Generalized Distribution Amplitudes and Gravitational Form Factors of Hadrons

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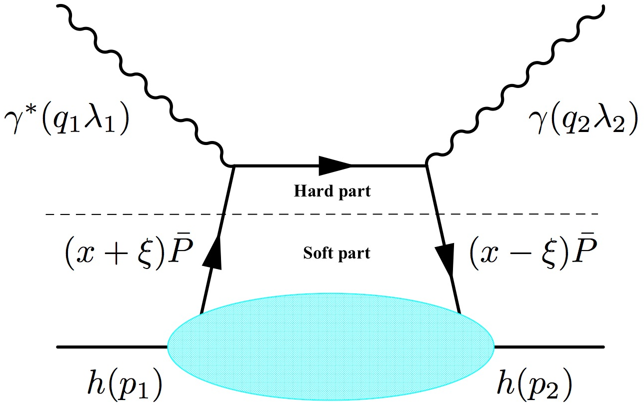
References:
C. Lorce, B. Pire and Qin-Tao Song, PRD106 (2022), 094030.
B. Pire and Qin-Tao Song, PRD 107 (2023), 114014.
B. Pire and Qin-Tao Song, PRD 109 (2024), 074016.
Qin-Tao Song, O. V. Teryaev and S. Yoshida, arXiv:2503.11316.

Outline

- GPDs, GDAs and gravitational FFs
- Kinematical higher-twist corrections in two-photon reactions
- Exotic hybrid mesons, the Θ_3 term (a new gravitational FF) and GDAs
- GDAs and gravitational FFs in the perturbative limit: existence of the Θ_3 term

Generalized Parton distributions (GPDs，广义部分子函数)



Deeply Virtual Compton Scattering
(DVCS, 深度虚康普顿散射)

$\gamma^* h \rightarrow \gamma h$

X.D. Ji, PRL 78(1997), 610.

GPDs

- Proton spin puzzle
- Energy momentum tensor (EMT) FFs of hadrons
- mass radius, mass distribution, pressure distribution and shear force distribution

Interesting, little known

↓

Recent Reviews:

M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A 33 (2018) no.26, 1830025.
V. D. Burkert, L. Elouadrhiri, F. Girod, C. Lorce, P. Schweitzer and P. Shanahan, Rev. Mod. Phys. 95 (2023), 041002.

EMT form factors and mass radius of pions (or unstable hadrons)?

The GPDs of pions cannot be accessed by DVCS, since there is currently no such a facility.

DVCS : $\gamma^* + \pi \to \gamma + \pi$

How to obtain EMT form factors of pions

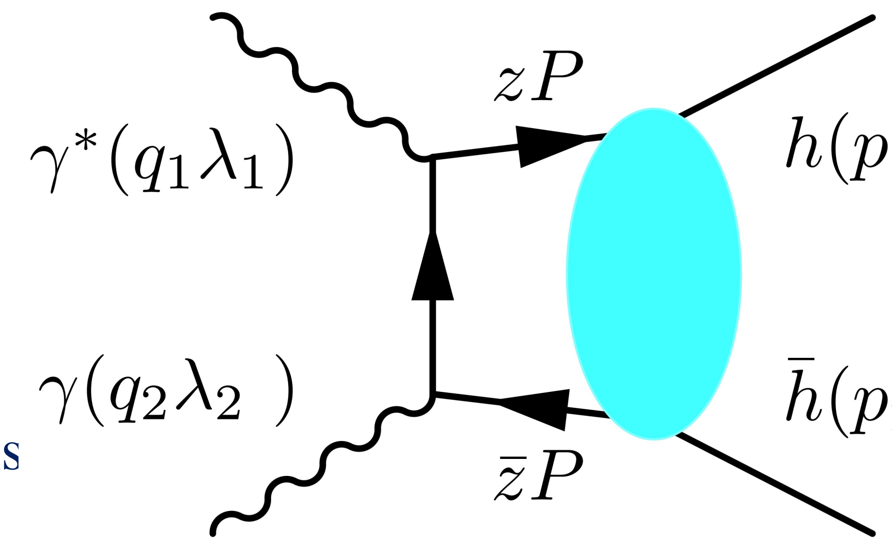
Option 1: Model calculations of EMT form factors.

Option 2: EMT form factors can be obtained from generalized distribution amplitudes (GDAs) of pions

GDAs in $\gamma^* + \gamma \to h + \bar{h}$:

Hard part: $\gamma^* + \gamma \to q + \bar{q}$

Soft part: $q + \bar{q} \to h + \bar{h}$, GDAs



Quark GDA of a scalar meson is defined as:

