南京航空航天大学

核科学与技术系

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Sketching the Parton Images of Light and Heavy Mesons

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Outline

- Parton images in terms of distribution functions/amplitudes.
- Sketching the parton image of $q\bar{q}$, $Q\bar{Q}$ and $q\bar{Q}$ mesons.
- Beyond sketching, demonstrated with pion twist-2 and -3 PDFs.
- Summary and Outlook.

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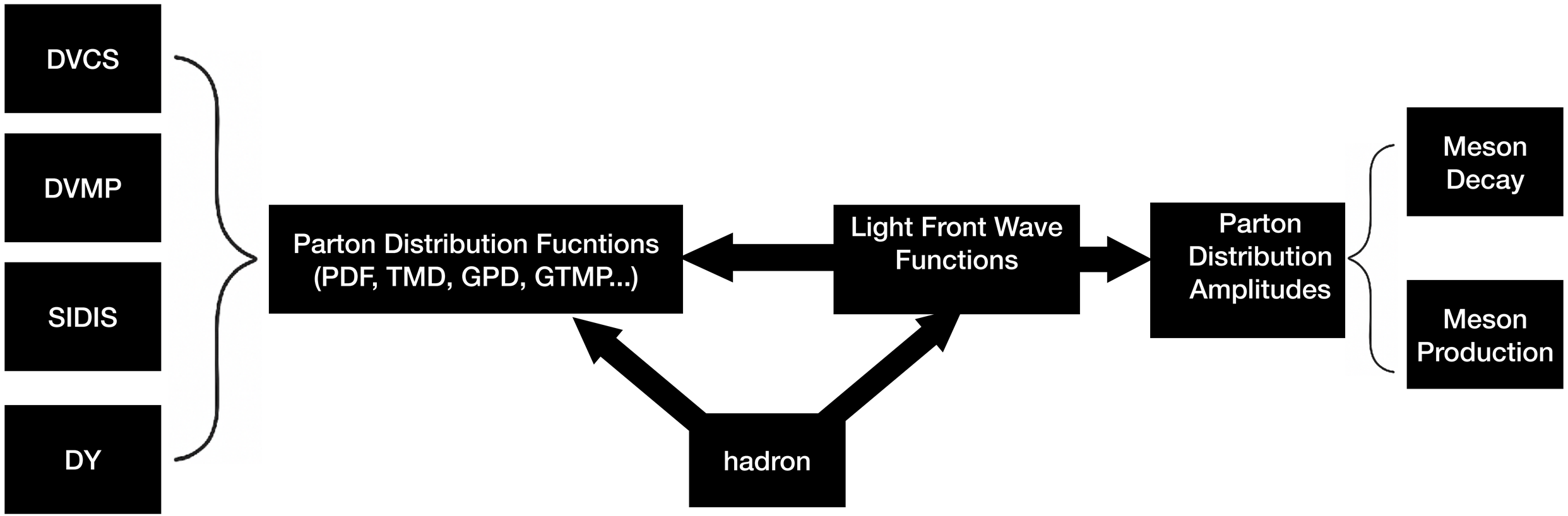
Parton image of hadrons



Light-Front Wave Functions

$$|M\rangle = \phi_2 |q\bar{q}\rangle + \phi_3 |q\bar{q}g\rangle + \phi_4 |q\bar{q}gg\rangle + \dots$$

$$|M\rangle = \sum_{\lambda_1,\lambda_2} \int \frac{d^2\boldsymbol{k}_T}{(2\pi)^3} \frac{dx}{2\sqrt{x\bar{x}}} \frac{\delta_{ij}}{\sqrt{3}} \Phi_{\lambda_1,\lambda_2}(x,\boldsymbol{k}_T)\, b^\dagger_{f,\lambda_1,i}(x,\boldsymbol{k}_T)\, d^\dagger_{h,\lambda_2,j}(\bar{x},\bar{\boldsymbol{k}}_T)|0\rangle + \phi_3 |q\bar{q}g\rangle + \dots$$

$q\bar{q}$-Light-Front Wave Function

- LFWFs are essentially transition amplitudes in the language of relativistic QFT.
- LFWFs encode complete parton information of hadrons.
- The convergence of Fock-state expansion can be slow, especially for light hadrons.
- The $q\bar{q}$-LFWFs provide dominant contribution in exclusive meson production/decay
- The $q\bar{q}$-LFWFs are useful in sketching parton images of mesons.
- LFWFs can be obtained by diagonalizing light front Hamiltonian, with complexities from higher Fock-states.

A new formula to obtain $q\bar{q}$-LFWFs

Bethe-Salpeter wave function

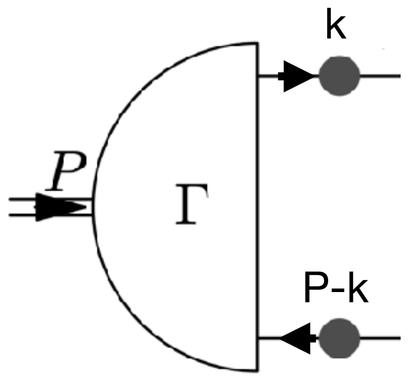
$$\phi_i(x,\vec{k}_T) \sim \int dk^- dk^+ \delta(xP^+ - k^+) \mathrm{Tr}[\Gamma_i \chi(k,P)]$$

LFWF

Setting light front time $\xi^+ = 0$

set $k^+ = xP^+$

spin configurations



(C.S., Y. Xie, M Li, X. Chen, et al, PRD(L) 2021)

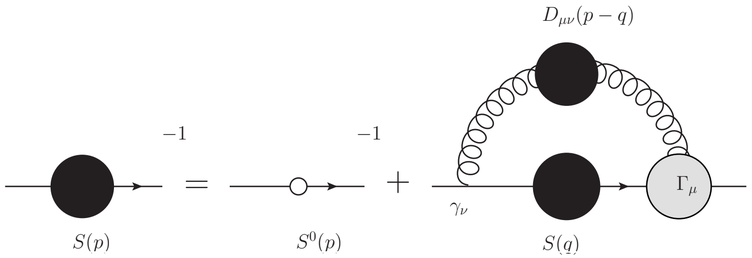
- LFWFs and BS WFs are both transition amplitudes, hence the connection can be built.
- $q\bar{q}$ LFWFs from various kinds of hadrons/particles can be extracted.
- $q\bar{q}$ LFWFs with all possible quark spin configurations can be extracted.
- $q\bar{q}$ LFWFs can be extracted from many Fock-states embedded.

$$|h\rangle = \phi_2 |q\bar{q}\rangle + \phi_3 |q\bar{q}g\rangle + \phi_4 |q\bar{q}gg\rangle \ldots$$

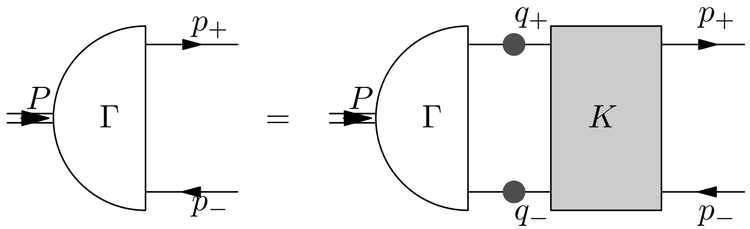
Bethe-Salpeter Wave Function From DSE

- The Bethe-Salpter wave function is solved by aligning quark gap equation and meson BS equation, incorporating nonperturbative quark-gluon dynamics.

