Determining the Baryon LCDA on Lattice QCD

华俊（华南师范大学，LPC Collaboration）

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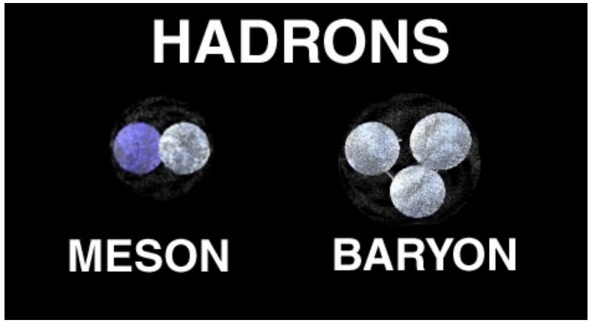
第七届全国重味物理与量子色动力学研讨会

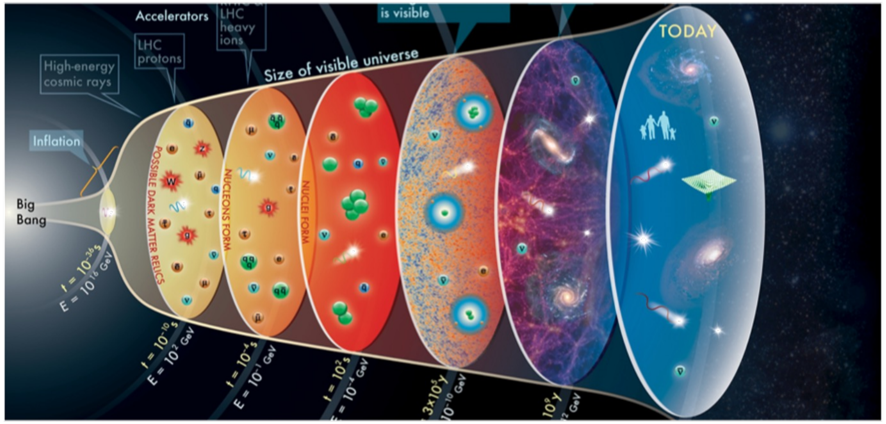
Phys.Rev.D 111, 034510 (2025) & more…

CONTENTS

Motivation

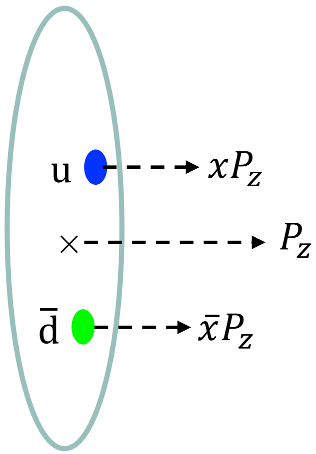
- The visible matter of the Universe is mainly made of baryons.
- Baryons play an important role in the evolution of the Universe, such as baryogenesis and big-bang nucleosythesis.



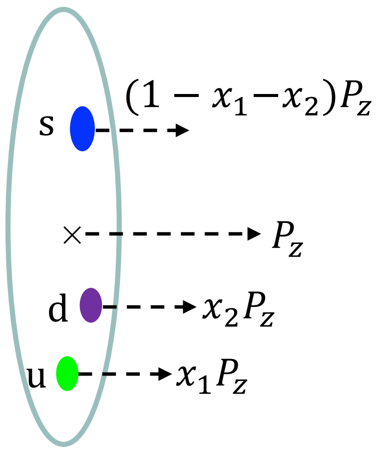


Motivation

Light meson: $\pi/K\ldots$



Baryon: Λ, proton…



- **CKM matrix**
- **CP violation**
- **New physics …**

- Sakharov conditions for Baryogenesis:
  1) Baryon number violation
  2) C and CP violation
  3) Out of thermal equilibrium

- CPV well established in K, B and D mesons
- Firstly observed in $\Lambda_b \to pK^-\pi^+\pi^-$