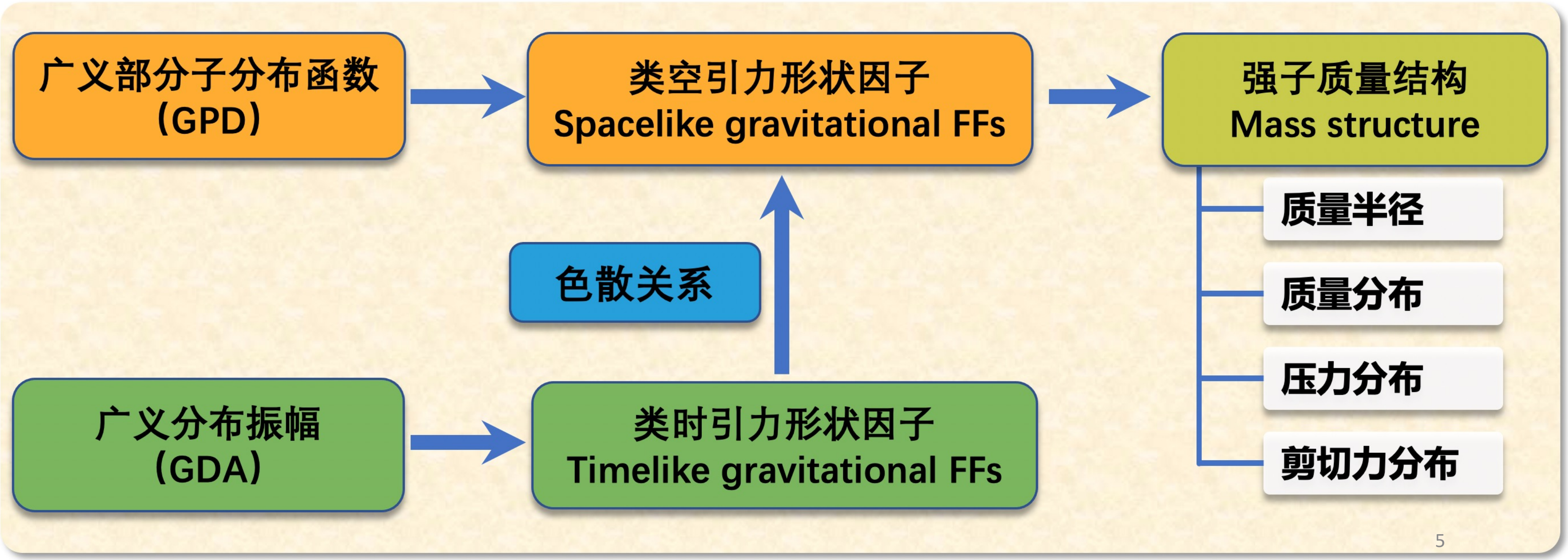
$$\Phi(z,\cos\theta,s) = \int \frac{dx^-}{2\pi} e^{-iP^+x^-} \langle h(p)\bar{h}(p')|\bar{q}(x^-)\gamma^+ q(0)|0\rangle$$

M. Diehl, T. Gousset, B. Pire and O. Teryaev, PRL **81** (1998) 1782.
M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.
M. V. Polyakov, NPB **555** (1999) 231.

GDAs are also important inputs for decays of B mesons.

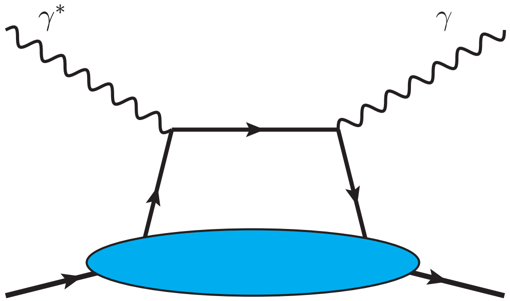
W. F. Wang, H. N. Li, W. Wang and C. D. Lu, PRD 91 (2015), 094024.
Y. Li, A. J. Ma, W. F. Wang and Z. J. Xiao, PRD 95 (2017), 056008.
M. K. Jia, C. Q. Zhang, J. M. Li and Z. Rui, PRD 104 (2021), 073001.
J. W. Zhang, B. Y. Cui, X. G. Wu, H. B. Fu and Y. H. Chen, PRD 110(2024), 036015.
…

From GPDs and GDAs to hadron gravitational FFs:

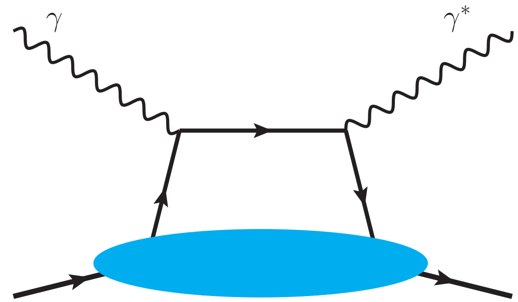


GPDs and GDAs are measured in two-photon reactions

GPDs:

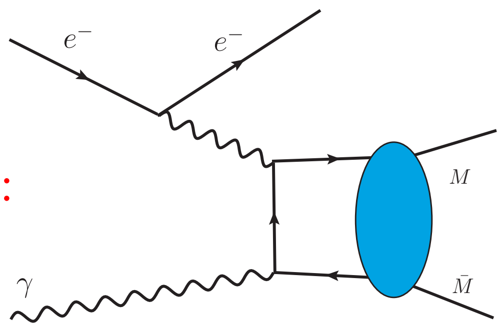


DVCS@JLAB, Compass,
EIC-US, EicC



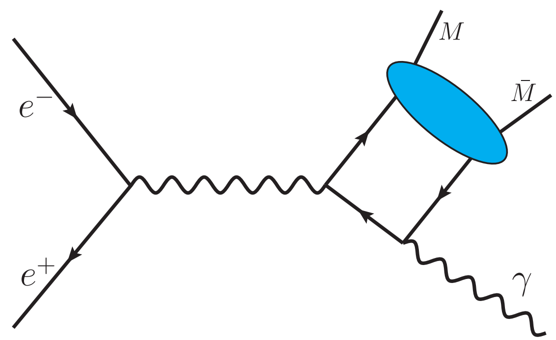
Timelike Compton scattering(TCS)
@JLAB, EIC-US, EicC
First measurement of TCS: PRL 127 (2021), 262501

GDAs:



$e^- + \gamma \rightarrow e^- + \mathrm{M} + \bar{\mathrm{M}}$
@Belle and Belle II

