The Shape Function of Heavy Meson in QCD and HQET

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第七届全国重味物理与量子色动力学研讨会

Outline

- Flavor Physics and Heavy Meson Profile
- Shape Functions and Factorization Formula
- Summary and Outlook

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Heavy Flavor Physics

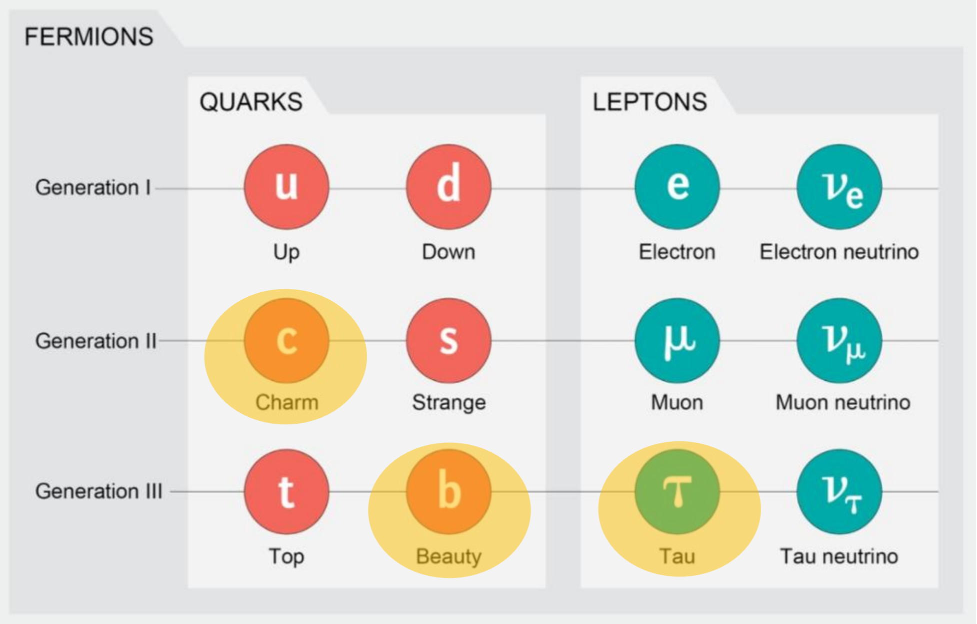
- The charm quark was predicted by Glashow, Iliopoulos and Maiani in 1970.

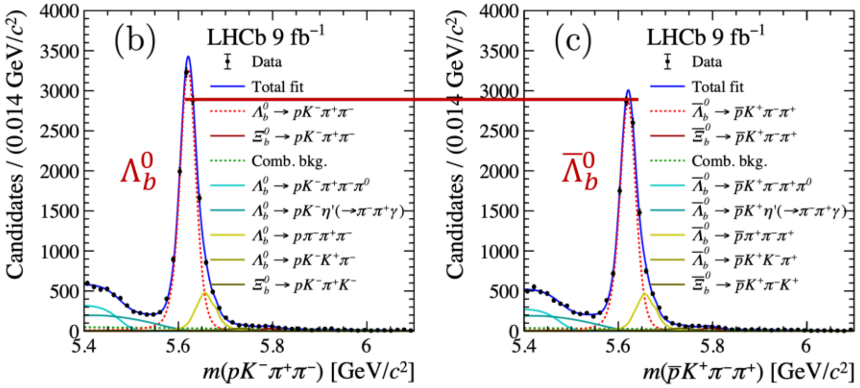
1976

- The bottom quark was first predicted in 1973 by Kobayashi and Maskawa to explain CP violation.

2008

- Heavy Flavor Physics: b, c, τ.

