QED corrections to $B_u \to \tau\nu$ at Subleading Power

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Based on:

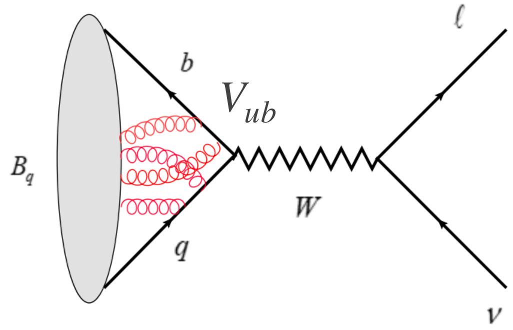
QED corrections to $B \to \tau\,\nu$ at Subleading Power

in preparation with **Y.L. Shen (沈月龙) , C. Wang (王超) and Y.B. Wei (魏焰冰)**

Why $B_u \to \ell\nu$?

be interesting for several reasons:

- **Determination of $|V_{ub}|$** largely unaffected by hadronic uncertainties



$$\Gamma \sim m_\ell^2 f_{B_u}^2 |V_{ub}|^2$$

- Helicity suppression offers sensitive **probe of (pseudo)scalar new interactions**

$$\bar{\ell}\gamma^\mu P_L \nu \to \frac{m_\ell}{m_b} \bar{\ell} P_L \nu$$

- Testing Lepton Flavor Universality in charged currents

$$B_u \to \ell\nu, \quad \ell = \mu, \tau$$

Why do we need to know the QED corrections?

- QCD matrix element is known with **<1% accuracy**

$$\langle 0|\,\bar{u}\gamma^\mu\gamma_5 b\,|B^-(p)\rangle = i f_{B_u} p^\mu \quad \text{with} \quad f_{B_u} = (189.4 \pm 1.4)\,\text{MeV}$$

[FNAL/MILC 2017]

QED corrections can be of similar magnitude or even larger, due to presence of **large logarithms** $\alpha\ln(m_b^2/m_\ell^2)$ and $\alpha\ln(m_\ell/E_\gamma)\ln(m_b/m_\ell)$

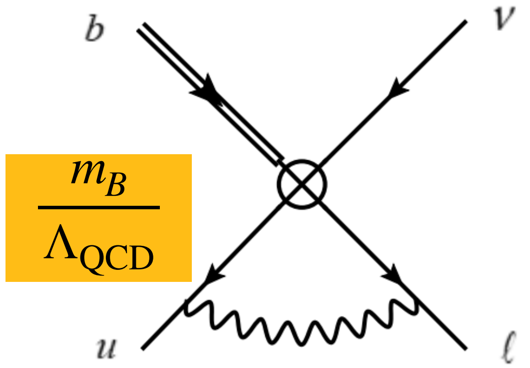
$\rightarrow$ **compete with QCD uncertainties**

- Belle II will measure the τ, μ channels with **5 − 7 % uncertainty**

[Belle II Physics Book]

$\rightarrow$ **compete with experimental uncertainties**

- scale $\Lambda_{\rm QCD} < \mu < m_b$ gives rise to more intricate effect, as virtual photons can **resolve the structure of *B* meson**

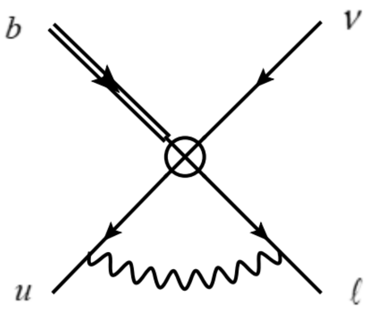


$\longrightarrow$ new effects: power enhanced effects

$B_s \to \mu^+\mu^-$ *[M. Beneke, etc. 17 & 19]*

$B_s \to \tau^+\tau^-$ *[Y.K.Huang, Y.L.Shen, X.C.Zhao, SHZ 23]*

Main challenges in formulating a factorization theorem



1. Quark current $\bar{u}\,\gamma^\mu P_L\, b$ is not gauge invariant under QED

$$\bar{u}\,\gamma^\mu P_L\, b \quad \longrightarrow \quad \bar{u}\,\gamma^\mu P_L\, b\, S_{n_-}^{(\ell)\dagger}$$

add a **Wilson line** $S_{n_-}^{(\ell)\dagger}$ to account for soft photon interactions with charged lepton $\rightarrow$ anomalous dimension sensitive to **IR regulators**

2. Beyond leading power convolutions have **endpoint divergences**

[Feldmann, Gubernari, Huber, Neubert, Seitz 2022; Hurth, Neubert, Szafron 2023]

cannot be dealt with using standard renormalization techniques and require appropriate subtractions.

e.g. "refactorization-based subtraction (RBS) scheme" in $B_u \rightarrow \mu\,\nu$

QED corrections for $B_u \to \tau \nu$

A muti-scale process

focus on $B_u \to \tau\,\nu$ new scales appear in the present of QED effects

m_W

$m_b \sim \mu_h$

$m_\tau \sim \mu_{hc}$

$\Lambda_{\rm QCD} \sim \mu_s, \mu_{sc}$

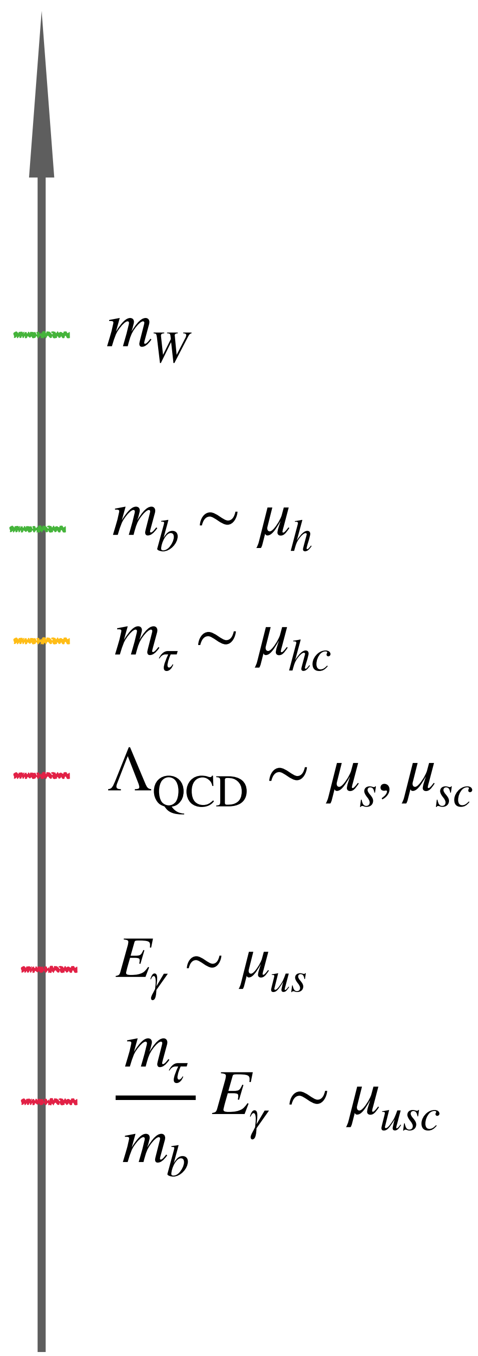
- relevant modes for virtual QED corrections

$E_\gamma \sim \mu_{us}$

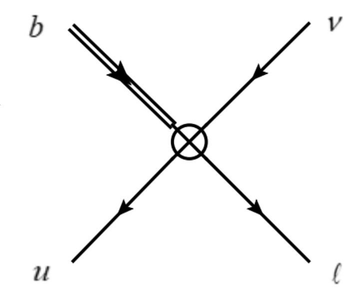
$\frac{m_\tau}{m_b} E_\gamma \sim \mu_{usc}$

- relevant modes for real QED corrections

A muti-scale process



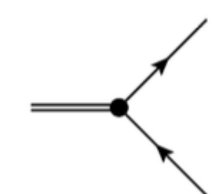
- QED for $\mu > m_b$ included in Effective weak Hamiltonian



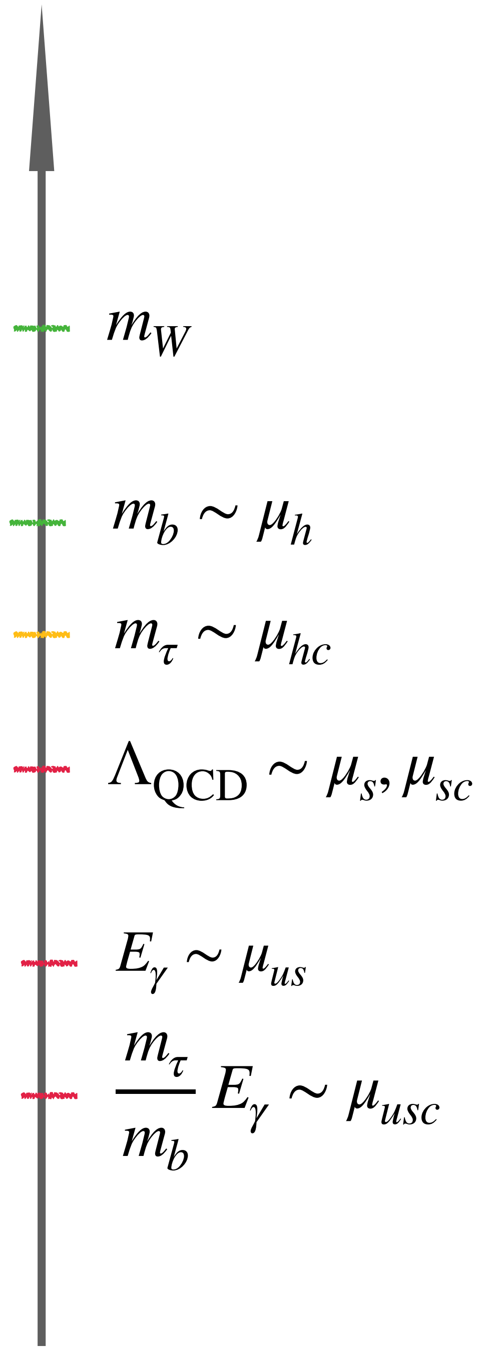
$$\mathcal{L}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} K_{\rm EW}(\mu) V_{ub} \, (\bar{u}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu_\ell)$$

- Ultrasoft photons $\mu \ll \Lambda_{\rm QCD}$ see B meson as point-like particle

[Isidori, Nabeebaccus, Zwicky 2020; Zwicky 2021; Dai, Kim, Leibovich 2021]



A muti-scale process



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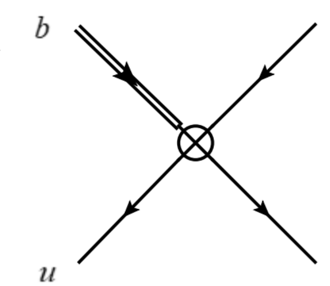

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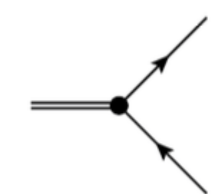
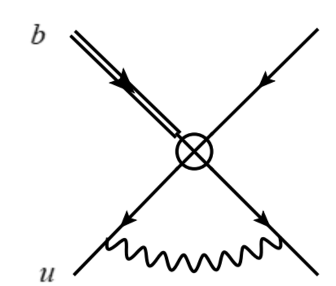


- Intermediate scale $\Lambda_{\rm QCD} < \mu < m_b$, virtual photons can **resolve the structure of B meson**



different scale hierarchies require different effective field-theory constructions

$B_u \to \mu \nu$ *[M.Neubert, etc 2023]*

focus on $B_u \to \tau \nu$

needs EFTs

Turning a muti-scale into a product of single scale

- Identifying the appropriate EFT description at each scale.
- Performing a step-by-step matching between each EFT.
- Deriving a factorization theorem to break this multi-scale problem into a convolution of single-scale objects.
- Using the renormalisation group to evaluate each object at its natural scale and run it to a common scale to resum logarithms.