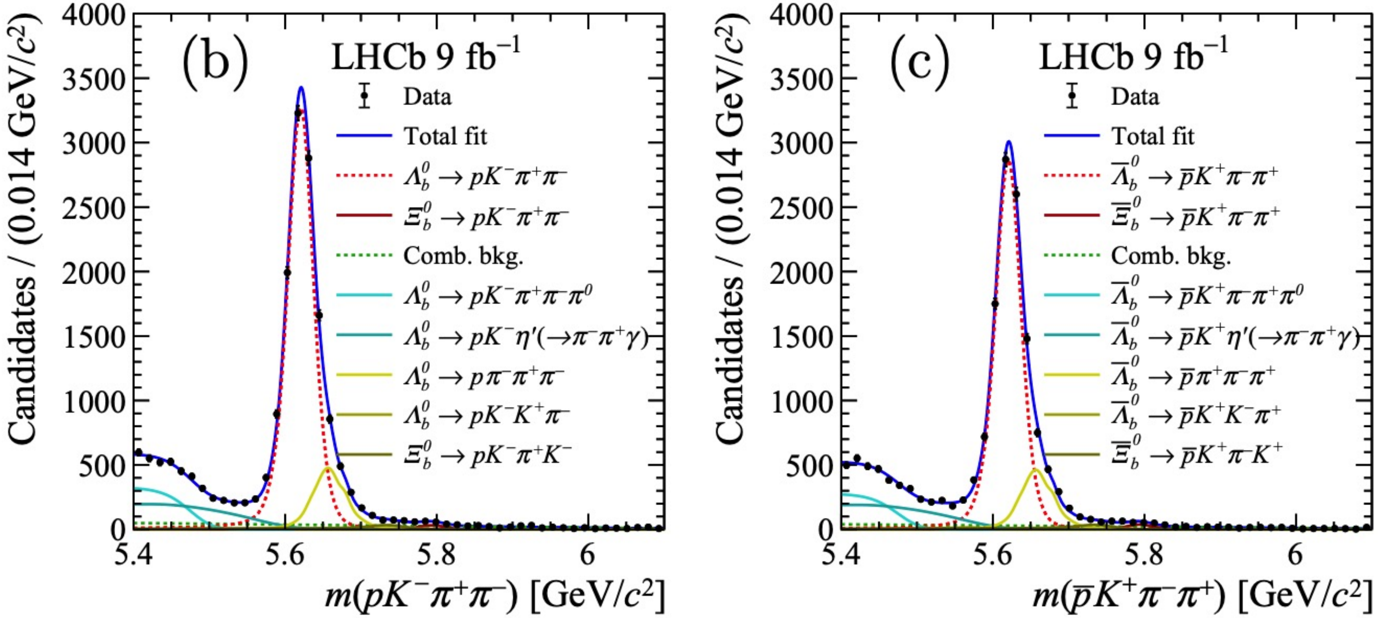
Motivation

- **First observation of baryon CP violation !**

$$\Lambda_b^0 \to pK^-\pi^+\pi^-$$

$$\mathcal{A}_{CP} = (2.45 \pm 0.46 \pm 0.10)\%$$

5.2σ



LHCb, arXiv: 2503.16954

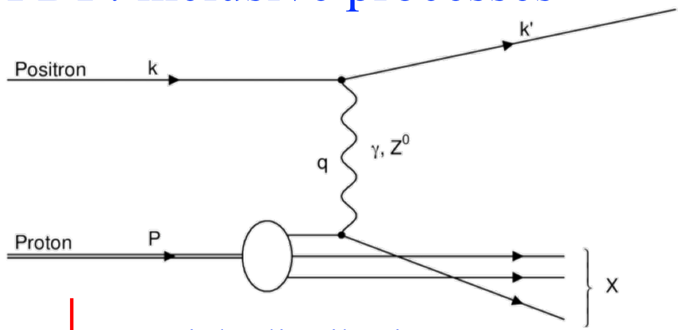
Theoretical calculation: *J.J.Han, et.al. 2409.02821*

See Y.X.Zhang's talk

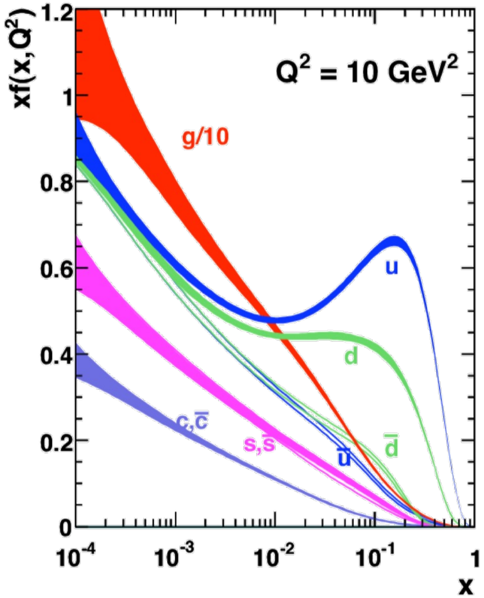
See F.S.Yu and J.J Han's talk

Motivation

PDF: inclusive processes



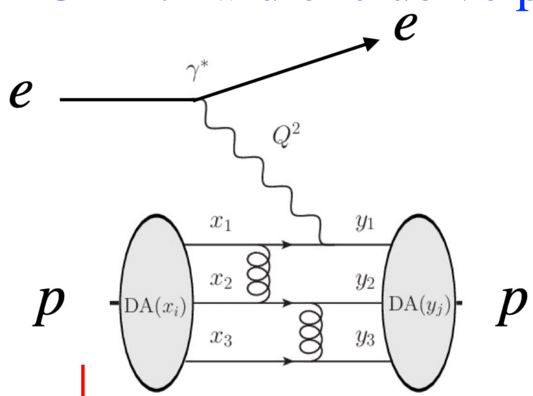
↓ 1-particle distributions



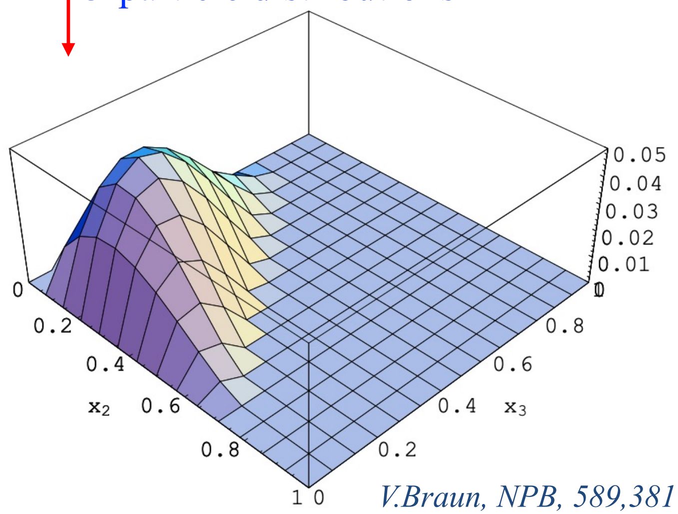
Complementary

⟷

LCDA: hard exclusive processes



↓ 3-particle distributions



V.Braun, NPB, 589,381(2000)

Motivation

- **Light Baryon LCDAs: (1980s - now)**

  - **Asymptotic LCDAs**

    Chernyak, Zhitnitsky, 1983

  - **Sum rules**

    King, Sachrajda, 1987; Chernyak, Ogloblin, Zhitnitsky 1989; Stefanis, Bergmann, 1993; Braun, Fries, Stein 2000; Braun, Lenz, Wittmann 2006;

