Improved global determination of two-meson distribution amplitudes from multi-body B decays

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arXiv:2501.15150

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第七届重味物理与QCD会议

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O u t l i n e

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02 Framework

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01 Motivation

Why multi-body B decays?

01 They can help us to test the factorization approach, which have been used in two-body decays successfully.

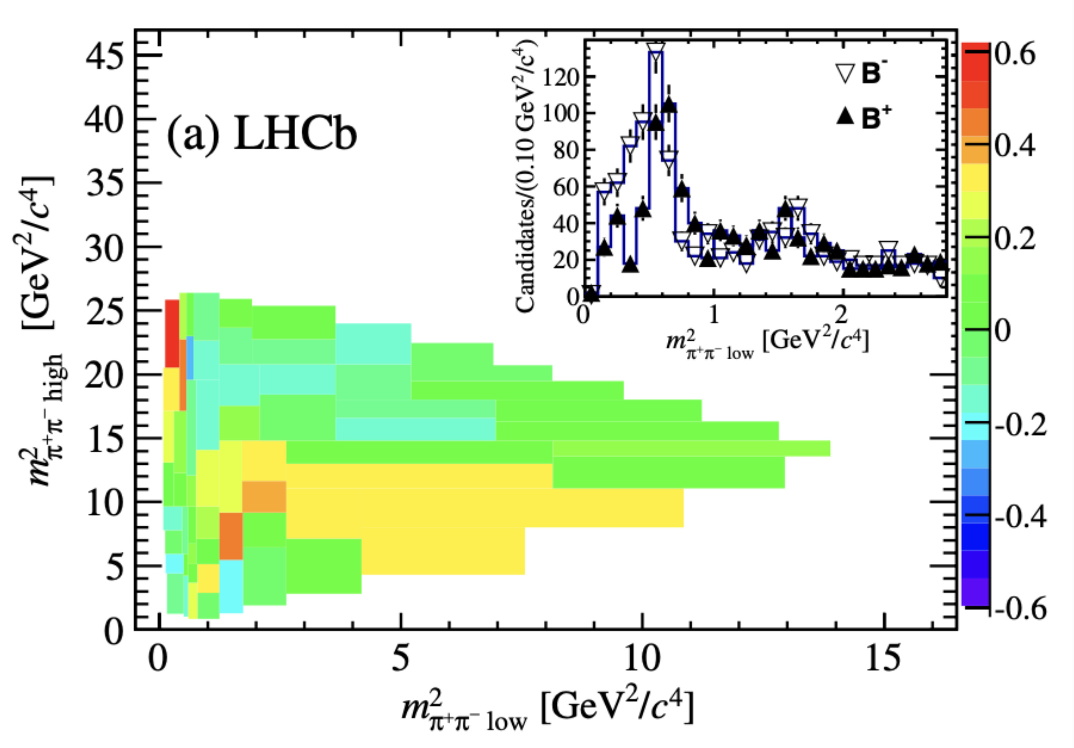
02 They also help us to test SM by measuring the CKM phases.

03 They offer us opportunities to study the line-shape of intermediate states.

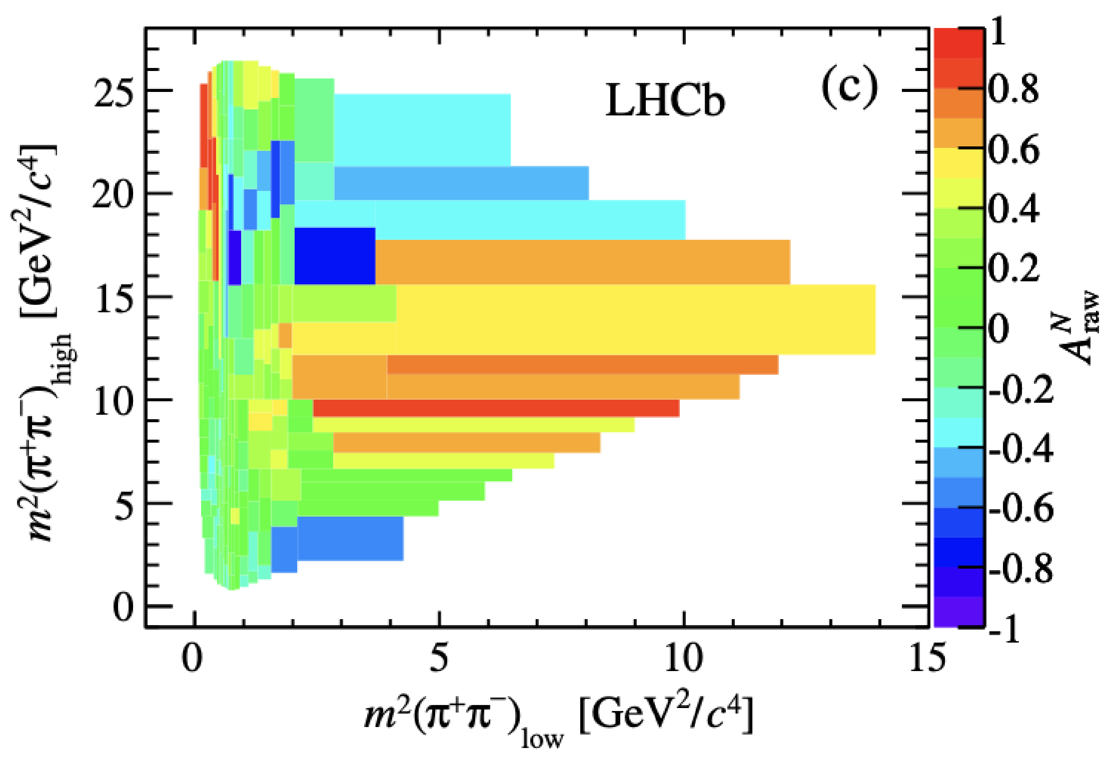
04

They offer one of the most promising avenues for probing CP violation (CPV)

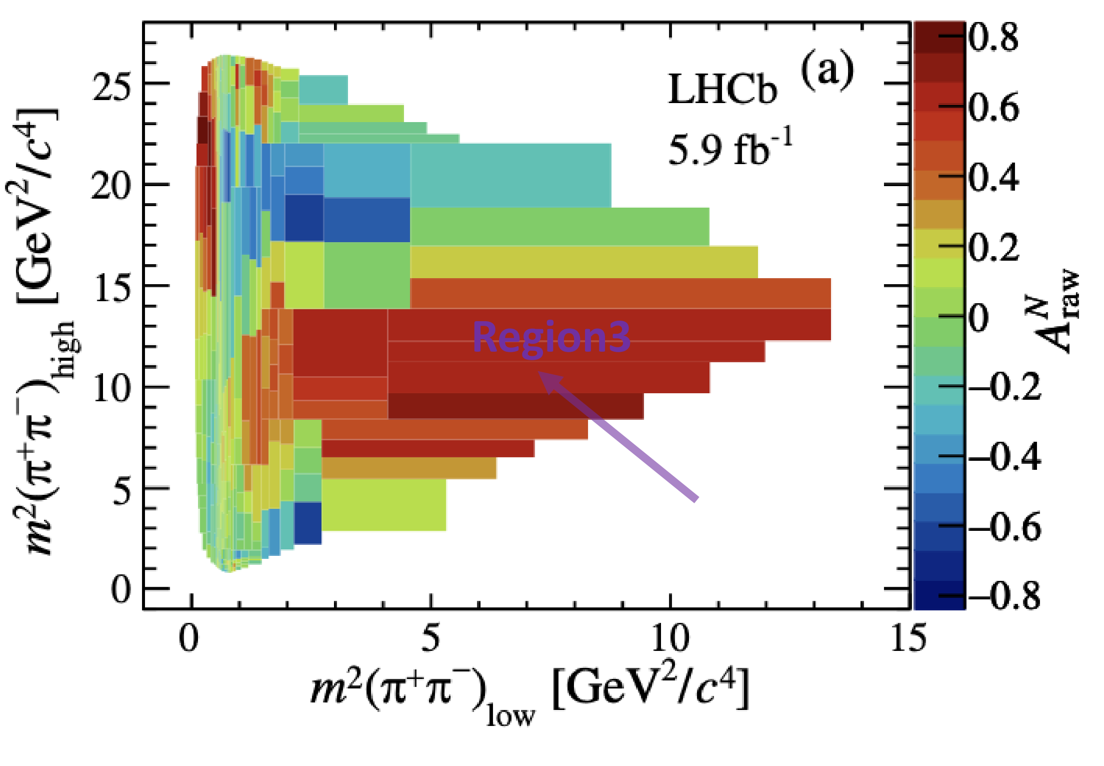
- Localized A_{CP} in three-body decays $B \to 3\pi, 3K, KK\pi, K\pi\pi$