In this talk, we focus on the **virtual QED corrections**

From Fermi theory to HQET $\times$ SCET$_{\mathrm{I}}$

m_W

$m_b \sim \mu_h$ ☑ Fermi Theory

$m_\tau \sim \mu_{hc}$ ☐ …

$\Lambda_{\mathrm{QCD}} \sim \mu_s$

$E_\gamma \sim \mu_{us}$

$\dfrac{m_\tau}{m_b} E_\gamma \sim \mu_{usc}$

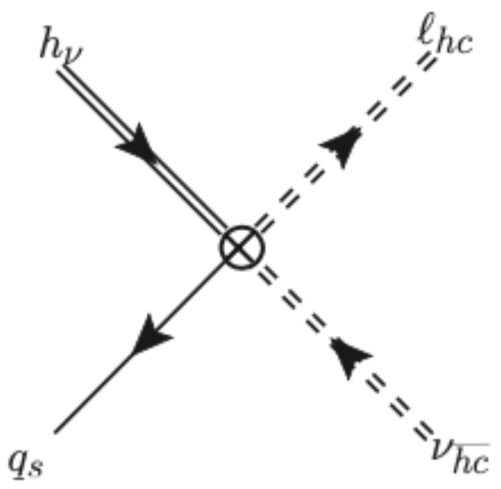
$$\mathscr{L}_{\mathrm{eff}} = -\frac{G_F}{\sqrt{2}} V_{ub} [\bar{q}\, \gamma_\mu (1-\gamma_5)\, b][\bar{\ell}\, \gamma^\mu (1-\gamma_5)\, \nu]$$

$$\equiv -\frac{G_F}{\sqrt{2}} V_{ub}\, Q_1 \,,$$

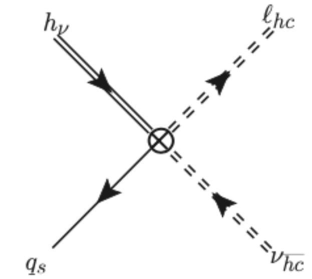
Fermi theory → HQET × SCET$_\mathrm{I}$

- The ***b* quark** can be described by a soft HQET field

$$b(x) \to e^{-im_b v\cdot x}(1 + \mathcal{O}(\lambda^2))\, h_v(x)$$



Fermi theory → HQET × SCET_I



- The ***b* quark** can be described by a soft HQET field

$$b(x) \to e^{-im_b v\cdot x}(1 + \mathcal{O}(\lambda^2))\, h_v(x)$$

- **Leptonic** fields can have large momenta, but small invariant mass, needs SCET

Relevant modes $p \sim (n_+p, n_-p, p_\perp)$

with expansion parameters:

$$\lambda^2 = \frac{\Lambda_{\text{QCD}}}{m_b}$$

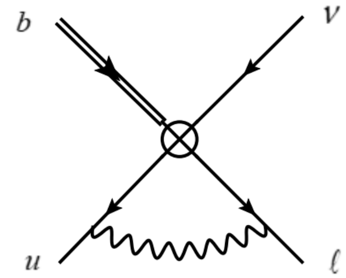
- Hard-collinear $p \sim (1, \lambda^2, \lambda)$

  →given by the lepton virtuality

- Soft $p \sim (\lambda^2, \lambda^2, \lambda^2)$

  →given by the spectator virtuality

Fermi theory → HQET × SCET$_\text{I}$



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In SCET$_\text{I}$, subleading power description for the different modes of **the spectator and the lepton** :

$$q(x) \to \left(1 + \frac{i\,D_\perp}{in_+D_C}\frac{\not{n}_+}{2}\right)\xi_C^{(q)} + \left(1 + \frac{1}{in\_D_s}Q_q\;A_{C\perp}\frac{\not{n}_-}{2}\right)q_s(x)$$

$$\ell(x) \to \left(1 + \frac{i\,D_\perp + m_\ell}{in_+D_C}\frac{\not{n}_+}{2}\right)\xi_C^{(\ell)}(x) + \left(1 + \frac{1}{in\_D_s}Q_q\;A_{C\perp}\frac{\not{n}_-}{2}\right)\ell_s(x)$$

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→given by the spectator virtuality

fields with power counting parameter

$$h_v, q_s \sim \lambda^3$$

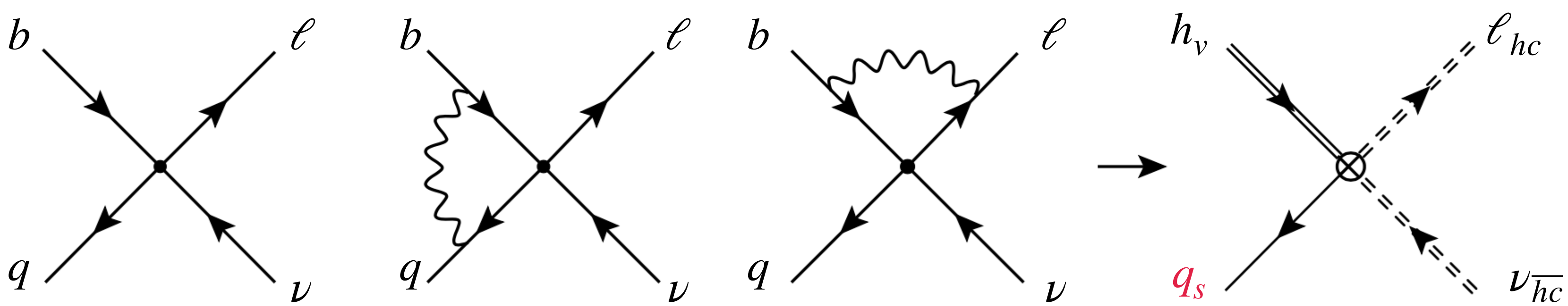
$$\ell_{hc}, \nu_{\overline{hc}}, \chi_{hc}^{(q)} \sim \lambda$$

$$\mathcal{A}_{hc}^\perp \sim \lambda$$

Construction of $\mathrm{HQET} \times \mathrm{SCET_I}$ operator

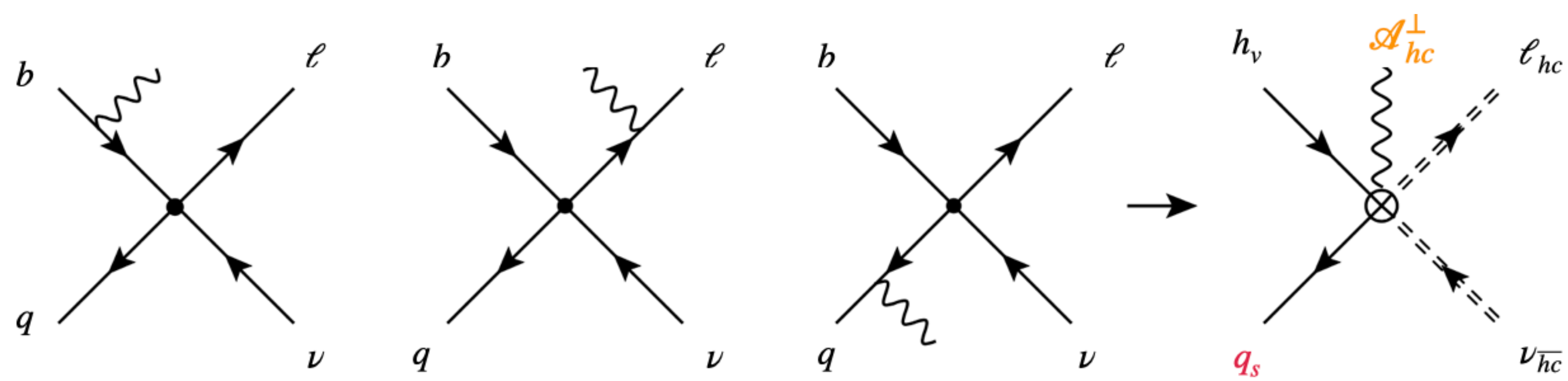
- **three classes operator are relevant**

1. local operator with soft spectator $\mathcal{O}_A$



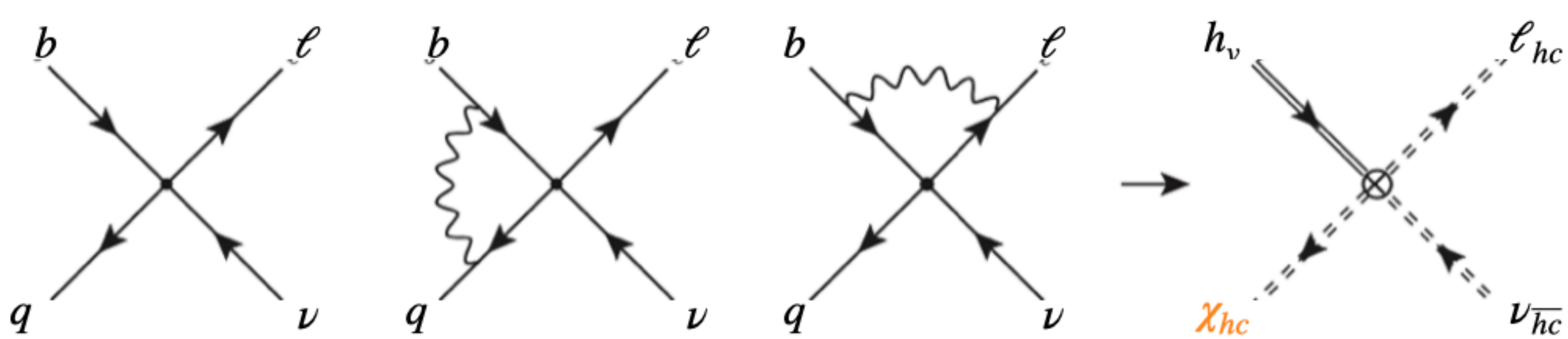
$$\mathcal{O}_A = [\bar{q}_s \ldots h_v]\,[\bar{\ell}_{hc} \ldots \nu_{\overline{hc}}]$$