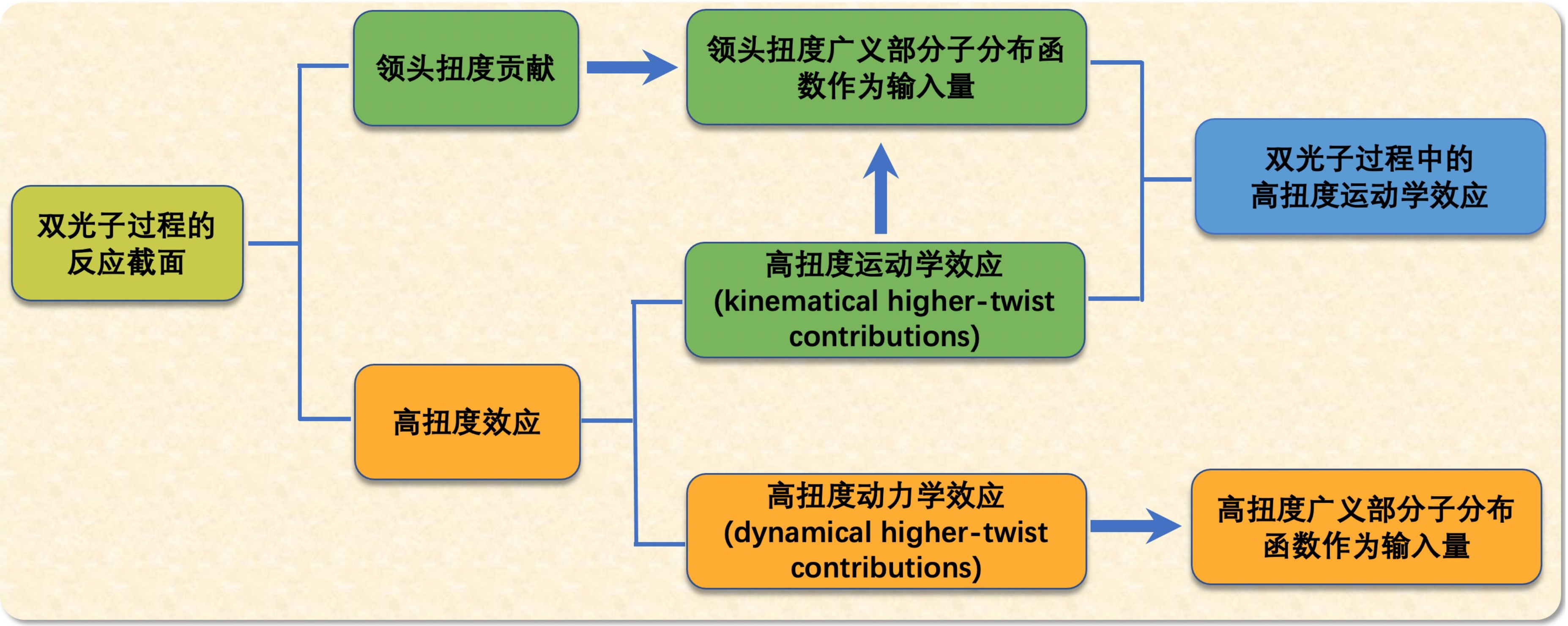
M. Masuda et al. [Belle], PRD 93 (2016), 032003.
M. Masuda et al. [Belle], PRD 97 (2018), 052003.



$\mathrm{e^+e^-} \rightarrow \gamma^* \rightarrow \mathrm{M} + \bar{\mathrm{M}} + \gamma$@BESIII
@STCF

Higher-twist contributions of order s/Q^2 and m^2/Q^2 are important in the measurements.
Kinematical higher-twist corrections are discussed in the first part of this talk.

Kinematical higher-twist contributions



- Higher-twist corrections: leading and higher-twist GPDs (GDAs).
- Kinematical higher-twist corrections: leading-twist GPDs(GDAs)!
- Higher-order corrections of α_s: leading-twist quark and gluon GPDs (GDAs).

V. Braun and A. Manashov, PRL 107(2011), 202001;JHEP 01 (2012), 085; PPNP 67 (2012), 162–167.
V. Braun, Y. Ji and A. Manashov, JHEP 01(2023), 078

The kinematical corrections are included in recent DVCS measurements.

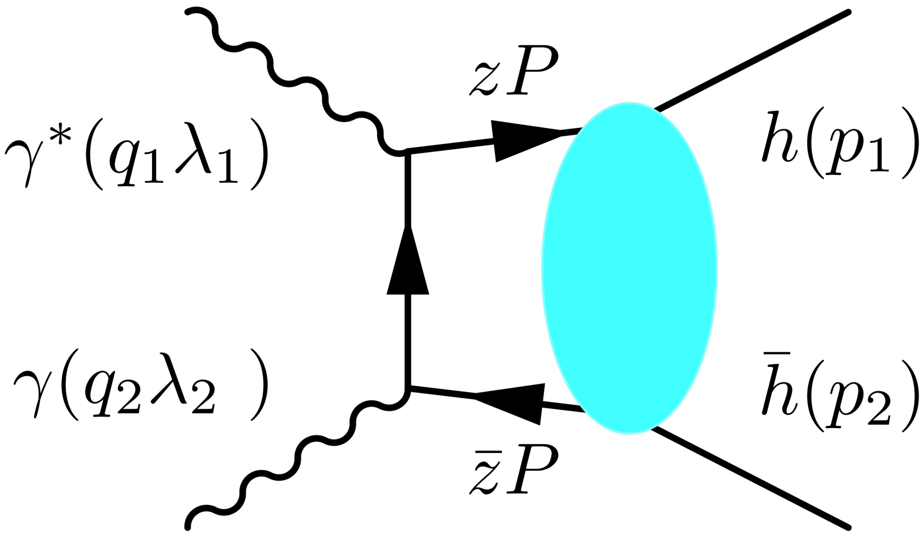
F. Georges et al. [Jefferson Lab Hall A], PRL. 128 (2022), 252002.
M. Defurne et al., Nature Communication 8(2017), 1408.
M. Defurne et al., Hall A collaboration, PRC92 (2015) no.5, 055202

