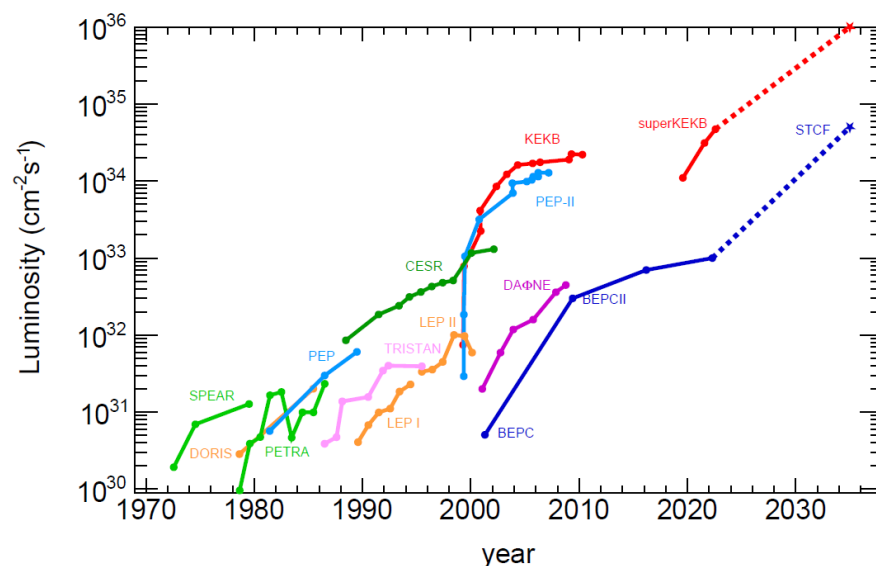


Inclusive hadron production at BESIII

鄢文标(中国科学技术大学)



Leading Quark TMDFFs

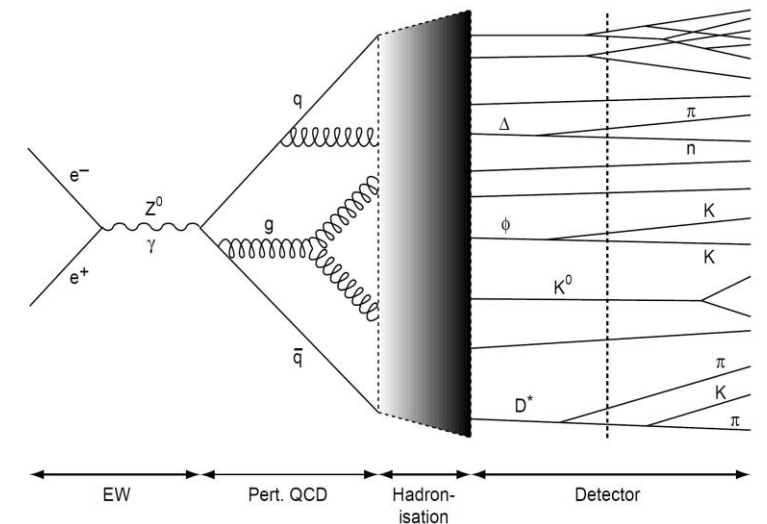
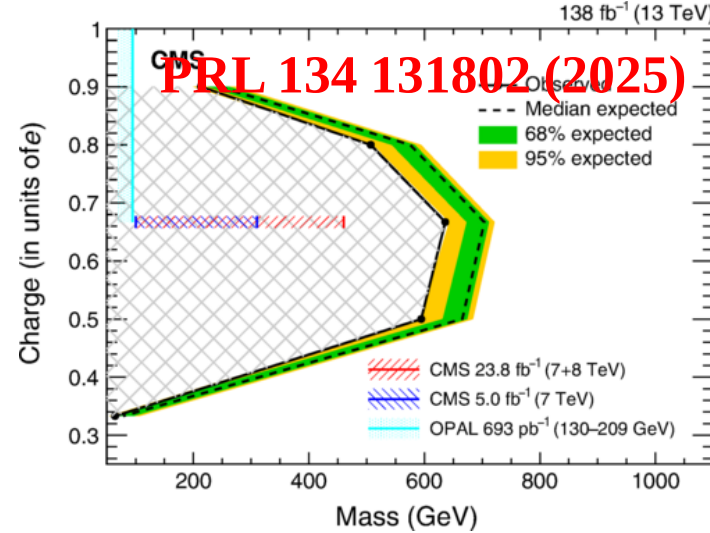
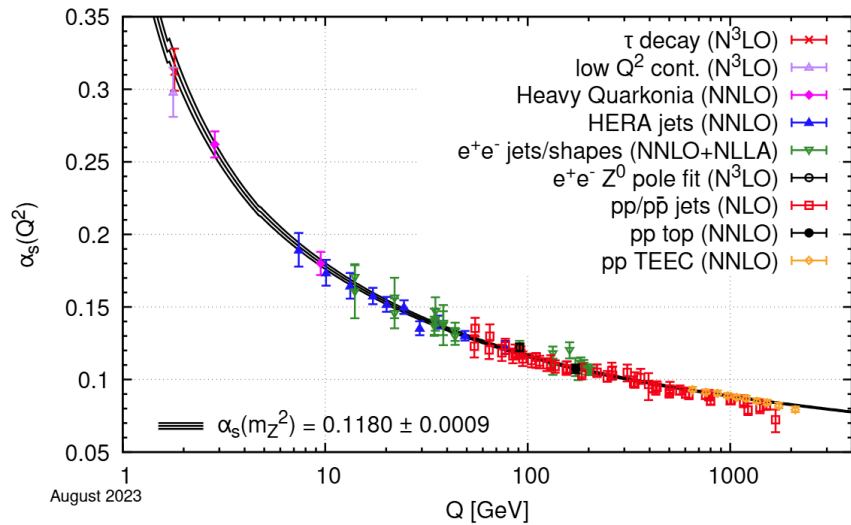
○ → Hadron Spin ⊙ → Quark Spin

	Quark Polarization		
	Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons	$D_1 = \odot$ Unpolarized		$H_1^\perp = \odot - \odot$ Collins
Polarized Hadrons		$G_1 = \odot \rightarrow \odot$ Helicity	$H_{1L}^\perp = \odot \rightarrow \odot$
	$D_{1T}^\perp = \odot - \odot$ Polarizing FF	$G_{1T}^\perp = \odot \rightarrow \odot$	$H_{1T}^\perp = \odot - \odot$ Transversity

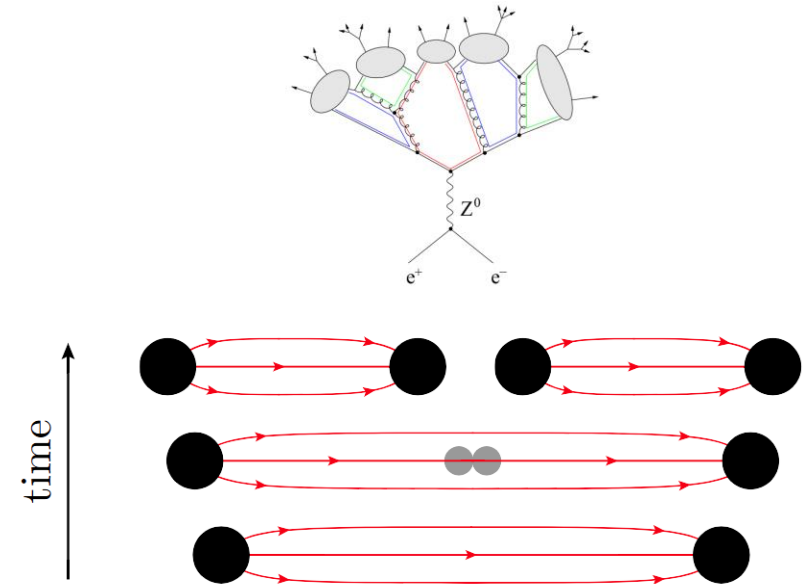
arXiv:2304.03302

第七届全国重味物理与量子色动力学研讨会，2025.04.19，南京

QCD: Asymptotic freedom & Confinement

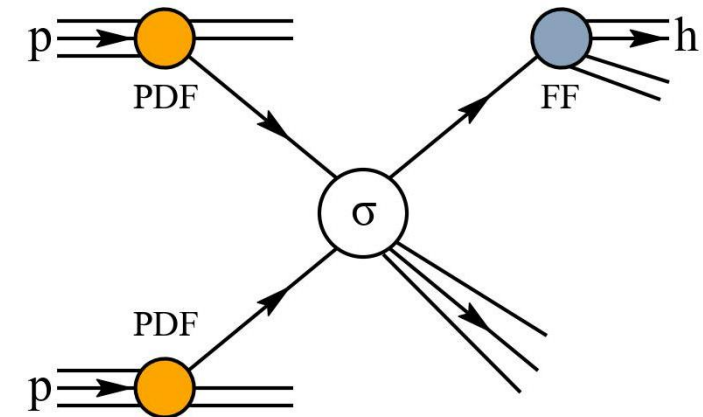
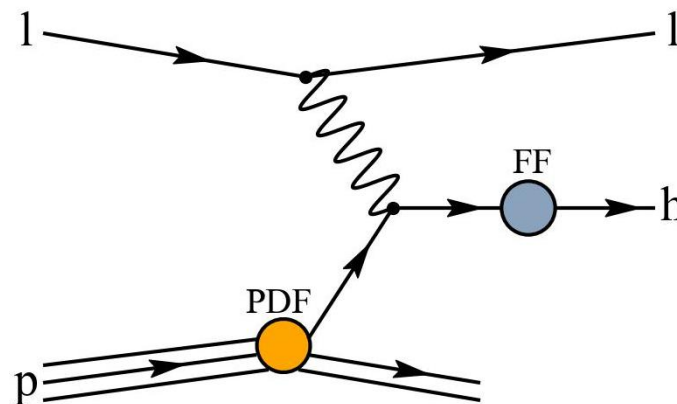
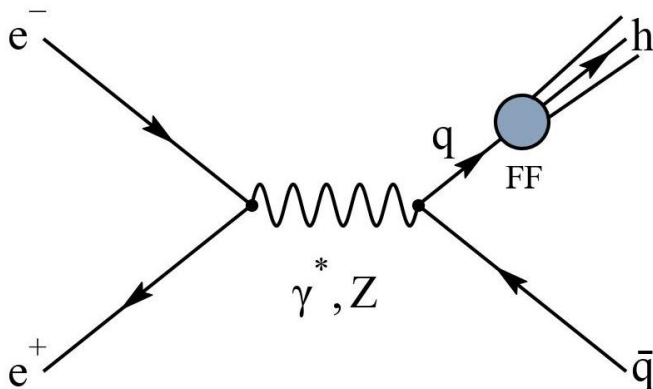
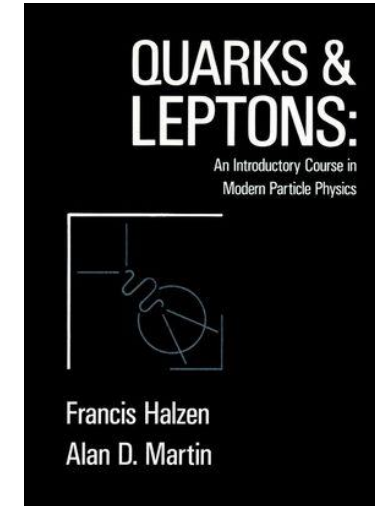


