# Precision Calculations of $B \to K^*$ Form Factors in Light-cone Sum Rules

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# Outline

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- Introduction to $B \to K^{*}\nu\bar{\nu}$ decay
- Factorization formula of $B \to K^{*}$ form factors in LCSR
- Sub-leading power corrections to $B \to K^{*}$ form factors
- Numerical applications

# Introduction

Sensitive probes of new physics: $b \to s + \ell\bar{\ell}, \nu\bar{\nu}$. Semileptonic $B$ anomalies

- $P_5'$ in $B \to K^* \mu^+\mu^-$,

$$P'^{[4.0,6.0]}_{5\ \mathrm{SM}} = -0.72 \pm 0.08 \quad P'^{[4.0,6.0]}_{5\ \mathrm{LHCb}} = -0.439 \pm 0.111 \pm 0.036 \ (1.9\sigma),$$
$$P'^{[6.0,8.0]}_{5\ \mathrm{SM}} = -0.81 \pm 0.08 \quad P'^{[6.0,8.0]}_{5\ \mathrm{LHCb}} = -0.583 \pm 0.090 \pm 0.030 \ (1.9\sigma).$$

[arXiv:1207.2753]

- $\mathcal{BR}$ in low $q^2$ region,

$$\mathcal{B}^{[4.0,5.0],\mathrm{SM}}_{B^+\to K^+\mu^+\mu^-} = (0.37 \pm 0.03) \times 10^{-7}, \qquad \mathcal{B}^{[4.0,5.0],\mathrm{LHCb}}_{B^+\to K^+\mu^+\mu^-} = (0.22 \pm 0.02) \times 10^{-7} \ (4.4\sigma),$$
$$\mathcal{B}^{[5.0,6.0],\mathrm{SM}}_{B^+\to K^+\mu^+\mu^-} = (0.37 \pm 0.03) \times 10^{-7}, \qquad \mathcal{B}^{[5.0,6.0],\mathrm{LHCb}}_{B^+\to K^+\mu^+\mu^-} = (0.23 \pm 0.02) \times 10^{-7} \ (4.0\sigma).$$

[arXiv:2207.12468]

- $\mathcal{BR}(B^+ \to K^+\nu\bar{\nu})$, one of the most cleanest channels in the SM

$$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})_{\text{Belle II}} = \left[23 \pm 5(\text{stat})^{+5}_{-4}(\text{syst})\right] \times 10^{-6}$$

362 $\mathrm{fb}^{-1}$

[arXiv:2311.14647]

2.7$\sigma$

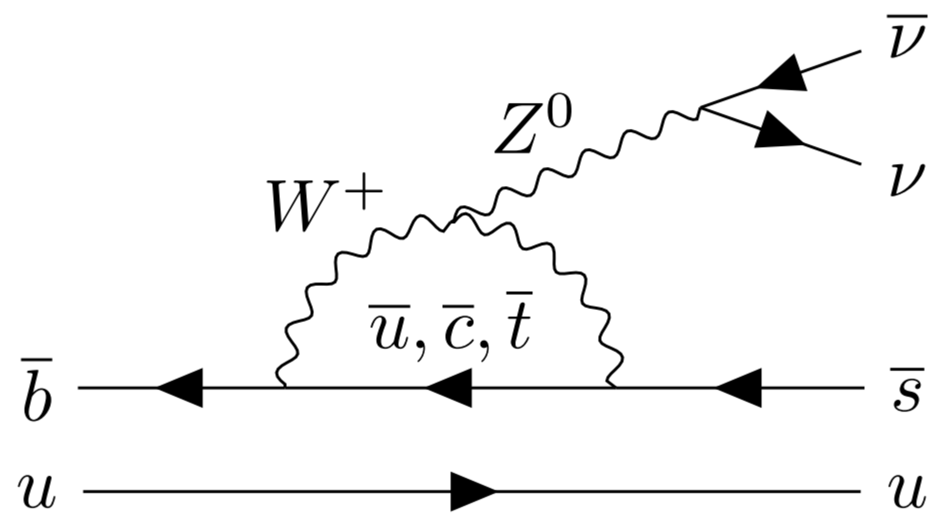
$$\mathcal{B}(B^+ \to K^+\nu\bar{\nu}) = (5.58 \pm 0.37) \times 10^{-6}$$

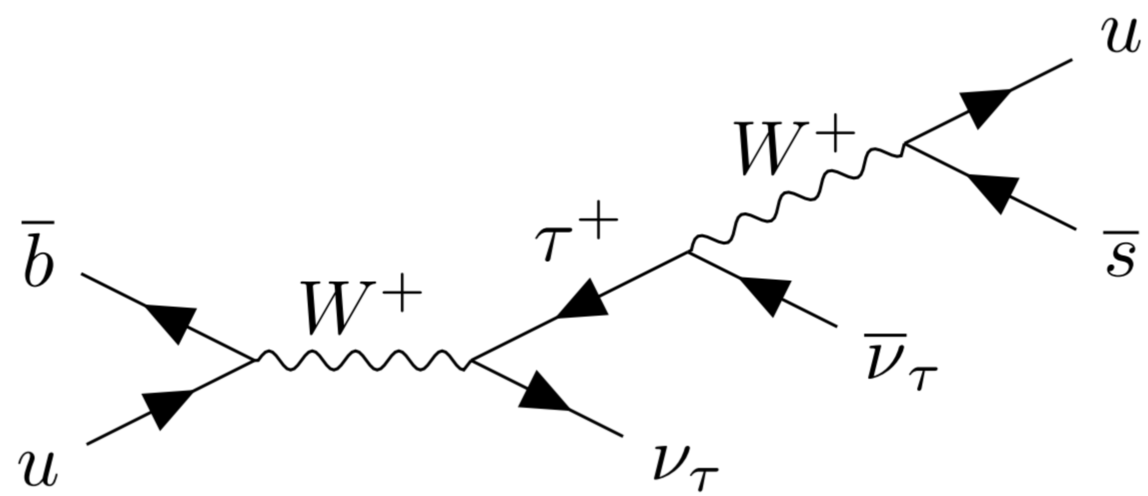
10 $a\mathrm{b}^{-1} \sim 5\sigma$

- $\mathcal{BR}(B \to K^* \nu\bar{\nu})$, one of the most cleanest channels in the SM

Short distance QCD and EW effects are under control.

Do not suffer from hadronic effects beyond the form factors.




LQCD calculations of $B \to K^*$ form factors have been performed.

[R.R. Horgan, Zhaofeng Liu el al. PRD 2014]

LCSR for $B \to K^*$ form factors can be derived at NLL.

[Jing Gao, Yue-Long Shen el al. PRD 2020]

# Factorization formula in LCSR

7 form factors in QCD can be reduced to 4 effective form factors in SCET.

