NANJING UNIVERSITY

Distribution Amplitudes and Two-Photon Transition Form Factors in Quarkonium

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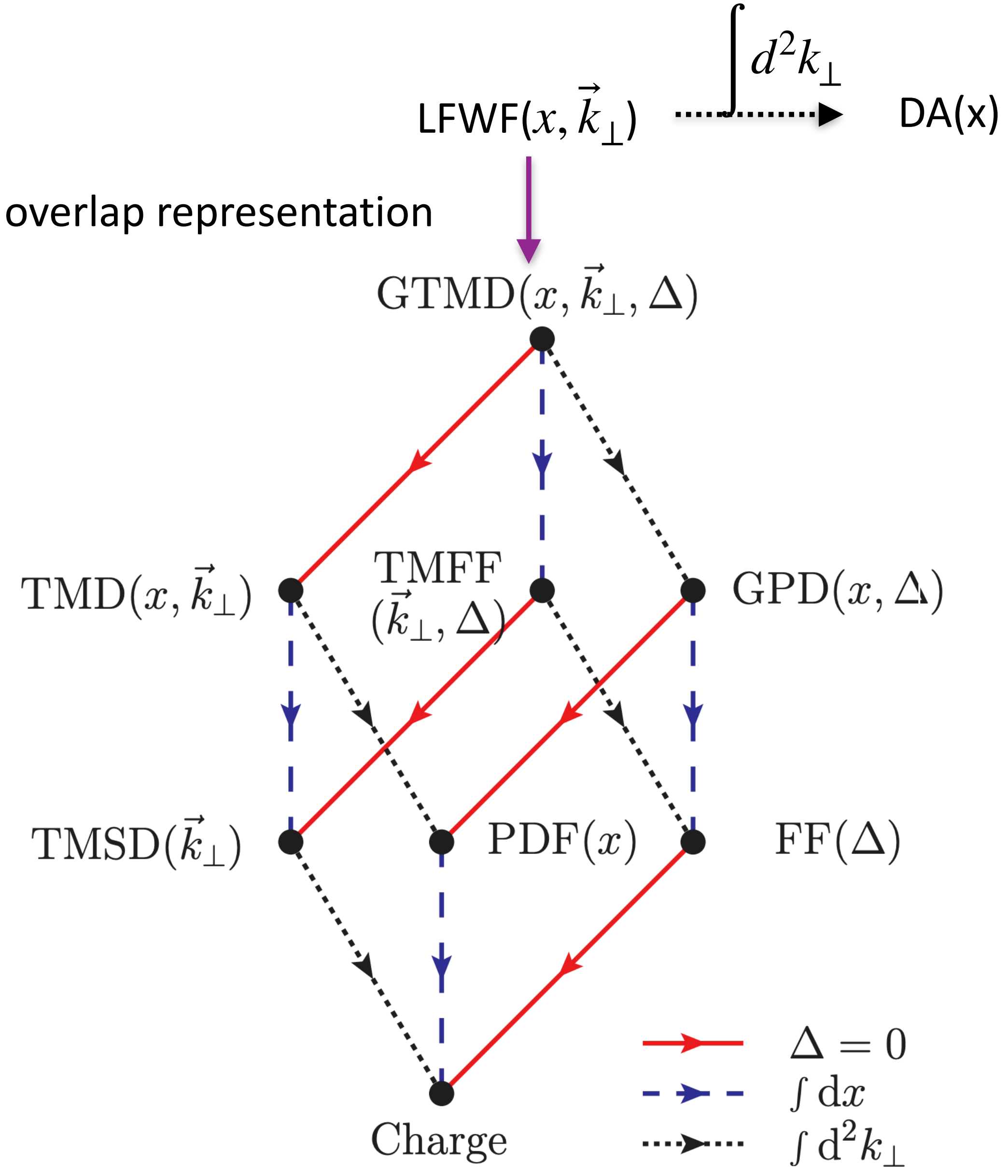
Distribution Amplitude (DA)

➢ Meson's dressed-valence-quark parton distribution amplitude (DA).

➢ Physics Goals:

✓Nearest thing in quantum field theory to Schrödinger wave function.

✓DA is 1D projection of hadron's light-front wave function, obtained by integration $\sim \int d^2k_\perp \, \psi(x, k_\perp)$



Lorce, Pasquini, Vanderhaeghen, JHEP 05 (2011) 041

Distribution Amplitude (DA) of pseudoscalar meson

- The matrix element and the light-front wave function:

$$\langle 0|\psi(-z)\gamma_5\gamma\cdot n\psi(z)|PS(P)\rangle = f_{PS} n\cdot P\int_0^1 dx\, e^{-i(2x-1)z\cdot P}\varphi_{PS}(x,z_\perp^2)\,,$$

- The matrix element and the Bethe-Salpeter wave function:

$$\langle 0|\psi(-z)\gamma_5\gamma\cdot n\psi(z)|PS(P)\rangle = Z_2\mathrm{tr}_{CD}\int_{dk}^{\Lambda} e^{-iz\cdot k-iz\cdot(k-P)}\gamma_5\gamma\cdot n\chi_{PS}(k,k-P)\,,$$

- The light-front wave function and the Bethe-Salpeter wave function:

$$f_{PS}\varphi_{PS}(x,k_\perp^2) = Z_2\mathrm{tr}_{CD}\int^{\Lambda}\frac{dk_3dk_4}{(2\pi)^2}\delta(xn\cdot P-n\cdot k)\gamma_5\gamma\cdot n\chi_{PS}(k,k-P)\,,$$

- The parton distribution amplitude (DA) and the light front wave function

$$\varphi_{PS}(x,\zeta) = \int_0^{\zeta}\frac{d^2k_\perp}{16\pi^3}\varphi_{PS}(x,k_\perp)\,.$$

Distribution Amplitude (DA) of pseudoscalar meson

- Projection of the meson's Bethe-Salpeter wave function onto the light-front

$$f_{PS}\,\varphi_{PS}(x) = tr_{CD} Z_2 \int_{dk}^{\Lambda} \delta(n \cdot k_+ - x\, n \cdot P)\, \gamma_5 \gamma \cdot n\, \chi_{PS}(k;P)\,,$$

- where: n is a light-like four-vector, $n^2 = 0$; and P is the meson's four-momentum, $P^2 = -m_{PS}^2$ and $n \cdot P = -m_{PS}$, with m_{PS} being the pseudoscalar meson's mass. $\chi_{PS}(k,P)$ is the Bethe-Salpeter wave function.