- QCD coupling constant $\alpha_s(Q)$
 - ✓ High Q: asymptotic freedom, perturbative QCD
 - ✓ Low Q: **non-perturbative QCD**
- Confinement: partons do not exist as free particles, but are always confined in hadron
- Essence of confinement ? Why & **how** ?
 - ✓ **Hadronization models** & **Fragmentation function**
 - ✓ **LPHD**: Local Parton Hadron Duality hypothesis



Fragmentation function: integrated $D_1^h(z)$

- Fragmentation function $D_q^h(z)$: probability that hadron h is found in the debris of a parton carrying a fraction z of parton's energy
- Consequence of confinement
- FF: QCD first principle (NOT YET)
 - ✓ FF evolution function: DGLAP
 - ✓ Fitting: parametrization & experimental data
 - ✓ Universality: e^+e^- , DIS, pp, $p\bar{p}$ data
- FFs contribute to **virtually all processes**



FFs with quark/hadron polarization

Hadron polarization	Quark polarization @ PPNP 91 136 (2016)		
	Unpolarized	Longitudinally	Transversely
Unpolarized	D_1^h		$H_1^{\perp h}$
Longitudinally		G_1^h	$H_{1L}^{\perp h}$
Transversely	$D_{1T}^{\perp h}$	G_{1T}^h	$H_1^h \quad H_{1T}^{\perp h}$

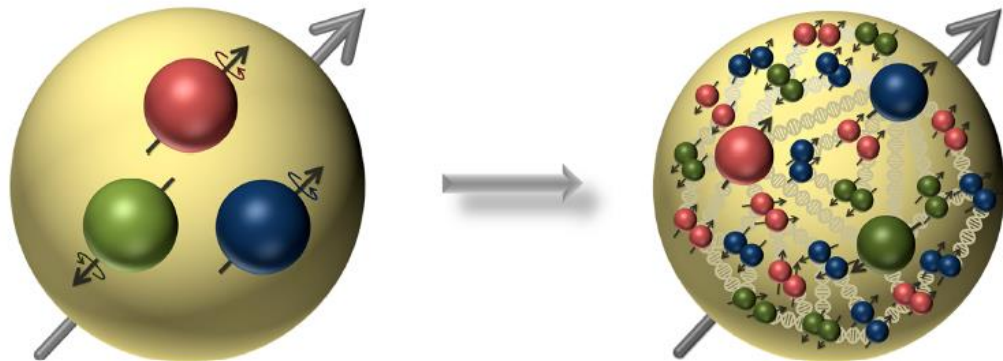
PLB396 (1993) 161



$$D_{hq\uparrow}(z, P_{h\perp}) = D_1^q(z, P_{h\perp}^2) + H_1^{\perp q}(z, P_{h\perp}^2) \frac{(\hat{\mathbf{k}} \times \mathbf{P}_{h\perp}) \cdot \mathbf{S}_q}{zM_h}$$

- Theoretically many more, in particular with **polarized hadrons** in the final state and **transverse momentum dependence (TMD)**

FFs for EIC & EicC

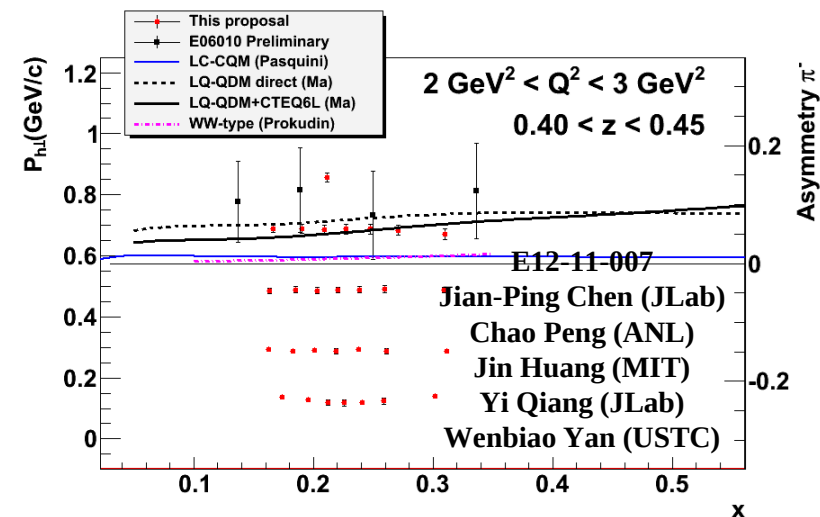
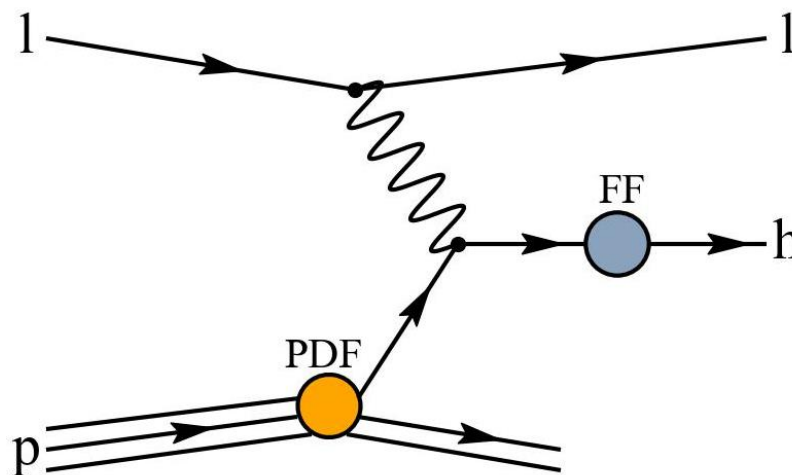
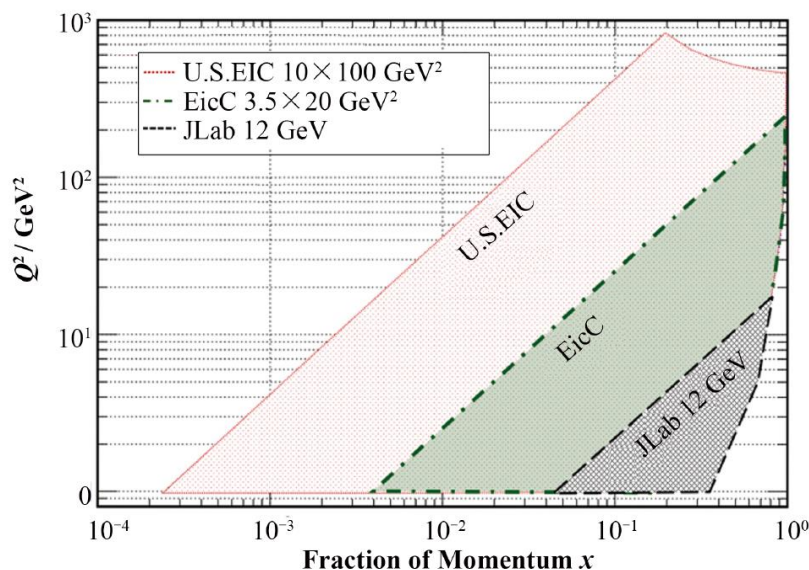


Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

arXiv:2304.03302

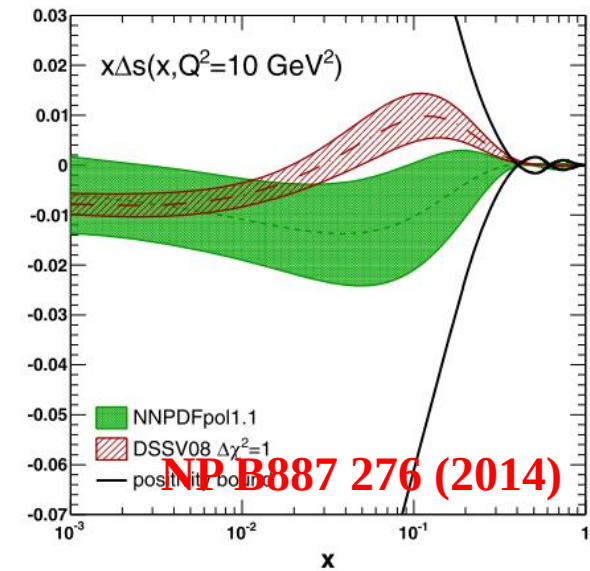
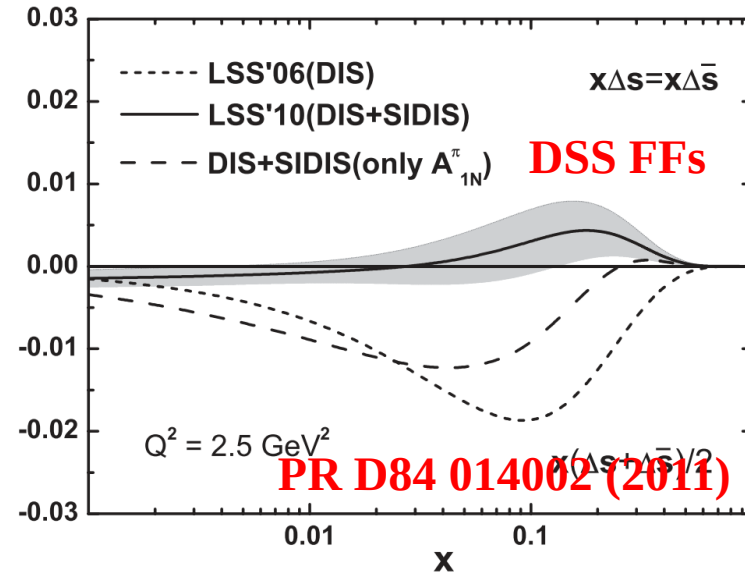
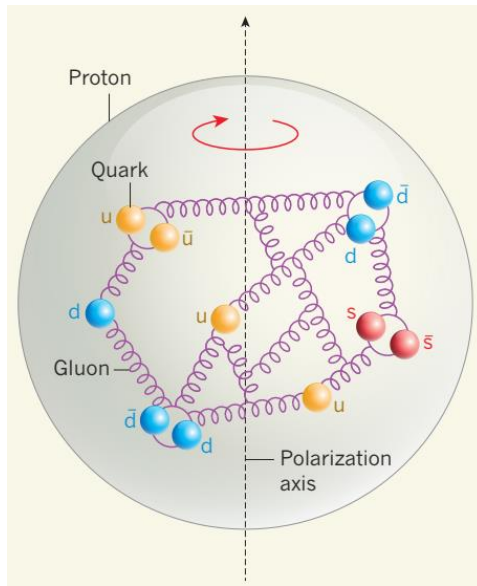
TMD Handbook

