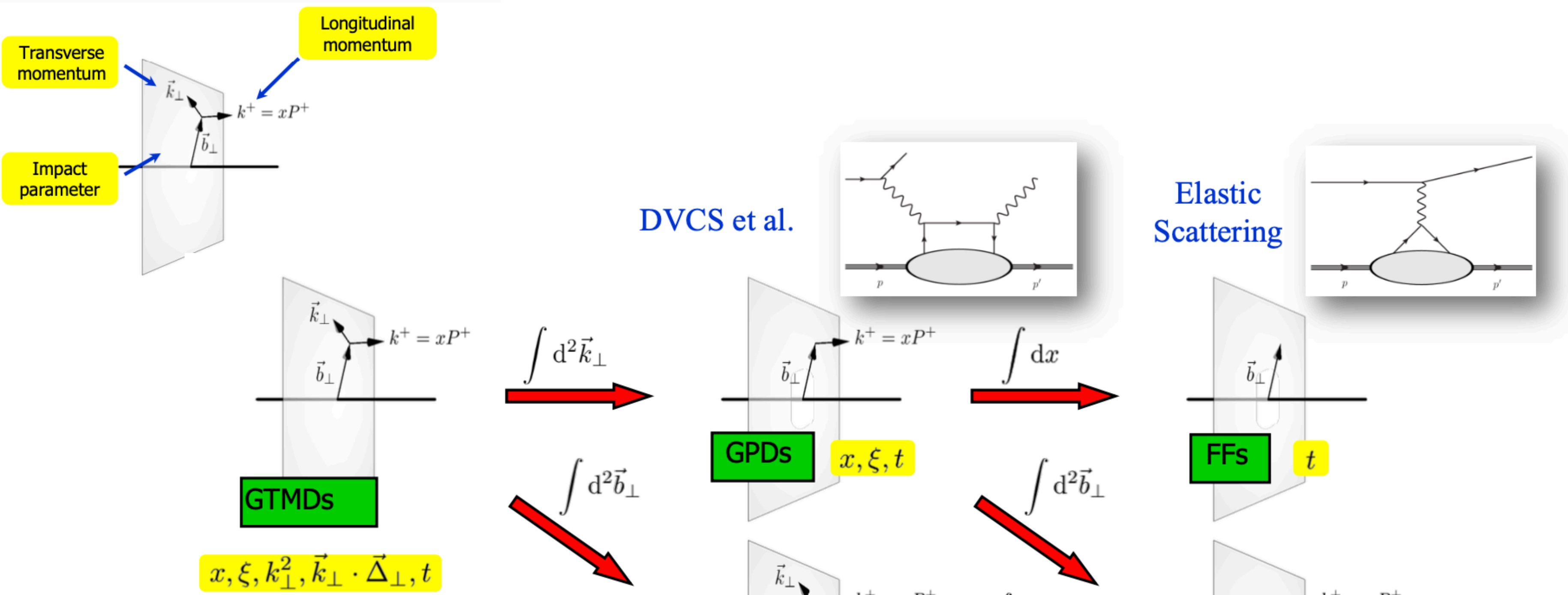
Progress on Hadron Structure from Lattice QCD

