RESONANCE SUBTRACTION IN HIGH ENERGY PROCESSES

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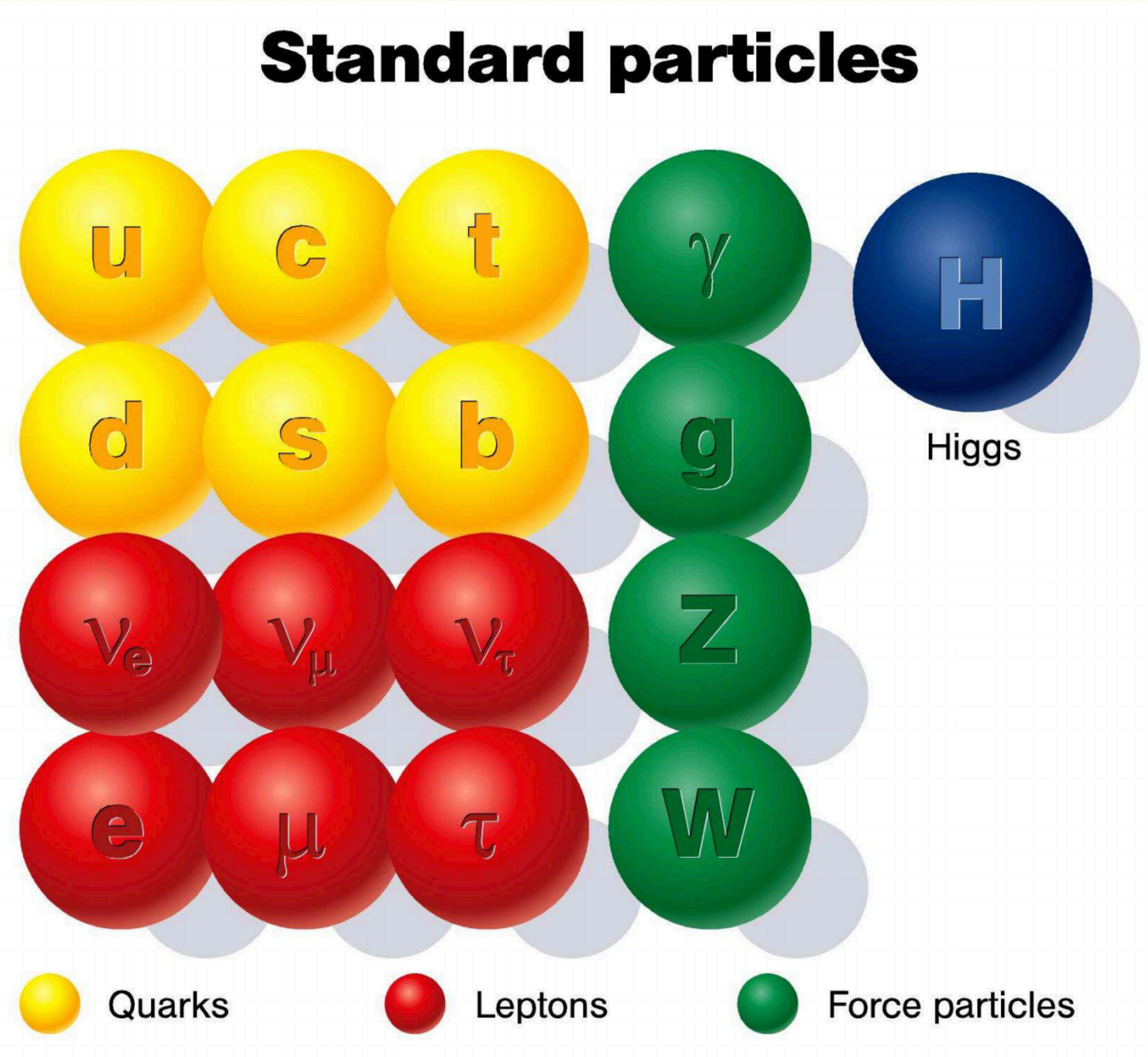
OUTLINE

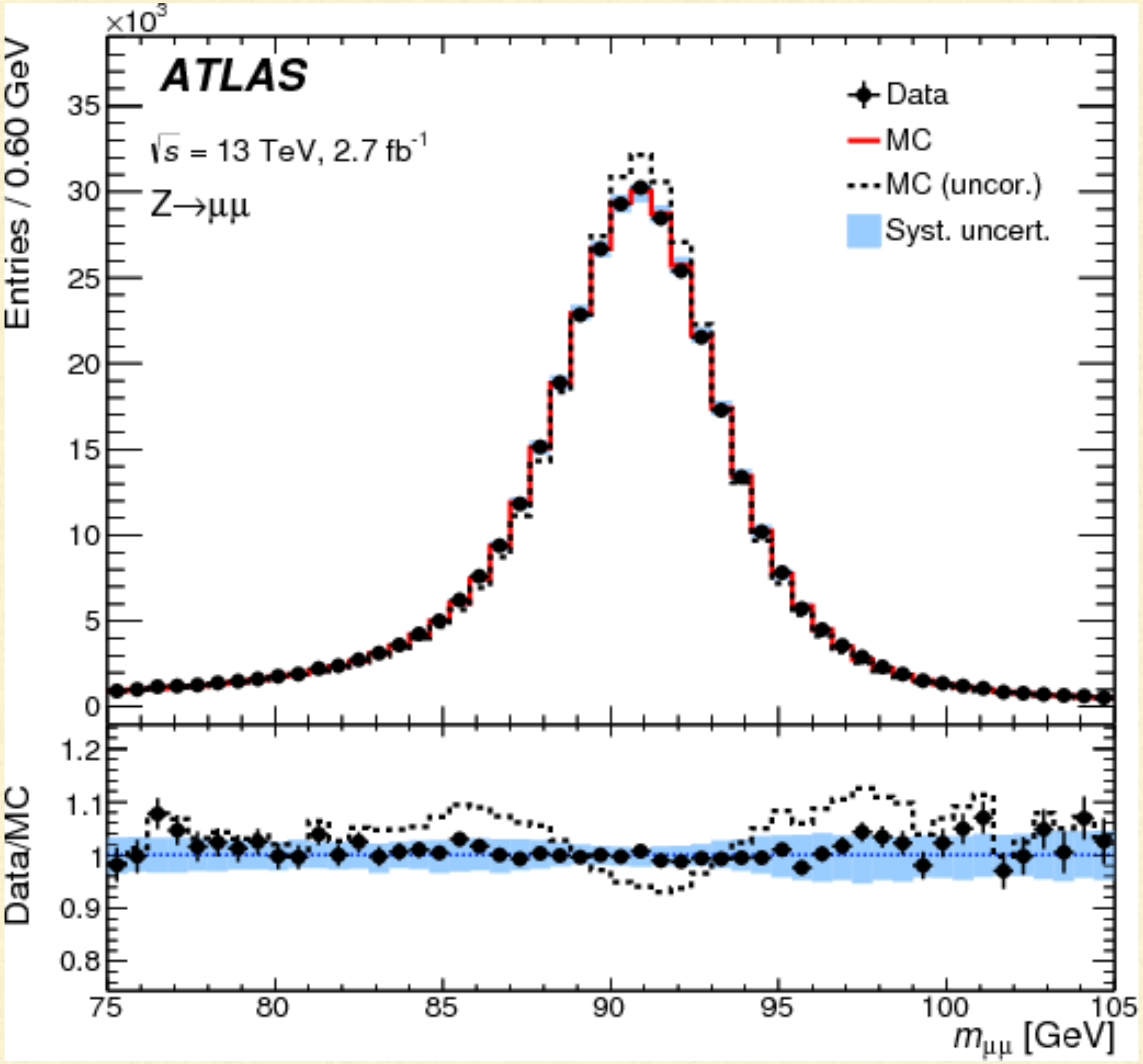
- Unstable particles
- Subtraction of $t\bar{t}$ resonant contributions in $tW^-\bar{b}$ production
- Summary

UNSTABLE PARTICLE



Most of the elementary particles are unstable!