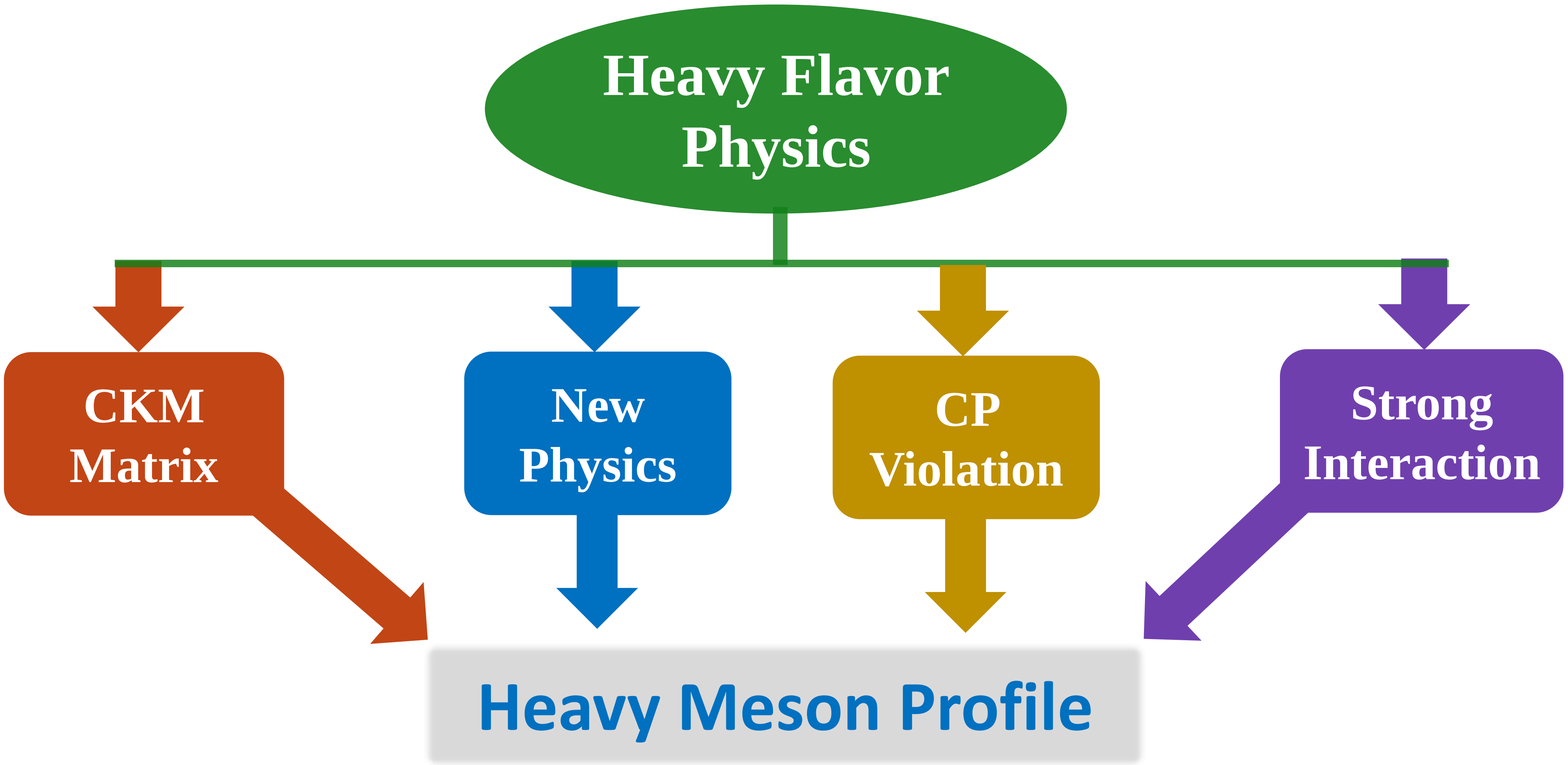




$$A_{CP} = (2.45 \pm 0.46 \pm 0.10)\%$$

Flavor Physics Plays a Very Important Role in Particle Physics

Heavy Meson Profile



Heavy Meson Profile Plays a Very Important Role in Flavor Physics

Heavy Quark Effective Theory

➢ The Lagrangian of HQET.

$$\mathcal{L}_{\text{eff}} = \bar{h}_v iv \cdot D h_v + \frac{1}{2m_Q} \sum_{n=0}^{\infty} \bar{h}_v i \not{D}_\perp \left(-\frac{iv \cdot D}{2m_Q} \right)^n i \not{D}_\perp h_v \,.$$

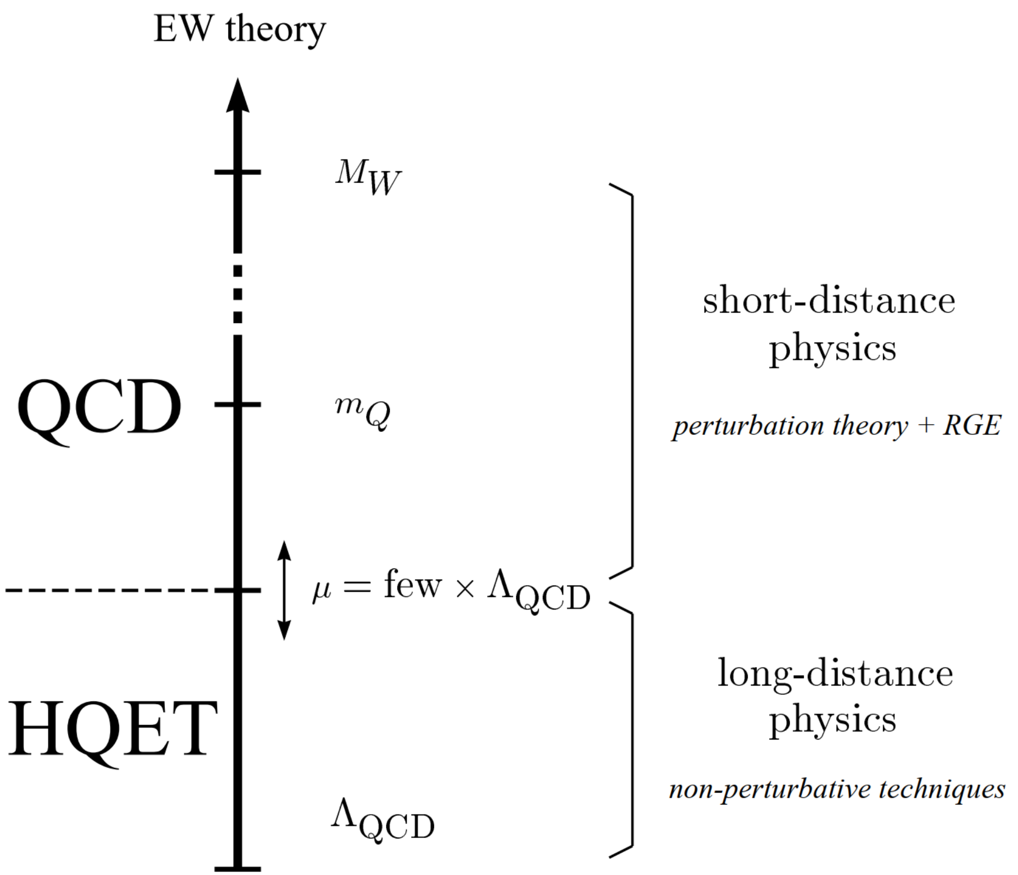


Figure 5: Philosophy of the heavy-quark effective theory.

[Neubert, Subnucl.Ser 34, 98-165 (1997)]

HQET is Constructed to Describe Heavy Flavor Physics

Heavy Quark Effective Theory

- Using HQET, observables can be written schematically as series.

$$\text{Observable} = \sum_{n=0}^{\infty} \sum_{j} c_n^j(\mu) \frac{\langle O_n^j(\mu) \rangle}{m_Q^n} .$$

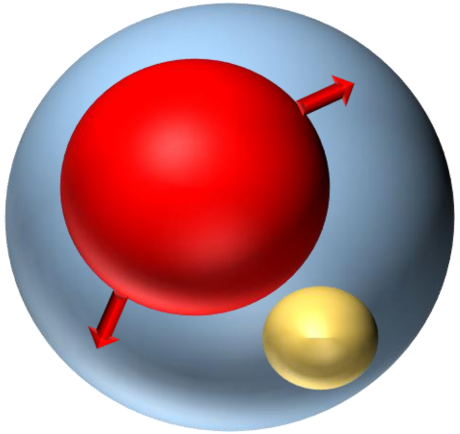
Wilson coefficients

- **Decay constant**
- **Shape function (SF)**
- **Light-cone distribution amplitude (LCDA)**
- **Form factor**