Renaud Boussarie¹, Matthias Burkardt², Martha Constantinou³, William Detmold⁴, Markus Ebert^{4,5}, Michael Engelhardt², Sean Fleming⁶, Leonard Gamberg⁷, Xiangdong Ji⁸, Zhong-Bo Kang⁹, Christopher Lee¹⁰, Keh-Fei Liu¹¹, Simonetta Liuti¹², Thomas Mehen¹³, Andreas Metz³, John Negele⁴, Daniel Pitonyak¹⁴, Alexei Prokudin^{7,16}, Jian-Wei Qiu^{16,17}, Abha Rajan^{12,18}, Marc Schlegel^{2,19}, Phiala Shanahan⁴, Peter Schweitzer²⁰, Iain W. Stewart⁴, Andrey Tarasov^{21,22}, Raju Venugopalan¹⁸, Ivan Vitev¹⁰, Feng Yuan²³, Yong Zhao^{24,4,18}

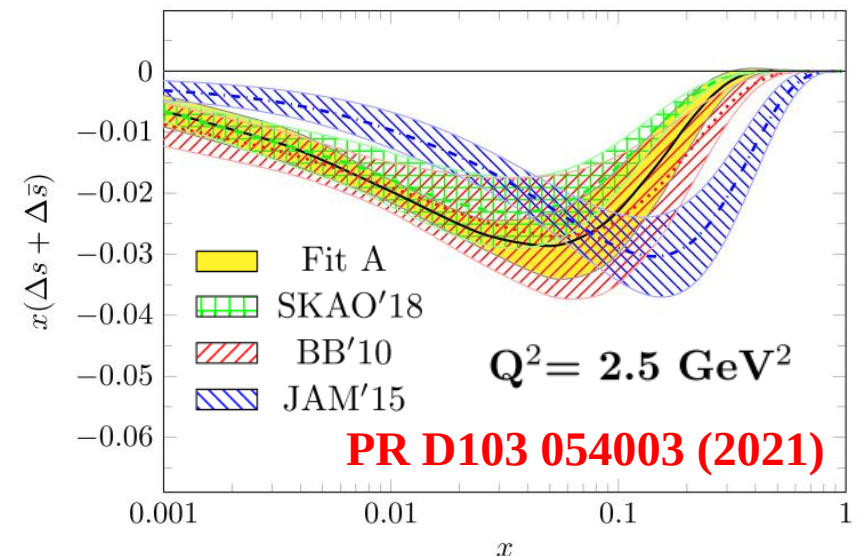


Precise knowledge of FFs will be crucial

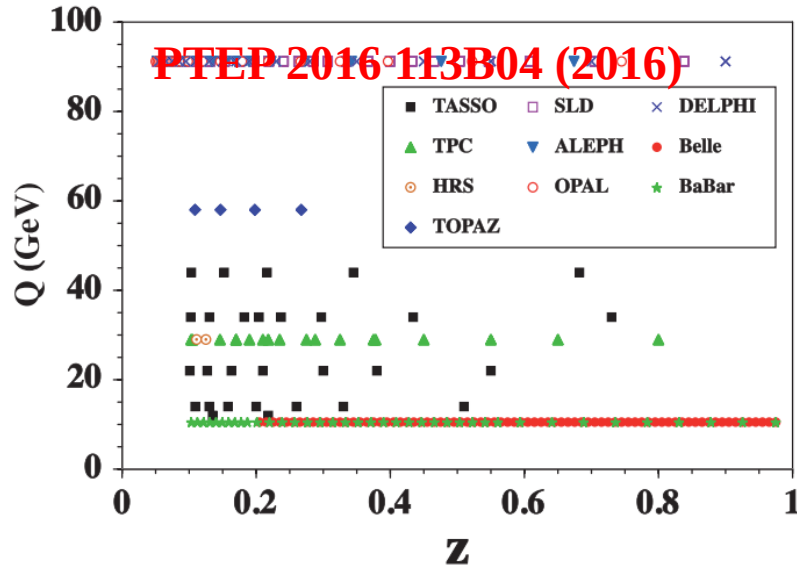
Strange quark polarization puzzle



- Strange quark density function: $\Delta s(x) + \Delta \bar{s}(x)$
 - ✓ Inclusive DIS: only proton PDF
 - a. **negative** for all values of x
 - ✓ Semi-inclusive DIS: proton PDF & kaon FF
 - a. DSS FFs: **positive** for most of measured x
 - b. NNPDF FF: **negative**
 - c. JAM FFs: **negative**
- Reliable FFs knowledge ? Need more efforts

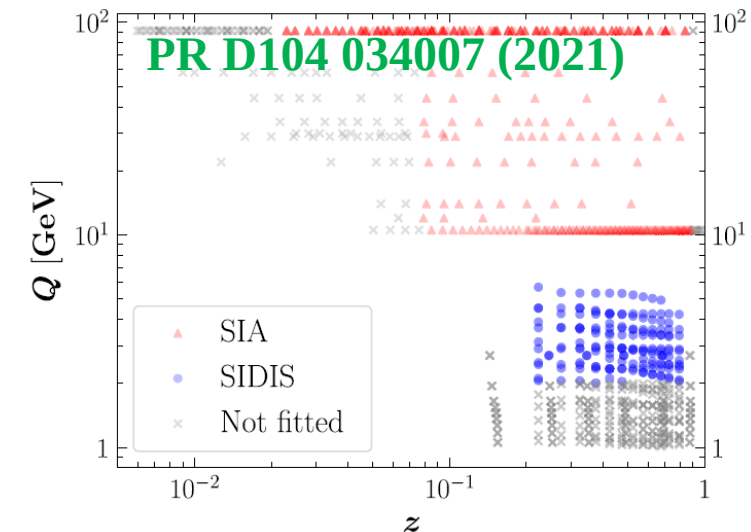


Used data set @ unpolarized FFs fitting

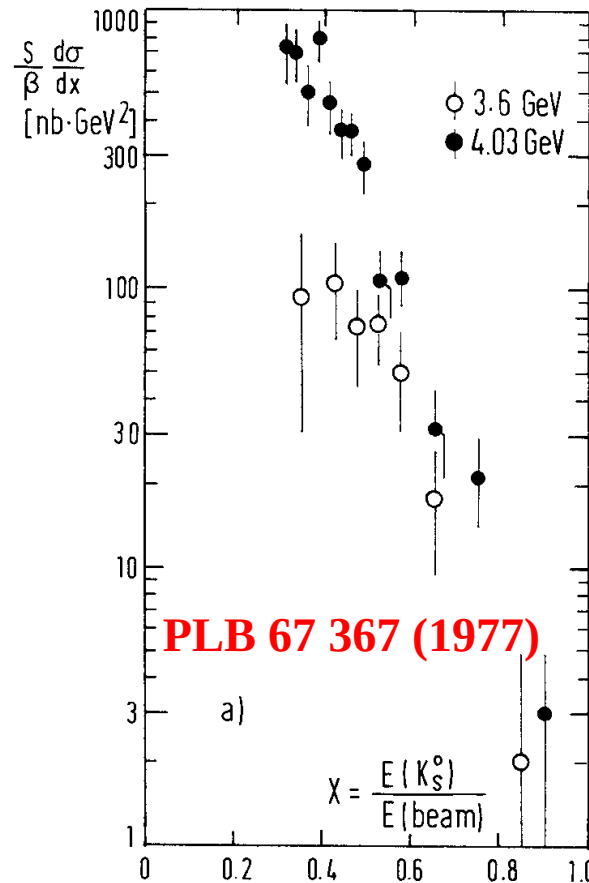
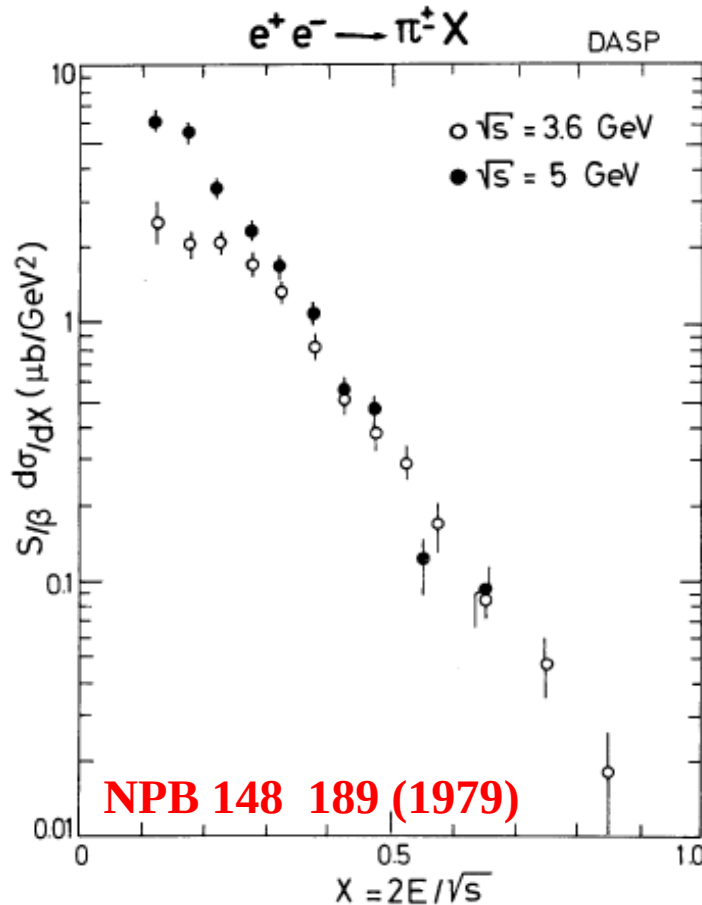