2. local operator with soft spectator and hard-collinear photon $\mathcal{O}_B$



$$\mathcal{O}_B = [\bar{q}_s \ldots h_v]\,[\bar{\ell}_{hc} \ldots \nu_{\overline{hc}}]\;\mathcal{A}_{hc}^{\perp}$$

3. nonlocal operator with hard-collinear spectator $\mathcal{O}_C$



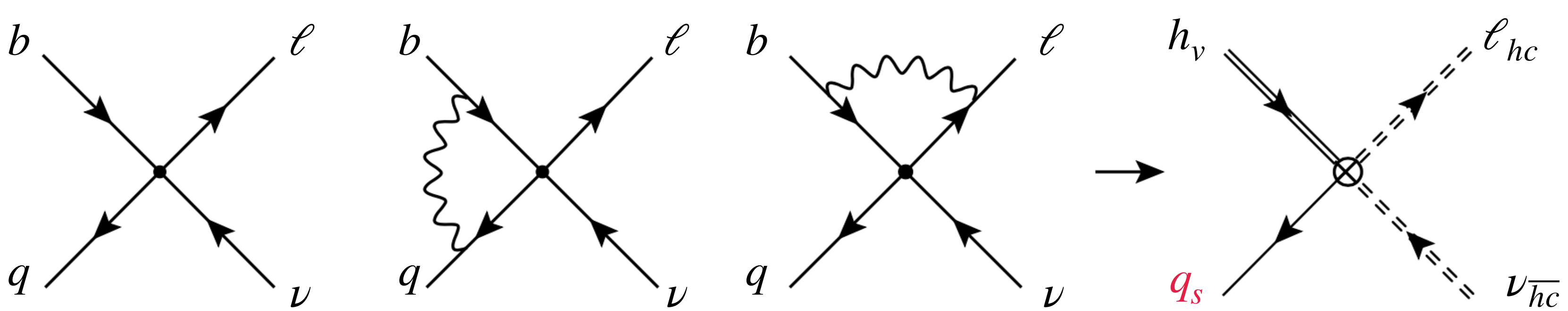
$$\mathcal{O}_C = [\;\chi_{hc} \;\; \ldots h_v]\,[\bar{\ell}_{hc} \ldots \nu_{\overline{hc}}]$$

Construction of $\mathrm{HQET} \times \mathrm{SCET_I}$ operator

1. local operator with soft spectator $\mathcal{O}_A$

only two irreducible Dirac structures

$$[\bar{\ell}_{hc}\,\Gamma_\ell\, P_L \nu_{\overline{hc}}] \quad \text{with} \quad \Gamma_\ell = 1, \gamma_\mu^\perp$$



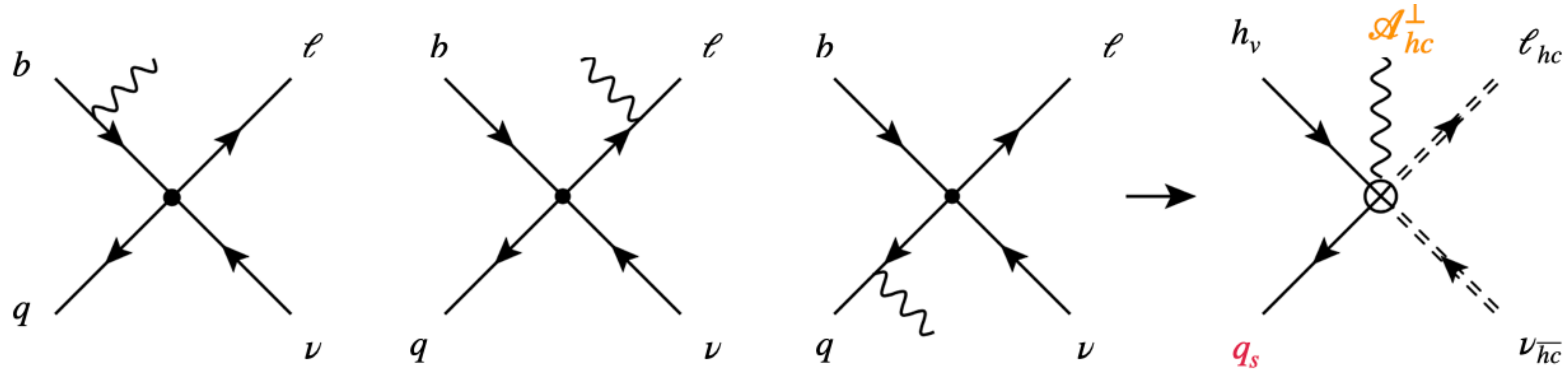
$$\mathcal{O}_A^{(\lambda)} = [\bar{q}_s \ldots h_v]\,[\bar{\chi}_{hc}^{(\ell)} \ldots \chi_{\overline{hc}}^{(\nu)}]$$

$$\mathcal{O}_{A,1}^{(9)} = m_\ell\,[\bar{q}_s \frac{\not{n}_+}{2} P_L\, h_v]\,[\bar{\ell}_{hc} \frac{1}{i\, n_+ \overleftarrow{\partial}_{hc}} P_L\, \nu_{\overline{hc}}]$$

$$\mathcal{O}_{A,2}^{(8)} = [\bar{q}_s \gamma_{\mu\perp} P_L\, h_v]\,[\bar{\ell}_{hc} \gamma_\perp^\mu P_L\, \nu_{\overline{hc}}]$$

Construction of $\mathrm{HQET} \times \mathrm{SCET_I}$ operator

2. local operator with soft spectator and hard-collinear photon $\mathcal{O}_B$



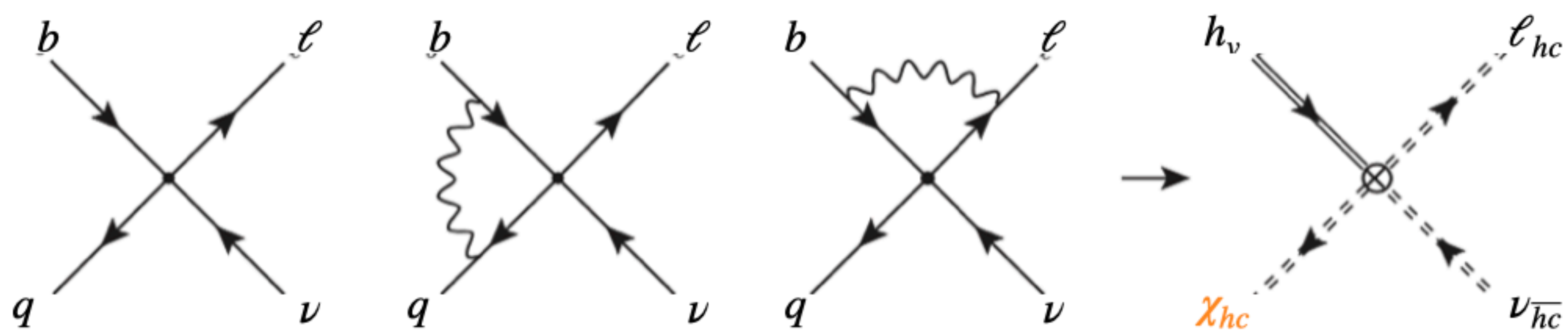
$$\mathcal{O}_B^{(\lambda)} = [\bar{q}_s \ldots h_v]\,[\bar{\ell}_{hc} \ldots \nu_{\overline{hc}}]\;\; \mathcal{A}_{hc}^{\perp}$$

$$\mathcal{O}_{B,1}^{(9)} = \frac{1}{i\, n_+ \partial_{hc}} [\bar{q}_s \frac{\not{n}_-}{2} \gamma_{\mu\perp} \;\not{\mathcal{A}}_{hc\perp}^{(b)} P_L\, h_v]\,[\bar{\ell}_{hc}\, \gamma_\perp^\mu P_L\, \nu_{\overline{hc}}]$$

$$\mathcal{O}_{B,2}^{(9)} = [\bar{q}_s \;\not{\mathcal{A}}_{hc\perp}^{(q)} \frac{1}{i\, n_+ \overleftarrow{\partial}_{hc}} \frac{\not{n}_+}{2} \gamma_{\mu\perp} P_L\, h_v]\,[\bar{\ell}_{hc}\; \gamma_\perp^\mu P_L\, \nu_{\overline{hc}}]$$

Construction of HQET × SCET$_\text{I}$ operator

3. nonlocal operator with hard-collinear spectator $\mathcal{O}_C$



$$\mathcal{O}_C = [\ \chi_{hc} \ \ldots h_v]\,[\bar{\ell}_{hc} \ldots \nu_{\overline{hc}}]$$

$$\mathcal{O}^{(6)}_{C,1}(s,t) = [\bar{\chi}^{(q)}_{hc}(sn_+)\,\gamma_{\mu\perp}\,P_L\,h_v(0)]\,[\bar{\ell}_{hc}(tn_+)\,\gamma^{\mu}_{\perp}\,P_L\,\nu_{\overline{hc}}(0)]$$

$$\mathcal{O}^{(7)}_{C,2}(s,t) = m_\ell\,[\bar{\chi}^{(q)}_{hc}(sn_+)\,\frac{\not{n}_+}{2}\,P_L\,h_v(0)]\,[\bar{\ell}_{hc}(tn_+)\,\frac{1}{i\,n_+\overleftarrow{\partial}_{hc}}\,P_L\,\nu_{\overline{hc}}(0)]$$

C operators are **power-enhanced** with respect to A and B ones, but hard-collinear quark needs to be converted to a soft field through SCET$_\text{I}$ **power-suppressed** soft-collinear interactions.