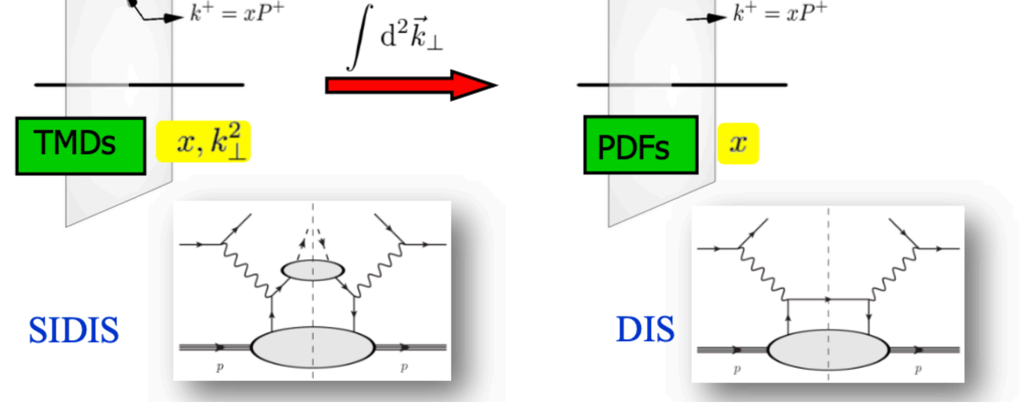
Jianhui Zhang

香港中文大學（深圳）
The Chinese University of Hong Kong, Shenzhen

第七届全国重味物理与量子色动力学研讨会，2025.04.20，南京

Towards hadron tomography

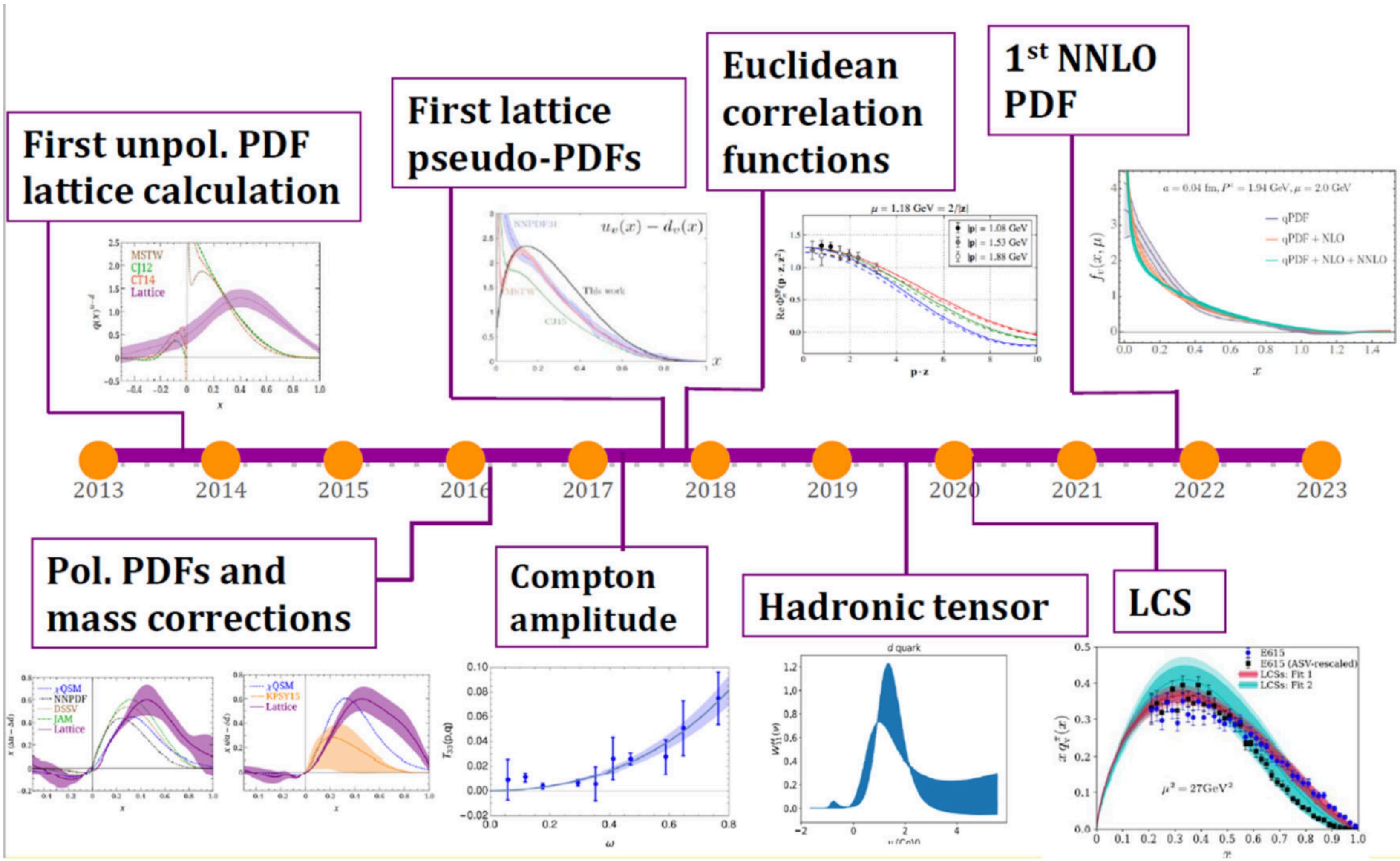




- Many phenomenological analyses have been done through global fits to various experimental data

Towards hadron tomography

- Lattice QCD can provide important complementary information
- — **x-dependent partonic structure**

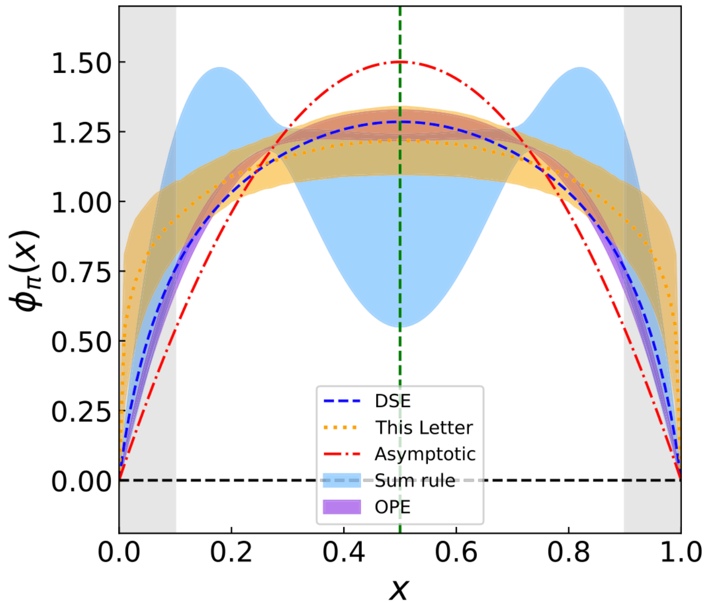


H-W Lin, FBS 23’

