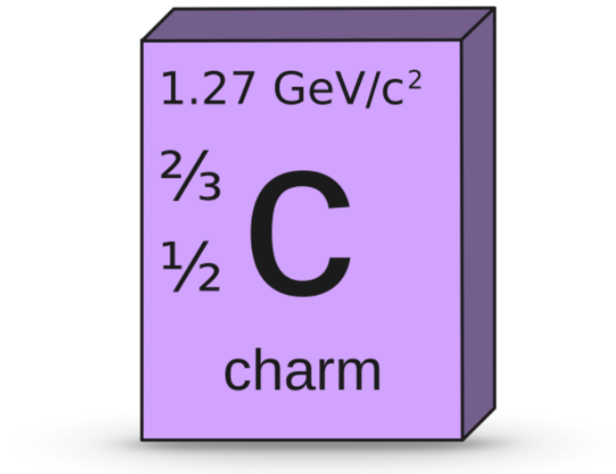

Progress on inclusive charm decays

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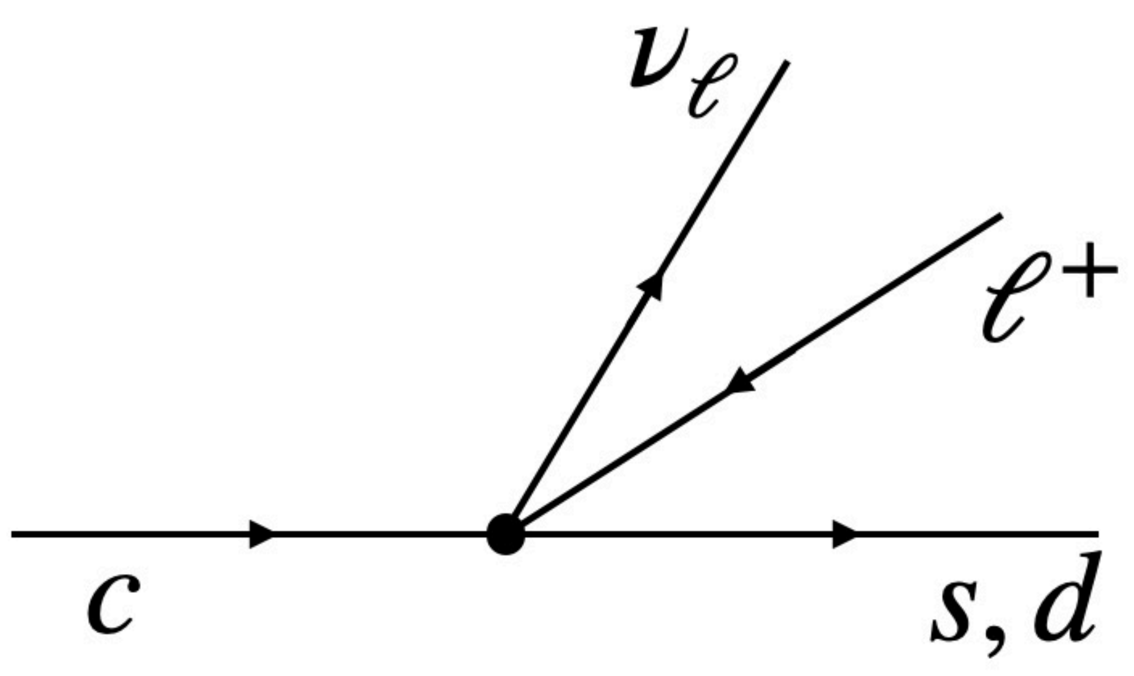
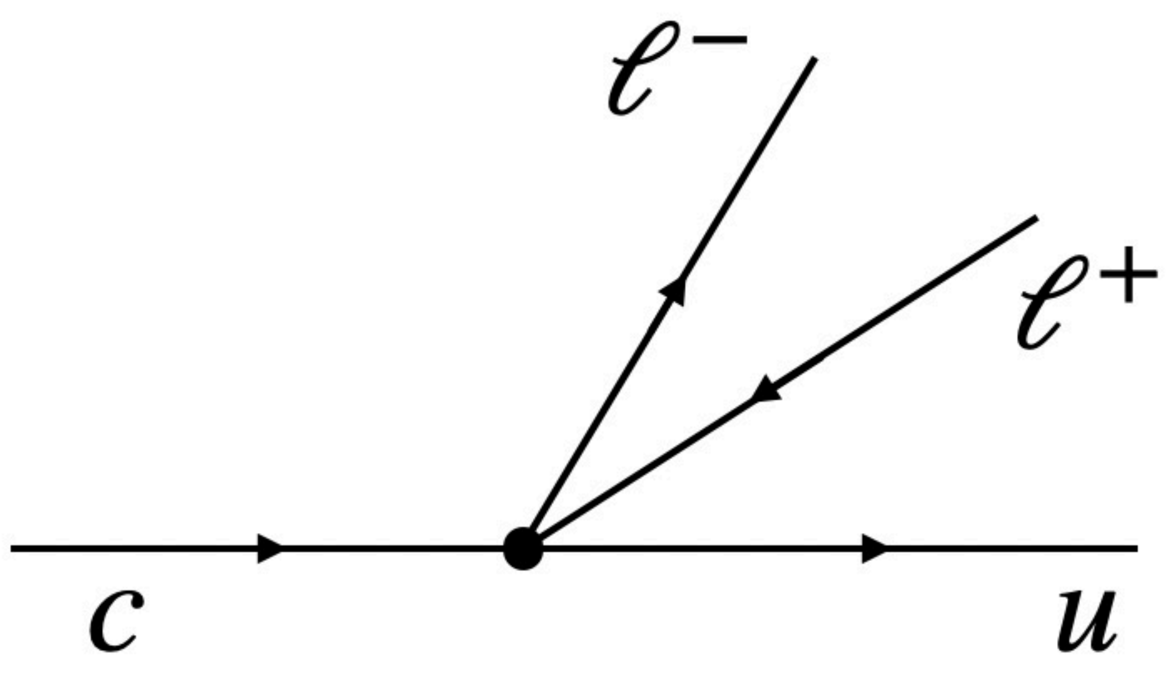
第七届重味物理与量子色动力学研讨会

南京师范大学，2025年04月18-22日

Semi-inclusive charm decays

- **Experimental detection of partial final state particles**
  - ➡ $D \to e^+X$ ($D \to e^+\nu_e X$, only e^+ is detected)
- **Sum of a group of exclusive channels**
  - ➡ $D^0 \to e^+X_s = D \to e^+\nu_e K^-, e^+\nu_e K^-\pi^0, e^+\nu_e \bar{K}^0\pi^-, \ldots$
  - ➡ $D^0 \to e^+X_d = D \to e^+\nu_e \pi^-, e^+\nu_e \pi^-\pi^0, e^+\nu_e \pi^-\pi^+\pi^-, \ldots$





Why inclusive charm decays?

- ❖ **As weak decays of heavy hadrons**
  - ➡ Probe new physics
  - ➡ Understand QCD
- ❖ **<u>Compared to exclusive decays</u>**
  - ➡ **Better theoretical control**

More important with stronger experiment (BESIII, STCF)

- ❖ **<u>Compared to beauty decays</u>**
  - ➡ **Special to new dynamics attached with up-type quarks**
  - ➡ **More sensitive to power corrections**

Determination by charm, application in beauty

Why inclusive charm decays?

❖ **Resolve (or at least give hints to) current flavor puzzles/anomalies**

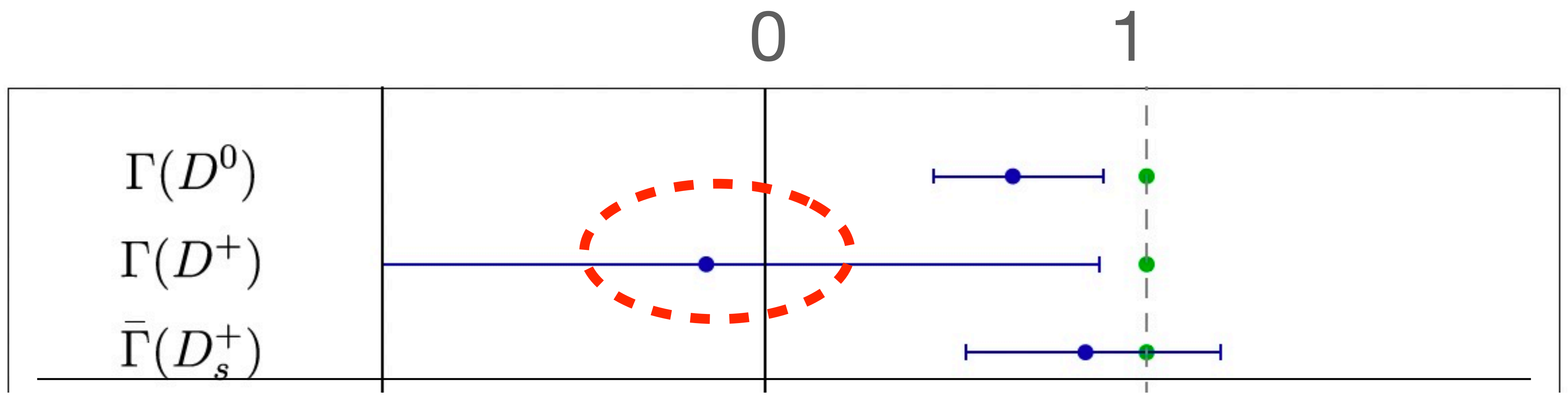
➡ Puzzles in charmed hadron lifetimes: theory vs experiment