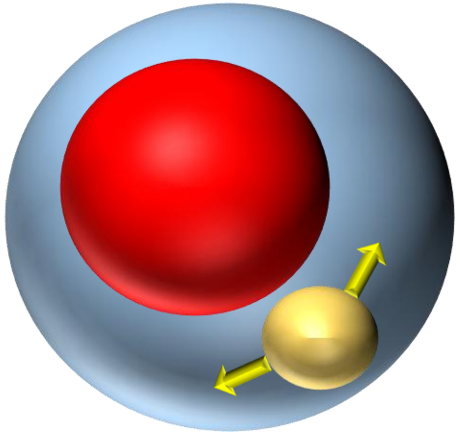
SF and LCDA are Crucial to Describe Heavy Meson Profile

Shape Function and Light-Cone Distribution Amplitude

A heavy flavor meson consists of a pair of heavy and light quarks.



- SF characterizes the momentum distribution function of the heavy quark.



- LCDA describes the momentum distribution amplitude of the light quark.

Together, they provide the most essential information about the profile of heavy mesons.

The “$|V_{ub}|$ Puzzle”

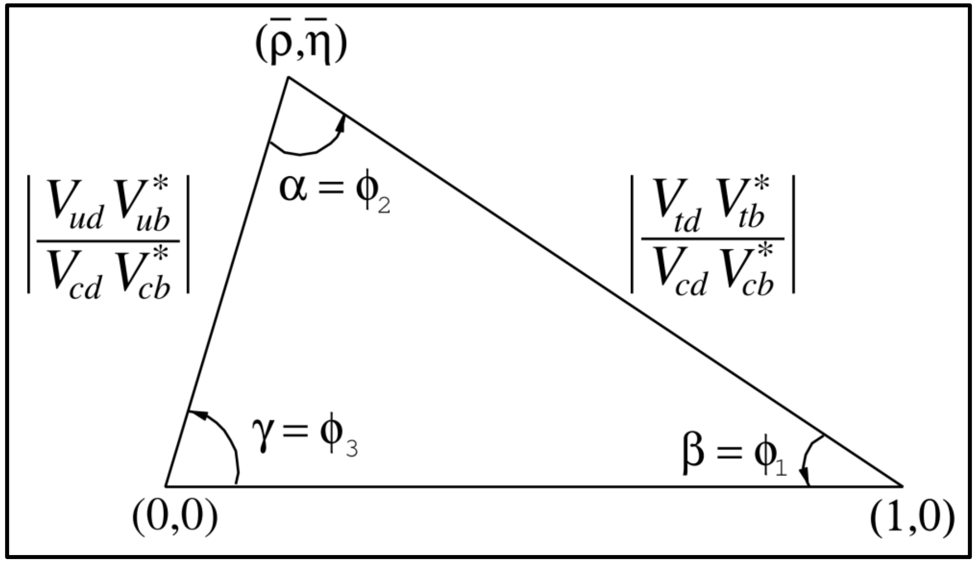
- The $|V_{ub}|$ tension

$$|V_{ub}| = (4.13 \pm 0.12\,^{+\,0.13}_{-\,0.14} \pm 0.18) \times 10^{-3} \quad \text{(inclusive)},$$ via $B \to X_u\, \ell\, \nu$ [PDG (2024)]

$$|V_{ub}| = (3.70 \pm 0.10 \pm 0.12) \times 10^{-3} \quad \text{(exclusive)},$$ via $B \to \pi\, \ell\, \nu$ [PDG (2024)]

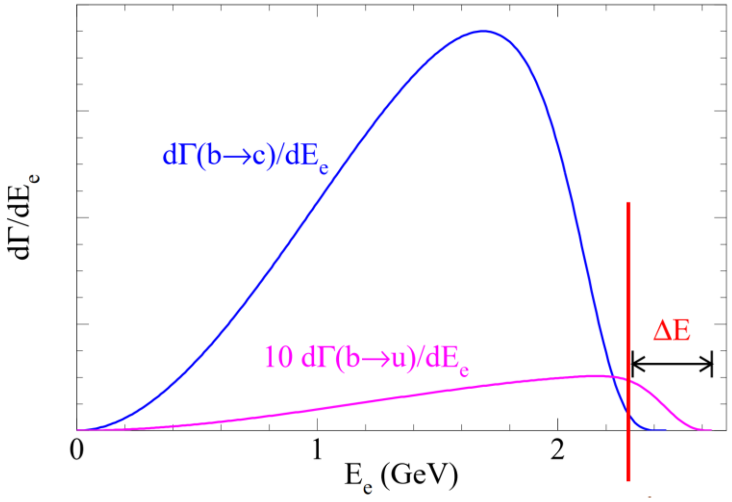
- The values derived from $|V_{ub}|/|V_{cb}|$ ratio measurements.

$$|V_{ub}| = (3.43 \pm 0.32) \times 10^{-3} \qquad (\mathrm{B_s},\ \Lambda_\mathrm{b}),$$



The "$|V_{ub}|$ Puzzle"

➢ The **inclusive** process: $B \to X_u \, \ell \, \nu$

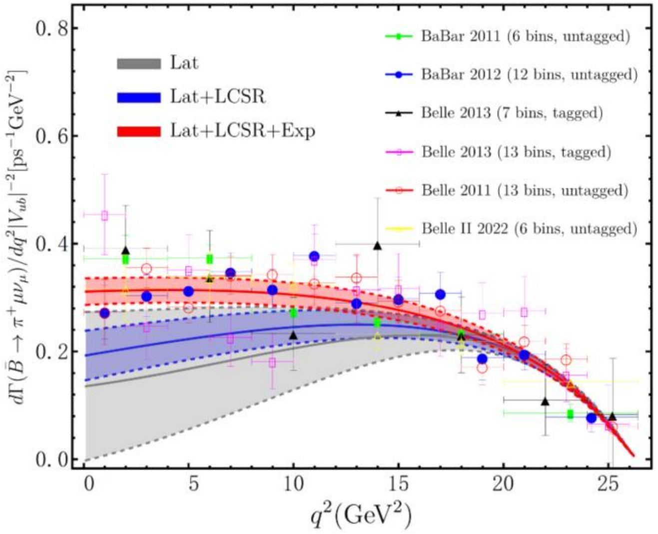


$$\frac{d\Gamma}{dq^2\, dE_e\, dE_\nu} = \frac{|V_{jb}|^2 \, G_F^2}{2\pi^3} \left[W_1 q^2 + W_2 \left(2E_e E_\nu - \tfrac{1}{2} q^2 \right) + W_3 q^2 \left(E_e - E_\nu \right) \right] ,$$