Kinematical higher-twist corrections in $\gamma^* + \gamma \to \mathrm{M} + \overline{\mathrm{M}}$ (Belle) and $\gamma^* \to \mathrm{M} + \overline{\mathrm{M}} + \gamma$ (BESIII)

Kinematical contributions in $\gamma^* + \gamma \rightarrow M + \bar{M}$

We can also calculte the amplitudes of $\gamma^* + \gamma \rightarrow M + \bar{M}$ by using the operator results of the kinematical contributions in two electromagnetic currents.

$$T_{\mu\nu} = \mathrm{T}\{j_\mu(z_1 x) j_\nu(z_2 x)\}$$



Helicity amplitudes of a scalar meson:

$$A_{\lambda_1\lambda_2} = T_{\mu\nu}\epsilon^\mu(\lambda_1)\epsilon^\nu(\lambda_2)$$

There are three independent helicity amplitudes: A_{++}, A_{0+} and A_{+-}.

Leading twist amplitude: A_{++}
Higher twist amplitudes: A_{0+} and A_{+-}.

M. Diehl, T. Gousset, B. Pire and O. Teryaev, PRL **81** (1998) 1782.
M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.
M. V. Polyakov, NPB **555** (1999) 231.

Helicity amplitudes (up to twist 4):

$$A^{(0)} = \chi \left\{ \left(1 - \frac{s}{2Q^2}\right) \int_0^1 dz\, \frac{\Phi(z,\eta,s)}{1-z} - \frac{s}{Q^2} \int_0^1 dz\, \frac{\Phi(z,\eta,s)}{z} \ln(1-z) \right.$$
$$\left. - \left(\frac{2s}{Q^2}\eta + \frac{\Delta_T^2}{\beta_0^2 Q^2}\frac{\partial}{\partial\eta}\right) \frac{\partial}{\partial\eta} \int_0^1 dz\, \frac{\Phi(z,\eta,s)}{z} \left[\frac{\ln(1-z)}{2} + \mathrm{Li}_2(1-z) - \mathrm{Li}_2(1)\right] \right\},$$

$$A^{(1)} = \frac{2\chi}{\beta_0 Q}\frac{\partial}{\partial\eta} \int_0^1 dz\, \Phi(z,\eta,s)\, \frac{\ln(1-z)}{z},$$

$$A^{(2)} = -\frac{2\chi}{\beta_0^2 Q^2}\frac{\partial^2}{\partial\eta^2} \int_0^1 dz\, \Phi(z,\eta,s)\, \frac{2z-1}{z} \ln(1-z),$$

$\eta = \cos\theta$

C. Lorce, B. Pire and Qin-Tao Song, PRD 106 (2022) , 094030

$A_{++} = A^{(0)}$

$A_{0+} = -A^{(1)} \Delta \cdot \epsilon(-)$ ⟶ $\propto \Delta_T$

$A_{-+} = -A^{(2)} [\Delta \cdot \epsilon(-)]^2$ ⟶ $\propto (\Delta_T)^2$

Δ is the relative momentum of final meson pair.

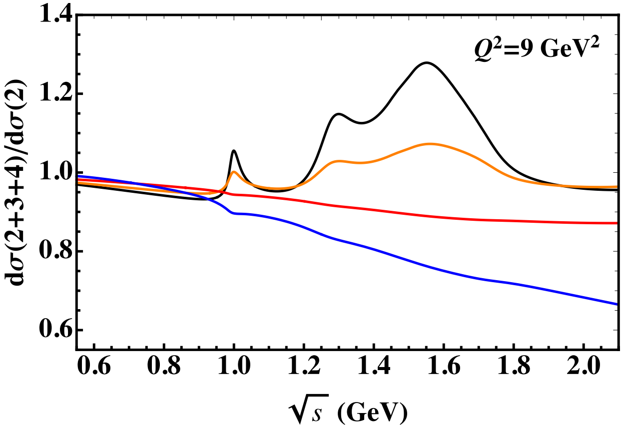
Asymptotic form of pion GDAs:

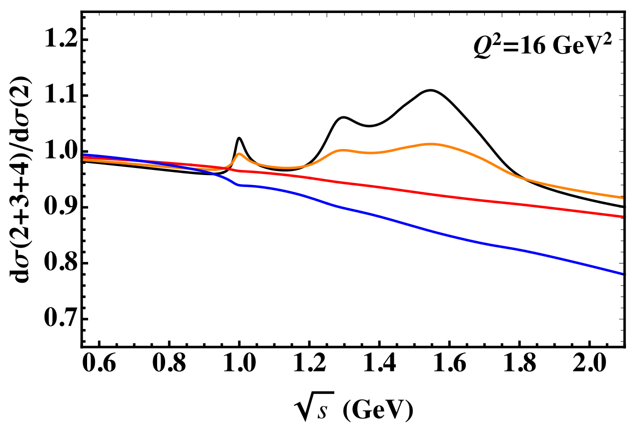
$$\Phi(z, \cos\theta, s) = 18z(1-z)(2z-1)[\tilde{B}_{10}(s) + \tilde{B}_{12}(s)\, P_2(\cos\theta)]$$

The nonvanishing helicity-flip amplitudes A_{0+} and A_{+-} indicate the existence of the D-wave GDAs.