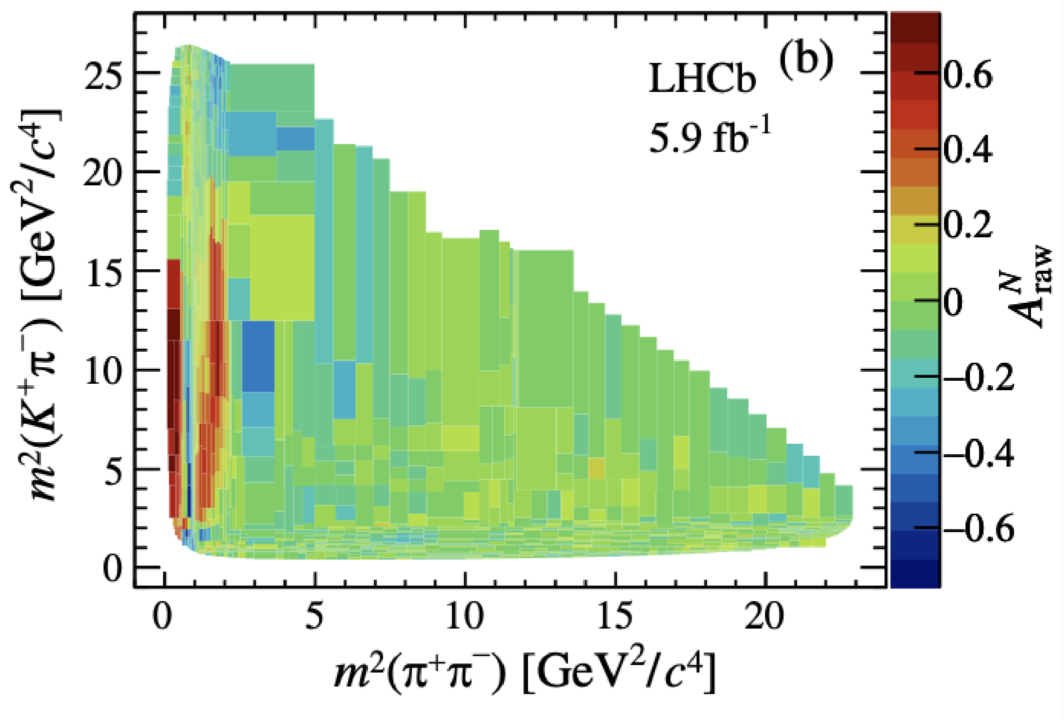


Phys. Rev. Lett.112, 011801(2014)



Phys. Rev. D 90,112004(2014)





$B^\pm \to \pi^\pm\pi^+\pi^-$	A_{CP}
Region1	$+0.303 \pm 0.009 \pm 0.004 \pm 0.003$
Region2	$-0.284 \pm 0.017 \pm 0.007 \pm 0.003$
Region3	$\mathbf{+0.745 \pm 0.027 \pm 0.018 \pm 0.003}$

Phys. ReV. D 108,012008(2023)

- Significant A_{CP}:
  $B \to 3\pi, 3K$ for the first time
- Large Localized A_{CP}

Large amounts of three- and four-body decays have been reported by LHCb, Belle, *BABAR*, CDF, D0 *etc*. Collaborations. (Most with BFs ~ 10^{-5})

- three-body B decays

$$B^+ \to \pi^+(\rho^0 \to)\pi\pi, B^+ \to K^+(\rho^0 \to)\pi\pi, B^0 \to \pi^0(\rho^0 \to)\pi\pi,$$
$$B^+ \to \pi^0(K^{*+} \to)K\pi, B^0 \to \pi^-(K^{*+} \to)K\pi, B_s^0 \to K^\pm(K^{*\mp} \to)K\pi,$$
$$B^0 \to K^0(\phi \to)KK \cdots\cdots$$

- four-body B decays

$$B_s^0 \to \phi\bar{K}^{*0} \to (KK)(K\pi), B_{(s)}^0 \to K^{*0}\bar{K}^{*0} \to (K\pi)(K\pi),$$
$$B^0 \to \rho^0 K^{*0} \to (\pi\pi)(K\pi), B_s^0 \to \phi\phi \to (KK)(KK),$$
$$B^0 \to \rho^+\rho^- \to (\pi\pi)(\pi\pi) \cdots\cdots$$

Why improved global fitting for a two-hadron DA?

Two-meson DAs were derived in recent global investigations of three- and four body B meson decays based on the PQCD formalism(Phys.Rev.D 104,096014 (2021), Phys.Rev.D 105, 093001 (2022), Eur.Phys.J.C 83,974 (2023)).

- $\pi\pi$ two-meson DA ($a^0_{2\rho}, a^s_{2\rho}, a^t_{2\rho}$)

$$\begin{cases} B \to P\rho \to P(\pi\pi), P = \pi, K \\ B^0 \to \rho^+\rho^- \to (\pi\pi)(\pi\pi) \end{cases}$$

- $K\pi$ two-meson DA ($a^{||}_{1K^{*0}}, a^{||}_{2K^{*0}}$)

$$B_{(s)} \to PK^* \to P(K\pi), P = \pi, K$$

- KK two-meson DA ($a^0_{2\phi}, a^T_{2\phi}$)

$$\begin{cases} B \to P\phi \to P(KK), P = \pi, K \\ B^0_s \to \phi\phi \to (KK)(KK) \end{cases}$$

The Gegenbauer moments in the Kπ and KK DAs, being slightly higher than unity, are not favored in view of the convergence of the Gegenbauer expansion.

$a^{||}_{2K^{*0}}$=1.19$\pm$0.10 (Phys.Rev.D 104,096014 (2021))

$a^T_{2\phi}$=1.48$\pm$0.07 (Phys.Rev.D 105, 093001 (2022))

We introduce additional parameters to compensate the possible discrepancy between two theoretical treatments of the hadronic matrix elements for vacuum transition to meson pairs. (arXiv:1011.0960)

$$N_{\pi\pi}, N_{K\pi}, N_{KK}$$

polarization fractions of the four-body $B \to V_1V_2 \to (P_1P_2)(P_3P_4)$ decays

$$f_0(B_s^0 \to K^{*0}\overline{K}^{*0} \to (K\pi)(K\pi)) = \begin{cases} (63.6^{+2.7+3.3+1.0}_{-4.2-3.9-1.0})\% & \text{PQCD (JHEP05(2021)082)} \\ (24 \pm 4)\% & \text{PDG} \end{cases}$$

▪Flavor-changing neutral-current transitions are very sensitive to new physics (NP) contributions, like $\bar{B}_s^0 \to K^{*0}\overline{K}^{*0}$ decay controlled by $b \to s$ transition.

▪ To attain definite predictions, nonperturbative inputs of two-meson DAs must be known to high precision. More four-body B meson decays should be included in the global fit.

02 Framework

In the standard model,
low-energy effective Hamiltonian is given:

Fermi coupling constant

Local four-quark operators

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\{\sum_{q=u,c} V_{qb}V_{qD}^{\star}[C_1 O_1^q + C_2 O_2^q] - V_{tb}V_{tD}^{\star}[\sum_{i=3}^{10,7\gamma,8g} C_i O_i]\}$$

CKM matrix elements

Wilson coefficient

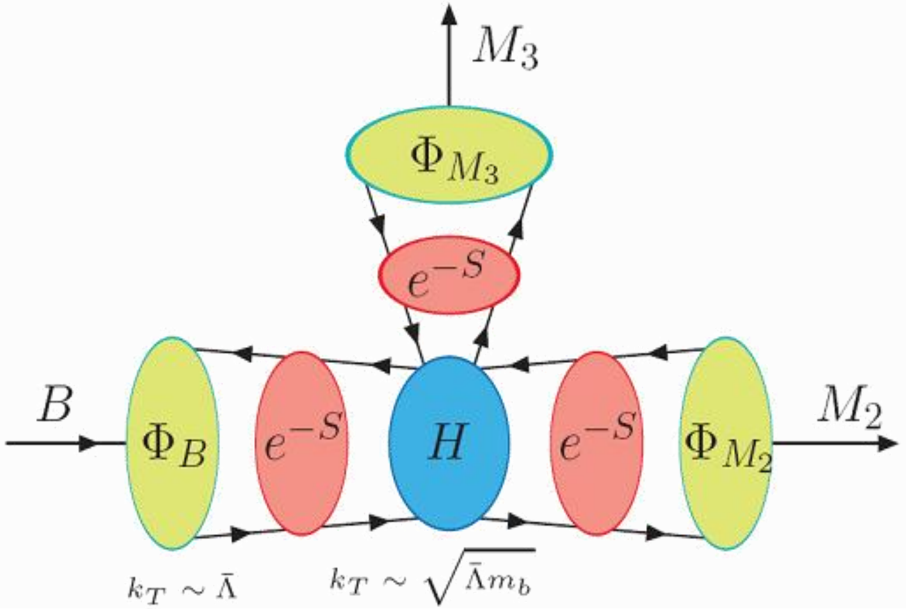
For two-body nonleptonic B meson decays,
the decay amplitude can be written as:

$$\mathcal{A}(B \to M_1 M_2) = \langle M_1 M_2 \mid \mathcal{H}_{eff} \mid B\rangle = \frac{G_F}{\sqrt{2}} \sum_i V_i C_i(\mu) \langle M_1 M_2 \mid O_i(\mu) \mid B\rangle$$

Theoretical approach

Factorization Approach	H.Y. Cheng, C.K. Chua, Y. Li,…
QCD Factorization	A. Furman, B.El Bennich, R. Kaminski, T. Mannel, X.H. Guo, Y.D. Yang, Z.H. Zhang,…
PQCD	H.n. Li, C.D. Lü, Z.J. Xiao, W. Wang, W.F. Wang, Z.Rui, Y.Li, Z.T.Zou ...
Symmetry	X.G. Xiao, G.N. Li, D. Xu, J.L. Rosner, M. Gronau,…
Sum rules	Ulf-G. Meißner (Light-cone), A. Khodjamirian (QCD)

k_T factorization: the full amplitudes of two-body decay $B \to M_1 M_2$ can be factorized as



References:
PPNP51-85(2003);
arXiv:0707.1294;
0907.4940;
1406.7689

$$A = \phi_B \otimes C \otimes H \otimes J_t \otimes S \otimes \phi_{M_2} \otimes \phi_{M_3}$$

Hard Kernel	Wave Function	Sudakov Factor	Jet Function	Wilson Coefficients

k_T resummation

Threshold resummation

Quasi-two-body decays in PQCD

Multi-body B decays

- More complicated strong dynamics than two-body ones.
- Receive both resonant and nonresonant contributions.
- Suffer substantial final-state interactions.
- A factorization formalism in full phase space is not yet available.