$$W^{\mu\nu} = \sum_{i,j=1}^{3} H_{ij}(\bar{n} \cdot p) \, \mathrm{tr} \left(\bar{\Gamma}_j^\mu \frac{\not{p}_-}{2} \Gamma_i^\nu \frac{1+\not{v}}{2} \right) \int d\omega \, J(p_\omega^2) \, S(\omega)$$

[Neubert et.al, NPB, 699 (2004)]

➢ The **exclusive** process: $B \to \pi \, \ell \, \nu$



$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |p_\pi|^3 |f_+(q^2)|^2,$$

$$f_{B\pi}^+(0) + f_{B\pi}^-(0) = \frac{f_B}{f_\pi \, m_B} \int_0^{s_0^\pi} ds e^{-s/M^2} \left[\frac{m_B^2}{m_B^2 - s} \phi_+^B(s/m_B) - 2 \frac{s}{m_B^2 - s} \phi_-^B(s/m_B) + 2 \frac{m_B^3}{(m_B^2 - s)^2} \overline{\Phi}_\pm^B(s/m_B) \right] ,$$

[Khodjamirian et.al, PRD, 75 (2007)]

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Definition on Shape Functions

➢ The definition on SF defined in HQET

$$S^{\mathrm{HQET}}(\omega,\mu)=\int_{-\infty}^{+\infty}\frac{dt}{2\pi}e^{i\omega v^+ t}\frac{\langle B(v)|\bar{h}_v(0)\,W(0,tn_+)\,h_v(tn_+)|B(v)\rangle}{\langle B(v)|\bar{h}_v(0)\,h_v(0)|B(v)\rangle}\,.$$

Here h_v is the heavy quark field defined in HQET, the variable has support $-\infty<\omega<\bar{\Lambda}$, with $\bar{\Lambda}=m_B-m_b$.

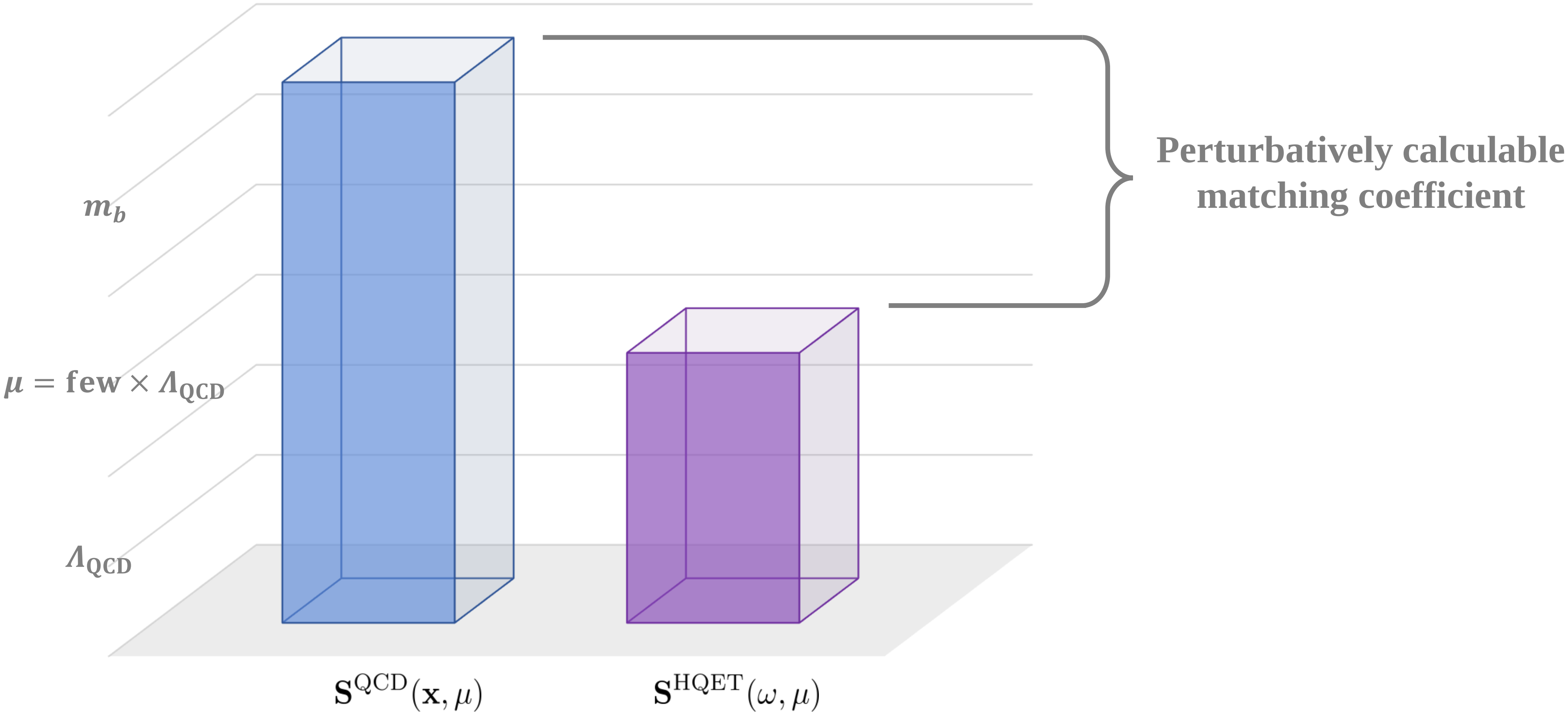
➢ The definition on SF defined in QCD

$$S^{\mathrm{QCD}}(x,\mu)=\int_{-\infty}^{+\infty}\frac{dz^-}{2\pi}e^{-ixp_B^+z^-}\frac{\langle B(p_B)|\bar{b}(0)\,\Gamma\,W(0,z)\,b(z)|B(p_B)\rangle}{\langle B(p_B)|\bar{b}(0)\,\Gamma b(0)|B(p_B)\rangle}\,.$$

Here the heavy quark field is defined in QCD, the variable has support $0<x<1$.