Experiment	Process	$\mathcal{L}[\text{pb}^{-1}]$	$Q^2[\text{GeV}^2]$	Final states
TPC [288–291]	e^+e^-	140	2	$\pi^\pm, K^\pm, p/\bar{p}$
TASSO [292–294]	e^+e^-	34	34,44	$\pi^\pm, K^\pm, p/\bar{p}, K_S^0, \Lambda/\bar{\Lambda}$
SLD [299,300]	e^+e^-	20	M_Z	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
ALEPH [301,302]	e^+e^-	800	M_Z	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
DELPHI [303–306]	e^+e^-	800	M_Z	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
OPAL [307–310]	e^+e^-	800	M_Z	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
H1 [311–313]	$e + p$	500	27.5(e) + 920(p)	$h^\pm, K_S^0$
ZEUS [314–316]	$e + p$	500	27.5(e) + 920(p)	$h^\pm$
BELLE [317,318]	e^+e^-	10^6	Near 10.58	$\pi^\pm, K^\pm, p/\bar{p}$
BaBar [319,320]	e^+e^-	$557 \cdot 10^3$	Near 10.58	$\pi^\pm, K^\pm, \eta, p/\bar{p}$
HERMES [321,322]	$e + p(d)$	272 (p) 329 (d)	27.6 fixed target	$\pi^{\pm,0}, K^\pm$
COMPASS [323]	$\mu + p(d)$	775	160 GeV fixed target	$h^\pm$
PHENIX [324–326]	pp	16×10^{-3}	62.4	$\pi^{\pm,0}, \eta$
STAR [327–329]	pp	2.5	200	$\pi^{\pm,0}, \eta, p/\bar{p}, K_S^0, \Lambda/\bar{\Lambda}$
ALICE [330–332]	pp	8	200	π^0, η
TOPAZ [333–335]	e^+e^-	6×10^{-3}	7×10^3	$\pi^\pm, K^\pm, K_S^0$
CDF [338,339]	$p + \bar{p}$	278	52–61.4	$h^\pm, K_S^0, \Lambda^0$
		n/a	630 (1800)	

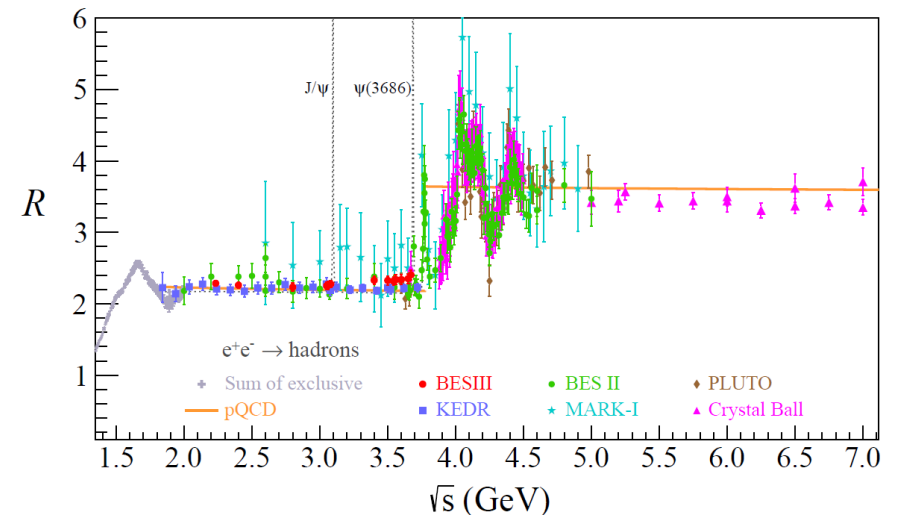
- Updated HKNS FFs @ 2016
 - ✓ only e^+e^- ($\sqrt{s} > 10$ GeV) data sets
- MAPFF1.0 FFs @ 2021
 - ✓ e^+e^- ($\sqrt{s} > 10$ GeV) and SIDIS data sets
- Data set at $\sqrt{s} < 10$ GeV e^+e^- collision ?



Inclusive π & K data of e^+e^- at low energy

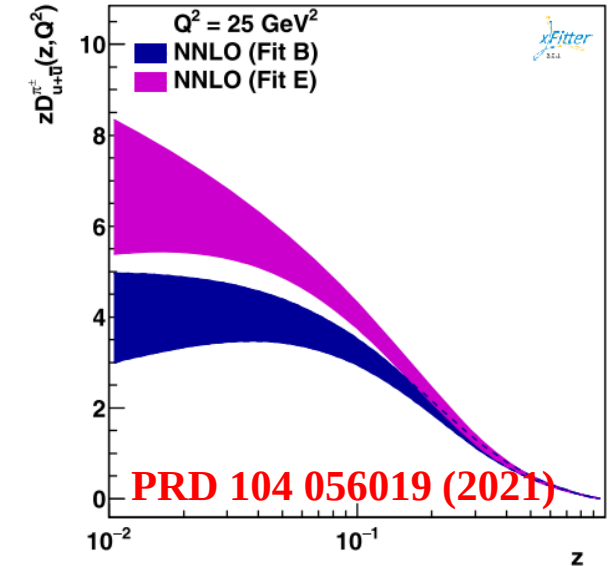
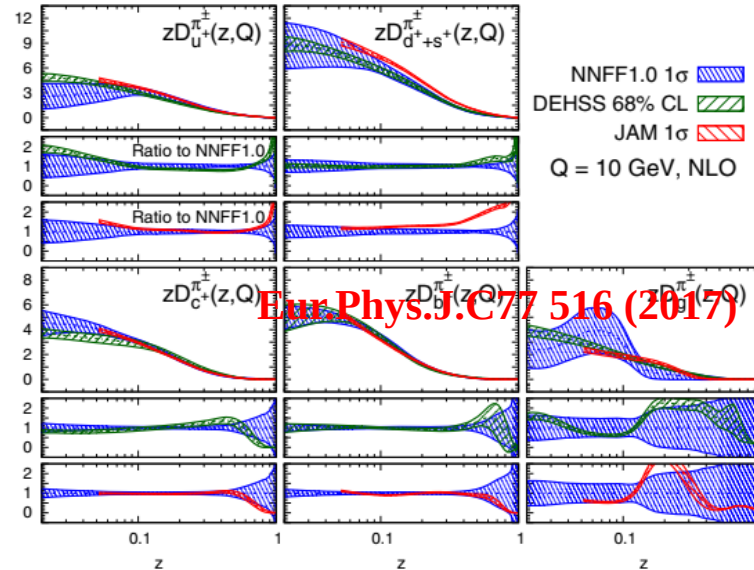
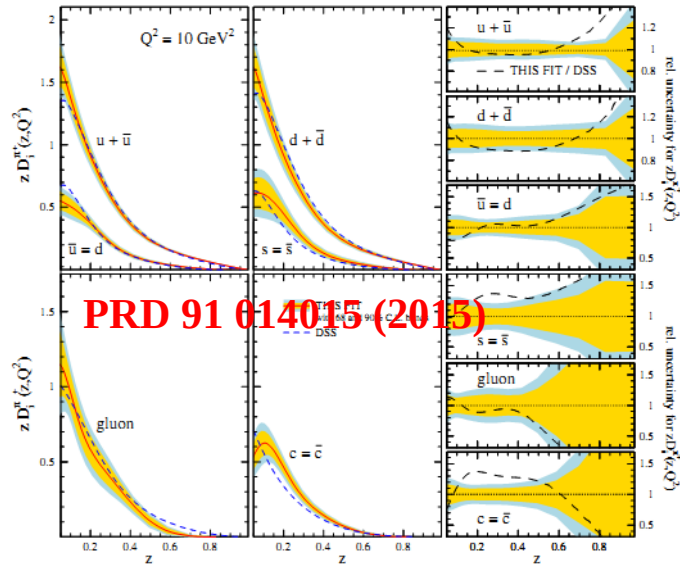


- Charged π @ DASP
 - ✓ about 46 years ago
 - ✓ stat. uncertainty: 18%
- K_S^0 @ PLUTO
 - ✓ about 48 years ago
 - ✓ stat. uncertainty: 18-41%
- Precision data ? TMD FFs ?

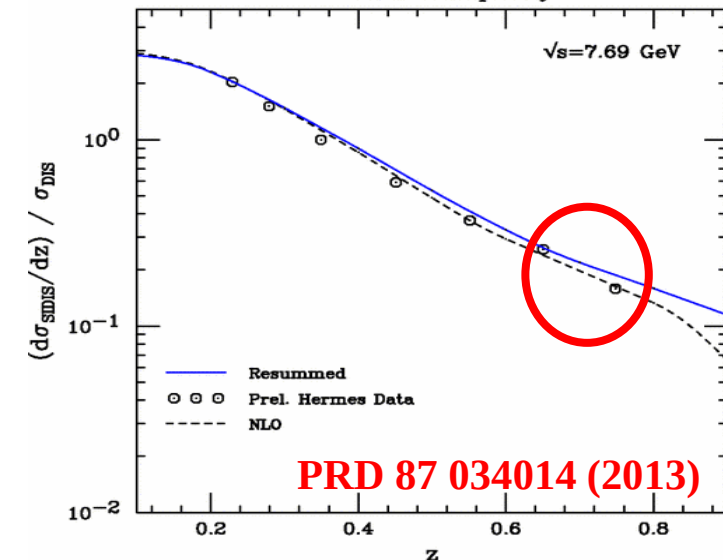


BESIII R scan data $\Rightarrow$ Precision measurement ?