- Mellin moments of the distribution; viz. $\langle x^m \rangle := \int_0^1 dx\, x^m \varphi_{PS}(x)$

- given by

$$f_{PS}(n \cdot P)^{m+1} \langle x^m \rangle = tr_{CD} Z_2 \int_{dk}^{\Lambda} (n \cdot k_+)^m\, \gamma_5 \gamma \cdot n\, \chi_{PS}(k;P)\,.$$

DA scale evolution

Efremov, Radyushkin, Phys. Lett. B 94, 245 (1980). Lepage, Brodsky, Phys. Rev. D 22, 2157 (1980).

- The equation describing the τ -evolution of $\varphi(x,\tau)$ is known and has the solution

$$\varphi_{PS}(x;\tau) = \varphi_{PS}^{asy}(x)\left[1 + \sum_{j=2,4,\ldots}^{\infty} a_j^{3/2}(\tau)\, C_j^{(3/2)}(x-\bar{x})\right]$$

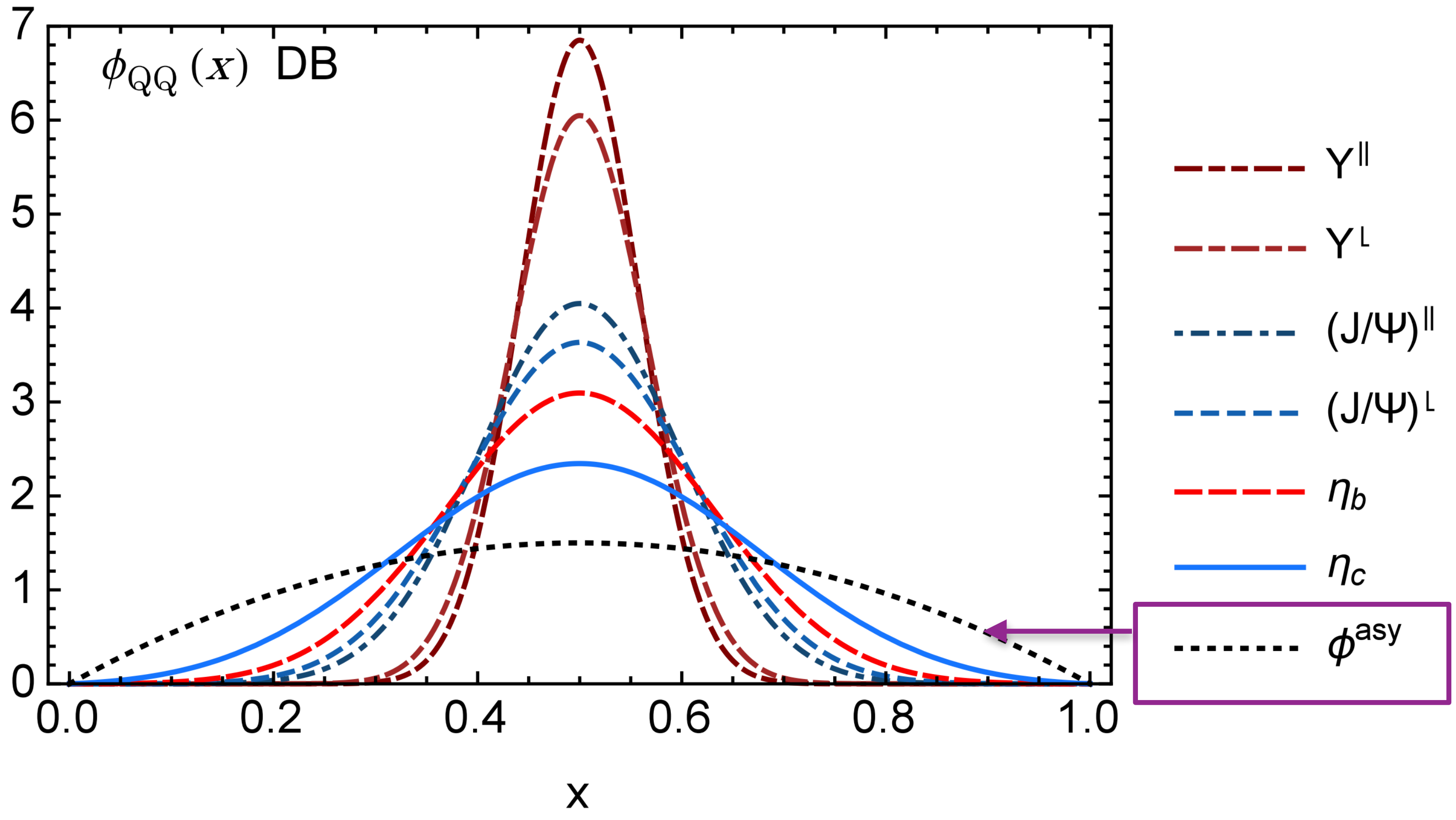
- where $\varphi_{PS}^{\rm asy}(x) = 6x(1-x)$. The expansion coefficients $\{a_j^{3/2}, j=1,\ldots,\infty\}$ evolve logarithmically with τ: they vanish as $\tau \to 0$, and at leading-logarithmic accuracy the moments evolve from $\tau = 1/[2\,\text{GeV}] \to \tau$ as:

$$a_j^{3/2}(\tau) = a_j^{3/2}(\tau_2)\left[\frac{\alpha_s(\tau_2)}{\alpha_s(\tau)}\right]^{\gamma_j^{(0)}/\beta_0},$$

- where the one-loop strong running-coupling is given as $\alpha_s(Q^2) \overset{Q^2>10\,\Lambda_{QCD}^2}{\approx} \dfrac{4\pi}{\beta_0\,\ln[Q^2/\Lambda_{\rm QCD}^2]}$,

- with anomalous dimension $\gamma_j^{(0)} = C_F\left[3 + \dfrac{2}{(j+1)(j+2)} - 4\displaystyle\sum_{k=1}^{j+1}\frac{1}{k}\right]$, $C_F = 4/3$.