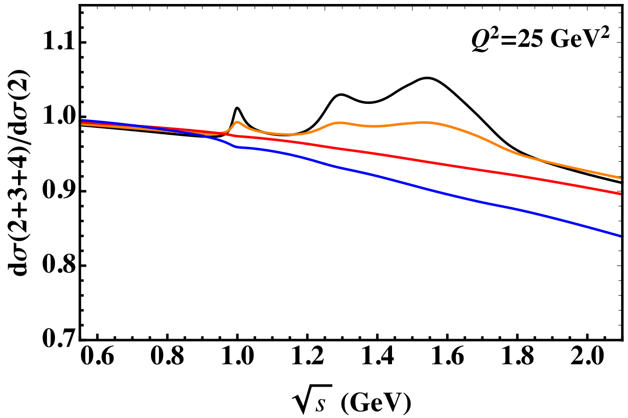
Ratios are estimated with the asymptotic π π GDA

Ratio=(twist 2+twist 3+twist 4)/ twist 2





Kinematics are chosen according to Belle(II)



Both types of π π GDAs indicate that the higher-twist kinematical contributions cannot be neglected if $s > 1$ GeV^2

GDAs ⟶ Timelike EMT form factors

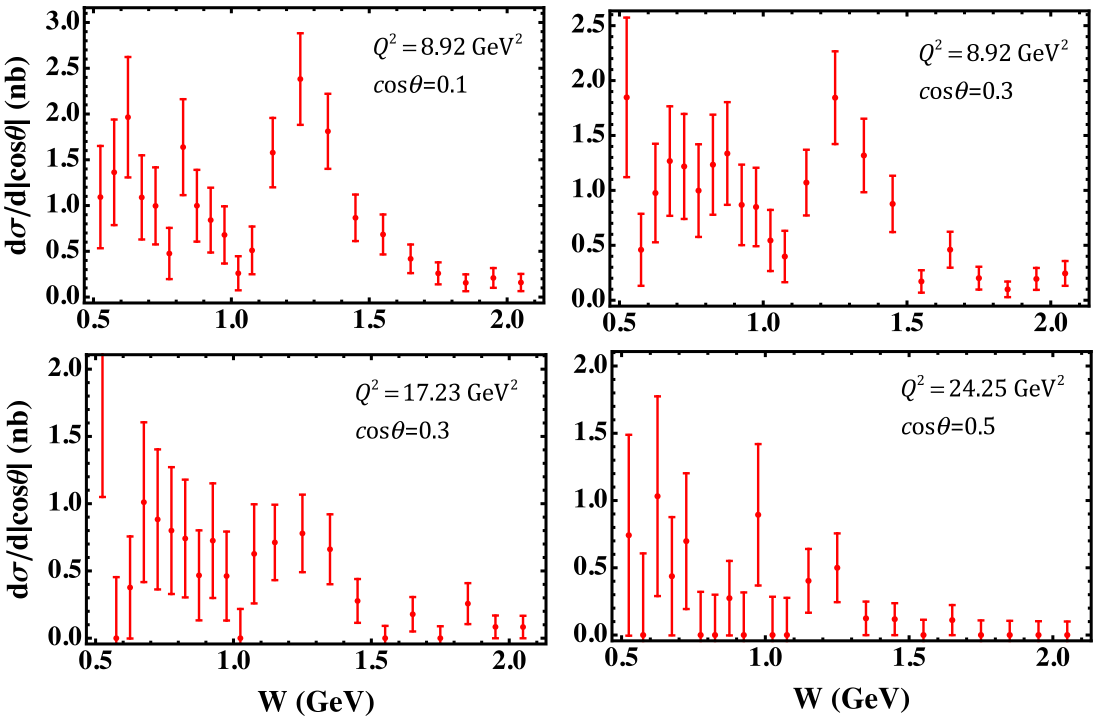
$\Lambda \geq 3$ GeV^2 is necessary for pion EMT form factor, PRD 97 (2018) 014020.

Dispersion relation: Spacelike form factor t<0

$$F(t) = \int_{4m^2}^{\Lambda} \frac{ds}{\pi} \frac{\text{Im}[F(s)]}{s - t - i\varepsilon}$$

Timelike form factor s>0

Future measurements of $\gamma^* + \gamma \rightarrow M + \bar{M}$ at Belle II



Belle measurements on $\gamma^*\gamma \rightarrow \pi^0\pi^0$ in 2016

$8\ \mathrm{GeV}^2 < Q^2 < 24\ \mathrm{GeV}^2$

$0.2\ \mathrm{GeV}^2 < s < 4\ \mathrm{GeV}^2$

$\sim s/Q^2$, $\sim m^2/Q^2$ kinematical corrections

M. Masuda et al. [Belle], PRD 93 (2016), 032003.

First extraction of pion GDAs and EMT FFs from experimental data

S. Kumano, Qin-Tao Song and O. Teryaev, PRD **97** (2018) 014020.

The errors are large, and statistical errors are dominant, however, this situation can be improved by Belle II.

Luminosity: $2\times 10^{34}\ \mathrm{cm}^{-2}s^{-1} \rightarrow 8\times 10^{35}\ \mathrm{cm}^{-2}s^{-1}$

Previous measurements at Belle focused on EM FFs, however, the extraction of EMT FFs will be the main physical target for future measurements at Belle II. See talk of Dr. Masuda at Joint Meeting the APS and JPS 2023.

