Next-to-leading order QCD corrections to $B_c^* \to J/\psi$ form factors

陶伟
河南师范大学

Based on
Qin Chang, Wei Tao, Zhen-Jun Xiao, Ruilin Zhu,
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Outline

1. Introduction
2. Calculation Procedure
3. Results
4. Summary

B_c^* semileptonic decay and form factors

➢ $c\bar{b}$ meson

- the only meson containing two different heavy flavors
- Bc(1S) discovered in 1998 through $B_c \to J/\psi + l\nu_l$
- Bc*(1S) not yet observed due to difficulty in detecting γ of $B_c^* \to B_c + \gamma$
- Bc*(1S) weak decays (e.g. $B_c^* \to J/\psi + l\nu_l$) help search for Bc*(1S)

[CDF, PRL(1998)]
[CMS, PRL(2019)]
[LHCb, PRL(2019)]

➢ Form factors (FFs)

- Related to decay widths and branching ratios
- Lattice results available for $B_c \to J/\psi\,(\eta_c)$ (axial-)vector form factors, not for B_c^*
- $B_c^* \to J/\psi$ form factors known only by light-front quark model (LFQM)

[HPQCD, 1611.01987]
[HPQCD, PRD(2020)]

[Q.Chang,L.T.Wang,X.N.Li, JHEP(2019)]
[Q.Chang,et al., AHEP(2020)]
[S.Y.Wang,et al., CPC(2024)]

Why NRQCD higher-order calculations

[G.T.Bodwin,E.Braaten,G.P.Lepage,PRD(1995)]

➢ Non-Relativistic Quantum Chromodynamics (NRQCD)

- Form factor = short-distance coefficient ✖ wavefunction at origin

perturbative expansion in α_s — nonperturbative

- To test perturbative expansion convergence and renormalization scale dependence in NRQCD
- To obtain more precise theoretical predictions and test the Standard Model
- To study new physics by calculating (axial-)tensor form factors

Review NRQCD calculations for $c\bar{b} \to c\bar{c}$ form factors

- 2007, first one-loop for $B_c \to \eta_c$ vector and tensor FFs
  [G. Bell, 0705.3133]

- Since 2011, next-to-leading order (NLO) QCD corrections to $B_c \to J/\psi\ (\eta_c)$ (axial-)vector and (axial-)tensor FFs
  [C.F.Qiao,P.Sun, JHEP(2012)]
  [C.F.Qiao,R.Zhu, PRD(2013)]
  [W.Tao,Z.J.Xiao,R.Zhu, PRD(2022)]

- From 2017 onward, relativistic corrections for Bc decaying into S(P)-wave charmonium FFs
  [R.Zhu,et al., PRD(2017)]
  [R.Zhu, NPB(2018)]
  [D.Shen,et al., IJMPA(2021)]

- 2024, leading order (LO) for $B_c^* \to J/\psi$ (axial-)vector FFs
  [Y.Geng,M.Cao,R.Zhu, PRD(2024)]

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Definition for $B_c^* \to J/\psi$ form factors

$$\begin{aligned}&\left\langle J/\psi\left(\epsilon', p'\right)\left|\bar{b}\gamma_\mu c\right| B_c^*(\epsilon, p)\right\rangle \\ =&-\left(\epsilon\cdot\epsilon'^*\right)\left[P_\mu V_1\left(q^2\right)-q_\mu V_2\left(q^2\right)\right]-\left(\epsilon\cdot q\right)\epsilon'^*_\mu V_3\left(q^2\right) \\ &+\left(\epsilon'^*\cdot q\right)\epsilon_\mu V_4\left(q^2\right)+\left(\epsilon\cdot q\right)\left(\epsilon'^*\cdot q\right)\left[\left(\frac{P_\mu}{M^2-M'^2}\right.\right. \\ &\left.\left.-\frac{q_\mu}{q^2}\right)V_5\left(q^2\right)+\frac{q_\mu}{q^2}V_6\left(q^2\right)\right],\end{aligned}$$

$$\begin{aligned}&\left\langle J/\psi\left(\epsilon', p'\right)\left|\bar{b}\gamma_\mu\gamma_5 c\right| B_c^*(\epsilon, p)\right\rangle \\ =&i\varepsilon_{\mu\nu\alpha\beta}\epsilon^\alpha\epsilon'^{*\beta}\left[P^\nu A_1\left(q^2\right)-q^\nu A_2\left(q^2\right)\right] \\ &+\frac{i\varepsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta}{M^2-M'^2}\left[\epsilon'^*\cdot q\epsilon^\nu A_3\left(q^2\right)-\epsilon\cdot q\epsilon'^{*\nu}A_4\left(q^2\right)\right] \\ &-\frac{i\varepsilon_{\rho\nu\alpha\beta}\epsilon^\alpha\epsilon'^{*\beta}P^\nu q^\rho}{M^2-M'^2}\left[P_\mu A_5\left(q^2\right)-q_\mu A_6\left(q^2\right)\right].\end{aligned}$$