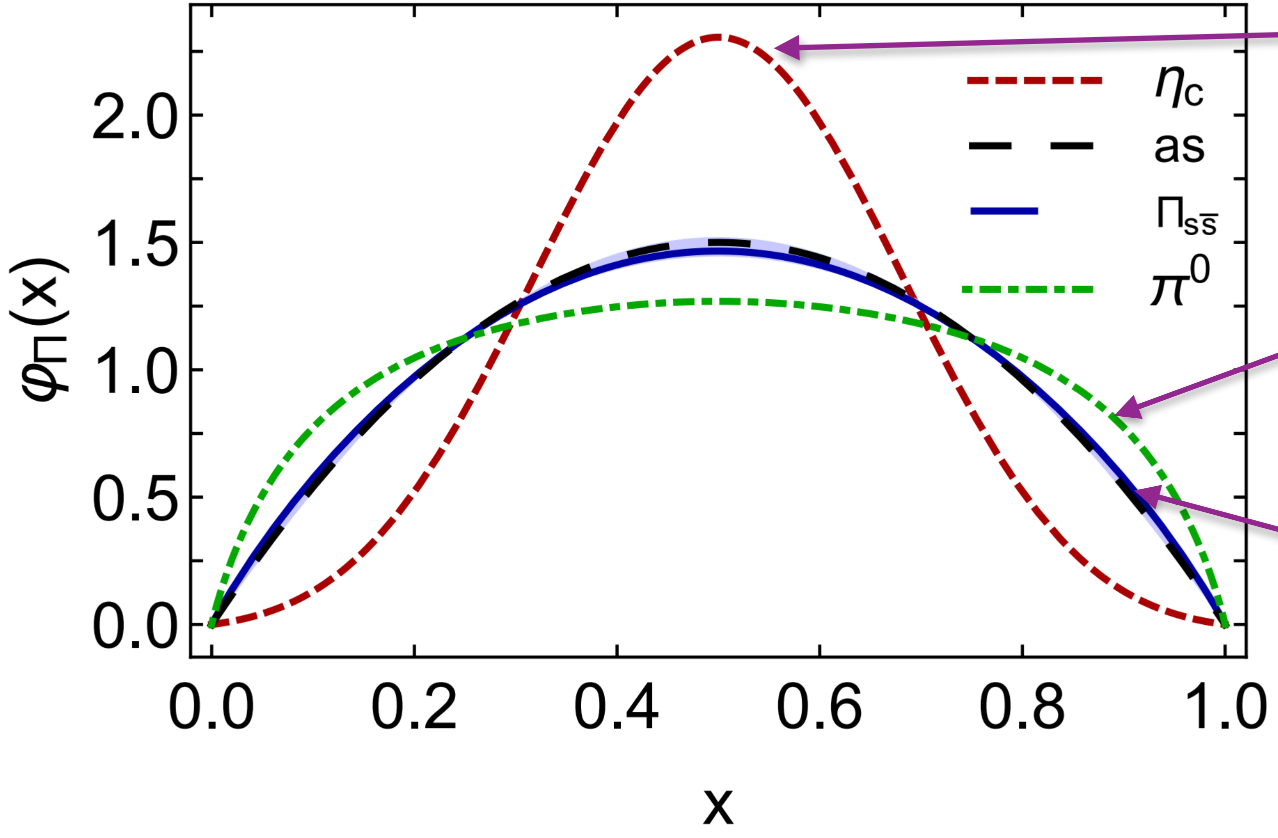
$\eta_c, \eta_b, J/\psi, \Upsilon$ twist-2 DAs



Ding, Gao, Chang, Liu, Roberts. Phys. Lett. B 753 (2016) 330-335.

- Heavy-quarkonia DAs are piecewise convex-concave-convex, much narrower than the asymptotic distribution $6x(1-x)$, but deviate noticeably from $\delta(x-1/2)$.
  see Yang Li's talk
- Peak heights and widths in this case show a natural ordering.