Hard function at $\mu \sim m_b$

- m_W
- $m_b \sim \mu_h$ ☑ Fermi Theory
- $m_\tau \sim \mu_{hc}$ ☑ $\mathrm{HQET} \times \mathrm{SCET_I}$
- $\Lambda_{\rm QCD} \sim \mu_s$ ☐ …

Hard function $H^{(0)}_{A,1}$, $H^{(1)}_{A,1}$, $H^{(0)}_{B,(1,2,3)}$, $H^{(0)}_{\chi,(1,2)}$

B and C operators only at tree level matching

$$H^{(1)}_{A,1} = -\frac{\alpha_{\rm em}}{4\pi} 2\,Q_\ell Q_b \left[(L - 2\ln s - s - 1)\,L - \frac{1}{2}L^2 + 2\ln^2 s + \frac{s^2-2}{s-1}\ln s + 2\,\mathrm{Li}_2(1-s) - 2s - 1 + \frac{\pi^2}{12}\right] + \frac{\alpha_{\rm em}}{4\pi}\,12\,Q_\ell(2\,Q_b - Q_\ell)$$

evanescent operator contribution

with $L = \ln(\mu^2/m_b^2)$, $s = n\cdot p_\ell/m_b \sim 1$

$$Q_1 = [\bar u\,\gamma^\mu\,P_L\,b]\,[\bar\ell\,\gamma_\mu\,P_L\,\nu]$$

$$E_1 = [\bar u\,\gamma^\mu\,\gamma^\nu\,\gamma^\rho\,P_L\,b]\,[\bar\ell\,\gamma_\mu\gamma_\nu\gamma_\rho P_L\,\nu] - 16\,Q_1$$

$\text{SCET}_\text{I} \to \text{bHLET}$ $\quad \mu \sim m_b \Lambda_{\text{QCD}}$

lower the virtuality to remove the hard-collinear mode to reach to bHLET

- heavy tau filed become to a soft-collinear (sc) field in boosted HLET after integrating m_τ

$$\ell_{hc} \to \mathrm{e}^{-i m_\ell v_\ell \cdot x} \left(1 + b \frac{\not{n}_+}{2}\right) \ell_{sc}$$

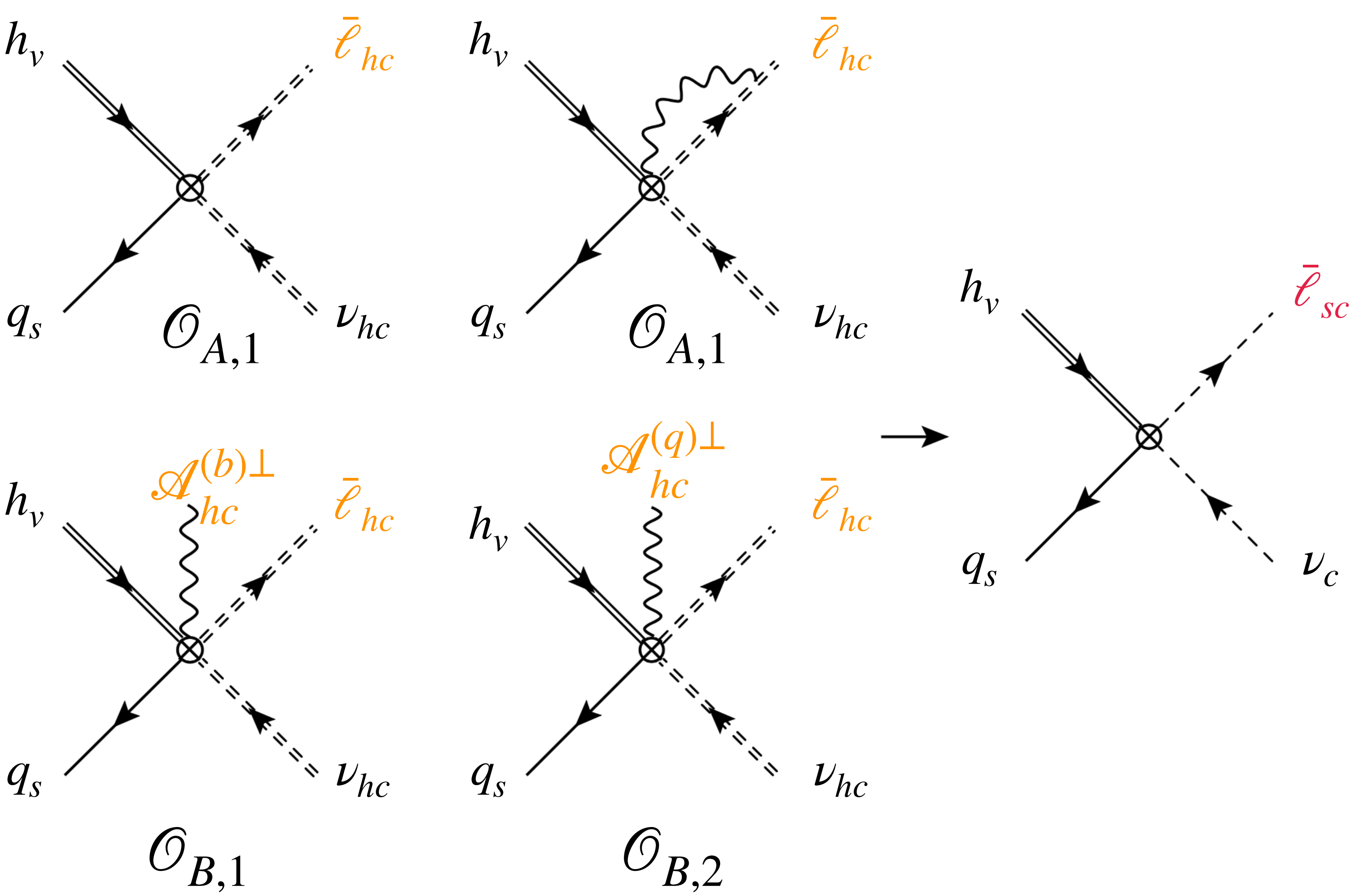
boosted parameters:

$$b = \frac{m_\tau}{m_b} \sim \lambda \qquad \lambda^2 = \frac{\Lambda_{\text{QCD}}}{m_b}$$

- Soft-collinear $\quad p \sim \lambda^2 \left(\frac{1}{b}, b, 1\right)$

$\to$soft scale to τ boosted in the B frame

$$p' \sim m_\tau (\lambda^2, \lambda^2, \lambda^2)$$



$$\mathcal{J}_m^{A,B} = \left[\bar{q}_s \frac{\not{n}_+}{2} P_L h_v\right] S_{n_-}^{(\ell)\dagger} \left[\bar{\ell}_{sc} P_L \nu_{\bar{c}}\right]$$

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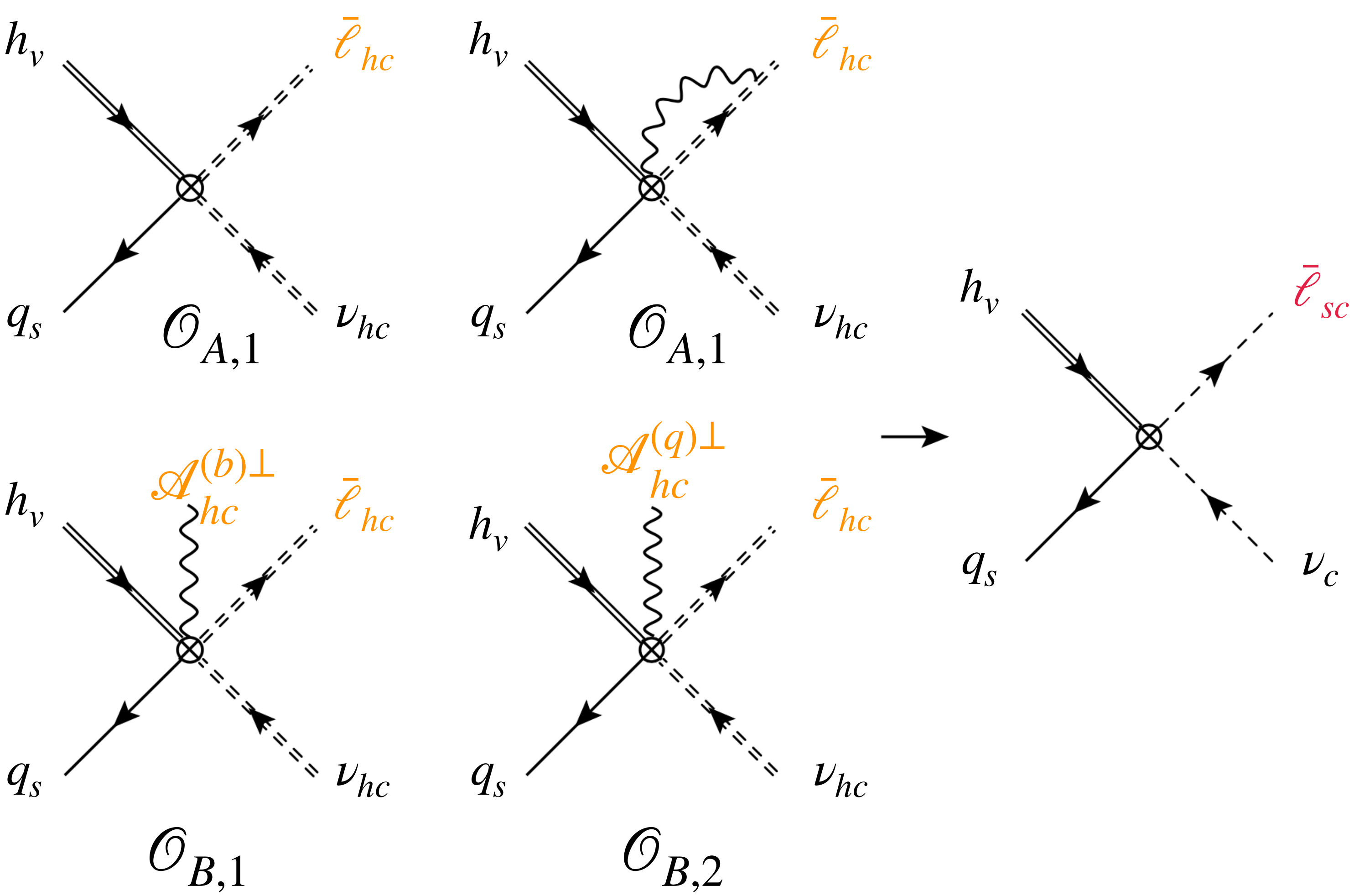


$$\mathcal{J}_m^{A,B} = [\bar{q}_s \frac{\not{n}_+}{2} P_L h_v]\, S_{n_-}^{(\ell)\dagger} [\bar{\ell}_{sc} P_L \nu_{\bar{c}}]$$

$$J_{A,1}^{(0)}(\mu) = \frac{m_\tau}{m_B}, \quad J_{A,1}^{(1)}(\mu) = \frac{\alpha_{\mathrm{em}}}{4\pi} Q_\ell Q_b \frac{1}{2} b \left(\frac{1}{2} \ln^2 \frac{\mu^2}{m_\tau^2} + \frac{\pi^2}{12}\right)$$

$$J_{B,1}^{(1)}(\mu) = -\frac{1}{(4\pi)^2} e\, Q_\ell \frac{m_\ell}{m_B} \left(2 \ln \frac{\mu^2}{m_\tau^2} + 1\right)$$

$$J_{B,2}^{(1)}(\mu) = -\frac{1}{(4\pi)^2} e\, Q_\ell\, 2 \frac{m_\ell}{m_B} \left(2 \ln \frac{\mu^2}{m_\tau^2} + 1\right)$$

