



SIDIS in the Target Fragmentation Region

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JHEP 08 (2024) 227,
PRD 108 (2023) 094015.



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➤ Twist-3 contributions for TFR SIDIS

➤ One-loop contributions for TFR SIDIS

➤ Connecting NEEC with fracture function

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➤ Introduction

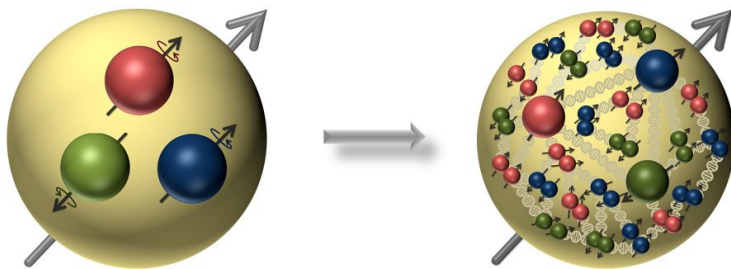
➤ Twist-3 contributions for TFR SIDIS

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■ Probing nucleon structure

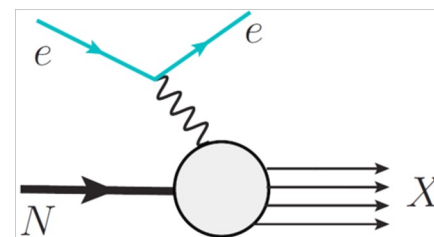


Images taken from Front. Phys. 16(6), 64701

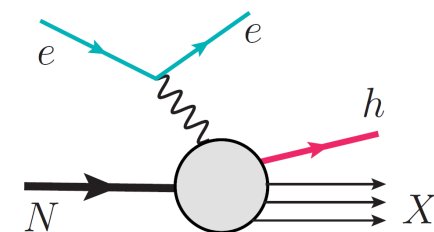
Why nucleon structure important?

- Fundamental components of matter
- Properties of strong interaction
- Inputs for describing high energy reactions
- Prerequisite for new physics researches
-

Lepton-nucleon deep inelastic scattering (DIS)



Inclusive DIS

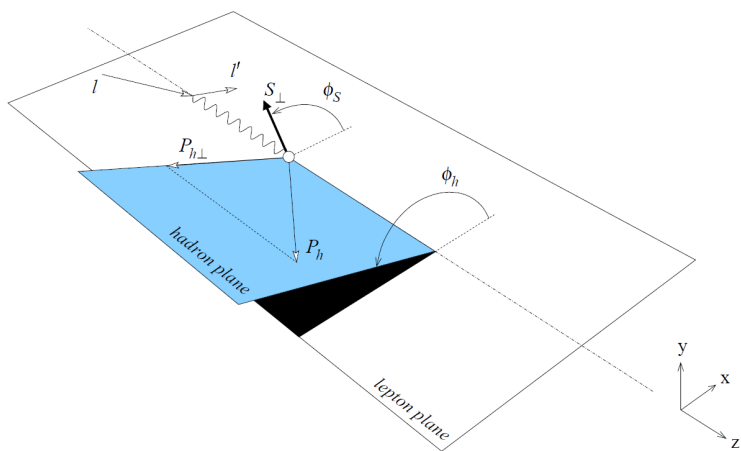


Semi-inclusive DIS



- Three-dimensional imaging of the nucleon
- Accessing fragmentation functions
- Azimuthal and/or spin asymmetries
- Flavor separation
-

■ Semi-inclusive DIS (SIDIS)

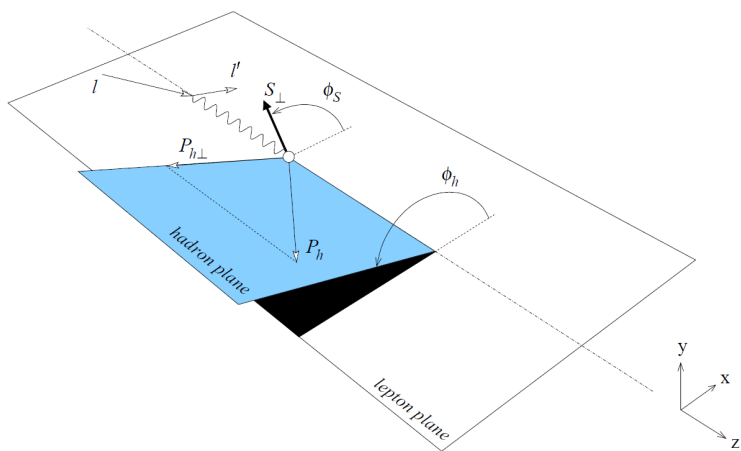


A. Bacchetta et al., JHEP 02, 093 (2007)

18 structure functions for SIDIS
with polarized lepton and nucleon

$$\begin{aligned} \frac{d\sigma}{dx dy d\psi dz d\phi_h dP_{h\perp}^2} = & \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x} \right) \\ & \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} \right. \\ & + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} \\ & + S_{\parallel} \left[\sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\ & + S_{\parallel} \lambda_e \left[\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\ & + |S_{\perp}| \left[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \\ & + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\ & + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \left. \right] \\ & + |S_{\perp}| \lambda_e \left[\sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} \right. \\ & \left. \left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \right\} \end{aligned}$$

■ Semi-inclusive DIS (SIDIS)



A. Bacchetta et al., JHEP 02, 093 (2007)

$$P_{h\perp} \sim M \ll Q$$

TMD factorization applies.