$s\bar{s}$ pseudoscalar meson twist-2 DA

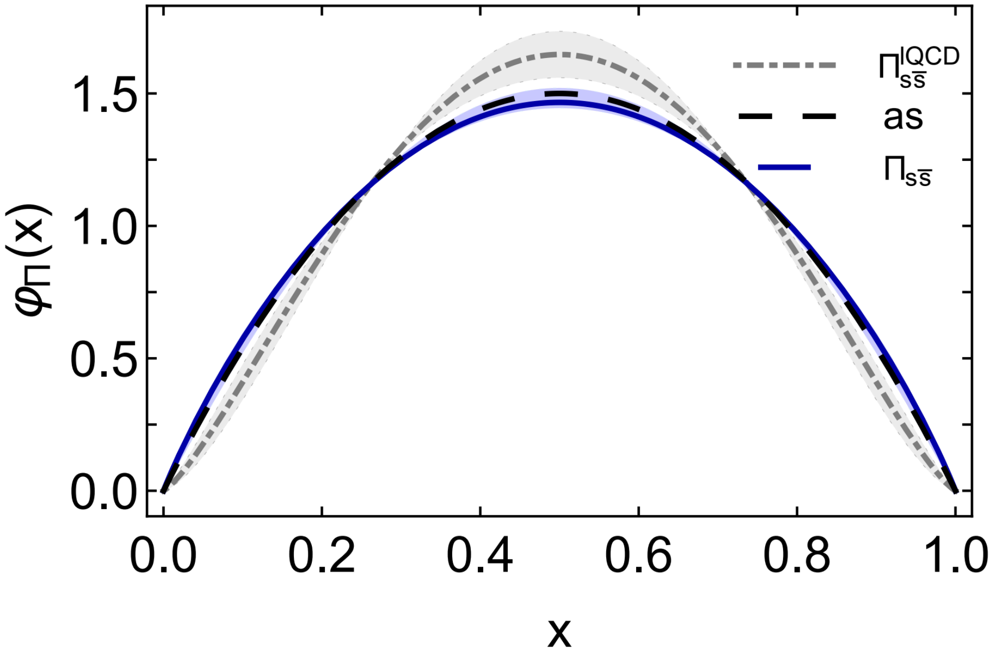


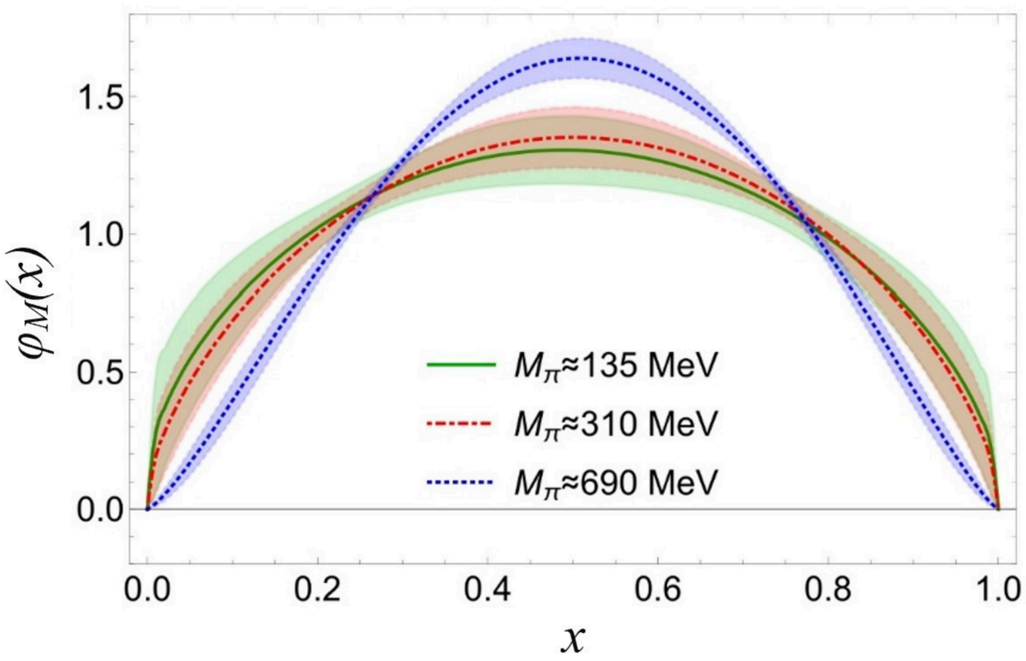
Roberts, Richards, Horn, Chang. Prog. Part. Nucl. Phys. 120 (2021) 103883

- DA of η_c meson, $c\bar{c}$, is much narrower than φ_{asy} , which feels the current quark mass effect strongly.
- DA of pion is broad and concave, largely formed by the mechanism of emergence of hadron mass (EHM). see Jianhui Zhang's talk
- $\Pi_{ss}(s\bar{s})$ system lies at the boundary: $\Pi_{ss}(x) \approx \varphi_{asy}(x)$, EHM and current quark mass effect are playing a roughly equal role.

$s\bar{s}$ pseudoscalar meson twist-2 DA

Zhang, Honkala, Lin and Chen, Phys. Rev. D 102, 094519 (2020).





- ➢ Meson mass: both continuum method and Lattice QCD calculations have delivered a bound state mass $m_{s\bar{s}} = 0.69$ GeV.
- ➢ DA: both continuum and lattice QCD agree upon the existence and value of m_{cr}, for which $\varphi_{q_{m_{cr}}\bar{q}_{m_{cr}}}(x,\zeta) \approx \varphi_{as}(x)$.

Response of DAs to increasing the current quark mass

EHM dominate

The binding and confinement mechanisms is fully nonperturbative. DA of $u\bar{d}$ meson is largely formed by the mechanism of emergence of hadron mass (EHM).

strange quark

DAs of $c\bar{c}$ and $b\bar{b}$ mesons (with large quark masses) are much narrower than φ_{asy} , and feel the current quark mass effect strongly.

Current quark mass effect dominate

$\Pi_{ss}(s\bar{s})$ system lies at the boundary:

EHM and current quark mass effect are playing a roughly equal role.

Charge-neutral pseudoscalar meson two-photon transition form factors

- Perturbative QCD predicts two-photon transition form factor

see Yang Li's talk

$$\exists Q_0 > \Lambda_{QCD} \mid Q^2 G^q_{PS}(Q^2) \overset{Q^2 > Q_0^2}{\approx} 4\pi^2 f^q_{PS} e_q^2 w^q_\varphi(Q^2)\,, \qquad w_\varphi(Q^2) = \int_0^1 dx \, \frac{1}{x} \varphi^q_{PS}(x)\,,$$