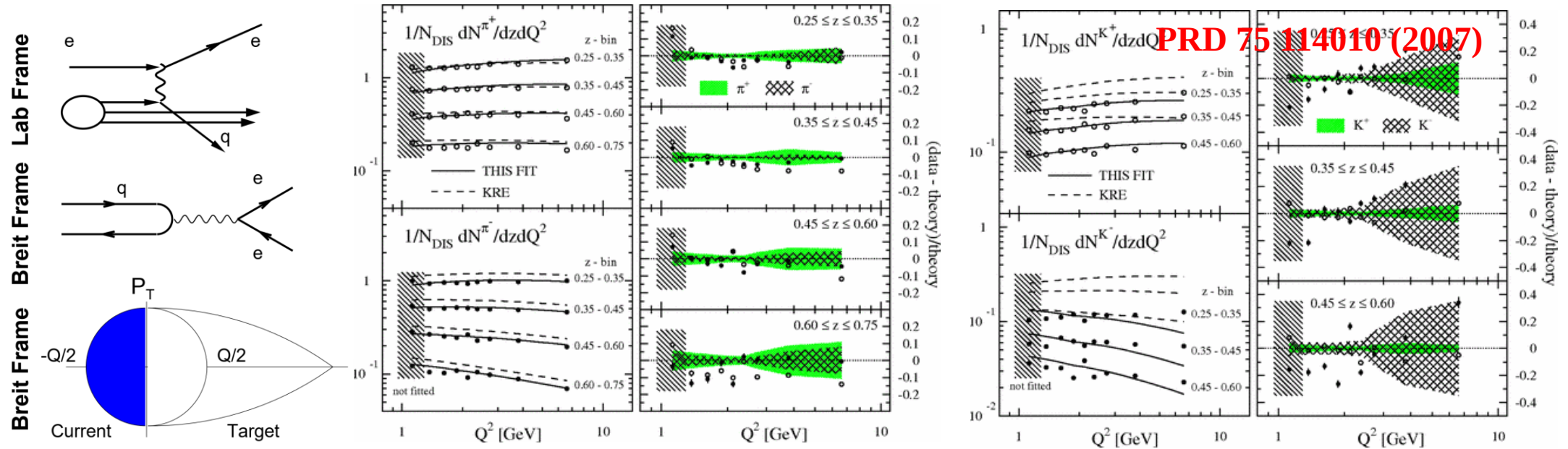
Pion FF: Best known FF



- For $z \geq 0.8$, uncertainty rapidly increase because of the lack of experimental data
- Xfitter: data at $\sqrt{s} > 10 \text{ GeV}$ e^+e^-
 - ✓ Low $\sqrt{s}$ e^+e^- data ?
- Large z re-summation
 - ✓ High z data ?

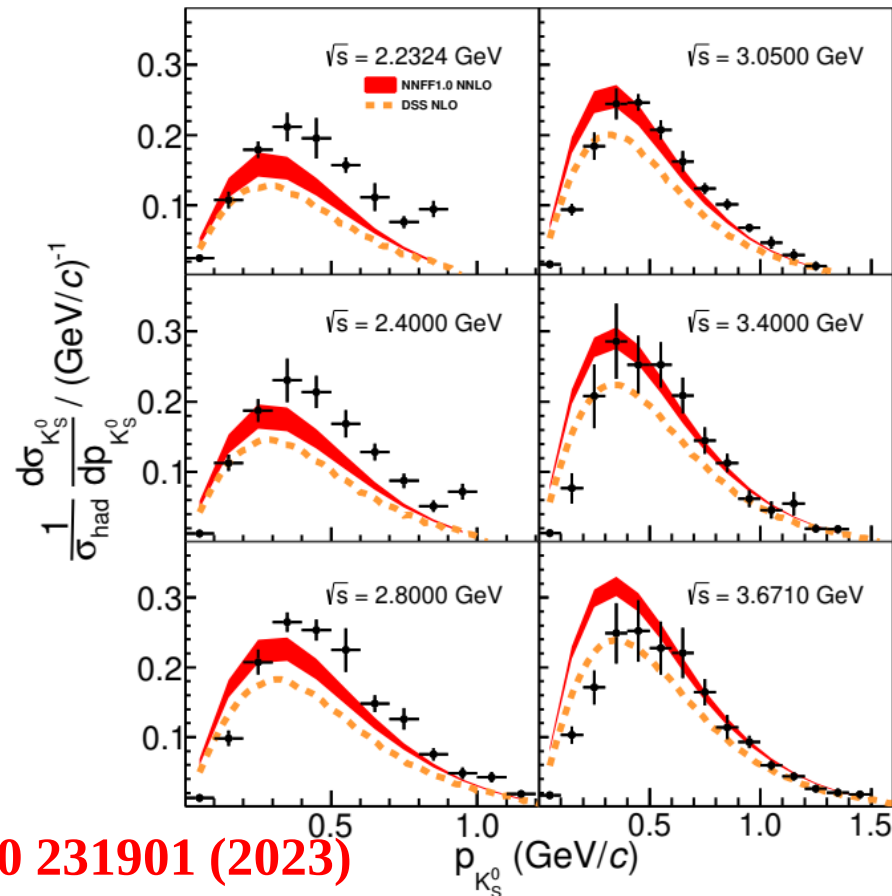
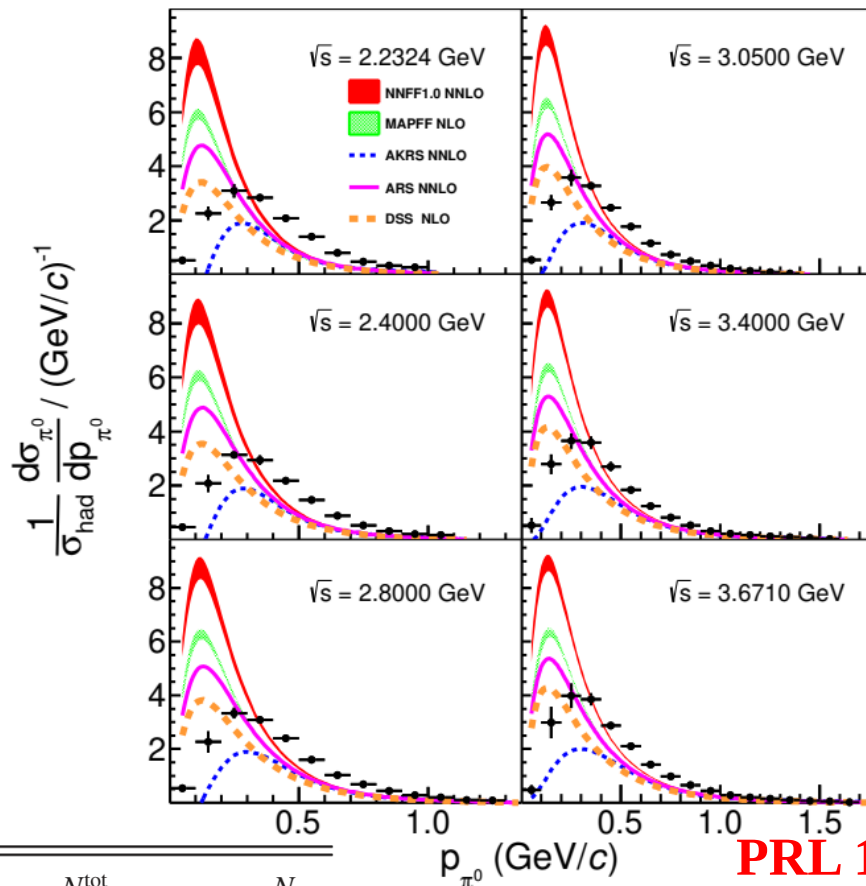


Pion FFs



- DIS @ Breit frame
 - ✓ Incoming quark scatters off photon and returns along same axis
- Current region of Breit frame is analogous to e^+e^-
 - ✓ Born level: DIS $Q = e^+e^- \sqrt{s}$
- DSS FFs: HERMES ep pion data at **10% level**
- e^+e^- data at low energy $\sqrt{s}$
 - ✓ FFs packages could describe e^+e^- data at **?% level**

$e^+e^- \rightarrow \pi^0/K_S^0 + X @ \text{BESIII}$

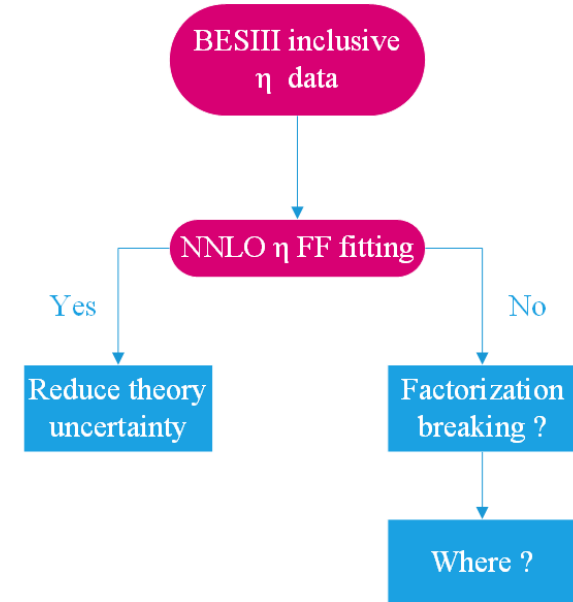
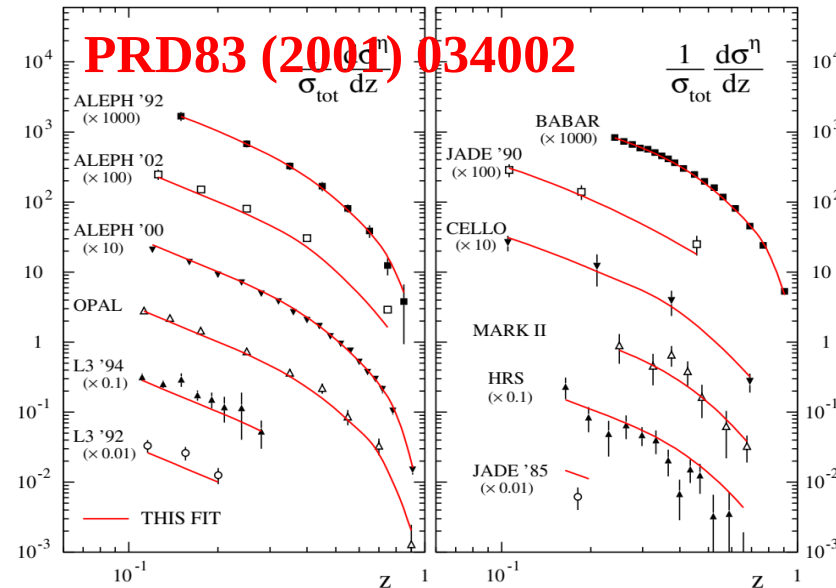
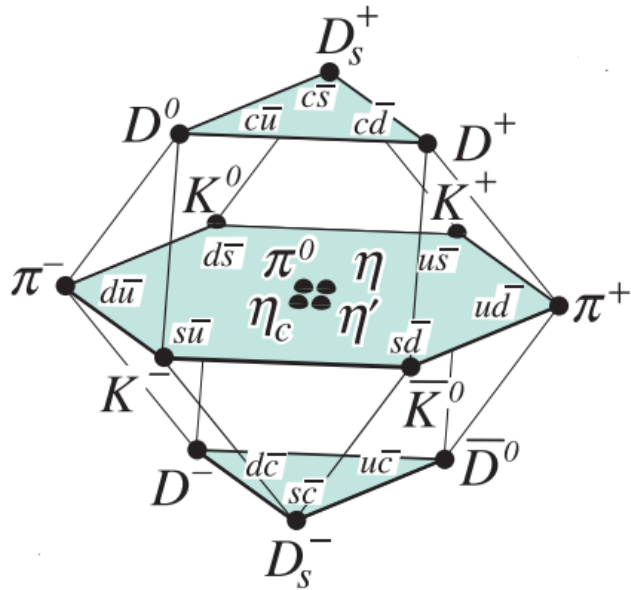