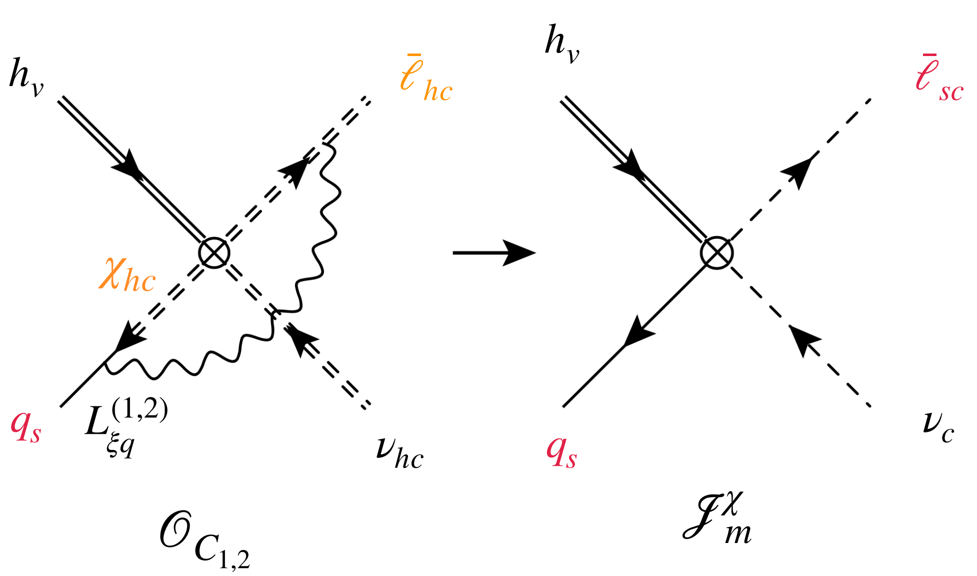
$\mathrm{SCET_I} \to \mathrm{HQET} \times \mathrm{bHLET}$

- $\chi_{hc}^{(q)} \to q_s$ by inserting the following NLP and NNLP interactions

$$\mathcal{O}_C = [\ \chi_{hc}^{(q)}\ \ldots h_v]\,[\bar{\ell}_{hc} \ldots \nu_{\overline{hc}}]$$

$$L_{\xi q}^{(1)}(x) = \bar{q}_s(x_-)[W_{\xi C} W_C]^\dagger(x)\, i\ \not{D}_{C\perp}\, \xi_C(x) + \mathrm{h.c.}$$

$$L_{\xi q}^{(2)}(x) = \bar{q}_s(x_-)\left[W_{\xi,hc} W_{hc}\right]^\dagger(x)\, \left(i\, n_- D_{hc} + i\ \not{D}_{hc\perp}(i\, n_+ D_{hc})^{-1} i\ \not{D}_{hc\perp}\right) \frac{\not{n}_+}{2}\, \xi_{hc}(x) + \ldots$$



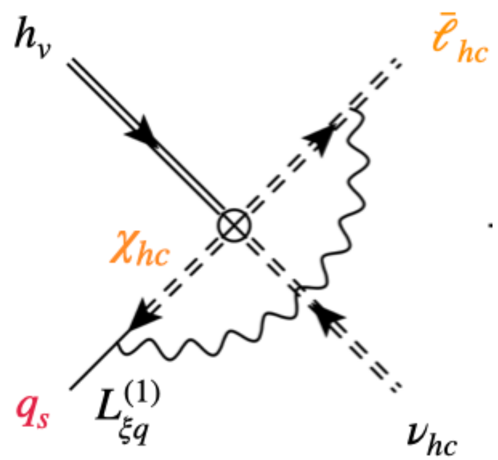
intermediate propagators introduce **non-local operators**

$$\mathcal{J}_m^\chi(v) = [\bar{q}_s(v\, n_-)\, Y(v\, n_-, 0)\, \frac{\not{n}_+}{2}\, P_L\, h_v(0)]\, S_{n_-}^{(\ell)\dagger}(0)\, [\bar{\ell}_{sc}(0)\, P_L\, \nu_{\bar{c}}(0)]$$

The helicity suppression be relaxed or not

$$\mathcal{J}^{\chi}_{m}(v) \; = \; [\bar{q}_s(v\, n_-)\, Y(v\, n_-,0)\, \frac{\not{n}_+}{2}\, P_L\, h_v(0)]\, S^{(\ell)\dagger}_{n_-}(0)\, [\bar{\ell}_{sc}(0)\, P_L\, \nu_{\bar{c}}(0)]$$

Nonlocal annihilation can **probe the meson structure**, and **possibly overcome** the **helicity suppression**



$$\langle 0 |\, q_s \, \frac{1}{in_-\partial_s} \ldots h_v | B \rangle \sim \frac{1}{\lambda_B} \sim \frac{1}{\Lambda_{\rm QCD}}$$

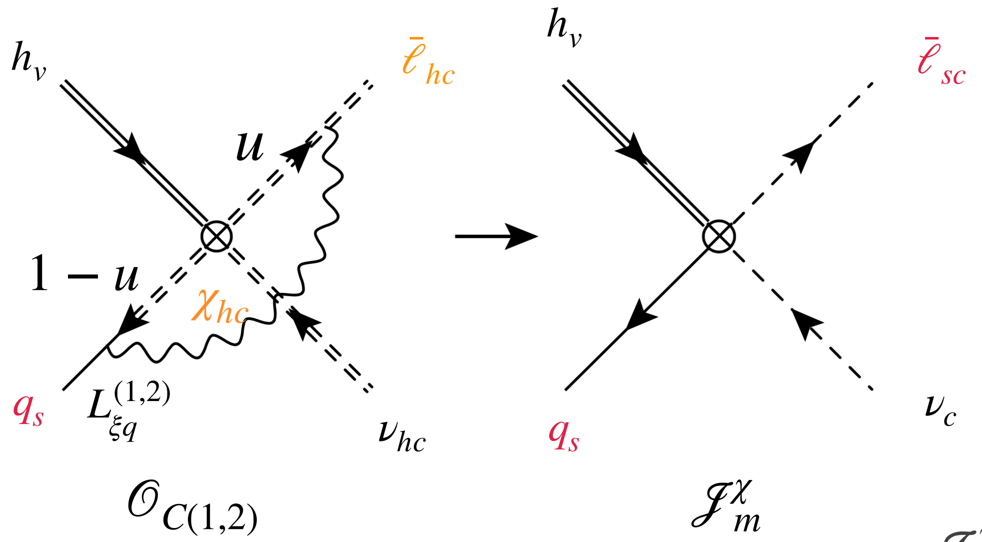
Happens for $B_s \to \ell^+ \ell^-$, but not for $B_u \to \ell\, \nu$ with left-handed currents

[Beneke, Bobeth, Szafron 2017 & 2019, Y.K.Huang, Y.L.Shen, X.C.Zhao, SHZ 2023]

$$[\bar{u}\, \frac{\not{n}_+}{2}\, \gamma^{\mu}_{\perp} \gamma^{\nu}_{\perp}\, P_L\, b][\bar{\ell}\, \gamma_{\perp\mu} \gamma_{\perp\nu}\, (\frac{v - a\,\gamma_5}{2})\, \nu] \; = \; 2\, (v \; - \; a)\, [\bar{u}\, \frac{\not{n}_+}{2}\, P_L\, b][\bar{\ell}\, P_R\, \nu]$$

$\longrightarrow$ No power enhancement in $B_u \to \ell\, \nu$!

$\mathrm{SCET_I} \to \mathrm{HQET} \times \mathrm{bHLET}$



$$\mathcal{O}^{(6)}_{C,1}(s,t) = [\bar{\chi}^{(q)}_{hc}(sn_+)\,\gamma_{\mu\perp}\,P_L\,h_v(0)]\,[\bar{\ell}_{hc}(tn_+)\,\gamma^{\mu}_{\perp}\,P_L\,\nu_{\overline{hc}}(0)]$$

$$\mathcal{O}^{(7)}_{C,2}(s,t) = m_\ell\,[\bar{\chi}^{(q)}_{hc}(sn_+)\,\frac{\not{n}_+}{2}\,P_L\,h_v(0)]\,[\bar{\ell}_{hc}(tn_+)\,\frac{1}{i\,n_+\overleftarrow{\partial}_{hc}}\,P_L\,\nu_{\overline{hc}}(0)]$$

$$\mathcal{J}^{\chi}_m(v) = [\bar{q}_s(v\,n_-)\,Y(v\,n_-,0)\,\frac{\not{n}_+}{2}\,P_L\,h_v(0)]\,S^{(\ell)\dagger}_{n_-}(0)\,[\bar{\ell}_{sc}(0)\,P_L\,\nu_{\bar{c}}(0)]$$

$$J^{(1)}_{\chi,1} = \frac{\alpha_{\rm em}}{4\pi}\,Q_\ell\,Q_u\,\frac{4\,m_\ell}{n_+p_\ell}\,u\left[\ln\frac{\mu^2}{\bar{u}^2\,m_b\,n_-p_\ell} - \frac{1+r}{r}\ln(1+r) + \frac{1}{2}\right]\theta(u)\theta(\bar{u})$$

$$J^{(1)}_{\chi,2} = \frac{\alpha_{\rm em}}{4\pi}\,Q_\ell\,Q_u\,\frac{2\,m_\ell}{n_+p_\ell}\left[\bar{u}\,\ln\frac{\mu^2}{\bar{u}^2\,m_b\,n_-p_\ell} - \frac{1+r}{r}\ln(1+r) + \frac{1}{2}\right]\theta(u)\theta(\bar{u})$$

$$r = \frac{u}{\bar{u}}\,\frac{\omega\,m_B}{m_\ell^2}$$

$\to$ **No endpoint div.** ($1/u \to \infty$, when u$\to$ 0) in $B_u \to \tau\,\nu$ when convoluting to hard function!

subtractions scheme independence

Factorization Formula

$m_b \sim \mu_h$ ☑ Fermi Theory

$m_\tau \sim \mu_{hc}$ ☑ HQET × SCET_I

$\Lambda_\text{QCD} \sim \mu_s$ ☑ HQET × bHLET

Hard function

$$H^{(0)}_{A,1}\,,H^{(1)}_{A,1}\,,H^{(0)}_{B,(1,2,3)}\,,H^{(0)}_{\chi,(1,2)}$$

Hard-collinear function

$$J^{(0)}_{A,1}\,,J^{(1)}_{A,1}\,,J^{(1)}_{B,(1,3)}\,,J^{(1)}_{\chi,(1,2)}$$

HQET × bHLET operators

$$\mathcal{J}^{A,B}_m = [\bar{q}_s \frac{\not{n}_+}{2} P_L\, h_v]\, S^{(\ell)\dagger}_{n_-}\, [\bar{\ell}_{sc}\, P_L\, \nu_{\bar{c}}]$$

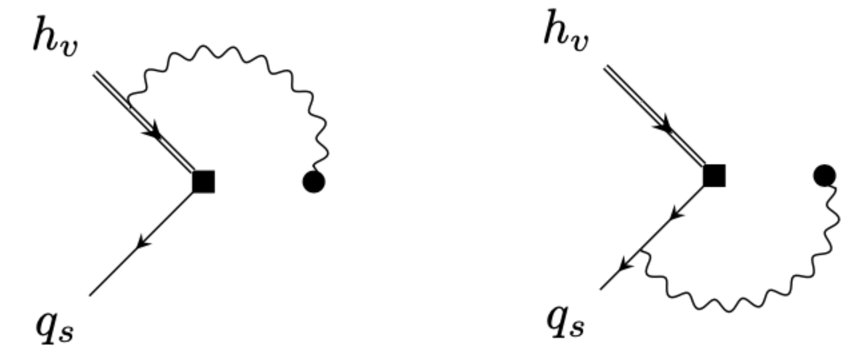
$$\mathcal{J}^{\chi}_m(v) = [\bar{q}_s(v\,n_-)\, Y(v\,n_-,0) \frac{\not{n}_+}{2} P_L\, h_v(0)]\, S^{(\ell)\dagger}_{n_-}(0)\, [\bar{\ell}_{sc}(0)\, P_L\, \nu_{\bar{c}}(0)]$$

$$A^\text{virtual}_{B\to\tau\nu} \sim H_{A,B}\, J_{A,B}\, \langle \tau^- \nu \,|\, \mathcal{J}^{A,B}_m \,|\, \bar{B}_u \rangle + \int_0^1 du\, H_\chi(u) \int_0^\infty d\omega\, J_\chi(u;\omega)\, \langle \tau^- \nu \,|\, \mathcal{J}^{\chi}_m \,|\, \bar{B}_u \rangle$$