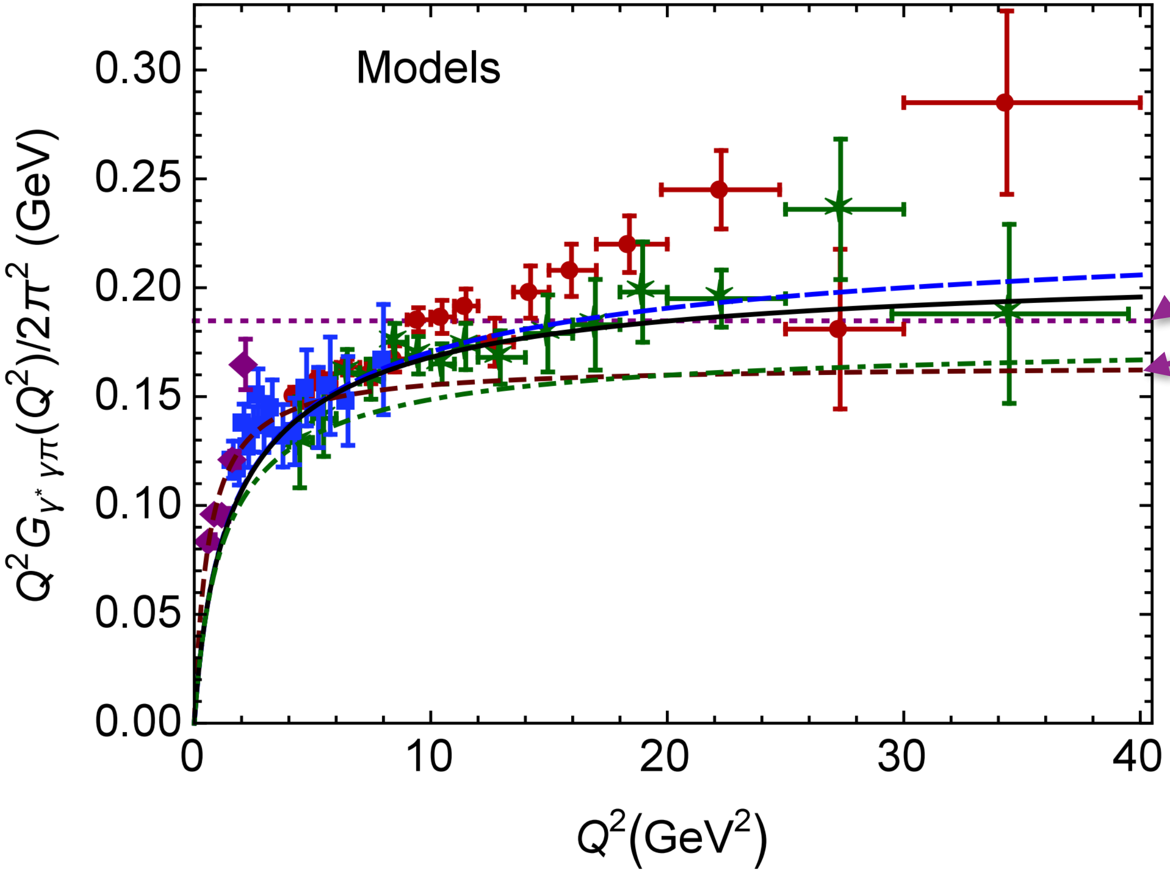
- f^q_{PS} is the leptonic decay constant, $\varphi^q_{PS}(x)$ is the dressed-valence-quark q-parton contribution to meson's distribution amplitude (DA). The value of Q_0 is not predicted by pQCD.

Lepage and Brodsky, Phys. Rev. D 22, 2157 (1980); Phys. Lett. B 87, 359 (1979). Efremov and Radyushkin, Phys. Lett. B 94, 245 (1980).
Farrar and Jackson, Phys. Rev. Lett. 43, 246 (1979).

- Asymptotic DA at $\Lambda^2_{\rm QCD}/Q^2 \simeq 0$, i.e., very large values of Q^2, $\quad \varphi^q_{PS}(x) = 6x(1-x)\,,$

- then the inverse moment $w_\varphi = 3$, and $\quad Q^2 G^q_{PS}(Q^2) \overset{Q^2 \to \infty}{\approx} 12\pi^2 f^q_{PS} e_q^2\,,$