PRL 130 231901 (2023)

$\sqrt{s}$ (GeV)	$\mathcal{L}$ (pb $^{-1}$)	$N_{\text{had}}^{\text{tot}}$	N_{bkg}
2.2324	2.645	83227	2041
2.4000	3.415	96627	2331
2.8000	3.753	83802	2075
3.0500	14.89	283822	7719
3.4000	1.733	32202	843
3.6710	4.628	75253	6461

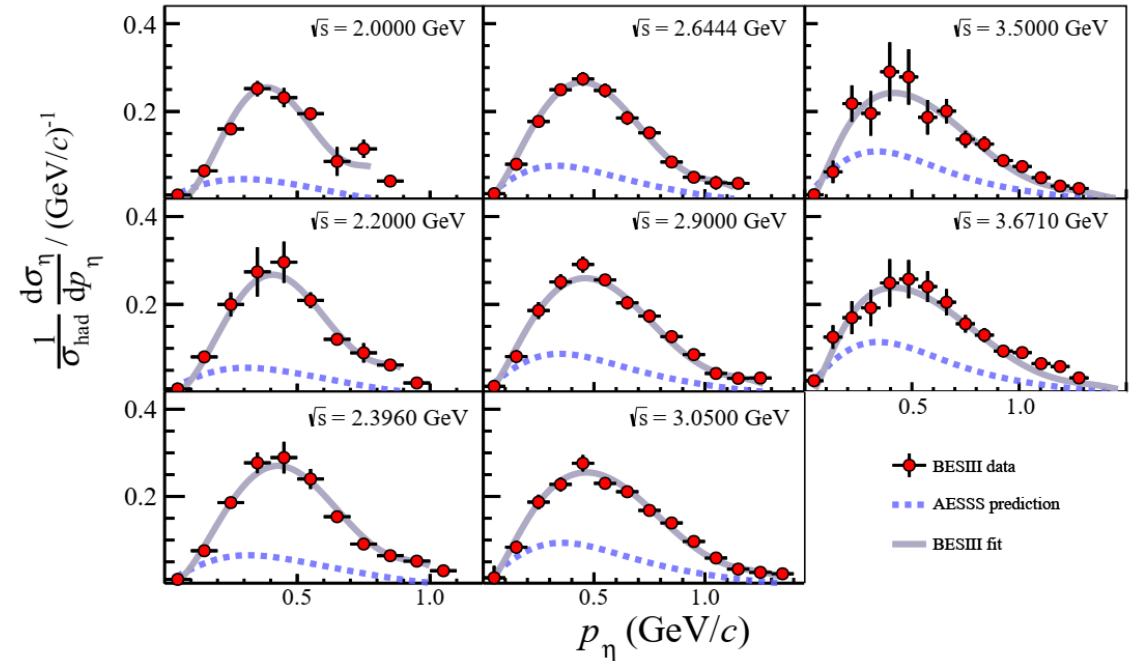
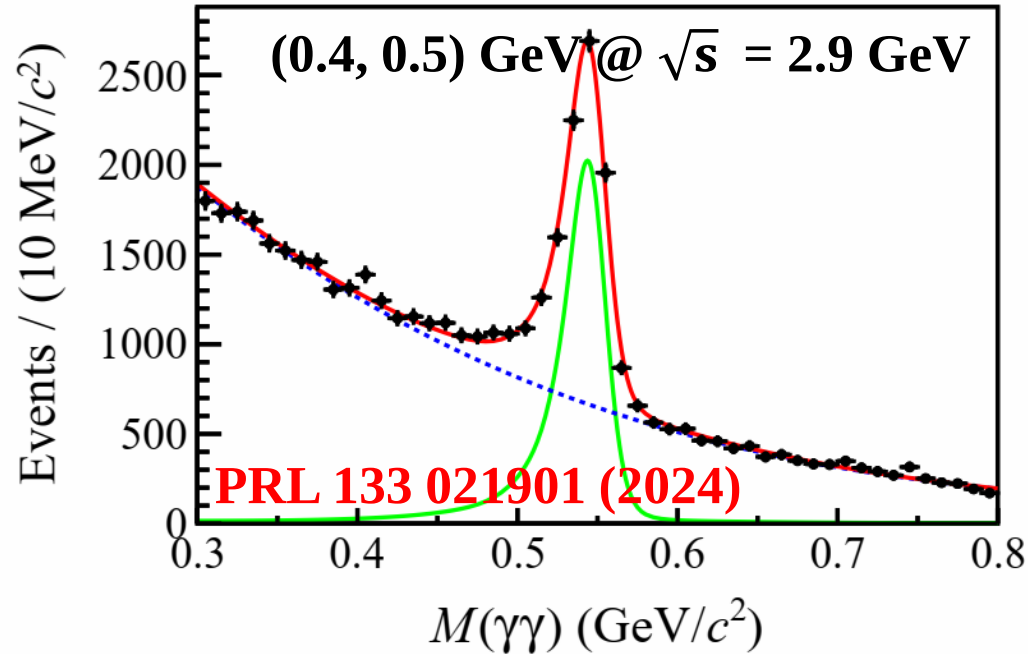
- Dominated uncertainty: MC generator
- inclusive π^0 production: **surprise**
- inclusive K_S^0 production: **not so bad**

η FF



- η : a Goldstone boson due to spontaneous breaking of QCD chiral symmetry
- η FF @ NLO: data at $\sqrt{s} > 10$ GeV e^+e^- collision
 - ✓ Missing theory uncertainty
- Theory improvement:
 - ✓ NNLO accuracy, hadron mass correction & higher twist
- BESIII results and its possible impact ?

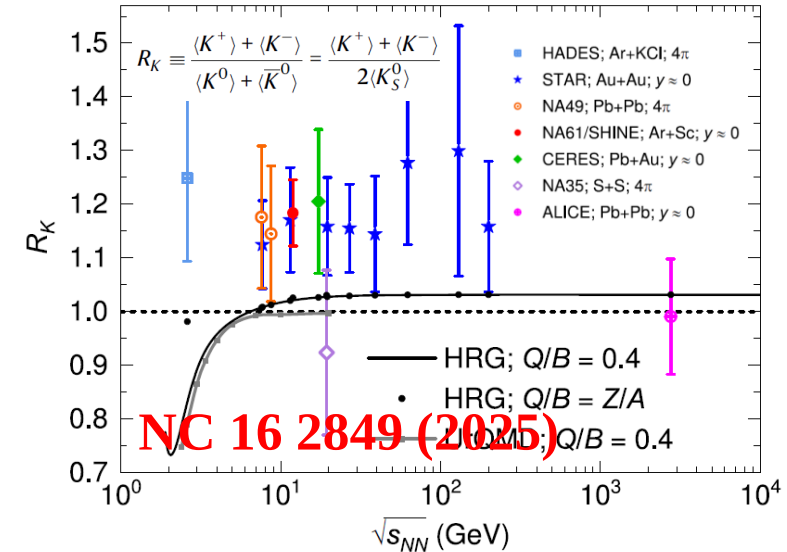
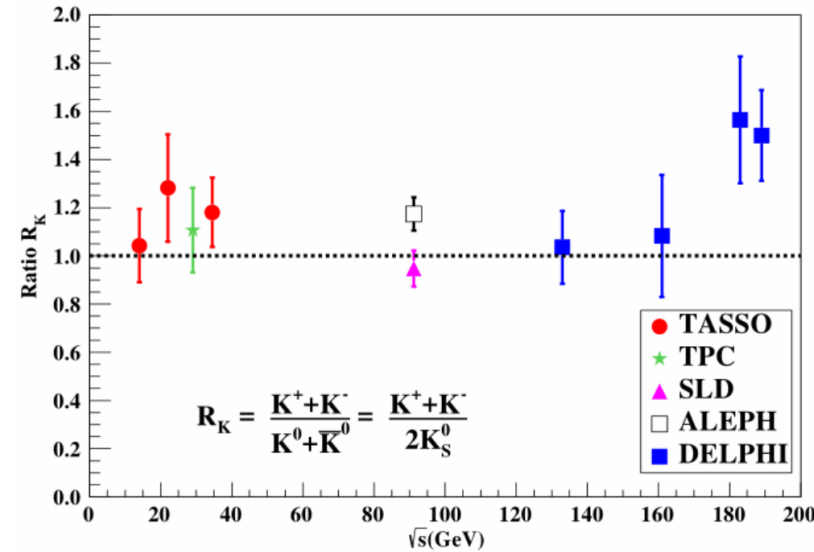
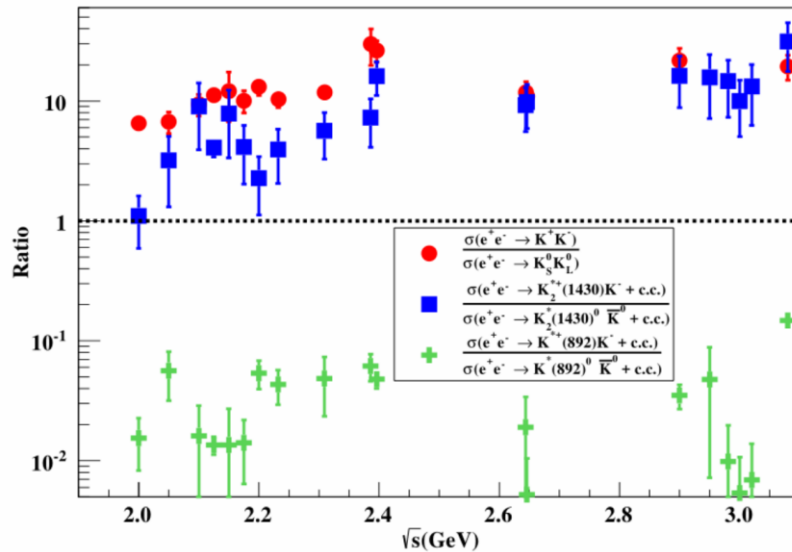
$e^+e^- \rightarrow \eta + X @ \text{BESIII}$