DECAY OF UNSTABLE PARTICLE



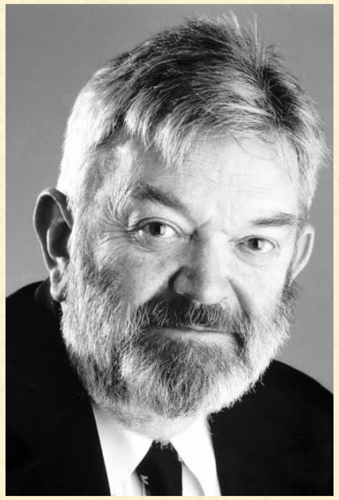
Unstable particles appear as resonances, constructed from decay products.

$$M_Z = 91.1880 \pm 0.0020 \text{ GeV}$$

$$\Gamma_Z = 2.4955 \pm 0.0023 \text{ GeV}$$

"Relativistic Field Theory of Unstable Particles", Matthews and Salam, 1958

"Unitarity and causality in a renormalizable field theory with unstable particles", 1963



Abdus Salam

M.J.G. Veltman

- In standard perturbation theory, the S-matrix element requires stable asymptotic states.
- One can put the resonance in the intermediate state with the propagator $\sim 1/(p^2 - M^2)$. However, near the resonance phase space, it leads to a singularity.
- Imaginary parts of the self-energy corrections have to be Dyson summed, $\sim \frac{1}{p^2 - M^2 + \Sigma(p^2)} \sim \frac{1}{p^2 - M^2 + iM\Gamma}$

- The simplest way is to employ the narrow-width approximation. The resonance is taken as stable intermediate state.

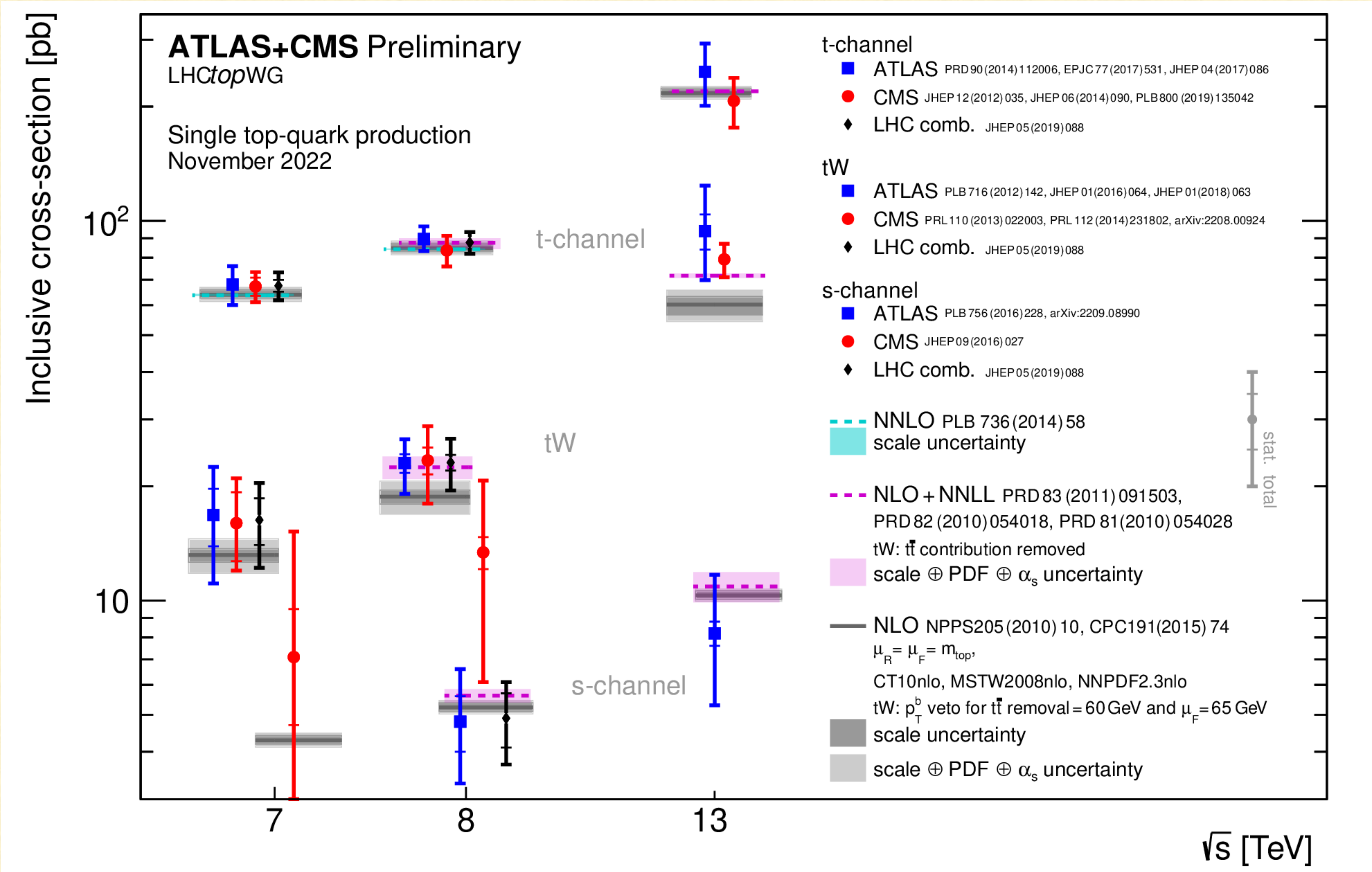
$$\frac{1}{|p^2 - M^2 + iM\Gamma|^2} \sim \frac{\pi}{M\Gamma}\delta(p^2 - M^2)$$

- NWA is useful to give an approximate calculation of radiative corrections, neglecting off-shell effects of $\mathcal{O}(\Gamma/M)$.

- Beyond LO, we need to consider mass renormalization.
- On-shell scheme: $M_{OS}^2 - M^2 + Re[\Sigma(M_{OS}^2)] = 0$
- Complex-mass scheme (gauge independent):
  $\mu^2 - M^2 + \Sigma(\mu^2) = 0,\ \mu^2 = M_{pole}^2 + iM_{pole}\Gamma_{pole}$
- Historically, the W and Z masses and widths were determined in the OS scheme. The conversion forms are
  $$M_{pole}^2 = \frac{M_{OS}^2}{1+r},\ \Gamma_{pole}^2 = \frac{\Gamma_{OS}^2}{1+r},\ r = \Gamma_{OS}^2/M_{OS}^2$$
- $M_{W,pole} \approx M_{W,OS} - 27\ \text{MeV},\ M_{Z,pole} \approx M_{Z,OS} - 34\ \text{MeV}$