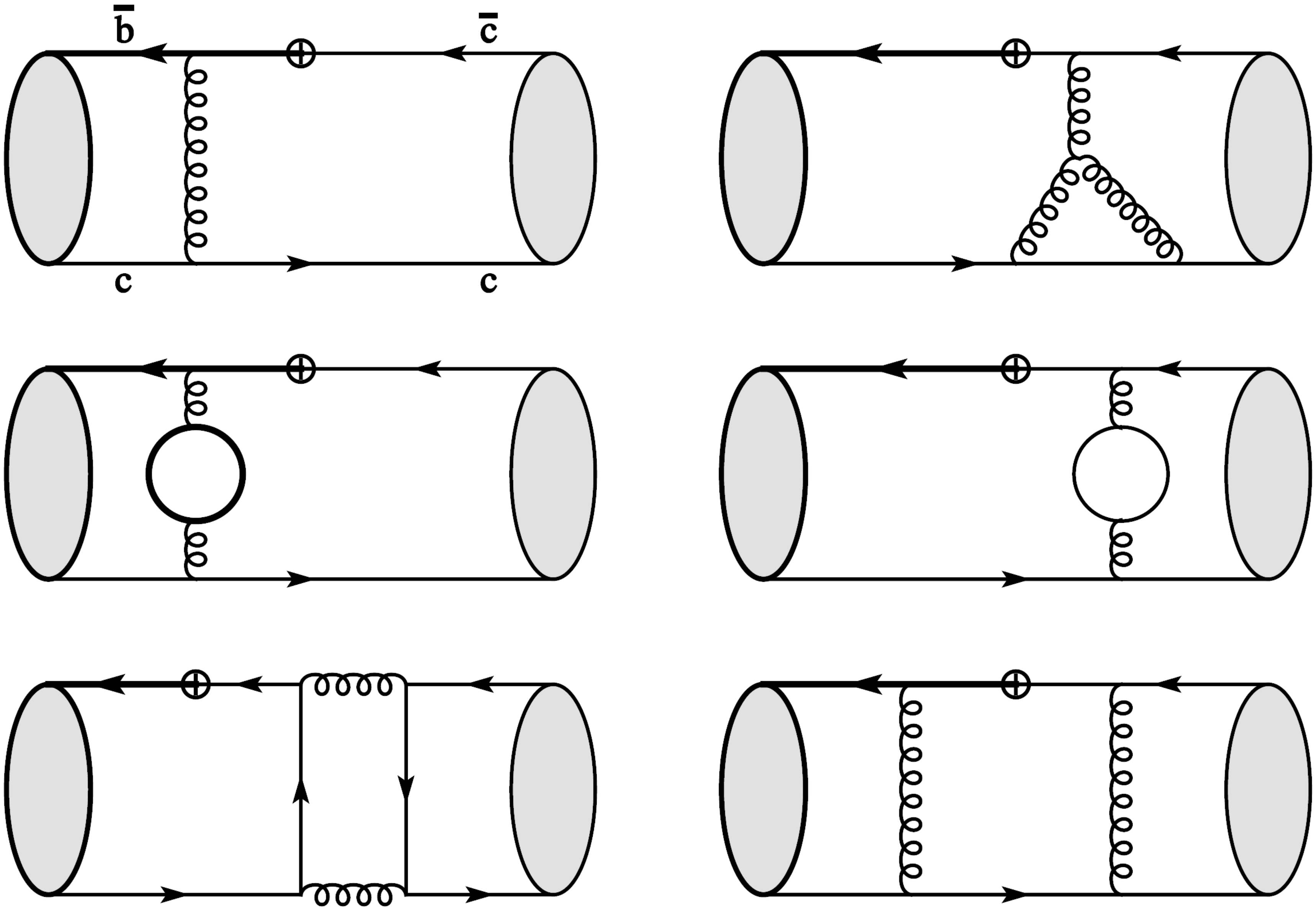
Independent FFs:
$V_{1,2,3,4,5,6}$, $A_{1,2,3,4}$,
$T_{2,3,4,6}$, $T'_{2,3,4}$