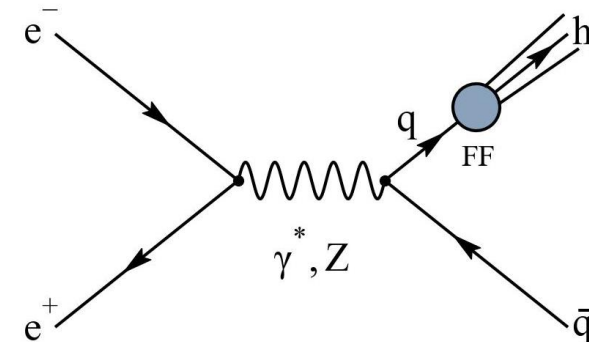
$\sqrt{s}$ (GeV)	$\mathcal{L}$ (pb $^{-1}$)	$N_{\text{had}}^{\text{tot}}$	N_{bkg}
2.0000	10.074	$350\,298 \pm 591$	8722 ± 93
2.2000	13.699	$445\,019 \pm 666$	$10\,737 \pm 103$
2.3960	66.869	$1\,869\,906 \pm 1365$	$47\,550 \pm 218$
2.6444	33.722	$817\,528 \pm 902$	$21\,042 \pm 145$
2.9000	105.253	$2\,197\,328 \pm 1478$	$56\,841 \pm 238$
3.0500	14.893	$283\,822 \pm 531$	7719 ± 87
3.5000	3.633	$62\,670 \pm 249$	1691 ± 41
3.6710	4.628	$75\,253 \pm 273$	6461 ± 80

- PRD83 (2001) 034002 prediction vs. BESIII data
✓ tension !
- BESIII fit: detail @ PRD 111 034030 (2025)
✓ $\sqrt{s} > 10$ GeV e^+e^- data + **BESIII data**
✓ NNLO accuracy, hadron mass correction & higher twist

Isospin symmetry



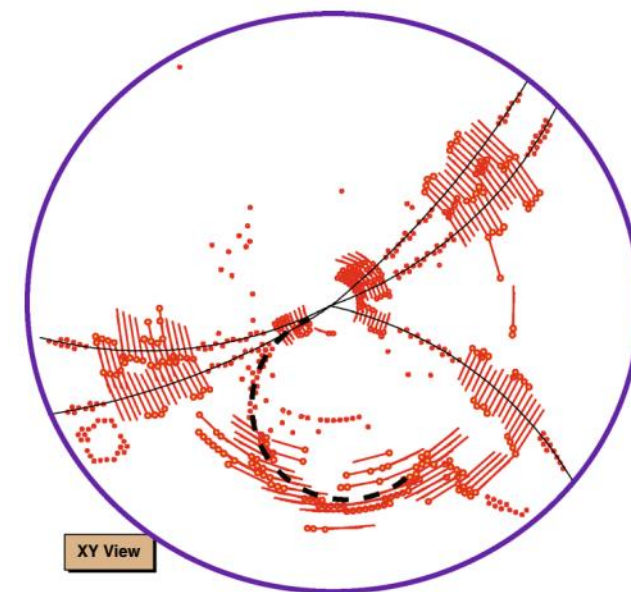
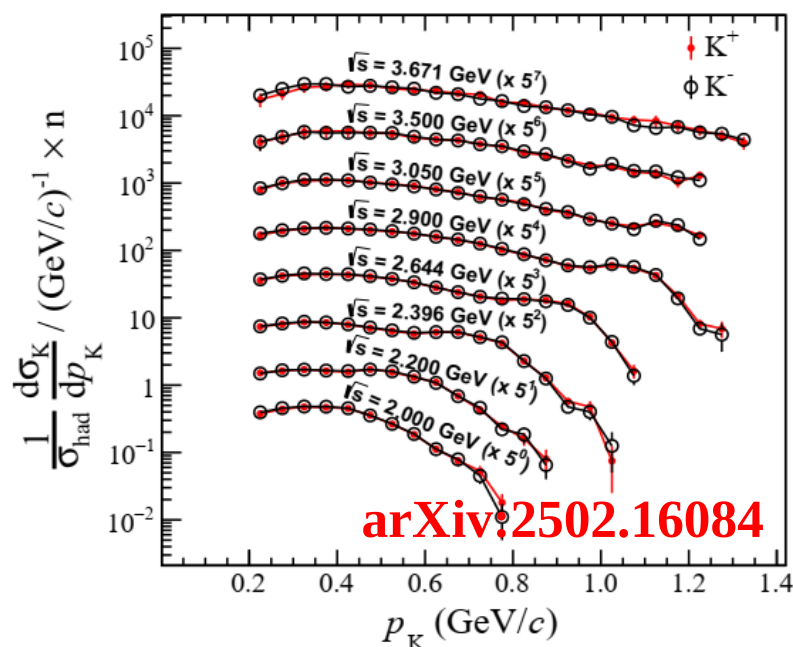
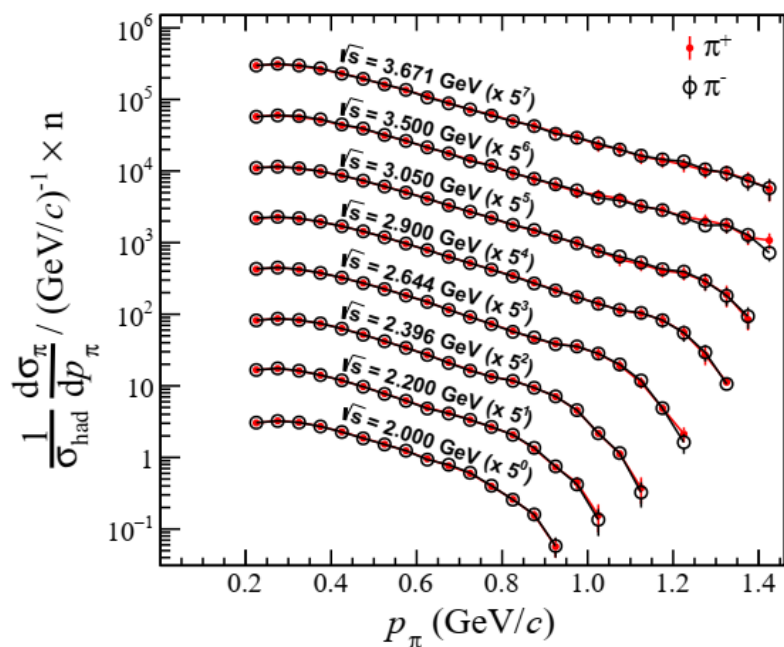
- $e^+e^- \rightarrow K^*K$: charged vs. neutral channel
 - ✓ Isospin symmetry violation
- Inclusive kaon production @ e^+e^-
 - ✓ Consistent with one ? precision !!!
- Inclusive kaon production @ AA
 - ✓ Large isospin symmetry breaking
- SU(2) isospin symmetry @ FFs



$$D_i^{\pi^0}(z, Q) = \frac{1}{2} D_j^{\pi^\pm}(z, Q) \quad \text{@NPB 803 42 (2008)}$$

$$D_i^{K_S^0}(z, Q) = \frac{1}{2} D_j^{K^\pm}(z, Q) \quad \begin{cases} \text{if } i = d(u), j = u(d) \\ \text{otherwise } i = j \end{cases}$$

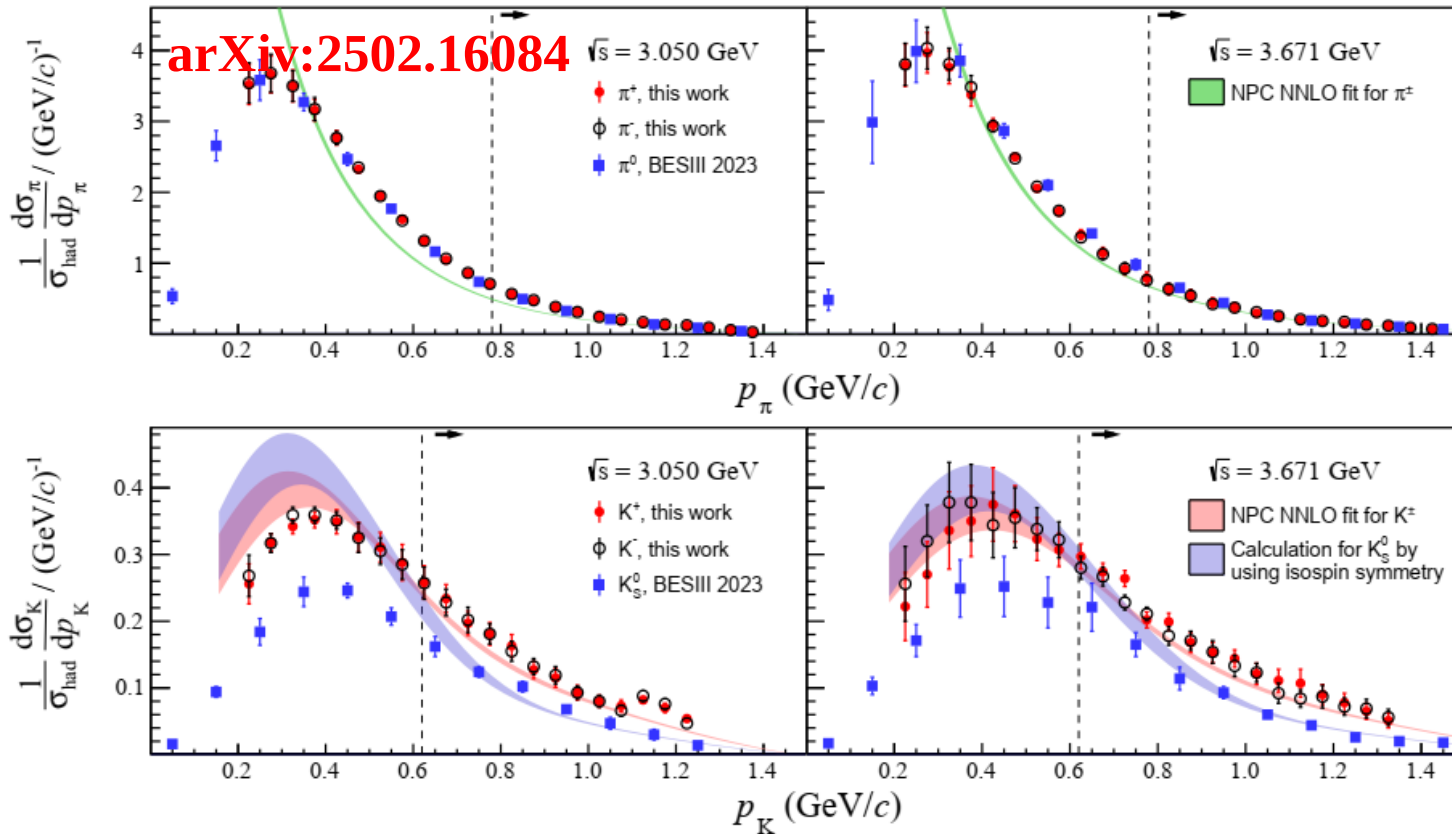
$e^+e^- \rightarrow \pi^\pm/K^\pm + X$ @ BESIII