Structure functions $\sim$ TMD PDFs $\otimes$ TMD FFs

$$F_{UU,T} \sim f_1 \otimes D_1$$

Number density

$$F_{LL} \sim g_{1L} \otimes D_1$$

Helicity

$$F_{UT,T}^{\sin(\phi_h - \phi_S)} \sim f_{1T}^\perp \otimes D_1$$

Sivers

$$F_{LT}^{\cos(\phi_h - \phi_S)} \sim g_{1T} \otimes D_1$$

Worm-gear L-T

$$F_{UT}^{\sin(\phi_h + \phi_S)} \sim h_1 \otimes H_1^\perp$$

Transversity

$$F_{UU}^{\cos 2\phi_h} \sim h_1^\perp \otimes H_1^\perp$$

Boer-Mulders

$$F_{UL}^{\sin 2\phi_h} \sim h_{1L}^\perp \otimes H_1^\perp$$

Worm-gear T-L

$$F_{UT}^{\sin(3\phi_h - \phi_S)} \sim h_{1T}^\perp \otimes H_1^\perp$$

Pretzelosity

8 leading twist structure functions



The diagram shows the (q_T, y) plane with several regions of QCD factorization:

- Collinear factorization** (yellow box): Located at high q_T , where the hard scale is $\frac{m}{Q}, \frac{m}{q_T}$.
- Hard Transverse Momentum - Current Region** (pink region): Located at high q_T and moderate y , where the hard scale is $\frac{m}{Q}, \frac{q_T}{Q}$.
- TMD - Current Region** (red region): Located at low q_T and moderate y , where the hard scale is $e^{y_{B,b}}, \frac{m}{Q}, \frac{q_T}{Q}$.
- Soft/Central Region** (green region): Located at low q_T and low y , where the hard scale is $\frac{m}{Q}, \frac{q_T}{Q}$.
- Target Region** (blue region): Located at low q_T and high y , where the hard scale is $e^{-y_{B,b}}, \frac{m}{Q}, \frac{q_T}{Q}$.

A red question mark is present in the Target Region. A dashed vertical line is labeled y_P . A horizontal axis is labeled y . A vertical axis is labeled q_T . A blue label $P_{h\perp} =$ is on the right. A diagram on the left shows a scattering process with an incoming electron e , an outgoing electron e' , a hard scale P_h , and a target P .

M. Boggione et al. JHEP 10 (2019) 122

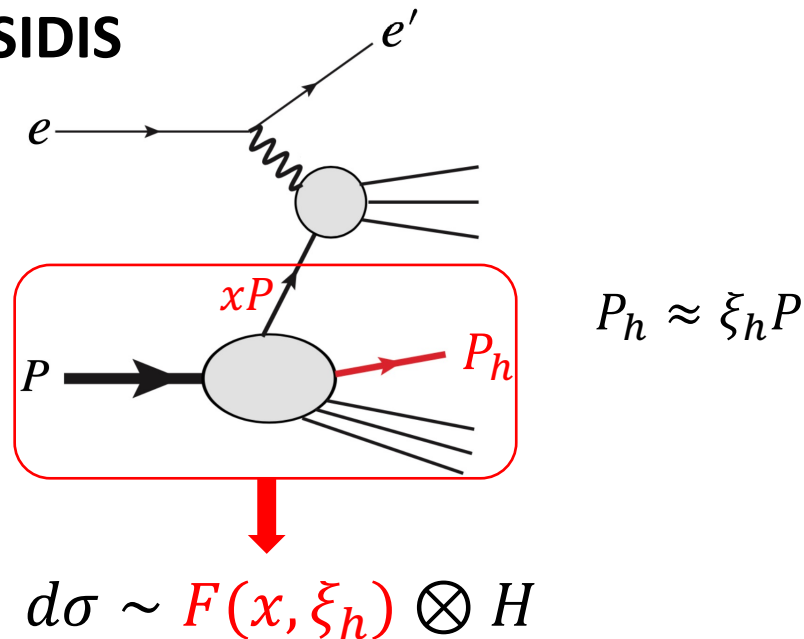
■ Fracture functions and target fragmentation region (TFR) SIDIS

- **Fracture function: $F(x, \xi_h)$**

Parton distributions in the presence of an almost collinear particle observed in the final state.

