粲偶素辐射跃迁研究

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中国科学技术大学近代物理系

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Acknowledgements

In collaboration with:
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Graduate students: Zhiguo Wang (USTC)
Undergraduate: Qi'ao Li (USTC)

Based on:

- Z. Wang, M. Li, YL, J.P. Vary, PRD 109, L031902 (2024)
- YL, M. Li, J.P. Vary, PRD 105, L071901 (2022)
- M. Li, YL, P. Maris, J.P. Vary, PRD 98, 034024 (2018)

Wave functions available on Mendeley Data, doi: 10.17632/cjs4ykv8cv.2

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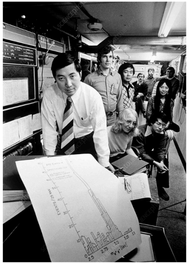
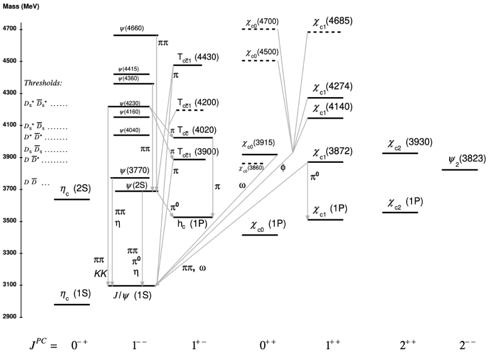
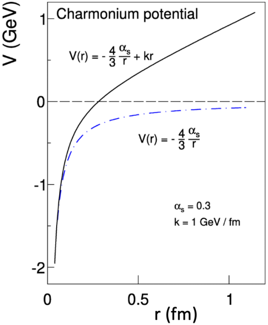
Charmonium



November revolution





- Theoretically a hard problem: multiscale, multi-physics

$$\Lambda_{\text{QCD}} \lesssim \alpha_s^2 m_c < \alpha_s m_c < m_c$$

- Physically a simple system: nonrelativistic ($v_c \ll 1$), perturbative ($\alpha_s \ll 1$)

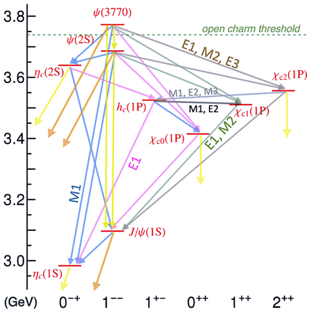
$$v_c^2 \sim 0.3,\ \alpha_s(m_c) \sim (0.3 - 0.6)$$

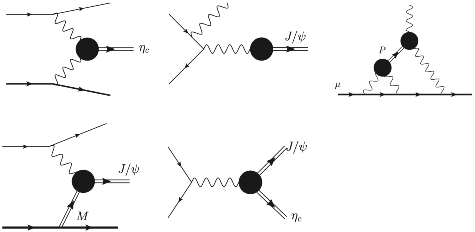
- Potential model, NRQCD, Lattice QCD, ...

[Review: Brambilla:2010cs]

Radiative transitions

[Reviews: Eichten:2007qx, Chernyak:2014wra]





Radiative widths (real γ) and radiative transition form factors (virtual γ^*)

- Golden channel for hadron identification & discovery w. J, P, C, gauge symmetry selection rules
- Clean probes to charmonium structures -- important for theories

Experimental measurements

- Extensive measurements of radiative widths [Review of particle physics 2024]
- TFFs: relatively scarce, $F_{\eta_c\gamma}(Q^2)$, $F_{\chi_{cJ}\gamma}(Q^2)$ [BaBar:2010siw, Belle:2017xsz]

Physical pictures

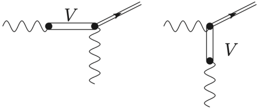
[e.g., Sakurai:1960ju, Lepage:1980fj]

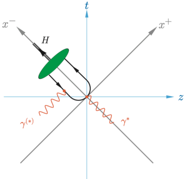
$$i\mathcal{M}_{H\to\gamma\gamma} = \varepsilon_\mu \varepsilon_\nu^* \int \mathrm{d}^4 x e^{iq\cdot z} \langle 0 | J^\mu(z) J^\nu(0) | H \rangle$$

$$\mathcal{M} \sim \frac{\Psi_V(r=0)}{M_V^2 + Q^2}$$

$$\mathcal{M} \sim \int dx\, T_H(x, Q^2)\phi(x,\mu)$$

$Q^2 \to 0$ intermediate Q^2? $Q^2 \to \infty$





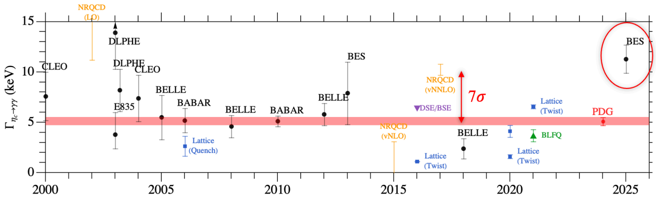
Vector Meson Dominance

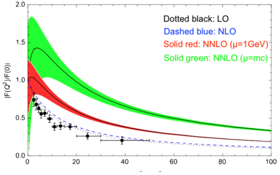
Light Cone Dominance

Intermediate Q^2: need non-perturbative methods that bridge the low & high Q^2 regimes