➡ V_{cb}, V_{ub} puzzles: inclusive vs exclusive

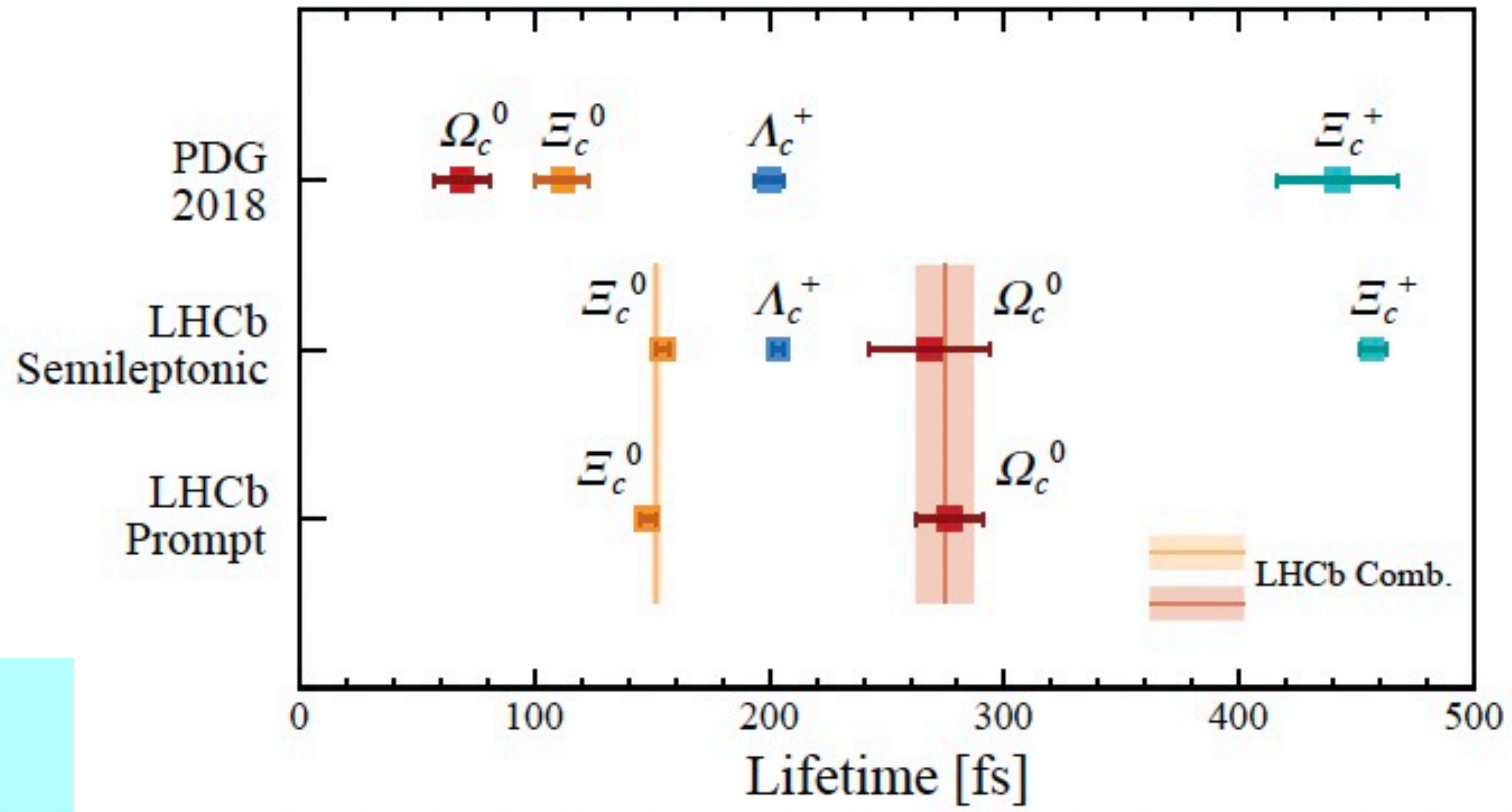
➡ $b \to s$ anomalies: P_5' in $B \to K^* \ell \ell$

Why inclusive charm decays?

❖ **Flavor puzzle 1.** Charmed hadron lifetimes: theory vs experiment



[Lenz et al, ’22]



$$\mathcal{O}(1/m_c^3) \Rightarrow \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\boldsymbol{\Omega_c^0}),$$
$$\mathcal{O}(1/m_c^4) \Rightarrow \tau(\boldsymbol{\Omega_c^0}) > \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0),$$
$$\mathcal{O}(1/m_c^4) \text{ with } \alpha \Rightarrow \tau(\Xi_c^+) > \tau(\boldsymbol{\Omega_c^0}) > \tau(\Lambda_c^+) > \tau(\Xi_c^0).$$

[Cheng, ’21]

✦ **Key issue:** Nonperturbative power corrections with large/unknown uncertainties

✦ **Solution**: Extraction in the inclusive decay **spectrum**

➡ Spectrum and lifetime share **identical** HQE parameters

Again a more precise experimental determination of μ_π^2 from fits to semileptonic D^+, D^0 and D_s^+ meson decays – as it was done for the B^+ and B^0 decays – would be very desirable.

[Lenz et al, ’22]

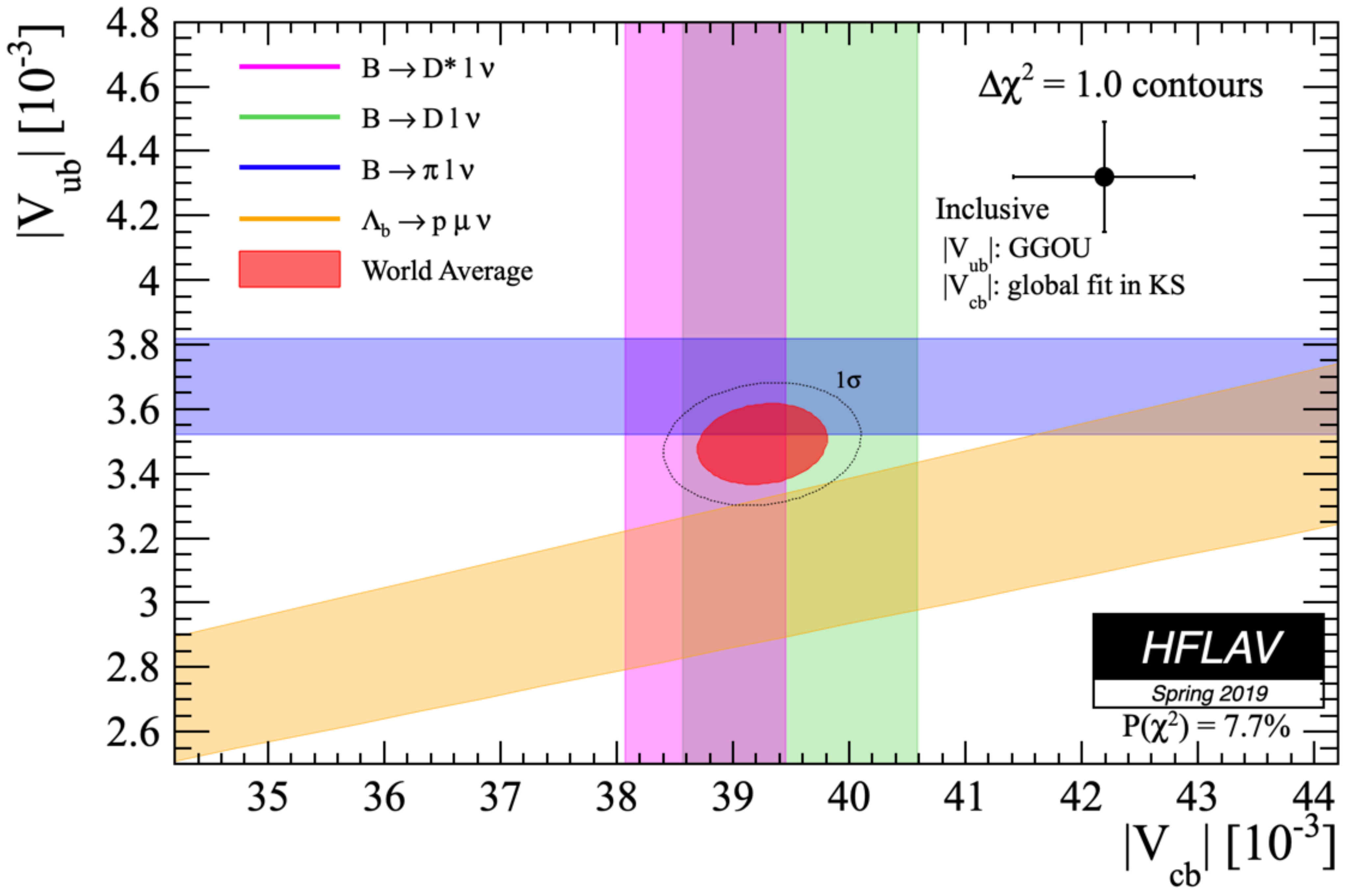
Why inclusive charm decays?

❖ **Flavor puzzle 2.** V_{cb}, V_{ub}: inclusive vs exclusive

✦ **Key issue:** Systematic uncertainties from theoretical inclusive and exclusive frameworks?

✦ **Solution**: Test V_{cd}, V_{cs}: inclusive vs exclusive

➡ Await the first inclusive values



Why inclusive charm decays?

❖ **Flavor puzzle 3.** $b \to s$ anomalies: P_5' in $B \to K^*\ell\ell$

✦ **Key issue:** First-principle calculation of long-distance penguin in this channel is still missing

➡ Only achieved in $B \to \gamma\gamma$

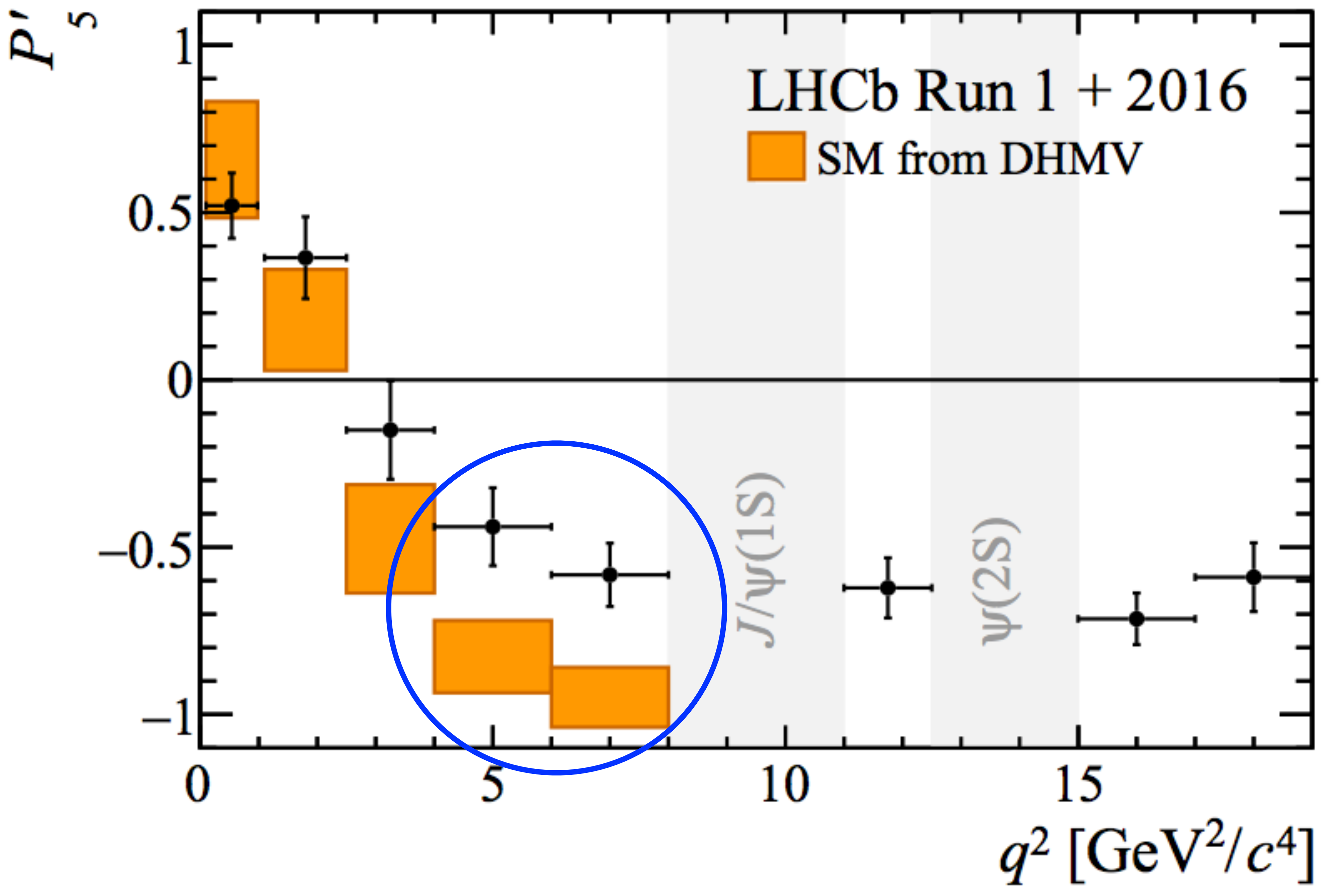
[**QQ**,Shen,Wang,Wang, PRL, '23]