$$\begin{aligned}&\left\langle J/\psi\left(\epsilon', p'\right)\left|\bar{b}\sigma_{\mu\nu}q^\nu c\right| B_c^*(\epsilon, p)\right\rangle \\ =&-i\left(\epsilon\cdot\epsilon'^*\right)\left[P_\mu T_1\left(q^2\right)-q_\mu T_2\left(q^2\right)\right]\left(M+M'\right) \\ &-i\left[\left(\epsilon\cdot q\right)\epsilon'^*_\mu T_3\left(q^2\right)-\left(\epsilon'^*\cdot q\right)\epsilon_\mu T_4\left(q^2\right)\right]\left(M+M'\right) \\ &+i\frac{\left(\epsilon\cdot q\right)\left(\epsilon'^*\cdot q\right)}{M+M'}\left[P_\mu T_5\left(q^2\right)+q_\mu T_6\left(q^2\right)\right],\end{aligned}$$

$$\begin{aligned}&\left\langle J/\psi\left(\epsilon', p'\right)\left|\bar{b}\sigma_{\mu\nu}\gamma_5 q^\nu c\right| B_c^*(\epsilon, p)\right\rangle \\ =&-\varepsilon_{\mu\nu\alpha\beta}\epsilon^\alpha\epsilon'^{*\beta}\left[P^\nu T'_1\left(q^2\right)+q^\nu T'_2\left(q^2\right)\right]\left(M+M'\right) \\ &+\frac{\varepsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta}{M+M'}\left[\epsilon'^*\cdot q\epsilon^\nu T'_3\left(q^2\right)+\epsilon\cdot q\epsilon'^{*\nu}T'_4\left(q^2\right)\right] \\ &-\frac{\varepsilon_{\rho\nu\alpha\beta}\epsilon^\alpha\epsilon'^{*\beta}P^\nu q^\rho}{M+M'}\left[P_\mu T'_5\left(q^2\right)-q_\mu T'_6\left(q^2\right)\right].\end{aligned}$$

- $P = p + p'$
- $q = p - p'$: transfer momentum

Step 1: generate Feynman diagrams & amplitudes



Step 2: amplitude simplification

- Dirac matrix simplification, index contraction, color algebra simplification, and trace calculation
- γ_5 scheme for the trace of a fermion chain containing γ_5

➢ Naïve γ_5 scheme when containing 0/2 γ_5 [V.Shtabovenko,R.Mertig,F.Orellana, CPC(2025)]

$$\gamma_5\gamma_\mu + \gamma_\mu\gamma_5 = 0, \gamma_5^2 = \mathbf{1} \text{ and cyclicity}$$

➢ Fixed reading point γ_5 scheme when containing 1/3 γ_5

- the fermion chain contains current vertex $\Gamma = \gamma_\mu\gamma_5$ or $\sigma_{\mu\nu}\gamma_5$

$$\mathrm{Trace}\,(a\cdot\Gamma\cdot b) \to \mathrm{Trace}\left(\frac{\Gamma\cdot b\cdot a + b\cdot a\cdot\Gamma}{2}\right)$$

- otherwise

[J.G.Korner,D.Kreimer,K.Schilcher, ZPC(1992)]
[S.A.Larin, PLB(1993)]
[S.Moch,J.A.M.Vermaseren,A.Vogt, PLB(2015)]

$$\mathrm{Trace}\,(a\cdot\gamma_5\cdot b) \to \mathrm{Trace}\,(b\cdot a\cdot\gamma_5)$$

2.4 Step 3: Express amplitudes as $A_0, B_0, C_{0,1}, D_0$ & calculate them

amplitudes

Consisting of scalar products of momenta

→ FeynCalc TID →

$A_0, B_0, C_{0,1}, D_0, E_0$

IBP reduction

[K.G.Chetyrkin,F.V.Tkachov, NPB(1981)]
[H.H.Patel, CPC(2017)]

↓ Package-X ↓

Li_2, Log, Sqrt

← Series: expanding in small m_c and taking the leading-order terms ←

hierarchical heavy quark limit

Step 4: Renormalization

- One-loop diagrams
- Tree diagrams inserted with one $\mathcal{O}(\alpha_s^1)$ counterterm vertex

- QCD coupling $\overline{\mathbf{MS}}$ renormalization constant
- QCD heavy quark field (mass) **OS** renormalization constant
- QCD heavy flavor-changing current **OS** renormalization constant Z_J^{OS}

$$Z_v^{\mathrm{OS}} = Z_a^{\mathrm{OS}} = 1$$

[W.Tao,Z.J.Xiao, JHEP(2023)]
[W.Tao,Z.J.Xiao, JHEP(2024)]

$$Z_t^{\mathrm{OS}} = Z_{t5}^{\mathrm{OS}} = 1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{1}{\epsilon} - \frac{2x\log x}{1+x} + 2\log y + \mathcal{O}(\epsilon)\right) + \mathcal{O}(\alpha_s^2)$$

$$x = \frac{m_c}{m_b}, \qquad y = \frac{\mu}{m_b}, \qquad s = \frac{1}{1 - \frac{q^2}{m_b^2}}$$

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Analytical results for $B_c^* \to J/\psi$ form factors