Comparison Between Shape Functions in QCD and HQET

➢ The typical energy scales possessed by S^{QCD} and S^{HQET} are different.



➢ The SFs should be divided by different regions: "peak region" and "tail region".

Factorization Formula

- The factorization formula between S^{QCD} and S^{HQET}.

$$S^{\mathrm{QCD}}(x,\mu) = \begin{cases} Z_{\mathrm{peak}}(x,\omega,\mu) \otimes S^{\mathrm{HQET}}(\omega,\mu)\,, & x \sim 1 - \Lambda_{\mathrm{QCD}}/m_b \quad \text{(peak region)} \\ Z_{\mathrm{tail}}(x,\mu)\,, & x \sim \Lambda_{\mathrm{QCD}}/m_b \quad \text{(tail region)} \end{cases}$$

- Expand the shape functions and matching coefficient,

$$S^{\mathrm{QCD}}(x,\mu) = S^{\mathrm{QCD}(0)}(x,\mu) + \frac{\alpha_s C_F}{2\pi} S^{\mathrm{QCD}(1)}(x,\mu) + \mathcal{O}(\alpha_s^2)\,,$$

$$S^{\mathrm{HQET}}(\omega,\mu) = S^{\mathrm{HQET}(0)}(\omega,\mu) + \frac{\alpha_s C_F}{2\pi} S^{\mathrm{HQET}(1)}(\omega,\mu) + \mathcal{O}(\alpha_s^2)\,.$$

$$Z_{\mathrm{peak}}(x,\omega,\mu) = Z^{(0)}_{\mathrm{peak}}(x,\omega,\mu) + \frac{\alpha_s C_F}{2\pi} Z^{(1)}_{\mathrm{peak}}(x,\omega,\mu) + \mathcal{O}(\alpha_s^2)\,,$$

$$Z_{\mathrm{tail}}(x,\mu) = Z^{(0)}_{\mathrm{tail}}(x,\omega,\mu) + \frac{\alpha_s C_F}{2\pi} Z^{(1)}_{\mathrm{tail}}(x,\mu) + \mathcal{O}(\alpha_s^2)\,.$$

Factorization Formula

- Identifying momentums of the B-meson and b-quark as $p_B^+ = m_B v^+,\ p_b^+ = m_b v^+ + k^+$.

- **At tree level:**

$$\delta(x m_B v^+ - m_b v^+ - k^+) = \int_{-\infty}^{\bar{\Lambda}} d\omega \, Z_{\text{peak}}^{(0)}(x, \omega, \mu) \delta(\omega v^+ - k^+) \,.$$

With

$$Z_{\text{peak}}^{(0)}(x, \omega, \mu) = \delta(\omega v^+ + m_b v^+ - x m_B v^+) \,.$$

$$Z_{\text{tail}}^{(0)}(x, \omega, \mu) = 0 \,.$$

- Therefore the shape function defined in QCD has a typical support in a small region close to the endpoint

$$x = \frac{\omega + m_b}{m_B} \sim 1 \,.$$

Factorization Formula

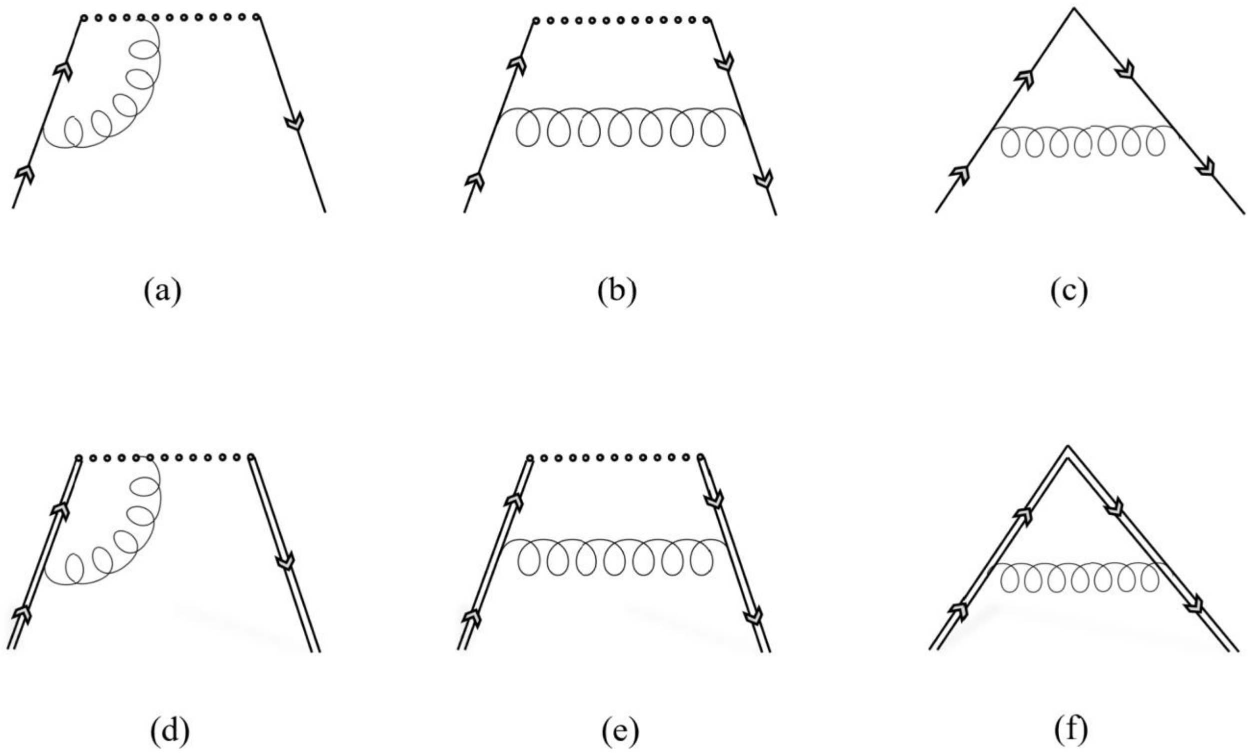
➢ **At one-loop level:**

1. This necessitates a region-separated calculation.

2. Work in the space-time dimension $d = 4 - 2\epsilon$ and use $\overline{\text{MS}}$ scheme.

3. Keep $v \cdot k$ non-zero and use plus function

$$F\left(\omega, k^{+}\right) = \left[F\left(\omega, k^{+}\right)\right]_{\oplus} + \delta\left(\omega - k^{+}\right) \int_{0}^{\Lambda} dt\, F(\omega, t)\,.$$



