CPAs corresponding to the imaginary parts of the interference terms in cascade decays of heavy hadrons

张振华
南华大学
email: zhenhua_zhang@163.com
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In collaboration with 祁敬娟 杨健宇

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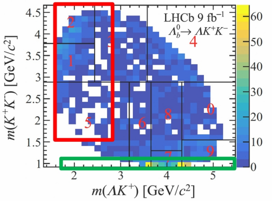
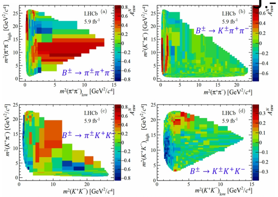
1. background and motivation

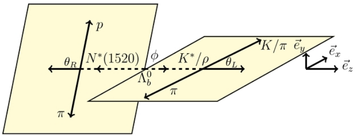
- CPV confirmed in decays of Λ_b is found by LHCb in four-body decays.
- While large regional CPA is found in some multi-body decays of bottom mesons, the overall CPA is smaller, hence cancellations occur.
- plenty CPV observables in multi-body decays
  T-odd observables, regional CPAs, partial-wave CP asymmetries,

From Yanxi Zhang's slides





J.-P. Wang, Q. Qin, F.-S. Yu, 2211.07331



CPV observables usually originates from the real parts if the interfering terms, and is proportional to $\sin\delta\sin\phi$

$$\mathcal{A} = A_1 + A_2$$

$$A_{CP} \equiv |\mathcal{A}|^2 - |\overline{\mathcal{A}}|^2 \sim \Re(A_1 A_2^*) - \Re(\overline{A_1 A_2}^*) \sim \sin\delta\sin\phi$$

How about constructing an observable originates from the imaginary parts of the interfering terms:

$$\Im(A_1 A_2^*) - \Im(\overline{A_1 A_2}^*) \sim \cos\delta\sin\phi$$

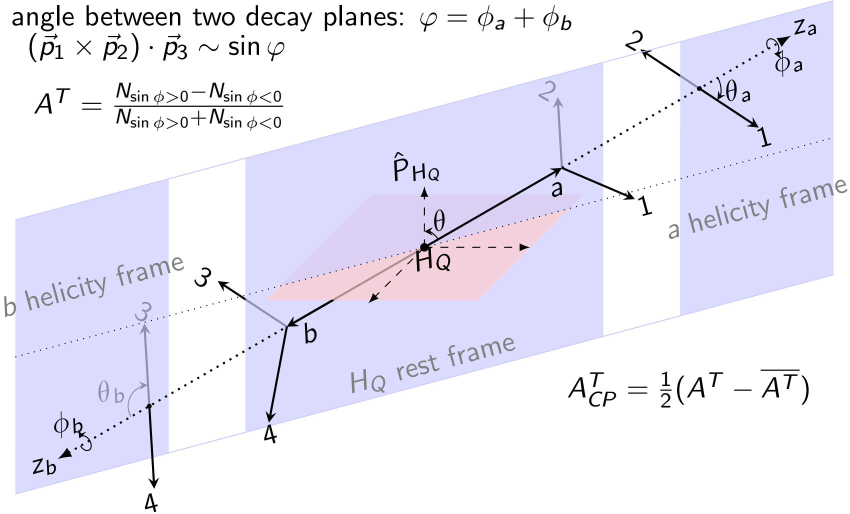
example: CPV induced by TPA in four-body decays

Triple-Product Asymmetry A^T

angle between two decay planes: $\varphi = \phi_a + \phi_b$

$(\vec{p}_1 \times \vec{p}_2) \cdot \vec{p}_3 \sim \sin\varphi$

$$A^T = \frac{N_{\sin\phi>0} - N_{\sin\phi<0}}{N_{\sin\phi>0} + N_{\sin\phi<0}}$$



$$A^T_{CP} = \frac{1}{2}(A^T - \overline{A^T})$$

TPA induced CP asymmetries

TP-CPA is proportional to the cosine of a strong phase δ:

$$\Im(A_1 A2^*) - \Im(\overline{A_1 A_2}^*) \sim \sin\phi_{\text{weak}} \cos\delta,$$

Why?

$$\mathcal{A} = \sum_m A_m e^{im\varphi}$$

$$\begin{aligned} |\mathcal{A}|^2 &\sim \Re(A_m A_{m'}^* e^{i(m-m')\varphi}) \\ &\sim \Re(A_m A_{m'}^*) \cos[(m-m')\varphi] \\ &\quad + \Im(A_m A_{m'}^*) \sin[(m-m')\varphi] \end{aligned}$$

If $m - m' \neq 0$ (suppose $m - m' = 1$), one can construct CPA observables such as TPA induced CPA.