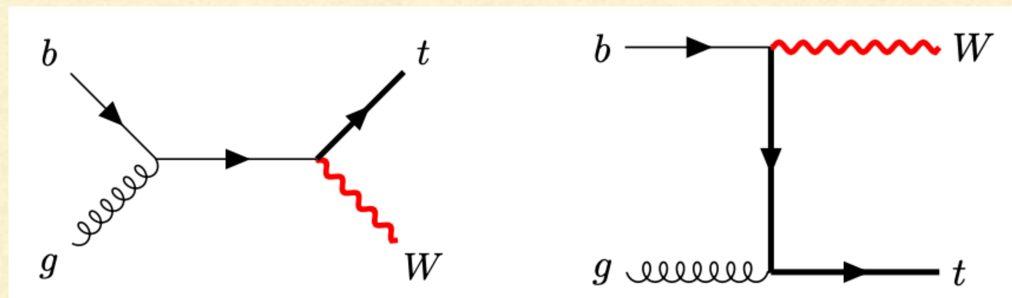
PLB 206,539, NPB 500,255

tW PRODUCTION

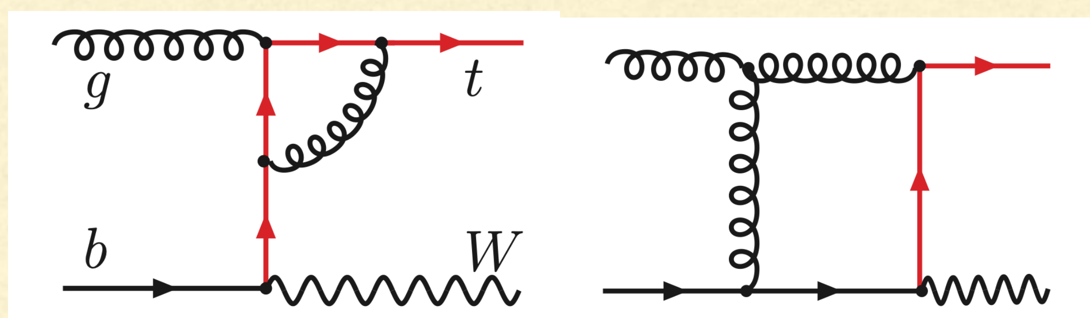


tW PRODUCTION

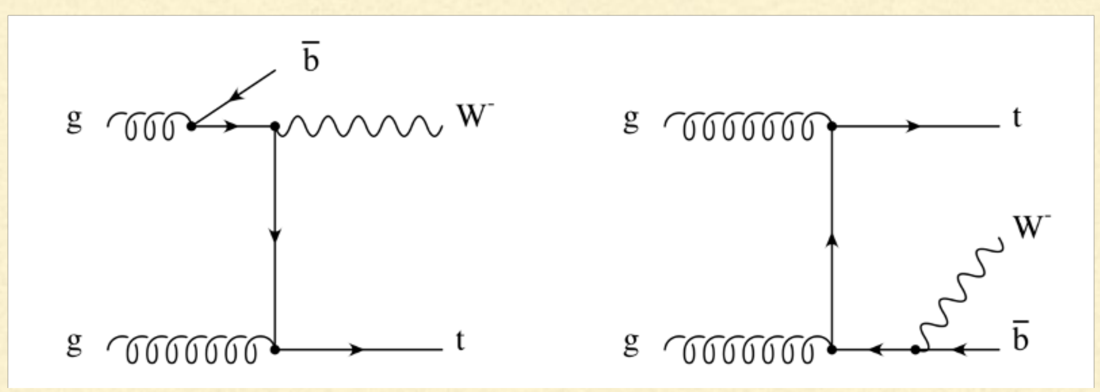
LO



NLO



Giele, arXiv:hep-ph/9511449
Zhu, arXiv:hep-ph/0109269
Campbell, Tramontano, hep-ph/0606289
Cao, arXiv:0801.1539
Kant et al arXiv:1406.4403

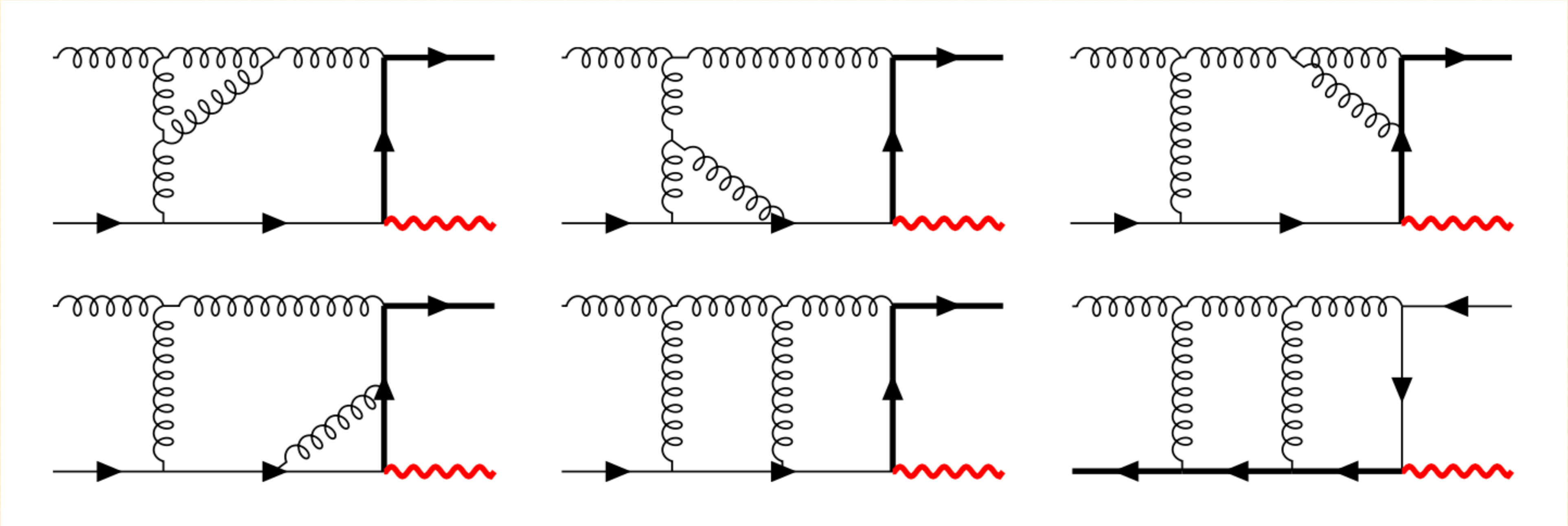


Frixione et al 2008
Hollik et al 2012
Campbell et al 2005

Top resonance appears!

tW PRODUCTION

- Toward NNLO



Chen, Dong, Li, Li, JW, Wang, arXiv:2208.08786, 2212.07190

tW PRODUCTION

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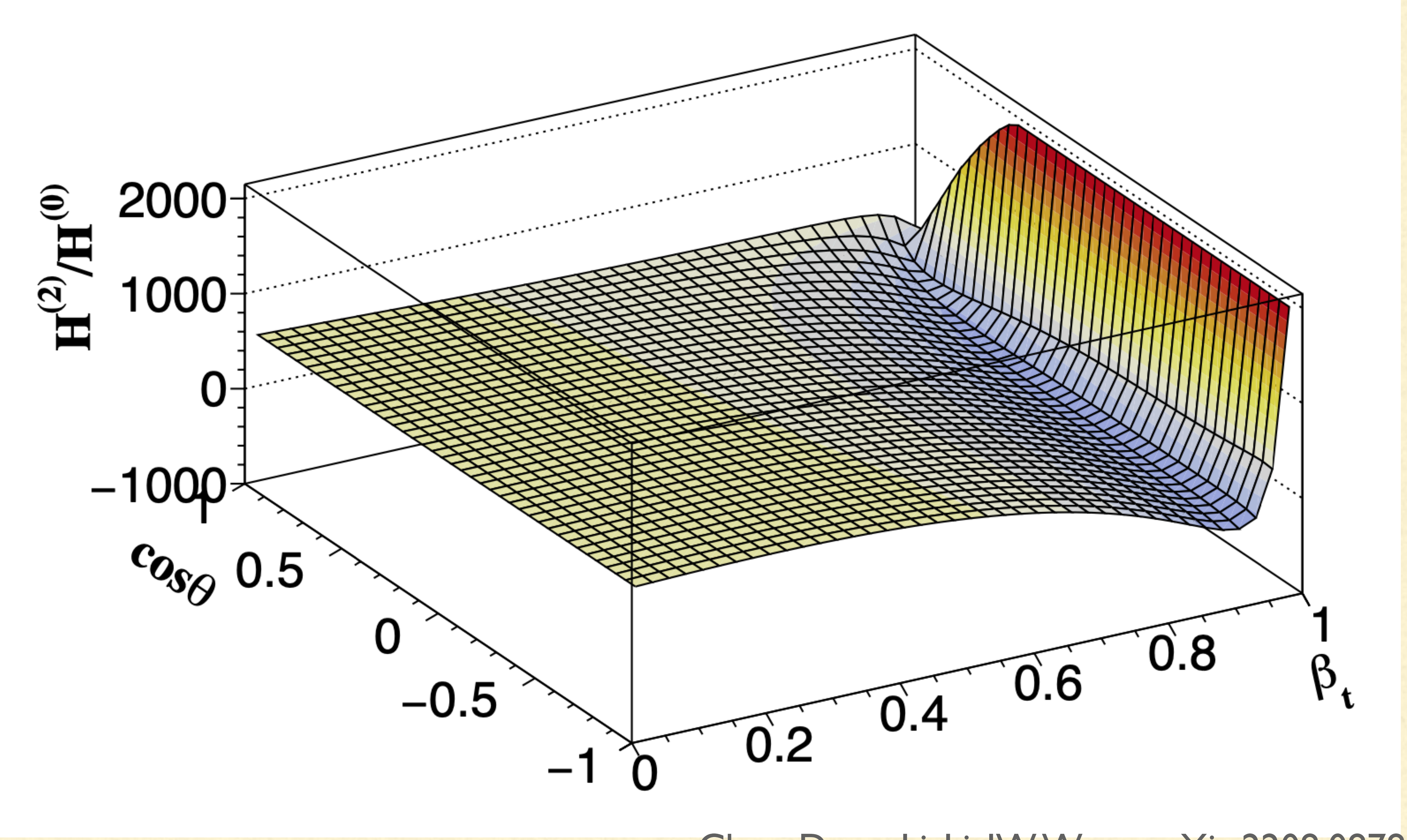