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Physics Letters B 561 (2003) 258–265

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Three-body nonleptonic *B* decays in perturbative QCD

Chuan-Hung Chen, Hsiang-Nan Li

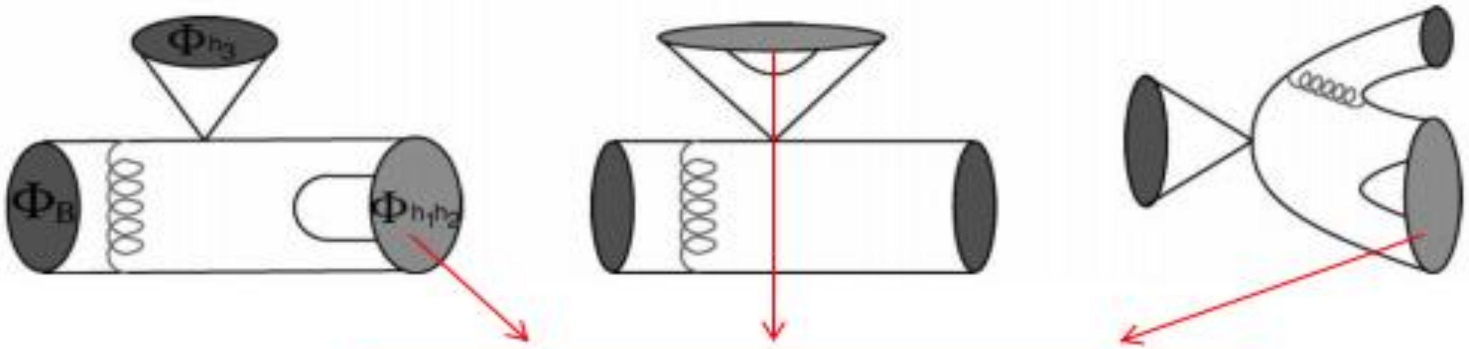
Abstract

We develop perturbative QCD formalism for three-body nonleptonic *B* meson decays. Leading contributions are identified by defining power counting rules for various topologies of amplitudes. The analysis is simplified into the one for two-body decays by introducing two-meson distribution amplitudes. This formalism predicts both nonresonant and resonant contributions, and can be generalized to baryonic decays.

Physics Letters B 561 (2003) 258–265

Three-body nonleptonic B decays in perturbative QCD

Chuan-Hung Chen, Hsiang-Nan Li



two-meson distribution amplitudes

TMDA

$$B \to h_1 h_2 h_3$$

$$\Rightarrow \mathcal{M} = \Phi_B \otimes H \otimes \boxed{\Phi_{h_1 h_2}} \otimes \Phi_{h_3}$$

P-wave two-meson DAs for both longitudinal and transverse polarizations

Longitudinal component

$$\Phi_P^L(z,\zeta,\omega) = \frac{1}{\sqrt{2N_c}}\left[\omega\not\epsilon_p\phi_P^0(z,\omega^2)+\omega\phi_P^s(z,\omega^2)+\frac{\not p_1\not p_2-\not p_2\not p_1}{\omega(2\zeta-1)}\phi_P^t(z,\omega^2)\right](2\zeta-1)\,,$$

Transverse component

$$\Phi_P^T(z,\zeta,\omega) = \frac{1}{\sqrt{2N_c}}\left[\gamma_5\not\epsilon_T\not p\phi_P^T(z,\omega^2)+\omega\gamma_5\not\epsilon_T\phi_P^a(z,\omega^2)+i\omega\frac{\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\epsilon_{T\nu}p_\rho n_{-\sigma}}{p\cdot n_-}\phi_P^v(z,\omega^2)\right]\cdot\sqrt{\zeta(1-\zeta)+\alpha}\,,$$

$P=\pi\pi$ pair

time-like form factors (GS)

Phys.Lett.B 763,29 (2016)

Phys.Rev.D 95,056008(2013)

Phys.Rev.D 86,032013(2012)

$$\frac{F^{\perp}_{\pi\pi}(\omega^2)}{F^{||}_{\pi\pi}(\omega^2)} \sim \frac{f^T_\rho}{f_\rho}$$

$$\phi^0_{\pi\pi}(z,\omega^2) = \frac{3F^{||}_{\pi\pi}(\omega^2)}{\sqrt{2N_c}} z(1-z)\left[1 + a^0_{2\rho}\frac{3}{2}(5t^2-1)\right],$$

$$\phi^s_{\pi\pi}(z,\omega^2) = \frac{3F^{\perp}_{\pi\pi}(\omega^2)}{2\sqrt{2N_c}} t\left[1 + a^s_{2\rho}(10z^2-10z+1)\right],$$

$$\phi^t_{\pi\pi}(z,\omega^2) = \frac{3F^{\perp}_{\pi\pi}(\omega^2)}{2\sqrt{2N_c}} t^2\left[1 + a^t_{2\rho}\frac{3}{2}(5t^2-1)\right],$$

$$\phi^T_{\pi\pi}(z,\omega^2) = \frac{3F^{\perp}_{\pi\pi}(\omega^2)}{\sqrt{2N_c}} z(1-z)\left[1 + a^T_{2\rho}\frac{3}{2}(5t^2-1)\right],$$

Phys. Rev. D 98 ,113003(2018)

$$\phi^a_{\pi\pi}(z,\omega^2) = \frac{3F^{||}_{\pi\pi}(\omega^2)}{4\sqrt{2N_c}} t[1 + a^a_{2\rho}(10z^2-10z+1)],$$

$$\phi^v_{\pi\pi}(z,\omega^2) = \frac{3F^{||}_{\pi\pi}(\omega^2)}{8\sqrt{2N_c}} [1 + t^2 + a^v_{2\rho}(3t^2-1)],$$

P=$K\pi$ pair

time-like form factors (RBW)

Eur.Phys. J.C 79,37(2019)
Eur.Phys.J.C 78,1019(2018)

$$\phi^0_{K\pi}(z,\omega^2) = \frac{3F^{||}_{K\pi}(\omega^2)}{\sqrt{2N_c}} z(1-z)\left[1 + a^{||}_{1K^*} 3t + a^{||}_{2K^*}\frac{3}{2}(5t^2-1)\right] ,$$

$$\phi^s_{K\pi}(z,\omega^2) = \frac{3F^{\perp}_{K\pi}(\omega^2)}{2\sqrt{2N_c}} t ,$$

$$\phi^t_{K\pi}(z,\omega^2) = \frac{3F^{\perp}_{K\pi}(\omega^2)}{2\sqrt{2N_c}} t^2 ,$$

$$\phi^T_{K\pi}(z,\omega^2) = \frac{3F^{\perp}_{K\pi}(\omega^2)}{\sqrt{2N_c}} z(1-z)\left[1 + a^{\perp}_{1K^*} 3t + a^{\perp}_{2K^*}\frac{3}{2}(5t^2-1)\right] ,$$

$$\phi^a_{K\pi}(z,\omega^2) = \frac{3F^{||}_{K\pi}(\omega^2)}{4\sqrt{2N_c}} t ,$$

$$\phi^v_{K\pi}(z,\omega^2) = \frac{3F^{||}_{K\pi}(\omega^2)}{8\sqrt{2N_c}} (1+t^2) ,$$

P=KK pair

time-like form factors (RBW)

Eur.Phys. J.C 79,37(2019)
Eur.Phys.J.C 78,1019(2018)

$$\phi^0_{KK}(z,\omega^2) = \frac{3F^{\parallel}_{KK}(\omega^2)}{\sqrt{2N_c}} z(1-z)\left[1 + a^0_{2\phi}\frac{3}{2}(5t^2-1)\right] ,$$

$$\phi^s_{KK}(z,\omega^2) = \frac{3F^{\perp}_{KK}(\omega^2)}{2\sqrt{2N_c}} t ,$$

$$\phi^t_{KK}(z,\omega^2) = \frac{3F^{\perp}_{KK}(\omega^2)}{2\sqrt{2N_c}} t^2 ,$$

$$\phi^T_{KK}(z,\omega^2) = \frac{3F^{\perp}_{KK}(\omega^2)}{\sqrt{2N_c}} z(1-z)\left[1 + a^T_{2\phi}\frac{3}{2}(5t^2-1)\right] ,$$

$$\phi^a_{KK}(z,\omega^2) = \frac{3F^{\parallel}_{KK}(\omega^2)}{4\sqrt{2N_c}} t ,$$

$$\phi^v_{KK}(z,\omega^2) = \frac{3F^{\parallel}_{KK}(\omega^2)}{8\sqrt{2N_c}} (1+t^2) ,$$

P-wave Parametrization

Factorizable contribution $\frac{G_F}{\sqrt{2}} \boxed{\langle P_1(p_1)P_2(p_2)|J_\mu|0\rangle} \langle P_3(p_3)|J^\mu|B(p_B)\rangle$

- time-like meson form factors Phys. Rev. D 58, 094009(1998)

$$\langle P_1(p_1)P_2(p_2)|J_\mu|0\rangle = \left[(p_1-p_2)_\mu - \frac{m_{P_1}^2 - m_{P_2}^2}{p^2} p_\mu\right] F_1^{P_1P_2}(p^2) + \frac{m_{P_1}^2 - m_{P_2}^2}{p^2} p_\mu F_0^{P_1P_2}(p^2)$$