Factorization Formula

- The matching function at one-loop

$$Z_{\rm tail}^{(1)}(x,\mu)=\frac{1}{m_b v^+}\frac{1+x^2}{1-x}\left[-1+\ln\frac{\mu^2}{(1-x)^2 m_b^2}\right].$$

$$Z_{\rm peak}^{(1)}(x,\omega,\mu)=\left(\frac{1}{2}\ln^2\frac{\mu^2}{m_b^2}-\frac{3}{2}\ln\frac{\mu^2}{m_b^2}+\frac{\pi^2}{12}-2\right)\delta(x m_B v^+-m_b v^+-\omega v^+)\,.$$

1. This result implies the validity of the factorization formula.

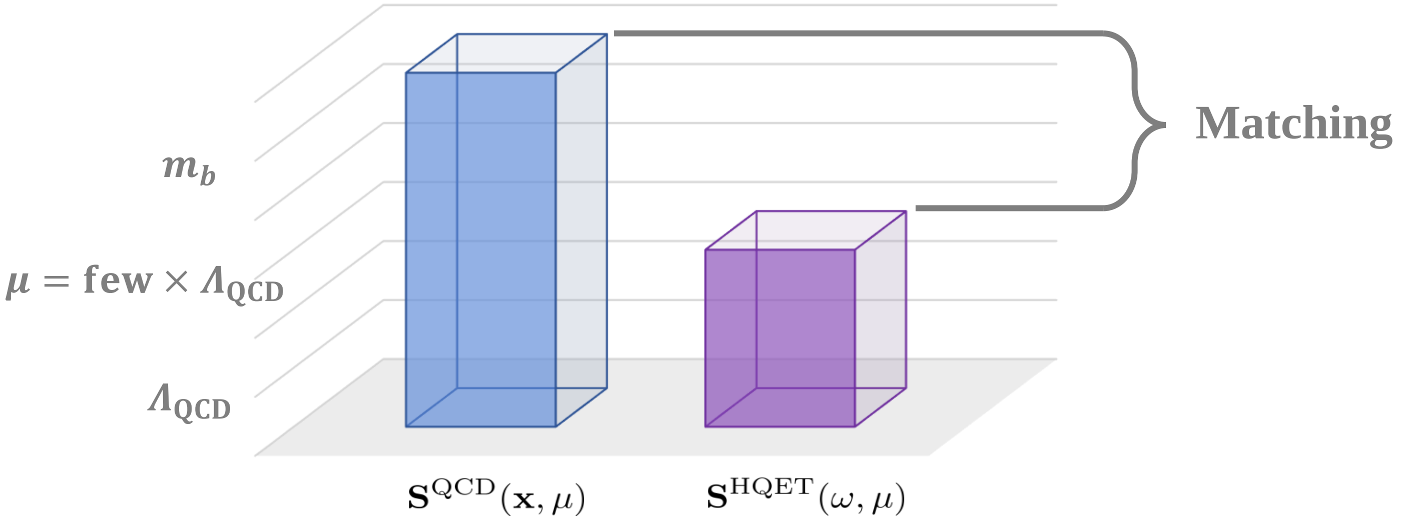
2. The plus distributions cancel out in the matching, yielding remarkably simple form of factorization formula.

3. The “refactorization framework” developed in this work is in a similar spirit to LCDAs defined in QCD and HQET.

[Yu Jia, De-Shan Yang et.al, PRL, 125 (2020)] [Beneke, Yan-Bing Wei et.al, JHEP, 09 (2023)]