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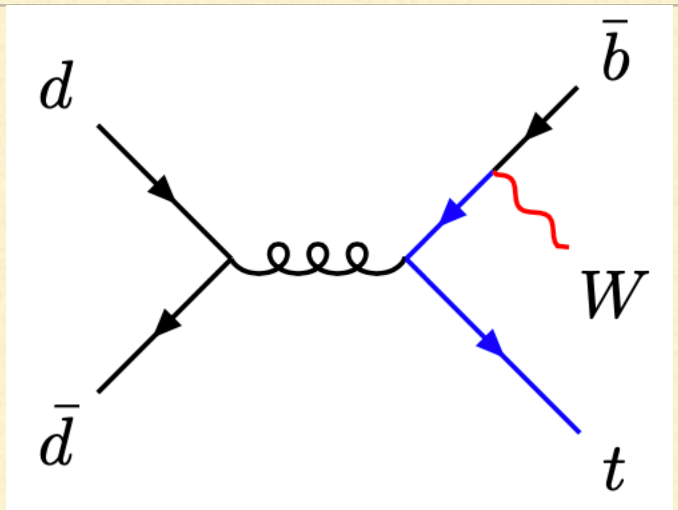
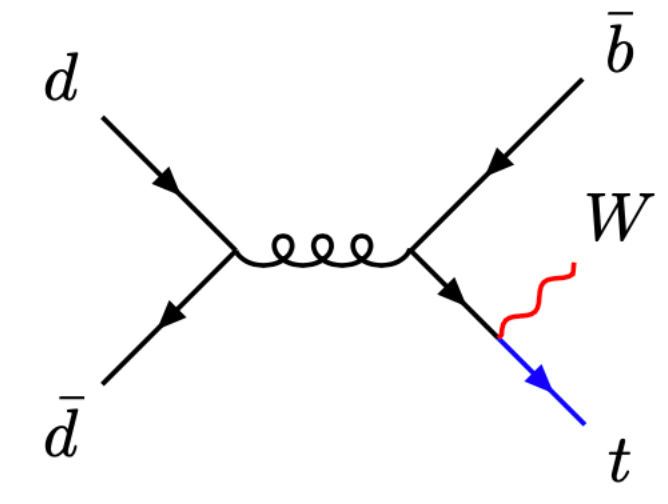
SUBTRACTION OF $t\bar{t}$



Double resonance



Single resonance

$$|\mathscr{M}_{1t}^{(0)}|^2 + 2\,\mathbf{Re}[\mathscr{M}_{1t}^{(0)}\mathscr{M}_{2t}^{(0)*}] + |\mathscr{M}_{2t}^{(0)}|^2$$

DR1: $$|\mathscr{M}_{1t}^{(0)}|^2 + 2\,\mathbf{Re}[\mathscr{M}_{1t}^{(0)}\mathscr{M}_{2t}^{(0)*}] + |\mathscr{M}_{2t}^{(0)}|^2$$ 0805.3067

DR2: $$|\mathscr{M}_{1t}^{(0)}|^2 + 2\,\mathbf{Re}[\mathscr{M}_{1t}^{(0)}\mathscr{M}_{2t}^{(0)*}] + |\mathscr{M}_{2t}^{(0)}|^2$$ 1207.1071

SUBTRACTION OF $t\bar{t}$

DS $$\left(|\mathscr{M}_{tW\bar{b}}|^2_{\text{LO}} \right)_{\text{Sub}} = |\mathscr{M}^{(0)}_{1t} + \mathscr{M}^{(0)}_{2t}|^2 - \mathscr{R}_{\text{LO}}$$

$$\mathscr{R}_{\text{LO}} = S(\{p_i\}, \{\tilde{p}_i\}) \cdot R_{\text{LO}}(\{\tilde{p}_i\})$$

Supp. Factor

Sub. Kernel

Reshuffled momenta to preserve gauge inv.

SUBTRACTION OF $t\bar{t}$

Diagram Subtraction (gauge invariant):

0805.3067

$$\begin{aligned}\left(|\mathcal{M}_{tW\bar{b}}|^2_{\rm LO}\right)_{\rm DS} &= \left[|\mathcal{M}^{(0)}_{1t} + \mathcal{M}^{(0)}_{2t}|^2 - S_1 \cdot (R_{\rm LO})_1\right]_{\rm Reg} \\ &= |\mathcal{M}^{(0)}_{1t} + \mathcal{M}^{(0)}_{2t}|^2_{\rm Reg} - \frac{(m_t\Gamma_t)^2}{\Delta^2 + (m_t\Gamma_t)^2} \cdot \frac{\widetilde{N}}{\widetilde{\Delta}^2 + (m_t\Gamma_t)^2} \\ &= |\mathcal{M}^{(0)}_{1t} + \mathcal{M}^{(0)}_{2t}|^2_{\rm Reg} - \frac{\widetilde{N}}{\Delta^2 + (m_t\Gamma_t)^2},\end{aligned}$$

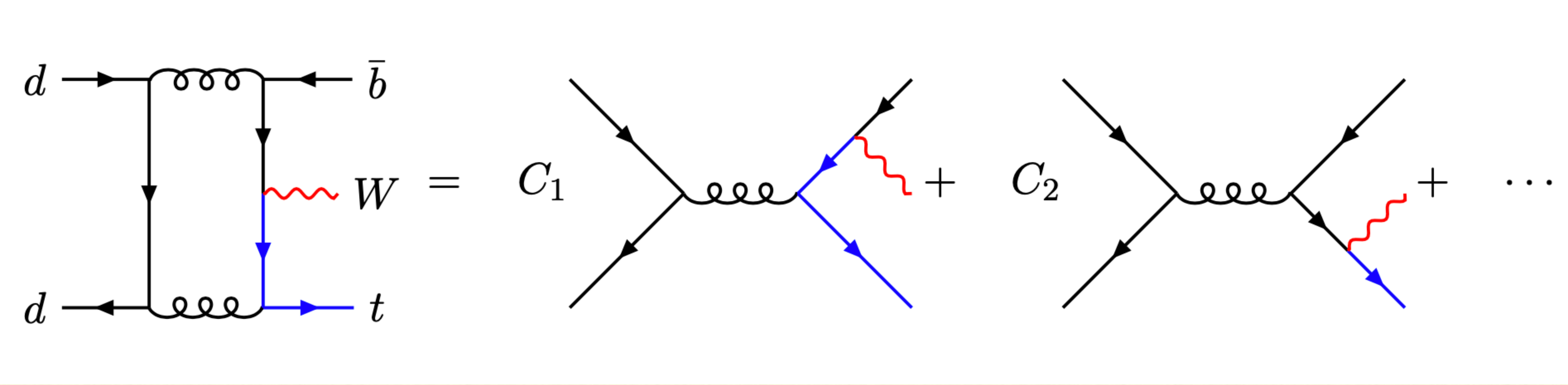
$$(R_{\rm LO})_1 = |\widetilde{\mathscr{M}^{(0)}_{2t}}|^2 \equiv |\mathscr{M}^{(0)}_{2t}|^2 \Big|_{\{p_i\}\to\{\tilde{p}_i\}}$$

$$\Delta \equiv s_{W\bar{b}} - m_t^2 \equiv (p_3 + p_4)^2 - m_t^2$$

Reg: regulation with a width

SUBTRACTION OF $t\bar{t}$

At one-loop level, it is not possible to distinguish the double and single resonant diagrams.