(conditional probability, hybrid of PDFs and FFs)

L. Trentadue and G. Veneziano, Phys. Lett. B323, 201 (1994)



- TFR SIDIS with hadron transverse momentum

$$\frac{d\sigma^{\text{TFR}}}{dx_B dy d\zeta d^2\mathbf{P}_{h\perp} d\phi_S} = \frac{2\alpha_{\text{em}}^2}{Q^2 y} \left\{ \left(1 - y + \frac{y^2}{2}\right) \sum_a e_a^2 \left[M(x_B, \zeta, \mathbf{P}_{h\perp}^2) - |\mathbf{S}_{\perp}| \frac{|\mathbf{P}_{h\perp}|}{m_h} M_T^h(x_B, \zeta, \mathbf{P}_{h\perp}^2) \sin(\phi_h - \phi_S) \right] \right. \\ \left. + \lambda_l y \left(1 - \frac{y}{2}\right) \sum_a e_a^2 \left[S_{\parallel} \Delta M_L(x_B, \zeta, \mathbf{P}_{h\perp}^2) + |\mathbf{S}_{\perp}| \frac{|\mathbf{P}_{h\perp}|}{m_h} \Delta M_T^h(x_B, \zeta, \mathbf{P}_{h\perp}^2) \cos(\phi_h - \phi_S) \right] \right\}.$$

4 structure functions at twist-2

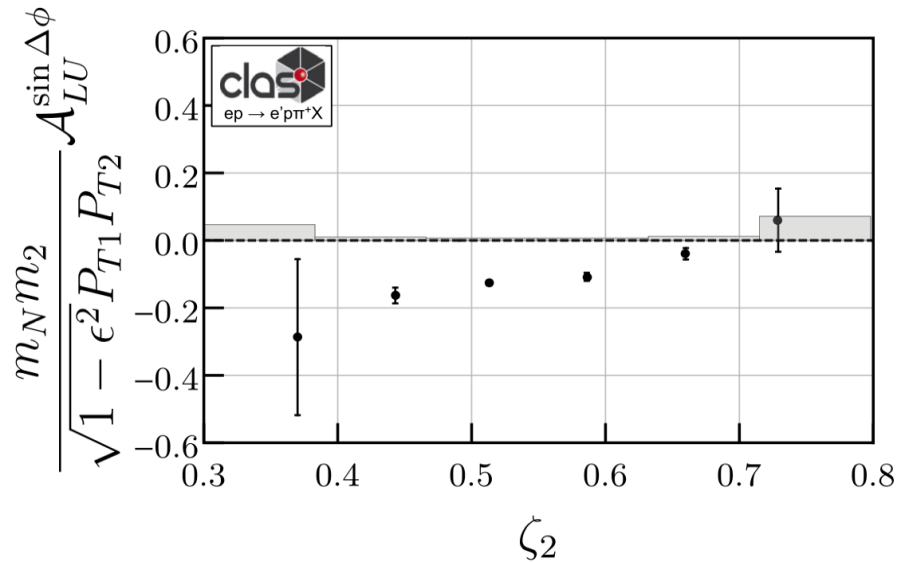
Anselmino, Barone, and Kotzinian, Phys. Lett. B699, 108 (2011); Phys. Lett. B706, 46 (2011).



Introduction

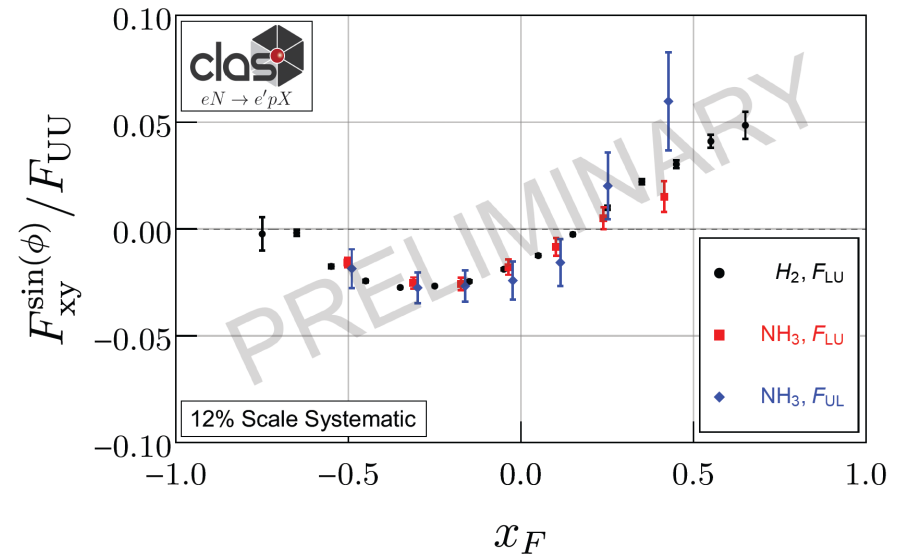
■ SIDIS in the target fragmentation region (TFR)

- Recent experimental studies on TFR hadron production



Beam-spin asymmetry for back-to-back dihadron production

H. Avakian et al., PRL 130, 022501 (2023)



Structure functions from CFR to TFR

A. Accardi et al., EPJA (2024) 60:173



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➤ **Twist-3 contributions for TFR SIDIS**

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■ Kinematics

$$e(l, \lambda_e) + h_A(P, S) \rightarrow e(l') + h(P_h) + X$$

$$\frac{d\sigma}{dx_B dy d\xi_h d\psi d^2 P_{h\perp}} = \frac{\alpha^2 y}{4\xi_h Q^4} L_{\mu\nu}(l, \lambda_e, l') W^{\mu\nu}(q, P, S, P_h)$$