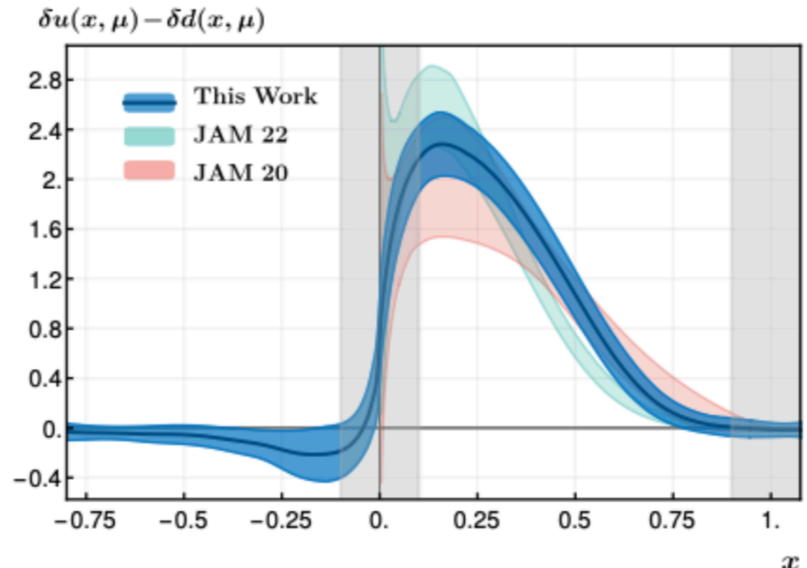
Towards hadron tomography

- Lattice QCD can provide important complementary information
- — **x-dependent partonic structure**

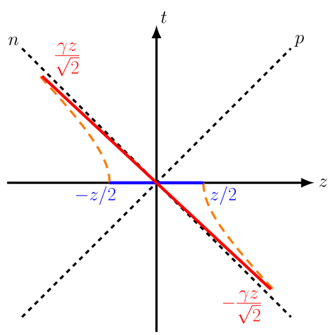
Example: 1D distributions



Hua, JHZ et al, PRL 22'



Yao, JHZ et al, PRL 23'



Large-momentum effective theory (LaMET)

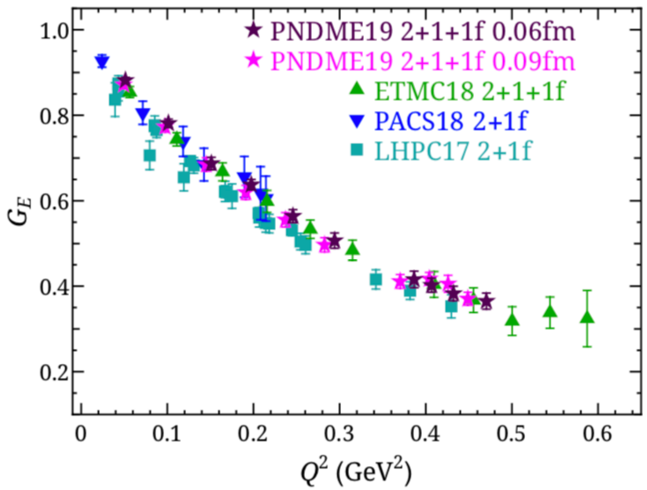
Ji, PRL 13' & SCPMA 14', Ji, JHZ et al, RMP 21'

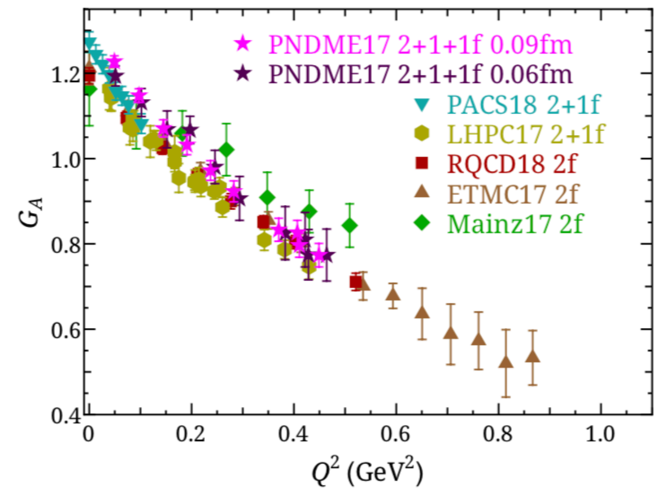
$$\tilde{q}(y, P^z) = C\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) \otimes q(x, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{(yP^z)^2}, \frac{\Lambda^2_{QCD}}{((1-y)P^z)^2}\right)$$