- Breit-Wigner (BW) propagators for intermediate P-wave resonances

$$\begin{aligned}\langle P_1(p_1)P_2(p_2)|J_\mu|0\rangle &= \sum_{\lambda=0,\pm1} \langle P_1(p_1)P_2(p_2)|V,\epsilon^\lambda\rangle \frac{1}{m_V^2 - s - im_V\Gamma(s)} \langle V,\epsilon^\lambda|J_\mu|0\rangle \\ &= \frac{g^{V\to P_1P_2}}{m_V^2 - s - im_V\Gamma(s)} \sum_{\lambda=0,\pm1} \epsilon^\lambda \cdot (p_1-p_2) \langle V,\epsilon^\lambda|J_\mu|0\rangle \\ &= \frac{g^{V\to P_1P_2}}{m_V^2 - s - im_V\Gamma(s)} \sum_{\lambda=0,\pm1} \epsilon^\lambda \cdot (p_1-p_2)\epsilon_\mu^\lambda f_V m_V,\end{aligned}$$

Phys. Rev. D 106 ,113004(2022)

Coupling strength

$$\frac{g^{V\to P_1P_2}}{m_V^2 - s - im_V\Gamma(s)} \cdot f_V m_V = N_P F_1^{P_1P_2}(p^2)$$

arXiv:1011.0960

The coefficient N_P has been introduced to remedy the possible theoretical mismatch between the meson form factors and the properties of the intermediate P-wave resonance.

$$\boxed{N_{\pi\pi} \approx 1.00, \quad N_{K\pi} \approx 1.40, \quad N_{KK} \approx 1.20}$$

The above three coefficients will be handled as free parameters, determined in our global fit

Physical observables

- three-body decays

$$\int d\mathcal{B} = \frac{\tau_B m_B}{256\pi^3} \int_{(\sqrt{r_1}+\sqrt{r_2})^2}^{1} d\eta \sqrt{(1-\eta)^2 - 2r_3(1+\eta) + r_3^2} \int_{\zeta_{\min}}^{\zeta_{\max}} d\zeta |\mathcal{A}|^2$$

with the bounds

$$\zeta_{\max,\min} = \frac{1}{2}\left[1 \pm \sqrt{1 - 2\frac{r_1+r_2}{\eta} + \frac{(r_1-r_2)^2}{\eta^2}}\right]$$

$$\mathcal{A}_{CP} = \frac{\mathcal{B}(\bar{B} \to \bar{f}) - \mathcal{B}(B \to f)}{\mathcal{B}(\bar{B} \to \bar{f}) + \mathcal{B}(B \to f)}$$

four-body decays

The differential rate for the decays

$$\frac{d^5\mathcal{B}}{d\Omega} = \frac{\tau_B k(\omega_1)k(\omega_2)k(\omega_1,\omega_2)}{16(2\pi)^6 m_B^2}|A|^2 \qquad \Omega \equiv \{\theta_1,\theta_2,\phi,\omega_1,\omega_2\}$$

$$k(\omega) = \frac{\sqrt{\lambda(\omega^2, m_{h_1}^2, m_{h_2}^2)}}{2\omega} \qquad \lambda(a,b,c) = a^2+b^2+c^2-2(ab+ac+bc)$$

$$k(\omega_1,\omega_2) = \frac{\sqrt{[m_B^2-(\omega_1+\omega_2)^2][m_B^2-(\omega_1-\omega_2)^2]}}{2m_B}$$

The transformation connecting the B meson rest frame and the meson pair rest frame leads to the relations between ζ and θ

$$2\zeta_i - 1 = \sqrt{1+4\alpha_i}\cos\theta_i,$$

The differential branching fraction

$$\frac{d^5\mathcal{B}}{d\zeta_1 d\zeta_2 d\omega_1 d\omega_2 d\varphi} = \frac{\tau_B k(\omega_1)k(\omega_2)k(\omega_1,\omega_2)}{4(2\pi)^6 m_B^2 \sqrt{1+4\alpha_1}\sqrt{1+4\alpha_2}}|A|^2$$

CP-averaged branching ratio and the direct CP asymmetry in each component $h = 0, \|, \perp$

$$\mathcal{B}_h^{avg} = \frac{1}{2}(\mathcal{B}_h + \bar{\mathcal{B}}_h), \quad \mathcal{A}_h^{\mathrm{dir}} = \frac{\bar{\mathcal{B}}_h - \mathcal{B}_h}{\bar{\mathcal{B}}_h + \mathcal{B}_h},$$

The total branching ratio and the overall direct CP asymmetry

$$\mathcal{B}_{\mathrm{total}} = \sum_h \mathcal{B}_h, \quad \mathcal{A}_{CP}^{\mathrm{dir}} = \frac{\sum_h \bar{\mathcal{B}}_h - \sum_h \mathcal{B}_h}{\sum_h \bar{\mathcal{B}}_h + \sum_h \mathcal{B}_h},$$

Polarization fractions f_h

$$f_h = \frac{\mathcal{B}_h}{\mathcal{B}_0 + \mathcal{B}_{||} + \mathcal{B}_\perp}$$

03 Numerical results

Global fit

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{v_i - v_i^{\text{th}}}{\delta v_i} \right)^2 .$$

experimental data： $v_i \pm \delta v_i$

(Measurements with significance larger than 3σ)

theoretical values: v_i^{th}

Three- and four-body decays included in our global fit

Three-body decays (Branching ratios)

$B^+ \to K^+(\rho^0 \to)\pi\pi$
$B^0 \to K^+(\rho^- \to)\pi\pi$
$B^+ \to K^0(\rho^+ \to)\pi\pi$
$B^0 \to K^0(\rho^0 \to)\pi\pi$
$B^+ \to \pi^+(\rho^0 \to)\pi\pi$
$B^0 \to \pi^\pm(\rho^\mp \to)\pi\pi$
$B^+ \to \pi^0(\rho^+ \to)\pi\pi$

$B^+ \to K^+(\bar{K}^{*0} \to)K\pi$
$B_s^0 \to K^+(K^{*-} \to)K\pi + c.c.$
$B_s^0 \to K^0(\bar{K}^{*0} \to)K\pi + c.c.$
$B^+ \to \pi^+(K^{*0} \to)K\pi$
$B^0 \to \pi^-(K^{*+} \to)K\pi$
$B^+ \to \pi^0(K^{*+} \to)K\pi$
$B^0 \to \pi^0(K^{*0} \to)K\pi$

$B^+ \to K^+(\phi \to)KK$
$B^0 \to K^0(\phi \to)KK$

Four-body decays (Branching ratios, polarization fractions)

$B^0 \to \rho^+\rho^-$
$B_s^0 \to \phi\phi$

$B^+ \to \rho^+K^{*0}$
$B^+ \to \rho^0K^{*+}$
$B^0 \to K^{*0}\bar{K}^{*0}$

longitudinal polarization fractions

$B^0 \to \phi K^{*0}$
$B^+ \to \phi K^{*+}$
$B_s^0 \to K^{*0}\bar{K}^{*0}$

longitudinal and transverse polarization fractions

A Gegenbauer-moment-independent database

$$
\begin{aligned}
|\mathcal{M}^L_{K^*\phi}|^2 &= M^L_{0K^*\phi} + a^0_{2\phi} M^L_{1K^*\phi} + a^{||}_{1K^*} M^L_{2K^*\phi} + a^{||}_{2K^*} M^L_{3K^*\phi} + (a^0_{2\phi})^2 M^L_{4K^*\phi} \\
&+ (a^{||}_{1K^*})^2 M^L_{5K^*\phi} + (a^{||}_{2K^*})^2 M^L_{6K^*\phi} + (a^0_{2\phi} a^{||}_{1K^*}) M^L_{7K^*\phi} + (a^0_{2\phi} a^{||}_{2K^*}) M^L_{8K^*\phi} \\
&+ (a^{||}_{1K^*} a^{||}_{2K^*}) M^L_{9K^*\phi} + (a^0_{2\phi})^2 a^{||}_{1K^*} M^L_{10K^*\phi} + (a^0_{2\phi})^2 a^{||}_{2K^*} M^L_{11K^*\phi} \\
&+ a^0_{2\phi} (a^{||}_{1K^*})^2 M^L_{12K^*\phi} + a^0_{2\phi} (a^{||}_{2K^*})^2 M^L_{13K^*\phi} + (a^0_{2\phi} a^{||}_{1K^*} a^{||}_{2K^*}) M^L_{14K^*\phi} \\
&+ (a^0_{2\phi} a^{||}_{1K^*})^2 M^L_{15K^*\phi} + (a^0_{2\phi} a^{||}_{2K^*})^2 M^L_{16K^*\phi} + (a^0_{2\phi})^2 a^{||}_{1K^*} a^{||}_{2K^*} M^L_{17K^*\phi},
\end{aligned}
$$

$M^L_{iK^*\phi}$: coefficients involving only Gegenbauer polynomials

Fitted Gegenbauer moments and parameters in the twist-2 and twist-3 two-meson DAs

χ^2/d.o.f=1.6

N_P	$N_{\pi\pi} = 1.05 \pm 0.04$	$N_{K\pi} = 1.48 \pm 0.03$	$N_{KK} = 1.22 \pm 0.03$	
$\pi\pi$	$a^0_{2\rho} = 0.16 \pm 0.10$	$a^s_{2\rho} = -0.11 \pm 0.14$	$a^t_{2\rho} = -0.21 \pm 0.04$	
$K\pi$	$a^{\|\|}_{1K^*} = 0.45 \pm 0.11$	$a^{\|\|}_{2K^*} = -0.75 \pm 0.08$	$a^{\perp}_{1K^*} = 0.61 \pm 0.21$	$a^{\perp}_{2K^*} = 0.45 \pm 0.06$
KK	$a^T_{2\phi} = 0.77 \pm 0.04$	$a^0_{2\phi} = -0.54 \pm 0.14$		

$a^{||}_{2K^{*0}}$=1.19$\pm$0.10 (Phys.Rev.D 104,096014 (2021))

$a^{T}_{2\phi}$=1.48$\pm$0.07 (Phys.Rev.D 105, 093001 (2022))

Discussions

- **Part I: $B \to P_1 P_2$ form factors**

- **Part II: Three-body decays $B \to P_3 (V \to) P_1 P_2$**

- **Part III : Four-body decays $B \to V_1 V_2 \to (P_1 P_2)(P_3 P_4)$**

Part I: $B \to P_1 P_2$ form factors

Testing The universality of the two-meson DAs in the PQCD factorization.