Theories to access the TFFs

- Potential model: large relativistic corrections [e.g. Babiarz:2019sfa]
- NRQCD: discrepancy at NNLO -- a crisis for NRQCD? [Feng:2015uha, Feng:2017hlu]
  - $\alpha_s \sim (0.3 - 0.6), v_c^2 \sim 0.3$: non-perturbative & relativistic effects
- Lattice QCD: tremendous progress in widths [Dudek:2006ut, Dudek:2006ej, Dudek:2009kk, Chen:2011kpa, Becirevic:2012dc, Donald:2012ga, CLQCD:2016ugl, CLQCD:2020njc, Liu:2020qfz, Zou:2021mgf, Meng:2021ecs, Colquhoun:2023zbc, Li:2023zig, Delaney:2023fsc, Meng:2024axn]
  - $am_c \sim 0.5$: $O(a^2)$ effects
  - QCD+QED: challenge for simulating γ^* on lattice
- Relativistic approaches: DSE, LFQM, LCSR, ... [e.g., Chen:2016bpj, Ryu:2018egt, Guo:2019xqa]





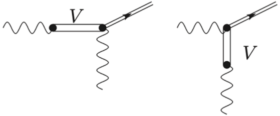
Light-front wave function representation of TFF [Li:2021ejv]

$$F_{P\gamma}(Q^2) = e_f^2 2\sqrt{2N_C}\int \frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int \frac{\mathrm{d}^2k_\perp}{(2\pi)^3}\frac{\psi^*_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp)}{k_\perp^2+m_f^2+x(1-x)Q^2} + \cdots\cdots$$

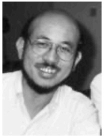
higher Fock sector contributions

$Q \to 0$

$$\sum_V \frac{e_f^2 f_V}{1+\frac{M_P}{M_V}}\frac{g_V(0)}{M_V^2+Q^2}$$

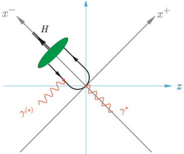


vector meson dominance



$Q \to \infty$

$$e_f^2 f_P \int_0^1 \mathrm{d}x \frac{\phi_P(x,\mu)}{x(1-x)Q^2+m_f^2}$$



light-cone dominance

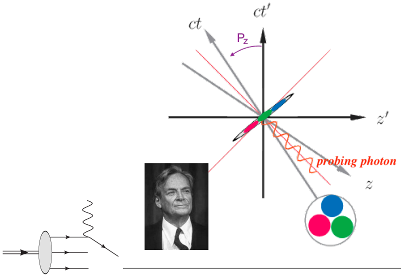
For charmonium, DSE calculations show that the non-valence contributions are small $\lesssim 5\%$ [Shi:2021nvg]

Physics on the light front

[Review: Brodsky:1997de]

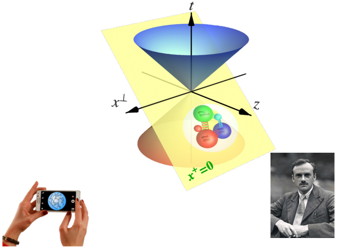
infinite momentum frame

$$P_z \to \infty$$



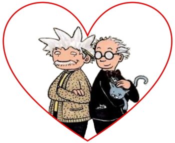
light front quantization

$$x^+ = 0$$

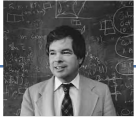
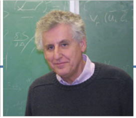
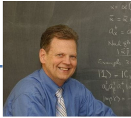


$$x^\pm = x^0 \pm x^3$$
$$P^\pm = P^0 \pm P^3$$
$$\underline{H}_{\mathrm{LC}} = P^+ \underline{P}^- - \vec{P}_\perp^2$$

$$i\frac{\partial}{\partial x^+}|\psi(x^+)\rangle = \frac{1}{2}\underline{P}^-|\psi(x^+)\rangle$$
$$\Downarrow$$
$$\underline{H}_{\mathrm{LC}}|\psi_h(P,j,m_j)\rangle = M_h^2|\psi_h(P,j,m_j)\rangle$$