TP-CPA searching

- $D^0 \to K^+K^-\pi^+\pi^-$, $D^+ \to K^+K^-\pi^+\pi^0$, $D^+_{(s)} \to K^+K^-\pi^+\pi^0$, $D^+_{(s)} \to K^+\pi^-\pi^+\pi^0$, $D^0 \to K^0_S K^0_S \pi^+\pi^-$, $\Lambda^0_b \to pK^-\pi^+\pi^-$, $\Lambda^0_b \to pK^-K^+K^-$, $\Xi^0_b \to pK^-K^-\pi^+$
- $\Lambda^0_b \to p\pi^-\pi^+\pi^-$: $a_P^{\hat{T}-\mathrm{odd}} = (-4.0 \pm 0.7 \pm 0.2)\%$.
- TPA induces CP asymmetry (TP-CPA) has never been observed yet.

Inspiration from TPA induced CPA

$$\mathcal{A} = \sum_m A_m e^{im\varphi}$$

$$|\mathcal{A}|^2 \sim \Re(A_m A^*_{m'} e^{i(m-m')\varphi}) \\ \sim \Re(A_m A^*_{m'}) \cos[(m-m')\varphi] \\ + \Im(A_m A^*_{m'}) \sin[(m-m')\varphi]$$

$$\mathcal{A} = \sum_k \mathcal{A}_k = \sum_k a_k b_k$$

$$|\mathcal{A}|^2 \sim \Re[(a_k b_k)(a^*_{k'} b^*_{k'})] \\ \sim \Re(a_k a^*_{k'})\Re(b_k b^*_{k'}) \\ + \Im(a_k a^*_{k'})\Im(b_k b^*_{k'})$$

non-zero $\Im(b_k b^*_{k'})$ provides opportunities for CPA corresponding to $\Im(a_k a^*_{k'})$.

2 interference of resonances in cascade decays

example 1: three-body decay with a resonance plus a smooth background term

decay amplitude:

$$\mathcal{M} = \frac{\mathcal{A}_r}{s_r} + \mathcal{B}, \qquad s_r = s - m_r^2 + \mathrm{i} m_r \Gamma_r$$

decay amplitude squared

$$\overline{|\mathcal{M}|^2} \approx \frac{|\mathcal{A}_r|^2}{|s_r|^2} + |\mathcal{B}_2|^2 + 2\Re\left(\frac{\mathcal{A}_1 \mathcal{B}_2^*}{s_r}\right),$$

The interfering term

$$\Re\left(\frac{\mathcal{A}_r\mathcal{B}^*}{s_r}\right) = \frac{\Re\left(\mathcal{A}_r\mathcal{B}^*\right)\left(s - m_r^2\right) + \Im\left(\mathcal{A}_r\mathcal{B}^*\right) m_r\Gamma_r}{\left|s_r\right|^2}.$$

a pair of complementary CPV observables

$$A_{CP} \equiv \frac{\int_{m_r{}^2-\Delta_-}^{m_r{}^2+\Delta_+}\left(\overline{|\mathcal{M}|^2} - \overline{|\overline{\mathcal{M}}|^2}\right) ds}{\int_{m_r{}^2-\Delta_-}^{m_r{}^2+\Delta_+}\left(\overline{|\mathcal{M}|^2} + \overline{|\overline{\mathcal{M}}|^2}\right) ds} \sim \sin\delta\sin\phi \qquad \text{mainly from } \Im\left(\mathcal{A}_r\mathcal{B}^*\right)$$

$$\tilde{A}_{CP} \equiv \frac{\int_{m_r{}^2-\Delta_-}^{m_r{}^2+\Delta_+}\left(\overline{|\mathcal{M}|^2} - \overline{|\overline{\mathcal{M}}|^2}\right) \operatorname{sgn}\left(s - m_2'{}^2\right) ds}{\int_{m_r{}^2-\Delta_-}^{m_r{}^2+\Delta_+}\left(\overline{|\mathcal{M}|^2} + \overline{|\overline{\mathcal{M}}|^2}\right) ds} \sim \cos\delta\sin\phi \quad \text{mainly } \Re\left(\mathcal{A}_r\mathcal{B}^*\right)$$

$$A_{CP}^2 + \tilde{A}_{CP}^2 \sim \sin^2\phi$$

example 2: three-body decay with two nearby resonance

three-body cascade decay $\mathbb{H} \to rc$, $r \to ab$

$$\mathcal{M} = \frac{\mathcal{A}_1}{s_1} + \frac{\mathcal{A}_2}{s_2}, \qquad s_r = s - m_r^2 + \mathrm{i} m_r \Gamma_r$$

decay amplitude squared

$$\overline{|\mathcal{M}|^2} \approx \frac{|\mathcal{A}_1|^2}{|s_1|^2} + \frac{|\mathcal{A}_2|^2}{|s_2|^2} + 2\Re\left(\frac{\mathcal{A}_1\mathcal{A}_2^*}{s_1 s_2^*}\right),$$