QCD: $V, A_0, A_1, A_{12}, T_1, T_2, T_{23}$

$$\langle V(p,\epsilon^*)|\bar{q}\gamma_\mu b|\bar{B}(p+q)\rangle = -\frac{2iV(q^2)}{m_B+m_V}\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho q^\sigma$$

SCET: $\xi_\perp, \xi_\parallel, \Xi_\perp, \Xi_\parallel$

$$\langle V(p,\epsilon^*)|(\bar{\xi}W_c)\gamma_5\gamma_{\mu\perp}h_v|\bar{B}_v\rangle = -n\cdot p(\epsilon^*_\mu - \epsilon^*\cdot v\bar{n}_\mu)\xi_\perp(n\cdot p)$$

Matching

$$(\bar{\psi}\Gamma_i Q)(0) = \int d\hat{s}\sum_j \tilde{C}^{(A0)}_{ij}(\hat{s})\,O^{(A0)}_j(s;0) + \int d\hat{s}\sum_j \tilde{C}^{(A1)}_{ij\mu}(\hat{s})\,O^{(A1)\mu}_j(s;0)$$
$$+\int d\hat{s}_1\int d\hat{s}_2\sum_j \tilde{C}^{(B1)}_{ij\mu}(\hat{s}_1,\hat{s}_2)\,O^{(B1)\mu}_j(s_1,s_2;0)+\ldots,$$

[arXiv:hep/ph-0508250]

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SCET: $\xi_\perp, \xi_\parallel, \Xi_\perp, \Xi_\parallel$

$$\langle V(p,\epsilon^*)|(\bar{\xi}W_c)\gamma_5\gamma_{\mu\perp}h_v|\bar{B}_v\rangle = -n\cdot p(\epsilon^*_\mu - \epsilon^*\cdot v\bar{n}_\mu)\xi_\perp(n\cdot p)$$

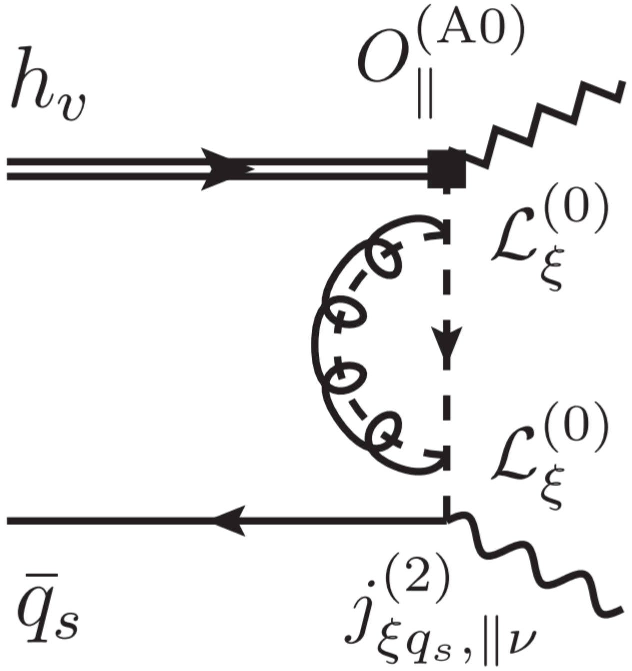
Matching

$$\frac{m_B}{m_B+m_V}V(n\cdot p) = C_V^{(\mathrm{A0})}\left(\frac{n\cdot p}{m_b},\mu\right)\xi_\perp(n\cdot p) + \int_0^1 d\tau\, C_V^{(\mathrm{B1})}\left(\frac{n\cdot p\bar{\tau}}{m_b},\frac{n\cdot p\tau}{m_b},\mu\right)\Xi_\perp(\tau,n\cdot p)$$

[arXiv:1907.11092]

$B$-meson LCSR for $\xi_{\parallel}$.

$$\Pi_{\nu,\parallel}(p,q) = \int d^4x\, e^{ip\cdot x} \langle 0|\mathrm{T}\{j_\nu(x), (\bar{\xi} W_c)(0)\gamma_5 h_v(0)\}|\bar{B}_v\rangle, \qquad j_\nu(x) = \bar{q}'(x)\gamma_\nu q(x).$$



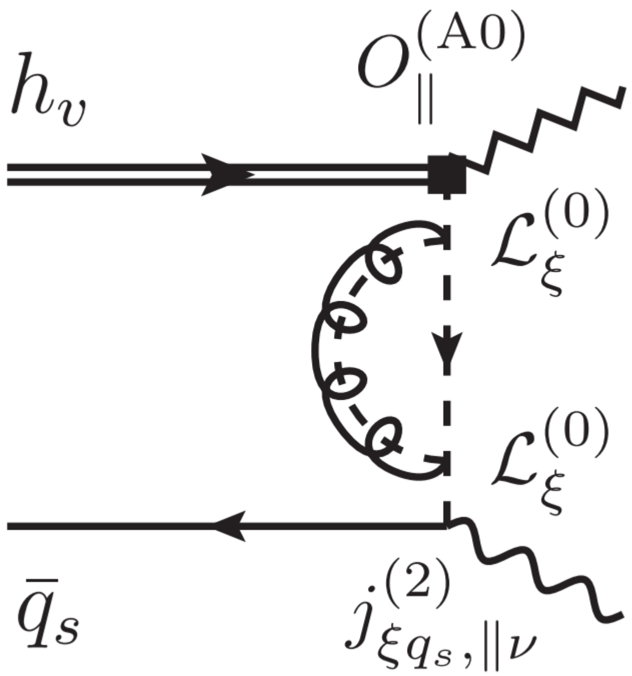

$$\Pi^A_{\nu,\parallel}(p,q) = \frac{\tilde{f}_B(\mu) m_B}{2} \int_0^{+\infty} d\omega\, J^{A,(0)}_{\parallel,-}\left(\frac{\mu^2}{n\cdot p\omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B^-(\omega,\mu)\bar{n}_\nu$$

$$J^{A,(0)}_{\parallel,-} = \frac{1}{\bar{n}\cdot p - \omega' + i0}$$

$$J^{A,(1)}_{\parallel,-} = J^{A,(0)}_{\parallel,-}\left\{1 + \frac{\alpha_s C_F}{4\pi}\left[\frac{4}{\epsilon^2} + \frac{1}{\epsilon}\left(4\ln\frac{\mu^2}{n\cdot p(\omega - \bar{n}\cdot p)} + 3\right) + 2\ln^2\frac{\mu^2}{n\cdot p(\omega - \bar{n}\cdot p)} + 3\ln\frac{\mu^2}{n\cdot p(\omega - \bar{n}\cdot p)} - \frac{\pi^2}{3} + 7\right]\right\},$$

$B$-meson LCSR for $\xi_{\parallel}$.

$$\Pi_{\nu,\parallel}(p,q) = \int d^4x\, e^{ip\cdot x}\langle 0|\mathrm{T}\{j_\nu(x), (\bar{\xi}W_c)(0)\gamma_5 h_v(0)\}|\bar{B}_v\rangle, \qquad j_\nu(x) = \bar{q}'(x)\gamma_\nu q(x).$$




$$\Pi_{\nu,\parallel}(p,q) = \left[-\frac{f_{V,\parallel} m_V}{m_V^2/n\cdot p - \bar{n}\cdot p - i0}\left(\frac{n\cdot p}{2m_V}\right)^2 \xi_{\parallel}(n\cdot p) + \int_{\omega_s}^{+\infty}\frac{d\omega'}{\omega' - \bar{n}\cdot p - i0}\rho_{\parallel}^h(\omega', n\cdot p)\right]\bar{n}_\nu,$$

$$\xi_{\parallel,\mathrm{NLO}}(n\cdot p) = 2\frac{\tilde{f}_B(\mu)}{f_{V,\parallel}}\frac{m_B m_V}{(n\cdot p)^2}\int_0^{\omega_s} d\omega' \exp\left[-\frac{n\cdot p\omega' - m_V^2}{n\cdot p\,\omega_M}\right][\phi^-_{B,\mathrm{eff}}(\omega',\mu) + \phi^+_{B,m}(\omega',\mu)].$$