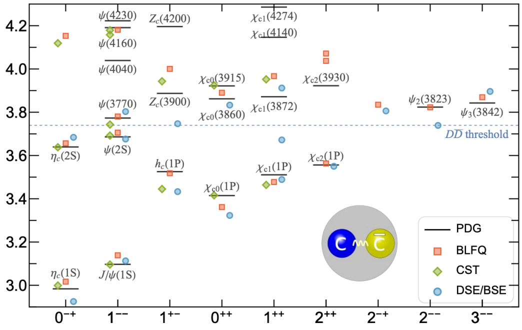


Charmonium from LFQCD

- The quantum many-body exponential wall
- Wilson's program: low-energy effective Hamiltonian from similarity RG [Wilson:1994fk]
- BLFQ: starting from semi-classical confinement $U^{(0)}$ from AdS/QCD [Brodsky:2003px, Vary:2009gt]

$$H_{\text{LFQCD}} \xrightarrow{\text{SRG}} H_{\text{eff}} = \sum_i \frac{p_{i\perp}^2 + m_i^2}{x_i} + U_i^{(0)} + \sum_{ij} \alpha_s U_{ij}^{(1)} + \sum_{ij} \alpha_s^2 U_{ij}^{(2)} + \cdots$$



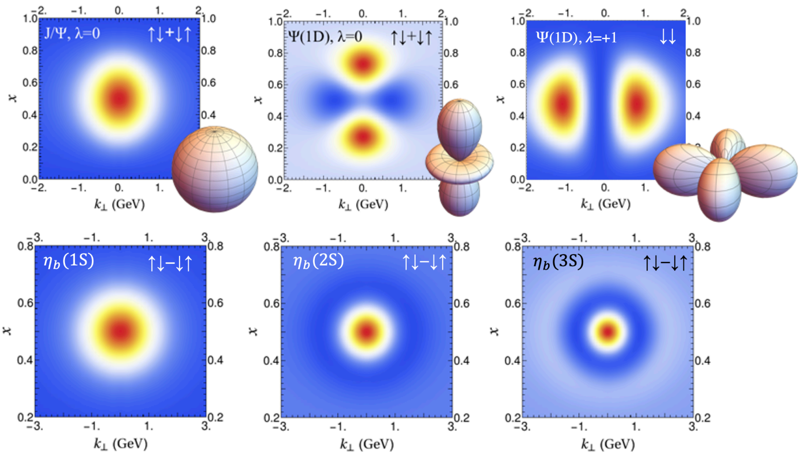
free parameters: m_c, κ

BLFQ: [Li:2017mlw]
CST: [Leitao:2017mlx]
DSE: [Fischer:2014cfa]

Light-front wave functions

angular
excitation
s

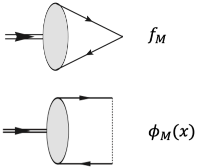
radial
excitation
s



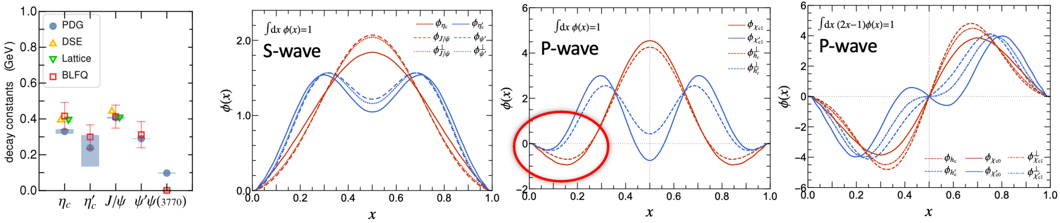
[LFWFs published on Mendeley Data, doi: 10.17632/cjs4ykv8cv.2]

Light-cone distribution amplitudes

$$\frac{f_M}{2\sqrt{2N_c}}\phi_M(x) = \frac{1}{\sqrt{x(1-x)}}\int \frac{d^2k_\perp}{2(2\pi)^3}\psi_M^{L=0}(x,\vec{k}_\perp)$$



- In NR limit, $\phi_S = \phi_A^\perp = \phi_h = \phi_{\text{odd}}$, $\phi_A = \phi_h^\perp = \phi_{\text{even}}$
- Leading-twist WF $\psi_{\uparrow\downarrow-\downarrow\uparrow/A}$ is purely relativistic



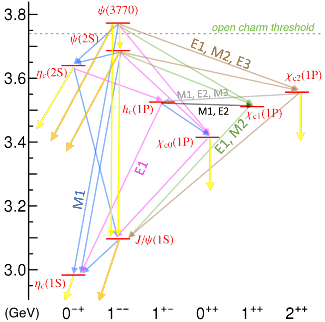
Radiative widths

[Li:2017mlw, Li:2018uif, Li:2021ejv, Wang:2023nhb]

$$\Gamma(i \to \gamma f) = \frac{1}{16\pi} \frac{M_i^2 - M_f^2}{M_i^3} \frac{1}{2J_i + 1} \sum_{\lambda_\gamma, s_i, s_f} \left| \langle f | J^\mu(0) | i \rangle \right|^2, \quad f = [c\bar{c}], n\gamma$$