Quark DSE:

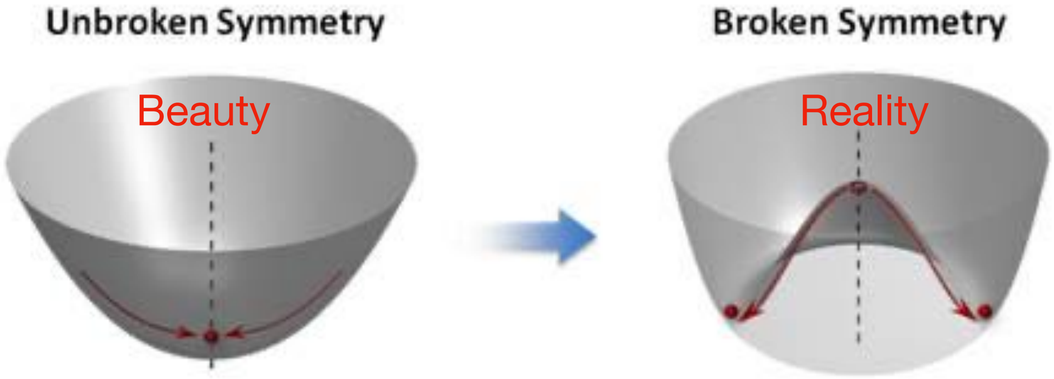


Meson BSE:



- QCD's dynamical chiral symmetry breaking.

$$SU_V(3) \otimes \cancel{SU_A(3)}$$



- A broken symmetry ≠ Absence of a symmetry.

- DCSB has consequences in BS wave functions. Truncate the DSEs wisely!

$$f_\pi E_\pi(k;0) = B(k^2)$$ (P. Maris, C.D. Roberts and P. C. Tandy, PLB1998)

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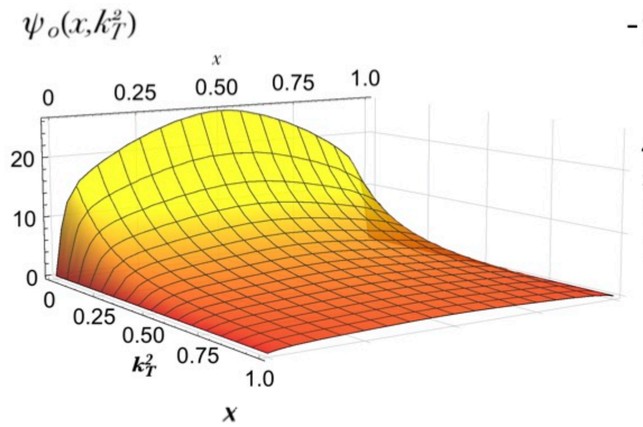
Light and Heavy 0^- Mesons

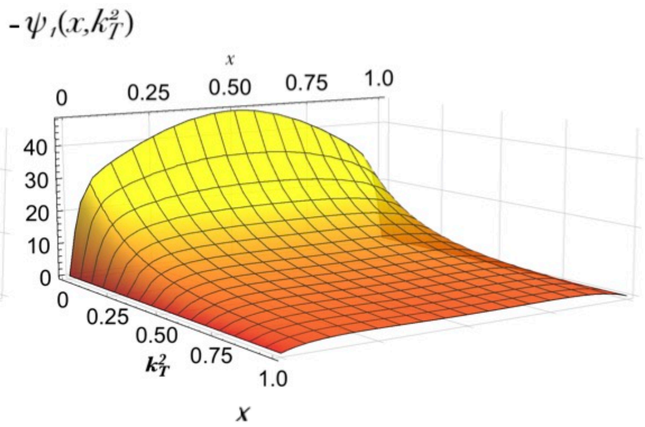
$\Lambda = \lambda + \lambda' + L_z$

s-wave $|L_z| = 0$

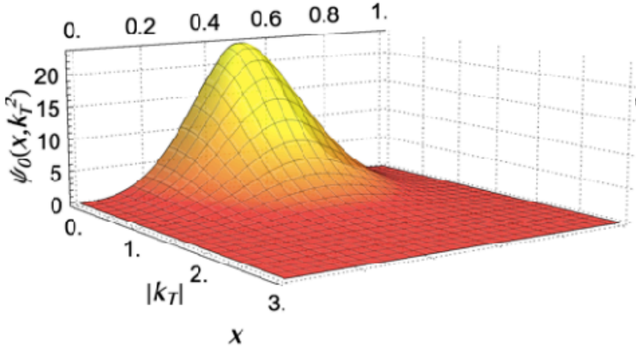
p-wave $|L_z| = 1$

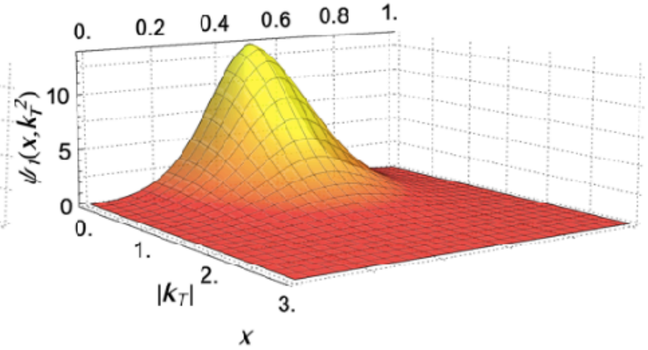
π



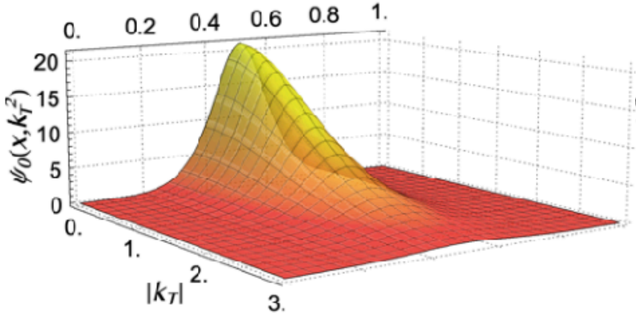


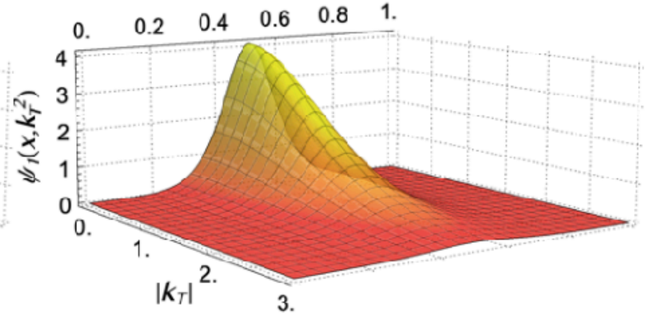
η_c



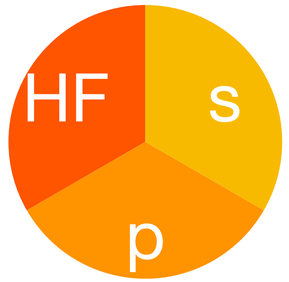


η_b

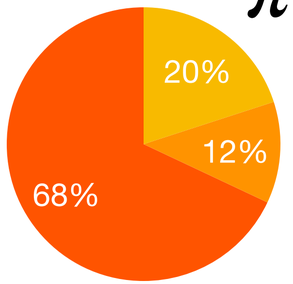




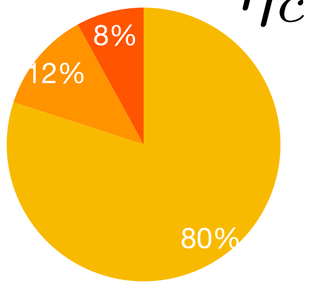
HF=Higher Fock states



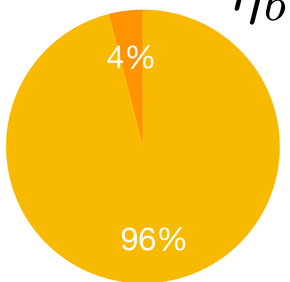
π



η_c



η_b



- $\Phi^{\Lambda}_{\lambda,\lambda'}(x, \overrightarrow{k}_T)$
- p-wave components exists due to relativity.
- A strong indication of considerable higher Fock-states in light mesons.