Kinematical higher-twist corrections in

$\mathrm{e}^+\mathrm{e}^- \to \gamma^* \to M_1 + M_2 + \gamma$ and $e + \gamma \to e + M_1 + M_2$

Exotic hybrid mesons

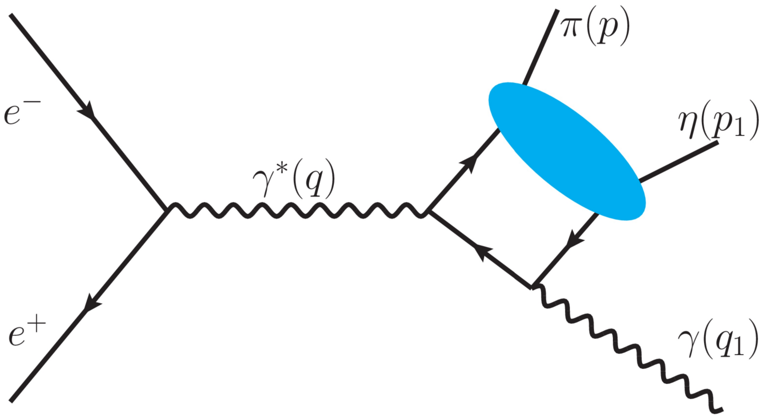
One can search for the candidates of the hybrid mesons from the P-wave of $M_1 M_2$ in $\gamma^* \to M_1 + M_2 + \gamma$ and $\gamma^* + \gamma \to M_1 + M_2$.

$M_1 M_2$: $\pi\eta, \pi\eta'$

Isovector hybrid mesons
$I^G(J^{PC}) = 1^-(1^{-+})$
$\pi_1(1400), \pi_1(1600)$

$M_1 M_2$: $\eta\eta'$

Isoscalar hybrid mesons
$I^G(J^{PC}) = 0^+(1^{-+})$
$\eta_1(1855)$



$\gamma^* \to \pi + \eta + \gamma$ at BESIII

The exotic quantum number($J^{PC} = 1^{-+}$) does not exit in quark model.

$\eta_1(1855)$ was observed by BESIII in $J/\psi \to \eta + \eta' + \gamma$ recently.

M. Ablikim et al. [BESIII], PRL 129 (2022), 192002.
M. Ablikim et al. [BESIII], PRD 106 (2022), 072012.

$J/\psi \to \gamma^*$: $\gamma^* \to \eta + \eta' + \gamma$ can be also measured by BESIII.

B. Pire and Q. T. Song, PRD 107 (2023), 114014.

"Shear viscosity" term (a new gravitational FF)

If the hybrid mesons are observed in $\gamma^* \to M_1 + M_2 + \gamma$ and $\gamma^* + \gamma \to M_1 + M_2$, it will indicate the existence of a new EMT FF.

$$\langle M_2(p_2) M_1(p_1) | T_q^{\mu\nu} | 0 \rangle \sim \Theta_3(s) P^\mu \Delta^\nu$$

O. Teryaev, JPS Conf. Proc. 37(2022), 020406.

The Θ_3 term could exist in matrix element of EMT.

Its sum over quarks and gluons should be zero which is a consequence of the conserved EMT, however, it will exist for a single flavor q on condition that there is P-wave GDA.

The study of the Θ_3 term and hybrid mesons by GDAs at BESIII and Belle II.