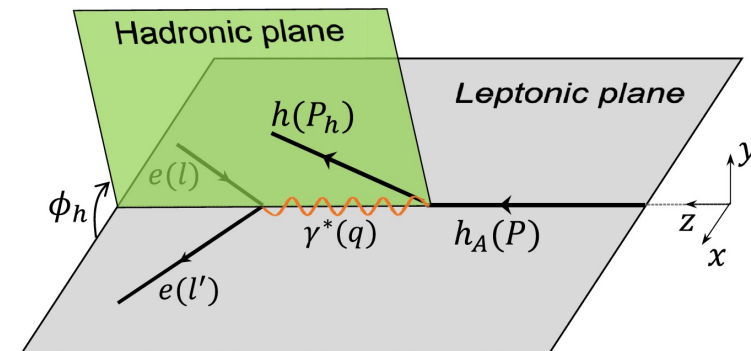
$$L^{\mu\nu}(l, \lambda_e, l') = 2(l^\mu l'^\nu + l^\nu l'^\mu - l \cdot l' g^{\mu\nu}) + 2i\lambda_e \epsilon^{\mu\nu\rho\sigma} l_\rho l'_\sigma$$

$$W^{\mu\nu}(q, P, S, P_h) = \sum_X \int \frac{d^4 x}{(2\pi)^4} e^{iq \cdot x} \langle S; h_A | J^\mu(x) | hX \rangle \langle Xh | J^\nu(0) | h_A; S \rangle$$

$$P^\mu \approx (P^+, 0, 0, 0) \quad l^\mu = \left(\frac{1-y}{y} x_B P^+, \frac{Q^2}{2x_B y P^+}, \frac{Q\sqrt{1-y}}{y}, 0 \right)$$

$$P_h^\mu = (P_h^+, P_h^-, \vec{P}_{h\perp}) \quad q^\mu = \left(-x_B P^+, \frac{Q^2}{2x_B P^+}, 0, 0 \right)$$

$$S^\mu = \left(\frac{S_L P^+}{M}, -\frac{S_L M}{2P^+}, \vec{S}_\perp \right)$$



$$Q^2 = -q^2,$$

$$x_B = \frac{Q^2}{2P \cdot q},$$

$$y = \frac{P \cdot q}{P \cdot l},$$

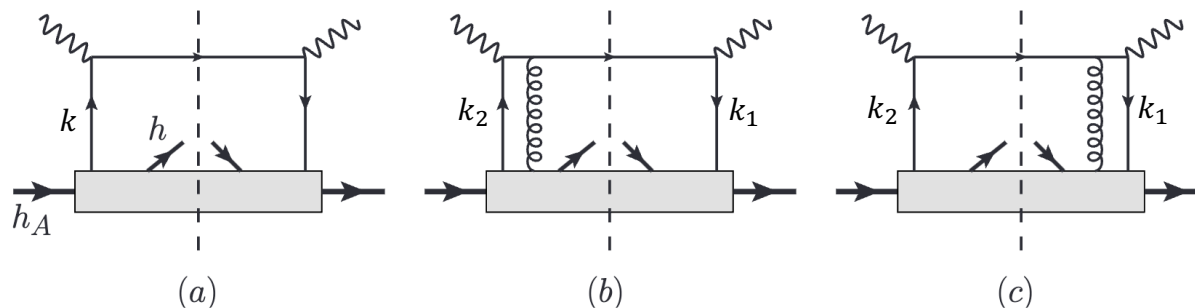
$$z_h = \frac{P \cdot P_h}{P \cdot q} \ll 1.$$

$$\xi_h = \frac{P_h \cdot q}{P \cdot q} \approx \frac{P_h^+}{P^+},$$

Twist-3 contributions for TFR SIDIS

■ Hadronic tensor

Hadronic tensor at the tree level



....

Multiple gluon scattering in the final state

$$W^{\mu\nu}|_a = \int \frac{d^3k}{(2\pi)^3} \left[(\gamma^\mu (\not{k} + \not{q}) \gamma^\nu)_{ij} 2\pi\delta((k+q)^2) \right] \sum_X \int \frac{d^3\eta}{(2\pi)^4} e^{-ik\cdot\eta} \langle h_A | \bar{\psi}_i(\eta) | hX \rangle \langle Xh | \psi_j(0) | h_A \rangle,$$

$$W^{\mu\nu}|_b = \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \left[\left(\gamma^\mu (\not{k}_1 + \not{q}) \gamma_\alpha \frac{i(\not{k}_2 + \not{q})}{(k_2 + q)^2 + i\epsilon} \gamma^\nu \right)_{ij} 2\pi\delta((k_1 + q)^2) \right] \\ \times (-ig_s) \sum_X \int \frac{d^3\eta d^3\eta_1}{(2\pi)^4} e^{-ik_1\cdot\eta} e^{i(k_1 - k_2)\cdot\eta_1} \langle h_A | \bar{\psi}_i(\eta) | hX \rangle \langle Xh | G^\alpha(\eta_1) \psi_j(0) | h_A \rangle,$$