JHEP12(2019)083

$$i\langle P_1(p_1)P_2(p_2)|\bar{q}\gamma^\mu b|B(p_B)\rangle = F_\perp(\omega^2, Q^2)k_\perp^\mu$$

where the vector $k_\perp^\mu$ has been specified in Ref. [75].

TABLE VII: PQCD predictions for the $B_{(s)} \to P_1 P_2$ transition form factors $F_\perp(m_R^2, 0)$ with $R = \rho, K^*, \phi$, whose theoretical errors arise from the same sources as in Table II, but are added in quadrature. The results from light-cone sum rules (LCSR) [69–75] are listed for comparison.

Decay modes	This work	LCSR [69]	LCSR [70]	LCSR [71]	LCSR [72]	LCSR [73]	LCSR [74]	LCSR [75]
$B \to \pi\pi$	39^{+9}_{-10}	29 ± 15	34 ± 11	35 ± 3	43 ± 12	34 ± 4	$\cdots$	$\cdots$
$B \to K\pi$	113^{+17}_{-16}	79 ± 26	93 ± 26	81 ± 10	$\cdots$	98 ± 11	86 ± 43	62 ± 36
$B_s \to K\pi$	62^{+14}_{-10}	$\cdots$	$\cdots$	72 ± 7	$\cdots$	75 ± 9	$\cdots$	$\cdots$
$B_s \to KK$	905^{+128}_{-132}	$\cdots$	$\cdots$	1080 ± 92	$\cdots$	1121 ± 136	$\cdots$	$\cdots$

Part II: Three-body decays $\mathbf{B \to P_3(V \to)P_1P_2}$

- $B \to P_3\rho \to P_3(\pi\pi)$ branching ratios

Channels	Results	Data
$B^+ \to K^+(\rho^0 \to)\pi\pi$	$2.90^{+0.35+0.50+1.38}_{-0.27-0.31-0.71}$	3.7 ± 0.5 †
$B^+ \to K^0(\rho^+ \to)\pi\pi$	$6.77^{+0.89+0.99+3.42}_{-0.82-0.84-1.94}$	$7.3^{+1.0}_{-1.2}$ †
$B^0 \to K^+(\rho^- \to)\pi\pi$	$8.31^{+1.43+1.26+3.89}_{-1.12-1.59-2.16}$	7.0 ± 0.9 †
$B^0 \to K^0(\rho^0 \to)\pi\pi$	$3.53^{+0.47+0.48+0.94}_{-0.37-0.43-0.72}$	3.4 ± 1.1 †
$B^0_s \to K^-(\rho^+ \to)\pi\pi$	$16.9^{+4.8+0.2+0.9}_{-5.9-0.1-2.4}$	$\cdots$
$B^0_s \to \bar{K}^0(\rho^0 \to)\pi\pi$	$0.18^{+0.02+0.02+0.02}_{-0.02-0.02-0.01}$	$\cdots$
$B^+ \to \pi^+(\rho^0 \to)\pi\pi$	$7.05^{+1.72+0.66+0.30}_{-1.26-0.57-0.35}$	8.3 ± 1.2 †
$B^+ \to \pi^0(\rho^+ \to)\pi\pi$	$10.07^{+4.20+0.48+0.14}_{-2.80-0.43-0.10}$	$10.6^{+1.2}_{-1.3}$ †
$B^0 \to \pi^0(\rho^0 \to)\pi\pi$	$0.04^{+0.01+0.01+0.02}_{-0.00-0.01-0.01}$	2.0 ± 0.5
$B^0 \to \pi^\pm(\rho^\mp \to)\pi\pi$	$30.49^{+8.91+2.35+1.26}_{-6.08-2.08-1.03}$	23.0 ± 2.3 †
$B^0_s \to \pi^+(\rho^- \to)\pi\pi$	$0.17^{+0.01+0.04+0.05}_{-0.01-0.03-0.03}$	$\cdots$
$B^0_s \to \pi^-(\rho^+ \to)\pi\pi$	$0.12^{+0.01+0.02+0.00}_{-0.01-0.02-0.01}$	$\cdots$
$B^0_s \to \pi^0(\rho^0 \to)\pi\pi$	$0.16^{+0.01+0.02+0.04}_{-0.01-0.01-0.03}$	$\cdots$

$B \to P_3 K^* \to P_3(K\pi)$ branching ratios

Channels	Results	Data
$B^+ \to K^+(\bar{K}^{*0} \to)K\pi$	$0.59^{+0.19+0.03+0.12}_{-0.13-0.02-0.08}$	0.59 ± 0.08 †
$B^+ \to \bar{K}^0(K^{*+} \to)K\pi$	$0.21^{+0.05+0.03+0.07}_{-0.04-0.03-0.04}$	$\cdots$
$B^0 \to K^\pm(K^{*\mp} \to)K\pi$	$0.51^{+0.01+0.15+0.01}_{-0.02-0.11-0.02}$	< 0.4
$B^0 \to \overset{(-)}{K}{}^0(\overset{(-)}{K}{}^{*0} \to)K\pi$	$0.60^{+0.17+0.04+0.12}_{-0.12-0.04-0.08}$	< 0.96
$B_s^0 \to K^\pm(K^{*\mp} \to)K\pi$	$9.35^{+2.11+0.50+1.92}_{-1.42-0.91-1.24}$	19 ± 5 †
$B_s^0 \to \overset{(-)}{K}{}^0(\overset{(-)}{K}{}^{*0} \to)K\pi$	$9.31^{+2.28+0.60+2.07}_{-1.49-0.48-1.34}$	20 ± 6 †
$B^+ \to \pi^+(K^{*0} \to)K\pi$	$8.89^{+2.60+0.31+2.31}_{-1.90-0.32-1.60}$	10.1 ± 0.8 †
$B^+ \to \pi^0(K^{*+} \to)K\pi$	$5.84^{+1.75+0.23+1.17}_{-1.27-0.21-0.83}$	6.8 ± 0.9 †
$B^0 \to \pi^-(K^{*+} \to)K\pi$	$7.40^{+2.20+0.17+1.60}_{-1.50-0.12-1.20}$	7.5 ± 0.4 †
$B^0 \to \pi^0(K^{*0} \to)K\pi$	$2.80^{+0.76+0.10+0.78}_{-0.51-0.07-0.53}$	3.3 ± 0.6 †
$B_s^0 \to \pi^+(K^{*-} \to)K\pi$	$3.63^{+1.50+0.76+0.17}_{-1.01-0.76-0.14}$	2.9 ± 1.1
$B_s^0 \to \pi^0(\bar{K}^{*0} \to)K\pi$	$0.11^{+0.02+0.01+0.02}_{-0.02-0.01-0.01}$	$\cdots$

Contributions from the subleading Gegenbauer moments in twist-3 Kπ DAs

$$\phi^s_{K\pi}(z,\omega^2) = \frac{3F^{\perp}_{K\pi}(\omega^2)}{2\sqrt{2N_c}}[t(1+a^s_{1K^*}t) - a^s_{1K^*}2z(1-z)]\,,$$

$$\phi^t_{K\pi}(z,\omega^2) = \frac{3F^{\perp}_{K\pi}(\omega^2)}{2\sqrt{2N_c}}t[t + a^t_{1K^*}(3t^2-1)]\,.$$

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TABLE V: Fitted Gegenbauer moments for the twist-2 and twist-3 two-meson DAs with the inclusion of the $a^{s(t)}_{1K^*}$ in the twist-3 $K\pi$ DA $\phi^{s(t)}_{K\pi}$.