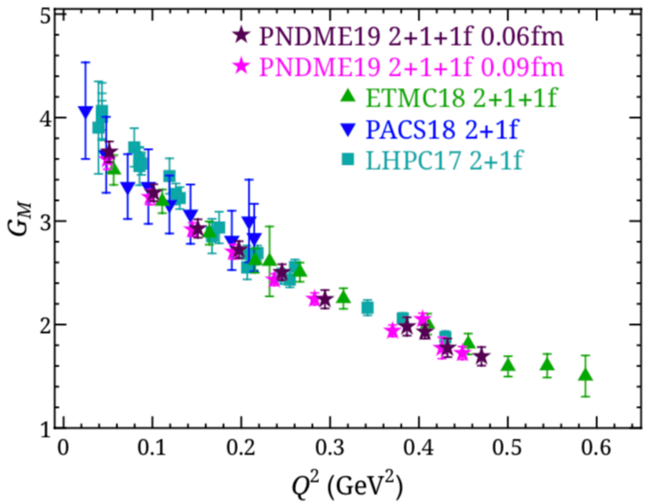
Towards hadron tomography

- Lattice QCD can provide important complementary information
- — **moments (traditional approach)**

Example: Form factors







$$\langle N(p_f)|V_\mu^+(x)|N(p_i)\rangle =$$
$$\bar{u}^N\left[\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{2M_N}F_2(q^2)\right]u_N e^{iq\cdot x}$$

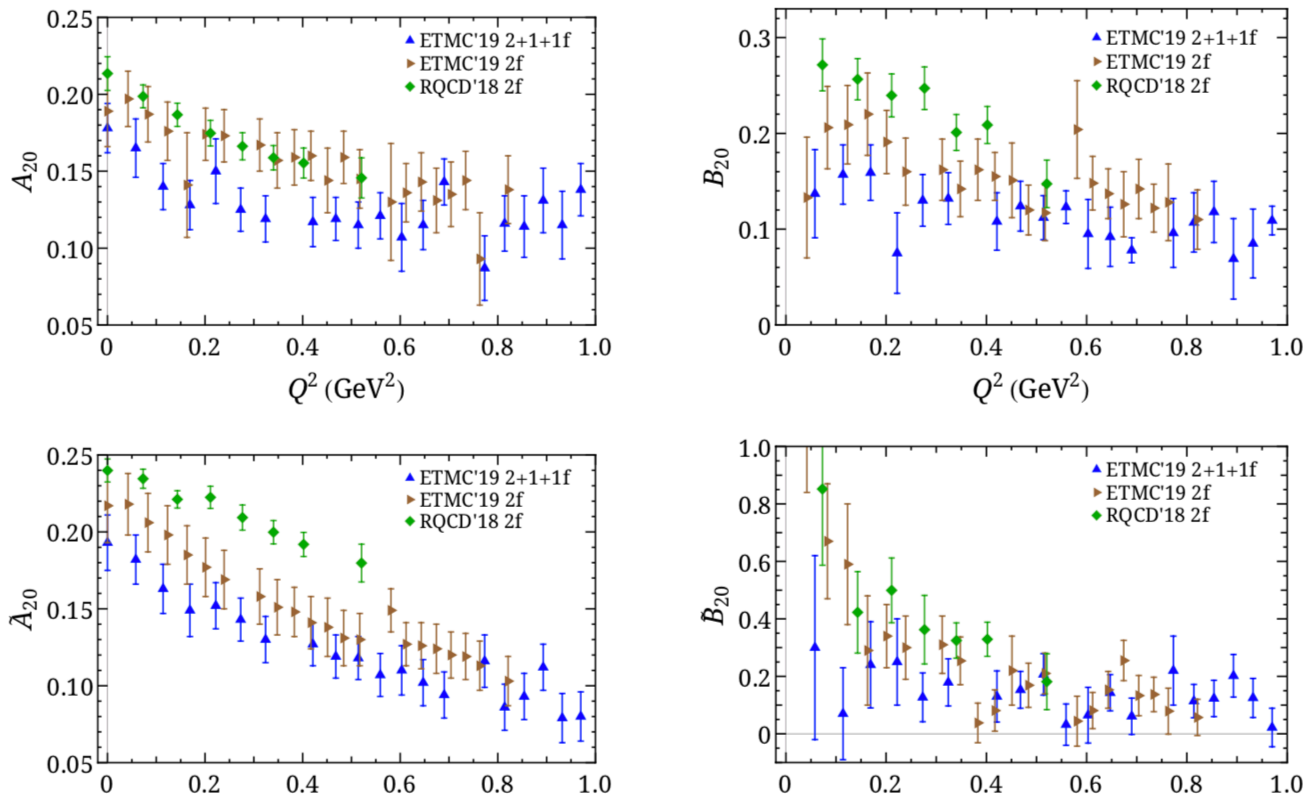
$$\langle N(p_f)|A_\mu^+(x)|N(p_i)\rangle =$$
$$\bar{u}_N\left[\gamma_\mu\gamma_5 G_A(q^2) + iq_\mu\gamma_5 G_P(q^2)\right]u_N e^{iq\cdot x}$$