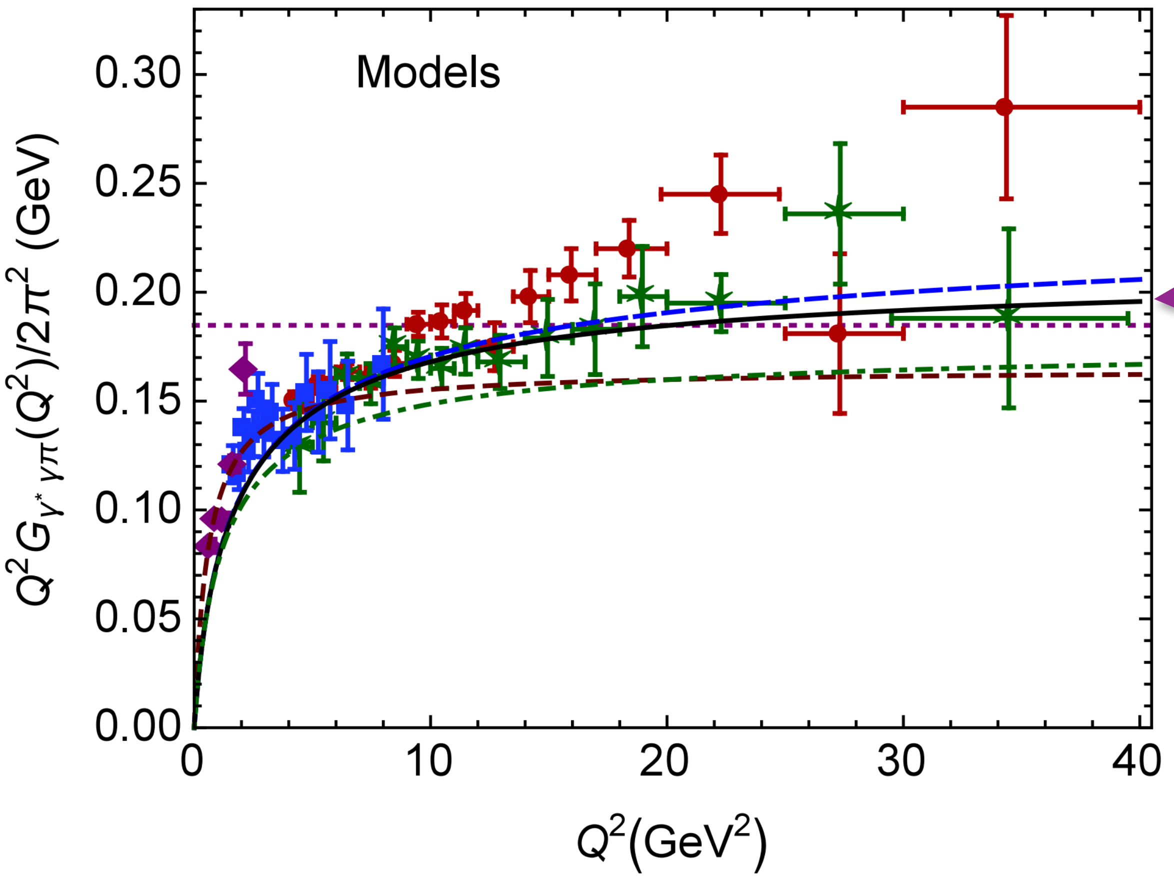
Pion two-photon transition form factor

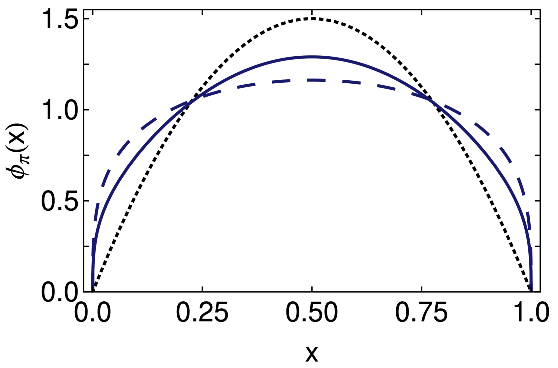


- Dotted (purple) – asymptotic limit: $Q^2 G_\pi(Q^2) \overset{Q^2\to\infty}{\approx} 4\pi^2 f_\pi$,
- Dashed (brown) – vector meson dominance (VMD) result: $2f_\pi G(Q^2) = m_\rho^2/(m_\rho^2 + Q^2)$, it is a reasonable approximation on $Q^2 \approx 0$, but it approaches an asymptotic limit of $m_\rho^2/(2f_\pi)$, which is just 90% of the result associated with the asymptotic limit.

Raya, Chang, Bashir, Cobos-Martinez, Gutierrez-Guerrero, Roberts, Tandy, Phys. Rev. D 93 (2016) 7, 074017.

Pion two-photon transition form factor

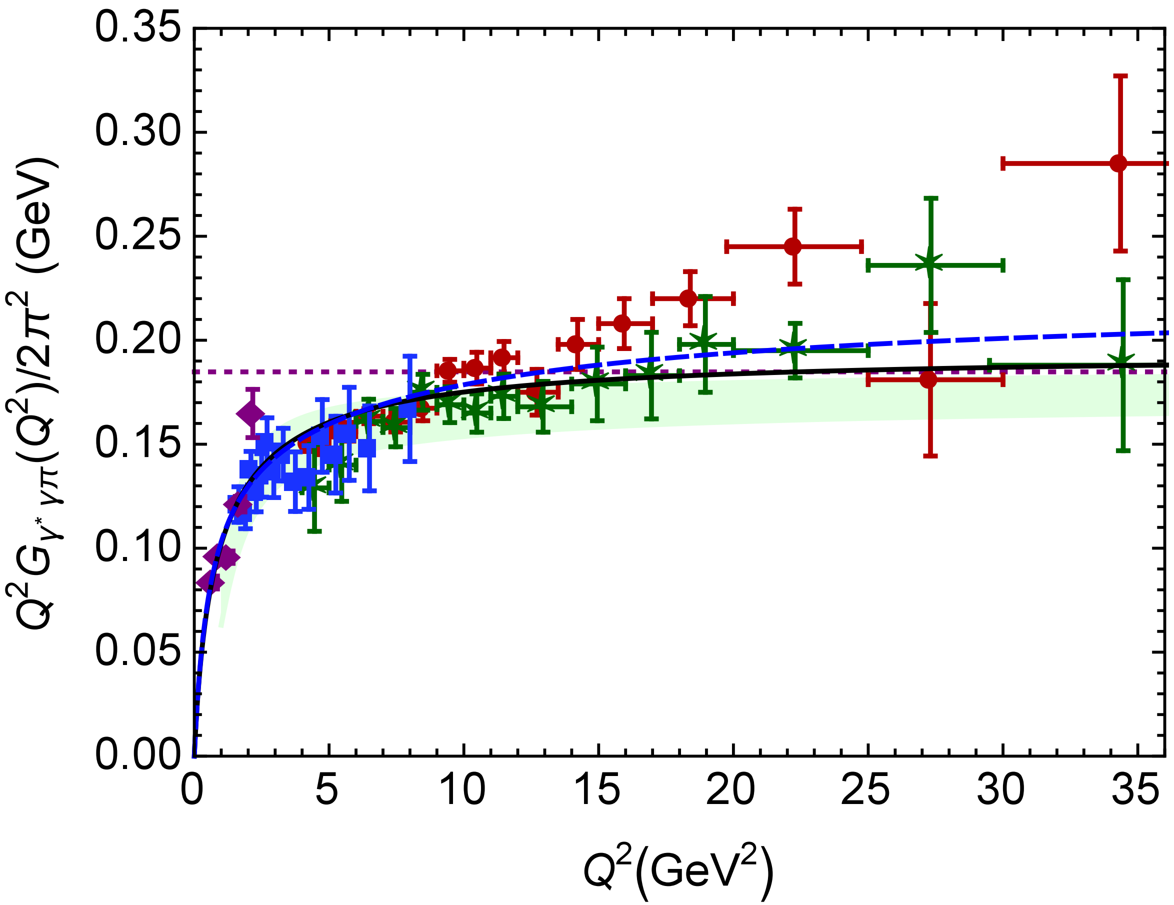




Chang, Cloet, Cobos-Martinez, Roberts, Schmidt, Tandy. Phys. Rev. Lett. 110 (2013) 13, 132001.