	$a^0_{2\rho}$	$a^s_{2\rho}$	$a^t_{2\rho}$			
fit	0.09 ± 0.12	-0.08 ± 0.14	-0.23 ± 0.04			
	$a^{\|\|}_{1K^*}$	$a^{\|\|}_{2K^*}$	$a^s_{1K^*}$	$a^t_{1K^*}$	$a^{\perp}_{1K^*}$	$a^{\perp}_{2K^*}$
fit	0.60 ± 0.16	-0.66 ± 0.11	0.17 ± 0.12	0.02 ± 0.11	0.56 ± 0.23	0.46 ± 0.07
	$a^0_{2\phi}$	$a^T_{2\phi}$	$N_{\pi\pi}$	$N_{K\pi}$	N_{KK}	
fit	-0.57 ± 0.15	0.77 ± 0.04	1.07 ± 0.05	1.45 ± 0.04	1.22 ± 0.03	

Channels	Results	Data
$B^0_s \to K^{\pm}(K^{*\mp} \to)K\pi$	$11.87^{+1.23}_{-0.93}$	19 ± 5
$B^0_s \to \overset{(-)}{K}{}^0(\overset{(-)}{K}{}^{*0} \to)K\pi$	$11.79^{+1.31}_{-1.04}$	20 ± 6

- $B \to P_3\phi \to P_3(KK)$ branching ratios

[illegible]	$[illegible]_{-0.02-0.01-0.01}$	
$B^+ \to K^+(\phi \to)KK$	$9.45^{+3.63+0.36+2.91}_{-2.43-0.22-2.17}$	$8.8^{+0.7}_{-0.6}$ †
$B^0 \to K^0(\phi \to)KK$	$8.63^{+3.30+0.33+2.66}_{-2.28-0.24-2.02}$	7.3 ± 0.7 †
$B_s^0 \to \bar{K}^0(\phi \to)KK$	$0.043^{+0.008+0.013+0.000}_{-0.006-0.002-0.013}$	$\cdots$
$B^+ \to \pi^+(\phi \to)KK$	$0.011^{+0.004+0.002+0.002}_{-0.003-0.002-0.002}$	0.032 ± 0.015
$B^0 \to \pi^0(\phi \to)KK$	$0.005^{+0.002+0.001+0.001}_{-0.001-0.001-0.001}$	< 0.015
$B_s^0 \to \pi^0(\phi \to)KK$	$0.11^{+0.04+0.01+0.01}_{-0.03-0.01-0.01}$	$\cdots$

$B \to P_3(V \to)P_1P_2$ *CP* asymmetries

Modes	Results	Data
$B^+ \to K^+(\rho^0 \to)\pi\pi$	$62.4^{+11.6}_{-13.5}$	16 ± 2
$B^+ \to K^0(\rho^+ \to)\pi\pi$	$9.3^{+1.2}_{-2.2}$	-3 ± 15
$B^0 \to K^+(\rho^- \to)\pi\pi$	$44.1^{+7.2}_{-7.3}$	20 ± 11
$B^0 \to K^0(\rho^0 \to)\pi\pi$	$1.5^{+2.4}_{-2.2}$	-4 ± 20
$B_s^0 \to K^-(\rho^+ \to)\pi\pi$	$19.6^{+4.4}_{-5.2}$	$\cdots$
$B_s^0 \to \bar{K}^0(\rho^0 \to)\pi\pi$	$-36.4^{+26.8}_{-17.3}$	$\cdots$
$B^+ \to \pi^+(\rho^0 \to)\pi\pi$	$-34.1^{+7.3}_{-9.8}$	0.9 ± 1.9
$B^+ \to \pi^0(\rho^+ \to)\pi\pi$	$23.1^{+7.0}_{-6.1}$	2 ± 11
$B^0 \to \pi^+(\rho^- \to)\pi\pi$	$-26.5^{+5.4}_{-5.7}$	-8 ± 8
$B^0 \to \pi^-(\rho^+ \to)\pi\pi$	$9.5^{+2.3}_{-2.6}$	13 ± 6
$B^0 \to \pi^0(\rho^0 \to)\pi\pi$	$16.7^{+23.5}_{-18.9}$	27 ± 24
$B_s^0 \to \pi^+(\rho^- \to)\pi\pi$	$-37.9^{+19.1}_{-13.1}$	$\cdots$
$B_s^0 \to \pi^-(\rho^+ \to)\pi\pi$	$-66.2^{+7.0}_{-4.5}$	$\cdots$
$B_s^0 \to \pi^0(\rho^0 \to)\pi\pi$	$-40.0^{+8.8}_{-4.4}$	$\cdots$
$B^+ \to K^+(\bar{K}^{*0} \to)K\pi$	$-4.0^{+12.2}_{-10.3}$	4 ± 5
$B^+ \to \bar{K}^0(K^{*+} \to)K\pi$	$-61.9^{+4.2}_{-19.2}$	$\cdots$
$B^0 \to K^+(K^{*-} \to)K\pi$	$18.4^{+8.9}_{-6.3}$	$\cdots$
$B^0 \to K^-(K^{*+} \to)K\pi$	$14.3^{+4.6}_{-9.9}$	$\cdots$
$B_s^0 \to K^+(K^{*-} \to)K\pi$	$11.5^{+7.2}_{-8.8}$	$\cdots$
$B_s^0 \to K^-(K^{*+} \to)K\pi$	$-7.3^{+8.0}_{-7.9}$	$\cdots$
$B^+ \to \pi^+(K^{*0} \to)K\pi$	$-2.3^{+1.2}_{-0.5}$	-4 ± 9
$B^+ \to \pi^0(K^{*+} \to)K\pi$	$-3.5^{+4.0}_{-3.9}$	-39 ± 21
$B^0 \to \pi^-(K^{*+} \to)K\pi$	$-15.6^{+5.9}_{-6.4}$	-27 ± 4
$B^0 \to \pi^0(K^{*0} \to)K\pi$	$-13.5^{+1.6}_{-1.8}$	-15 ± 13
$B_s^0 \to \pi^+(K^{*-} \to)K\pi$	$-18.1^{+5.7}_{-5.9}$	$\cdots$
$B_s^0 \to \pi^0(\bar{K}^{*0} \to)K\pi$	$-22.3^{+15.7}_{-17.8}$	$\cdots$
$B^+ \to K^+(\phi \to)KK$	$0.7^{+1.2}_{-1.8}$	2.4 ± 2.8
$B_s^0 \to \pi^0(\phi \to)KK$	$31.2^{+3.7}_{-3.7}$	$\cdots$

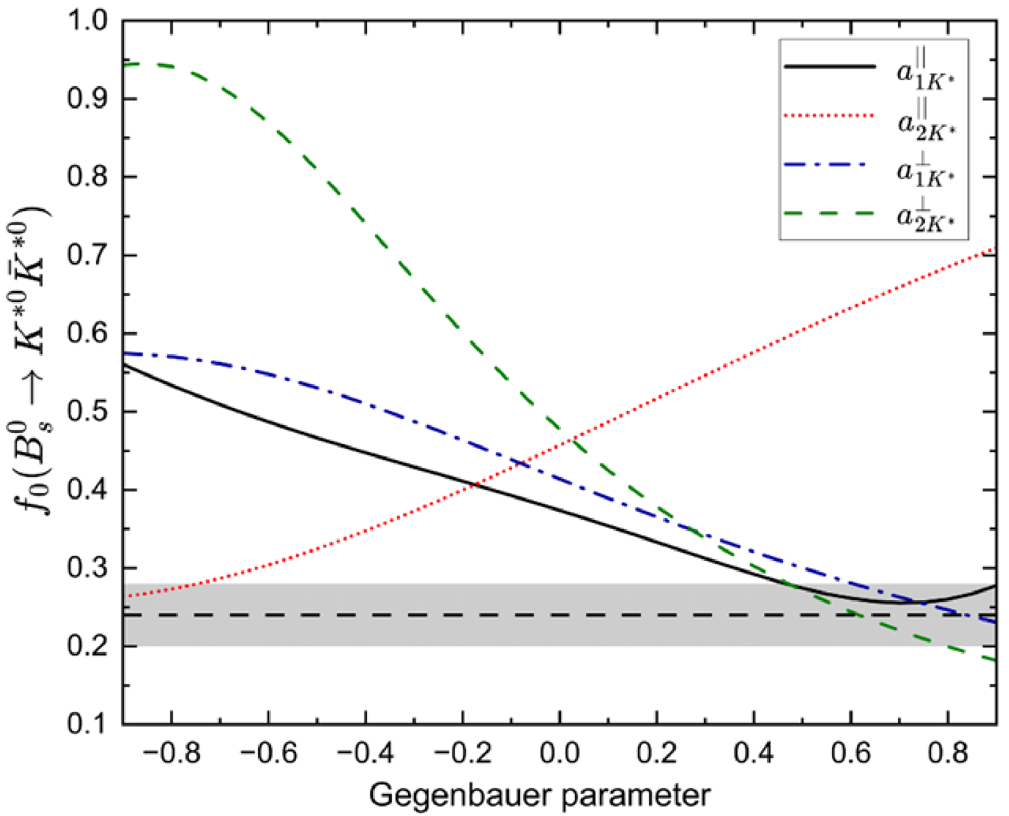
Part III : Four-body decays $B \to V_1 V_2 \to (P_1 P_2)(P_3 P_4)$