Phys.Rev.Lett. 122 (2019) 8, 082301
Phys.Rev.D 101 (2020) 7, 074014
Phys.Rev.D 104 (2021) 9, 094016

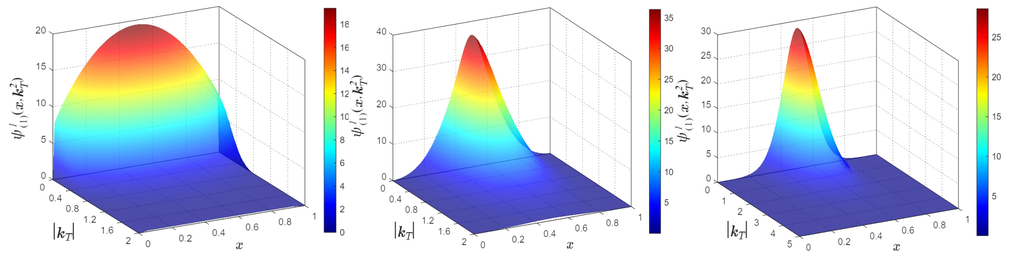
Light and Heavy 1^+ Mesons

Λ=1

ρ J/ψ Υ

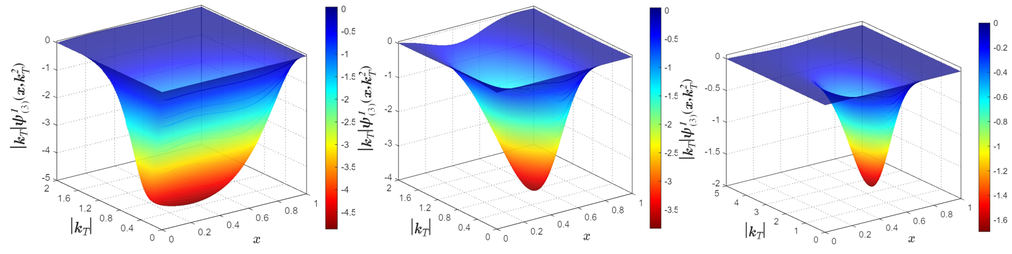
s-wave

$|L_z|$=0



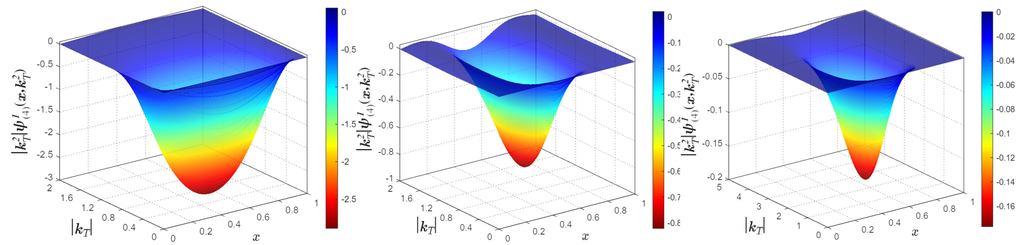
p-wave

$|L_z|$=1



d-wave

$|L_z|$=2

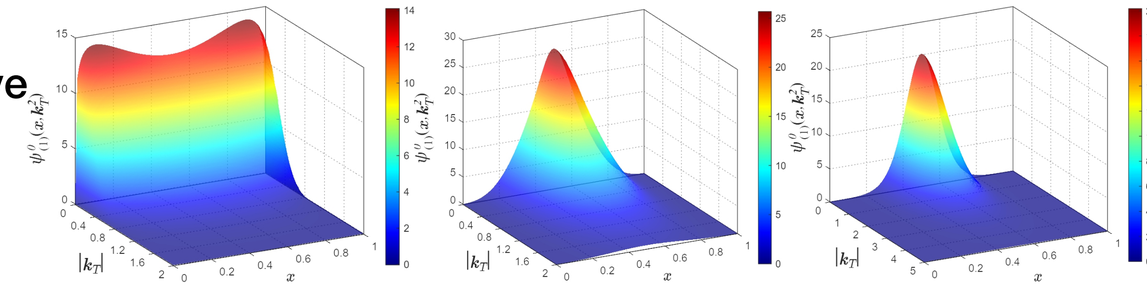


Λ=0

ρ J/ψ Υ

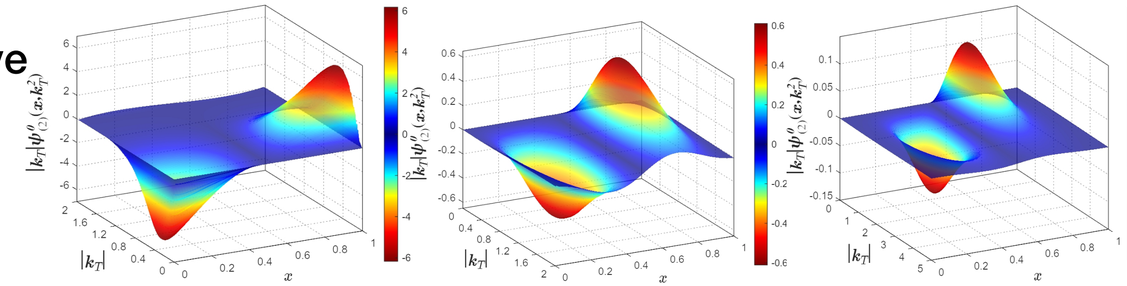
s-wave

$|L_z|$=0

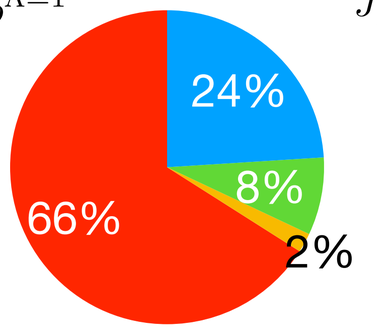


p-wave

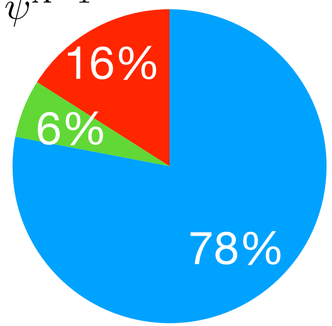
$|L_z|$=1



$\rho^{\Lambda=1}$



$J/\psi^{\Lambda=1}$



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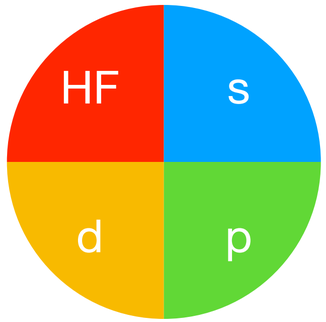
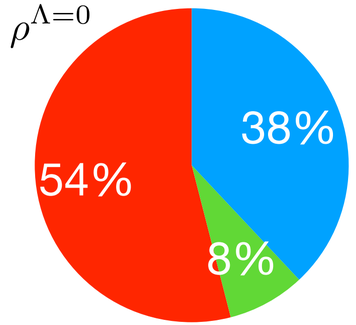
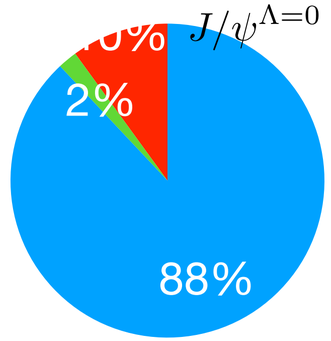