  - **Model parametrization** —— **Model**

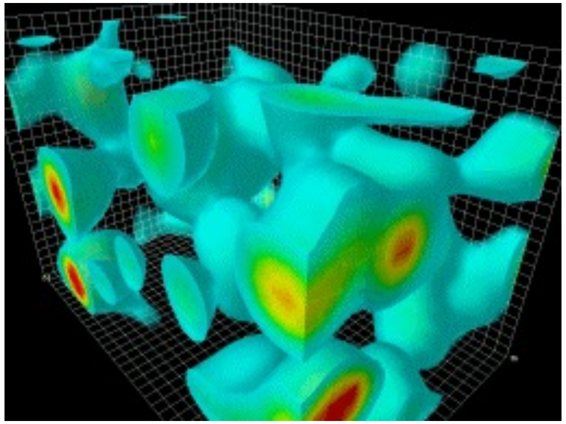
    Bell, Feldmann, Wang, Matthew 2013

  - **Lattice with OPE**

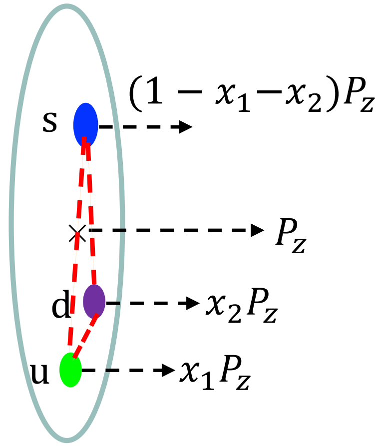
    QCDSF collaboration, 2008, 2009; RQCD collaboration, 2016, 2019(2025) —— **Only first moment**

  - **Baryon LCDA from LaMET**

    Phys.Rev.D 111 (2025) 3, 034510 ...... coming soon



Motivation

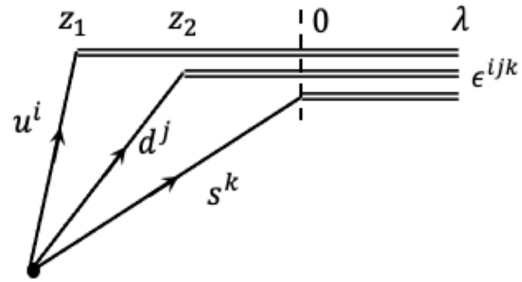


- **Definition of baryon LCDA:**

$$M_L(\xi_1,\xi_2;P)=\langle 0|\epsilon^{ijk} f_\alpha^{i'}(\xi_1 n)W_{i'i}(\xi_1,\xi_0)g_\beta^{j'}(\xi_2 n)W_{j'j}(\xi_2,\xi_0)h_\gamma^k(\xi_0 n)|B(P,\lambda)\rangle$$

$$\Phi(x_1,x_2,\mu)=\int\frac{\mathrm{d}P^+\xi_1}{2\pi}\frac{\mathrm{d}P^+\xi_2}{2\pi}\mathrm{e}^{\mathrm{i}x_1P^+\xi_1+\mathrm{i}x_2P^+\xi_2}\frac{M_L(\xi_1,\xi_2;P,\mu)}{M_L(0,0;P,\mu)}$$

C.Han JHEP 07019 (2024)
V.L.C & I.R.Z NPB 24652(1984)
G.R.Farrar et.al. NPB 311585(1989)



- **Leading twist octet baryon LCDA:**

$$\langle 0|f_\alpha(z_1n)g_\beta(z_2n)h_\gamma(z_3n)|B(P_B,\lambda)\rangle$$
$$=\frac{1}{4}f_V\left[(\not{P}_BC)_{\alpha\beta}(\gamma_5u_B)_\gamma V^B(z_in\cdot P_B)+(\not{P}_B\gamma_5C)_{\alpha\beta}(u_B)_\gamma A^B(z_in\cdot P_B)\right]$$
$$+\frac{1}{4}f_T(i\sigma_{\mu\nu}P_B^\nu C)_{\alpha\beta}(\gamma^\mu\gamma_5u_B)_\gamma T^B(z_in\cdot P_B),$$

Octet	n	p	Λ
fgh	d d u	u u d	u d s

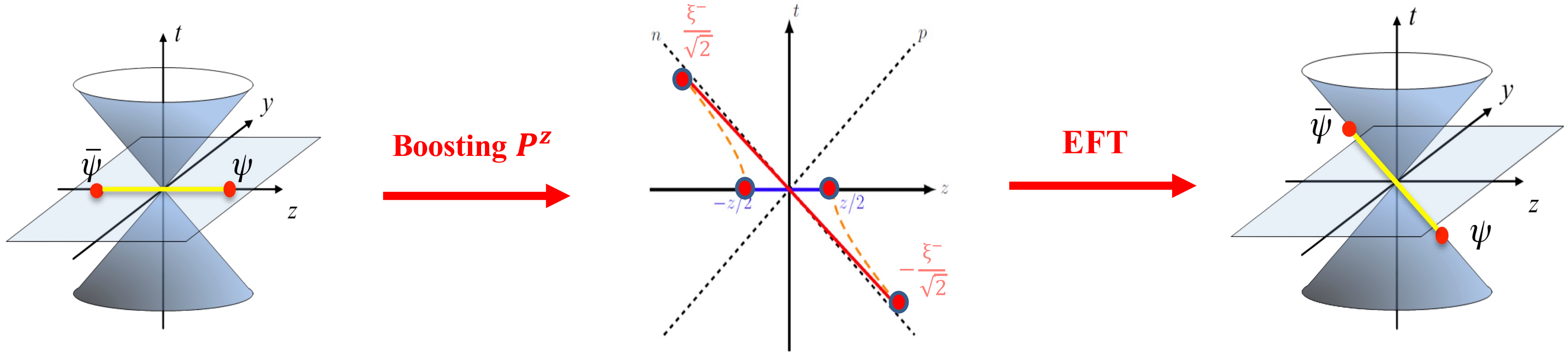
Quasi DA Method on Lattice

- **Based on CLQCD Ensembles**
  - Three lattice spacing for $a \to 0$ limit
  - Three momentum for $P_z \to \infty$ limit

Ensemble	Volume	Lattice spacing	π mass	measurement	P^z
C24P29	$24^3 \times 72$	0.105 fm	293 MeV	864*4*…	2.45, 2.94, 3.43 GeV
F32P30	$32^3 \times 96$	0.077 fm	303 MeV	777*4*…	2.50, 3.00, 3.50 GeV
H48P32	$48^3 \times 144$	0.052 fm	317 MeV	550*6*…	2.34, 2.80, 3.28 GeV