- LO

$$V_1 = \frac{16\sqrt{2}\pi\alpha_s C_F s^2(1+x)^{\frac{5}{2}}\Psi_{B_c^*}(0)\Psi_{J/\psi}(0)}{m_b^3 x^{\frac{3}{2}}(1+s(x-2)x)^2},$$

$$V_2 = A_2 = T_2 = \frac{1-x}{1+x}V_1 = \frac{2(1+x)}{1+3x}T_6,$$

$$T_3 = \frac{1+x}{2x}T_4 = \frac{-1+s(4+10x+3x^2)}{2s(1+x)(1+3x)}V_1,$$

$$V_1 = \frac{V_3}{2} = A_1 = \frac{1+x}{4x}V_4 = \frac{s(1+x)(1+3x)}{1+4sx+3sx^2}T_2',$$

$$T_4' = \frac{1+x}{2}T_3' = \frac{1+3x}{2(1+x)}V_1,$$

$$V_5 = V_6 = A_3 = A_4 = 0,$$

$\Psi_{B_c^*(J/\psi)}(0)$: $B_c^*(J/\psi)$ wavefunction at origin

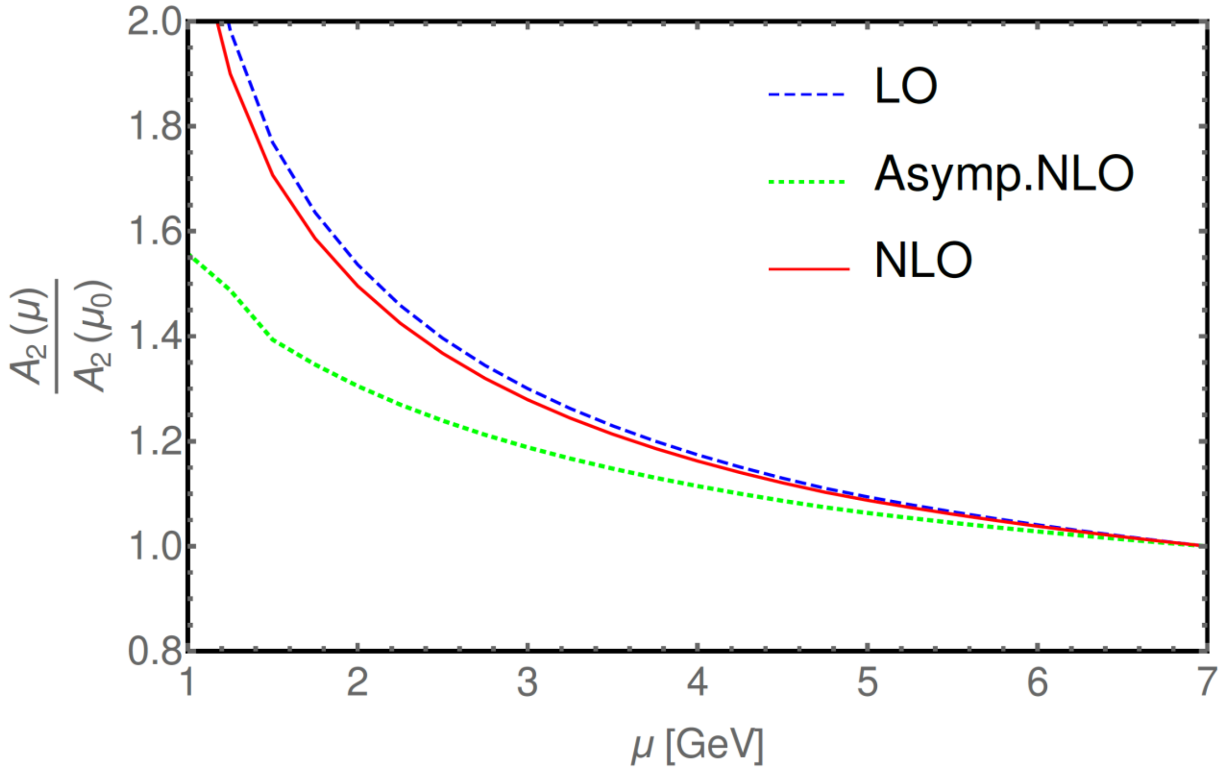
$n_f = n_b + n_c + n_l$

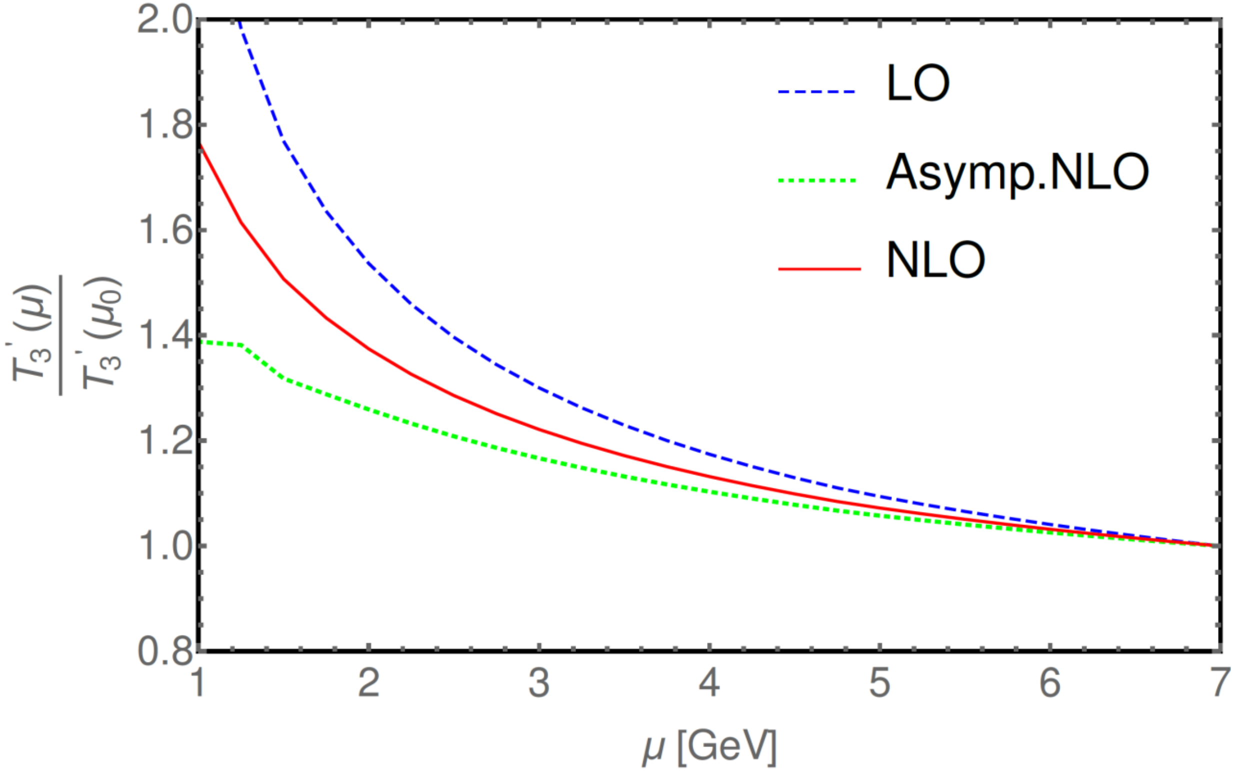
- Asymptotic NLO