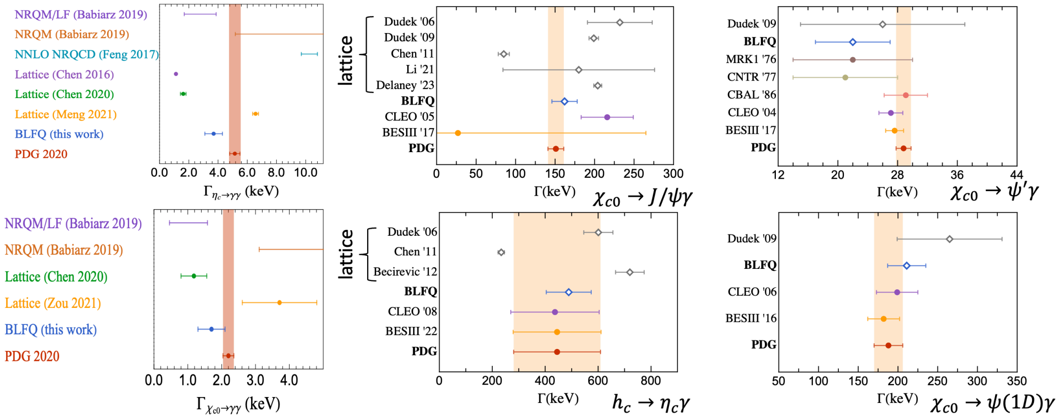
decay width (keV)		Γ_{ee}	$\Gamma_{\gamma\gamma}$	$\Gamma_{\eta_c\gamma}$	$\Gamma_{J/\psi\gamma}$	$\Gamma_{\chi_{c0}\gamma}$	$\Gamma_{\chi_{c1}\gamma}$	$\Gamma_{h_c\gamma}$	$\Gamma_{\chi_{c2}\gamma}$	$\Gamma_{\eta_c'\gamma}$	$\Gamma_{\psi'\gamma}$
η_c	PDG	-	5.15(35)								
	BLFQ	-	3.7(6)								
J/ψ	PDG	5.53(10)	-	1.6(4)							
	BLFQ	5.7(1.9)	-	2.72(5)							
χ_{c0}	PDG	-	2.20(16)	-	151(10)						
	BLFQ	-	1.7(4)	-	162(16)						
χ_{c1}	PDG	-	-	-	288(16)	-					
	BLFQ	-	-	-	in progress	-					
h_c	PDG	-	-	445(164)	-						
	BLFQ	-	-	489(85)		in progress	in progress				
χ_{c2}	PDG	-	0.56(5)	-	374(20)	-	-				
	BLFQ	-	0.70(13)	-	in progress	-	-	in progress			
η_c'	PDG	-	2.1(1.6)	-	<195	-	-		-		
	BLFQ	-	1.9(4)	-	0.16(23)	-	-	26.4(1.1)	-		
ψ'	PDG	2.33(81)	-	1.00(16)	-	28.8(1.0)	28.7(1.1)	-	28(1)	0.23(17)	
	BLFQ	2.7(1.3)	-	2.9(2.0)	-	22(5)	in progress	-	in progress	0.165(12)	
$\psi(1D)$	PDG	0.261(21)	-	<19	-	188(18)	80(7)	-	<17	<24.9	-
	BLFQ	0.051(19)	-	0.136(2)	-	211(24)	in progress	-	in progress	0.0017(24)	-

- M1
- E1
- E1, M2
- M1, E2
- E1,M2,E3
- M1,E2,M3



Radiative widths

[Li:2017mlw, Li:2018uif, Li:2021ejv, Wang:2023nhb]



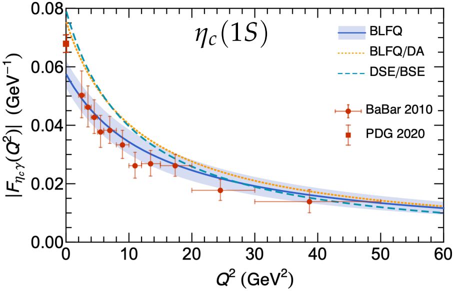
Two-photon TFF: $\eta_c \to \gamma\gamma^*$

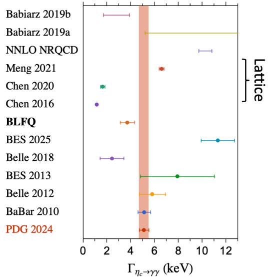
[Chen:2016bp, Babiarz:2019sfa, Meng:2021ecs, Li:2021ejv]

$$\epsilon^*_\mu(q_1)\mathcal{M}^{\mu\nu} = \langle\gamma(q_1,\lambda_1)|J^\nu(0)|P\rangle = \epsilon^*_\mu(q_1)4\pi\alpha_{\rm em}\varepsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}F_{P\gamma\gamma}(q_1^2,q_2^2),$$