$\rho^{\Lambda=0}$

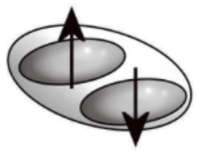


$J/\psi^{\Lambda=0}$



Phys.Rev.D 104 (2021) 9, L091902
Phys.Rev.D 106 (2022) 1, 014026
Phys.Rev.D 107 (2023) 7, 074009

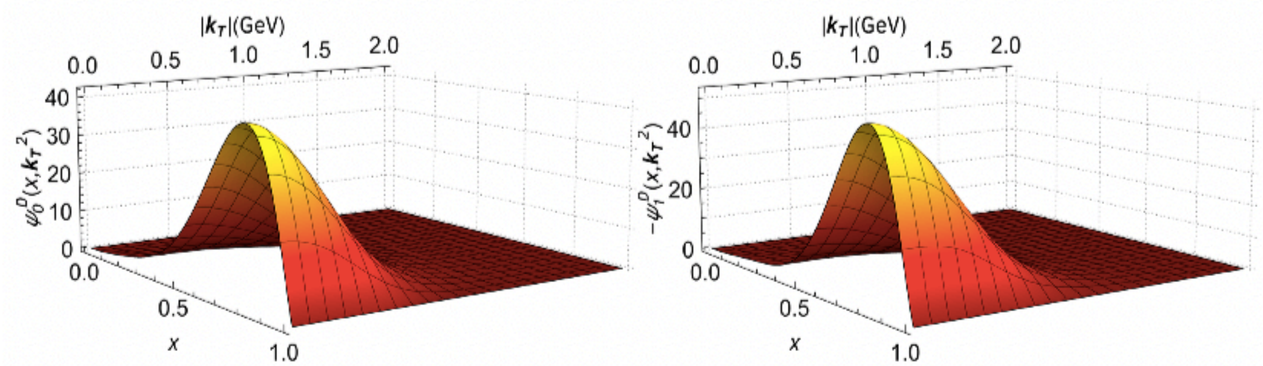
LFWFs of 0^- heavy flavor asymmetric Meson