$$\frac{V_1^{\rm NLO}}{V_1^{\rm LO}} = 1 + \frac{\alpha_s}{4\pi}\Bigg\{\left(\frac{11C_A}{3} - \frac{2}{3}n_f\right)\ln\frac{2sy^2}{x} - \frac{10}{9}n_f + \left(\frac{2\ln s}{3} - \frac{2\ln x}{3} + \frac{10}{9} + \frac{2\ln 2}{3}\right)n_b$$
$$- C_A\left[\frac{\ln^2 x}{2} + \left(\ln s + 2\ln 2 + \frac{3}{2}\right)\ln x + \frac{1}{2}\ln^2 s + \left(\frac{3}{2} + 2\ln 2\right)\ln s + 2\ln^2 2 + \frac{3\ln 2}{2}\right.$$
$$\left. - \frac{1}{9}(67 - 3\pi^2)\right] + C_F\Big[2{\rm Li}_2(1-s) + \ln^2 x + (2\ln s + 10\ln 2 - 5)\ln x + 2\ln^2 s$$
$$+ (10\ln 2 - 2)\ln s + 7\ln^2 2 + 9\ln 2 + \frac{1}{3}(\pi^2 - 51)\Big]\Bigg\},$$

3.2 Renormalization scale dependence of form factors

- At maximum recoil point ($q^2 = 0$)
- $\mu_0 = 7$ GeV
- $m_b = 4.75$ GeV
- $m_c = 1.5$ GeV