The Two-Step Matching Scheme for Shape Function

➢ **Two-step factorization to access heavy meson LCDA.**



The Two-Step Matching Scheme for Shape Function

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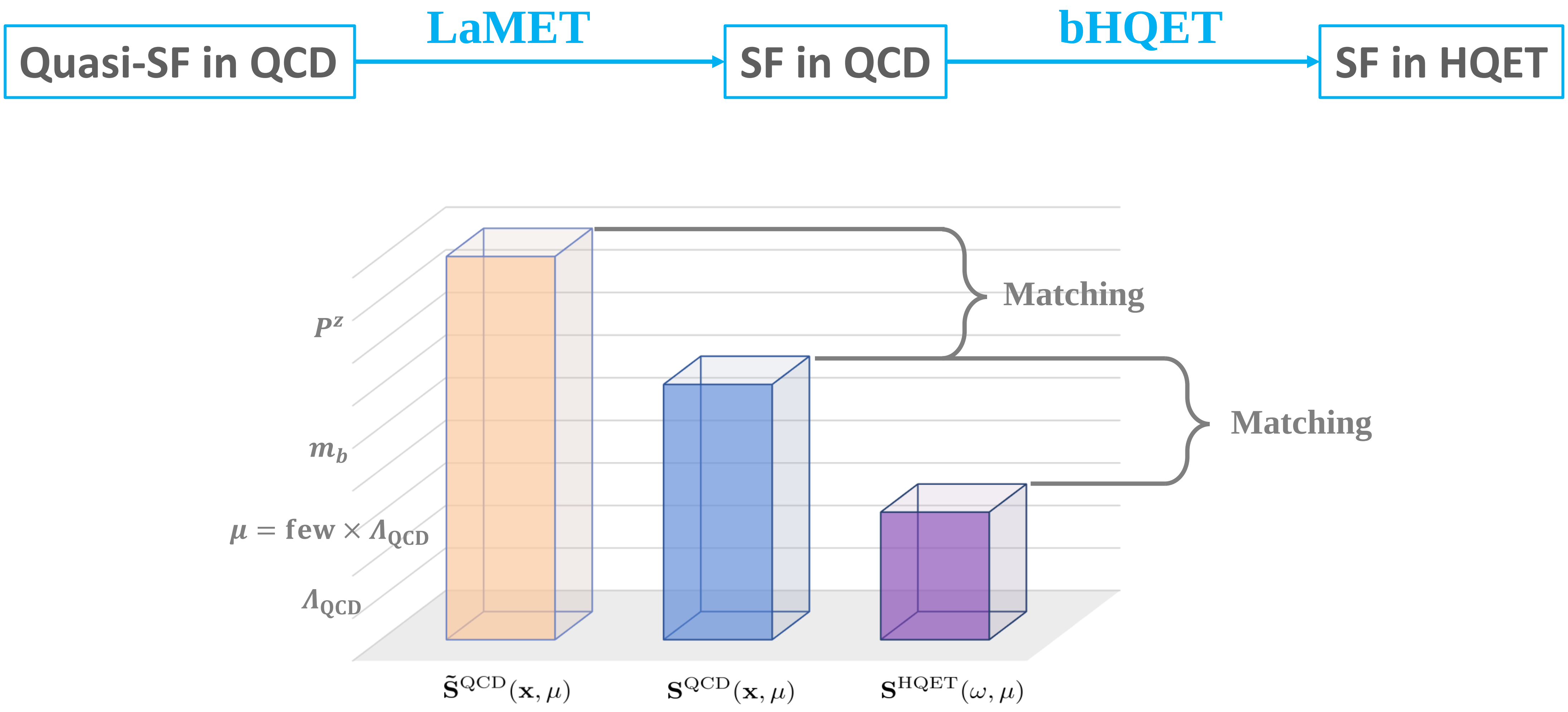


Determining QCD Shape Function from Model

- It is instructive to understand the characteristic feature of QCD shape function and beneficial for future lattice simulations.

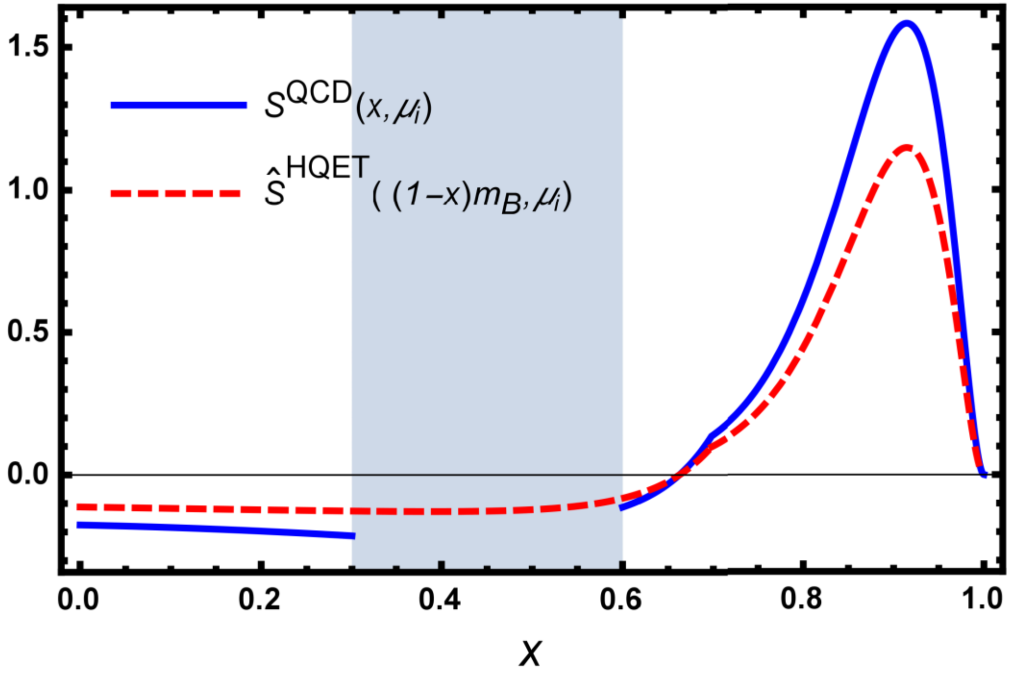
- Taking advantage of a widely adopted model of HQET shape function at the soft scale $\mu_i = 1.5$ GeV,

[Neubert et.al, NPB, 699 (2004)]

$$\hat{S}^{\rm HQET}(\hat{\omega},\mu) = \frac{N}{A}\left(\frac{\hat{\omega}}{A}\right)^{b-1}\exp\left(-b\frac{\hat{\omega}}{A}\right) - \frac{\alpha_s C_F}{\pi}\frac{\theta(\hat{\omega}-A-\mu/\sqrt{e})}{\hat{\omega}-A}\left(2\ln\frac{\hat{\omega}-A}{\mu}+1\right) .$$

$$N = \left[1-\frac{\alpha_s C_F}{\pi}\left(\frac{\pi^2}{24}-\frac{1}{4}\right)\right]\frac{b^b}{\Gamma(b)} .$$

with $\hat{\omega} = \bar{\Lambda} - \omega \geq 0$.