Dong, Li, Li, JW, JHEP 01(2015), 158

SUBTRACTION OF $t\bar{t}$

We propose a new subtraction scheme, Power Subtraction.

$$|\mathcal{M}_{tW\bar{b}}|^2_{\mathrm{LO}} = \frac{B^{(2)}}{\Delta^2} + \frac{B^{(1)}}{\Delta} + B^{(0)} + \cdots$$

Subtraction kernel:

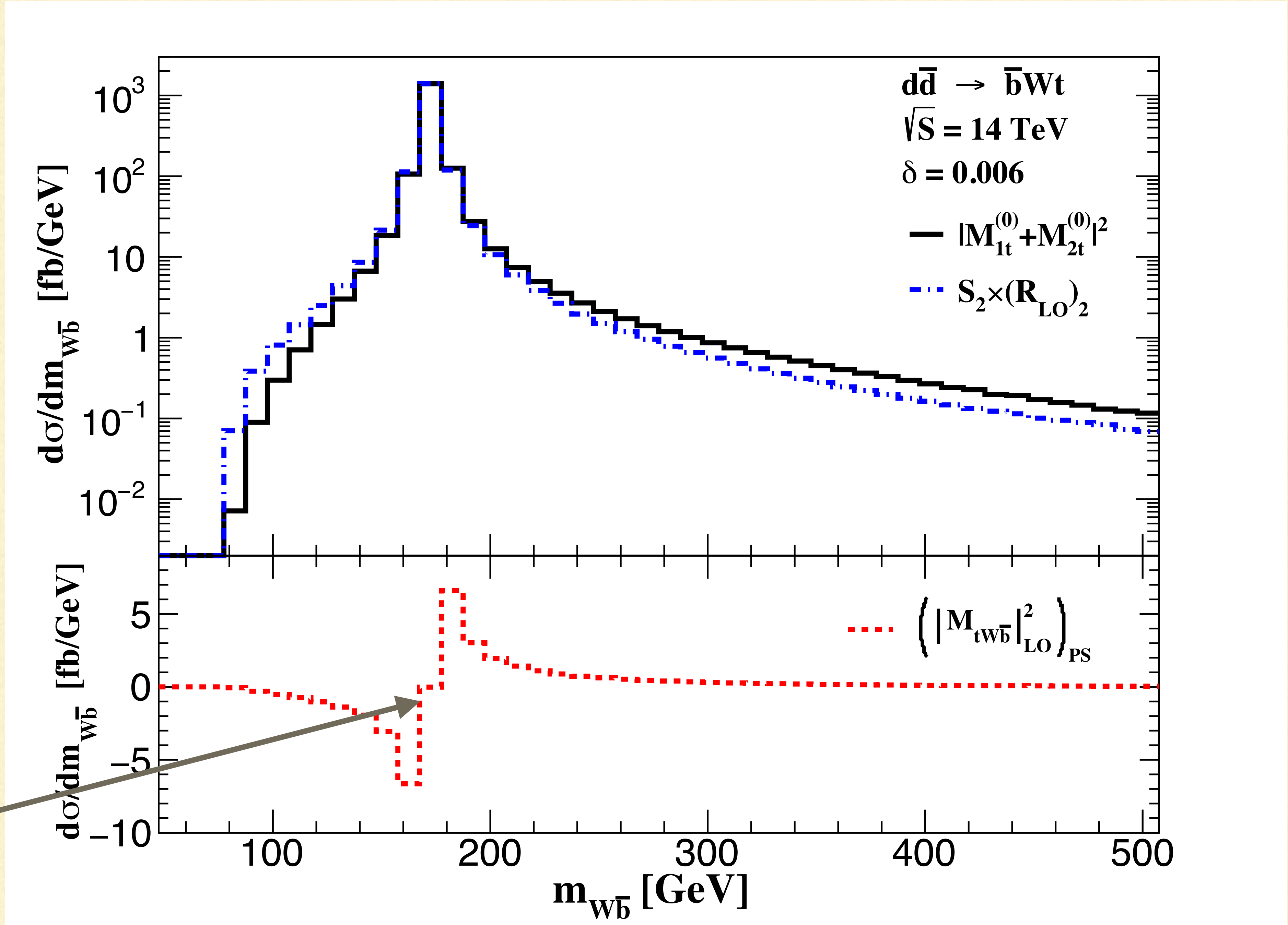
$$(R_{\mathrm{LO}})_2 = \frac{\widetilde{B^{(2)}}}{\widetilde{\Delta}^2}$$

Suppression factor:

$$S_2 = \frac{\widetilde{\Delta}^2}{\Delta^2}$$

$$\left(|\mathcal{M}_{tW\bar{b}}|^2_{\mathrm{LO}} \right)_{\mathrm{PS}} = |\mathcal{M}^{(0)}_{1t} + \mathcal{M}^{(0)}_{2t}|^2 - \frac{\widetilde{B^{(2)}}}{\Delta^2}$$

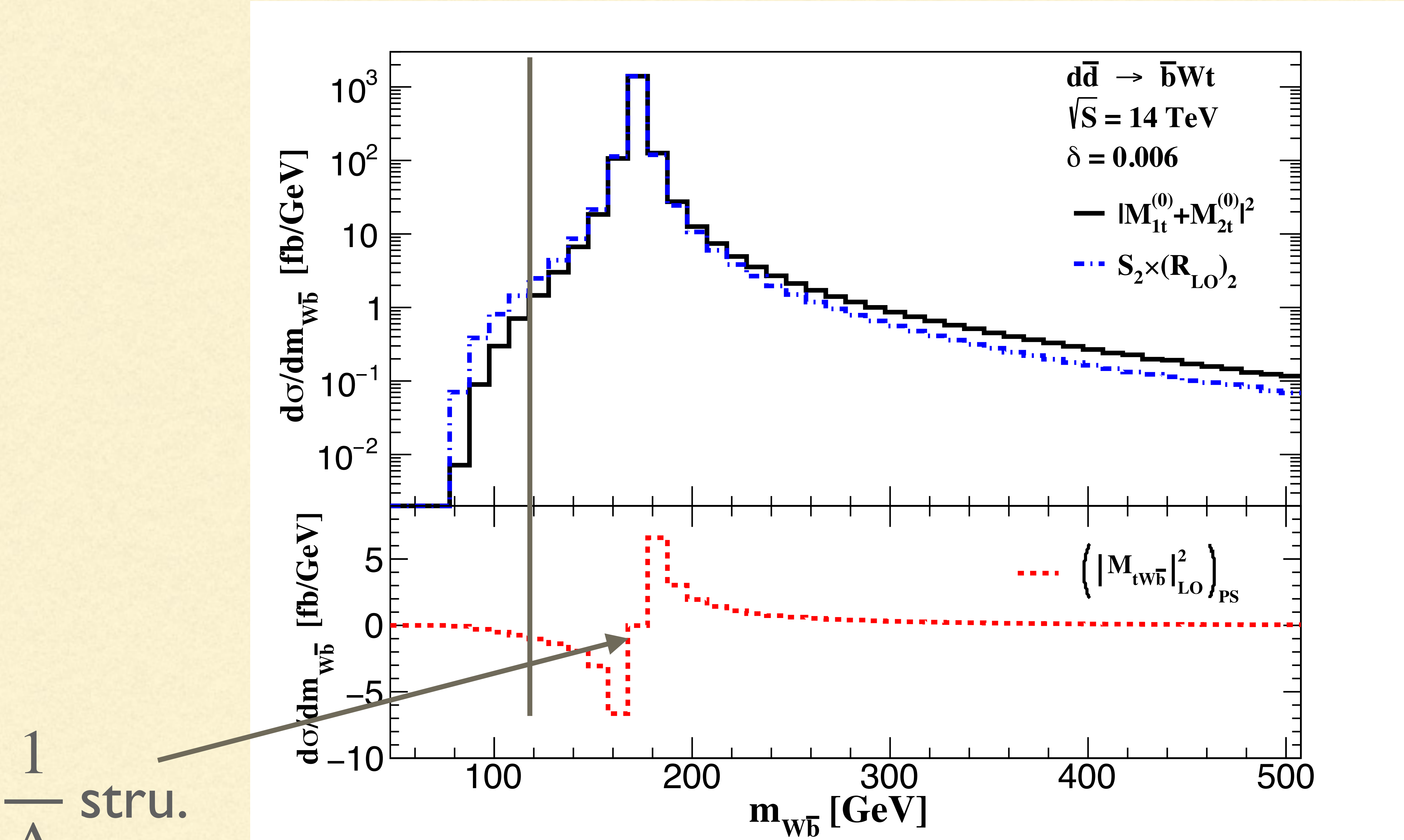
To ensure num. stability: $|\sqrt{s_{W\bar{b}}} - m_t|/m_t > \delta,\ \delta = (10^{-2}, 10^{-6})$



$\frac{1}{\Delta}$ stru.