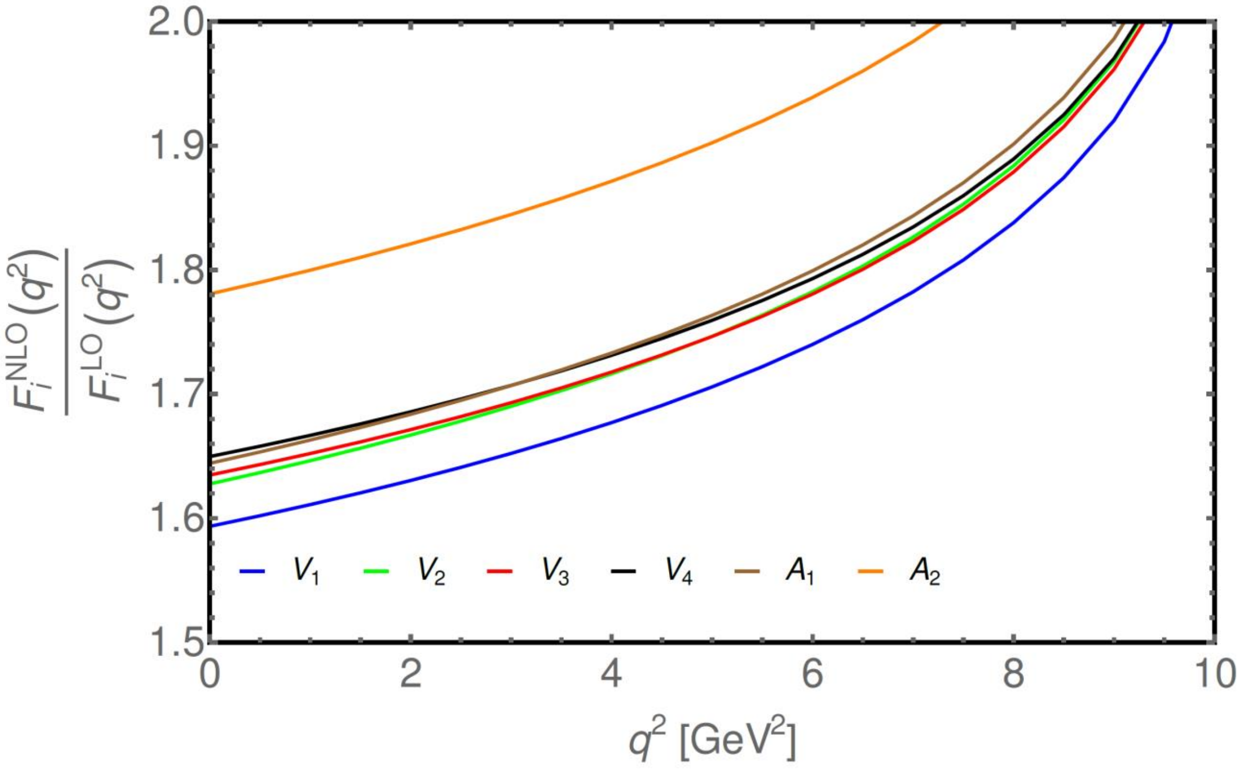


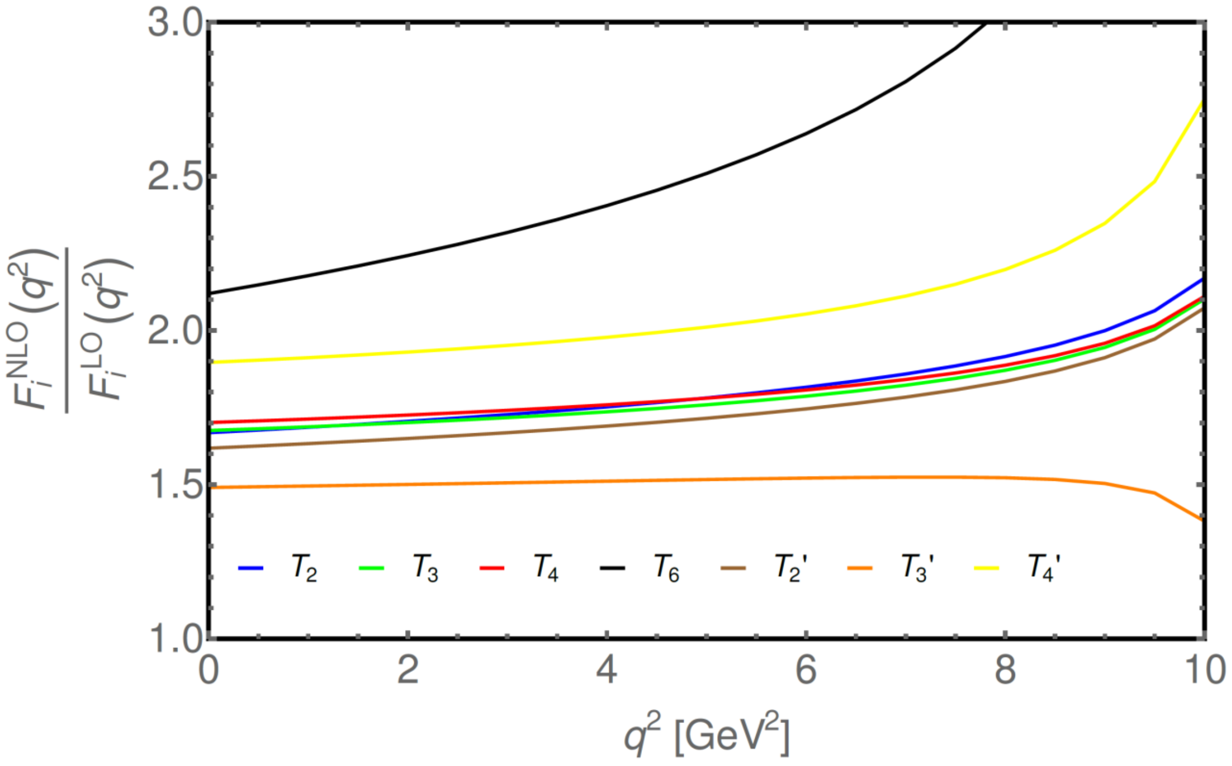


NLO corrections reduce the renormalization scale dependence

q^2 dependence of NLO to LO form factor ratio

$\mu = 3$ GeV





- NLO corrections are both significant and convergent in low q^2 region
- The convergence breaks down in high q^2 region

NRQCD+Lattice predictions for $B_c^* \to J/\psi$ form factors at $q^2 = 0$

$$F_{i,\mathrm{NRQCD+Lattice}}^{B_c^* \to J/\psi}(q^2) = \frac{1}{4}\sum_{j=1}^{4} \frac{F_{i,\mathrm{NRQCD}}^{B_c^* \to J/\psi}(q^2)}{F_{j,\mathrm{NRQCD}}^{B_c \to J/\psi}(q^2)} F_{j,\mathrm{Lattice}}^{B_c \to J/\psi}(q^2)$$

$$\Psi_{B_c^*}(0) \approx \Psi_{B_c}(0)$$

$$F_j^{B_c \to J/\psi} \in \{V, A_{0,1,2}\}^{B_c \to J/\psi}$$

The second uncertainties from lattice data dominate over the first uncertainties from $\mu = 3^{+4}_{-1.5}$ GeV

[HPQCD, PRD(2020)]
[Q.Chang,L.T.Wang,X.N.Li, JHEP(2019)]
[Q.Chang,et al., AHEP(2020)]

	NRQCD+Lattice	LFQM [7, 8]
V_1	$0.4320^{+0.0030}_{-0.0048} \pm 0.0448$	$0.56^{+0.01+0.17}_{-0.01-0.17}$