- Branching ratios and polarization fractions

Channels	Results $\mathcal{B}(10^{-6})$	Results $f_0(\%)$	Results $f_\perp(\%)$	Data $\mathcal{B}(10^{-6})$	Data $f_0(\%)$	Data $f_\perp(\%)$
$B^+ \to \rho^+\rho^0$	$12.7^{+4.5}_{-3.3}$	$97.7^{+0.8}_{-0.9}$	$1.3^{+0.5}_{-0.4}$	24 ± 1.9	95.0 ± 1.6	$\cdots$
$B^0 \to \rho^+\rho^-$	$27.0^{+10.7}_{-7.5}$	$92.2^{+3.6}_{-4.3}$	$4.5^{+2.3}_{-1.9}$	$27.7 \pm 1.9^{\dagger}$	$99.0^{+2.1}_{-1.9}{}^{\dagger}$	$\cdots$
$B^0 \to \rho^0\rho^0$	$0.35^{+0.12}_{-0.07}$	$37.9^{+9.4}_{-3.2}$	$33.9^{+2.6}_{-5.7}$	0.96 ± 0.15	71^{+8}_{-9}	$\cdots$
$B_s^0 \to \rho^+\rho^-$	$1.35^{+0.84}_{-0.43}$	$99.4^{+0.6}_{-0.7}$	$0.3^{+0.1}_{-0.2}$	$\cdots$	$\cdots$	$\cdots$
$B_s^0 \to \rho^0\rho^0$	$0.68^{+0.42}_{-0.22}$	$99.4^{+0.6}_{-0.7}$	$0.3^{+0.1}_{-0.2}$	< 320	$\cdots$	$\cdots$
$B^+ \to \rho^0 K^{*+}$	$6.92^{+2.06}_{-3.21}$	$57.8^{+15.1}_{-12.8}$	$16.9^{+6.2}_{-6.9}$	$4.6 \pm 1.1^{\dagger}$	$78 \pm 12^{\dagger}$	$\cdots$
$B^+ \to \rho^+ K^{*0}$	$11.1^{+5.81}_{-3.82}$	$48.7^{+17.5}_{-13.4}$	$25.7^{+7.6}_{-8.5}$	$9.2 \pm 1.5^{\dagger}$	$48 \pm 8^{\dagger}$	$\cdots$
$B^0 \to \rho^- K^{*+}$	$9.91^{+4.84}_{-3.29}$	$48.0^{+16.5}_{-12.9}$	$26.2^{+6.3}_{-8.0}$	10.3 ± 2.6	38 ± 13	$\cdots$
$B^0 \to \rho^0 K^{*0}$	$4.35^{+2.40}_{-1.59}$	$33.2^{+10.9}_{-9.9}$	$41.2^{+4.6}_{-7.8}$	3.9 ± 1.3	17.3 ± 2.6	40 ± 4
$B_s^0 \to \rho^0 \bar{K}^{*0}$	$0.35^{+0.11}_{-0.06}$	$59.4^{+9.4}_{-9.6}$	$21.5^{+4.9}_{-4.8}$	< 767	$\cdots$	$\cdots$
$B_s^0 \to \rho^+ K^{*-}$	$12.1^{+4.4}_{-3.9}$	$89.4^{+1.5}_{-2.3}$	$5.3^{+1.2}_{-0.9}$	$\cdots$	$\cdots$	$\cdots$
$B^+ \to \rho^+\phi$	$0.025^{+0.013}_{-0.009}$	$87.4^{+4.9}_{-7.2}$	$5.8^{+2.9}_{-1.9}$	< 3.0	$\cdots$	$\cdots$
$B^0 \to \rho^0\phi$	$0.012^{+0.006}_{-0.004}$	$87.4^{+4.9}_{-7.2}$	$5.8^{+2.9}_{-1.9}$	< 0.33	$\cdots$	$\cdots$
$B_s^0 \to \rho^0\phi$	$0.20^{+0.09}_{-0.06}$	$82.9^{+2.7}_{-1.9}$	$8.9^{+1.1}_{-1.4}$	0.27 ± 0.08	$\cdots$	$\cdots$
$B^0 \to \phi\phi$	$0.015^{+0.004}_{-0.003}$	$98.6^{+0.7}_{-2.0}$	$0.01^{+0.01}_{-0.00}$	< 0.027	$\cdots$	$\cdots$
$B_s^0 \to \phi\phi$	$16.6^{+6.6}_{-4.8}$	$38.7^{+10.6}_{-10.3}$	$30.9^{+5.1}_{-5.5}$	$18.5 \pm 1.4^{\dagger}$	$37.9 \pm 0.8^{\dagger}$	$31.0 \pm 0.6^{\dagger}$
$B^+ \to \phi K^{*+}$	$11.5^{+4.4}_{-3.9}$	$54.6^{+4.6}_{-9.1}$	$23.1^{+4.5}_{-2.2}$	$10 \pm 2^{\dagger}$	$55.0 \pm 5.0^{\dagger}$	$20.0 \pm 5.0^{\dagger}$
$B^0 \to \phi K^{*0}$	$10.4^{+4.6}_{-3.5}$	$51.9^{+6.3}_{-8.7}$	$24.5^{+4.2}_{-3.1}$	$10.00 \pm 0.50^{\dagger}$	$49.7 \pm 1.7^{\dagger}$	$22.4 \pm 1.5^{\dagger}$
$B_s^0 \to \phi \bar{K}^{*0}$	$0.29^{+0.17}_{-0.10}$	$62.3^{+11.9}_{-13.2}$	$25.2^{+8.8}_{-8.2}$	1.14 ± 0.30	51 ± 17	$\cdots$
$B^+ \to K^{*+}\bar{K}^{*0}$	$0.71^{+0.34}_{-0.16}$	$83.5^{+5.0}_{-3.8}$	$8.5^{+1.5}_{-3.3}$	0.91 ± 0.29	82^{+15}_{-21}	$\cdots$
$B^0 \to K^{*+}K^{*-}$	$1.24^{+0.38}_{-0.32}$	~ 100	~ 0	< 2.0	$\cdots$	$\cdots$
$B^0 \to K^{*0}\bar{K}^{*0}$	$0.60^{+0.22}_{-0.14}$	$81.1^{+3.2}_{-5.2}$	$9.6^{+2.7}_{-1.7}$	$0.83 \pm 0.24^{\dagger}$	$74 \pm 5^{\dagger}$	$\cdots$
$B_s^0 \to K^{*+}K^{*-}$	$13.7^{+5.4}_{-3.7}$	$32.3^{+10.5}_{-10.6}$	$33.9^{+5.3}_{-5.2}$	$\cdots$	$\cdots$	$\cdots$
$B_s^0 \to K^{*0}\bar{K}^{*0}$	$13.2^{+5.2}_{-3.7}$	$28.2^{+8.8}_{-9.5}$	$35.7^{+4.7}_{-4.2}$	$11.1 \pm 2.7^{\dagger}$	$24 \pm 4^{\dagger}$	$38 \pm 12^{\dagger}$

longitudinal polarization fraction of the $B_s^0 \to K^{*0}\overline{K}^{*0}$ decay

Channels	Results			Data		
	$\mathcal{B}(10^{-6})$	$f_0(\%)$	$f_\perp(\%)$	$\mathcal{B}(10^{-6})$	$f_0(\%)$	$f_\perp(\%)$
$B_s^0 \to K^{*0}\bar{K}^{*0}$	$13.2^{+5.2}_{-3.7}$	$28.2^{+8.8}_{-9.5}$	$35.7^{+4.7}_{-4.2}$	11.1 ± 2.7 †	24 ± 4 †	38 ± 12 †



PQCD (JHEP05(2021)082)

$$f_0(B_s^0 \to K^{*0}\overline{K}^{*0}) = (63.6^{+2.7+3.3+1.0}_{-4.2-3.9-1.0})\%$$

$$a^{||}_{1K^*} = a^{\perp}_{1K^*} = 0.31 \pm 0.16$$

$$a^{||}_{2K^*} = a^{\perp}_{2K^*} = 1.188 \pm 0.098$$

A new observable $L_{K^*\bar{K}^*}$ defined as the ratio of the longitudinal branching ratios of $B_s^0 \to K^{*0}\bar{K}^{*0}$ versus $B^0 \to K^{*0}\bar{K}^{*0}$

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$$L_{K^{*0}\bar{K}^{*0}} = \frac{\mathcal{B}(B_s^0 \to K^{*0}\bar{K}^{*0})\boxed{g(B^0 \to K^{*0}\bar{K}^{*0})}\boxed{f_L(B_s^0 \to K^{*0}\bar{K}^{*0})}}{\mathcal{B}(B^0 \to K^{*0}\bar{K}^{*0})\boxed{g(B_s^0 \to K^{*0}\bar{K}^{*0})}\boxed{f_L(B^0 \to K^{*0}\bar{K}^{*0})}},$$

phase-space factor

longitudinal polarization fractions

$L_{K^*\bar{K}^*}$	exp	PQCD (2209.13389)	QCDF (2011.07867)	This work
	4.43 ± 0.92	$12.7^{+5.6}_{-3.2}$	$19.5^{+9.3}_{-6.8}$	$7.7^{+4.9}_{-3.8}$