Quasi DA Method on Lattice

- **Large momentum effective theory (LaMET)**

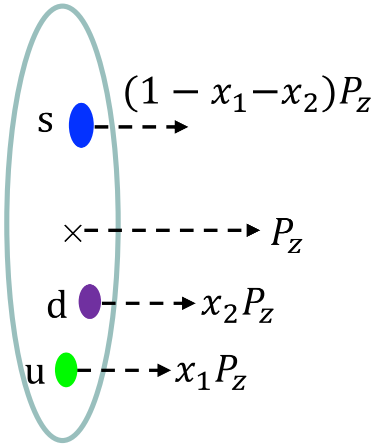


- **Factorization in LaMET**

$$\tilde{\phi}\left(x_1, x_2, P^z, \mu\right) = \int_0^1 dy_1 \int_0^{1-y_1} dy_2 C\left(x_1, x_2, y_1, y_2, P^z, \mu\right) \phi\left(y_1, y_2\right) + \mathcal{O}\left(\frac{1}{(x_1 P^z)^2}, \frac{1}{(x_2 P^z)^2}, \frac{1}{[(1-x_1-x_2)P^z]^2}\right)$$

Quasi-DA | Matching kernel | LCDA | High power correction

Quasi DA Method on Lattice



- **Quasi-DA on Euclidean lattice:**

$$M\left(z_1,z_2;P^z\right)=\langle 0|\epsilon^{ijk}f_\alpha^{i'}\left(z_1\right)W_{i'i}\left(z_1,0\right)g_\beta^{j'}\left(z_2\right)W_{j'j}\left(z_2,0\right)h_\gamma^k(0)\left|B\left(P^z,\lambda\right)\right\rangle$$

$$\widetilde{\Phi}\left(x_1,x_2,P^z,\mu\right)=\left(P^z\right)^2\int\frac{\mathrm{d}z_1}{2\pi}\frac{\mathrm{d}z_2}{2\pi}\mathrm{e}^{-x_1P^zz_1-x_2P^zz_2}\frac{M\left(z_1,z_2;P^z,\mu\right)}{M\left(0,0;P^z,\mu\right)}$$

- **For octet baryon, the leading twist V, A, T:**

$$\left\langle 0\left|f^T\left(z_1n\right)\left(C\not{n}\right)g\left(z_2n\right)h\left(z_3n\right)\right|B\right\rangle=-f_VV^B(z_in\cdot P_B)P_B^+\gamma_5u_B,$$

$$\left\langle 0\left|f^T\left(z_1n\right)\left(C\gamma_5\not{n}\right)g\left(z_2n\right)h\left(z_3n\right)\right|B\right\rangle=f_VA^B(z_in\cdot P_B)P_B^+u_B,$$

$$\left\langle 0\left|f^T\left(z_1n\right)\left(iC\sigma_{\mu\nu}n^\nu\right)g\left(z_2n\right)\gamma^\mu h\left(z_3n\right)\right|B\right\rangle=2f_TT^B(z_in\cdot P_B)P_B^+\gamma_5u_B,$$

☐ **A first implement can be found in Phys.Rev.D 111, 034510 (2025)**

Improvement on Simulation

- **Improvement on signal to noise ratio**
  - ☐ Improve Structure of the source interpolator
  - ☐ Combination of different projection operator

- **Improvement on frame work**
  - ☐ Renormalization scheme
  - ☐ Matching scheme

Improvement on Simulation： Source interpolator

- **Two point correlation of baryon**

$$\mathcal{O}_{Sink} = (u(x,t,z_1,z_2)\Gamma d(x,t,0,0))s(x,t,0,0)$$

$$C_2(z_a, z_2; t, \vec{P}) = \int \mathrm{d}^3x e^{-\mathrm{i}\vec{P}\cdot\vec{x}} \langle 0 | \mathcal{O}_{\mathrm{Sink}}(\vec{x}, t; z_1, z_2) \bar{\mathcal{O}}_{\mathrm{Src}}(0,0;0,0) T | 0 \rangle$$

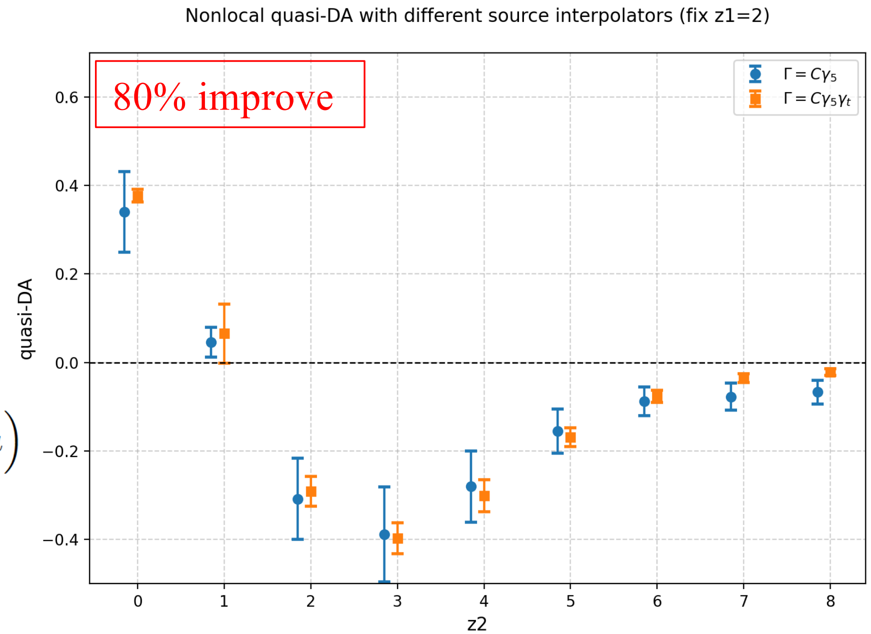
Determined by twist — Up to choice

Original interpolator:

$$\mathcal{O}^{\Lambda} = \frac{1}{\sqrt{6}}\Big((u^T C\gamma_5 d)\ s + (u^T C\gamma_5 s)\ d + (s^T C\gamma_5 d)\ u\Big)$$

Improved interpolator (for large P_z):

$$\mathcal{O}^{\Lambda} = \frac{1}{\sqrt{6}}\Big((u^T C\gamma_5\gamma_{t/z} d)\ s + (u^T C\gamma_5\gamma_{t/z} s)\ d + (s^T C\gamma_5\gamma_{t/z} d)\ u\Big)$$