V_2	$0.2295^{+0.0003}_{-0.0004} \pm 0.0238$	$0.33^{+0.01+0.05}_{-0.01-0.04}$
V_3	$0.8865^{+0.0001}_{+0.0003} \pm 0.0919$	$1.17^{+0.02+0.23}_{-0.02-0.29}$
V_4	$0.4294^{-0.0009}_{+0.0018} \pm 0.0445$	$0.65^{+0.01+0.20}_{-0.01-0.19}$
V_5	$0.1303^{-0.0338}_{+0.0569} \pm 0.0135$	$0.20^{+0.00+0.02}_{-0.00-0.02}$
V_6	$0.1303^{-0.0338}_{+0.0569} \pm 0.0135$	$0.20^{+0.00+0.02}_{-0.00-0.02}$
A_1	$0.4458^{-0.0006}_{+0.0013} \pm 0.0462$	$0.54^{+0.01+0.16}_{-0.01-0.17}$
A_2	$0.2510^{-0.0053}_{+0.0090} \pm 0.0260$	$0.35^{+0.00}_{-0.00}$
A_3	$0.0942^{-0.0244}_{+0.0411} \pm 0.0098$	$0.13^{+0.00+0.03}_{-0.00-0.02}$
A_4	$0.1092^{-0.0284}_{+0.0477} \pm 0.0113$	$0.14^{+0.00+0.02}_{-0.00-0.02}$
T_2	$0.2352^{-0.0012}_{+0.0021} \pm 0.0244$	–
T_3	$0.5724^{-0.0035}_{+0.0062} \pm 0.0593$	–
T_4	$0.2790^{-0.0028}_{+0.0049} \pm 0.0289$	–
T_6	$0.2211^{-0.0131}_{+0.0221} \pm 0.0229$	–
T_2'	$0.4387^{+0.0012}_{-0.0018} \pm 0.0455$	–
T_3'	$0.4546^{+0.0115}_{-0.0190} \pm 0.0471$	–
T_4'	$0.3805^{-0.0136}_{+0.0230} \pm 0.0394$	–