Schemes	DR1	DR2	DS	PS
$\sigma_{\rm LO}$(fb)	63.56(1)	−45.03(1)	56.7(2)	56.1(1)

Dong, Li, Li, JW, JHEP 01(2015), 158



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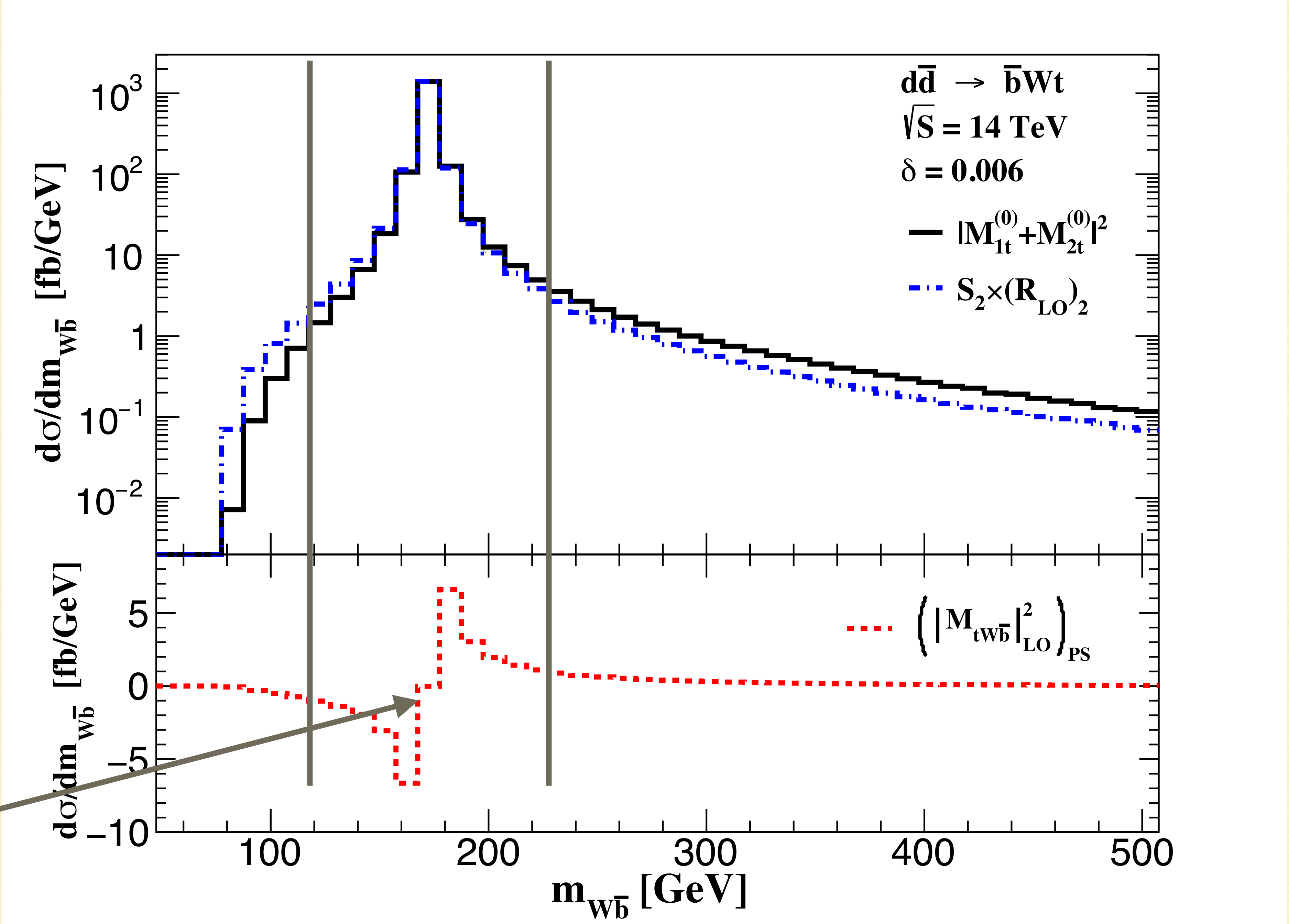

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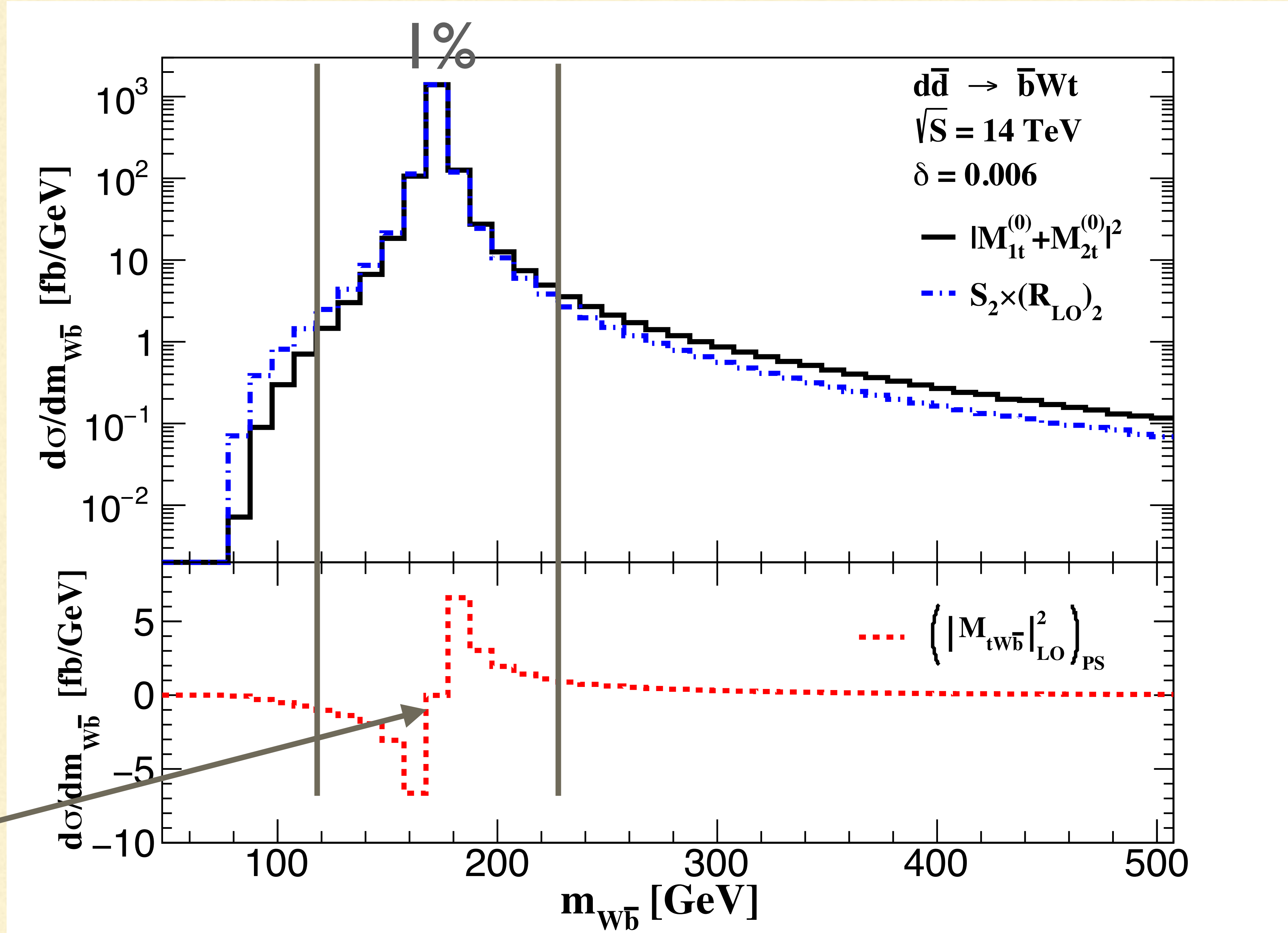

