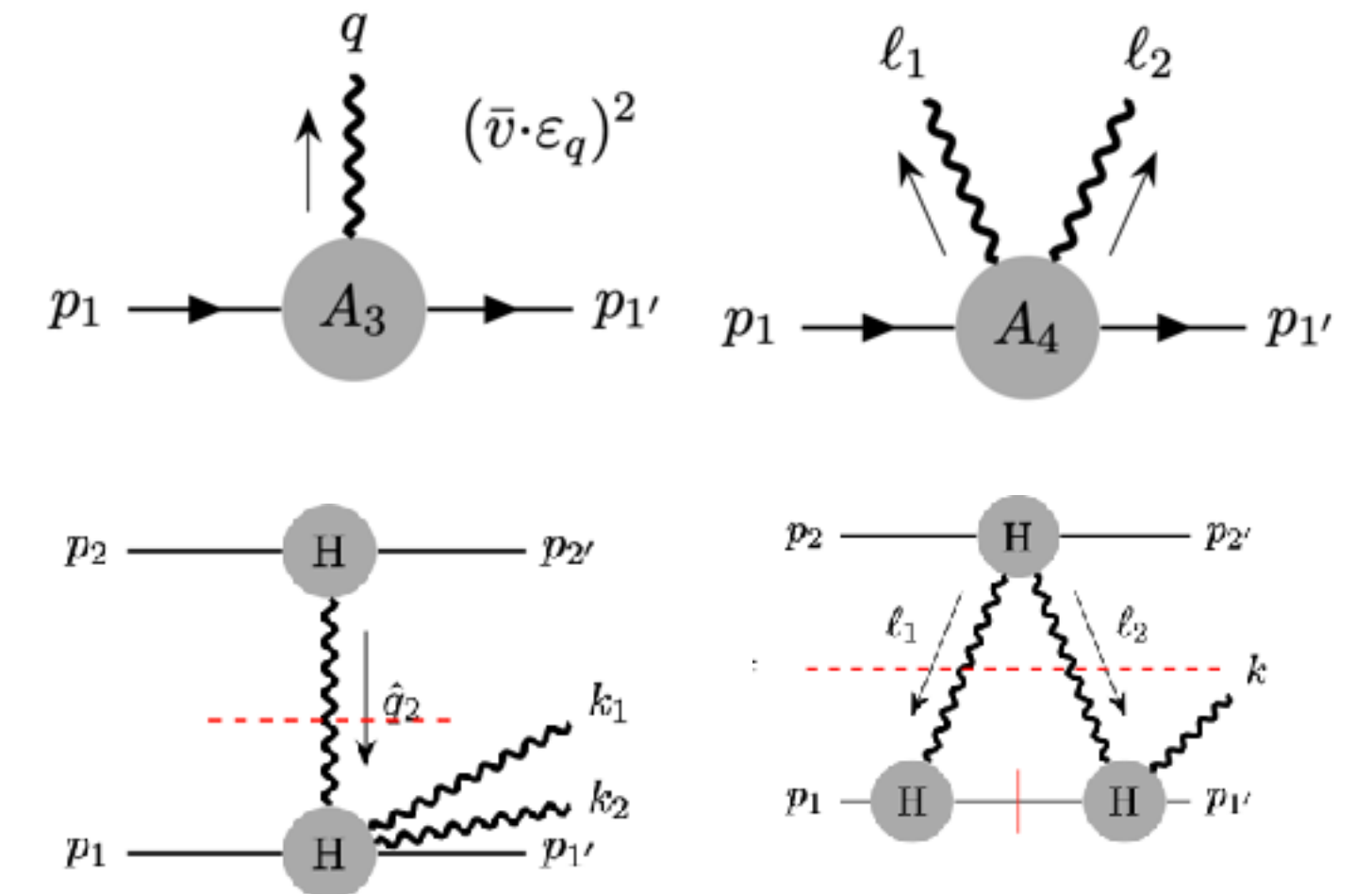
Heavy black hole effective theory

dim	Operators		
5	$N^\dagger \sigma \cdot BN$	$N^\dagger \sigma \cdot B\chi$	$\chi^\dagger \sigma \cdot B\chi$
6	$N^\dagger \sigma \cdot B\partial_t N$	$N^\dagger \sigma \cdot B\partial_t \chi$	$\chi^\dagger \sigma \cdot B\partial_t \chi$
	$N^\dagger \sigma \cdot \partial_t BN$	$N^\dagger \sigma \cdot \partial_t B\chi$	$\chi^\dagger \sigma \cdot \partial_t B\chi$