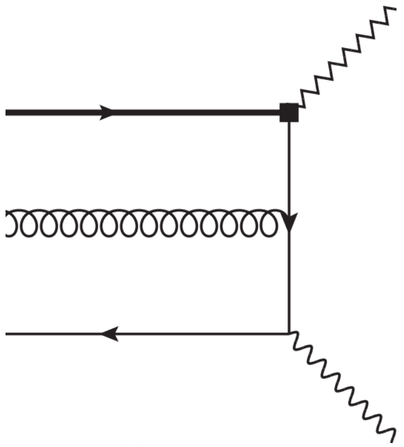
# Sub-leading power corrections

Power corrections are important in $B$ decays, $\lambda = \Lambda/m_b$

$\lambda \sim \mathcal{O}(\alpha_s) \sim 20-30\% \Longrightarrow$ NLP at leading order + NLO at leading power

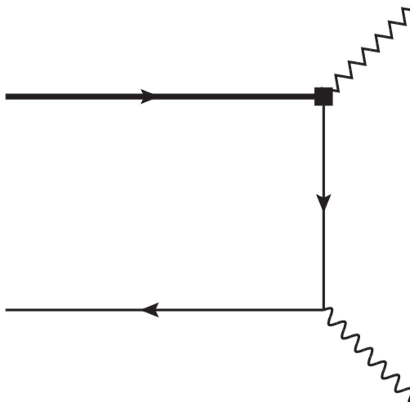
- Higher Fock state

$$\langle 0|\bar{q}_\alpha(\tau_1\,\bar{n})\,g_s\,G_{\mu\nu}(\tau_2\,\bar{n})\,h_{v\,\beta}(0)|\bar{B}_v\rangle$$



- Quark propagator expansion

$$\frac{(\not{p}-\not{k})+m_q}{(p-k)^2-m_q^2+i0} = \underbrace{\frac{1}{\bar{n}\cdot(p-k)}\,\frac{\not{n}}{2}}_{\text{LP}} + \underbrace{\frac{1}{(p-k)^2}\left[\bar{n}\cdot p\,\frac{\not{\bar{n}}}{2} - \not{k} + \frac{n\cdot k\;\bar{n}\cdot p}{\bar{n}\cdot(p-k)}\,\frac{\not{n}}{2}\right]}_{\text{NLP}}$$



- Heavy quark expansion in HQET

$$b(x) = \left[1 + \frac{i\,\overrightarrow{\not{D}}_\top}{2\,m_b} + \;\ldots\;\right] h_v(x)$$

# Sub-leading power corrections

Power corrections are important in $B$ decays, $\lambda = \Lambda/m_b$

$\lambda \sim \mathcal{O}(\alpha_s) \sim 20-30\%$ $\Longrightarrow$ NLP at leading order + NLO at leading power

- Quark propagator expansion

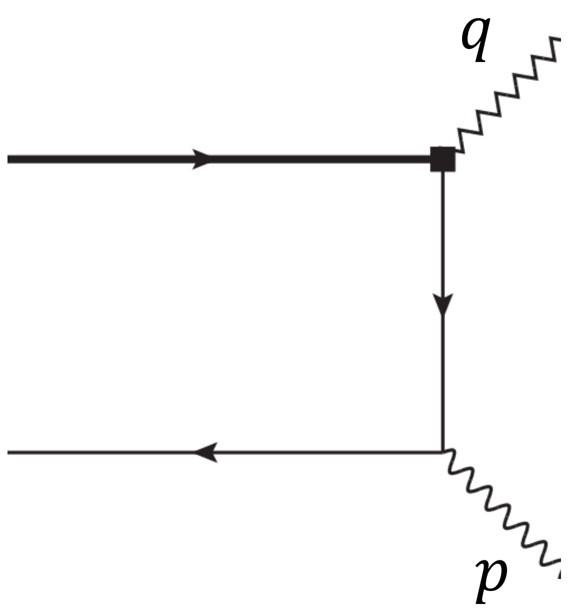
$$f_V \exp\left[-\frac{m_V^2}{n\cdot p\,\omega_M}\right]\mathcal{F}_{i,\mathrm{NLP}}^{\mathrm{QPE}}(q^2) = \frac{\tilde{f}_B(\mu)m_B}{(n\cdot p)^2}\left\{\kappa_i\left(f_{2,1}[\boldsymbol{\eta}_1] - f_{3,2}[\boldsymbol{\eta}_2]\right)\right.$$
$$\left. + \tilde{\kappa}_i\left(f_{2,1}[\boldsymbol{\eta}_3] - f_{3,2}[\boldsymbol{\eta}_4] - f_{2,2}[\boldsymbol{\eta}_5] - f_{3,3}[\boldsymbol{\eta}_6]\right)\right\},$$

$$\kappa_i \in \left\{1, -1, \frac{n\cdot q}{m_B}, -\frac{n\cdot q}{m_B}, -\frac{2m_V\, n\cdot q}{m_B\, n\cdot p}, \frac{2m_V\, n\cdot q}{m_B\, n\cdot p}, \frac{2m_V}{n\cdot p}\right\};$$

$$\tilde{\kappa}_i \in \left\{1, 1, \frac{\bar{n}\cdot q}{m_B}, \frac{\bar{n}\cdot q}{m_B}, \frac{2m_V}{n\cdot p}, \frac{2m_V}{n\cdot p}, \frac{2m_V}{n\cdot p}\right\},$$

# Numerical applications

$$\Pi^{(a)}_{\mu,\|}(p,q) = \int d^4x e^{ip\cdot x}\langle 0|T\{\bar{q}'(x)\frac{\not{n}}{2}q(x), \bar{q}(0)\Gamma^{(a)}_{\mu}b(0)\}|\bar{B}(p+q)\rangle$$




- Partonic level: factorization formula for $\Pi^{a}_{\mu}(p,q)$

- Hadronic level: $\sum_n |n\rangle\langle n| \quad \Rightarrow \quad f_V F^{i}_{B\to V} + \cdots$