SCET_I operators with soft spectator (A-type and B-type)

SCET_I operators with hc spectator (C-type)

Generalized decay constant and LCDA



Soft photon decoupling from lepton

- **Modified** B-meson decay constant and LCDA

$$\mathcal{J}_m^{A,B} = [\bar{q}_s \frac{\not{n}_+}{2} P_L h_v] S_{n_-}^{(\ell)\dagger} [\bar{\ell}_{sc} P_L \nu_{\bar{c}}]$$

$$\mathcal{J}_m^{\chi}(v) = [\bar{q}_s(v n_-) Y(v n_-,0) \frac{\not{n}_+}{2} P_L h_v(0)] S_{n_-}^{(\ell)\dagger}(0) [\bar{\ell}_{sc}(0) P_L \nu_{\bar{c}}(0)]$$

Additional QED soft Wilson lines

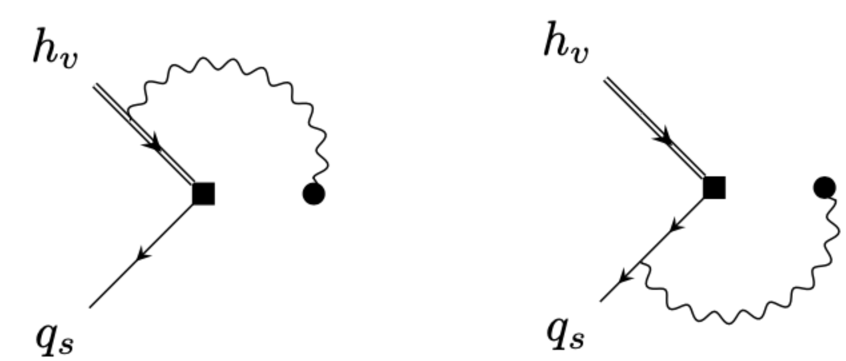
$$S_r^{(i)}(x) = \exp\left[-i\, e\, Q_i \int_0^\infty ds\, r \cdot A_s(x + s \cdot r)\right]$$

- Factorization anomaly

$$\mathcal{J}_S^{A,B} = [\bar{q}_s \frac{\not{n}_+}{2} P_L h_v] S_{n_-}^{(\ell)\dagger}$$

$$\mathcal{J}_S^{\chi}(v) = [\bar{q}_s(v n_-) Y(v n_-,0) \frac{\not{n}_+}{2} P_L h_v(0)] S_{n_-}^{(\ell)\dagger}(0)$$

Generalized decay constant and LCDA



Soft photon decoupling from lepton

- **Modified** B-meson decay constant and LCDA

$$\mathcal{J}_m^{A,B} = [\bar{q}_s \frac{\not{n}_+}{2} P_L h_v] \, S_{n_-}^{(\ell)\dagger} \, [\bar{\ell}_{sc} P_L \nu_{\bar{c}}]$$

$$\mathcal{J}_m^{\chi}(v) = [\bar{q}_s(v n_-) \, Y(v n_-, 0) \frac{\not{n}_+}{2} P_L h_v(0)] \, S_{n_-}^{(\ell)\dagger}(0) \, [\bar{\ell}_{sc}(0) P_L \nu_{\bar{c}}(0)]$$

Additional QED soft Wilson lines

$$S_r^{(i)}(x) = \exp\left[-i e Q_i \int_0^\infty ds \, r \cdot A_s(x + s \cdot r)\right]$$

- Factorization anomaly

$$\mathcal{J}_S^{A,B} = [\bar{q}_s \frac{\not{n}_+}{2} P_L h_v] \, S_{n_-}^{(\ell)\dagger}$$

$$\mathcal{J}_S^{\chi}(v) = [\bar{q}_s(v n_-) \, Y(v n_-, 0) \frac{\not{n}_+}{2} P_L h_v(0)] \, S_{n_-}^{(\ell)\dagger}(0)$$

- Refactorization

$$\mathcal{F}_B \equiv \frac{\langle 0 | \mathcal{J}_S^{A,B} | B \rangle}{\langle 0 | [S_{v_B}^{(B)}(0) \, S_{n_-}^{(\ell)\dagger}(0)] | 0 \rangle}$$

$$\mathcal{F}_B \Phi_B(v) \equiv \frac{\langle 0 | \mathcal{J}_S^{\chi}(v) | B \rangle}{\langle 0 | [S_{v_B}^{(B)}(0) \, S_{n_-}^{(\ell)\dagger}(0)] | 0 \rangle}$$

- For $\alpha_{\rm em} \to 0$, $\mathcal{F}_B$ and $\mathcal{F}_B \Phi_B$ reduces to the standard HQET decay constant and LCDA
- For $\alpha_{\rm em} \neq 0$, high order $\mathcal{F}_B$ and $\mathcal{F}_B \Phi_B$ are **new nonperturbative hadronic parameters**.

Lattice determination ? QCD SR estimate ?

Decay amplitude including virtual QED corrections at NLP+NLO

$$i\,\mathcal{A}^{\text{virtual}} = -\frac{i}{4}\, m_{B_u}\, \bar{u}_{sc}\left(p_\ell\right)\; P_L\, v_{\bar{c}}\left(p_\nu\right)\;\left[\sum_{i=A,1}^{B,(1,3)} H_i(\mu)\, J_i(\mu)\, \mathcal{F}_{B_u}(\mu)\; + \right.$$

$$\left. \sum_{j=\chi,1}^{\chi,2} \int_0^1 du\, H_j(u,\mu)\, \int_0^\infty d\omega\, J_j(u;\omega,\mu)\, \mathcal{F}_{B_u}(\mu)\, \Phi_+(\omega,\mu) \right]$$

e.g.

$$H_{A,1}^{(1)} = -\frac{\alpha_{\text{em}}}{4\pi} 2\, Q_\ell Q_b \left[\left(\ln\frac{\mu^2}{m_b^2} - 2\ln s - s - 1\right) \ln\frac{\mu^2}{m_b^2} - \frac{1}{2}\ln^2\frac{\mu^2}{m_b^2} + 2\ln^2 s + \frac{s^2-2}{s-1}\ln s \right.$$

$$\left. + 2\,\text{Li}_2(1-s) - 2s - 1 + \frac{\pi^2}{12} \right] + \frac{\alpha_{\text{em}}}{4\pi}\, 12\, Q_\ell (2\, Q_b - Q_\ell)$$

$$J_{A,1}^{(0)}(\mu) = \frac{m_\ell}{n_+ p_\ell}$$

large double logarithms at $\mu = hc$.

$$J_{A,1}^{(1)}(\mu) = \frac{\alpha_{\text{em}}}{4\pi}\, Q_\ell\, Q_b\, \frac{1}{2}\, b \left(\frac{1}{2}\ln^2\frac{\mu^2}{m_\tau^2} + \frac{\pi^2}{12} \right)$$

Numerical prediction (preliminary)

- **The non-radiative QED corrections to branching fraction of $B_u \to \tau\nu$ for central values of the parameters**

$$\mathrm{Br}^{(0)}(B_u \to \tau\nu) = \left(7.28_{\text{(LO)}} - 0.07_{\text{(NLO)}}\right) \times 10^{-5}$$

NLP+**NLO**+**LL** QED virtual correction changes the branching fraction by: $\sim 1\ \%$

compete with QCD uncertainties <1% accuracy

$$\langle 0|\, \bar{u}\gamma^{\mu}\gamma_5 b\, |B^-(p)\rangle = i f_{B_u} p^{\mu} \quad \text{with} \quad f_{B_u} = (189.4 \pm 1.4)\,\mathrm{MeV}$$

- **The measured branching fraction by Belle II is [arXiv:2520.04885]**

$$\mathrm{Br}^{(exp)}(B_u \to \tau\nu) = \left(1.24 \pm 0.41(stat.) \pm 0.19(syst.)\right) \times 10^{-4}$$

Summary

Subleading power factorization formula for QED corrections to $B_u \to \tau\nu$ derived in SCET, HQET and bHLET

- no endpoint divergences in this factorization at NLP (with tau mass)

$\longrightarrow$ **subtractions scheme independence**

- **Structure depended QED corrections** arising from hard, hard-collinear photons exchange $\to$ important source of **large logarithmic corrections**

- Structure depended QED corrections from leptonic field decoupling produce **generalized *B* decay constant and LCDA** $\to$ new hadronic parameters

- **NLP+NLO+LL** QED virtual correction changes the branching fraction by $\sim 1\,\%$

Thank you

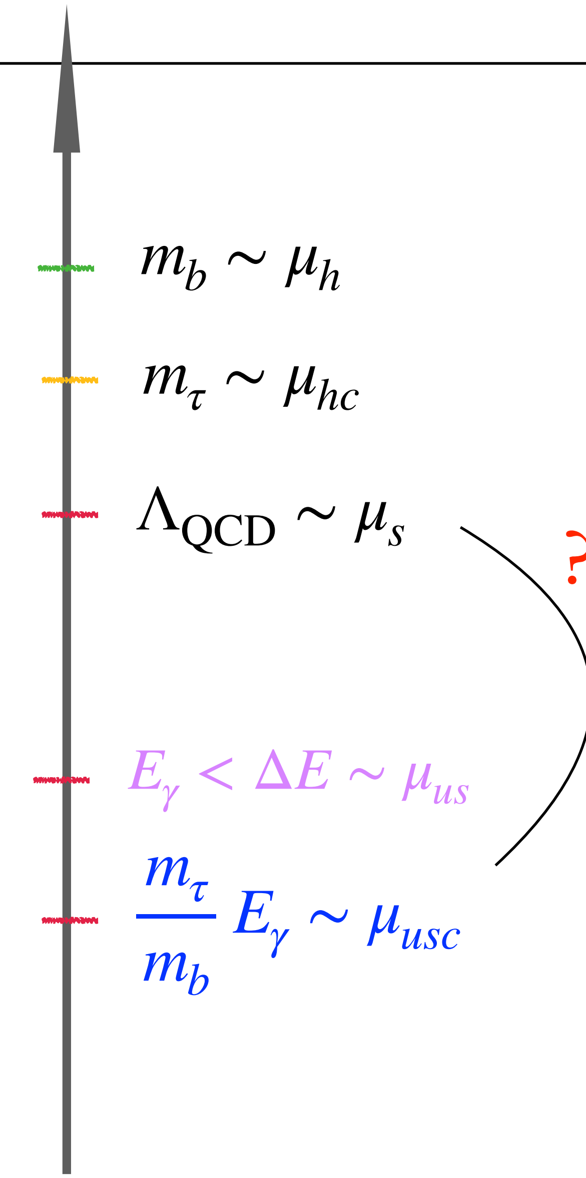
Backup slides

HQET × bHLET → **Low-energy theory ($\mu < \mu_s, \mu_{sc}$)**

power parameters: $\lambda_E^2 = \frac{E_\gamma}{m_b} \sim \lambda^4$

- Ultra-soft $p \sim (\lambda_E^2, \lambda_E^2, \lambda_E^2)$
- Ultra-soft-collinear $p \sim \lambda_E^2 (1, b^2, b)$