- Direct CP asymmetries

Modes	$\mathcal{A}^0_{\rm CP}$	$\mathcal{A}^{\parallel}_{\rm CP}$	$\mathcal{A}^{\perp}_{\rm CP}$	$\mathcal{A}^{\rm dir}_{\rm CP}$
$B^+ \to \rho^+\rho^0 \to (\pi^+\pi^0)(\pi^+\pi^-)$	$0.4^{+0.2}_{-0.1}(97.7\%)$	$-0.1^{+0.4}_{-0.6}(1.0\%)$	$0.5^{+0.2}_{-0.6}(1.3\%)$	$0.4^{+0.2}_{-0.1}$
Data	$\cdots$	$\cdots$	$\cdots$	-5 ± 5
$B^0 \to \rho^+\rho^- \to (\pi^+\pi^0)(\pi^-\pi^0)$	$-3.7^{+0.9}_{-2.0}(92.2\%)$	$43.4^{+11.7}_{-17.7}(3.3\%)$	$38.4^{+12.0}_{-16.9}(4.5\%)$	$-0.3^{+2.9}_{-2.4}$
Data	$\cdots$	$\cdots$	$\cdots$	0 ± 9
$B^0 \to \rho^0\rho^0 \to (\pi^+\pi^-)(\pi^+\pi^-)$	$61.7^{+12.3}_{-14.8}(37.9\%)$	$64.9^{+6.1}_{-8.4}(28.2\%)$	$76.9^{+8.8}_{-9.0}(33.9\%)$	$67.8^{+7.1}_{-9.9}$
Data	$\cdots$	$\cdots$	$\cdots$	20 ± 90
$B^0_s \to \rho^+\rho^- \to (\pi^+\pi^0)(\pi^-\pi^0)$	$5.1^{+4.3}_{-2.5}(99.4\%)$	$4.3^{+4.6}_{-3.9}(0.3\%)$	$5.9^{+8.6}_{-5.3}(0.3\%)$	$5.1^{+4.2}_{-2.6}$
$B^0_s \to \rho^0\rho^0 \to (\pi^+\pi^-)(\pi^+\pi^-)$	$5.1^{+4.3}_{-2.5}(99.4\%)$	$4.3^{+4.6}_{-3.9}(0.3\%)$	$5.9^{+8.6}_{-5.3}(0.3\%)$	$5.1^{+4.2}_{-2.6}$
$B^+ \to \rho^+K^{*0} \to (\pi^+\pi^0)(K^+\pi^-)$	$-0.2^{+1.9}_{-3.4}(48.7\%)$	$1.6^{+0.9}_{-1.3}(25.6\%)$	$1.7^{+0.2}_{-0.6}(25.7\%)$	$0.8^{+0.8}_{-1.8}$
Data	$\cdots$	$\cdots$	$\cdots$	-1 ± 16
$B^+ \to \rho^0K^{*+} \to (\pi^+\pi^-)(K^0\pi^+)$	$32.8^{+2.7}_{-8.5}(57.8\%)$	$0.8^{+4.4}_{-4.6}(25.3\%)$	$-56.2^{+9.7}_{-9.4}(16.9\%)$	$9.6^{+7.1}_{-7.7}$
Data	$\cdots$	$\cdots$	$\cdots$	31 ± 13
$B^0 \to \rho^0K^{*0} \to (\pi^+\pi^-)(K^+\pi^-)$	$0.1^{+5.3}_{-7.3}(33.2\%)$	$-34.5^{+9.2}_{-12.2}(25.6\%)$	$12.8^{+0.8}_{-1.4}(41.2\%)$	$-3.5^{+2.2}_{-2.7}$
Data	$\cdots$	$\cdots$	$\cdots$	-6 ± 9
$B^0 \to \rho^-K^{*+} \to (\pi^-\pi^0)(K^0\pi^+)$	$57.2^{+5.8}_{-10.1}(48.0\%)$	$-26.5^{+5.3}_{-4.7}(25.8\%)$	$-30.7^{+4.6}_{-4.9}(26.2\%)$	$12.6^{+12.0}_{-9.5}$
Data	$\cdots$	$\cdots$	$\cdots$	21 ± 15
$B^0_s \to \rho^+K^{*-} \to (\pi^+\pi^0)(\bar{K}^0\pi^-)$	$-14.2^{+3.4}_{-3.8}(89.4\%)$	$73.7^{+12.5}_{-15.4}(5.3\%)$	$75.2^{+11.5}_{-14.1}(5.3\%)$	$-4.7^{+3.1}_{-2.9}$
$B^0_s \to \rho^0\bar{K}^{*0} \to (\pi^+\pi^-)(K^-\pi^+)$	$21.2^{+19.6}_{-16.5}(59.4\%)$	$73.2^{+15.4}_{-25.0}(19.1\%)$	$81.8^{+11.6}_{-21.8}(21.5\%)$	$44.2^{+11.5}_{-13.9}$
$B^0_s \to \rho^0\phi \to (\pi^+\pi^-)(K^+K^-)$	$-2.4^{+9.0}_{-6.9}(82.9\%)$	$-18.3^{+5.2}_{-4.2}(8.2\%)$	$-16.8^{+4.0}_{-2.9}(8.9\%)$	$-5.0^{+8.2}_{-5.8}$
$B^+ \to K^{*+}\phi \to (K^0\pi^+)(K^+K^-)$	$-5.7^{+11.2}_{-3.7}(54.6\%)$	$2.8^{+0.3}_{-2.9}(22.3\%)$	$-2.3^{+1.3}_{-1.0}(23.1\%)$	$-3.3^{+7.6}_{-1.6}$
$B^+ \to K^{*+}\bar{K}^{*0} \to (K^0\pi^+)(K^+\pi^-)$	$-21.5^{+8.7}_{-10.8}(83.5\%)$	$-9.0^{+1.4}_{-1.3}(8.0\%)$	$7.9^{+1.7}_{-1.5}(8.5\%)$	$-19.4^{+7.6}_{-9.2}$
$B^0 \to K^{*+}K^{*-} \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$19.1^{+2.8}_{-1.5}(\sim 100\%)$	$-29.5^{+17.3}_{-16.0}(\sim 0)$	$10.2^{+6.7}_{-5.1}(\sim 0)$	$19.1^{+2.8}_{-1.5}$
$B^0_s \to K^{*+}K^{*-} \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$33.1^{+7.2}_{-8.2}(32.3\%)$	$-17.4^{+4.5}_{-5.4}(33.8\%)$	$-16.6^{+4.6}_{-5.2}(33.9\%)$	$-0.9^{+1.6}_{-2.4}$

The total direct CP asymmetry can be well approximated by the weighted sum of the three asymmetries

$$\mathcal{A}_{CP}^{\rm dir} \approx f_0 \mathcal{A}_{CP}^0 + f_{||}\mathcal{A}_{CP}^{||} + f_{\perp}\mathcal{A}_{CP}^{\perp},$$

Modes	$\mathcal{A}_{\rm CP}^0$	$\mathcal{A}_{\rm CP}^{\parallel}$	$\mathcal{A}_{\rm CP}^{\perp}$	$\mathcal{A}_{\rm CP}^{\rm dir}$
$B^0 \to \rho^0 K^{*0} \to (\pi^+\pi^-)(K^+\pi^-)$	$0.1^{+5.3}_{-7.3}(33.2\%)$	$-34.5^{+9.2}_{-12.2}(25.6\%)$	$12.8^{+0.8}_{-1.4}(41.2\%)$	$-3.5^{+2.2}_{-2.7}$
Data	$\cdots$	$\cdots$	$\cdots$	-6 ± 9
$B^0 \to \rho^- K^{*+} \to (\pi^-\pi^0)(K^0\pi^+)$	$57.2^{+5.8}_{-10.1}(48.0\%)$	$-26.5^{+5.3}_{-4.7}(25.8\%)$	$-30.7^{+4.6}_{-4.9}(26.2\%)$	$12.6^{+12.0}_{-9.5}$
Data	$\cdots$	$\cdots$	$\cdots$	21 ± 15
$B_s^0 \to \rho^+ K^{*-} \to (\pi^+\pi^0)(\bar{K}^0\pi^-)$	$-14.2^{+3.4}_{-3.8}(89.4\%)$	$73.7^{+12.5}_{-15.4}(5.3\%)$	$75.2^{+11.5}_{-14.1}(5.3\%)$	$-4.7^{+3.1}_{-2.9}$
$B_s^0 \to \rho^0 \bar{K}^{*0} \to (\pi^+\pi^-)(K^-\pi^+)$	$21.2^{+19.6}_{-16.5}(59.4\%)$	$73.2^{+15.4}_{-25.0}(19.1\%)$	$81.8^{+11.6}_{-21.8}(21.5\%)$	$44.2^{+11.5}_{-13.9}$
$B_s^0 \to \rho^0 \phi \to (\pi^+\pi^-)(K^+K^-)$	$-2.4^{+9.0}_{-6.9}(82.9\%)$	$-18.3^{+5.2}_{-4.2}(8.2\%)$	$-16.8^{+4.0}_{-2.9}(8.9\%)$	$-5.0^{+8.2}_{-5.8}$
$B^+ \to K^{*+}\phi \to (K^0\pi^+)(K^+K^-)$	$-5.7^{+11.2}_{-3.7}(54.6\%)$	$2.8^{+0.3}_{-2.9}(22.3\%)$	$-2.3^{+1.3}_{-1.0}(23.1\%)$	$-3.3^{+7.6}_{-1.6}$
$B^+ \to K^{*+}\bar{K}^{*0} \to (K^0\pi^+)(K^+\pi^-)$	$-21.5^{+8.7}_{-10.8}(83.5\%)$	$-9.0^{+1.4}_{-1.3}(8.0\%)$	$7.9^{+1.7}_{-1.5}(8.5\%)$	$-19.4^{+7.6}_{-9.2}$
$B^0 \to K^{*+}K^{*-} \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$19.1^{+2.8}_{-1.5}(\sim 100\%)$	$-29.5^{+17.3}_{-16.0}(\sim 0)$	$10.2^{+6.7}_{-5.1}(\sim 0)$	$19.1^{+2.8}_{-1.5}$
$B_s^0 \to K^{*+}K^{*-} \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$33.1^{+7.2}_{-8.2}(32.3\%)$	$-17.4^{+4.5}_{-5.4}(33.8\%)$	$-16.6^{+4.6}_{-5.2}(33.9\%)$	$-0.9^{+1.6}_{-2.4}$

04 Summary and outlook

- We have improved the PQCD formalism of multi-body charmless hadronic B meson decays by resolving the possible inconsistency in the parametrization for P-wave resonances in two-meson DAs.

- The determination of the Gegenbauer moments in the two-meson DAs is then updated in the global fit of the improved PQCD factorization formulas at leading order to available data for branching ratios and polarization fractions of three- and four-body B decays. The convergence of the Gegenbauer expansion of the resultant two-meson DAs is manifest.

- The precision of the two-meson DAs does play a crucial role in accounting for the data, especially for the unexpected low longitudinal polarization fraction of the $B_s^0 \to K^{*0}\overline{K}^{*0}$ decay.

- The precision of the two-meson DAs can be further enhanced systematically, when higher-order and/or higher-power corrections to multi-body hadronic B-meson decays are taken into account. If a high-precision global investigation discloses notable tensions between theoretical results and experimental data, it may hint that NP effects are inevitable.

Thank you

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