Constantinou, JHZ et al, PPNP 21'

Towards hadron tomography

- Lattice QCD can provide important complementary information
- — **moments (traditional approach)**

Example: Form factors



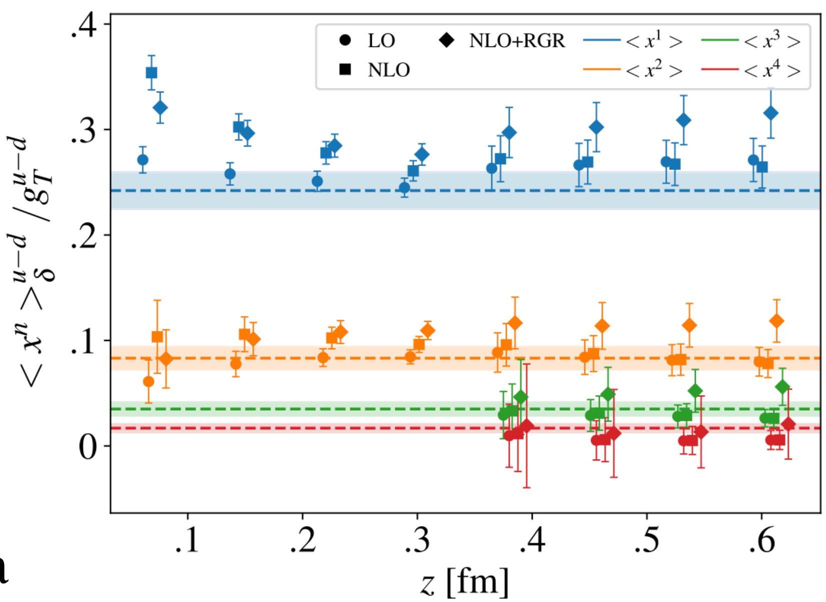
$$\langle N(p',s')|\mathcal{O}_V^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\frac{1}{2}\Big[A_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\,\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m_N} + C_{20}(q^2)\,\frac{1}{m_N}q^{\{\mu}q^{\nu\}}\Big]u_N(p,s),$$

$$\langle N(p',s')|\mathcal{O}_A^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\frac{i}{2}\Big[\tilde{A}_{20}(q^2)\,\gamma^{\{\mu}P^{\nu\}}\gamma^5 + \tilde{B}_{20}(q^2)\,\frac{q^{\{\mu}P^{\nu\}}}{2m_N}\gamma^5\Big]u_N(p,s),$$

Towards hadron tomography

- Lattice QCD can provide important complementary information
- — **moments (from short-distance factorization)**

Example: Nucleon quark transversity PDF



- Fits of a truncated OPE to lattice data up to large distances

Gao et al, PRD 24'

Short-distance factorization/Pseudo-distribution

Radyushkin, PRD 17'

$$\tilde{h}(\lambda = zP^z, z^2) = C\left(\alpha, z^2\mu^2\right) \otimes h(\alpha\lambda, \mu) + \mathcal{O}\left(z^2\Lambda_{QCD}^2\right)$$

Towards precision calculations of 1D structure

- Theory developments:
  - Unified framework for factorization of all leading-twist parton distributions **Yao, Ji, JHZ, JHEP 23'**

Coordinate space

$$\begin{pmatrix} O_q \\ O_g \end{pmatrix} = \begin{pmatrix} C_{qq} & C_{qg} \\ C_{gq} & C_{gg} \end{pmatrix} \otimes \begin{pmatrix} O_q^{l.t.} \\ O_g^{l.t.} \end{pmatrix} + h.t.$$

$$O_q(z_1, z_2) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta \Big[C_{qq}(\alpha, \beta, \mu^2 z_{12}^2) O_q^{l.t.}(z_{12}^{\alpha}, z_{21}^{\beta}) + C_{qg}(\alpha, \beta, \mu^2 z_{12}^2) O_g^{l.t.}(z_{12}^{\alpha}, z_{21}^{\beta}) + \widetilde{C}_{qq}(\alpha, \beta, \mu^2 z_{12}^2) O_q^{l.t.}(z_{21}^{\alpha}, z_{12}^{\beta}) + \widetilde{C}_{qg}(\alpha, \beta, \mu^2 z_{12}^2) O_g^{l.t.}(z_{21}^{\alpha}, z_{12}^{\beta}) \Big]$$

  - Applies to collinear PDFs, DAs, GPDs in appropriate kinematic limits
  - Coefficient function in coordinate space is **universal**