Improvement on Simulation： Projection operator

- **Two point correlation of baryon**

$$C_2(z_a, z_2; t, \vec{P}) = \int \mathrm{d}^3 x \mathrm{e}^{-\mathrm{i}\vec{P}\cdot\vec{x}} \langle 0 | \mathcal{O}_{\mathrm{Sink}}(\vec{x}, t; z_1, z_2) \bar{\mathcal{O}}_{\mathrm{Src}}(0,0;0,0) T | 0 \rangle$$

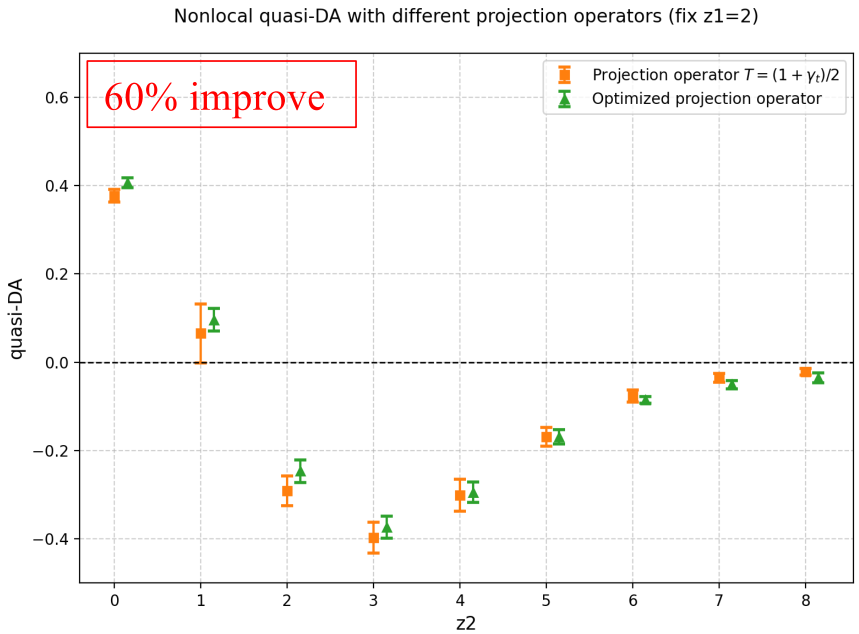
Projection

Original interpolator:

$$T = \frac{1+\gamma_t}{2}$$

Improved interpolator (for large P_z):

Combine $T = I, \gamma_t, \gamma_z$



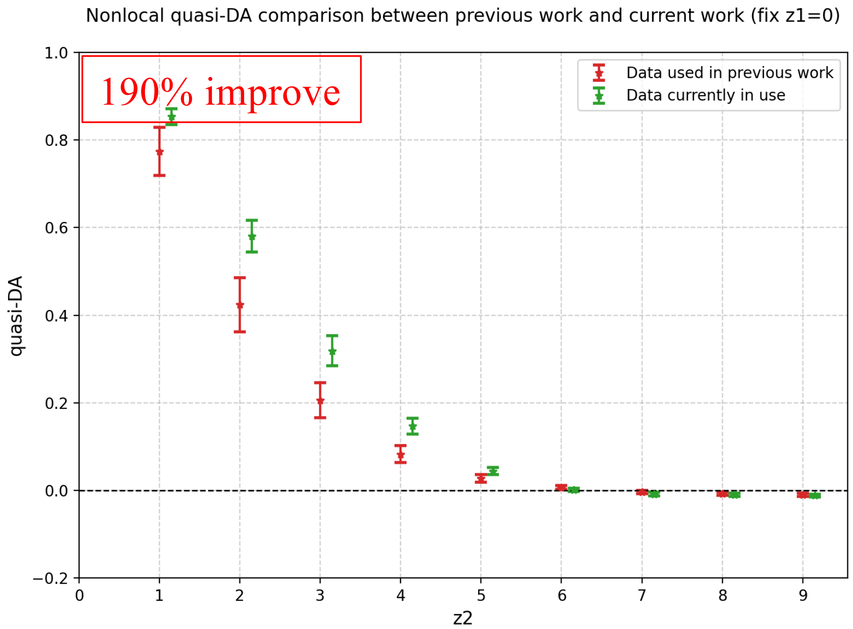
Improvement on Simulation： Total improvement

- **Two point correlation of baryon**

$$C_2(z_a, z_2; t, \vec{P}) = \int \mathrm{d}^3x \mathrm{e}^{-\mathrm{i}\vec{P}\cdot\vec{x}} \langle 0|\mathcal{O}_{\mathrm{Sink}}(\vec{x}, t; z_1, z_2)\bar{\mathcal{O}}_{\mathrm{Src}}(0, 0; 0, 0)T|0\rangle$$

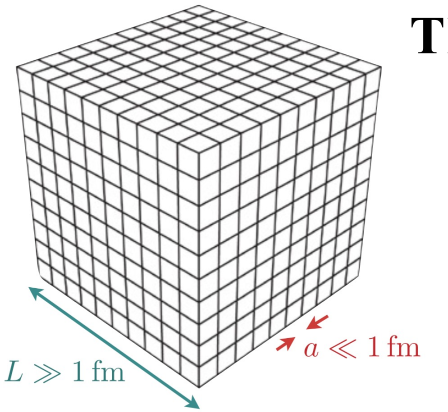
- ☐ Improve Structure of the source interpolator
- ☐ Combination of different projection operator
- ☐ Other lattice setups

Compare with PRD 111, 034510 (2025)



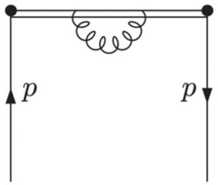
Improvement on Simulation: Renormalization