$$k_i \sim Q(1, \lambda^2, \lambda), \text{ with } \lambda \sim \frac{\Lambda_{QCD}}{Q}$$

$$W^{\mu\nu}|_c = \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \left[\left(\gamma^\mu \frac{i(\not{k}_1 + \not{q})}{(k_1 + q)^2 - i\epsilon} \gamma_\alpha (\not{k}_2 + \not{q}) \gamma^\nu \right)_{ij} 2\pi\delta((k_2 + q)^2) \right] \\ \times (-ig_s) \sum_X \int \frac{d^3\eta d^3\eta_1}{(2\pi)^4} e^{-ik_1\cdot\eta} e^{i(k_1 - k_2)\cdot\eta_1} \langle h_A | \bar{\psi}_i(\eta) G^\alpha(\eta_1) | hX \rangle \langle Xh | \psi_j(0) | h_A \rangle,$$

Collinear expansion
for the hard parts



Twist-3 contributions for TFR SIDIS

■ Hadronic tensor

$$W^{\mu\nu} = \xi_h (\gamma^\mu \gamma^+ \gamma^\nu)_{ij} \mathcal{M}_{ji}(x_B) + \left[\frac{-i\xi_h}{2q^-} (\gamma^\mu \gamma^+ \gamma_\perp^\alpha \gamma^- \gamma^\nu)_{ij} \mathcal{M}_{\partial,ji}^\alpha(x_B) + (\mu \leftrightarrow \nu)^* \right] + \left\{ \frac{-i\xi_h}{2q^-} (\gamma^\mu \gamma^+ \gamma_\perp^\alpha \gamma^- \gamma^\nu)_{ij} \int dx_2 \left[P \frac{1}{x_2 - x_B} - i\pi \delta(x_2 - x_B) \right] \mathcal{M}_{F,ji}^\alpha(x_B, x_2) + (\mu \leftrightarrow \nu)^* \right\}$$

twist-2 (points to $\mathcal{M}_{ji}(x_B)$)
 twist-3 (points to the curly bracketed term)

$$\mathcal{M}_{ij}(x) = \int \frac{d\eta^-}{2\xi_h(2\pi)^4} e^{-ixP^+\eta^-} \sum_X \langle h_A | \bar{\psi}_j(\eta^-) \mathcal{L}_n^\dagger(\eta^-) | hX \rangle \langle Xh | \mathcal{L}_n(0) \psi_i(0) | h_A \rangle,$$

$$\mathcal{M}_{\partial,ij}^\alpha(x) = \int \frac{d\eta^-}{2\xi_h(2\pi)^4} e^{-ixP^+\eta^-} \sum_X \langle h_A | \bar{\psi}_j(\eta^-) \mathcal{L}_n^\dagger(\eta^-) | hX \rangle \langle Xh | \partial_\perp^\alpha (\mathcal{L}_n \psi_i)(0) | h_A \rangle,$$

$$\mathcal{M}_{F,ij}^\alpha(x_1, x_2) = \int \frac{d\eta^- d\eta_1^-}{4\pi\xi_h(2\pi)^4} e^{-ix_1P^+\eta^- - i(x_2-x_1)P^+\eta_1^-} \sum_X \langle h_A | \bar{\psi}_j(\eta^-) | hX \rangle \langle Xh | g_s F^{+\alpha}(\eta_1^-) \psi_i(0) | h_A \rangle.$$

Gauge invariant matrix elements!

$$\mathcal{L}_n(x) = \mathcal{P} \exp \left\{ -ig_s \int_0^\infty d\lambda G^+(\lambda n + x) \right\}$$

$$g_s F^{+\alpha} = g_s [\partial^+ G_\perp^\alpha - \partial_\perp^\alpha G^+] + \mathcal{O}(g_s^2)$$



Twist-3 contributions for TFR SIDIS

■ Fracture functions defined via the correlation matrices

$$\mathcal{M}_{ij}(x) = \frac{(\gamma_\rho)_{ij}}{2N_c} \left[\bar{n}^\rho \left(\mathbf{u}_1 - \frac{P_{h\perp} \cdot \tilde{S}_\perp}{M} \mathbf{u}_{1T}^h \right) + \frac{1}{P_+} \left(P_{h\perp}^\rho \mathbf{u}^h - M \tilde{S}_\perp^\rho \mathbf{u}_T - S_L \tilde{P}_{h\perp}^\rho \mathbf{u}_L^h - \frac{P_{h\perp}^{\langle \rho} P_{h\perp}^{\beta \rangle}}{M} \tilde{S}_{\perp\beta} \mathbf{u}_T^h \right) \right] \\ - \frac{(\gamma_\rho \gamma_5)_{ij}}{2N_c} \left[\bar{n}^\rho \left(S_L \mathbf{l}_{1L} - \frac{P_{h\perp} \cdot S_\perp}{M} \mathbf{l}_{1T}^h \right) + \frac{1}{P_+} \left(\tilde{P}_{h\perp}^\rho \mathbf{l}^h + M S_\perp^\rho \mathbf{l}_T + S_L P_{h\perp}^\rho \mathbf{l}_L^h - \frac{P_{h\perp}^{\langle \rho} P_{h\perp}^{\beta \rangle}}{M} S_{\perp\beta} \mathbf{l}_T^h \right) \right] + \dots,$$