The interfering term

$$\Re\left(\frac{\mathcal{A}_1\mathcal{A}_2^*}{s_1 s_2^*}\right) = \frac{\Re\left(\mathcal{A}_1\mathcal{A}_2^*\right)\Re\left(s_1 s_2^*\right) + \Im\left(\mathcal{A}_1\mathcal{A}_2^*\right)\Im\left(s_1 s_2^*\right)}{|s_1 s_2|^2}.$$

The interfering term

$$\Re\left(\frac{\mathcal{A}_1\mathcal{A}_2^*}{s_1 s_2^*}\right) = \frac{\Re\left(\mathcal{A}_1\mathcal{A}_2^*\right)\Re\left(s_1 s_2^*\right) + \Im\left(\mathcal{A}_1\mathcal{A}_2^*\right)\Im\left(s_1 s_2^*\right)}{\left|s_1 s_2\right|^2}.$$

$$\Re\left(s_1 s_2^*\right) = m_1\Gamma_1 m_2\Gamma_2 + \left(s - m_1^2\right)\left(s - m_2^2\right)$$

$$\begin{aligned}\Im\left(s_1 s_2^*\right) &= \left(s - m_2^2\right) m_1\Gamma_1 - \left(s - m_1^2\right) m_2\Gamma_2 \\ &= m_1\Gamma_1\left(1 - \frac{m_2\Gamma_2}{m_1\Gamma_1}\right)\left(s - {m_2'}^2\right)\end{aligned}$$

where $m_2' = m_2\sqrt{(1 - \frac{m_1\Gamma_2}{m_2\Gamma_1})/(1 - \frac{m_2\Gamma_2}{m_1\Gamma_1})}$.
Now the difference between the behaviour of $\Re\left(s_1 s_2^*\right)$ and $\Im\left(s_1 s_2^*\right)$ is obvious: while the latter tends to change sign as s passes through ${m_2'}^2$, the former does not.

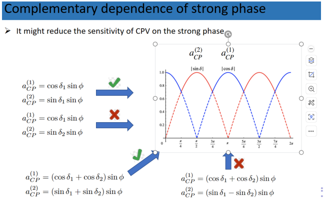
a pair of CPV observables

$$A_{CP} \equiv \frac{\int_{m_2'^2-\Delta_-}^{m_2'^2+\Delta_+}\left(\overline{|\mathcal{M}|^2}-\overline{|\overline{\mathcal{M}}|^2}\right)ds}{\int_{m_2'^2-\Delta_-}^{m_2'^2+\Delta_+}\left(\overline{|\mathcal{M}|^2}+\overline{|\overline{\mathcal{M}}|^2}\right)ds} \sim \sin\delta\sin\phi \qquad \text{mainly from } \Re\left(\mathcal{A}_1\mathcal{A}_2^*\right)$$

$$A_{CP}^{\Im} \equiv \frac{\int_{m_2'^2-\Delta_-}^{m_2'^2+\Delta_+}\left(\overline{|\mathcal{M}|^2}-\overline{|\overline{\mathcal{M}}|^2}\right)\ \mathrm{sgn}\left(s-m_2'^2\right)ds}{\int_{m_2'^2-\Delta_-}^{m_2'^2+\Delta_+}\left(\overline{|\mathcal{M}|^2}+\overline{|\overline{\mathcal{M}}|^2}\right)ds} \sim \cos\delta\sin\phi \quad \text{mainly } \Im\left(\mathcal{A}_1\mathcal{A}_2^*\right)$$

$$A_{CP}^2 + (A_{CP}^{\Im})^2 \sim \sin^2\phi$$

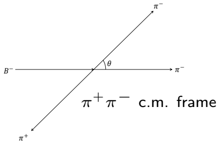
From Jian-Peng Wang's slides in 2023



3. Forward-Bacward Asymmetry induced CPA in $B^{\pm} \to \pi^{+}\pi^{-}\pi^{\pm}$

Forward-Bacward Asymmetry induced CPA in $B^{\pm} \to \pi^+\pi^-\pi^{\pm}$

Interfering of $\rho^0(770)$ with a S-wave results in FBA.