- **The challenge of renormalization in baryon quasi-DA**



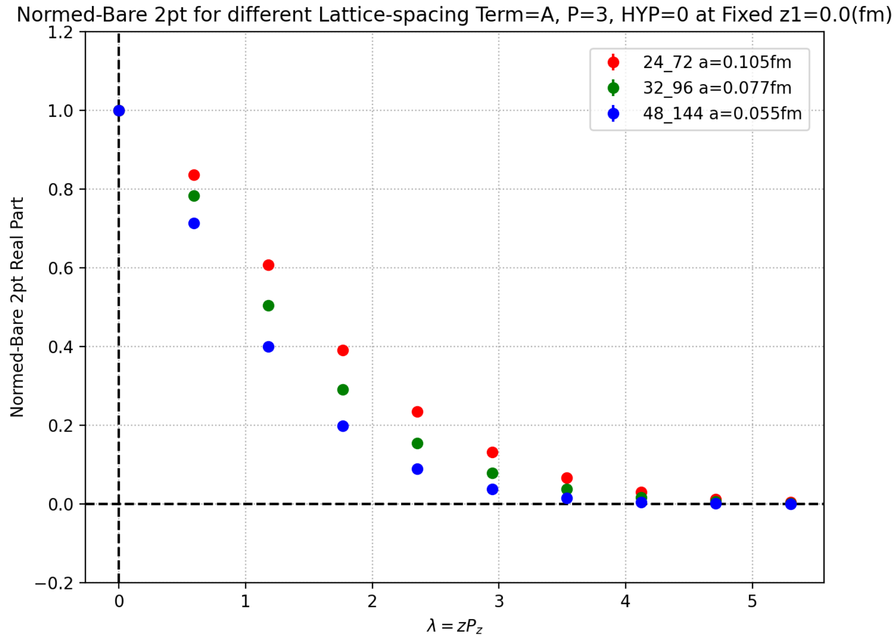
The lattice simulation introduce:

- ☐ $\frac{1}{a}$ UV cut
- ☐ Linear divergence (nonlocal)



$$M(z) \sim \exp\left(-\frac{C(\alpha)z}{a}\right) f(z)$$

Non-consistent results on different lattice spacing before renormalization:



Improvement on Simulation： Renormalization

- **The challenge of renormalization in baryon quasi-DA**

  - Self renormalization: 1) parameterize the matrix element to extract the linear divergence
    2) match with the $\overline{\text{MS}}$ perturbative matrix element

- Parameterized form:

$$\ln M\left(z_1, z_2; P_z = 0; a\right) = \boxed{\frac{k}{a \ln\left(a\Lambda_{\text{QCD}}\right)}\tilde{z}} + g\left(z_1, z_2\right) + f\left(z_1, z_2\right) a^2 + \frac{\gamma_0}{b_0} \ln \frac{\ln\left(1/a\Lambda_{\text{QCD}}\right)}{\ln\left(\mu/\Lambda_{\overline{\text{MS}}}\right)} + \ln\left[1 + \frac{d}{\ln\left(a\Lambda_{\text{QCD}}\right)}\right]$$

Linear divergence

- $\overline{\text{MS}}$ perturbative matrix element:

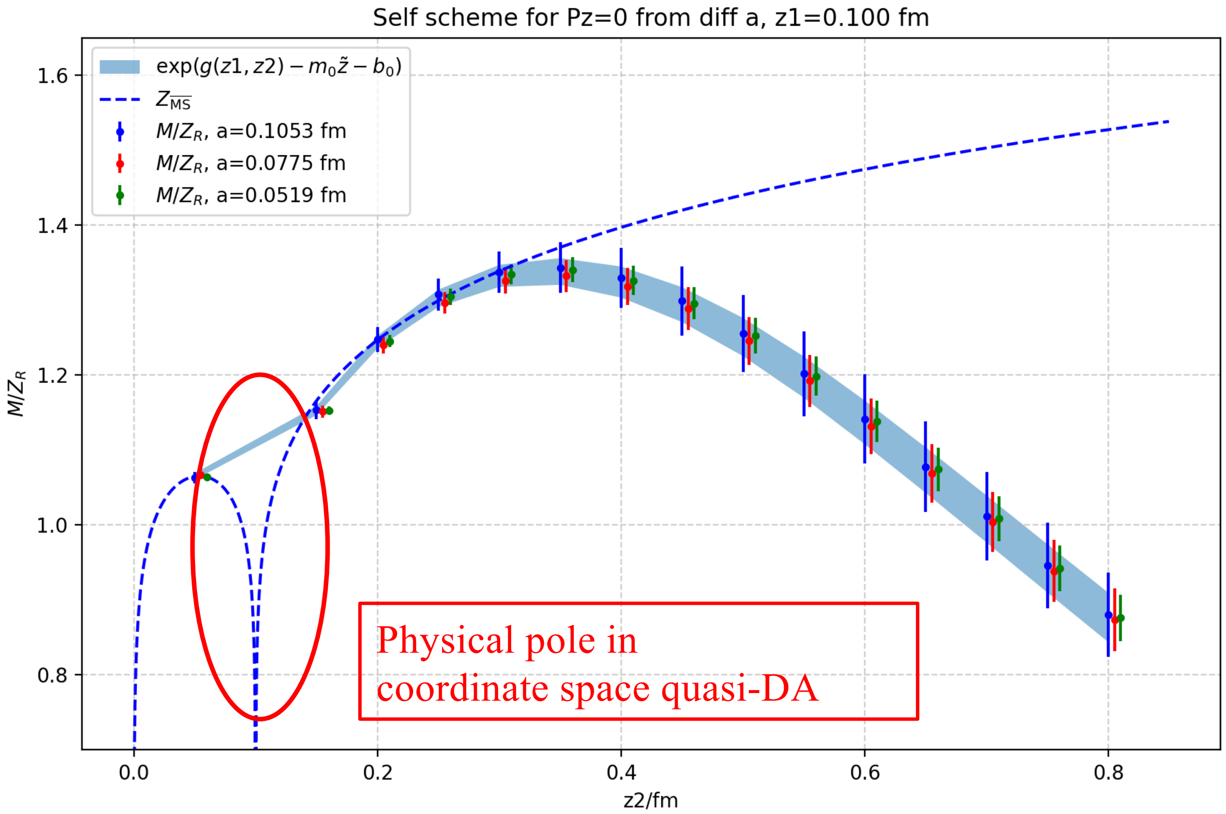
$$Z_{\overline{\text{MS}}}\left(z_1, z_2; \mu, \Lambda_{\overline{\text{MS}}}\right) = 1 + \frac{\alpha_s C_F}{2\pi}\left[\frac{7}{8}\ln\frac{z_1^2\mu^2 e^{2\gamma_E}}{4} + \frac{7}{8}\ln\frac{z_2^2\mu^2 e^{2\gamma_E}}{4} + \frac{3}{4}\ln\frac{\boxed{\left(z_1 - z_2\right)^2}\mu^2 e^{2\gamma_E}}{4} + 4\right]$$