$$\mathcal{M}_{\partial,ij}^\alpha(x) = \frac{(\gamma^-)_{ij}}{2N_c} i \left(-P_{h\perp}^\alpha \mathbf{u}_\partial^h + M \tilde{S}_\perp^\alpha \mathbf{u}_{\partial T} + S_L \tilde{P}_{h\perp}^\alpha \mathbf{u}_{\partial L}^h + \frac{P_{h\perp}^{\langle \alpha} P_{h\perp}^{\beta \rangle}}{M} \tilde{S}_{\perp\beta} \mathbf{u}_{\partial T}^h \right) \\ + \frac{(\gamma^- \gamma_5)_{ij}}{2N_c} i \left(\tilde{P}_{h\perp}^\alpha \mathbf{l}_\partial^h + M S_\perp^\alpha \mathbf{l}_{\partial T} + S_L P_{h\perp}^\alpha \mathbf{l}_{\partial L}^h - \frac{P_{h\perp}^{\langle \alpha} P_{h\perp}^{\beta \rangle}}{M} S_{\perp\beta} \mathbf{l}_{\partial T}^h \right) + \dots,$$

$$\mathcal{M}_{F,ij}^\alpha(x_1, x_2) = \frac{(\gamma^-)_{ij}}{2N_c} \left(P_{h\perp}^\alpha \mathbf{w}^h - M \tilde{S}_\perp^\alpha \mathbf{w}_T - S_L \tilde{P}_{h\perp}^\alpha \mathbf{w}_L^h - \frac{P_{h\perp}^{\langle \alpha} P_{h\perp}^{\beta \rangle}}{M} \tilde{S}_{\perp\beta} \mathbf{w}_T^h \right) \\ - \frac{(\gamma^- \gamma_5)_{ij}}{2N_c} i \left(\tilde{P}_{h\perp}^\alpha \mathbf{v}^h + M S_\perp^\alpha \mathbf{v}_T + S_L P_{h\perp}^\alpha \mathbf{v}_L^h - \frac{P_{h\perp}^{\langle \alpha} P_{h\perp}^{\beta \rangle}}{M} S_{\perp\beta} \mathbf{v}_T^h \right) + \dots,$$

Fracture functions
up to twist-3

Red: twist-2.

Green: twist-3

$$\tilde{a}_\perp^\mu \equiv \varepsilon_\perp^{\mu\nu} a_{\perp\nu}$$

$$P_{h\perp}^{\langle \alpha} P_{h\perp}^{\beta \rangle} \equiv P_{h\perp}^\alpha P_{h\perp}^\beta + g_\perp^{\alpha\beta} \vec{P}_{h\perp}^2 / 2$$

- Only chiral-even terms contribute
- Similar form and naming rules as TMDs, ($k_\perp \rightarrow P_{h\perp}$)



Twist-3 contributions for TFR SIDIS

■ Fracture functions defined via the correlation matrices

These twist-3 fracture functions are **not independent** from each other!

QCD equation of motion $i\gamma \cdot D\psi = 0$, gives:

$$\begin{aligned} & x[u_S^K(x) + il_S^K(x)] \\ &= u_{\partial S}^K(x) + il_{\partial S}^K(x) + i \int dy \left[P \frac{1}{y-x} - i\pi\delta(y-x) \right] [w_S^K(x, y) - v_S^K(x, y)] \end{aligned}$$

Four sets of equations

The relations take a **unified form**!

The hadronic tensor will be expressed by those fracture functions defined **only from the quark-quark correlator** $\mathcal{M}_{ij}$.

S	K
null	h
L	h
T	null
T	h



Twist-3 contributions for TFR SIDIS

■ Results for structure functions and azimuthal asymmetries

Structure functions

$$F_{UU,T} = x_B u_1, \quad F_{UT,T}^{\sin(\phi_h - \phi_S)} = \frac{|\vec{P}_{h\perp}|}{M} x_B u_{1T}^h,$$

$$F_{LL} = x_B l_{1L}, \quad F_{LT}^{\cos(\phi_h - \phi_S)} = \frac{|\vec{P}_{h\perp}|}{M} x_B l_{1T}^h.$$

Twist-2

Azimuthal/spin asymmetries

$$\langle \sin(\phi_h - \phi_S) \rangle_{UT} = \frac{|\vec{P}_{h\perp}|}{2M} \frac{u_{1T}^h}{u_1},$$

$$\langle \cos(\phi_h - \phi_S) \rangle_{LT} = \frac{|\vec{P}_{h\perp}| C(y)}{2M A(y)} \frac{l_{1T}^h}{u_1}.$$

$$F_{UU}^{\cos \phi_h} = -\frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 u^h, \quad F_{LU}^{\sin \phi_h} = \frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 l^h,$$

$$F_{UL}^{\sin \phi_h} = -\frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 u_L^h, \quad F_{LL}^{\cos \phi_h} = -\frac{2|\vec{P}_{h\perp}|}{Q} x_B^2 l_L^h,$$

$$F_{UT}^{\sin \phi_S} = -\frac{2M}{Q} x_B^2 u_T, \quad F_{LT}^{\cos \phi_S} = -\frac{2M}{Q} x_B^2 l_T,$$

$$F_{UT}^{\sin(2\phi_h - \phi_S)} = -\frac{\vec{P}_{h\perp}^2}{QM} x_B^2 u_T^h, \quad F_{LT}^{\cos(2\phi_h - \phi_S)} = -\frac{\vec{P}_{h\perp}^2}{QM} x_B^2 l_T^h.$$