NRQCD+Lattice+Z-series predictions in full q^2 range

m_R: masses of low-lying $c\bar{b}$ resonances

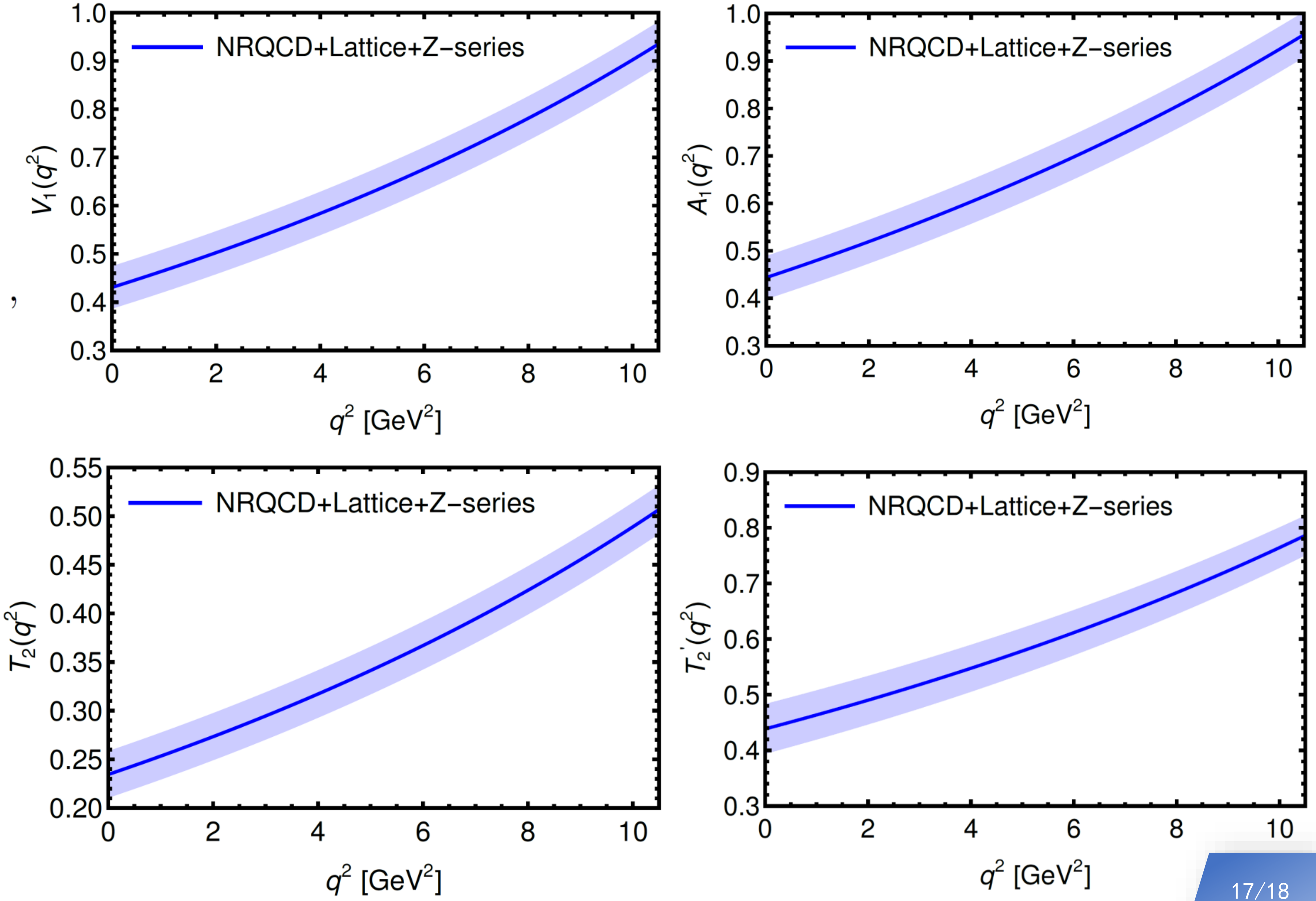
$$F_i(q^2) = \frac{1}{1-\frac{q^2}{m_R^2}} \sum_{n=0}^{N} \alpha_{i,n}\, z^n\left(q^2\right),$$

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}},$$

$$t_0 = t_+\left(1 - \sqrt{1 - \frac{t_-}{t_+}}\right),$$

$$t_\pm = \left(m_{B_c^*} \pm m_{J/\psi}\right)^2,$$

[C.G.Boyd,B.Grinstein,R.F.Lebed, PRD(1997)]
[D.Leljak,B.Melic,M.Patra, JHEP(2019)]



Summary

- Obtain complete and asymptotic analytical results for NLO QCD corrections to $B_c^* \to J/\psi$ (axial-)vector and (axial-)tensor form factors

- NLO corrections reduce renormalization scale dependence, and are both significant and convergent in low q^2 region

- Provide NRQCD+Lattice+Z-series predictions for $B_c^* \to J/\psi$ form factors across full physical q^2 range

Thank you!