Pole in coordinate space

Improvement on Simulation： Renormalization

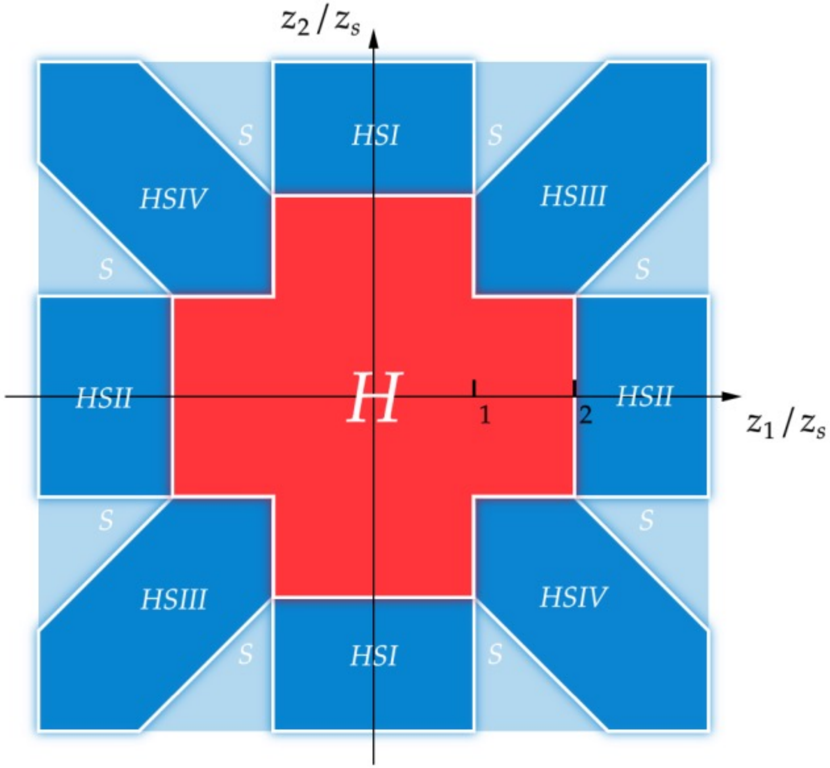
- **The challenge of renormalization in baryon quasi-DA**

  - Self renormalization:



Improvement on Simulation: Renormalization

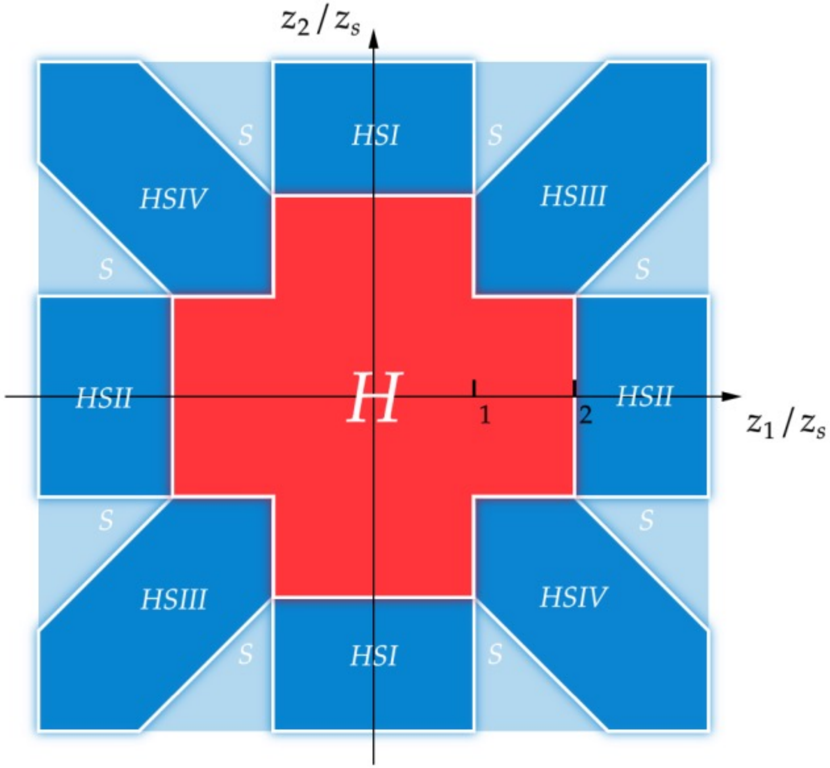
- **Solve the renormalization challenge with hybrid scheme**



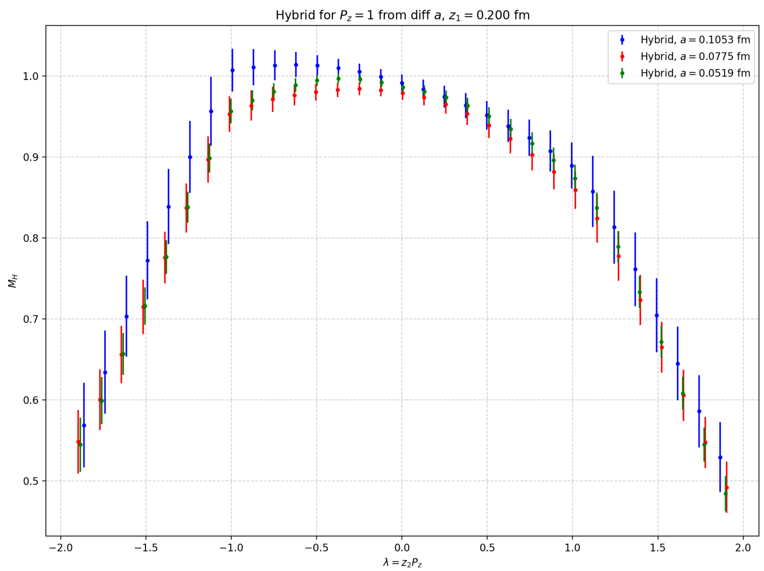
- H region: Ratio scheme
- HIS-HSIV regions:
  Ratio scheme for short z_i
  Self-renormalization for large z_i
- S regions: Self-renormalization