Twist-3

$$\langle \cos \phi_h \rangle_{UU} = -\frac{|\vec{P}_{h\perp}|}{Q} \frac{B(y)}{A(y)} \frac{x_B u^h}{u_1}, \quad \langle \sin \phi_h \rangle_{LU} = \frac{|\vec{P}_{h\perp}|}{Q} \frac{D(y)}{A(y)} \frac{x_B l^h}{u_1},$$

$$\langle \sin \phi_h \rangle_{UL} = -\frac{|\vec{P}_{h\perp}|}{Q} \frac{B(y)}{A(y)} \frac{x_B u_L^h}{u_1}, \quad \langle \cos \phi_h \rangle_{LL} = -\frac{|\vec{P}_{h\perp}|}{Q} \frac{D(y)}{A(y)} \frac{x_B l_L^h}{u_1},$$

$$\langle \sin \phi_S \rangle_{UT} = -\frac{M}{Q} \frac{B(y)}{A(y)} \frac{x_B u_T}{u_1}, \quad \langle \cos \phi_S \rangle_{LT} = -\frac{M}{Q} \frac{D(y)}{A(y)} \frac{x_B l_T}{u_1},$$

$$\langle \sin(2\phi_h - \phi_S) \rangle_{UT} = -\frac{\vec{P}_{h\perp}^2}{2MQ} \frac{B(y)}{A(y)} \frac{x_B u_T^h}{u_1}, \quad \langle \cos(2\phi_h - \phi_S) \rangle_{LT} = -\frac{\vec{P}_{h\perp}^2}{2MQ} \frac{D(y)}{A(y)} \frac{x_B l_T^h}{u_1}.$$



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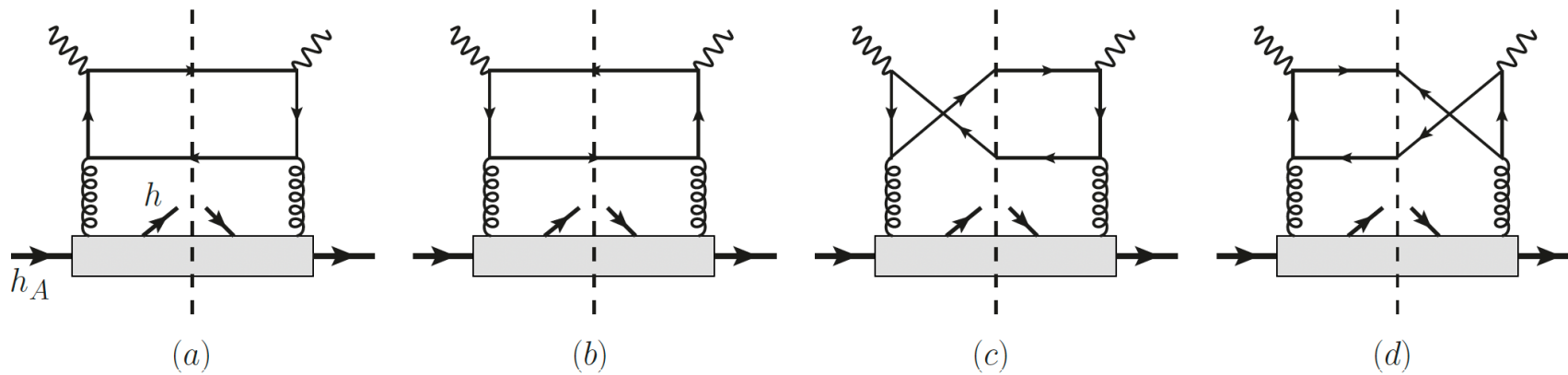
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■ Gluonic contribution



$$W^{\mu\nu}(q, P, S, P_h) = \alpha_s T_F \sum_f e_f^2 \int \frac{dx}{x} \int d\Phi_{k_1 k_2} H^{\mu\nu\alpha\beta}(k_g, k_1, k_2) \mathcal{M}_{G, \alpha\beta}(x, \xi_h, P_{h\perp}),$$

$$\begin{aligned} \mathcal{M}_G^{\alpha\beta}(x, \xi_h, P_{h\perp}) &= \frac{1}{2\xi_h(2\pi)^3} \frac{1}{xP^+} \int \frac{d\lambda}{2\pi} e^{-i\lambda x P^+} \sum_X \langle h_A(P) | (G^{+\alpha}(\lambda n) \mathcal{L}_n^\dagger(\lambda n))^a | X h(P_h) \rangle \\ &\quad \times \langle h(P_h) X | (\mathcal{L}_n(0) G^{+\beta}(0))^a | h_A(P) \rangle, \end{aligned}$$

$$\int d\Phi_{k_1 k_2} \equiv \int \frac{d^D k_1}{(2\pi)^D} \int \frac{d^D k_2}{(2\pi)^D} (2\pi) \delta(k_1^2) (2\pi) \delta(k_2^2) (2\pi)^4 \delta^{(4)}(q + k_g - k_1 - k_2) \theta(q^0 + k_g^0)$$