✦ **Solution:** Test FCNC inclusive decays

➡ Inclusive $B \to X_{d,s}\ell\ell$

[Huber,Hurth,Jenkins,Lunghi,**QQ**,Vos, JHEP, '19,'20,'24]

➡ Inclusive $D \to X_u\ell\ell$



Theoretical framework

❖ **Optical theorem**

$$\sum \langle D|H|X\rangle\langle X|H|D\rangle \propto \mathrm{Im}\int \mathrm{d^4x}\langle \mathrm{D}|\mathrm{T}\{\mathrm{H(x)H(0)}\}|\mathrm{D}\rangle$$

❖ **Operator product expansion (OPE)**

➡ Short distance $x \sim 1/m_c$

➡ Dynamical fluctuation in D meson $\sim \Lambda_{\mathrm{QCD}}$

$$T\{H(x)H(0)\} = \sum_n C_n(x)O_n(0) \to 1 + \frac{\Lambda_{\mathrm{QCD}}}{m_c} + \frac{\Lambda^2_{\mathrm{QCD}}}{m_c^2} + \dots$$

Systematic OPE in HQET.

Theoretical framework

- **Heavy quark effective theory**

$$h_v(x) \equiv e^{-im_c v\cdot x}\frac{1+\gamma\cdot v}{2}c(x) \qquad v=(1,0,0,0)$$

Subtract the big intrinsic momentum,

Leave only $\sim\Lambda_{\mathrm{QCD}}$ degrees of freedom.

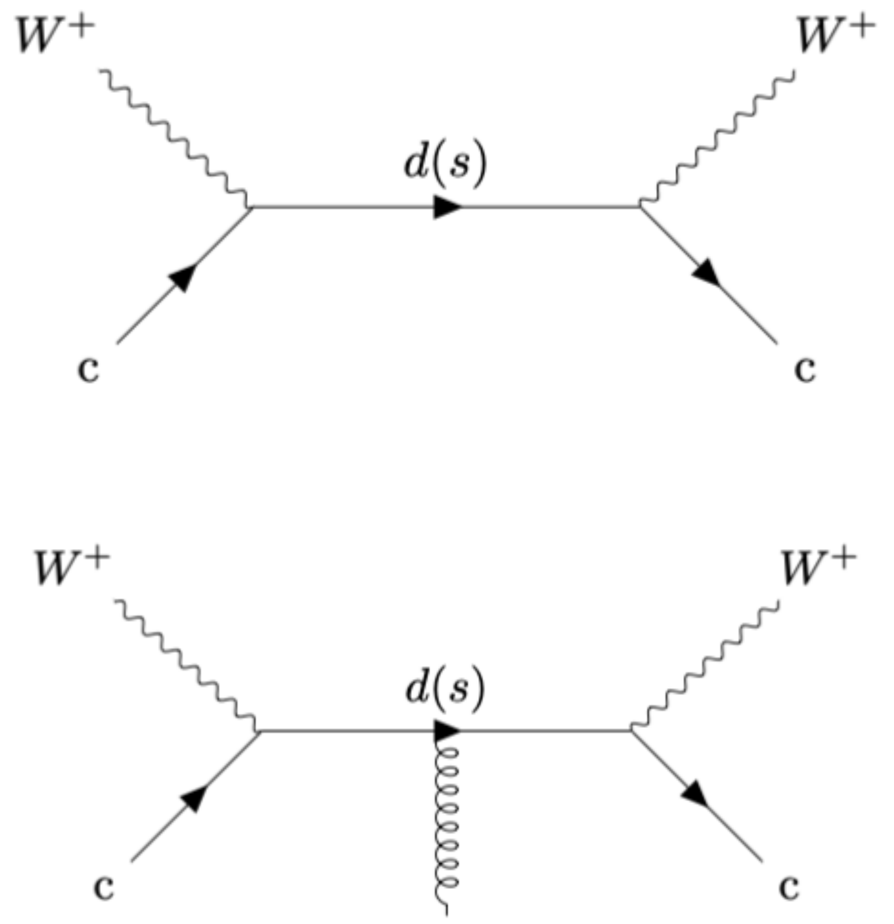
$$L \ni \bar{h}_v i v\cdot D h_v$$
$$-\bar{h}_v\frac{D_\perp^2}{2m_c}h_v - a(\mu)g\bar{h}_v\frac{\sigma\cdot G}{4m_c}h_v + \ldots$$

Similar to $\dfrac{m}{\sqrt{1-v^2}} = m + \dfrac{1}{2}mv^2 + \ldots$

Theoretical framework

❖ **OPE and Perturbative QCD:** Systematical expansion of $\alpha_s(m_c)$ and $\Lambda_{\rm QCD}/m_c$

$$\langle T\{H(x)H(0)\}\rangle = \sum_n C_n(x)\langle O_n(0)\rangle$$



- LO: $\alpha_s^0(m_c)$
- NLO: $\alpha_s(m_c)$
- NNLO: $\alpha_s^2(m_c)$
- ...

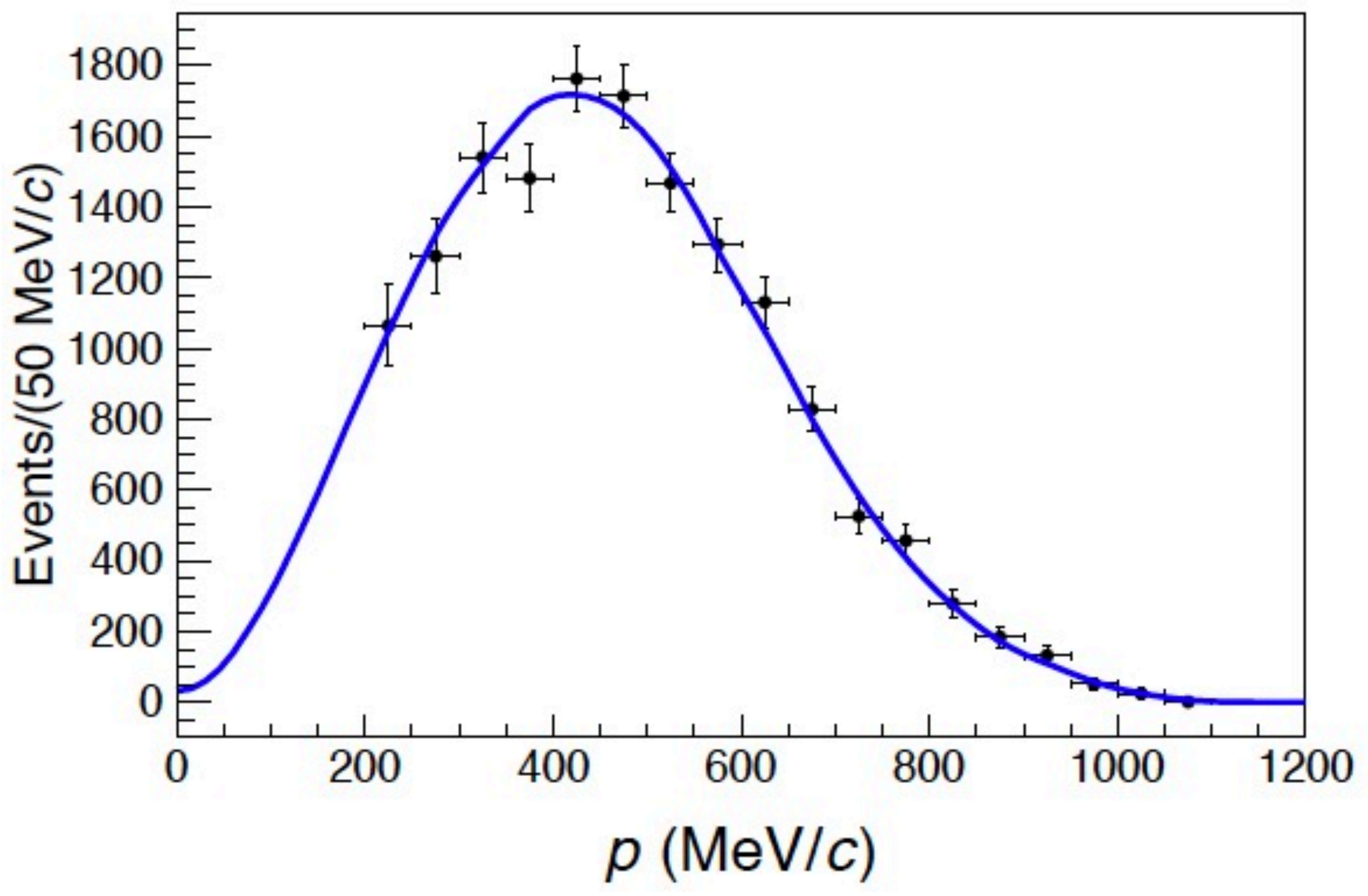
- Dim-3: $\bar{h}_v h_v$ $(\bar{c}\gamma^\mu c)$ partonic decay rate
- Dim-5: $\bar{h}_v D_\perp^2 h_v$, $g\bar{h}_v \sigma \cdot G h_v$.
- Dim-6: $\bar{h}_v D_\mu (v\cdot D) D^\mu h_v$, $(\bar{h}_v \Gamma_1 q)(\bar{q}\Gamma_2 h_v), \ldots$
- ...