→ultrosoft scale to τ boosted in the B frame



HQET × bHLET → **Low-energy theory** ($\mu < \mu_s, \mu_{sc}$)

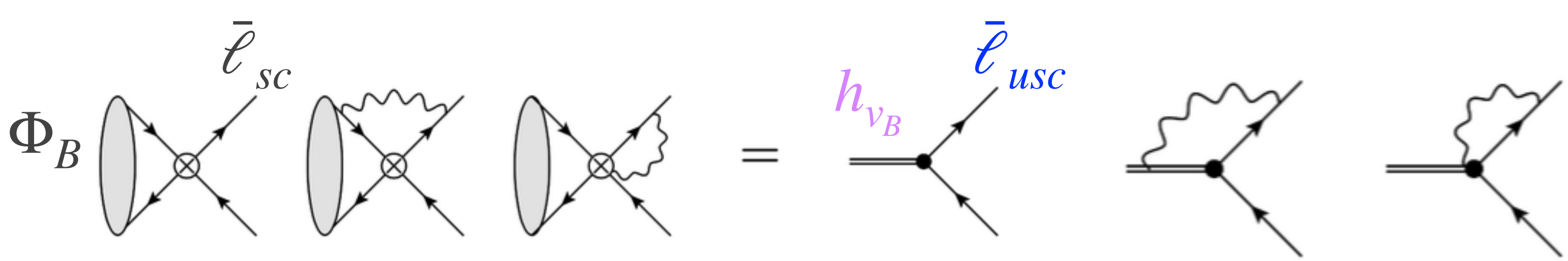
- $\mu < \Lambda_{\rm QCD}$, the hadronic B meson can be described as a heavy scalar effective theory (HSET)

$$\Phi_B(x) \to {\rm e}^{-im_B v_B\cdot x}\, h_{v_B}(x) \qquad m_B v_B \sim \mu_s$$

- $\mu < \mu_{sc}$, soft-coll. (sc) field in bHLET turned into ultra-soft-coll. one (usc) in bHLET-2

$$\ell_{sc} \to {\rm e}^{-im_\ell v'_\ell\cdot x}\, \ell_{usc} \qquad m_\ell v'_\ell \sim \mu_{sc}$$

- HQET × bHLET → HSET × $\text{bHLET}_{\rm II}$ $\quad \mu \sim \Lambda_{\rm QCD}$



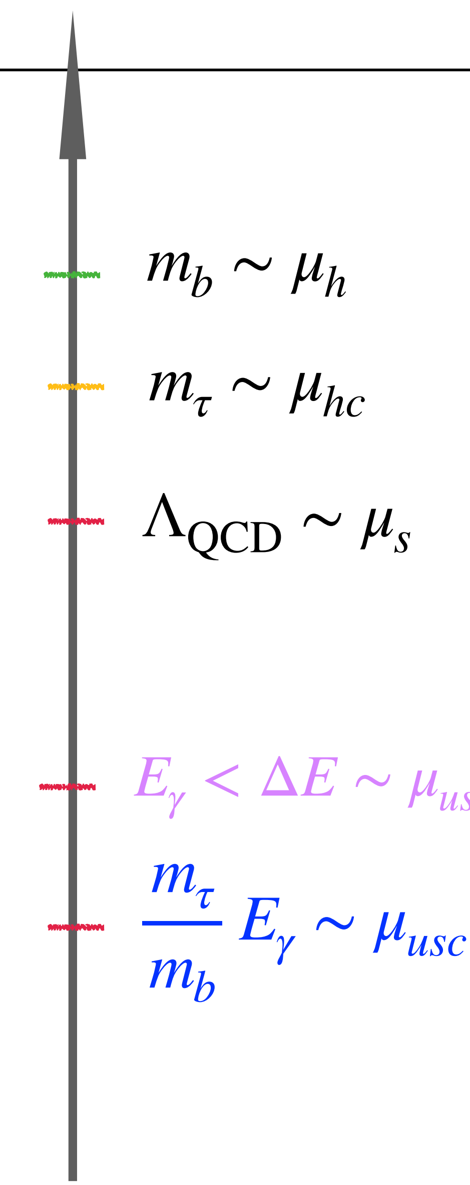
nonperturbative hadronic matrix element before decoupling

$$= y_B\, h_{v_B} [\bar\ell_{usc} P_L \nu_{\bar c}]$$

power parameters: $\lambda_E^2 = \dfrac{E_\gamma}{m_b} \sim \lambda^4$

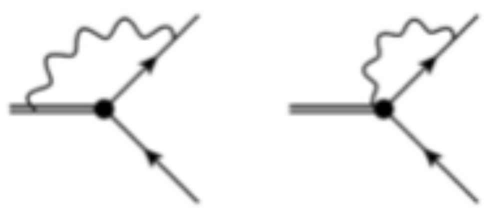
- Ultra-soft $\quad p \sim (\lambda_E^2, \lambda_E^2, \lambda_E^2)$
- Ultra-soft-collinear $\quad p \sim \lambda_E^2(1, b^2, b)$

→ultrosoft scale to τ boosted in the B frame



Real correction to $\mathrm{HSET} \times \mathrm{bHLET}_{\mathrm{II}}$

- all interactions of the B and the tauon with ultra-soft **and** ultra-soft-collinear photons can be decoupled into **Wilson lines** via field redefinitions



$$h_{v_B}(x) \to S_{v_B}^{(B)}\; C_{n_+}^{(B)}\; h_{v_B}^{(0)}(x)$$

$$\chi_{usc}^{(\ell)}(x) \to S_{n_-}^{(\ell)}\; C_{v'_\ell}^{(\ell)}\; \chi_{usc}^{(\ell,0)}(x)$$

- **Real corrections** are matrix elements of these Wilson lines

$$S(E_\gamma, \mu) = \int_0^\infty d\,\omega_{us} \int_0^\infty d\,\omega_{usc}\, \theta(\frac{E_\gamma}{2} - \omega_{us} - \omega_{usc})\;\; W_{us}(\omega_{us}, \mu)\;\; W_{usc}(\omega_{usc}, \mu)$$

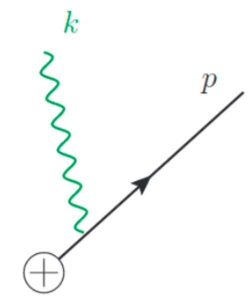
- Real emissions are factorized at the level of the decay rate

$$\Gamma[B_u \to \tau\nu] \sim |A^{\text{virtual}}_{B\to\tau\nu}|^2 \otimes S(E_\gamma, \mu)$$

$$A^{\text{virtual}}_{B\to\tau\nu} \sim H_i \otimes J_i \otimes S \otimes SC$$

- Ultra-soft photons (under the assumption that $\Delta E \ll \Lambda_{\rm QCD}$)

  Based on eikonal approximation,



$$\varepsilon_\mu(k)\bar{u}(p)\gamma^\mu \frac{\not{p}+\not{k}+m}{(k+p)^2-m^2} \to \frac{\varepsilon_\mu(k)p^\mu}{p\cdot k}\bar{u}(p)\,,$$

  note $k^\mu \ll p^\mu, m$

$$\delta_{\rm QED} \sim \frac{\alpha}{\pi}\ln^2\frac{m_B}{m_\ell}$$

  Large logarithmic enhancements can mimic lepton-flavor universality violation

The ultrasoft contribution $\mathcal{S}(v_\ell, v_{\bar{\ell}}, \Delta E)$ is

$$\mathcal{S}(v_\ell, v_{\bar{\ell}}, \Delta E) = \sum_{X_s}\left|\left\langle X_s \left| S^\dagger_{v_\ell}(0)\, S_{v_{\bar{\ell}}}(0)\right| 0\right\rangle\right|^2 \theta(\Delta E - E_{X_s})\,, \tag{5.5}$$

with the one-loop ultrasoft function for massive final particles given in [16] ,

$$S^{(1)}(v_\ell, v_{\bar{\ell}}, \Delta E) = 8\left(1+\frac{1}{2}\ln\frac{m_\tau^2}{m_B^2}\right)\ln\frac{\mu}{2\,\Delta E} - \left(2+\ln\frac{m_\tau^2}{m_B^2}\right)\ln\frac{m_\tau^2}{m_B^2} + 4 - \frac{2}{3}\pi^2, \tag{5.6}$$

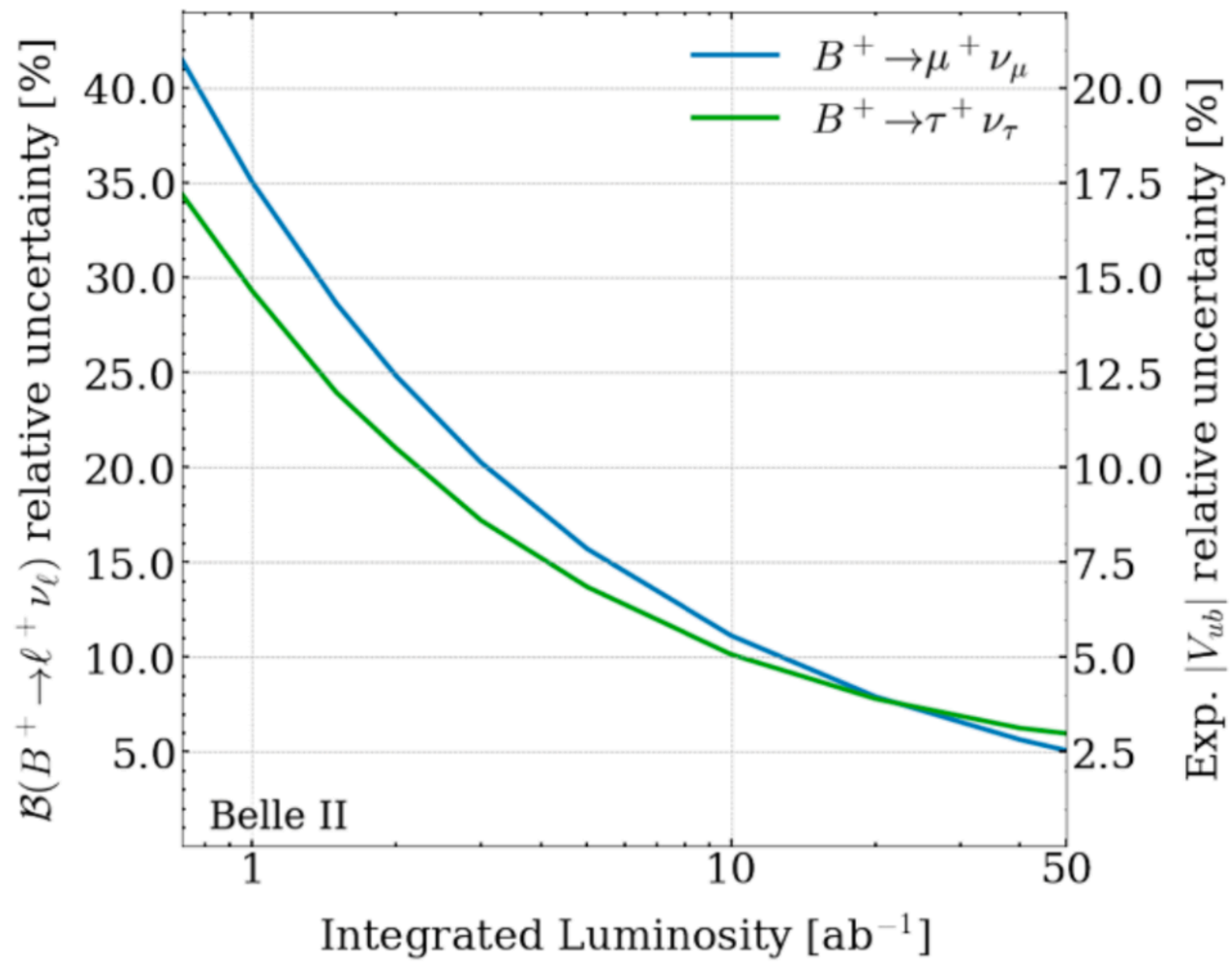
The resummed soft function can be achieved by using the QED exponentiation theorem as a approximate, e.g. full soft function can be considered as the exponent of the one-loop result,