Improvement on Simulation：Renormalization

- **Solve the renormalization challenge with hybrid scheme**



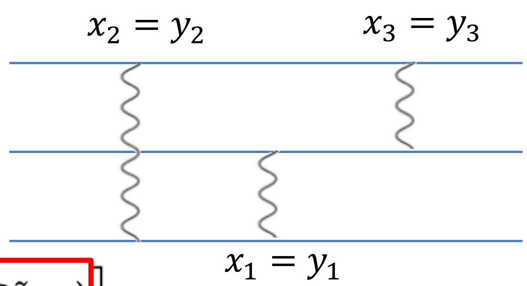
Renormalized quasi-DA in hybrid scheme



Improvement on Simulation： Matching

- **LaMET factorization for Baryon LCDA:**

$$\tilde{\phi}(x_1,x_2) = \int_0^1 dy_1 \int_0^{1-y_1} dy_2 C(x_1,x_2,y_1,y_2)\phi(y_1,y_2) + \mathcal{O}\left(\frac{1}{(x_1P^z)^2}, \frac{1}{(x_2P^z)^2}, \frac{1}{[(1-x_1-x_2)P^z]^2}\right)$$



$$\mathcal{C}_H^{(1)}(x_1,x_2,y_1,y_2,P^z,\mu) = \left[\mathcal{C}_{\overline{\rm MS}}^{(1)}(x_1,x_2,y_1,y_2,P^z,\mu) - \delta\mathcal{C}_H^{(1)}(x_1,x_2,y_1,y_2,P^z,\mu)\right]_\oplus$$

Correction from hybrid scheme

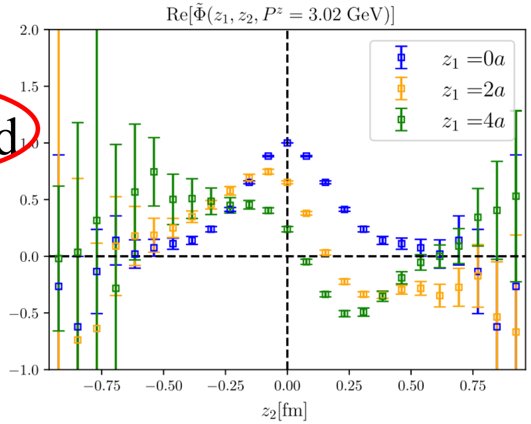
Double plus function:

C.Han et.al. JHEP 07 019 (2024)

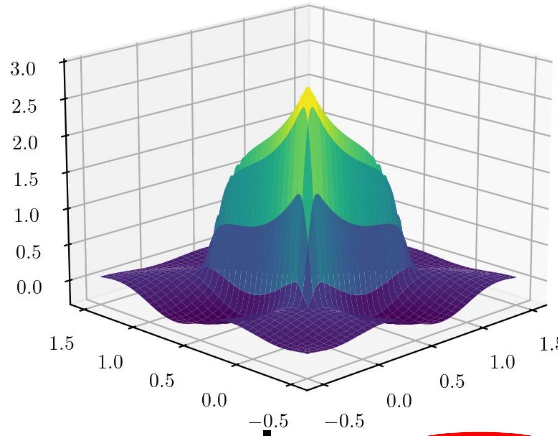
$$[g(x_1,x_2,y_1,y_2)]_\oplus = g(x_1,x_2,y_1,y_2) - \delta(x_1-y_1)\delta(x_2-y_2)\int dx_1' dx_2' g(x_1',x_2',y_1,y_2)$$

Improvement on Simulation

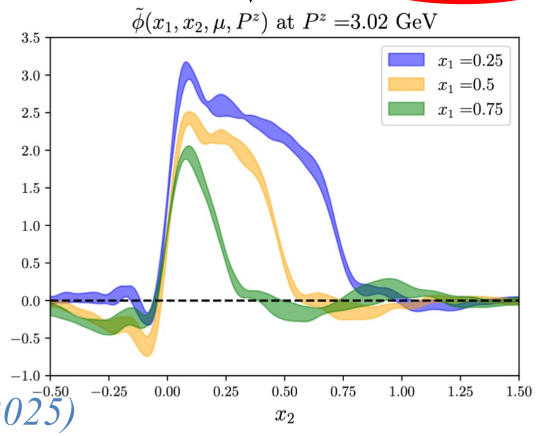
Renormalized Quasi-DA



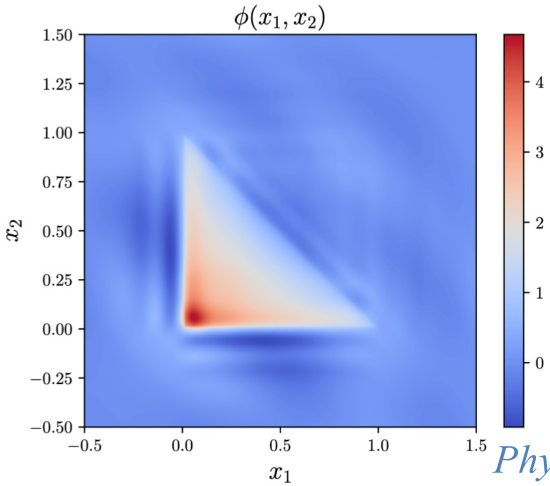
Incomplete FT



Matching



LCDA



All after improvement ?

Phys.Rev.D 111, 034510 (2025)

Summary

- We made the first attempt to implement the numerical computation of baryon LCDA in the LaMET framework.

- To improve and calculate all leading twist structure Proton and Lambda LCDA, we improve:
  - ☐ source interpolator
  - ☐ projection operator
  - ☐ renormalization scheme
  - ☐ matching scheme

  - ◆ Please stay tuned our results for all leading twists LCDA of Proton and Lambda
  - ◆ High twists will be the next

Thanks for Four Attention !