	Amplitudes		
	$[12^+][13^+]$	$[12^-][13^+]$	$[12^-][13^-]$
$(v \cdot k_2) \times$	$[12^+][13^+]$	$[12^-][13^+]$	$[12^-][13^-]$
$(v \cdot q_1) \times$	$[12^+][13^+]$	$[12^-][13^+]$	$[12^-][13^-]$

Linear RPI

dim	Amplitudes	Operators
5	$[12^+][13^+]$	$N^\dagger \boldsymbol{\sigma} \cdot \mathbf{B} N$
6	$[12^+][1 q_{1\perp} 3^+]$	$N^\dagger \epsilon^{ijk} \sigma^i [\nabla^j, B^k] N$
	$[12^+][1 k_{2\perp} 3^+]$	$N^\dagger \mathbf{B} \cdot \mathbf{D} N$
	$[13^+][1 k_{2\perp} 2^+]$	$N^\dagger \epsilon^{ijk} \sigma^i B^j D^k N$
7	$[1 q_{1\perp} 2^+][1 k_{2\perp} 3^+]$	$N^\dagger \epsilon^{ijk} \{D^i, [\partial^j B^k]\} N$
	$[1 q_{1\perp} 2^+][1 q_{1\perp} 3^+]$	$N^\dagger \boldsymbol{\sigma} \cdot \mathbf{D} \mathbf{B} \cdot \mathbf{D} N$
	$[1 k_{2\perp} 2^+][1 k_{2\perp} 3^+]$	$N^\dagger \mathbf{D} \cdot \mathbf{B} \boldsymbol{\sigma} \cdot \mathbf{D} N$
	$\text{tr}[q_{1\perp} q_{1\perp}][12^+][13^+]$	$N^\dagger [\partial^2, \boldsymbol{\sigma} \cdot \mathbf{B}] N$
	$\text{tr}[k_{2\perp} k_{2\perp}][12^+][13^+]$	$N^\dagger \boldsymbol{\sigma} \cdot \mathbf{B} D^2 N$
	$\text{tr}[q_{1\perp} k_{2\perp}][12^+][13^+]$	$N^\dagger \epsilon_{ijk} D^i \boldsymbol{\sigma} \cdot \mathbf{B} D^j N$



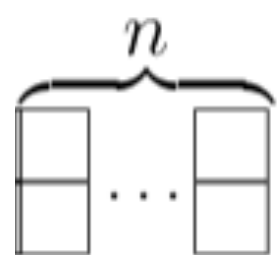
Jiang-Hao Yu (ITP-CAS)

Non-relativistic effective theory

The NR operator basis for **any spins** can be obtained, and perform spin-orbital partial wave decomposition

[Yong-Kang Li, Yi-Ning Wang, **J.H.Yu**, in préparation]