Momentum space

$$\mathbb{H}\left(x, \xi, \frac{\mu}{P_z}\right) = \int_{-1}^{1} dy\, \mathbb{C}\left(x_1, x_2, y_1, y_2; \frac{\mu}{P_z}\right) H(y, \xi, \mu) \qquad \mathbb{C}\left(x_1, x_2, y_1, y_2; \frac{\mu}{P_z}\right) = \begin{pmatrix} \mathbb{C}_{qq} & \mathbb{C}_{qg} \\ \mathbb{C}_{gq} & \mathbb{C}_{gg} \end{pmatrix}$$

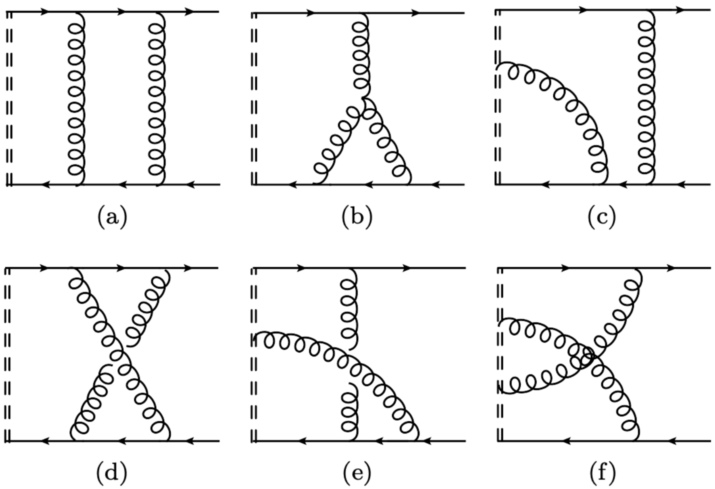
  - Results are available in a state-of-the-art scheme at NLO for both nonsinglet and singlet quark combinations and gluons

Towards precision calculations of 1D structure

- Theory developments:
  - NNLO results for light meson DAs **Ji, Yao, JHZ, arXiv: 2504.09367**
    - For unpol. quark PDF, see **Li et al, PRL 21', Chen et al, PRL 21', Cheng et al, 24'**

- Coordinate space integrals in general kinematics is highly non-trivial

- Non-planar diagrams yield new kinematic regions that do not show up at NLO



Coordinate space

$$\tilde{h}_R^{\rm ratio}(z_{12},\lambda) = \int_0^1 d\alpha d\beta\, C(\alpha,\beta,z_{12}^2\mu^2) h_R^{\overline{\rm MS}}(\alpha,\beta,\lambda,\mu) + h.t.,$$

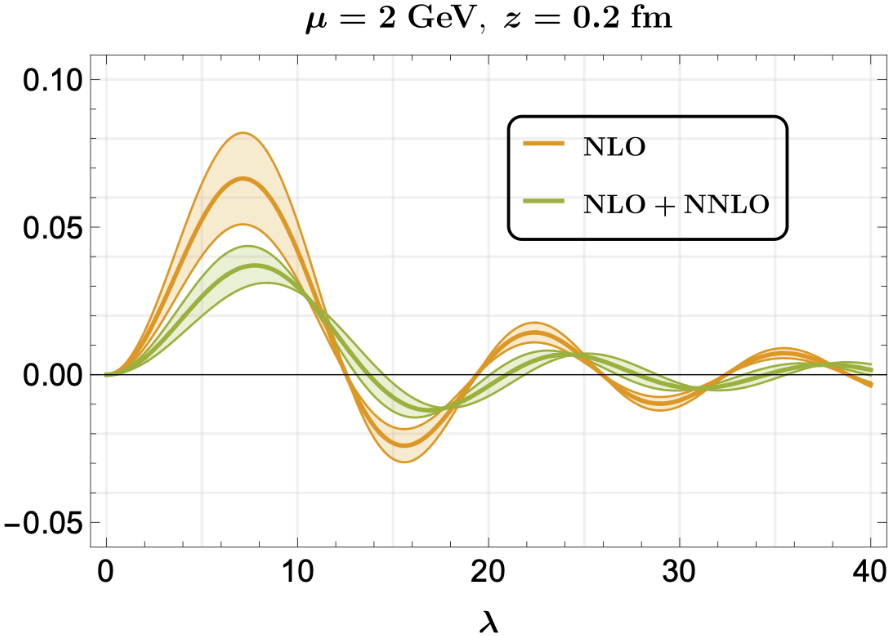
Momentum space

$$\mathbb{C}^{(2)}_{\rm ratio}(x,y,\mu/P_z) = \begin{cases} [h_1(x,y,\mu/P_z)]_+, & x<0<y<1 \\ [h_2(x,y,\mu/P_z)]_+, & 0<x<y<1 \\ [h_2(\bar{x},\bar{y},\mu/P_z)]_+, & 0<y<x<1 \\ [h_1(\bar{x},\bar{y},\mu/P_z)]_+, & 0<y<1<x \end{cases} + \begin{cases} [h_3(x,y,\mu/P_z)]_+, & x<0<\bar{y}<1 \\ [h_4(x,y,\mu/P_z)]_+, & 0<x<\bar{y}<1 \\ [h_4(\bar{x},\bar{y},\mu/P_z)]_+, & 0<\bar{y}<x<1 \\ [h_3(\bar{x},\bar{y},\mu/P_z)]_+, & 0<\bar{y}<1<x \end{cases}$$