Question: convergent expansion of $\alpha_s(m_c)$ and $\Lambda_{\rm QCD}/m_c$?

Theoretical results

❖ **Electron energy spectrum** $(y \equiv 2E_e/m_c)$

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} = 12(1-y)y^2\theta(1-y) + \frac{2\mu_\pi^2}{m_c^2}\left[-10y^3\theta(1-y) + 2\delta(1-y)\right] - \frac{2\mu_G^2}{3m_c^2}\left[6y^2(6-5y)\theta(1-y)\right] + \mathcal{O}(\alpha_s, \frac{\Lambda^3}{m_c^3})$$



❖ Up to finite power, the obtained spectrum is **NOT** the experimental spectrum

➡ Observables require integration over final states

➡ $\Gamma = \int \frac{d\Gamma}{dy}dy\,,\ \langle E_\ell^n \rangle = \frac{1}{\Gamma}\int \frac{d\Gamma}{dy}E_\ell^n\, dy$ (n=1,2,3,4) are the **observables**

➡ Shape function —— infinite power summation? [Neubert, '93]

Theoretical results

❖ Analytical results for total decay rate and energy moments (NNLO & $\Lambda_{\rm QCD}^3/m_c^3$)

NNLO numerical results provided by Long Chen

[Chen,Chen,Guan,Ma,'23]

NLO analytical integration

$$\Gamma_{D_i} = \sum_{q=d,s} \hat{\Gamma}_0 \left|V_{cq}\right|^2 m_c^5 \Big\{ 1 + \frac{\alpha_s}{\pi}\frac{2}{3}\left(\frac{25}{4}-\pi^2\right) + \frac{\alpha_s^2}{\pi^2}\left[\frac{\beta_0}{4}\frac{2}{3}\left(\frac{25}{4}-\pi^2\right)\log\left(\frac{\mu^2}{m_c^2}\right) + 2.14690 n_l - 29.88311\right]$$

$$- 8\rho\delta_{sq} - \frac{1}{2}\frac{\mu_\pi^2(D_i)}{m_c^2} - \frac{3}{2}\frac{\mu_G^2(D_i)}{m_c^2} + 6\frac{\rho_D^3(D_i)}{m_c^3} + \ldots \Big\},$$

Dim-5, $\Lambda_{\rm QCD}^2/m_c^2$

Dim-6, $\Lambda_{\rm QCD}^3/m_c^3$

Mass scheme

❖ **Pole mass scheme** (suffering from renormalon)

$$\Gamma/\Gamma_{\rm LO} = 1 - 0.77\alpha_s - 2.38\alpha_s^2 - 10.73\alpha_s^3 \approx 1 - 30\% - 36\% - 62\%$$

Not convergent, negative at NNNLO!

❖ $\overline{\rm MS}$ **mass scheme**

$$\Gamma/\Gamma_{\rm LO} = 1 + 1.35\alpha_s + 3.02\alpha_s^2 + 7.69\alpha_s^3 \approx 1 + 52\% + 46\% + 44\%$$

Convergent, but very slowly!

$$\Gamma = m_c^5(\Gamma^{(0)} + \alpha_s\Gamma^{(1)} + \alpha_s^2\Gamma^{(2)}) = \left(\overline{m}_c(1 + \alpha_s m^{(1)} + \alpha_s^2 m^{(2)})\right)^5(\Gamma^{(0)} + \alpha_s\Gamma^{(1)} + \alpha_s^2\Gamma^{(2)})$$

❖ **1S mass scheme** (half of J/ψ mass) [Hoang,Ligeti,Manohar, '98]

$$\Gamma/\Gamma_{\rm LO} \approx 1 - 13.1\% - 4.8\% + 1.8\%$$

Answer: convergent expansion of $\alpha_s(m_c)$!

Theoretical results

❖ Analytical results for total decay rate and energy moments (NNLO & $\Lambda^3_{\rm QCD}/m_c^3$)

$$\langle E_e\rangle_{D_i} = \frac{\hat\Gamma_0}{\Gamma_{D_i}}\sum_{q=d,s}|V_{cq}|^2\, m_c^6\left[\frac{3}{10}+\frac{\alpha_s}{\pi}a_1^{(1)}+\frac{\alpha_s^2}{\pi^2}a_1^{(2)}-3\rho\delta_{sq}-\frac{1}{2}\frac{\mu_G^2(D_i)}{m_c^2}+\frac{139}{30}\frac{\rho_D^3\,(D_i)}{m_c^3}+\frac{3}{10}\frac{\rho_{LS}^3\,(D_i)}{m_c^3}+\ldots\right],$$

$$\langle E_e^2\rangle_{D_i} = \frac{\hat\Gamma_0}{\Gamma_{D_i}}\sum_{q=d,s}|V_{cq}|^2\, m_c^7\left[\frac{1}{10}+\frac{\alpha_s}{\pi}a_2^{(1)}+\frac{\alpha_s^2}{\pi^2}a_2^{(2)}-\frac{6}{5}\rho\delta_{sq}+\frac{1}{12}\frac{\mu_\pi^2(D_i)}{m_c^2}-\frac{11}{60}\frac{\mu_G^2(D_i)}{m_c^2}+\frac{17}{6}\frac{\rho_D^3\,(D_i)}{m_c^3}\right.$$
$$\left.+\frac{7}{30}\frac{\rho_{LS}^3\,(D_i)}{m_c^3}+\ldots\right],$$

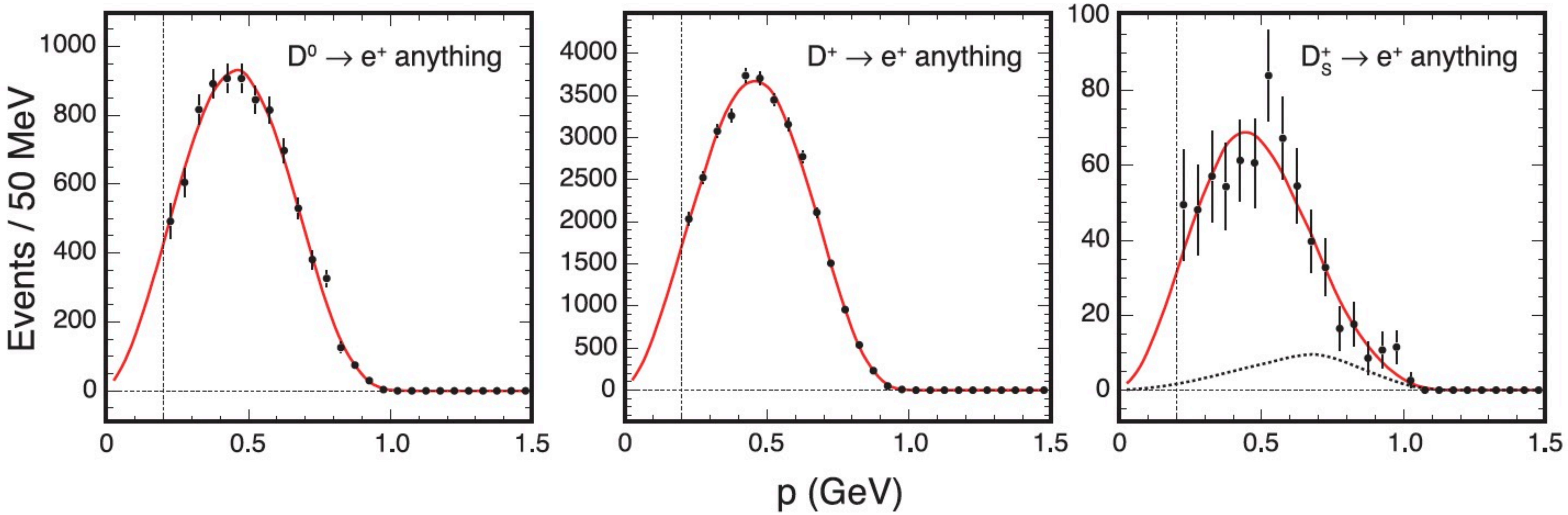
$$\langle E_e^3\rangle_{D_i} = \frac{\hat\Gamma_0}{\Gamma_{D_i}}\sum_{q=d,s}|V_{cq}|^2\, m_c^8\left[\frac{1}{28}+\frac{\alpha_s}{\pi}a_3^{(1)}+\frac{\alpha_s^2}{\pi^2}a_3^{(2)}-\frac{1}{2}\rho\delta_{sq}+\frac{1}{14}\frac{\mu_\pi^2\,(D_i)}{m_c^2}-\frac{1}{14}\frac{\mu_G^2\,(D_i)}{m_c^2}+\frac{223}{140}\frac{\rho_D^3\,(D_i)}{m_c^3}\right.$$
$$\left.+\frac{1}{7}\frac{\rho_{LS}^3\,(D_i)}{m_c^3}+\ldots\right],$$

$$\langle E_e^4\rangle_{D_i} = \frac{\hat\Gamma_0}{\Gamma_{D_i}}\sum_{q=d,s}|V_{cq}|^2\, m_c^9\left[\frac{3}{224}+\frac{\alpha_s}{\pi}a_4^{(1)}+\frac{\alpha_s^2}{\pi^2}a_4^{(2)}-\frac{3}{14}\rho\delta_{sq}+\frac{3}{64}\frac{\mu_\pi^2\,(D_i)}{m_c^2}-\frac{13}{448}\frac{\mu_G^2\,(D_i)}{m_c^2}+\frac{481}{560}\frac{\rho_D^3\,(D_i)}{m_c^3}\right.$$
$$\left.+\frac{9}{112}\frac{\rho_{LS}^3\,(D_i)}{m_c^3}+\ldots\right],$$