- Solid (black) curve: broad and concave concave DA which evolves with scale.
- At $Q^2 = 40\,\text{GeV}^2$, the inverse moment $w(Q^2 = 40\text{GeV}^2) = 3.163$. Recall that in the asymptotic limit, $w_\varphi = 3$.
- $G(Q^2)$ is monotonically increasing and concave and reaches a little above the asymptotic limit.
- The growth is logarithmically slow, however; and whilst the curve remains a line-width above the asymptotic limit on a large domain, logarithmic growth eventually becomes suppression, and the curve thereafter proceeds towards the QCD asymptotic limit from above.

Pion two-photon transition form factor

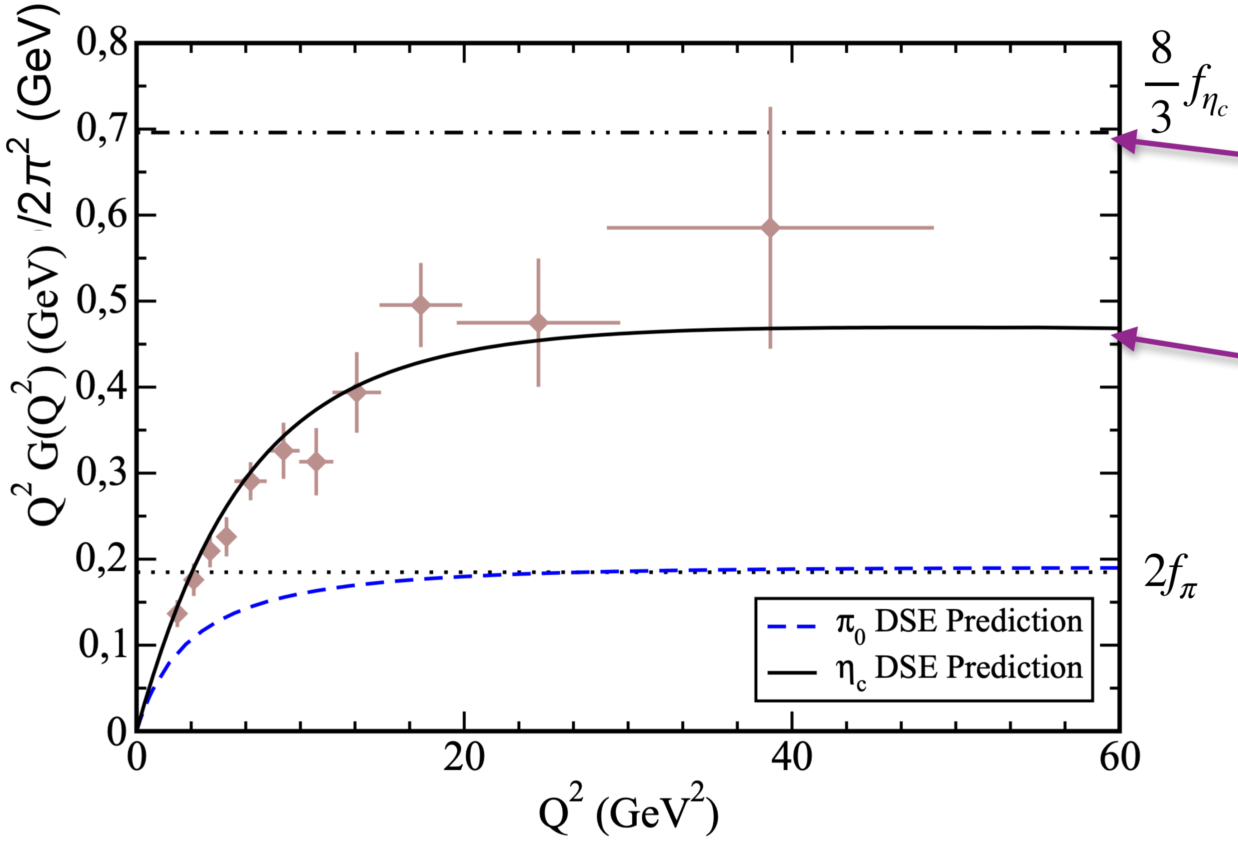


- Data: CELLO - diamonds (purple); CLEO - squares (blue); **BaBar** - circles (red); **Belle** - stars (green). Two available sets exhibit conflicting trends in their evolution with photon virtuality.
- CSMs results favor the Belle data, The situation may be clarified by upcoming data from BelleII.

BaBar Collaboration: B. Aubert et al., Phys. Rev. D 80, 052002 (2009).

Belle Collaboration: S. Uehara et al., Phys. Rev. D 86, 092007 (2012).