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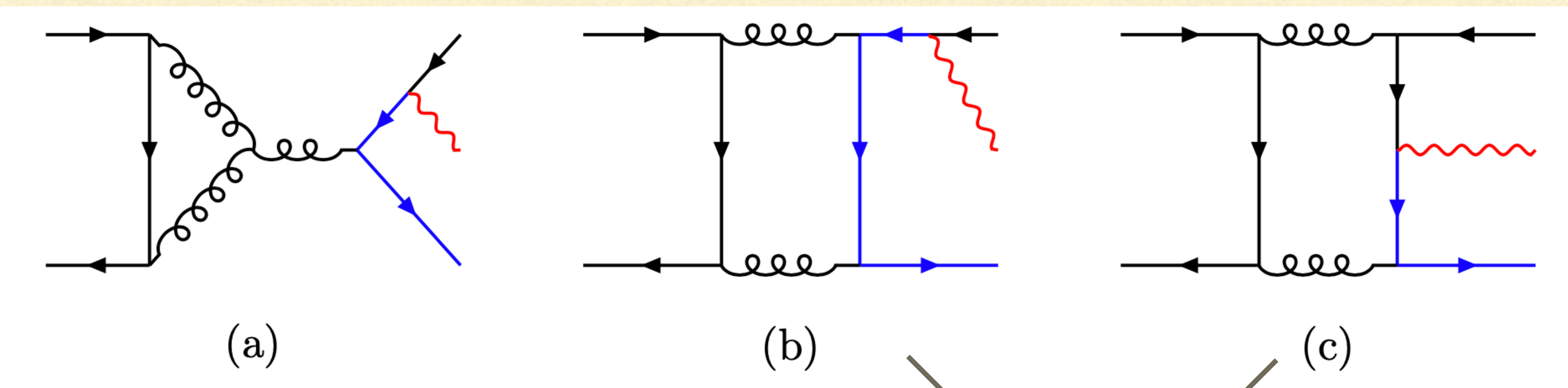
SUBTRACTION AT LOOP LEVEL

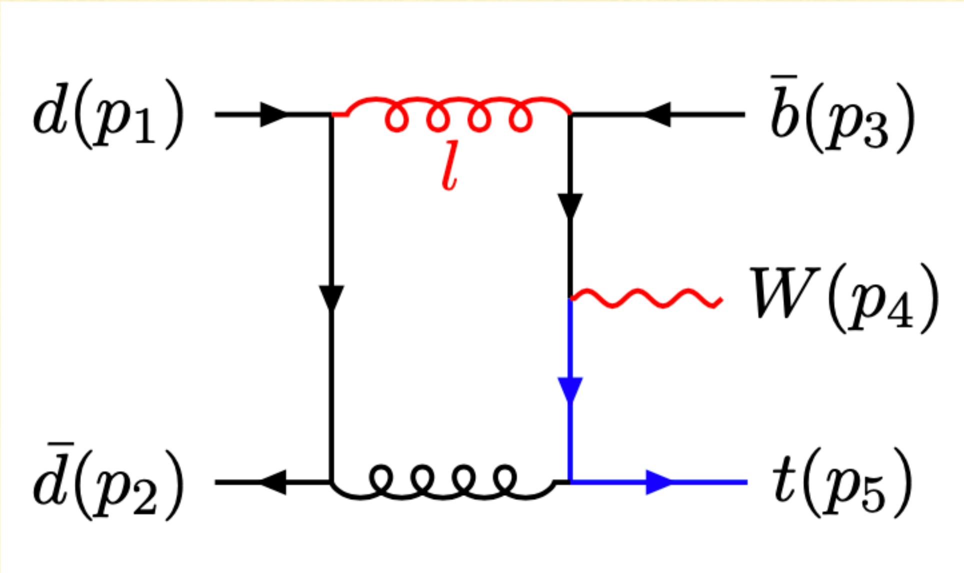
Expand the loop amplitudes:

$$V_{\text{Ren}} \equiv 2\,\mathbf{Re}[\mathscr{M}^{(0)*}\mathscr{M}^{(1)}_{\text{Ren}}] = \frac{C^{(2)}}{\Delta^2} + \frac{C^{(1)}}{\Delta} + C^{(0)} + \cdots$$



Taylor expansion

Contain $\log \Delta$



Expansion in Δ can be performed in the integrand using the EFT of unstable particles.

Beneke et al, hep-ph/0312331

Eikonal

Resonance prop.

$$I_s^{(a)} = \frac{\mu^{4-D}}{i\pi^{D/2} r_\Gamma} \int d^D l \frac{1}{(l^2 + i0)\,(-2l \cdot p_3 + i0)\,[-2l \cdot p_{34} + \Delta + i0]\,(-2l \cdot p_1 + i0)}$$

$$= -\frac{1}{2(p_1 \cdot p_3)\Delta} \left(\frac{-\Delta - i0}{\mu\, m_t} \right)^{-2\epsilon} \left[\frac{1}{\epsilon^2} - \frac{1}{\epsilon} \log \left(\frac{m_t^2 (p_1 \cdot p_3)}{2\,(p_1 \cdot p_{34})(p_3 \cdot p_{34})} \right) + \frac{1}{2} \log^2 \left(\frac{m_t^2 (p_1 \cdot p_3)}{2\,(p_1 \cdot p_{34})(p_3 \cdot p_{34})} \right) + \mathrm{Li}_2 \left(1 - \frac{m_t^2 (p_1 \cdot p_3)}{2\,(p_1 \cdot p_{34})(p_3 \cdot p_{34})} \right) + \frac{\pi^2}{2} \right]$$

Soft scale

Dong, Li, Li, JW, JHEP 01(2015), 158

No double $\log^2\Delta$ because of color conservation:

$$\frac{\alpha_s}{2\pi\epsilon^2}\left(\frac{-\Delta - i0}{\mu\, m_t}\right)^{-2\epsilon}\left[2\mathbf{T}_1\cdot\mathbf{T}_3 + 2\mathbf{T}_2\cdot\mathbf{T}_3 + \mathbf{T}_3\cdot\mathbf{T}_5 - \mathbf{T}_1\cdot\mathbf{T}_{\bar{t},f} - \mathbf{T}_2\cdot\mathbf{T}_{\bar{t},f} - \mathbf{T}_3\cdot\mathbf{T}_{\bar{t},i}\right]$$

$$= \frac{\alpha_s}{2\pi\epsilon^2}\left(\frac{-\Delta - i0}{\mu\, m_t}\right)^{-2\epsilon}\left[\mathbf{T}_1\cdot\mathbf{T}_3 + \mathbf{T}_2\cdot\mathbf{T}_3 + \mathbf{T}_3\cdot\mathbf{T}_5 + \mathbf{T}_3\cdot\mathbf{T}_{\bar{t},f}\right] = 0$$

The divergence structure:

$$I^{\text{div}}_{d\bar{d}\to\bar{b}Wt} = \frac{d_2}{\epsilon^2} + \frac{d_1}{\epsilon} + \frac{d_s}{\epsilon}\left(\frac{-\Delta - i0}{\mu\, m_t}\right)^{-2\epsilon} = \frac{d_2}{\epsilon^2} + \frac{d_1 + d_s}{\epsilon} - 2d_s\log\left(\frac{-\Delta - i0}{\mu\, m_t}\right)$$

Taking the limit $\Delta \to 0$ before expansion in ϵ

$$I^{\text{div}}_{d\bar{d}\to\bar{t}(\to\bar{b}W)t} = \frac{d_2}{\epsilon^2} + \frac{d_1}{\epsilon}$$

Therefore, the $\log\Delta$ term is predictable and gauge invariant.