Experimental status

CLEO measurements

$D^0 \to e^+ X$ $\quad$ $D^+ \to e^+ X$ $\quad$ $D_s^+ \to e^+ X$

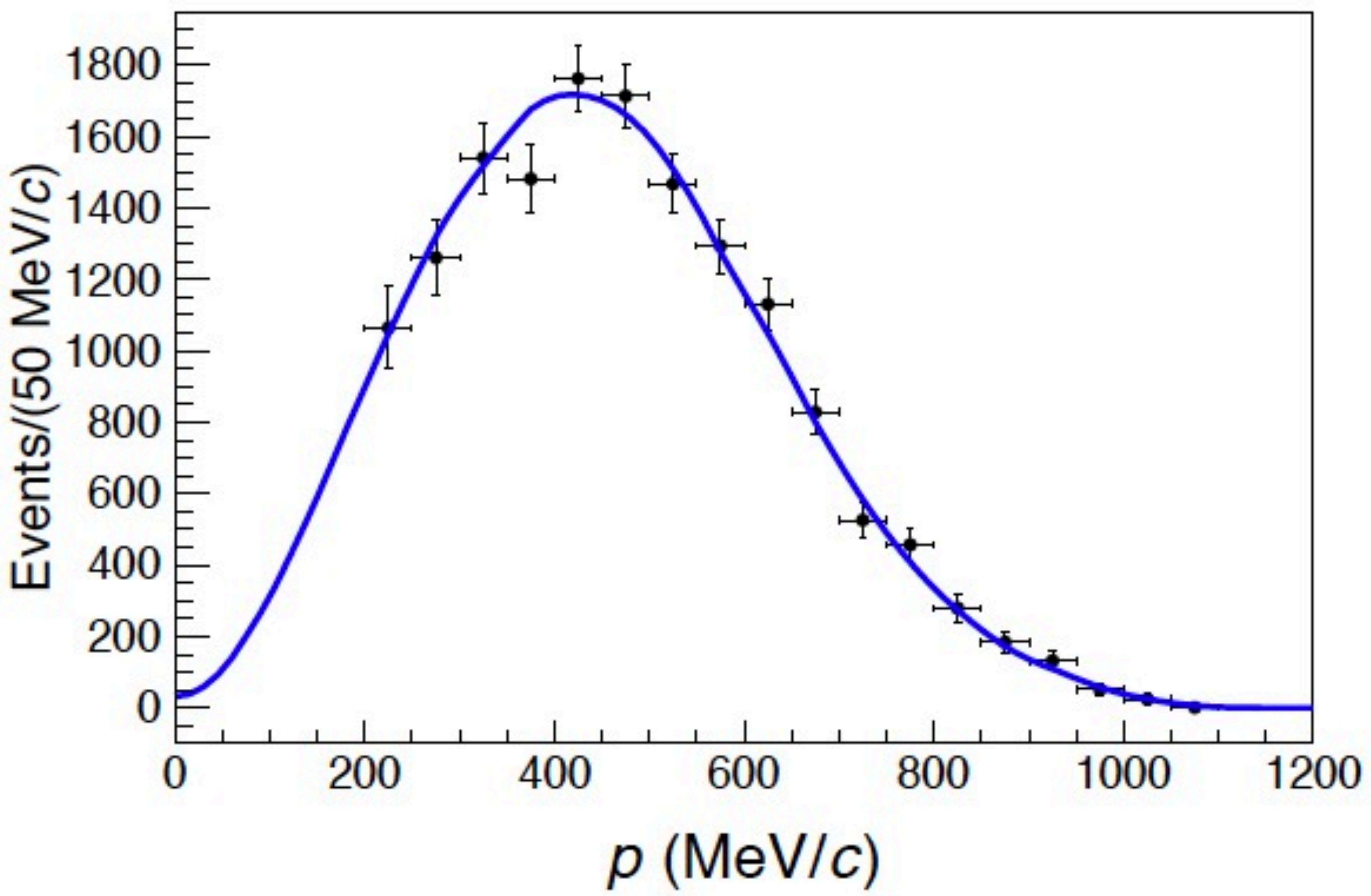


$$\mathcal{B}(D^0 \to X e^+ \nu_e) = (6.46 \pm 0.09 \pm 0.11)\%,$$
$$\mathcal{B}(D^+ \to X e^+ \nu_e) = (16.13 \pm 0.10 \pm 0.29)\%,$$
$$\mathcal{B}(D_s^+ \to X e^+ \nu_e) = (6.52 \pm 0.39 \pm 0.15)\%,$$

[CLEO, ’09]

BESIII measurements

$D_s^+ \to e^+ X$



$$B(D_s^+ \to X e^+ \nu_e) = (6.30 \pm 0.13 \pm 0.10)\ \%$$

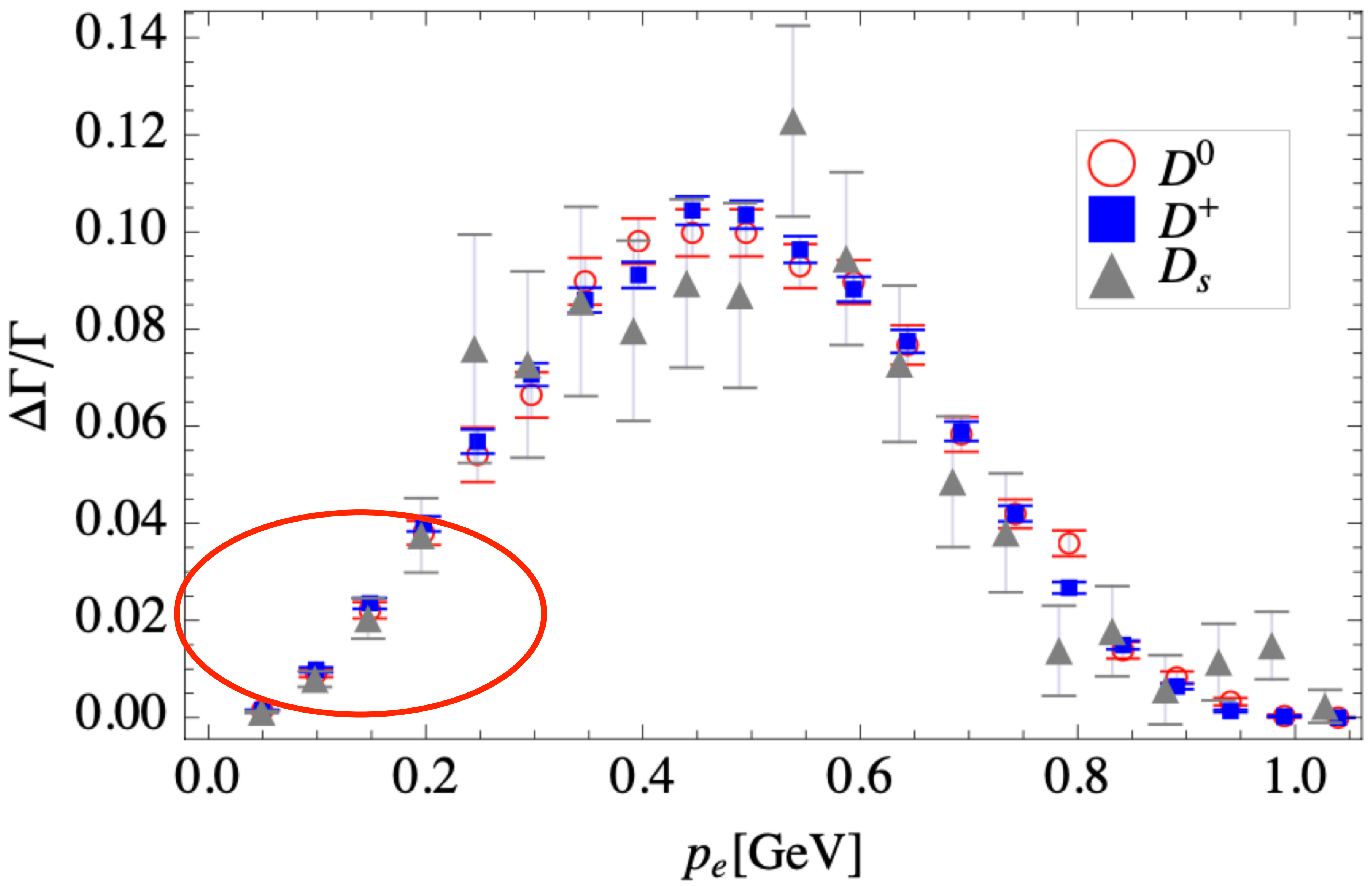
[BESIII, ’21]

2% precision!

Experimental data

$$\frac{d\Gamma}{dy} = ay^2(1+by)(1-y)$$

Original data → Extrapolation → **All-bin distribution** → Lorentz boost → **Rest frame** → MC Simulation → **Electron energy moments**



$$\langle E_e \rangle^{D_s}_{exp} = 0.437(6)\text{GeV}, \quad \langle E_e^2 \rangle^{D_s}_{exp} = 0.220(5)\text{GeV}^2$$
$$\langle E_e \rangle^{D^0}_{exp} = 0.462(5)\text{GeV}, \quad \langle E_e^2 \rangle^{D^0}_{exp} = 0.242(5)\text{GeV}^2$$
$$\langle E_e \rangle^{D^+}_{exp} = 0.455(4)\text{GeV}, \quad \langle E_e^2 \rangle^{D^+}_{exp} = 0.236(4)\text{GeV}^2$$
$$\langle E_e^3 \rangle^{D_s}_{exp} = 0.121(4)\text{GeV}^3, \quad \langle E_e^4 \rangle^{D_s}_{exp} = 0.072(3)\text{GeV}^4$$
$$\langle E_e^3 \rangle^{D^0}_{exp} = 0.138(4)\text{GeV}^3, \quad \langle E_e^4 \rangle^{D^0}_{exp} = 0.084(3)\text{GeV}^4$$
$$\langle E_e^3 \rangle^{D^+}_{exp} = 0.134(3)\text{GeV}^3, \quad \langle E_e^4 \rangle^{D^+}_{exp} = 0.081(3)\text{GeV}^4$$

Statistical uncertainties uncorrelated,
Systematic uncertainties fully correlated.