Diphoton width: $\Gamma_{\gamma\gamma} = \frac{\pi}{4}\alpha^2_{\rm em}M_P^3|F_{P\gamma\gamma}(0,0)|^2$, where, $F_{P\gamma}(Q^2) \equiv F_{P\gamma\gamma}(Q^2,0) = F_{P\gamma\gamma}(0,Q^2)$ is the single-tag TFF.

$$F_{P\gamma}(Q^2) = e_f^2 2\sqrt{2N_C}\int\frac{{\rm d}x}{2\sqrt{x(1-x)}}\int\frac{{\rm d}^2k_\perp}{(2\pi)^3}\frac{\psi^*_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp)}{k_\perp^2+m_f^2+x(1-x)Q^2}$$



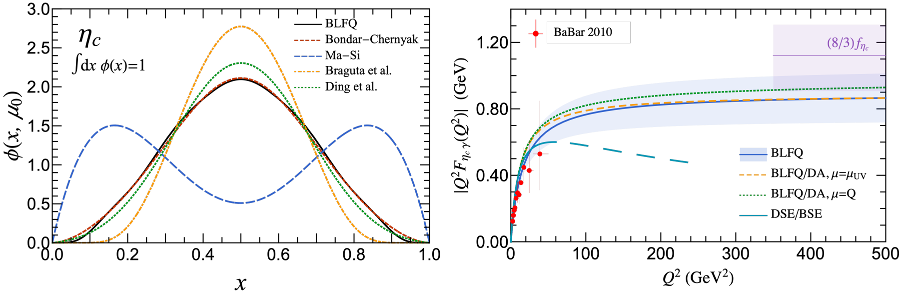


LCDA and large-Q^2 asymptotics

[Chen:2016bp, Li:2021ejv]

At large-Q^2, viz. $Q^2 + \langle m_f^2/x(1-x)\rangle \gg \langle k_\perp^2/x(1-x)\rangle$,

$$F_{P\gamma}(Q^2) \approx e_f^2 f_P \int_0^1 \mathrm{d}x \frac{\phi_P(x,\mu)}{x(1-x)Q^2 + m_f^2} \xrightarrow{Q\to\infty} \frac{6e_f^2 f_P}{Q^2}.$$



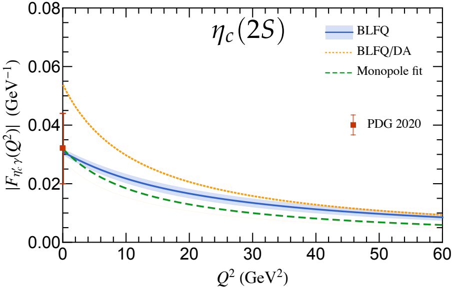
[LCDAs: Ma:2004qf, Bondar:2004sv, Braguta:2006wr, Ding:2015rkn]

Two-photon TFF: $\eta_c(2S) \to \gamma\gamma^*$

$$\epsilon^*_\mu(q_1)\mathcal{M}^{\mu\nu} = \langle\gamma(q_1,\lambda_1)|J^\nu(0)|P\rangle = \epsilon^*_\mu(q_1)4\pi\alpha_{\rm em}\varepsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}F_{P\gamma\gamma}(q_1^2,q_2^2),$$

Diphoton width: $\Gamma_{\gamma\gamma} = \frac{\pi}{4}\alpha^2_{\rm em}M_P^3\left|F_{P\gamma\gamma}(0,0)\right|^2$, where, $F_{P\gamma}(Q^2) \equiv F_{P\gamma\gamma}(Q^2,0) = F_{P\gamma\gamma}(0,Q^2)$ is the single-tag TFF.

$$F_{P\gamma}(Q^2) = e_f^2 2\sqrt{2N_C}\int\frac{{\rm d}x}{2\sqrt{x(1-x)}}\int\frac{{\rm d}^2k_\perp}{(2\pi)^3}\frac{\psi^*_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp)}{k_\perp^2+m_f^2+x(1-x)Q^2}$$

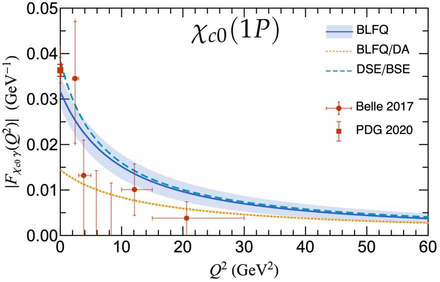


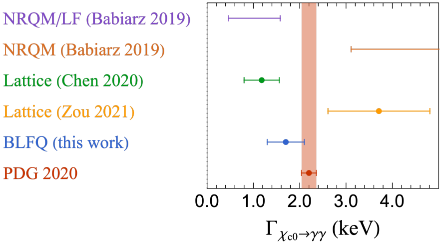
- No experimental measurement yet.
- A monopole fit using $\Lambda^2 = M^2_{\psi'}$ is included for comparison.