$$\mathcal{S}(v_\ell, v_{\bar{\ell}}, \Delta E) = \exp\left[\frac{\alpha_{\rm em}}{4\pi} Q_\ell^2\, S^{(1)}(v_\ell, v_{\bar{\ell}}, \Delta E)\right]. \tag{5.7}$$

$$\mathrm{Br}^{(0)}(B_u \to \tau\nu) = \left(7.28_{\rm (LO)} - 0.10_{\rm (ultrasoft)}\right) \times 10^{-5}$$

NLP+NLO+LL QED virtual + ultrasoft correction changes the branching fraction by: $\sim 1.3\,\%$

Backup slides



[Belle II Physics Book]

Figure 1: Projection of uncertainties on the branching fractions $\mathcal{B}(B^+ \to \mu^+ + \nu_\mu)$ and $\mathcal{B}(B^+ \to \tau^+ + \nu_\tau)$. The corresponding uncertainty on the experimental value of $|V_{ub}|$ is shown on the right-hand vertical axis.

TAB. I. Published results for $\mathcal{B}(B^+ \to \tau^+ \nu_\tau)$ by Belle, *BABAR* and the PDG average.

Experiment	Tag	$\mathcal{B}(10^{-4})$
Belle	Hadronic	$0.72^{+0.27}_{-0.25} \pm 0.11$
BABAR	Hadronic	$1.83^{+0.53}_{-0.49} \pm 0.24$
Belle	Semileptonic	$1.25 \pm 0.28 \pm 0.27$
BABAR	Semileptonic	$1.8 \pm 0.8 \pm 0.2$
PDG		1.09 ± 0.24

modes

- Relevant modes $k \sim (n_+k, n\_k, k_\perp)$ for **virtual** QED corrections:

Expansion parameters:

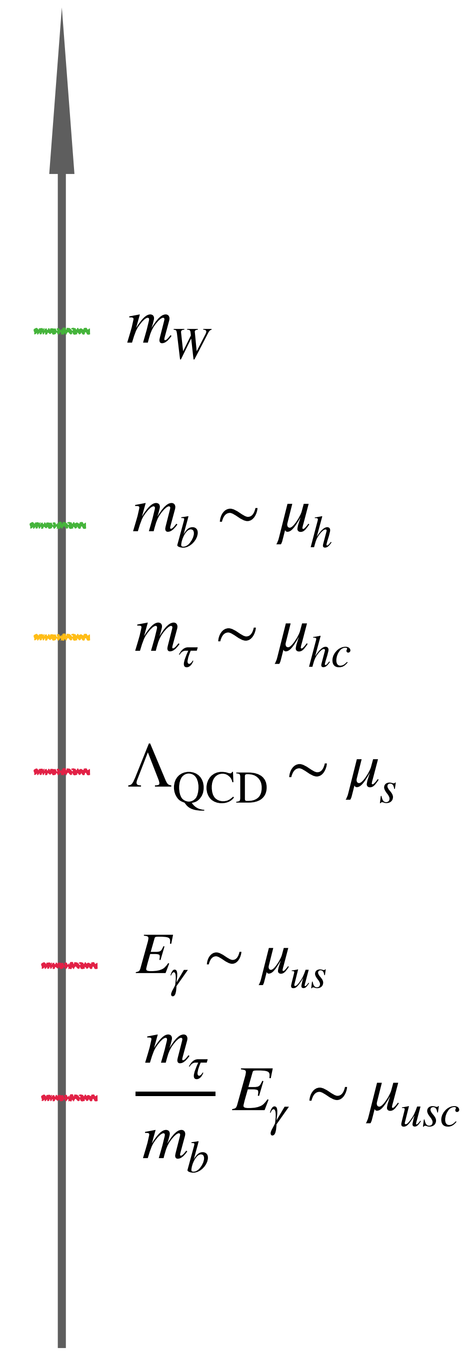
- Hard $(1, 1, 1)$
- Hard-collinear $(1, \lambda^2, \lambda)$
- Soft $(\lambda^2, \lambda^2, \lambda^2)$
- Soft-collinear $\lambda^2(1, b^2, b) \sim (\lambda^2, \lambda^4, \lambda^3)$

$$\lambda^2 = \frac{\Lambda_{\mathrm{QCD}}}{m_b}$$

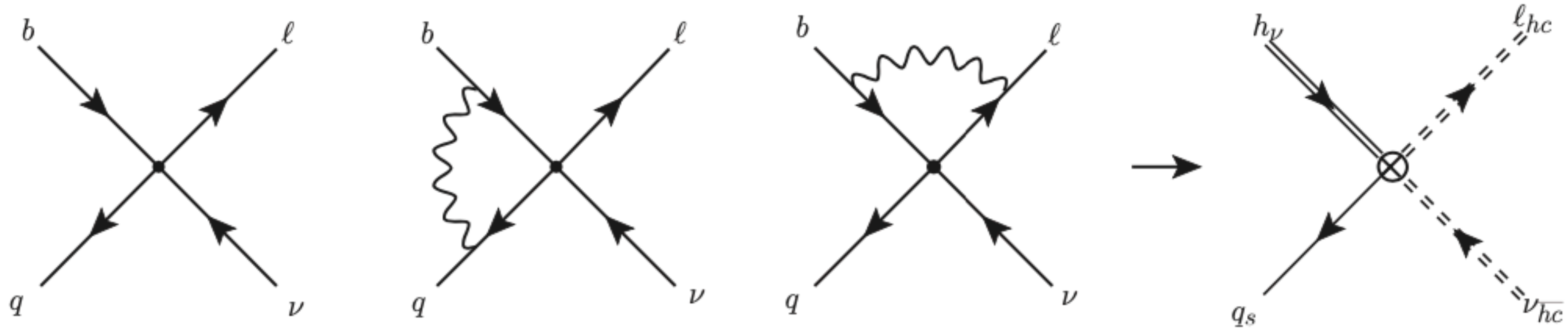
$$b = \frac{m_\tau}{m_b} \sim \lambda$$

- Relevant modes for **real** QED corrections:
  - ultrasoft $(\lambda_E^2, \lambda_E^2, \lambda_E^2)$
  - ultrasoft soft-collinear $\lambda_E^2(1, b^2, b)$

$$\lambda_E^2 = \frac{E_\gamma}{m_b} \sim \lambda^4$$



Fermi theory $\to$ HQET $\times$ SCET$_{\rm I}$



$$Q_1 = [\bar{u}\,\gamma^\mu P_L\, b]\,[\bar{\ell}\,\gamma_\mu P_L\,\nu]$$

$$E_1 = [\bar{u}\,\gamma^\mu\gamma^\nu\gamma^\rho P_L\, b]\,[\bar{\ell}\,\gamma_\mu\gamma_\nu\gamma_\rho P_L\,\nu] - 16\,Q_1$$

evanescent operator

$$\mathcal{O}^{(9)}_{A,1} = m_\ell\,[\bar{q}_s \frac{\not{n}_+}{2} P_L\, h_v]\,[\bar{\ell}_{hc}\frac{1}{i\,n_+\overleftarrow{\partial}_{hc}} P_L\,\nu_{\overline{hc}}]$$

$$\mathcal{O}_E = [\bar{q}_s \frac{\not{n}_+}{2}\gamma^\mu_\perp\gamma^\nu_\perp P_L\, h_v]\,[\bar{\ell}_{hc}\gamma_{\perp\nu}\gamma_{\perp\mu} P_L\,\nu_{\overline{hc}}]$$

$$\langle Q_1\rangle = \langle \mathcal{O}_{A,1}\rangle$$

$$\langle E_1\rangle = 6\,(D-4)\,\langle\mathcal{O}_{A,1}\rangle - 3\,\langle\mathcal{O}_E\rangle \sim \mathcal{O}(\epsilon)\, A^{(0)}_{E1,A1}$$

$$H^{(1)}_{A,1} = A^{(1)}_{1,(A,1)} + Z^{(1)}_{ext}\,A^{(0)}_{1,(A,1)} + Z^{(1)}_{(A,1)j}\,A^{(0)}_{j,(A,1)} - H^{(0)}_{A,1}\,(\mu_b)\;Z^{(1)}_{(A,1)(A,1)}$$

$$Z_{A1,E1} = \frac{1}{2\,\epsilon}\,Q_\ell\,(Q_\ell + 2\,Q_u)$$

$$Y(x,y) = \exp\left[i\, e\, Q_q \int_y^x dz_\mu\, A_s^\mu(z)\right] \mathcal{P}\exp\left[i\, g_s \int_y^x dz_\mu\, G_s^\mu(z)\right],$$

$$Y_\pm(x) = \exp\left[-i\, e\, Q_\ell \int_0^\infty ds\, n_\mp A_s\,(x + s n_\mp)\right].$$

$$S_r^{\prime(i)}(x) = \exp\left[-i\, e\, Q_i \int_0^\infty ds\, r \cdot A_{us}\,(x + s \cdot r)\right]$$

$$C_r^{\prime(i)}(x) = \exp\left[-i\, e\, Q_i \int_0^\infty ds\, r \cdot A_{usc}\,(x + s \cdot r)\right]$$