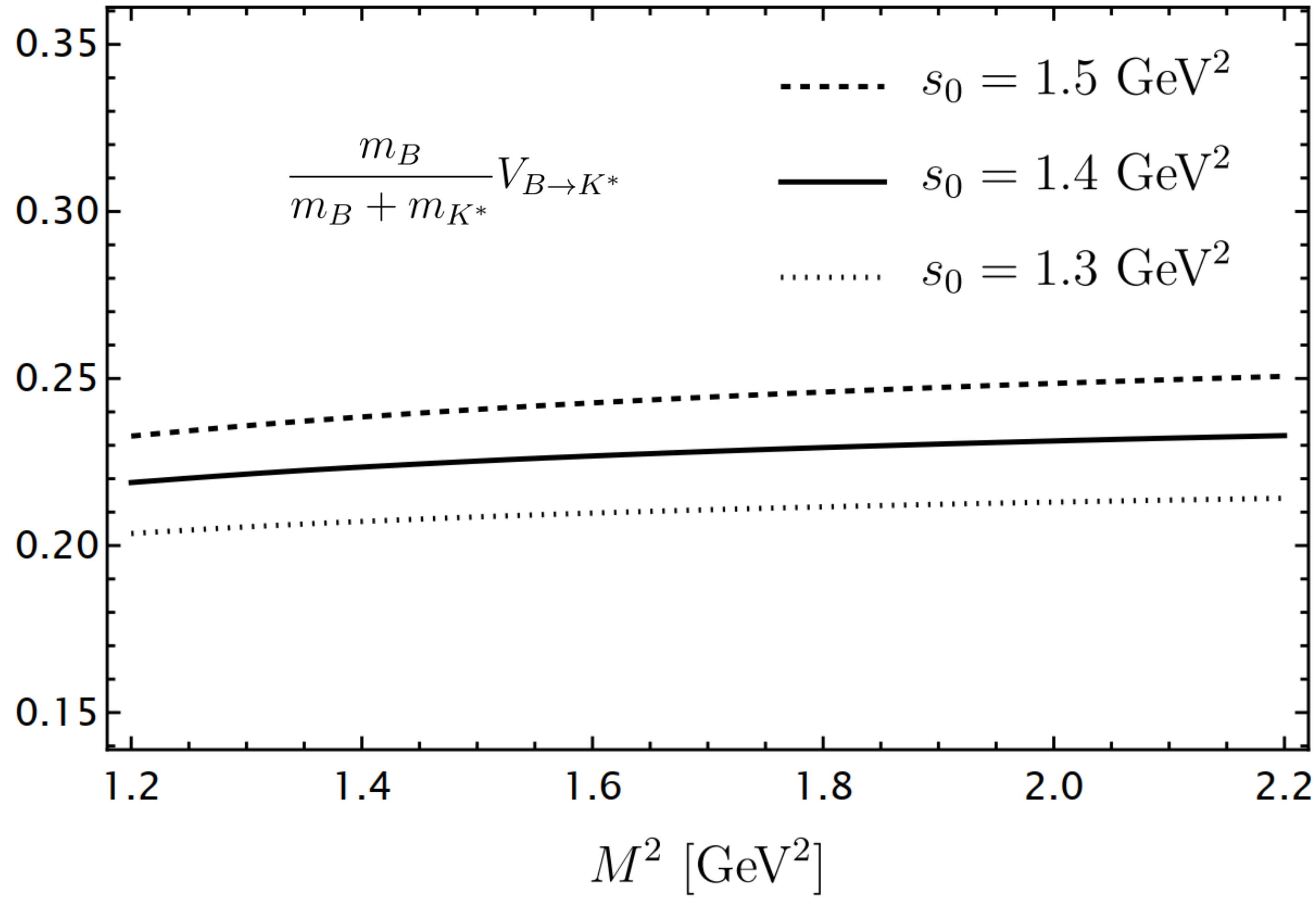
- Parton-hadron duality above $s_0$ to obtain lowest lying hadronic parameter.

- Borel transformation to improve the stability of the results.

# Numerical applications

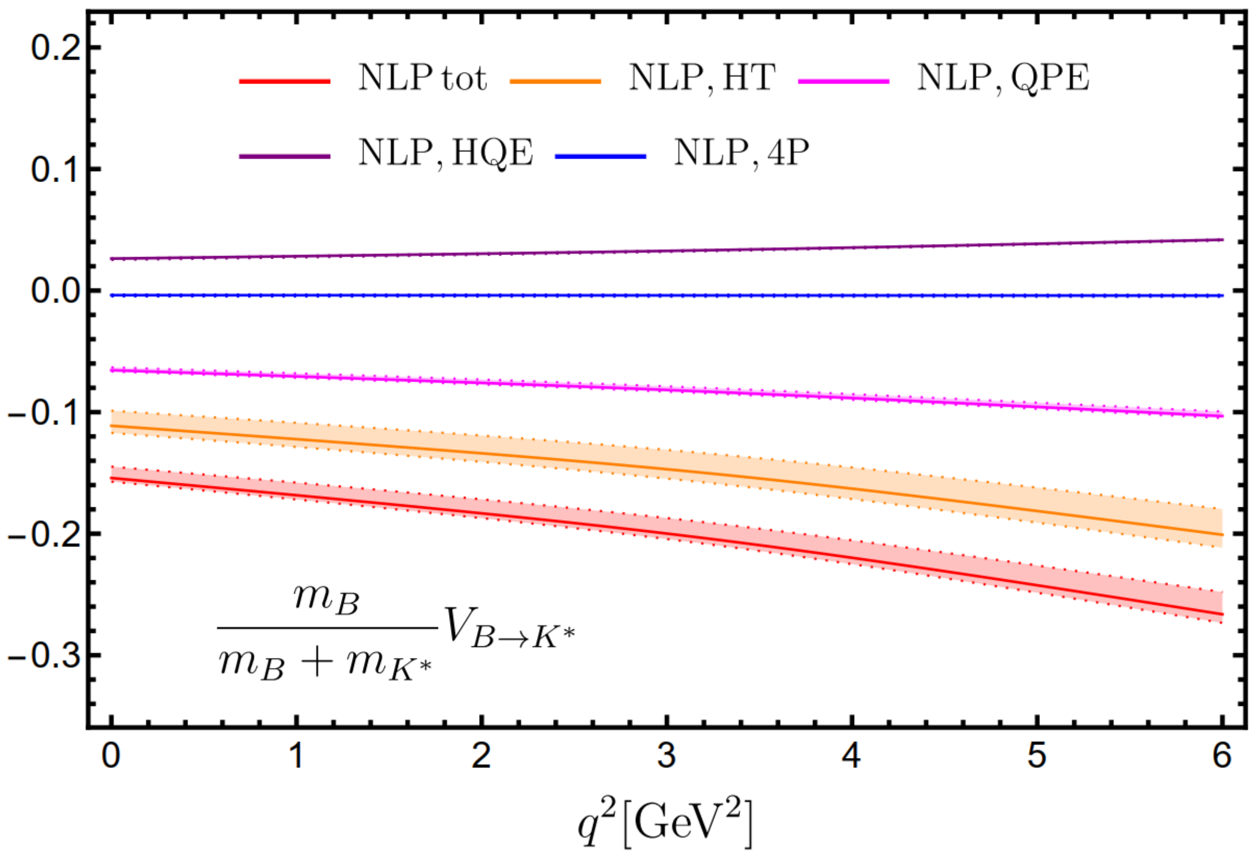
Inputs:

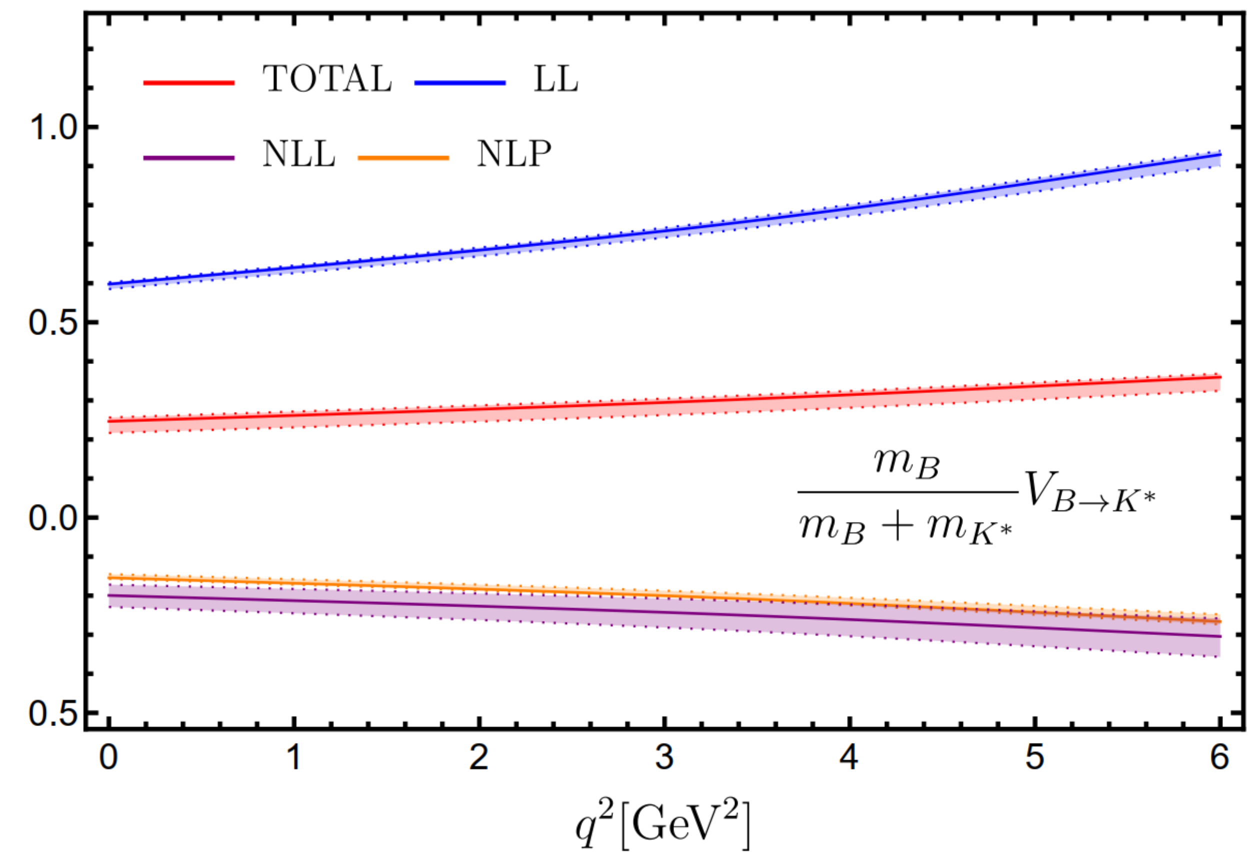
B-LCDAs
Borel mass $M^2$
Effective threshold $s_0$
scale $\mu, \nu$




- Uncertainty from Borel mass ~ 5%, from effective threshold ~ 6%

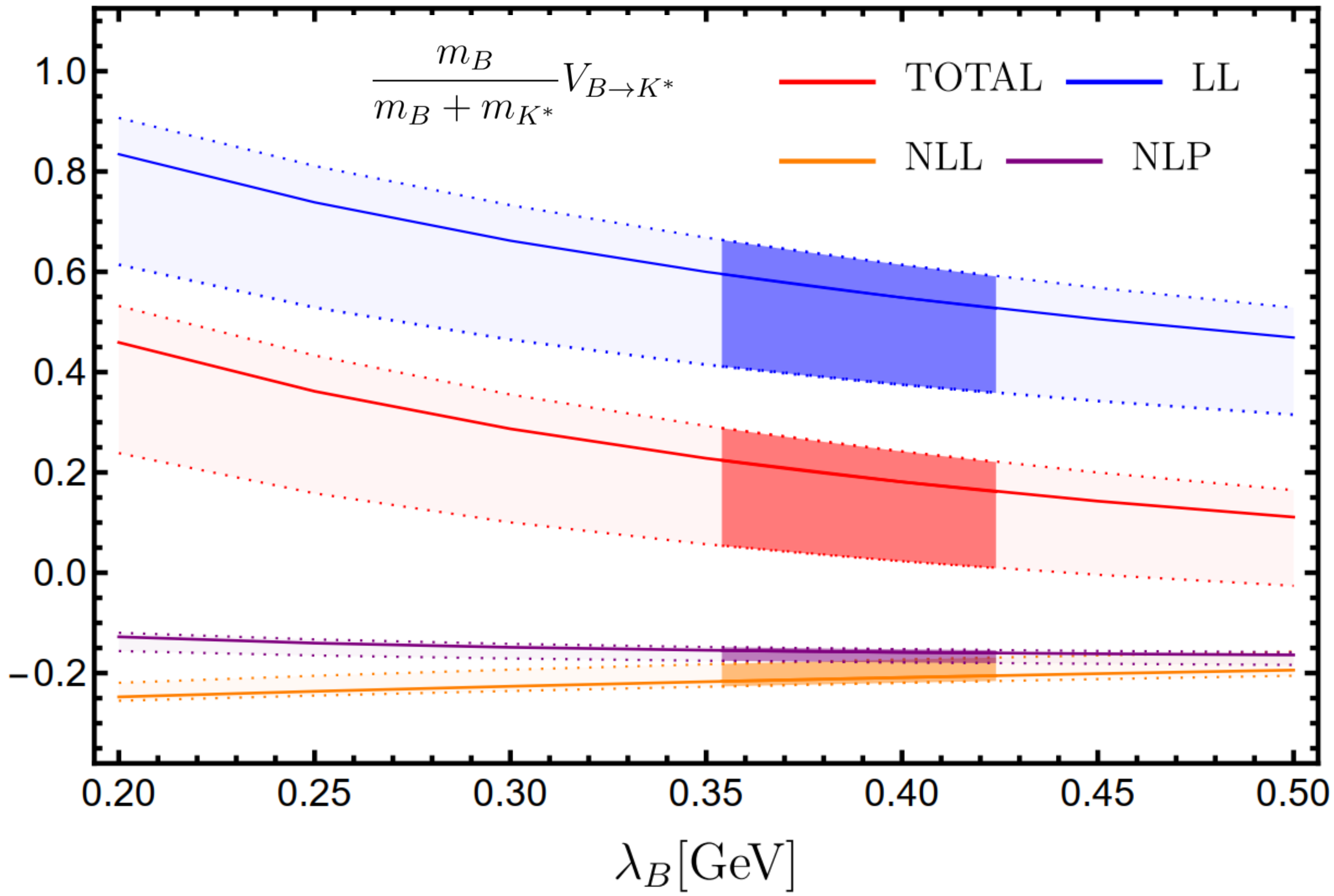
# Numerical applications




- NLP contribution : higher twist LCDA + quark propagator expansion dominate
- NLO contribution ~ 25%, NLP contribution ~ 25%