Two-photon TFF: $\chi_{c0} \to \gamma\gamma^*$ [Chen:2016bp, Babiarz:2019sfa, Zou:2021mgf, CLQCD:2020njc, Li:2021ejv]

$$\mathcal{M}^{\mu\nu}_{S\to\gamma\gamma} = 4\pi\alpha_{\rm em}\Big\{\big[(q_1\cdot q_2)g^{\mu\nu} - q_2^\mu q_1^\nu\big]F_1^S(q_1^2,q_2^2) + \frac{1}{M_S^2}\big[q_1^2 q_2^2 g^{\mu\nu} + (q_1\cdot q_2)q_1^\mu q_2^\nu - q_1^2 q_2^\mu q_2^\nu - q_2^2 q_1^\mu q_1^\nu\big]F_2^S(q_1^2,q_2^2)\Big\}$$

Width $\Gamma_{\gamma\gamma} = \frac{\pi\alpha_{\rm em}^2}{4}M_S^3|F_{S\gamma}(0)|^2$, where $F_{S\gamma}(q^2) = F_1^S(q^2,0) = F_1^S(0,q^2)$ is the single-tag TFF. Belle provides the first measurement of the TFF, albeit with limited statistics. [Belle:2017xsz]

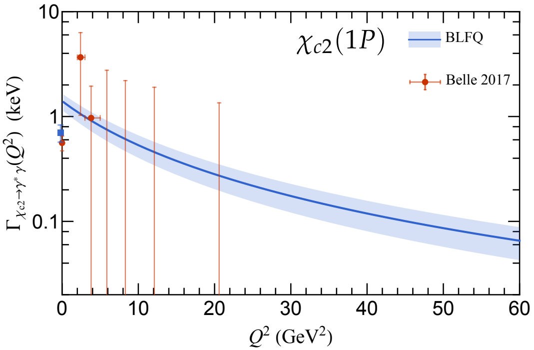




Two-photon TFF: $\chi_{c2} \to \gamma\gamma^*$

$$\Gamma_{T\to\gamma\gamma} = \frac{\pi\alpha_{\text{em}}^2}{5M_T}\left(\left|\mathcal{M}_{++;0}\right|^2 + \left|\mathcal{M}_{+-;2}\right|^2\right).$$

Belle provides the first measurement of the Q^2 dependent width, albeit with limited statistics. [Belle:2017xsz]



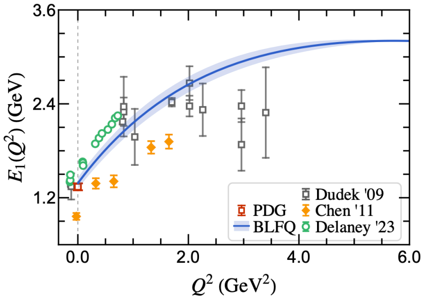
E1 TFF: $\chi_{c0} \to J/\psi\gamma^*$

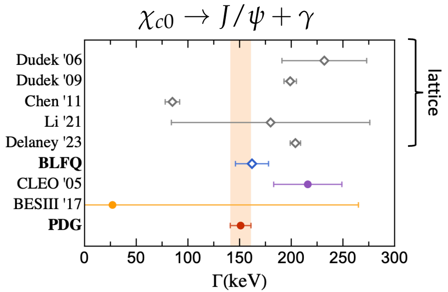
[Dudek:2009kk, Chen:2011kpa, Delaney:2023fsc, Wang:2023nhb]

$$\langle V(p',\lambda')|J^\mu(0)|S(p)\rangle = E_1(Q^2)\Big[e^{\mu *}_{\lambda'}(p') - \frac{e^*_{\lambda'}\cdot p}{(p\cdot p')^2 - M_S^2 M_V^2}\big(p'^\mu(p\cdot p') - M_V^2 p^\mu\big)\Big]$$
$$+ C_1(Q^2)\frac{M_V}{Q(p\cdot p')^2 - M_S^2M_V^2}(e^*_{\lambda'}\cdot p)\Big[(p\cdot p')(p+p')^\mu - M_S^2 p'^\mu - M_V^2 p^\mu\Big],$$

Note that there are two TFFs, and E_1 is related to the radiative width: $\Gamma = \frac{2\alpha_{\rm em}}{9}\frac{M_S^2 - M_V^2}{M_S^3}|E_1(0)|^2$.