Global fit

❖ Global fit in two mass schemes, each with **Scenario 1** ($\Lambda^2_{\rm QCD}/m_c^2$) and **Scenario 2** ($\Lambda^3_{\rm QCD}/m_c^3$)

$\overline{\rm MS}$ scheme	χ^2/d.o.f.	D_i	$\mu_\pi^2/{\rm GeV}^2$	$\mu_G^2/{\rm GeV}^2$	$\rho_{\rm D}^3/{\rm GeV}^3$	$\rho_{LS}^3/{\rm GeV}^3$
Scenario 1	4.5	$D^{0,+}$	0.09 ± 0.01	0.27 ± 0.14	-	-
		D_s	0.09 ± 0.02	0.39 ± 0.12	-	-
Scenario 2	2.1	$D^{0,+}$	0.11 ± 0.02	0.26 ± 0.14	-0.002 ± 0.002	0.003 ± 0.002
		D_s	0.12 ± 0.02	0.38 ± 0.13	-0.003 ± 0.002	0.005 ± 0.002

1S scheme	χ^2/d.o.f.	D_i	$\mu_\pi^2/{\rm GeV}^2$	$\mu_G^2/{\rm GeV}^2$	$\rho_{\rm D}^3/{\rm GeV}^3$	$\rho_{LS}^3/{\rm GeV}^3$
Scenario 1	4.9	$D^{0,+}$	0.04 ± 0.01	0.33 ± 0.02	-	-
		D_s	0.06 ± 0.02	0.44 ± 0.02	-	-
Scenario 2	0.33	$D^{0,+}$	0.09 ± 0.02	0.32 ± 0.02	-0.003 ± 0.002	0.004 ± 0.002
		D_s	0.11 ± 0.02	0.43 ± 0.02	-0.004 ± 0.002	0.005 ± 0.002

Reliable perturbative calculation ensures a good fit!

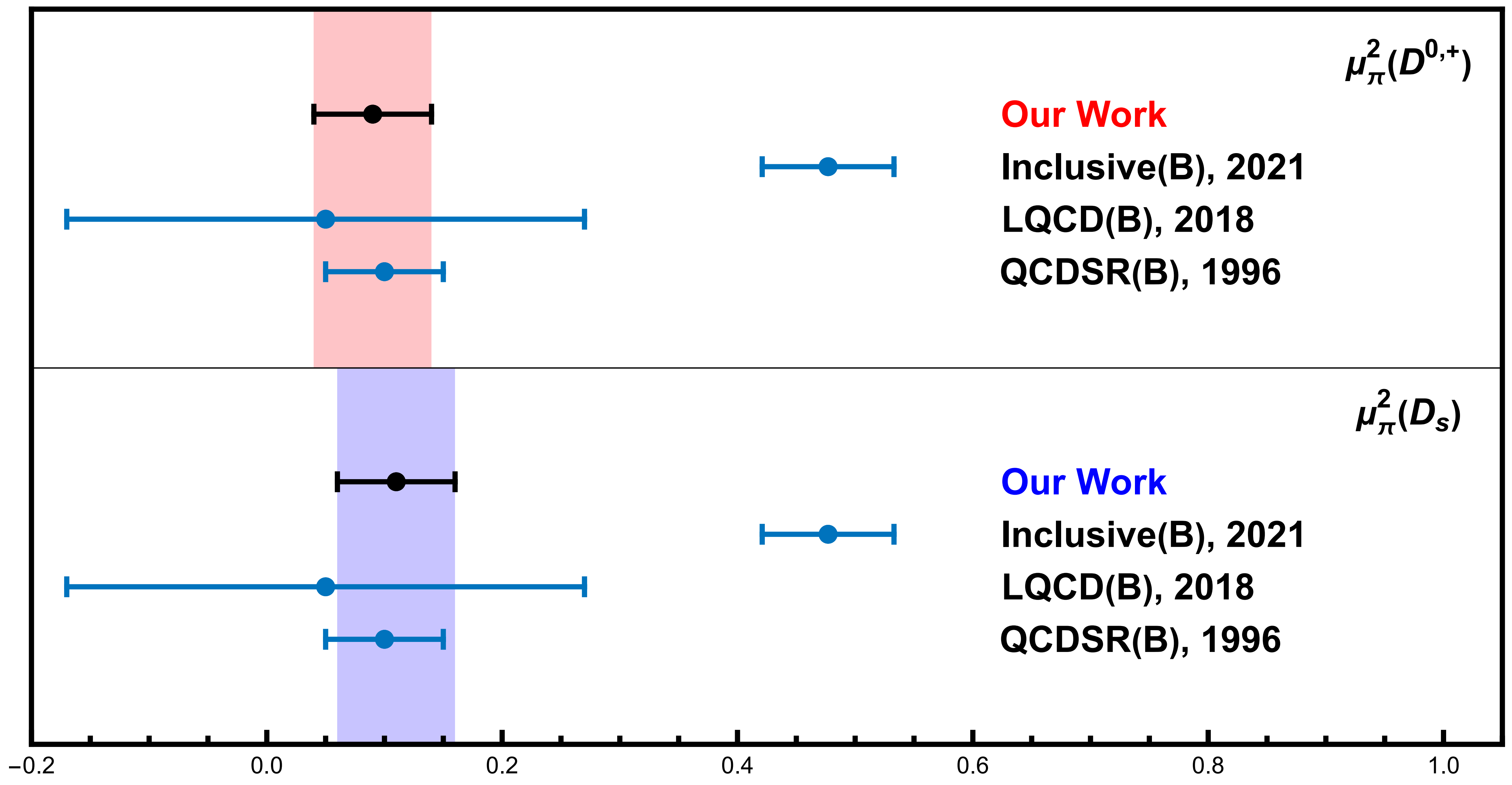
Differences between Scenarios 1 and 2 as systematic uncertainties.

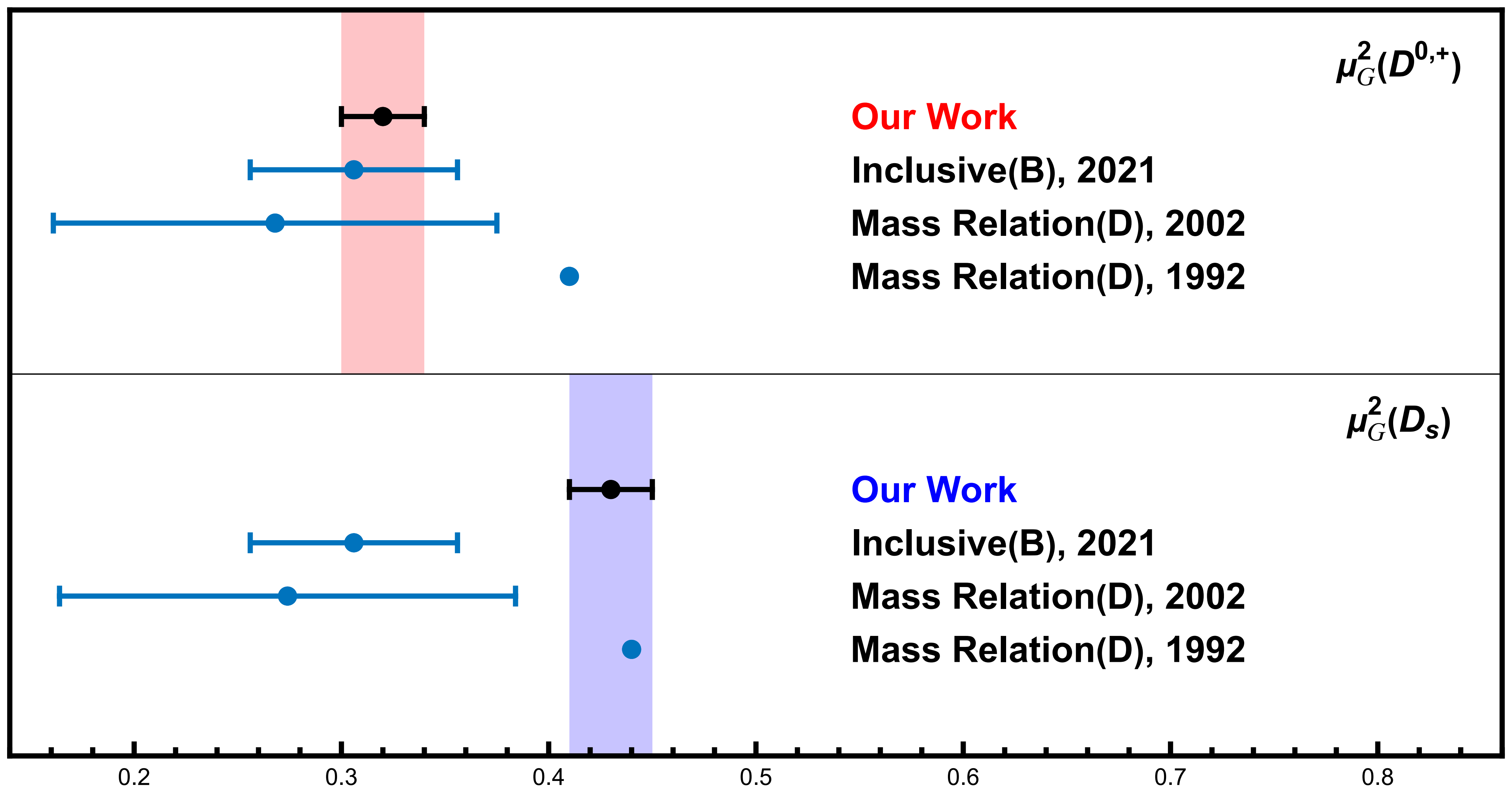
Global fit

❖ Extracted nonperturbative HQET parameters

$$\mu_\pi^2(D^{0,+}) = (0.09 \pm 0.05)\mathrm{GeV}^2, \qquad \mu_\pi^2(D_s^+) = (0.11 \pm 0.05)\mathrm{GeV}^2,$$
$$\mu_G^2(D^{0,+}) = (0.32 \pm 0.02)\mathrm{GeV}^2, \qquad \mu_G^2(D_s^+) = (0.43 \pm 0.02)\mathrm{GeV}^2,$$
$$\rho_D^3(D^{0,+}) = (-0.003 \pm 0.002)\mathrm{GeV}^3, \qquad \rho_D^3(D_s^+) = (-0.004 \pm 0.002)\mathrm{GeV}^3,$$
$$\rho_{LS}^3(D^{0,+}) = (0.004 \pm 0.002)\mathrm{GeV}^3, \qquad \rho_{LS}^3(D_s^+) = (0.005 \pm 0.002)\mathrm{GeV}^3.$$

Sizable breaking effects of flavor SU(3) symmetry and heavy quark symmetry.