SU(2) x SU(N) Young diagram



NR 4 fermion operators

Pion-less nuclear EFT

Cold atom EFT

Spin-s dark matter operators

Spinning binary black hole

NR Fermion bilinear

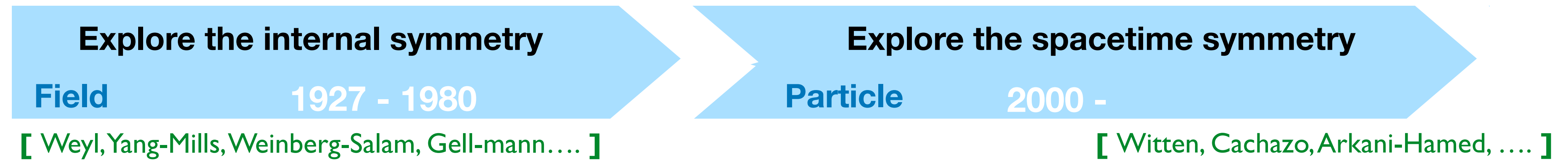
NR partial wave analysis

Integrate-out amplitudes		Operators
$[1^+ 2^+][3^+ 4^+]$	$[1^+ 3^+][2^+ 4^+]$	$(N^\dagger N)(N^\dagger N) \quad (N^\dagger \sigma N) \cdot (N^\dagger \sigma N)$
$[1^+ k_{2\perp} 2^+][3^+ k_{2\perp} 4^+]$	$[1^+ k_{2\perp} 3^+][2^+ k_{2\perp} 4^+]$	$:(N^\dagger \vec{\sigma} \cdot \vec{\nabla} \vec{\nabla}^i N^\dagger)(N \sigma^i N) + h.c.$
$[1^+ k_{2\perp} k_{2\perp} 2^+][3^+ 4^+]$	$[1^+ 3^+][2^+ k_{3\perp} k_{2\perp} 4^+]$	$:(N^\dagger \vec{\nabla}^2 N^\dagger)(N N) + h.c.$
$[1^+ k_{2\perp} k_{3\perp} 2^+][3^+ 4^+]$	$[1^+ k_{2\perp} k_{3\perp} 4^+][2^+ 3^+]$	$:(N^\dagger \vec{\sigma} \cdot \vec{\nabla} \sigma^i N^\dagger)(N \vec{\nabla}^i N) + h.c.$
$[1^+ k_{2\perp} 2^+][3^+ k_{3\perp} 4^+]$	$[1^+ 2^+][3^+ 4^+] tr[k_{2\perp} k_{3\perp}]$	$:(N^\dagger \vec{\sigma} \cdot \vec{\nabla} N^\dagger)(N \vec{\sigma} \cdot \vec{\nabla} N) + h.c.$
$[1^+ 3^+][2^+ 4^+] tr[k_{2\perp} k_{4\perp}]$	$[1^+ k_{2\perp} k_{4\perp} 3^+][2^+ 4^+]$	$:-2(N^\dagger N)(N^\dagger \vec{\nabla} \cdot \vec{\nabla} N) + h.c.$
$[1^+ k_{2\perp} k_{4\perp} 4^+][2^+ 3^+]$	$[1^+ k_{2\perp} 2^+][3^+ k_{4\perp} 4^+]$	$:(N^\dagger \sigma^i \sigma^j N)(N^\dagger \vec{\nabla}^i \vec{\nabla}^j N) + h.c.$