η_c two-photon transition form factor



Raya, Ding, Bashir, Chang, Roberts. Phys.Rev.D 95 (2017) 7, 074014

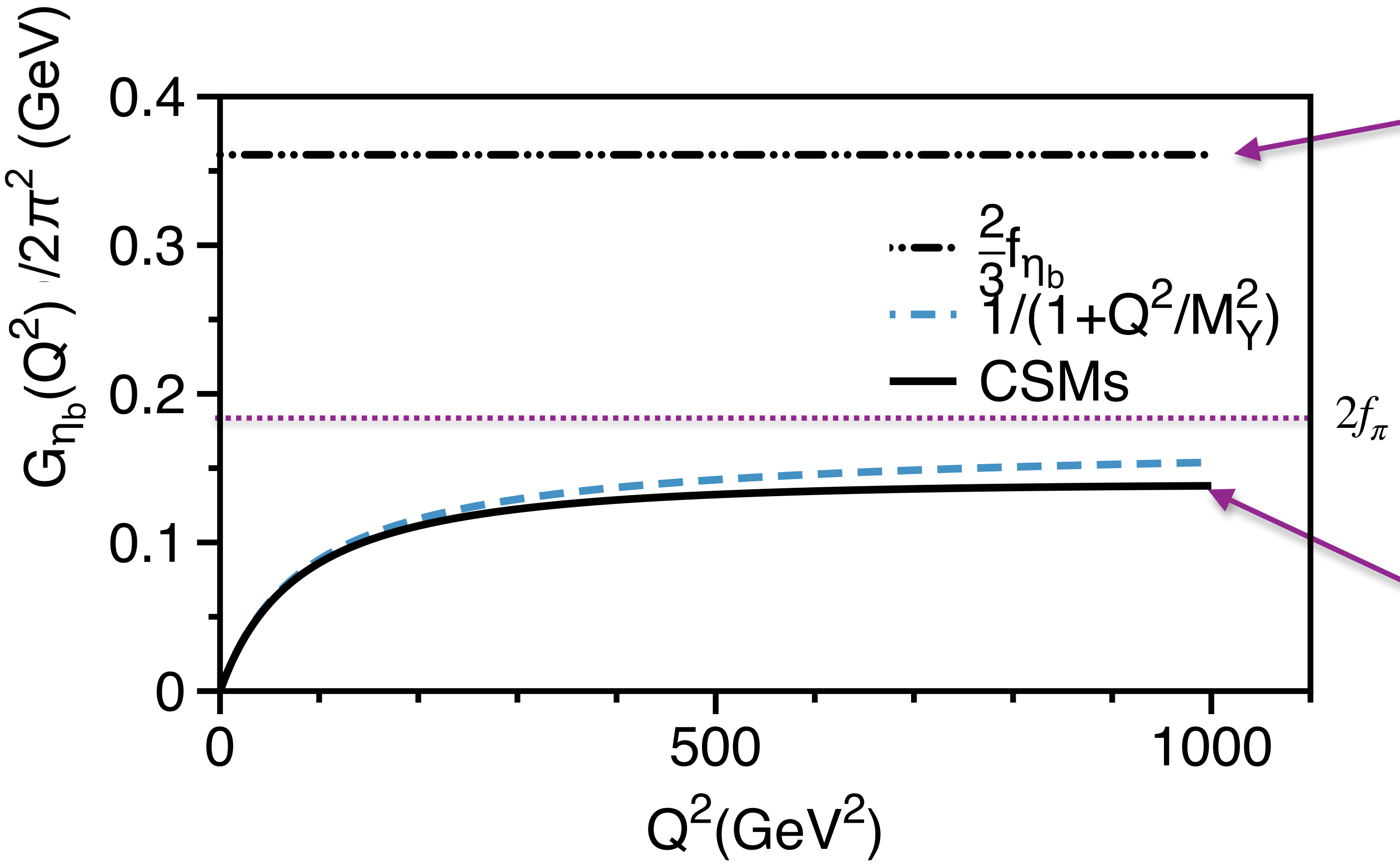
BaBar Collaboration: Phys.Rev.D 81 (2010) 052010

- Asymptotic limit: $Q^2 G_{\eta_c}(Q^2) \overset{Q^2\to\infty}{\approx} \frac{4}{3} 4\pi^2 f_{\eta_c}$,
- Asymptotic ratio:

$$\lim_{Q^2\to\infty} [R(Q^2) := G_{\eta_c}(Q^2)/G_\pi(Q^2)] = \frac{4f_{\eta_c}}{3f_\pi} \approx 3.73$$

- At $Q^2_{35} = 35\,\text{GeV}^2$, $\lim\limits_{Q^2\to\infty} [R(Q^2_{35}] = 2.4$
- Pion: $\lim\limits_{Q^2\to\infty} [Q^2 G_\pi(Q^2_{35})] \approx 2f_\pi$
- The reason of mismatch is $G_{\eta_c}(Q^2_{35})$
- At small Q^2, $\varphi_{\eta_c}(x)$ is much narrower than $6x(1-x)$, therefore $G_{\eta_c}(Q^2)$ should lie far below its asymptotic limit value
- Moreover, given that ERBL evolution is logarithmic, this must remain the case even at $Q^2 \geq 10^3$ GeV^2.

η_b two-photon transition form factor



Chen, Ding, Chang, Liu, Phys.Rev.D 95 (2017) 1, 016010

- Asymptotic limit:

$$Q^2 G_{\eta_b}(Q^2) \overset{Q^2\to\infty}{\approx} \frac{1}{3} 4\pi^2 f_{\eta_b},$$

- Asymptotic ratio:

$$\lim_{Q^2\to\infty} [R(Q^2) := G_{\eta_b}(Q^2)/G_\pi(Q^2)]$$

$$= \frac{f_{\eta_b}}{3f_\pi} \approx 1.94$$

- $G_{\eta_b}(Q^2)$ matches well with VMD model.
- At small Q^2, $\varphi_{\eta_b}(x)$ is much narrower than $6x(1-x)$, therefore $G_{\eta_b}(Q^2)$ should lie far below its asymptotic limit value.

Summary and Outlook

- Summary
  - ✓ Distribution amplitudes (DAs): pion, η_c and η_b.
  - ✓ DA of pion is brand and concave, DAs of η_c and η_b are narrower than 6x(1-x).
  - ✓ Two-photon transition form factors (TFFs): pion, η_c and η_b.
  - ✓ TFF of pion approaches its asymptotic limit from above, and TFFs of η_c and η_b approach their asymptotic limits from below.
- Outlook

  Hadron structure, such as Parton distribution function (DF), transverse momentum dependent distribution (TMD), generalized parton distribution (GPD), fragmentation function (FF), etc..


Thank you!