# Numerical applications




$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\,\phi_B^+(\omega,\mu)\,,$$

$$\frac{\widehat{\sigma}_n(\mu)}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\,\ln^n \frac{e^{-\gamma_E}\lambda_B(\mu)}{\omega}\,\phi_B^+(\omega,\mu)$$

| | |
|---|---|
| $\lambda_B$ | $0.35(15)$ GeV |
| | $\{0.7\,,6.0\}$ |
| $\{\widehat{\sigma}_1,\widehat{\sigma}_2\}$ | $\{0.0\,,\pi^2/6\}$ |
| | $\{-0.7\,,-6.0\}$ |

Newly obtained by LPC $\lambda_B = 0.389(39)$ GeV

[Xue-Ying Han el al. arxiv:2410.18654]

# Numerical applications

- $z$-series expansion $k = 0,1,2$

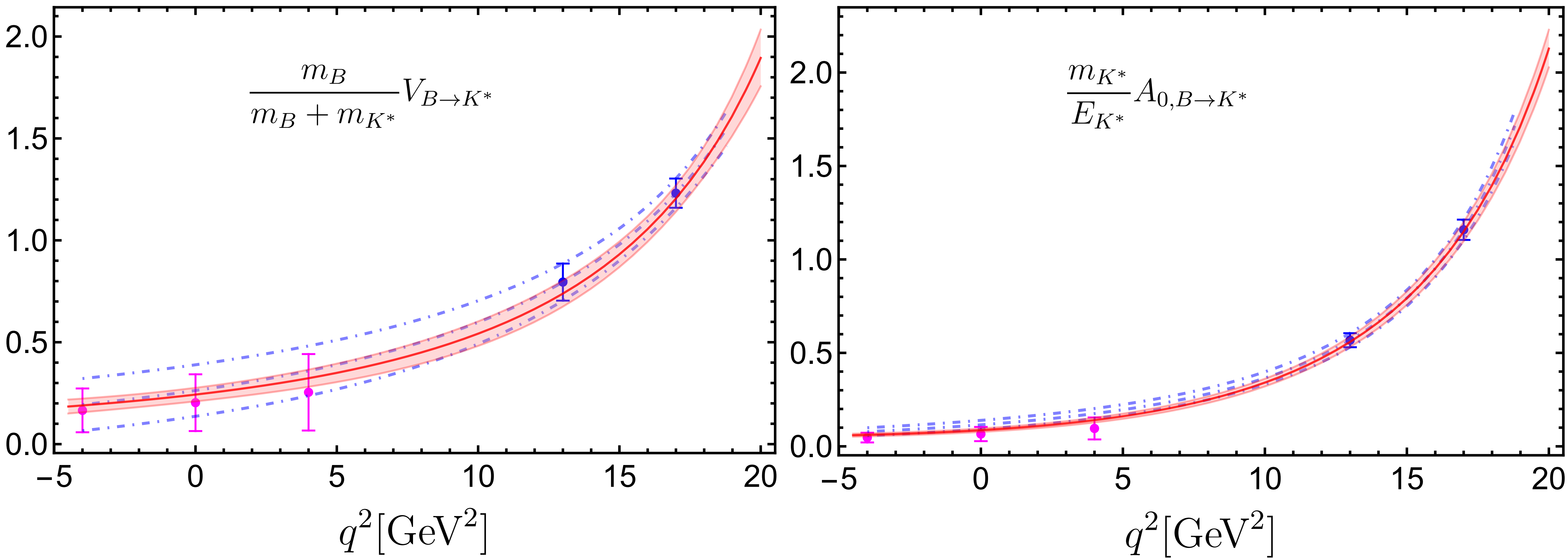
$$z(t) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}} \qquad t_\pm \equiv (m_B \pm m_V)^2$$

$|z_{\max}| < 0.1$

$$F_i(q^2) = P_i(q^2) \sum_k \alpha_k^i \left[z(q^2) - z(0)\right]^k$$

$$P_i(q^2) = (1 - q^2/m_{R,i}^2)^{-1}$$

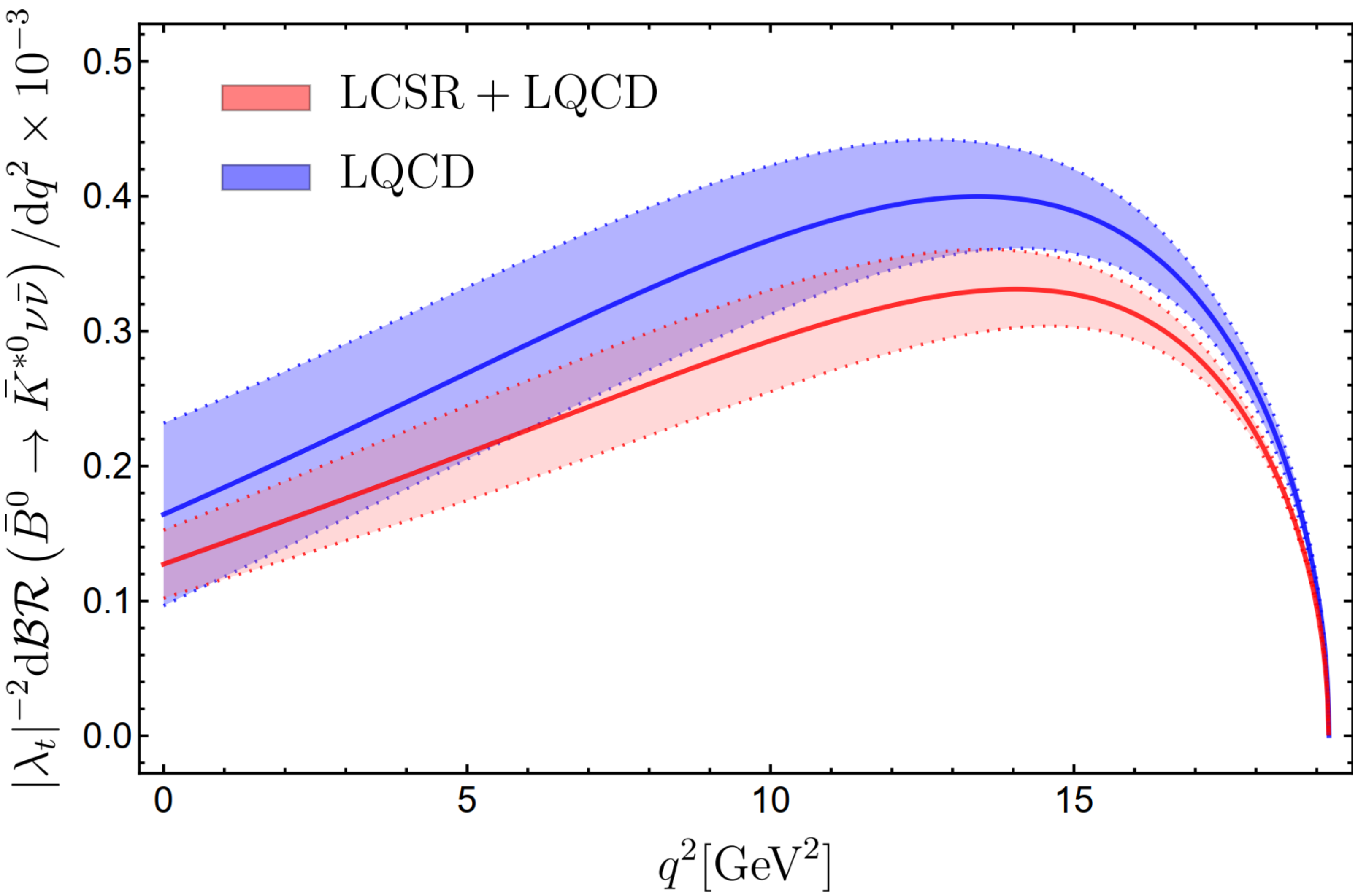
| $F_i$ | $J^P$ | $m_{R,i}^{b\to s}$ / GeV |
|---|---|---|
| $A_0$ | $0^-$ | 5.366 |
| $T_1, V$ | $1^-$ | 5.415 |
| $T_2, T_{23}, A_1, A_{12}$ | $1^+$ | 5.829 |