FBA induced CPA (FB-CPA):

$$A_{CP}^{FB} \equiv \frac{\int_{m_2'^2-\Delta_-}^{m_2'^2+\Delta_+} \int_{-1}^{+1} \left(\overline{|\mathcal{M}|^2} - \overline{|\overline{\mathcal{M}}|^2}\right) \, \mathrm{sgn}(c_\theta) \, dc_\theta \, ds}{\int_{m_2'^2-\Delta_-}^{m_2'^2+\Delta_+} \int_{-1}^{+1} \left(\overline{|\mathcal{M}|^2} + \overline{|\overline{\mathcal{M}}|^2}\right) dc_\theta \, ds}$$

Complementary CPV observable to FB-CPA:

$$A_{CP}^{FB,\Im} \equiv \frac{\int_{m_2'^2-\Delta_-}^{m_2'^2+\Delta_+} \int_{-1}^{+1} \left(\overline{|\mathcal{M}|^2} - \overline{|\overline{\mathcal{M}}|^2}\right) \, \mathrm{sign}(c_\theta) \; \mathrm{sgn}\left(s - m_2'^2\right) dc_\theta \, ds}{\int_{m_2'^2-\Delta_-}^{m_2'^2+\Delta_+} \int_{-1}^{+1} \left(\overline{|\mathcal{M}|^2} + \overline{|\overline{\mathcal{M}}|^2}\right) dc_\theta \, ds}$$

LHCb, PRD 101 (2020) 012006 [1909.05212]

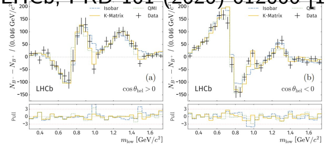


Figure 12: Raw difference in the number of B^- and B^+ candidates in the low $m_{\rm low}$ region, for (a) positive, and (b) negative cosine of the helicity angle. The pull distribution is shown below each fit projection.

$$A_{CP,k}^{FB} = \frac{(N_{B-} - N_{B+})_{\cos\theta_{\rm hel}>0,k} - (N_{B-} - N_{B+})_{\cos\theta_{\rm hel}<0,k}}{(N_{B-} + N_{B+})_{\cos\theta_{\rm hel}>0,k} + (N_{B-} + N_{B+})_{\cos\theta_{\rm hel}<0,k}}$$

$$A_{CP}^{FB,\rm ave} = \frac{\sum_{k=8}^{15}[(N_{B-} - N_{B+})_{\cos\theta_{\rm hel}>0,k} - (N_{B-} - N_{B+})_{\cos\theta_{\rm hel}<0,k}]}{\sum_{k=8}^{15}[(N_{B-} + N_{B+})_{\cos\theta_{\rm hel}>0,k} + (N_{B-} + N_{B+})_{\cos\theta_{\rm hel}<0,k}]} = (0.8 \pm 1.0)\%$$

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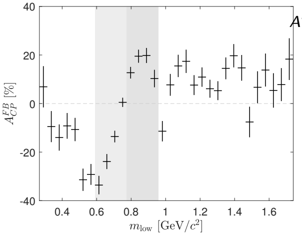

$$A_{CP}^{FB,\Im} = \frac{\left(\sum_{k=12}^{15} - \sum_{k=8}^{11}\right)\left[(N_{B-} - N_{B+})_{\cos\theta_{\rm hel}>0,k} - (N_{B-} - N_{B+})_{\cos\theta_{\rm hel}<0,k}\right]}{\sum_{k=8}^{15}\left[(N_{B-} + N_{B+})_{\cos\theta_{\rm hel}>0,k} + (N_{B-} + N_{B+})_{\cos\theta_{\rm hel}<0,k}\right]} = (13.2 \pm 1.0)\%$$

4 Summary and Outlook

Summary and Outlook

pair(s) of complementary CPA observables in multi-body decays

- overcome the cancellation problem of CPA
- more complete understanding of CPV induced by the interference of intermediate resonances
- provide us a more comprehensive method of study CPV in multi-body decays of heavy hadrons.

$\mathcal{Y}_{\sigma}^{j_a j_b}$	Ψ_0^{01}	Ψ_0^{10}	Ψ_0^{11}	Ψ_1^{11}	Φ_1^{11}	Ψ_1^{12}	Φ_1^{12}	stat.err.
$A_{CP}^{\mathcal{Y}_{\sigma}^{j_a j_b}}$	7.3	-6.1	-2.9	-3.1	-4.0	**9.5**	-0.8	2.07
$A_{CP}^{\mathcal{Y}_{\sigma}^{j_a j_b},\Im}$	**10.8**		**10.6**	0.4	-0.6			2.05

Thank you for your attentions!