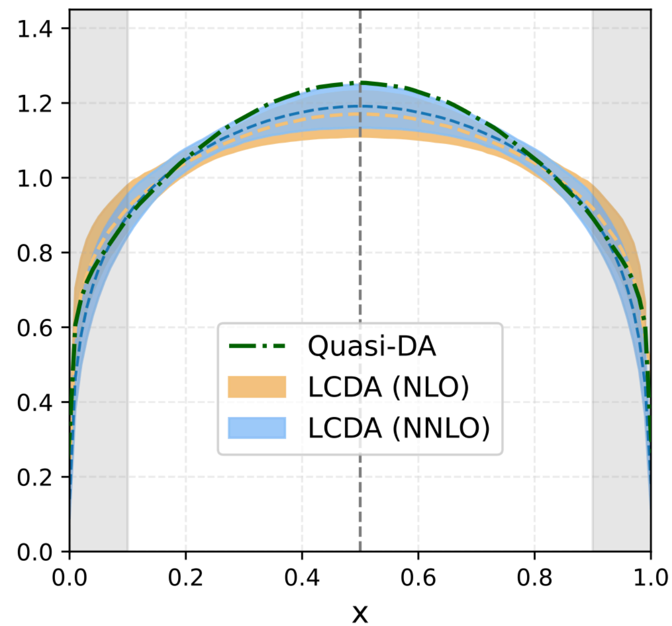
Towards precision calculations of 1D structure

- Theory developments:
  - NNLO results for light meson DAs Ji, Yao, JHZ, arXiv: 2504.09367

Impact of NNLO matching in pion DA



Based on FT of the asymptotic form $6x(1-x)$



Based on lattice data in Hua, JHZ et al, PRL 22'

NNLO corrections amount to ~10-30% of NLO correction

Complementarity: Moments

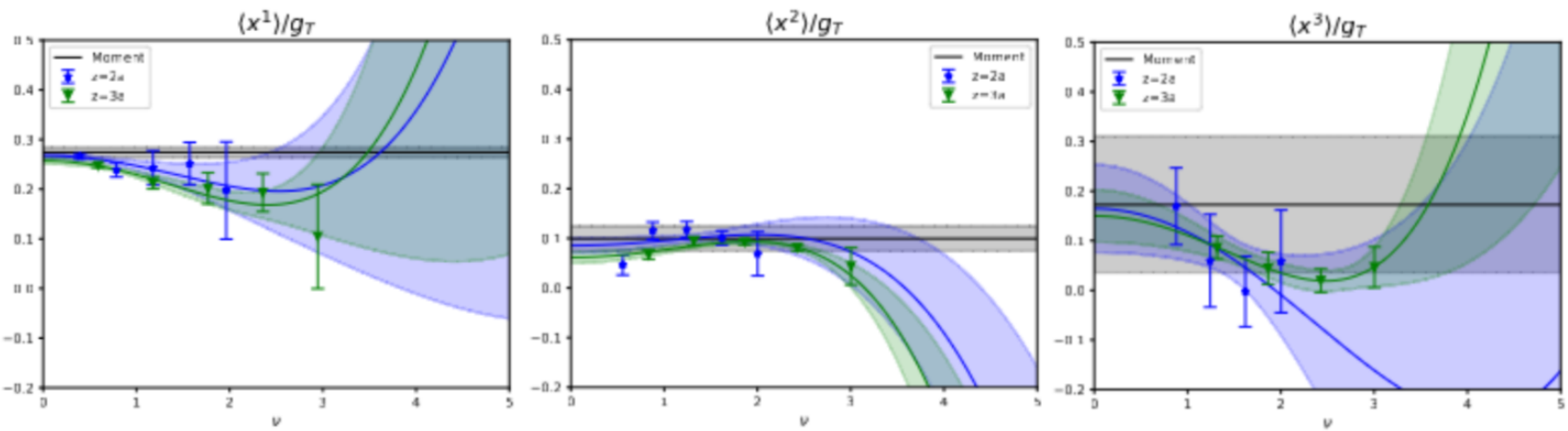
- Moments from momentum derivatives based on OPE