# Numerical applications

$$\frac{d\mathcal{B}}{dq^2}(B \to K^*\nu\bar{\nu}) = \mathcal{N}_{K^*}(q^2)|C_L^{\rm SM}|^2|\lambda_t|^2\mathcal{F}(q^2) \quad |\lambda_t| \times 10^3 = \begin{cases} 41.4 \pm 0.8\,, & (B \to X_c l\bar{\nu}) \\ 39.3 \pm 1.0\,, & (B \to D l\bar{\nu}) \\ 37.8 \pm 0.7\,, & (B \to D^* l\bar{\nu}) \end{cases}$$

$$\lambda_t = V_{tb}V_{ts}^*$$




$|\lambda_t|^{-2}\mathcal{BR}(\bar{B} \to \bar{K}^{0*}\nu\bar{\nu}) \times 10^3$

$= 4.76 \pm 0.56$

[this work]

$= 5.86 \pm 0.93$

[R.R. Horgan, el al. PRD 2014]

$= 5.85 \pm 0.58$

[R. Zwicky et al. JHEP 2016]

# Numerical applications



$$\frac{d\mathcal{B}}{dq^2}(B \to K^* \nu\bar{\nu}) = \mathcal{N}_{K^*}(q^2)|C_L^{\rm SM}|^2|\lambda_t|^2\mathcal{F}(q^2)$$

$|\lambda_t| \times 10^3 = 41.25 \pm 0.45$ UTfit

longitudinal polarization fractions $F_L$

| $[q_1^2, q_2^2]$ (in GeV$^2$) | $10^6 \times \Delta\mathcal{BR}^{B^0\to K^{*0}\nu_\ell\bar{\nu}_\ell}(q_1^2, q_2^2)$ | $10^6 \times \Delta\mathcal{BR}^{B^+\to K^{*+}\nu_\ell\bar{\nu}_\ell}(q_1^2, q_2^2)$ | $\Delta F_L(q_1^2, q_2^2)$ |
|---|---|---|---|
| $[0.0, 1.0]$ | 0.23(5) | 0.33(5) | 0.93(2) |
| $[1.0, 2.5]$ | 0.40(7) | 0.55(8) | 0.79(4) |
| $[2.5, 4.0]$ | 0.46(8) | 0.61(9) | 0.67(5) |
| $[4.0, 6.0]$ | 0.71(12) | 0.91(13) | 0.57(5) |
| $[6.0, 8.0]$ | 0.83(13) | 1.03(14) | 0.48(5) |
| $[8.0, 12.0]$ | 1.99(26) | 2.39(28) | 0.40(4) |
| $[12.0, 16.0]$ | 2.22(20) | 2.61(22) | 0.33(2) |
| $[16.0, (m_B - m_{K^*})^2]$ | 1.26(6) | 1.53(7) | 0.31(1) |
| $[0.0, (m_B - m_{K^*})^2]$ | 8.09(96) | 9.95(1.05) | 0.44(4) |

$$F_L(q^2) = \frac{H_{A_{12}}(q^2)}{H_V(q^2) + H_{A_1}(q^2) + H_{A_{12}}(q^2)}. \tag{83}$$

In addition, we introduce two $q^2$-binned observables for comparison with future high-luminosity Belle II data

$$\begin{aligned} \Delta\mathcal{BR}(q_1^2, q_2^2) &= \tau_B \int_{q_1^2}^{q_2^2} dq^2 \frac{d\Gamma(B \to K^* \nu_\ell \bar{\nu}_\ell)}{dq^2}, \\ \Delta F_L(q_1^2, q_2^2) &= \frac{\int_{q_1^2}^{q_2^2} dq^2 \lambda^{3/2}(m_B^2, m_{K^*}^2, q^2) H_{A_{12}}(q^2)}{\int_{q_1^2}^{q_2^2} dq^2 \lambda^{3/2}(m_B^2, m_{K^*}^2, q^2)[H_V(q^2) + H_{A_1}(q^2) + H_{A_{12}}(q^2)]}. \end{aligned} \tag{84}$$

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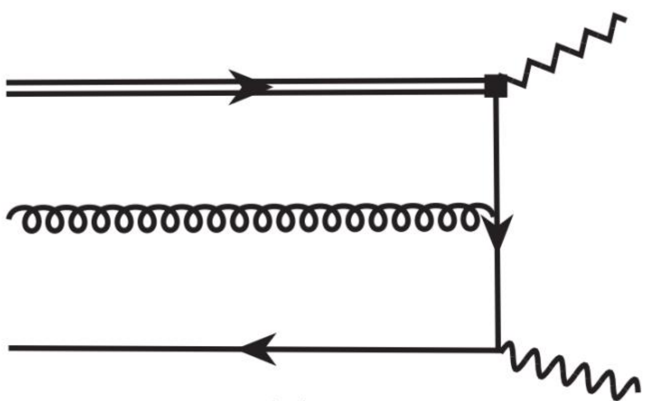
$$\langle 0|\bar{q}_\alpha(z_1\bar{n})g_s G^{\mu\nu}(z_2\bar{n})h_{v\beta}|0\rangle = \frac{\tilde{f}_B(\mu)m_B}{4}[(1+\not{v})\{(v_\mu\gamma_\nu - v_\nu\gamma_\mu)\left[\Psi_A(z_1,z_2,\mu) - \Psi_V(z_1,z_2,\mu)\right]$$
$$- i\sigma_{\mu\nu}\Psi_V(z_1,z_2,\mu) - (\bar{n}_\mu v_\nu - \bar{n}_\nu v_\mu)X_A(z_1,z_2,\mu) + (\bar{n}_\mu\gamma_\nu - \bar{n}_\nu\gamma_\mu)\left[W(z_1,z_2,\mu) + Y_A(z_1,z_2,\mu)\right]$$
$$+ i\epsilon_{\mu\nu\alpha\beta}\bar{n}^\alpha v^\beta\gamma_5\tilde{X}_A(z_1,z_2,\mu) - i\epsilon_{\mu\nu\alpha\beta}\bar{n}^\alpha\gamma^\beta\gamma_5\tilde{Y}_A(z_1,z_2,\mu)$$
$$- (\bar{n}_\mu v_\nu - \bar{n}_\nu v_\mu)\not{\bar{n}}W(z_1,z_2,\mu) + (\bar{n}_\mu\gamma_\nu - \bar{n}_\nu\gamma_\mu)\not{\bar{n}}Z(z_1,z_2,\mu)\}\gamma_5]_{\beta\alpha}, \quad (24)$$