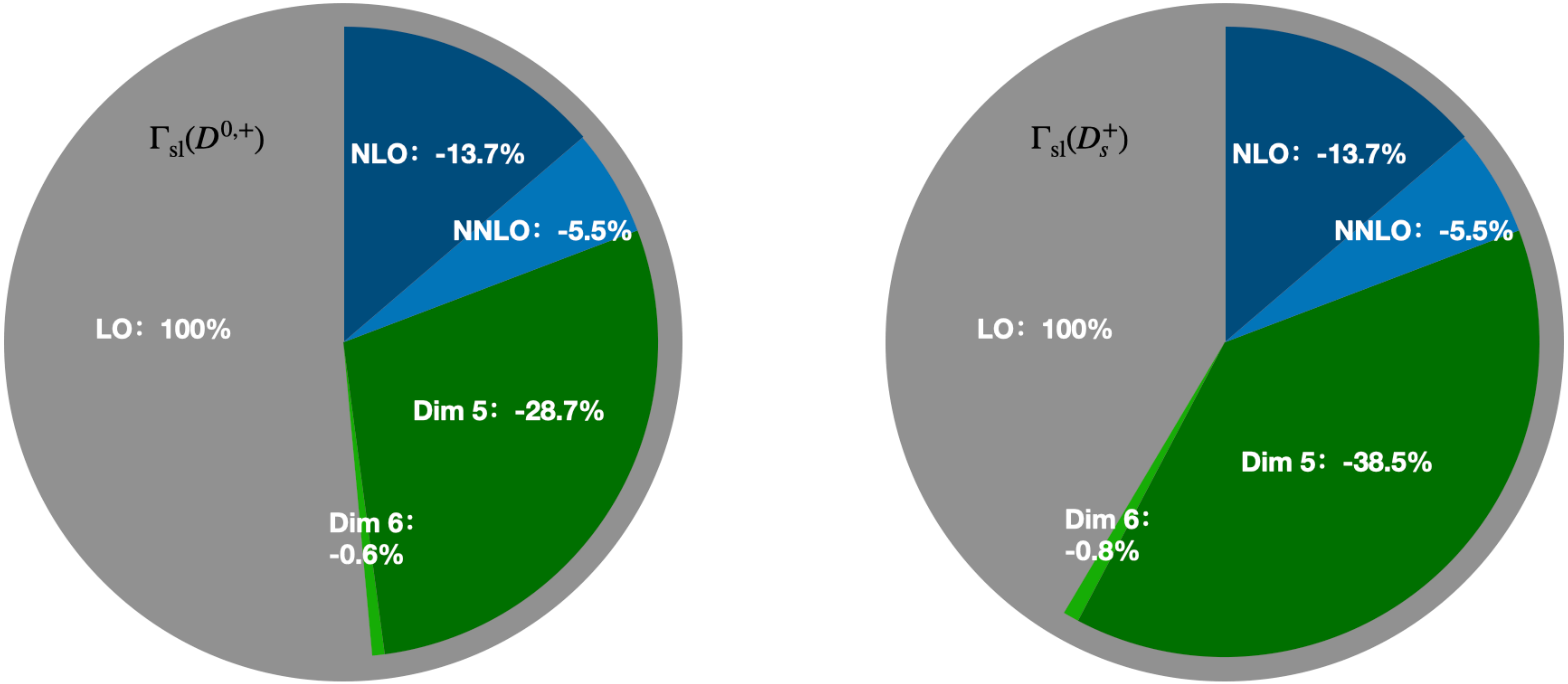




Inappropriate to use beauty parameters for charm!

Final comment: Convergence

❖ Various contributions to inclusive $D^{0,+}$ and D_s^+ decay widths



Convergent expansions of $\alpha_s(m_c)$ and Λ_{QCD}/m_c !

Summary and Prospect

- ❖ For the **first time**, the **HQE parameters** in inclusive charm decays are determined model independently, by data

- ❖ **α_s-expansion and heavy quark expansion** are **valid** in inclusive charm decays (with appropriate mass scheme chosen)

- ❖ **Possible improvements**

  - ➡ Include higher order radiative corrections, $\mathcal{O}(\alpha_s^3)$

  - ➡ Include higher power corrections, complete dimension-6 and -7 operator

  - ➡ Extend the study to charmed baryons

  - ➡ ……

Wishlist

- ❖ Measurements performed in the **rest frame** of charmed hadrons

- ❖ **Direct measurements** of $\langle E_e^n \rangle$, instead of the electron energy spectrum

- ❖ Measurements of q^2**-moments**, good for higher-dimensional operators

- ❖ Separate X_d, X_s, to give **first** inclusive measurements of V_{cd}, V_{cs}

Thank you!

Backup

Mass scheme transformation

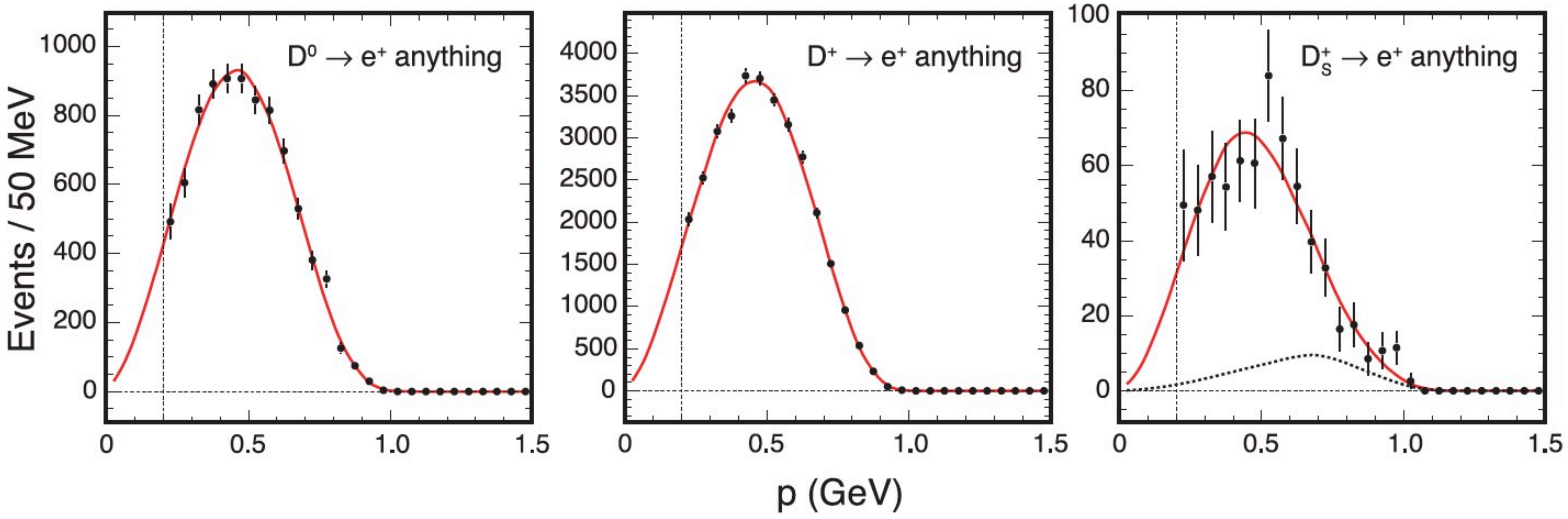
$$m_c = \overline{m}_c(\mu)\left[1 + \frac{\alpha_s(\mu)}{\pi}\left(\frac{4}{3} + \log\left(\frac{\mu^2}{\overline{m}_c^2}\right)\right) + \frac{\alpha_s^2(\mu)}{\pi^2}\frac{1}{288}\Big(112\pi^2 + 2905 + 16\pi^2\log(4) - 48\zeta(3)\right.$$
$$\left. - 12(2n_f - 45)\log^2\left(\frac{\mu^2}{\overline{m}_c^2}\right) - 4(26n_f - 519)\log\left(\frac{\mu^2}{\overline{m}_c^2}\right) - 2\left(71 + 8\pi^2\right)n_f\Big) + \mathcal{O}(\alpha_s^3)\right]$$

$$m_c = m_{c,1\mathrm{S}} + m_{c,1\mathrm{S}}\frac{\alpha_s(\mu)^2 C_F^2}{8}\left\{1 + \frac{\alpha_s}{\pi}\left[\left(-\log\left(\alpha_s(\mu)m_{c,1\mathrm{S}}C_F/\mu\right) + \frac{11}{6}\right)\beta_0 - 4 + \frac{\pi}{8}C_F\alpha_s\right] + \ldots\right\}$$