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Mass Dependence

[Shuai Zhao et.al, 2411.07101]

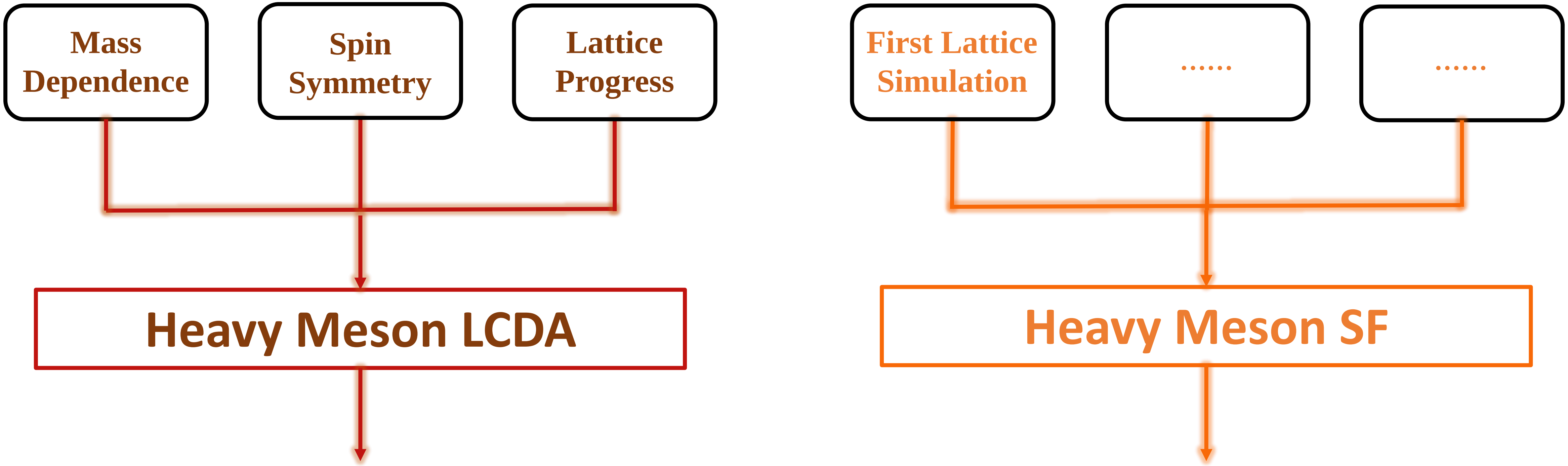
Spin Symmetry

[Jun Zeng et.al, PRD, 110 (2024)]

Lattice Progress

[Xue-Ying Han's talk]
[Qi-An Zhang's talk]

Heavy Meson LCDA


Mass Dependence
Spin Symmetry
Lattice Progress
Heavy Meson LCDA
First Lattice Simulation
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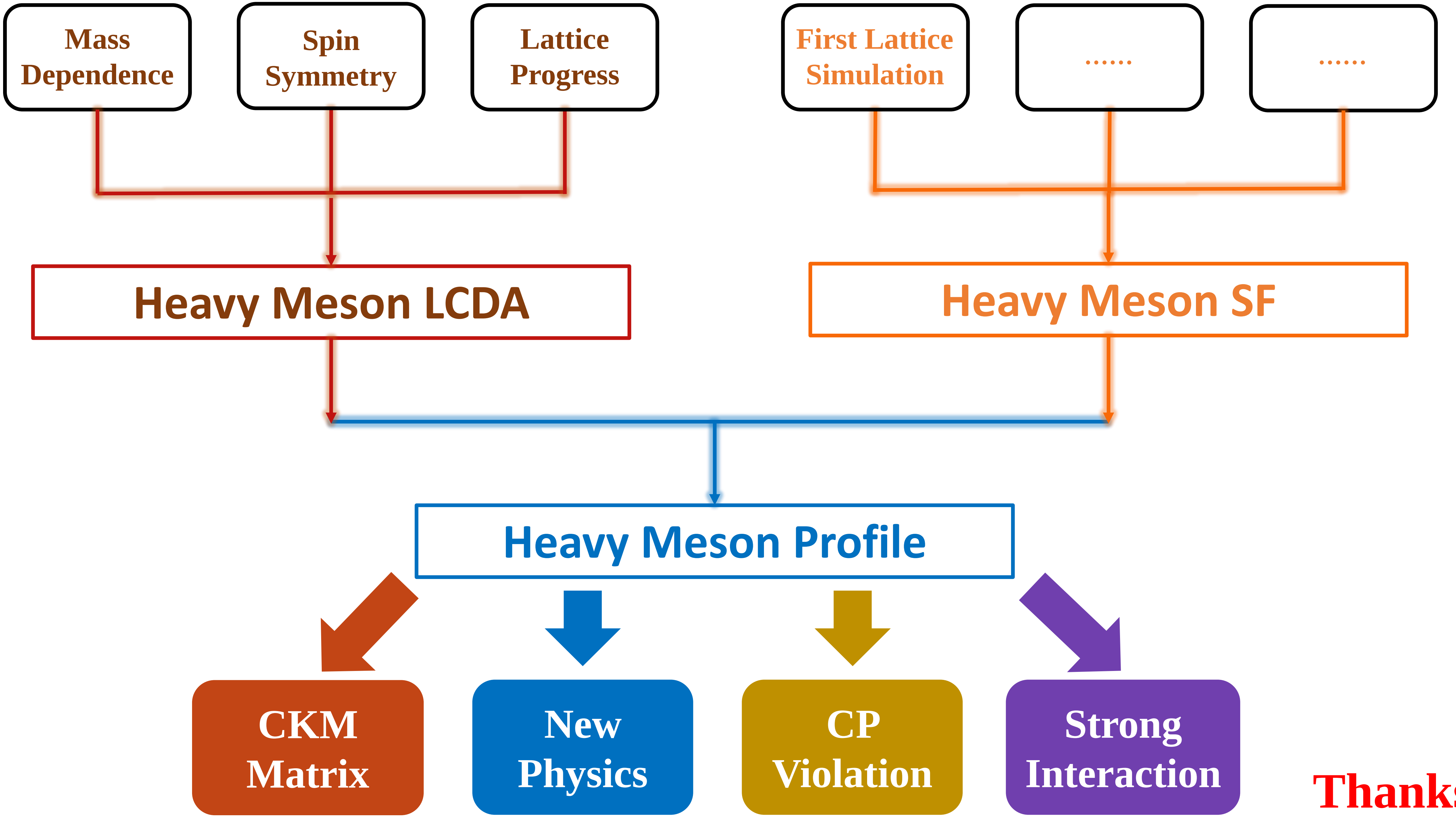
Heavy Meson SF


Mass Dependence
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……
Heavy Meson LCDA
Heavy Meson SF
Heavy Meson Profile
CKM Matrix
New Physics
CP Violation
Strong Interaction

Thanks!