$\gamma^* \to M_1 M_2 \gamma$
$\gamma^* \gamma \to M_1 M_2$

P wave $M_1 M_2$

The Θ_3 term exists

Resonances in P wave

Exotic (hybrid) mesons observed

Extracted from cross sections

GDAs and EMT FFs

GDAs and gravitational FFs in the perturbative limit: existence of the Θ_3 term

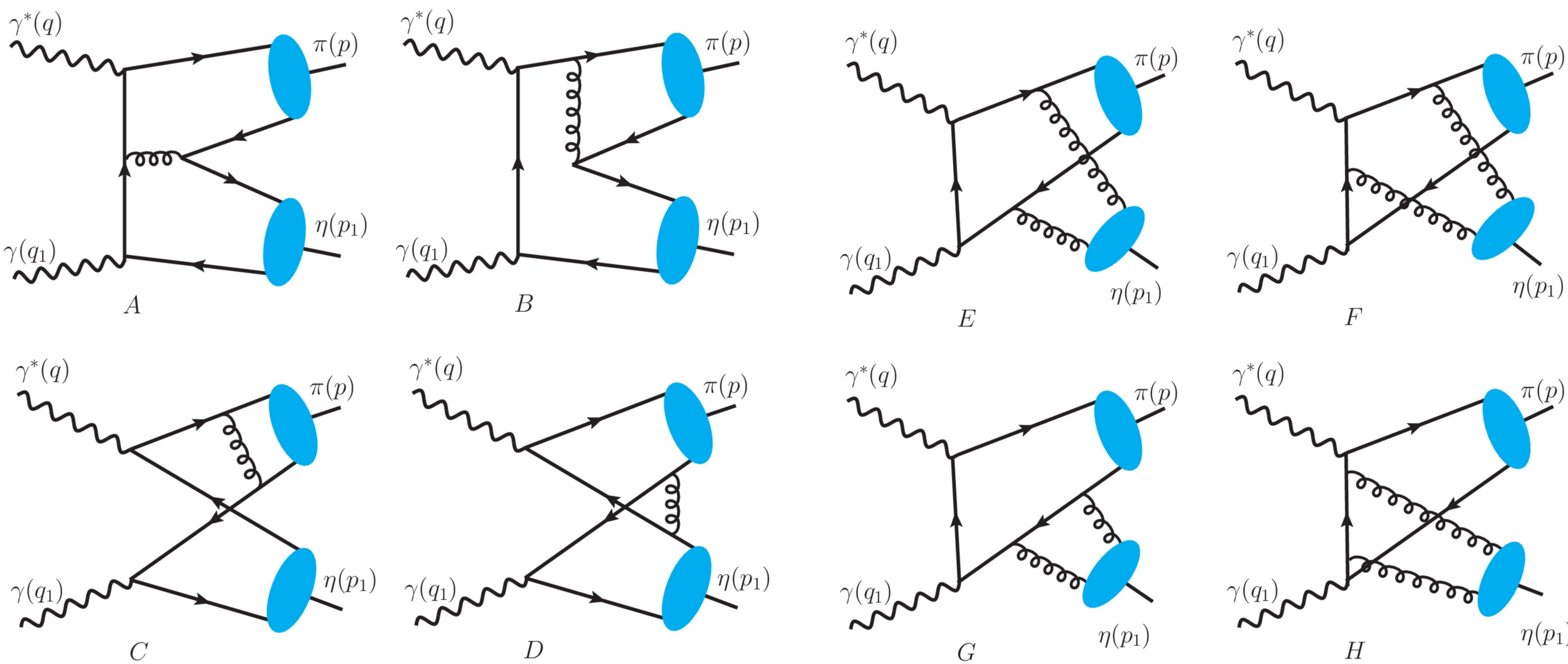
GDAs in the perturbative limit

Perturbative limit : $Q^2 \gg s \gg \Lambda^2_{\mathrm{QCD}}$

The two-pion GDA can be expressed in terms of pion DAs from the amplitude of $\gamma^* + \gamma \to \pi^+ + \pi^-$, and meson DAs are relatively well-known quantites.

M. Diehl, T. Feldmann, P. Kroll and C. Vogt, PRD 61 (2000), 074029

The relation between pion-eta GDA and meson DAs in $\gamma^* + \gamma \to \pi + \eta$



Feynman diagrams for $\gamma^* + \gamma \to \pi + \eta$ in perturbative limit

$$\hat{\Phi}^{1q}_{\pi\eta} = 2c\left\{\theta(z_\xi)\frac{\bar{\xi}}{z_\xi}\int_0^1 \frac{dx}{s}\frac{z+\bar{x}\xi}{z-x\xi}\frac{\phi^q_\pi(x)}{\bar{x}}\phi^q_\eta(\tfrac{\bar{z}}{\bar{\xi}}) + \theta(-z_\xi)\frac{\xi}{z_\xi}\int_0^1 \frac{dx}{s}\frac{\bar{z}+\bar{x}\bar{\xi}}{\bar{z}-x\bar{\xi}}\frac{\phi^q_\eta(x)}{\bar{x}}\phi^q_\pi(\tfrac{z}{\xi})\right\}$$

Quark DAs of mesons

$$\hat{\Phi}^{2q}_{\pi\eta} = -\frac{2c\tilde{c}\xi}{s}\left\{\int_{S_1}\frac{dy}{\bar{y}y}\frac{y^2-\rho_y z+\xi\bar{y}y}{(z-y-\xi\bar{y})(z-\bar{\xi}y)}\phi^g_\eta(y)\phi^q_\pi(x) - \frac{2\theta(-z_\xi)}{z_\xi}\int_0^1 dy\frac{\bar{y}}{y}\phi^g_\eta(y)\phi^q_\pi(\tfrac{z}{\xi})\right\}, \quad (22$$

Pion quark DA and eta gluon DA

GDAs in the perturbative limit: $\hat{\Phi}^q_{\pi\eta}(z,\xi,s) = \hat{\Phi}^{1q}_{\pi\eta}(z,\xi,s) + \hat{\Phi}^{2q}_{\pi\eta}(z,\xi,s)$

At current stage, there are no experimental facilities to measure $\gamma^* + \gamma \to \pi + \eta$ in the perturbative limit, and the obtained GDAs cannot be tested by experiment.

The process $\gamma^* \to \pi + \eta + \gamma$ can be measured at Belle II in the perturbative limit

$\gamma^* \to \pi + \eta + \gamma$: timelike photon
$\gamma^* + \gamma \to \pi + \eta$: spacelike photon
→ Test the universality of GDAs

D. Mueller, B. Pire, L. Szymanowski and J. Wagner, RD 86 (2012), 031502.

After a similar calculation, we find the GDAs are identical for these two processes, which verifies the universality of GDAs

Qin-Tao Song, O. V. Teryaev and S. Yoshida, arXiv:2503.11316.

From GDAs to gravitational FFs

$$\int_0^1 dz \rho_z \Phi^q_{\pi\eta}(z,\xi,s)$$
$$= \frac{2}{(P^+)^2} \left\langle \eta(p_1)\pi(p) \left| T_q^{++}(0) \right| 0 \right\rangle ,$$

The second moment of the GDA

$$\left\langle \eta(p_1)\pi(p) \left| T_q^{\mu\nu}(0) \right| 0 \right\rangle$$
$$= \frac{1}{2}\left[\Theta_1^q(s)(s g^{\mu\nu} - P^\mu P^\nu) + \Theta_2^q(s) \Delta^\mu \Delta^\nu + \Theta_3^q(s) P^{\{\mu} \Delta^{\nu\}} \right].$$

The Θ_3 term does not exist for $\pi\pi$, which breaks the conservation law of EMT for each quark flavor in its hadronic matrix element, and its existence has not been verified.