SUBTRACTION AT LOOP LEVEL

Infrared divergence needs to be subtracted too.

$$\mathscr{I}_{\text{NLO}} = \mathbf{I} \otimes |\mathscr{M}_{tW\bar{b}}|^2_{\text{LO}}$$

Power expansion:

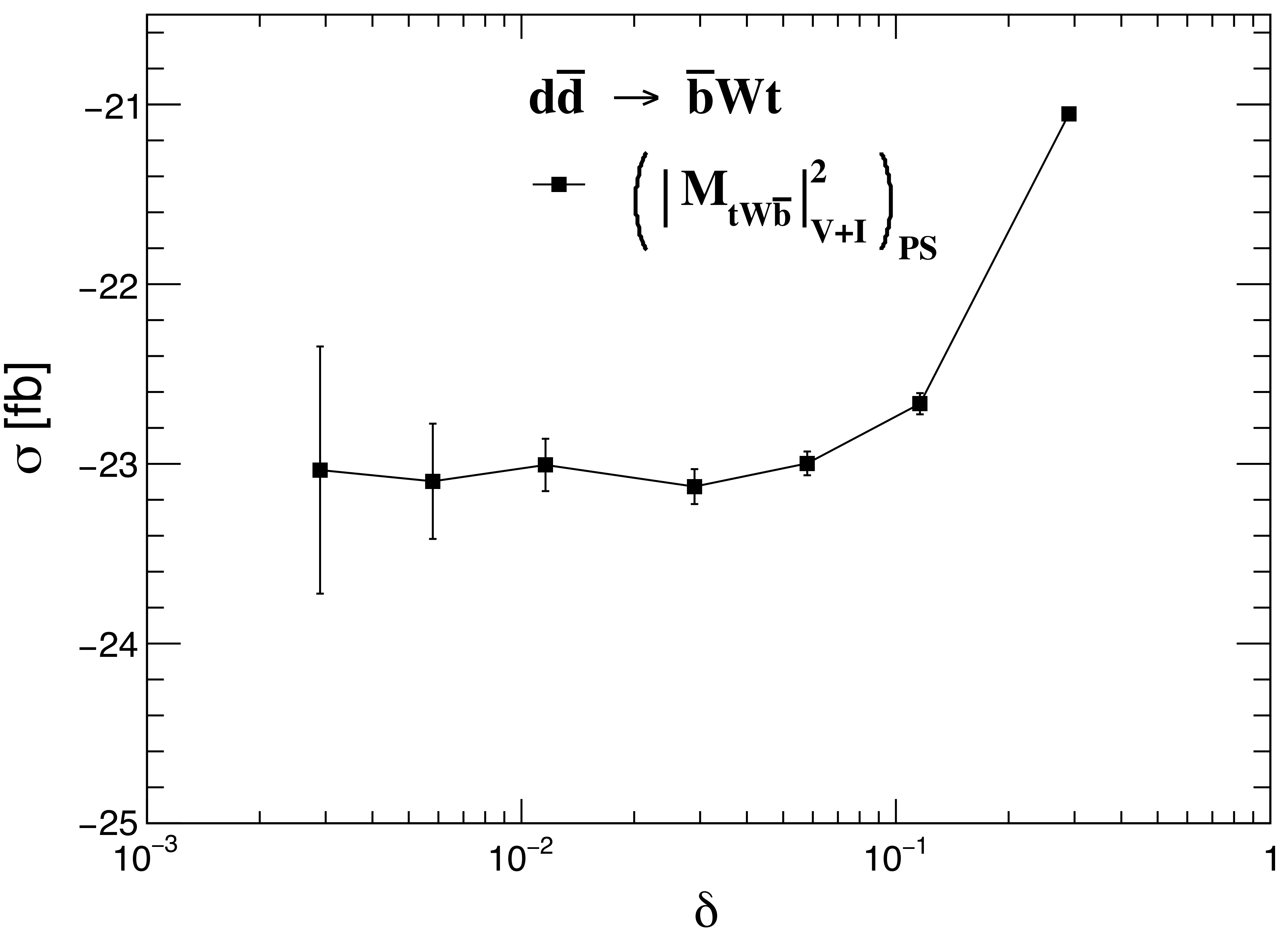
$$\mathscr{I}_{\text{NLO}} = \frac{I^{(2)}}{\Delta^2} + \frac{I^{(1)}}{\Delta} + I^{(0)} + \cdots$$

Subtraction kernel:

$$\left(R_{\text{V+I}}\right)_2 = \frac{\widetilde{C^{(2)}} + \widetilde{I^{(2)}}}{\widetilde{\Delta}^2}$$

Power Subtraction:

$$\left(|\mathscr{M}_{tW\bar{b}}|^2_{\text{V+I}}\right)_{\text{PS}} = V_{\text{Ren}} + \mathscr{I}_{\text{NLO}} - \frac{\widetilde{C^{(2)}} + \widetilde{I^{(2)}}}{\Delta^2}$$



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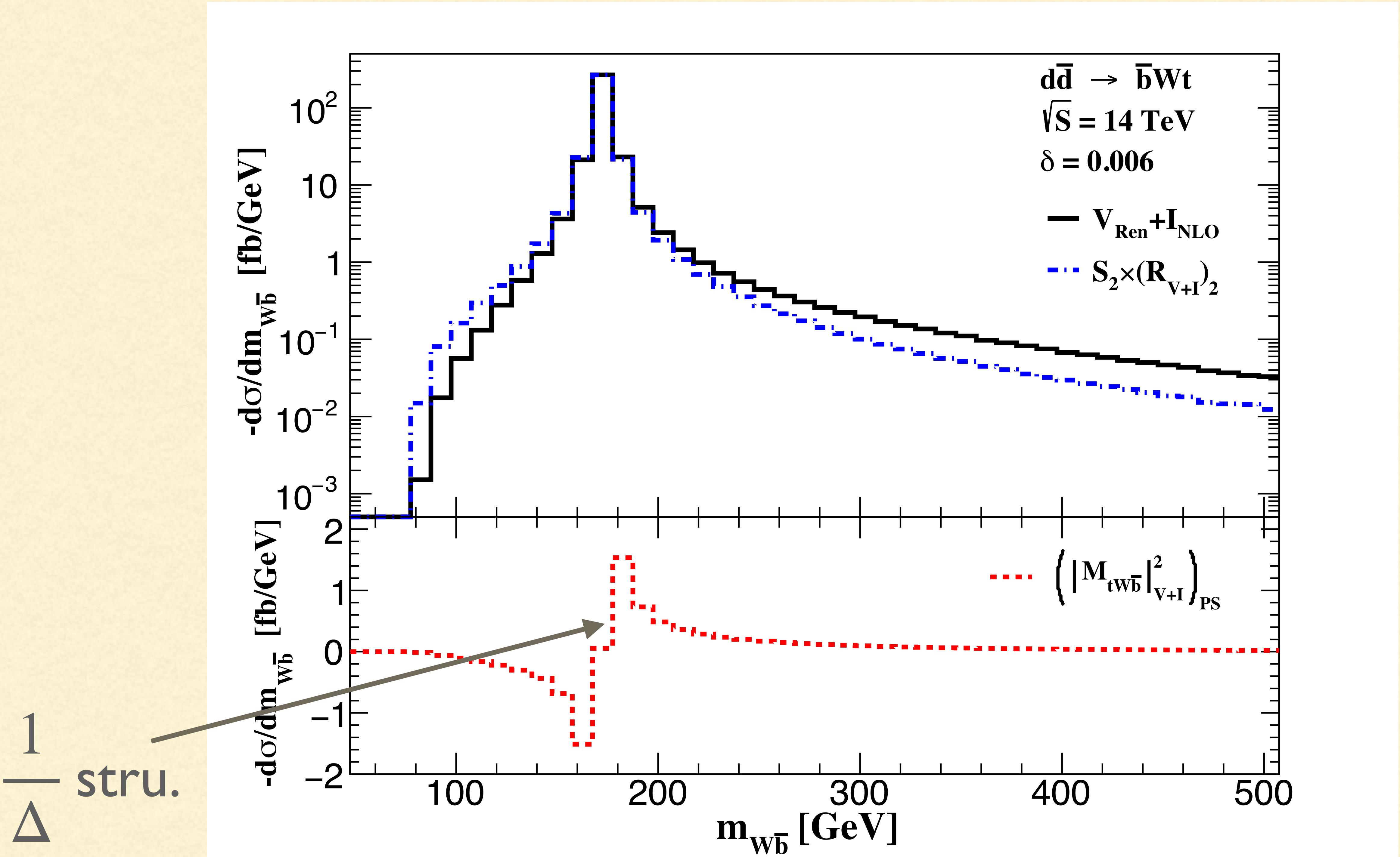

Scheme	σ_{LO}(fb)	σ_{V+I}(fb)	σ_{V+I}/σ_{LO}
PS	56.1(1)	-23.1(3)	-0.41

Dong, Li, Li, JW, JHEP 01(2015), 158

SUMMARY

- Resonances (unstable particles) can appear as not only signals, but also backgrounds.
- Subtraction of resonances needs to be implemented with caution.
- We propose the power subtraction at loop level, and apply it in $d\bar{d} \to tW\bar{b}$.
- This method may be applied in other high energy processes.

Thank you very much!