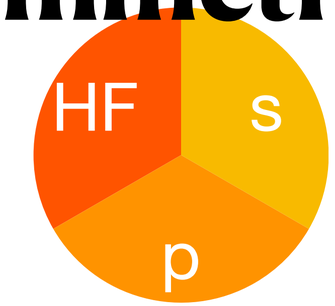


s-wave

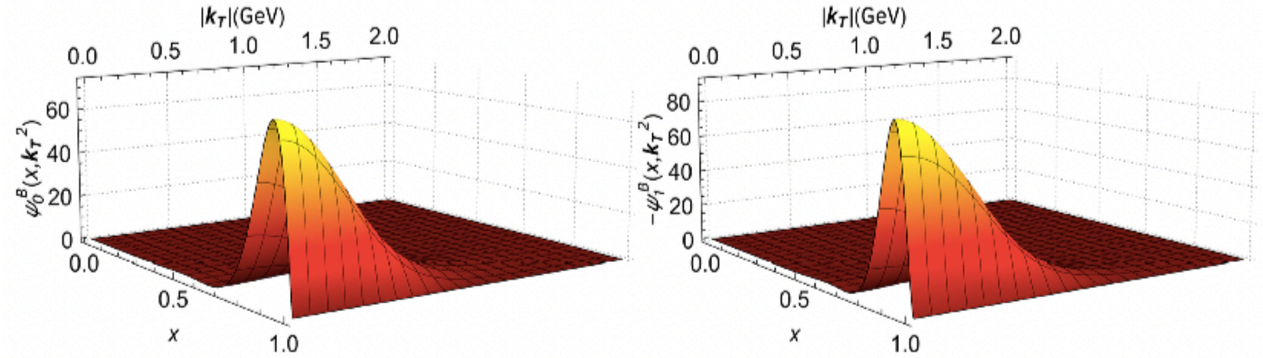
p-wave

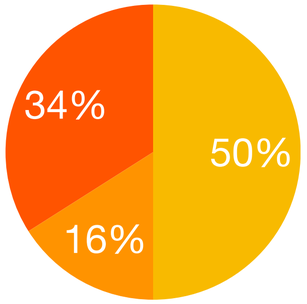
D



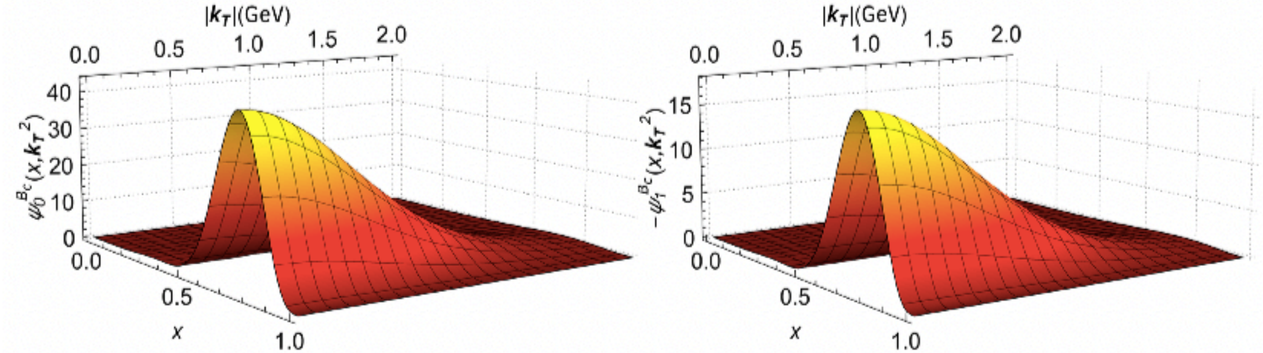


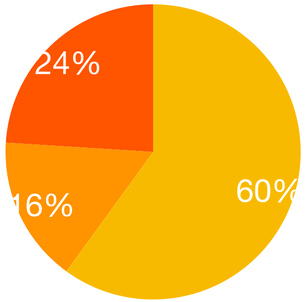
B

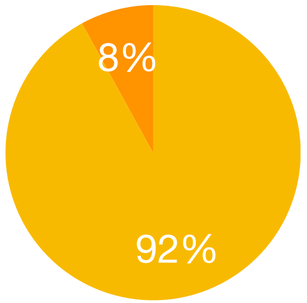




B_c





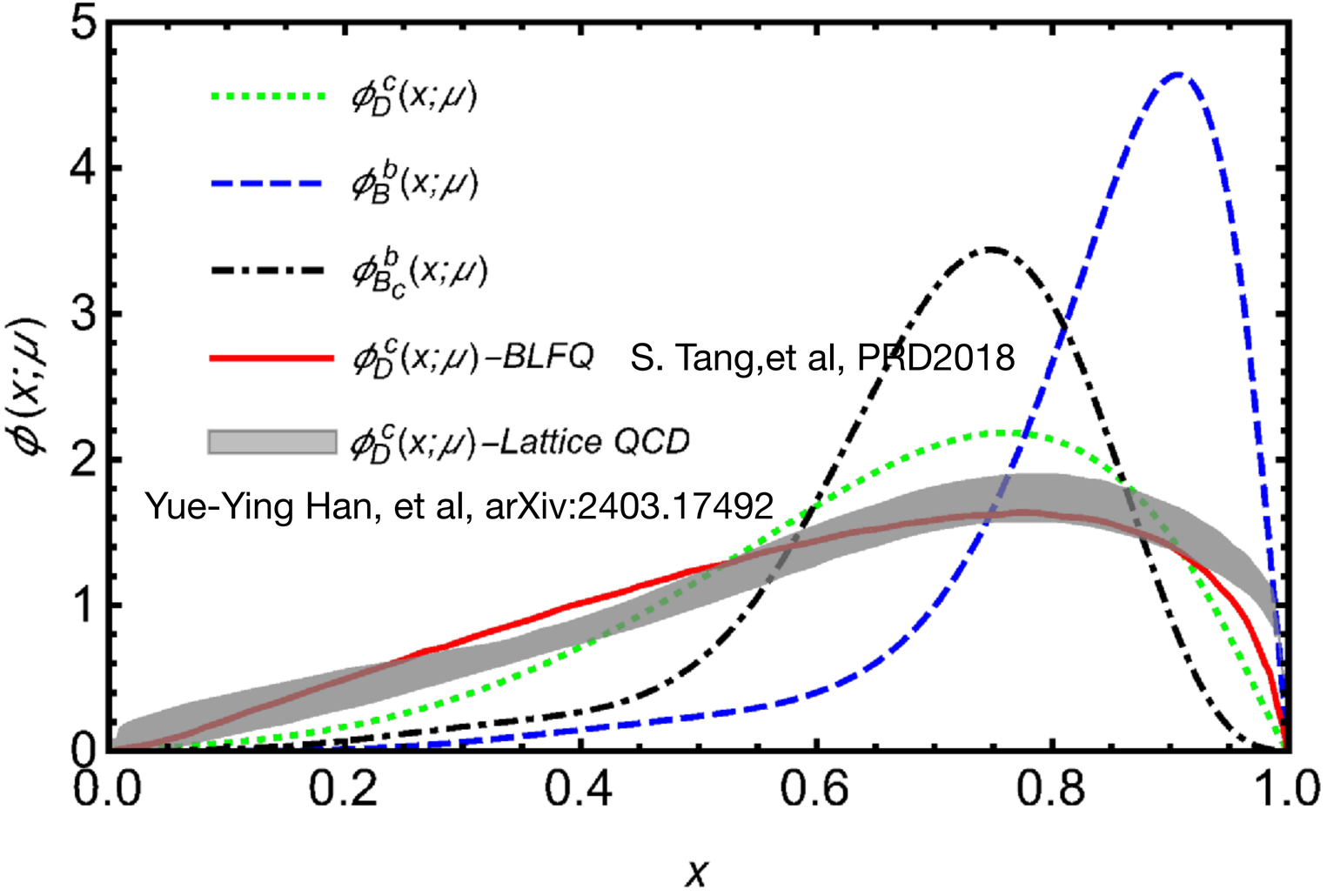


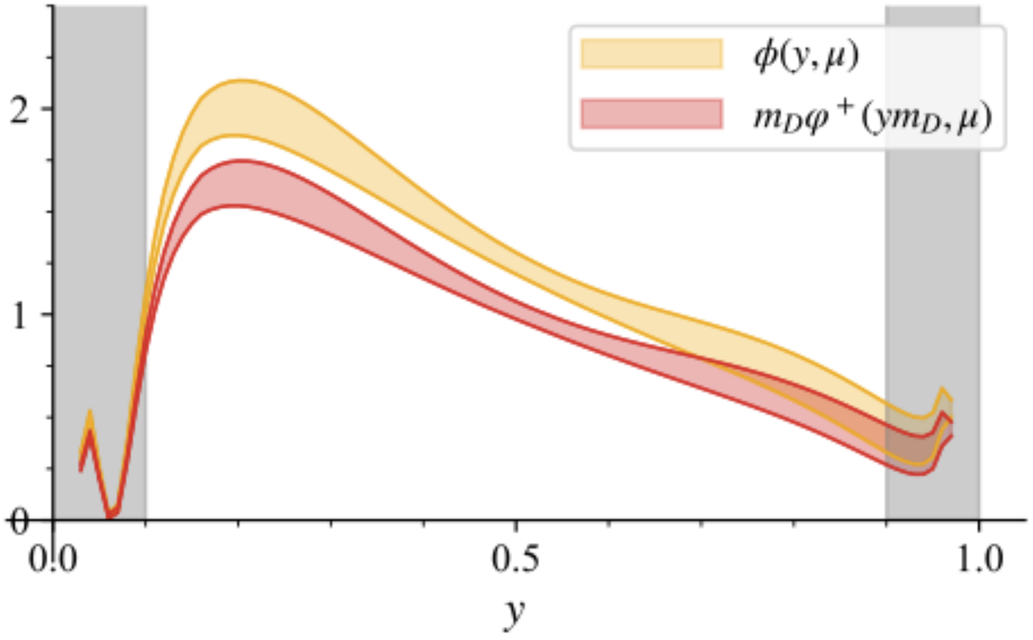
- Narrow x-distribution
- Narrow k_T-distribution
- Exhibiting a duality embodying characteristics from both light mesons and heavy quarkonium.

CS, P Liu, Y Du and W Jia,
Phys.Rev.D 110 (2024) 9, 094010

Parton Distribution Amplitude

$$\phi(x, Q^2) = \int^{Q^2} dk_T^2 \psi_0(x, k_T^2)$$

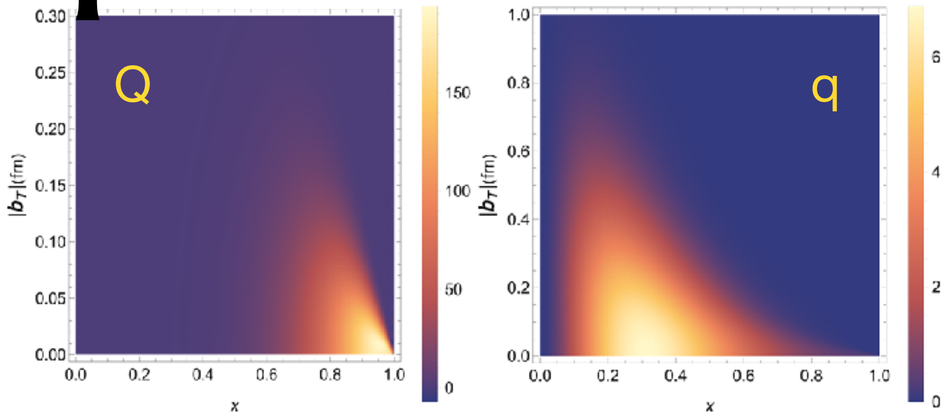




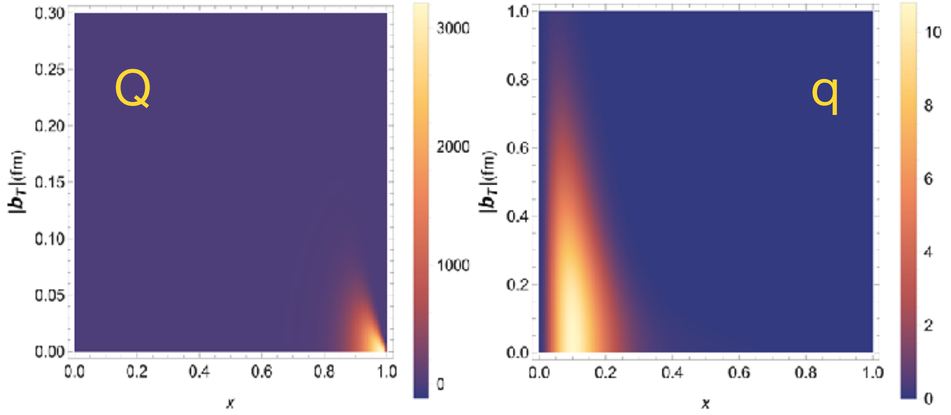
Lattice Parton Collaboration :arXiv:2410.18654