Experimental status

CLEO measurements

$D^0 \to e^+ X$ $D^+ \to e^+ X$ $D_s^+ \to e^+ X$

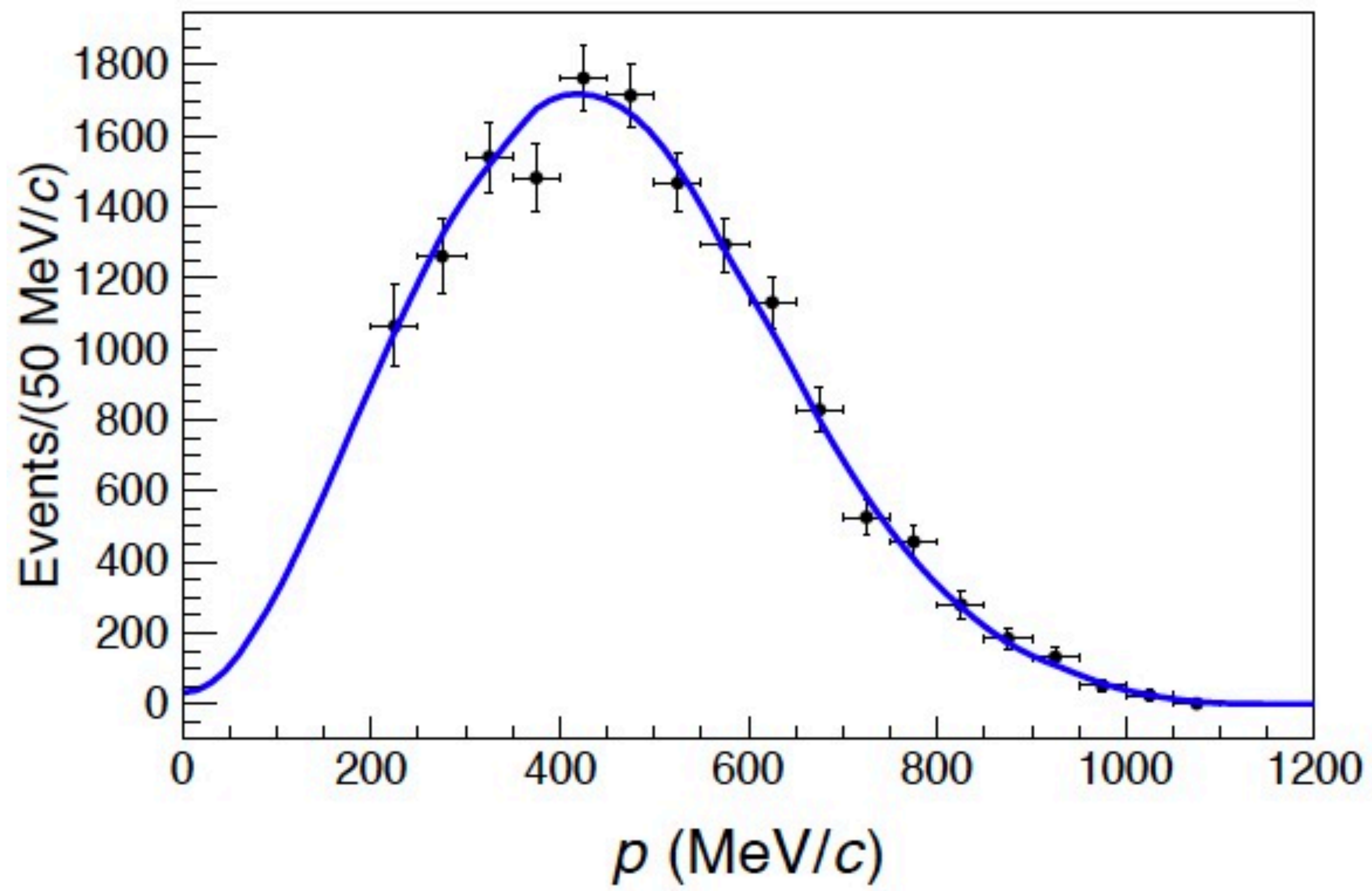


3.0×10^6 $D^0\bar{D}^0$ and 2.4×10^6 D^+D^- pairs, and is used to
ays. The latter data set contains 0.6×10^6 $D_s^{*\pm}D_s^{\mp}$ pairs,

[CLEO (818pb^{-1}($D^{0,\pm}$), 602pb^{-1}($D_s^{\pm}$)), '09]

BESIII measurements

$D_s^+ \to e^+ X$



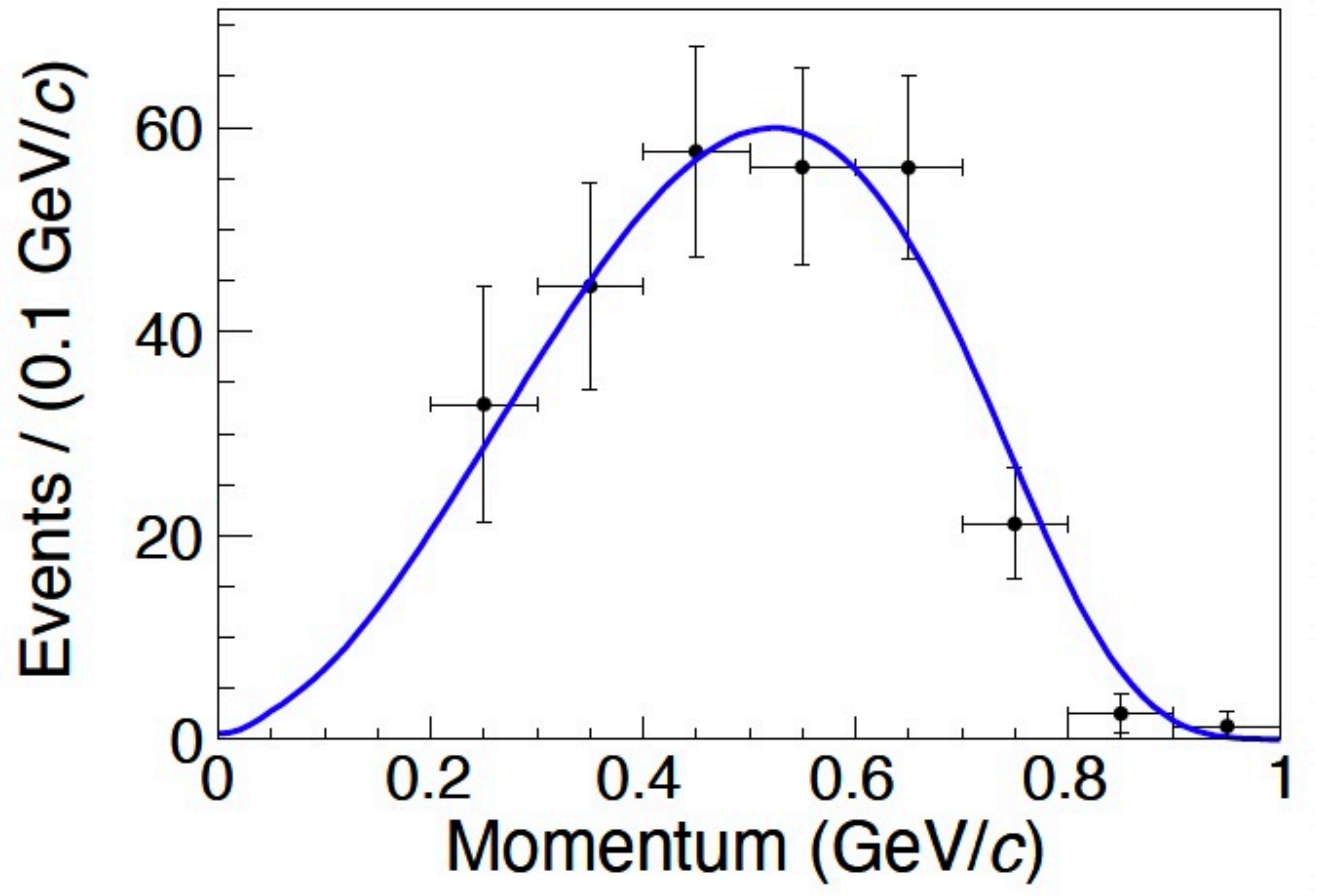
$E_{\rm cm}$ (MeV)	$\int \mathcal{L}\,dt$ (pb^{-1})	N_{D_s} ($\times10^6$)
4178	$3189.0\pm0.9\pm31.9$	6.4
4189	$526.7\pm0.1\pm2.2$	1.0
4199	$526.0\pm0.1\pm2.1$	1.0
4209	$517.1\pm0.1\pm1.8$	0.9
4219	$514.6\pm0.1\pm1.8$	0.8
4225 − 4230 [32]	$1047.3\pm0.1\pm10.2$ [33]	1.3

[BESIII, '21]

Experimental status

BESIII measurements

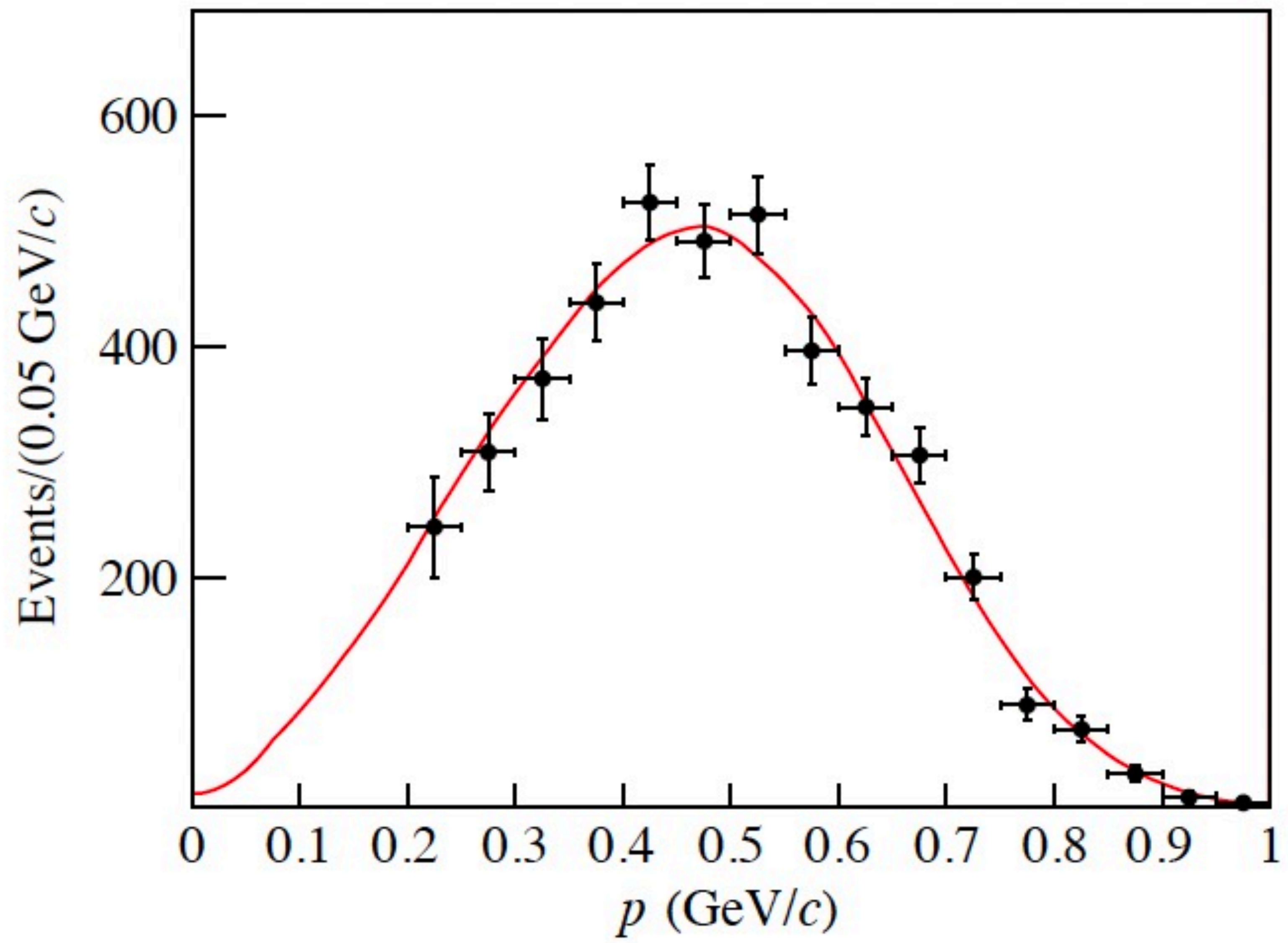
$\Lambda_c \to e^+X$



$$B(\Lambda_c^+ \to Xe^+\nu_e) = (3.95 \pm 0.34 \pm 0.09)\,\%$$

[BESIII (567 pb^{-1}), '18]

$\Lambda_c \to e^+X$



$$\mathcal{B}(\Lambda_c^+ \to Xe^+\nu_e) = (4.06 \pm 0.10_{\text{stat.}} \pm 0.09_{\text{syst}})\%$$

[BESIII (4.5 fb^{-1}), '23]