where $\epsilon_{0123} = -1$, and we also introduce three-particle HQET distribution amplitudes of definite collinear twist as follows

$$\begin{aligned}
\Phi_3 &= \Psi_A - \Psi_V, & \Phi_4 &= \Psi_A + \Psi_V, \\
\Psi_4 &= \Psi_A + X_A, & \tilde{\Psi}_4 &= \Psi_V - \tilde{X}_A, \\
\tilde{\Phi}_5 &= \Psi_A + \Psi_V + 2Y_A - 2\tilde{Y}_A + 2W, & \Psi_5 &= -\Psi_A + X_A - 2Y_A, \\
\tilde{\Psi}_5 &= -\Psi_V - \tilde{X}_A + 2\tilde{Y}_A, & \Phi_6 &= \Psi_A - \Psi_V + 2Y_A + 2W + 2\tilde{Y}_A - 4Z.
\end{aligned} \quad (25)$$

# Backup

$$\langle 0|T\{\bar{q}(x),q(0)\}|0\rangle \supset ig_s\int\frac{d^4l}{(2\pi)^4}e^{-il\cdot x}\int_0^1 du\left[\frac{ux_\mu\gamma_\nu}{l^2-m_q^2}-\frac{(\not{l}+m_q)\sigma_{\mu\nu}}{2(l^2-m_q^2)^2}\right]G^{\mu\nu}(ux),$$



(b)

$$\langle 0|(\bar{q}_sY_s)_\beta(x)(Y_s^\dagger h_v)_\alpha|\bar{B}_v\rangle = -i\frac{\tilde{f}_B(\mu)m_B}{4}\int_0^\infty d\omega e^{-i\omega v\cdot x}\left[\frac{1+\not{v}}{2}\left\{2\left[\phi_B^+(\omega,\mu)+x^2g_B^+(\omega,\mu)\right]\right.\right.$$
$$\left.\left.-\frac{\not{x}}{v\cdot x}\left[(\phi_B^+(\omega,\mu)-\phi_B^-(\omega,\mu))+x^2(g_B^+(\omega,\mu)-g_B^-(\omega,\mu))\right]\right\}\gamma_5\right]_{\alpha\beta},$$

# Backup

$$v_\rho \frac{\partial}{\partial x_\rho} \bar{q}(x)\Gamma[x,0]h_v(0) = v\cdot\partial\bar{q}(x)\Gamma[x,0]h_v(0) + i\int_0^1 du\bar{u}\bar{q}(x)[x,ux]x^\lambda g_s G_{\lambda\rho}(ux)[ux,0]v^\rho\Gamma h_v(0),$$

$$iv\cdot\partial\langle 0|\bar{q}(x)\Gamma[x,0]h_v(0)|\bar{B}_v\rangle = \bar{\Lambda}\langle 0|\bar{q}(x)\Gamma[x,0]h_v(0)|\bar{B}_v\rangle,$$

$$\frac{\partial}{\partial x_\rho}\bar{q}(x)\gamma_\rho\Gamma[x,0]h_v(0) = -i\int_0^1 du u\bar{q}(x)[x,ux]x^\lambda g_s G_{\lambda\rho}(ux)[ux,0]\gamma^\rho\Gamma h_v(0) + im_{q'}\bar{q}(x)\Gamma[x,0]h_v(0).$$

$$\Pi^{(a)}_{\mu,\|}(p,q) = \int d^4x e^{ip\cdot x}\langle 0|T\{\bar{q}'(x)\frac{\not{n}}{2}q(x),\bar{q}(0)\Gamma^{(a)}_\mu b(0)\}|\bar{B}(p+q)\rangle,$$

$$\Pi^{(a)}_{\delta\mu,\perp}(p,q) = \int d^4x e^{ip\cdot x}\langle 0|T\{\bar{q}'(x)\frac{\not{n}}{2}\gamma_{\delta\perp}q(x),\bar{q}(0)\Gamma^{(a)}_\mu b(0)\}|\bar{B}(p+q)\rangle.$$

# Backup

$$f_V^{\perp} \exp\left[-\frac{m_V^2}{n\cdot p\omega_M}\right]\left\{\mathcal{V}_{\rm NLP}^{\rm HT}\left(q^2\right), \mathcal{A}_{1,\rm NLP}^{\rm HT}\left(q^2\right), \mathcal{T}_{1,\rm NLP}^{\rm HT}\left(q^2\right), \mathcal{T}_{2,\rm NLP}^{\rm HT}\left(q^2\right)\right\}$$

$$= \frac{\tilde{f}_B(\mu)m_B}{(n\cdot p)^2}\left\{f_{2,1}[\boldsymbol{\tau}_1] + f_{3,2}[\boldsymbol{\tau}_2] - \kappa_i \frac{m_q}{n\cdot p} f_{3,2}[\boldsymbol{\tau}_2]\right\},$$

$$f_V^{\parallel} \exp\left[-\frac{m_V^2}{n\cdot p\omega_M}\right]\left\{\mathcal{A}_{0,\rm NLP}^{\rm HT}\left(q^2\right), \mathcal{A}_{12,\rm NLP}^{\rm HT}\left(q^2\right), \mathcal{T}_{23,\rm NLP}^{\rm HT}\left(q^2\right)\right\}$$

$$= \frac{2\tilde{f}_B(\mu)m_B m_V}{(n\cdot p)^3}\left\{f_{2,1}[\boldsymbol{\tau}_1] + f_{3,2}[\boldsymbol{\tau}_3] + \frac{m_q}{n\cdot p} f_{3,2}[\boldsymbol{\tau}_4] + \iota_i\left(f_{3,2}[\boldsymbol{\tau}_5] + \frac{m_q}{n\cdot p} f_{3,2}[-\boldsymbol{\tau}_3]\right)\right\},$$

e symmetry-breaking factors

$$\kappa_i \in \left\{+1, -1, \frac{n\cdot q}{\bar{n}\cdot q}, -\frac{n\cdot q}{\bar{n}\cdot q}\right\}, \quad \iota_i \in \left\{\frac{n\cdot q}{m_B}, -\frac{n\cdot q}{m_B}, -1\right\},$$

$$f_{2,1}[\phi(\omega)] = -\int_0^{\omega_s} d\omega e^{-\frac{\omega}{\omega_M}}\phi(\omega),$$

$$f_{2,2}[\phi(\omega)] = e^{-\frac{\omega_s}{\omega_M}}\phi(\omega_s) + \int_0^{\omega_s} d\omega \frac{e^{-\frac{\omega}{\omega_M}}}{\omega_M}\phi(\omega),$$