Impact Parameter Dependent GPD

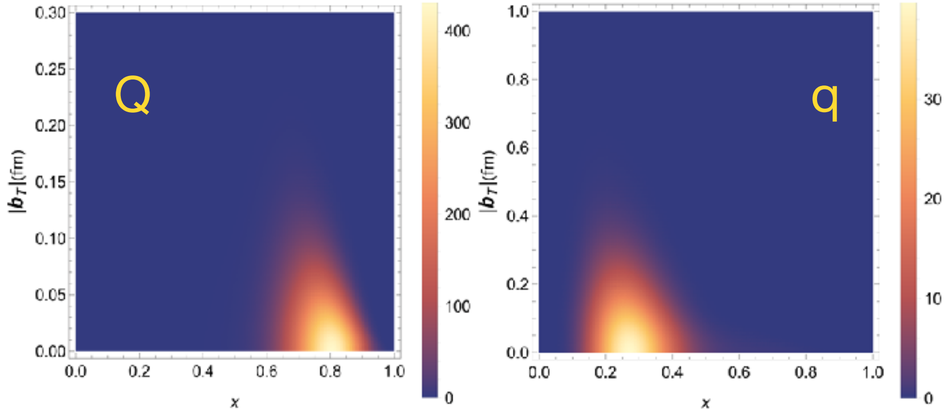
D



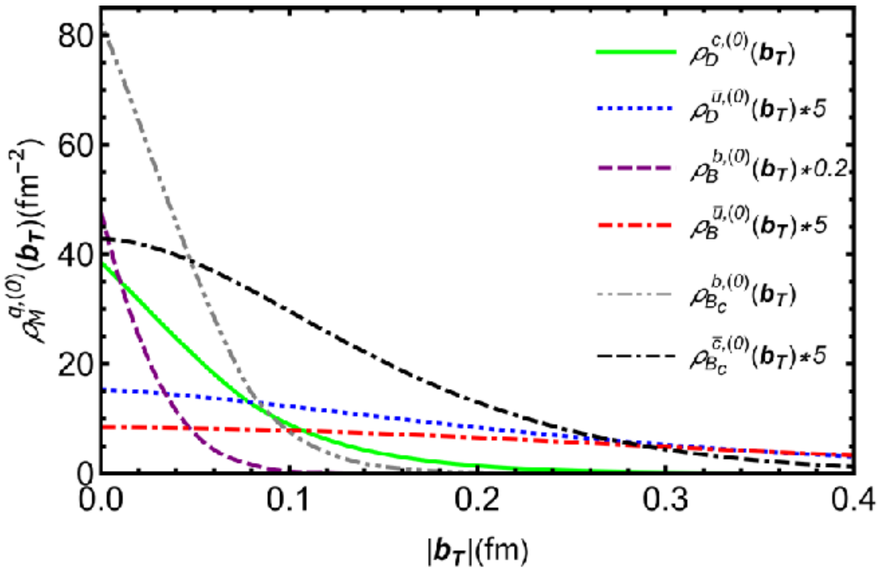
B

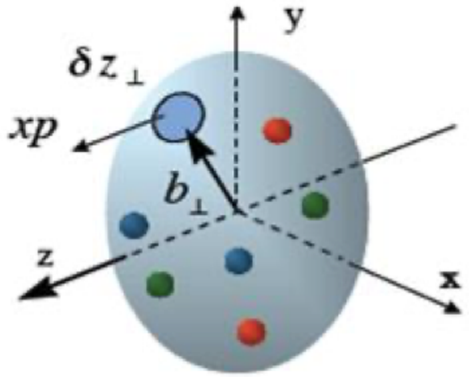


B_c



$$\rho_M^{q,(0)}(\boldsymbol{b_T}) = \int_0^1 dx \rho_M^q(x, \boldsymbol{b_T}^2)$$

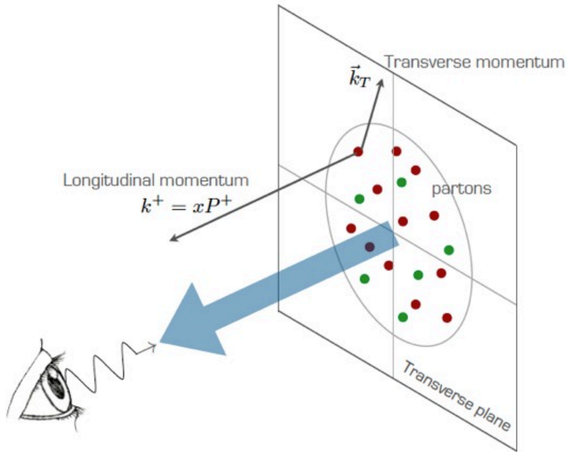




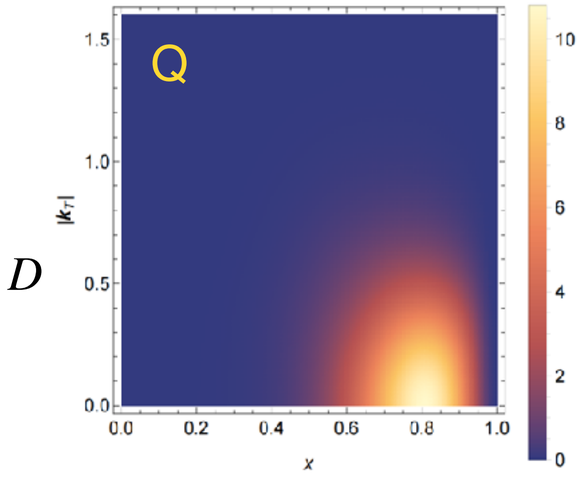
$\rho(x, |\vec{b}_T|)$

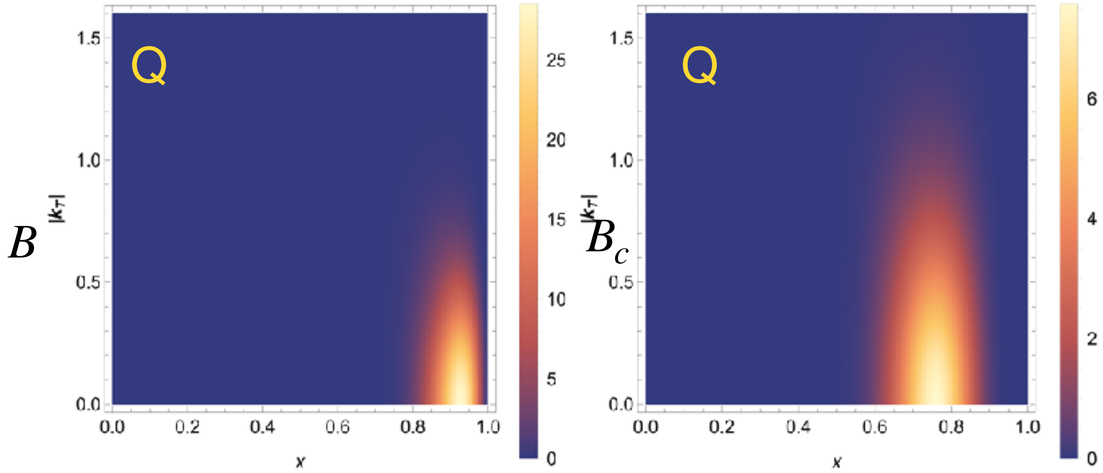
- Heavier quarks are spatially more centered, and carries most light-cone momentum.
- Lighter quarks are spread out, yet carries electric charge.
- In $B(b\bar{u})$, we find $\langle r^2_{E,\mathrm{LC}}\rangle_B = (0.14\ \mathrm{fm})^2 \ll \langle r^2_{c,LC}\rangle_{B^-} = (0.38\ \mathrm{fm})^2$

Unpolarized TMD PDF



$$f_1(x, \overrightarrow{k}_T^2)$$





- $\langle | \overrightarrow{k}_T | \rangle = \int dx d^2\overrightarrow{k}_T f_1^q(x, \overrightarrow{k}_T^2) | \overrightarrow{k}_T |$ for D, B and B_c mesons are 0.43, 0.42 and 0.65 GeV, as compared to 0.39, 0.65, 1.0 GeV for π, η_c and η_b.
- The mean transverse momentum inside $Q\bar{q}$ is close to that in $q\bar{q}$!