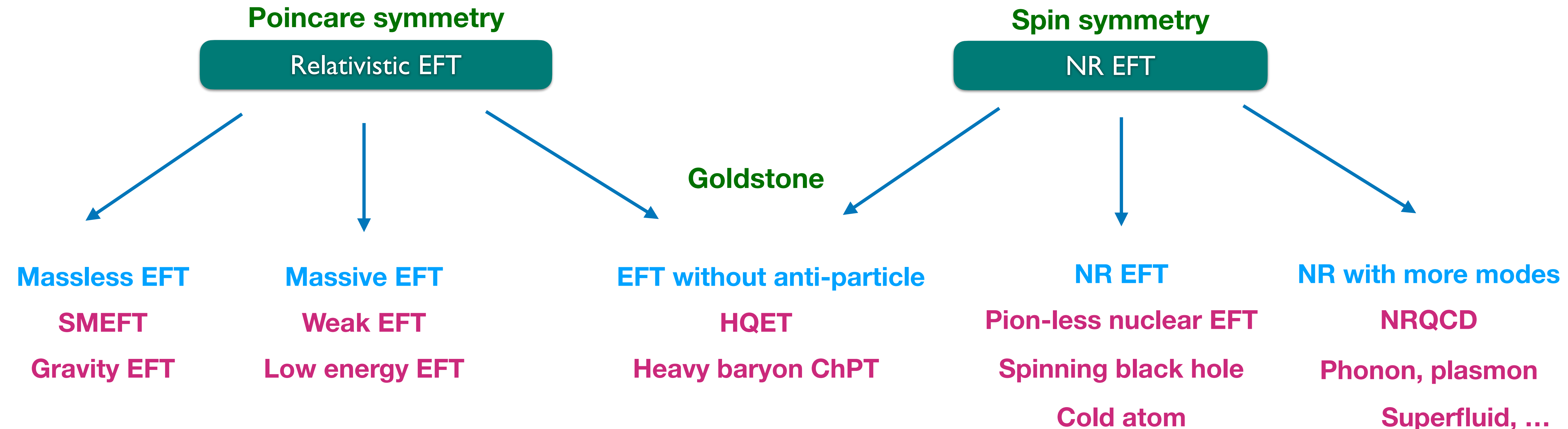
Young tableau	Amplitudes	Operators
$\begin{array}{ c c } \hline i_1 & i_3 \\ \hline i_2 & i_4 \\ \hline \end{array}$	$[1^+ 2^+][3^+ 4^+]$	$(N_1^\dagger N_2)(\xi_3^\dagger \xi_4)$
$\begin{array}{ c c } \hline i_1 & i_2 \\ \hline i_3 & i_4 \\ \hline \end{array}$	$[1^+ 3^+][2^+ 4^+]$	$(N_1^\dagger \xi_3^\dagger)(N_2 \xi_4)$
$\begin{array}{ c c c } \hline i_1 & i_2 & j_2 \\ \hline j_2 & i_3 & i_4 \\ \hline \end{array}$	$-[1^+ k_{2\perp} 4^+][2^+ 3^+]$	$-(N_1^\dagger \sigma \cdot \nabla_2 \xi_4)(N_2 \xi_3^\dagger)$
$\begin{array}{ c c c } \hline i_1 & i_2 & i_3 \\ \hline j_2 & j_2 & i_4 \\ \hline \end{array}$	$[1^+ k_{2\perp} 2^+][3^+ 4^+]$	$(N_1^\dagger \sigma \cdot \nabla_2 N_2)(\xi_3^\dagger \xi_4)$
$\begin{array}{ c c c } \hline i_1 & j_2 & j_2 \\ \hline i_2 & i_3 & i_4 \\ \hline \end{array}$	$[1^+ 2^+][3^+ k_{2\perp} 4^+]$	$(N_1^\dagger N_2)(\xi_3^\dagger \sigma \cdot \nabla_2 \xi_4)$
$\begin{array}{ c c c } \hline i_1 & i_2 & i_3 \\ \hline j_3 & j_3 & i_4 \\ \hline \end{array}$	$[1^+ k_{3\perp} 2^+][3^+ 4^+]$	$(N_1^\dagger \sigma \cdot \nabla_3 N_2)(\xi_3^\dagger \xi_4)$

	Amp	$^{2S+1}L_J$	C	P
NN	$[1^+ 2^+]$	1S_0	$\backslash$	+
$N \sigma^i N$	$ 1^+ \rangle^{(I)} 2^+ \rangle^{(J)}$	3S_1	$\backslash$	+
$N \vec{\nabla}^i N$	$q^{IJ} [1^+ 2^+]$	1P_0	$\backslash$	-
$N(\vec{\nabla} \cdot \sigma) N$	$[1^+ q 2^+]$	3P_0	$\backslash$	-
$N \vec{\nabla} \times \vec{\sigma} N$	$q_I^K 1^+ \rangle^{(I)} 2^+ \rangle^{(J)}$	3P_1	$\backslash$	-
$N(\vec{\nabla}^i \vec{\sigma}^j - \frac{\vec{\nabla} \cdot \sigma}{3} \delta^{ij}) N$	$q^{(KL)} 1^+ \rangle^{(I)} 2^+ \rangle^{(J)}$	3P_2	$\backslash$	-

Summary and outlook



The spacetime **symmetry** dictates interactions (**operator basis**)



Summary and outlook

[Witten, Cachazo, Arkani-Hamed,]

Pion-less nuclear EFT

Spinning black hole

Cold atom

NRQCD

Phonon, plasmon

Superfluid, ...

Spin symmetry

NR EFT

NR EFT

NR with more modes

Thanks for your attention!