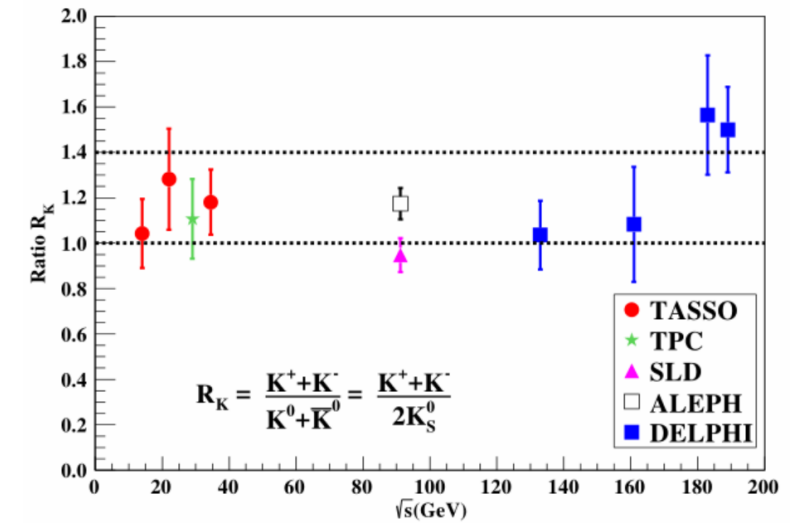
$\sqrt{s}$ (GeV)	$\mathcal{L}$ (pb $^{-1}$)	$N_{\text{had}}^{\text{tot}}$	N_{bkg}
2.0000	10.074	350298 ± 592	8722 ± 94
2.2000	13.699	445019 ± 668	10737 ± 104
2.3960	66.869	1869906 ± 1368	47550 ± 219
2.6444	33.722	817528 ± 905	21042 ± 146
2.9000	105.253	2197328 ± 1483	56841 ± 239
3.0500	14.893	283822 ± 533	7719 ± 88
3.5000	3.633	62670 ± 251	1691 ± 42
3.6710	4.628	75253 ± 275	6461 ± 81

- $z = 2p/\sqrt{s}$ coverage
 - ✓ pion: 0.13 – 0.95
 - ✓ kaon: 0.30 – 0.90
- Missing bins with momentum < 0.2 GeV
 - ✓ Low p_t : curling track, not reconstructed
- Dominated uncertainty: MC generator

$e^+e^- \rightarrow \pi^\pm/K^\pm + X$ @ BESIII



- Inclusive pion cross section
 - ✓ $\pi^+ \approx \pi^- \approx \pi^0$
 - ✓ Isospin symmetry
- Inclusive kaon cross section
 - ✓ $K^+ \approx K^- \approx 1.4 K_S$
 - ✓ **Large isospin violation ?**



- NPC NNLO: $\sqrt{s} > 3.0$ GeV & $E_h > 0.8$ GeV @ BESIII
- arXiv:2502.17837: Validity of QCD factorization and pQCD calculation at energy scales down to 3 GeV
- Test of isospin symmetry with FF fitting

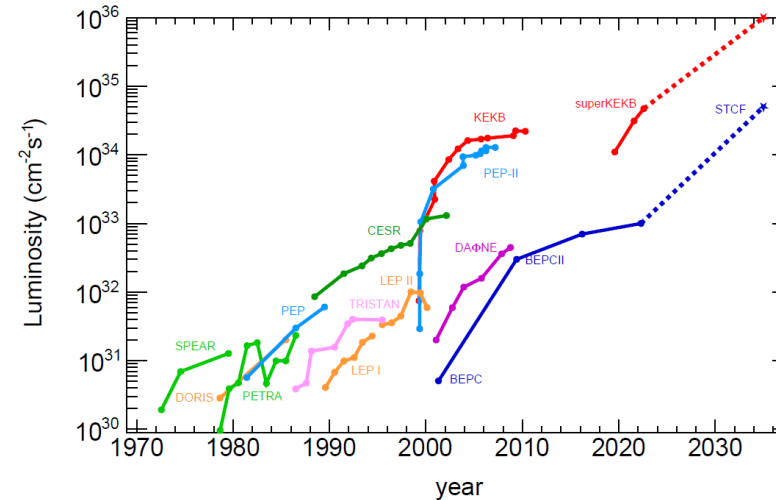
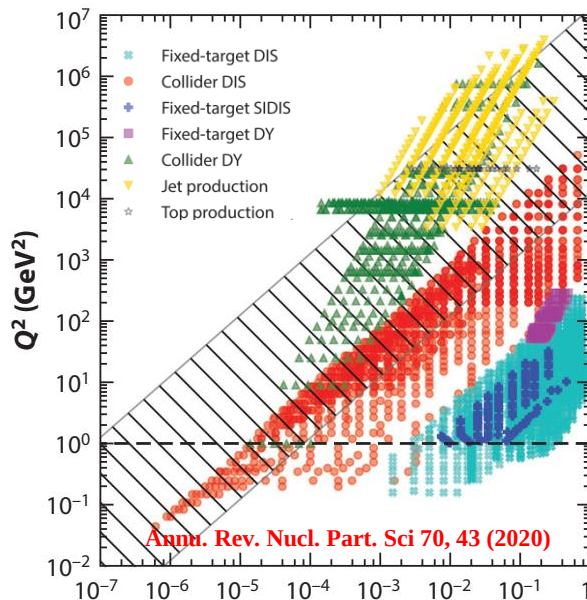
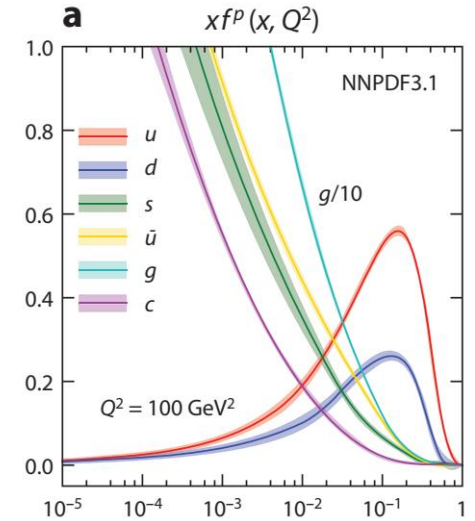
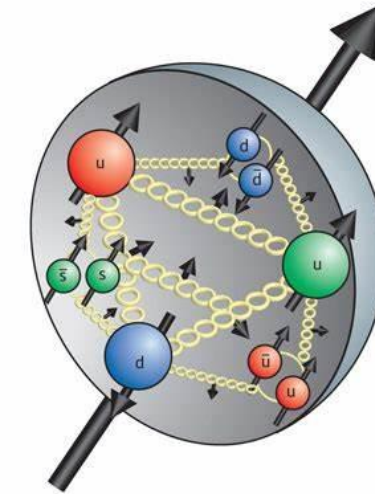
Summary

- The FF play a crucial role in describing hadronization process, and understanding of non-perturbative QCD dynamics.
- Inclusive π^0 & K_s : Large discrepancy with theory calculation. **PRL 130 231901 (2023)**
- Inclusive η : FF fitting with NNLO accuracy, hadron mass & higher twist correction, could describe BESIII data. **PRL 133 021901 (2024)**
- Inclusive π & K : Validity of QCD factorization and pQCD calculation at energy scales down to 3 GeV, test of isospin symmetry with FF fitting. **arXiv:2502.16084**
- Inclusive hadron production @ low energy e^+e^- annihilation: **use or useless ???**



Global PDF fits vs. Global FFs Fits

- Parton distribution functions(PDF)
 - ✓ Probability of parton inside a hadron
 - ✓ A key ingredient in hadron collider physics
- Aim: **high precision theory** + **wide range of data**
= **precise & accurate PDFs & FFs**
- Study of FFs < Study of PDFs
 - ✓ **Data set at $\sqrt{s} < 10 \text{ GeV } e^+e^-$???**



Experiment	Process	$\mathcal{L}[\text{pb}^{-1}]$	$Q^2[\text{GeV}^2]$	Final states
TPC [288–291]	e^+e^-	140	29	$\pi^\pm, K^\pm, p/\bar{p}$
TASSO [292–294]	e^+e^-	34	10	$\pi^\pm, K^\pm, p/\bar{p}, K_S^0, \Lambda/\bar{\Lambda}$
SLD [295–298]	e^+e^-	20	M_Z	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
ALEPH [301,302]	e^+e^-	800	M_Z	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
DELPHI [303–306]	e^+e^-	800	M_Z	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
OPAL [307–310]	e^+e^-	800	M_Z	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
H1 [311–313]	$e + p$	500	27.5(e) + 920(p)	$h^\pm, K_S^0$
ZEUS [314–316]	$e + p$	500	27.5(e) + 920(p)	$h^\pm$
BELLE [317,318]	e^+e^-	10^6	Near 10.58	$\pi^\pm, K^\pm, p/\bar{p}$
BaBar [319,320]	e^+e^-	$557 \cdot 10^3$	Near 10.58	$\pi^\pm, K^\pm, p, K_S^0, \Lambda/\bar{\Lambda}$
HERMES [321,322]	$e + p(d)$	272 (p) 329 (d)	27.6 fixed target	$\pi^\pm, K^\pm, \eta, p/\bar{p}$
COMPASS [323]	$\mu + p(d)$	775	160 GeV fixed target	$\pi^\pm, K^\pm$
PHENIX [324–326]	pp	16×10^{-3}	62.4	$h^\pm$
[327–329]	pp	2.5	200	$\pi^\pm, \eta$
STAR [330–332]	pp	128	510	$\pi^\pm, \eta$
[333–335]	pp	8	200	$\pi^\pm, \eta, p/\bar{p}, K_S^0, \Lambda/\bar{\Lambda}$
ALICE [336]	pp	6×10^{-3}	7×10^3	$\pi^\pm, \eta$
TOPAZ [337]	e^+e^-	278	52–61.4	$\pi^\pm, K^\pm, K_S^0$
CDF [338,339]	$p + \bar{p}$	n/a	630 (1800)	$h^\pm, K_S^0, \Lambda^0$

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