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- Summary and Outlook.

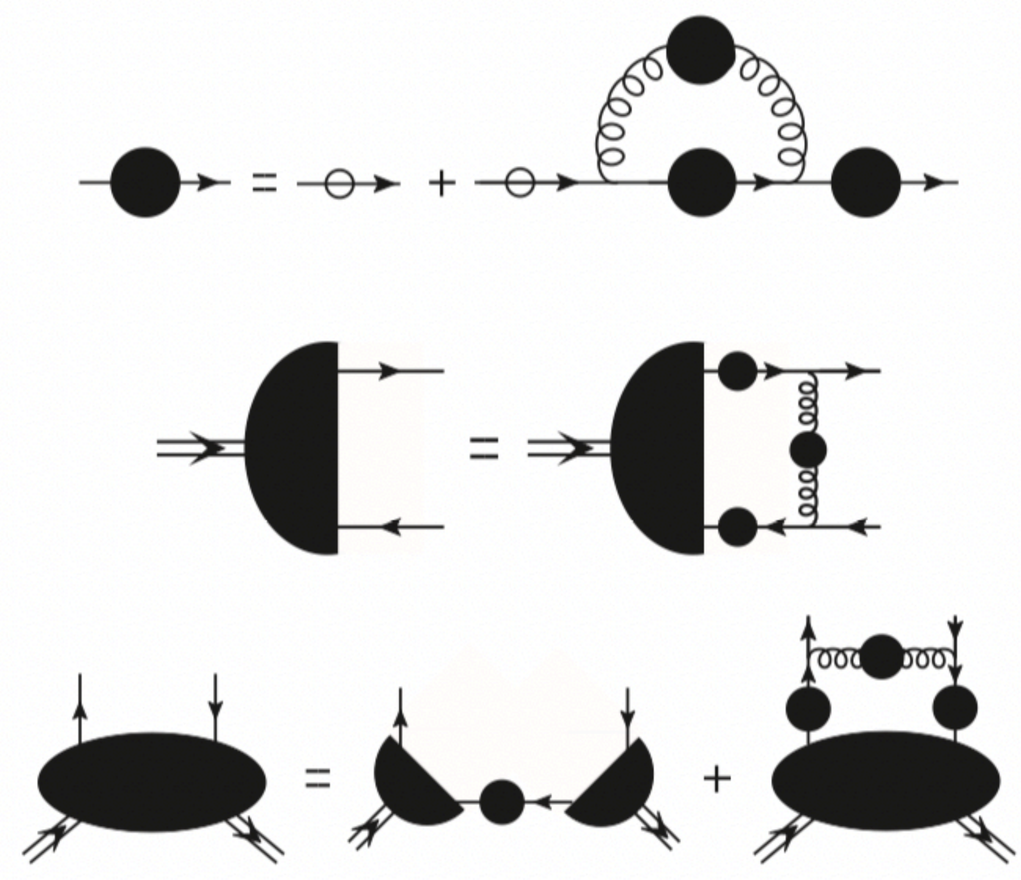
Going Beyond $q\bar{q}$ Fock-state

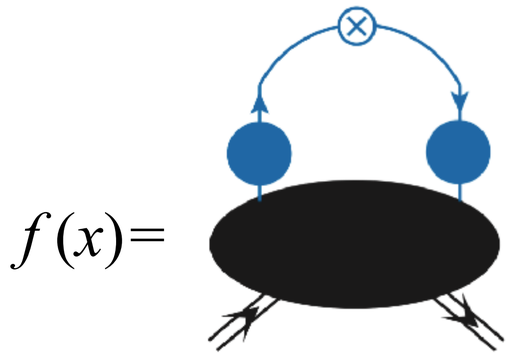
$$f(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle P|\bar{\psi}(0)\gamma^+\psi(\xi)|P\rangle\Big|_{\xi^+=0,\boldsymbol{\xi}_\perp=0}$$

- A covariant calculation in ordinary space-time can "effectively" include infinitely many higher Fock-states, such as $|q\bar{q}gg\ldots\rangle$.
- We develop the DSE for quark-quark correlation matrix, which is the mother function of various leading and subleading-twist PDFs.

$$\Phi_{ij}(k,P) = \int d^4\xi\ e^{ik\cdot\xi}\langle P|\bar{\psi}_j(0)\psi_i(\xi)|P\rangle.$$

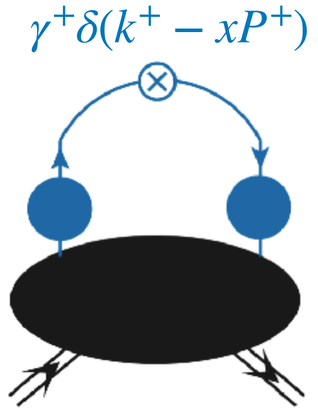
- Poincare, chiral and gauge symmetries are preserved.