The gravitation FFs are expressed in terms of meson DAs

$$\Theta_1^q = -\frac{c}{s}\int dx dy \left[\frac{1+\bar{x}+y}{\bar{x}y} \phi_\eta^q(y) + \frac{\tilde{c}y}{\bar{x}x} \phi_\eta^g(y) \right] \phi_\pi^q(x),$$
$$\Theta_2^q = -\frac{c}{s}\int dx dy \left[\frac{1+x+\bar{y}}{\bar{x}y} \phi_\eta^q(y) - \frac{\tilde{c}y}{\bar{x}x} \phi_\eta^g(y) \right] \phi_\pi^q(x),$$
$$\Theta_3^q = \frac{2c}{s}\int dx dy \left[\frac{x-\bar{y}}{\bar{x}y} \phi_\eta^q(y) + \frac{\tilde{c}y}{\bar{x}x} \phi_\eta^g(y) \right] \phi_\pi^q(x). \quad (31)$$

Gravitational FFs in the perturbative limit

Isospin symmetry: $\Theta_3^u(s) = -\Theta_3^d(s)$

The Θ_3 term vanishes when summing over quark flavors, and the conserved hadronic matrix elements of EMT is recovered.

$$\langle \eta(p_1)\pi(p) | \sum_{i=q} T_i^{\mu\nu}(0) | 0 \rangle P_\mu = 0.$$

We substitute the meson DAs into these FFs

$$\Theta_1^q = -\frac{c f_\pi^q}{2s}\Big\{6\left[5 + 4(a_2^\pi + a_2^\eta) + 3a_2^\pi a_2^\eta\right] f_\eta^q + \tilde{c}(1 + a_2^\pi) b_2^\eta f_\eta^1\Big\},$$

$$\Theta_2^q = -\frac{c f_\pi^q}{2s}\Big\{6\left[7 + 8(a_2^\pi + a_2^\eta) + 9a_2^\pi a_2^\eta\right] f_\eta^q - \tilde{c}(1 + a_2^\pi) b_2^\eta f_\eta^1\Big\},$$

$$\Theta_3^q = \frac{c f_\pi^q}{s} \sum_{i=1} \Big[6(a_{2i}^\pi - a_{2i}^\eta) f_\eta^q + \tilde{c}(1 + \sum_{j=1} a_{2j}^\pi) b_{2i}^\eta f_\eta^1\Big]$$

The second term will be nonzero provided that the gluon DA does not vanish

$$b_{2i}^\eta \neq 0$$

The first term arises only when the quark DA of the pion meson differs from that of the eta meson.

The gravitational FFs for the $\eta\eta'$ pair

$$\Theta_3^q = \frac{c f_\pi^q}{s} \sum_{i=1} \left[6(a_{2i}^\pi - a_{2i}^\eta) f_\eta^q + \tilde{c}(1 + \sum_{j=1} a_{2j}^\pi) b_{2i}^\eta f_\eta^1 \right]$$

I. Cloet, L. Chang, C.D.Roberts,et. al.,PRL111(2013), 092001.
J. Hua et. al., [Lattice Parton], PRL 129(2022) no.13, 132001.
T. Zhong, Z.H.Zhu, H.B.Fu et. al., PRD 104(2021), 016021.
C. Shi, M. Li, X. Chen and W.Jia,PRD 104(2021), 094016.
X. Gao, et. al., PRD 106(2022), 074505.
…

$$\mu_F^2 = 30\ \text{GeV}^2 \begin{cases} a_2^\pi \sim 0.16 \\ a_2^\eta \sim -0.03 \end{cases}$$

P. Kroll and K. Passek-Kumericki, J. Phys. G 40 (2013), 075005.

The eta gluon DA: $b_2^\eta \neq 0$

Yeo-Yie Charng, T. Kurimoto, and Hsiang-nan Li,PRD 74(2006), 074024.
S. Agaev, V. Braun, N. Offen, A. Porkert and A. Schäfer, PRD 90(2014), 074019.
P. Kroll and K. Passek-Kumericki,PRD 67(2003), 054017.

The existence of Θ_3 term ⟶ P-wave GDAs, exotic hybrid mesons in $\gamma^* \to M_1 + M_2 + \gamma$ and $\gamma^* + \gamma \to M_1 + M_2$.

For the $\eta\eta'$ pair

$$\Theta_1^q|_{\eta'\eta} = \Theta_1^q|_{\pi\eta} - \frac{c\tilde{c}}{s} \int dx dy \frac{x}{\bar{y}y} \phi_{\eta'}^g(x) \phi_\eta^q(y),$$

$$\Theta_2^q|_{\eta'\eta} = \Theta_2^q|_{\pi\eta} + \frac{c\tilde{c}}{s} \int dx dy \frac{x}{\bar{y}y} \phi_{\eta'}^g(x) \phi_\eta^q(y),$$

$$\Theta_3^q|_{\eta'\eta} = \Theta_3^q|_{\pi\eta} - \frac{2c\tilde{c}}{s} \int dx dy \frac{x}{\bar{y}y} \phi_{\eta'}^g(x) \phi_\eta^q(y),$$

Qin-Tao Song, O. V. Teryaev and S. Yoshida, arXiv:2503.11316.

Summary

- GDAs can be considered as an alternative way to investigate the EMT form factors of unstable hadrons such as pions.

- Kinematical higher-twist contributions are calculated for $\gamma^* + \gamma \rightarrow \mathrm{M} + \bar{\mathrm{M}}$ and $\gamma^* \rightarrow \mathrm{M} + \bar{\mathrm{M}} + \gamma$ from which the GDAs can be extracted. The numerical calculation indicates that kinematical contributions are significant for Belle (II) and BESIII (STCF).

- GDAs and gravitational FFs in the perturbative limit: existence of the Θ_3 term and the universality of GDAs are verified.

- The Θ_3 term indicate P-wave GDAs, thus, it is promising to search for exotic hybrid mesons in $\gamma^* \rightarrow M_1 + M_2 + \gamma$ and $\gamma^* + \gamma \rightarrow M_1 + M_2$.

Thank you very much