$$\tilde{h}_R(z^2,\lambda) = N\sum_{n=1}^{\infty}\frac{(-iz)^{n-1}}{(n-1)!}C^{(n-1)}(z^2\mu^2)$$
$$\times\langle P|n^t_{\mu_1}n_{\mu_2}...n_{\mu_n}O^{\mu_1\cdots\mu_n}(\mu)|P\rangle + p.c.$$
$$=\sum_{n=1}^{\infty}\frac{(-izP^z)^{n-1}}{(n-1)!}C^{(n-1)}(z^2\mu^2)\langle x^{n-1}\rangle + p.c.,$$

$$\langle x^{2k}\rangle = \frac{(2k)!}{k!(-z^2)^k}\frac{1}{C^{2k}(z^2\mu^2)}\frac{d}{d\zeta^k}Re\,\tilde{h}_R\Big|_{\zeta=0} + p.c.,$$
$$\langle x^{2k+1}\rangle = \frac{(2k+1)!}{k!(-z^2)^k}\frac{1}{C^{2k+1}(z^2\mu^2)}\frac{d}{d\zeta^k}\frac{Im\,\tilde{h}_R}{-i\lambda}\Big|_{\zeta=0} + p.c.,$$

Moments of nucleon quark transversity PDF

$\zeta = P_z^2$



Moments can be extracted order by order, to all orders in principle

Pang, JHZ, Zhao, 2412.19862

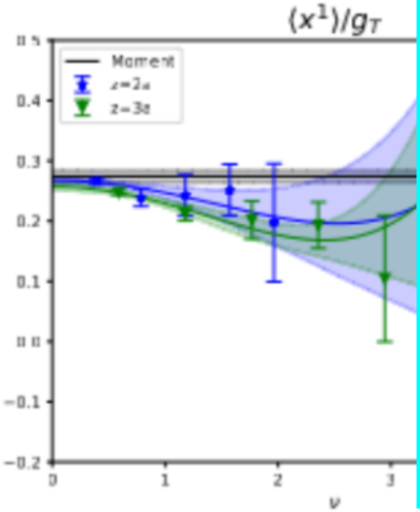
Complementarity: Moments

- Moments from momentum derivatives based on OPE

$$\tilde{h}_R(z^2,\lambda) = N\sum_{n=1}^{\infty}\frac{(-iz)^{n-1}}{(n-1)!}C^{(n-1)}(z^2\mu^2)$$
$$\times \langle P|n^t_{\mu_1}n_{\mu_2}...n_{\mu_n}O^{\mu_1\cdots\mu_n}(\mu)|P\rangle + p.c.$$
$$= \sum_{n=1}^{\infty}\frac{(-izP^z)^{n-1}}{(n-1)!}C^{(n-1)}(z^2\mu^2)\langle x^{n-1}\rangle + p.c.,$$

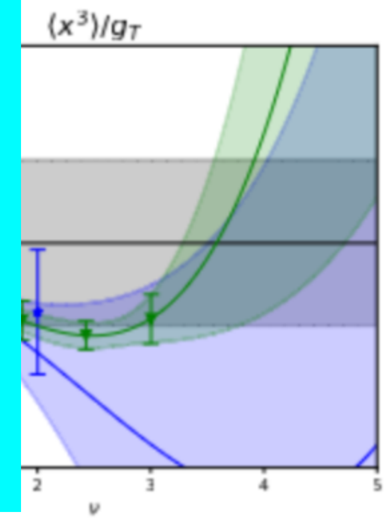
$$\langle x^{2k}\rangle = \frac{(2k)!}{k!(-z^2)^k}\frac{1}{C^{2k}(z^2\mu^2)}\frac{d}{d\zeta^k}Re\,\tilde{h}_R\Big|_{\zeta=0} + p.c.,$$
$$\langle x^{2k+1}\rangle = \frac{(2k+1)!}{k!(-z^2)^k}\frac{1}{C^{2k+1}(z^2\mu^2)}\frac{d}{d\zeta^k}\frac{Im\,\tilde{h}_R}{-i\lambda}\Big|_{\zeta=0} + p.c.,$$

$$\zeta = P_z^2$$



Advantages:
- No power divergent mixings
- No need of long distance correlations
- Differentiation w.r.t. momentum (square) rather than distance, computationally much cheaper
- Even/odd moments determined separately, easier to go to higher orders

Requirement for lattice setup:
- Large box size L, so that the step in momentum $\Delta P = 2\pi/L$ is small; no need of large momentum



Moments can be extracted order by order, to all orders in principle

Pang, JHZ, Zhao, 2412.19862

Summary

- Lattice QCD can provide complementary information to phenomenological analyses on hadron structure from 1D to 3D

- For simple 1D quantities such as collinear PDFs and DAs, lattice calculations have reached a stage where precision control becomes important
  - unpol. isovector quark PDF@NNLO complete + N^3LO (in coord. space)
  - Light meson DA@NNLO complete
  - GPDs@NNLO (coord. space complete, mom. space underway)

- Complementarity:
  - LaMET yields reliable predictions in the moderate x region
  - Moments from momentum derivatives of short-distance correlations

- A lot more to explore …