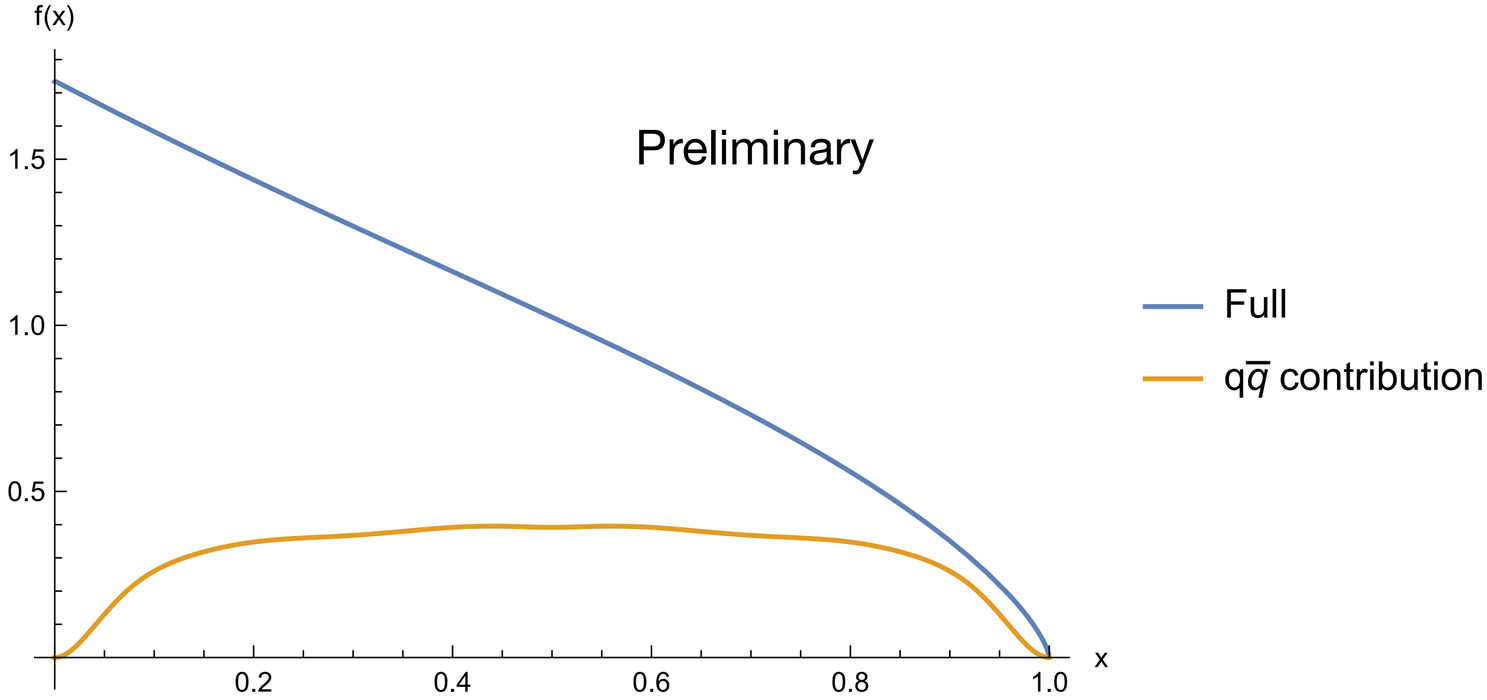


Pion twist-2 PDF

$$f(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle P|\bar{\psi}(0)\gamma^+\psi(\xi)|P\rangle\Big|_{\xi^+=0,\boldsymbol{\xi}_\perp=0}$$

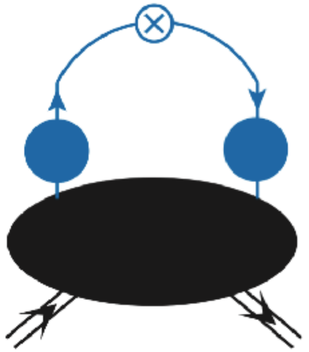
$f(x)=$ 

- $\langle x^0\rangle \equiv \int dx x^0 f(x) = 1$: quark number sum rule.
- $\langle x^1\rangle < 0.5$: gluon carries away some momentum.
- $q\bar{q}$ LFWF contribution dominate very large x region.
- Higher Fock-states provide large contribution in pion!



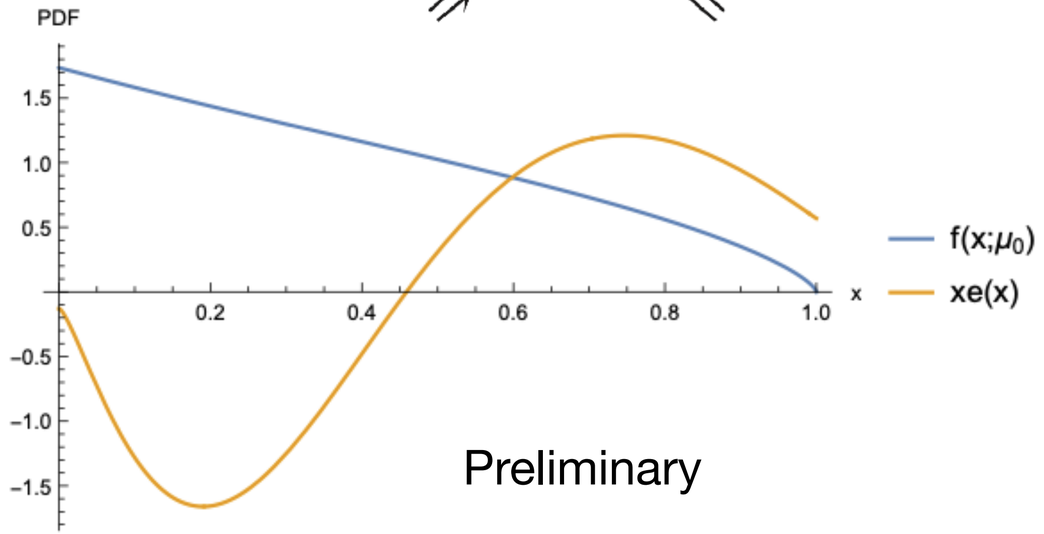
Pion twist-3 PDF

$$e(x) = \frac{P^+}{M}\int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle P|\bar{\psi}(0) I_4 \psi(\xi)|P\rangle\Big|_{\xi^+=0,\boldsymbol{\xi}_\perp=0}$$

$e\,(x)=$ 

- General QCD EOM yields the decomposition

$$e^q(x) = \frac{\sigma_{\pi N}}{2m_q}\delta(x) + e^q_{\rm m}(x) + e^q_{\rm tw3}(x).$$



Preliminary

- Strong indication of $\delta(x)$ is found in our nonperturbative computation!
- e_{tw3} is the genuine twist-3 PDF encoding quark-gluon-quark multi-parton distribution.

$$\bar{\psi}_q(0)\,\sigma^{\alpha\beta}\,n_\beta\,[0,v\lambda n]\,gG_{\alpha\nu}(v\lambda n)\,n^\nu\,[v\lambda n,u\lambda n]\,\psi_q(u\lambda n)$$

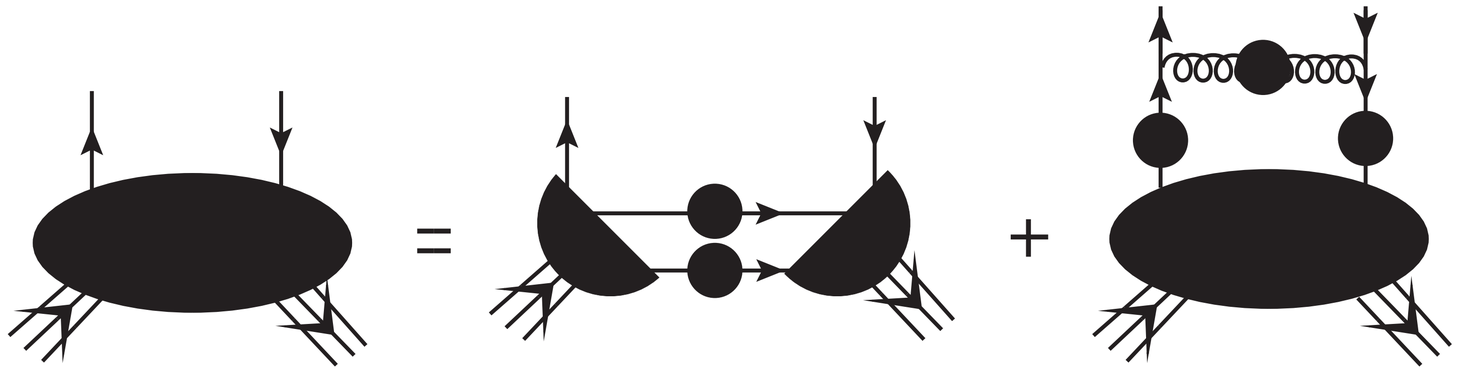
- e_{tw3} is comparable with twist-2 f(x) !

Summary

- We proposed a new formula to extract all LF-LFWFs from various mesons.
- The LF-LFWFs are used to sketching the parton image of light and heavy mesons.
- Improvement upon sketching is ongoing, with gluons taken into account.

Outlook

- More mesons $q\bar{q}$LFWFs (diffractive vector meson productions&gluon saturation).
- Nucleon.
- TMD PDFs.
- Gluon distribution.



Thank you very much for your attention!