$$E_1(0) = \frac{M_S^2 - M_V^2}{i\sqrt{2}}\sum_{s,\bar{s}}\int_0^1\frac{\mathrm{d}x}{4\pi}\int \mathrm{d}^2 r_\perp (r_x + i r_y)\psi^{(\lambda=+1)*}_{s\bar{s}/V}(x,\vec{r}_\perp)\psi_{s\bar{s}/S}(x,\vec{r}_\perp).$$





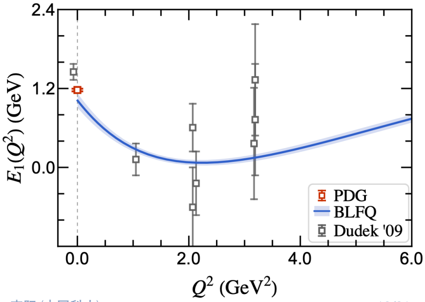
E1 TFF: $\psi(2S) \to \chi_{c0}\gamma^*$

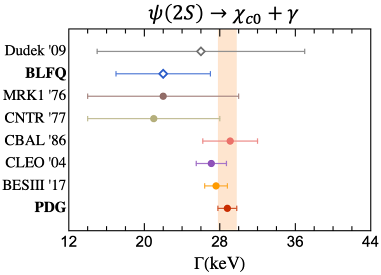
[Dudek:2009kk, Wang:2023nhb]

$$\langle V(p',\lambda')|J^\mu(0)|S(p)\rangle = E_1(Q^2)\Big[e^{\mu*}_{\lambda'}(p') - \frac{e^*_{\lambda'}\cdot p}{(p\cdot p')^2 - M_S^2 M_V^2}\big(p'^\mu(p\cdot p') - M_V^2 p^\mu\big)\Big]$$
$$+ C_1(Q^2)\frac{M_V}{Q(p\cdot p')^2 - M_S^2 M_V^2}(e^*_{\lambda'}\cdot p)\Big[(p\cdot p')(p+p')^\mu - M_S^2 p'^\mu - M_V^2 p^\mu\Big],$$

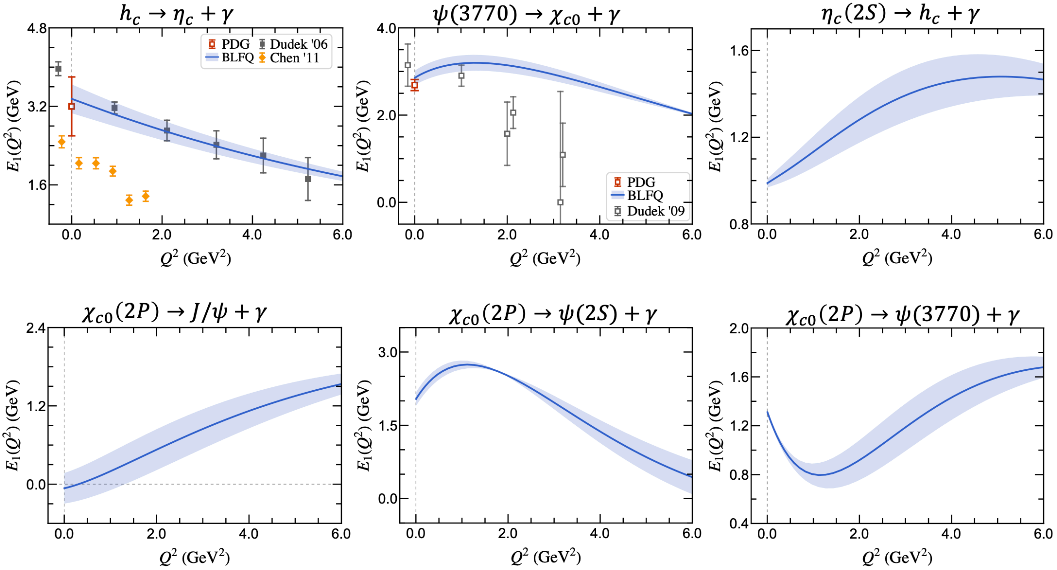
Note that there are two TFFs, and E_1 is related to the radiative width: $\Gamma = \frac{2\alpha_{em}}{27}\frac{M_V^2 - M_S^2}{M_V^3}|E_1(0)|^2$.

$$E_1(0) = \frac{M_V^2 - M_S^2}{i\sqrt{2}}\sum_{s,\bar{s}}\int_0^1 \frac{dx}{4\pi}\int d^2 r_\perp (r_x + i r_y)\psi^{(\lambda=+1)*}_{s\bar{s}/V}(x,\vec{r}_\perp)\psi_{s\bar{s}/S}(x,\vec{r}_\perp).$$



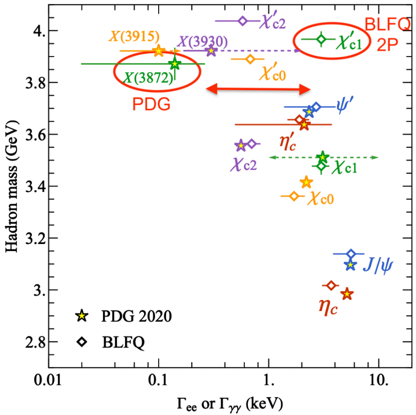


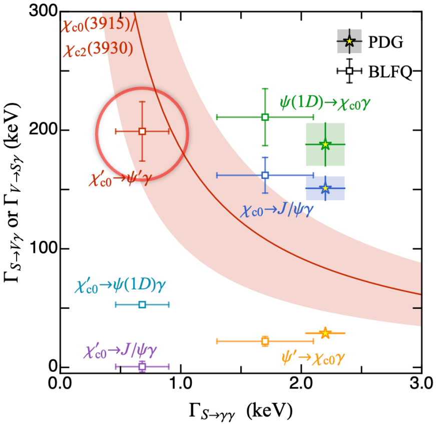
[Dudek:2006ej, Dudek:2009kk, Chen:2011kpa, Wang:2023nhb]



Implication to charmonium-like mesons

- $\Gamma_{ee}, \Gamma_{\gamma\gamma} \propto |\Psi(0)|^2$
- $\widetilde{\Gamma}_{\gamma\gamma}$ does not support $X(3872)$ as $\chi_{c1}(2P)$ [Belle:2020ndp, cf. Babiarz:2023ebe]
- $\Gamma_{R\to\psi'\gamma}\Gamma_{R\to\gamma\gamma}$ is compatible with $\chi_{c0}(2P)$ [Belle:2021nuv]





Summary

- We investigate charmonium radiative transitions using light-front Hamiltonian formalism that incorporates relativity, light-cone dominance and nonperturbative effects
- High-quality fit to charmonium spectrum and parameter free predictions on radiative widths and transition form factors which are in good agreement with experiments and lattice results whenever available
- Application to hadronic form factors, partonic distributions and hard exclusive charmonium production [Chen:2018vdw, Lan:2019img, Lappi:2020ufv, Xu:2024hfx, Hu:2024edc; see also X. Cao's talk on Apr. 21st]
- Future developments: High Fock sectors (e.g. gluons and sea), nonperturbative Hamiltonian renormalization, charmonium in strong external fields ... [Xu:2024sjt, Wen:2025dwy]

Thank you!

Light-front dynamics ≠ NR dynamics w. relativistic corrections

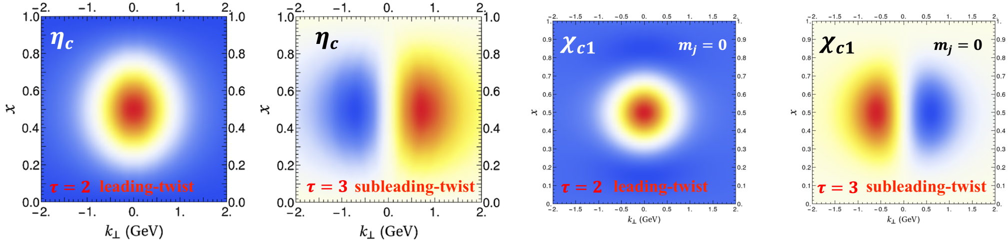
N.B. wave functions from non-relativistic dynamics with relativistic corrections in general are different from wave functions from relativistic dynamics, e.g. LFD.

- Parities in NRQM: $\mathsf{P} = (-1)^{L+1}$, $\mathsf{C} = (-1)^{L+S}$ are approximations since L is not a good quantum number

  NRQM + relativistic correction: spin-orbital coupling → partial wave mixing subject to parities

- Parities in LFD: $m_{\mathsf{P}} = (-1)^{L_z+S+1}$, $\mathsf{C} = (-1)^{L_z+S+\ell}$ are exact

  There exist leading-twist wavefunctions that are absent in NRQM (including relativistic corrections) due to parities



(a) $\psi_{\uparrow\downarrow-\downarrow\uparrow}(\vec{k}_\perp, x)$ (b) $\psi_{\downarrow\downarrow}(\vec{k}_\perp, x) = \psi^*_{\uparrow\uparrow}(\vec{k}_\perp, x)$ (c) $\psi_{\uparrow\downarrow-\downarrow\uparrow}(\vec{k}_\perp, x)$ (d) $\psi_{\downarrow\downarrow}(\vec{k}_\perp, x) = \psi^*_{\uparrow\uparrow}(\vec{k}_\perp, x)$