One-loop contributions for TFR SIDIS

■ Gluonic contribution

Gluon fracture functions

$$\begin{aligned}\mathcal{M}_G^{\alpha\beta} = & -\frac{1}{2-2\epsilon}g_{\perp}^{\alpha\beta}u_{1g}^{\textcolor{red}{h}} + \frac{1}{2M^2}\left(P_{h\perp}^{\alpha}P_{h\perp}^{\beta} + \frac{1}{2-2\epsilon}g_{\perp}^{\alpha\beta}P_{h\perp}^2\right)t_{1g}^{\textcolor{red}{h}} + S_L\left[i\frac{\varepsilon_{\perp}^{\alpha\beta}}{2}l_{1gL}^{\textcolor{red}{h}} + \frac{\tilde{P}_{h\perp}^{\{\alpha}P_{h\perp}^{\beta\}}}{4M^2}t_{1gL}^{\textcolor{red}{h}}\right] \\ & + \frac{g_{\perp}^{\alpha\beta}}{2-2\epsilon}\frac{P_{h\perp}\cdot\tilde{S}_{\perp}}{M}u_{1gT}^{\textcolor{red}{h}} + \frac{P_{h\perp}\cdot S_{\perp}}{M}\left[i\frac{\varepsilon_{\perp}^{\alpha\beta}}{2}l_{1gT}^{\textcolor{red}{h}} - \frac{\tilde{P}_{h\perp}^{\{\alpha}P_{h\perp}^{\beta\}}}{4M^2}t_{1gT}^{\textcolor{red}{h}}\right] + \frac{\tilde{P}_{h\perp}^{\{\alpha}S_{\perp}^{\beta\}} + \tilde{S}_{\perp}^{\{\alpha}P_{h\perp}^{\beta\}}}{8M}t_{1gT}^{\textcolor{red}{h}} + \dots\end{aligned}$$

The hard parts:

$$H_{(a)+(b)}^{\mu\nu\alpha\beta}(k_g, k_1, k_2) = \text{Tr}\left[k_1\gamma^{\nu}\frac{(\not{k}_g - \not{k}_2)}{(k_g - k_2)^2}\gamma^{\beta}\not{k}_2\gamma^{\alpha}\frac{(\not{k}_g - \not{k}_2)}{(k_g - k_2)^2}\gamma^{\mu}\right] + (k_1 \leftrightarrow k_2),$$

$$H_{(c)+(d)}^{\mu\nu\alpha\beta}(k_g, k_1, k_2) = \text{Tr}\left[k_1\gamma^{\nu}\frac{(\not{k}_g - \not{k}_2)}{(k_g - k_2)^2}\gamma^{\beta}\not{k}_2\gamma^{\mu}\frac{(\not{k}_1 - \not{k}_g)}{(k_1 - k_g)^2}\gamma^{\alpha}\right] + (k_1 \leftrightarrow k_2).$$



One-loop contributions for TFR SIDIS

■ Gluonic contribution

$$F_{UU}^{\cos 2\phi_h} = -\frac{\alpha_s T_F}{2\pi} \frac{P_{h\perp}^2}{2M^2} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} z^2 t_{1g}^h(x_B/z, \xi_h, P_{h\perp}),$$

$$F_{UL}^{\sin 2\phi_h} = \frac{\alpha_s T_F}{2\pi} \frac{P_{h\perp}^2}{2M^2} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} z^2 t_{1gL}^h(x_B/z, \xi_h, P_{h\perp}),$$

$$F_{UT}^{\sin(3\phi_h - \phi_s)} = \frac{\alpha_s T_F}{2\pi} \frac{P_{h\perp}^3}{4M^3} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} z^2 t_{1gT}^{hh}(x_B/z, \xi_h, P_{h\perp}),$$

$$F_{UT}^{\sin(\phi_h + \phi_s)} = \frac{\alpha_s T_F}{2\pi} \frac{P_{h\perp}}{2M} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} z^2 \left[t_{1gT}^h(x_B/z, \xi_h, P_{h\perp}) + \frac{P_{h\perp}^2}{2M^2} t_{1gT}^{hh}(x_B/z, \xi_h, P_{h\perp}) \right].$$

Generated **uniquely** by gluon fracture functions!



One-loop contributions for TFR SIDIS

■ Quark and gluon contribution

$$F_{UU,L} = \frac{\alpha_s}{2\pi} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} \left[4T_F z \bar{z} u_{1g}(x_B/z, \xi_h, P_{h\perp}) + 2C_F z u_1(x_B/z, \xi_h, P_{h\perp}) \right],$$

$$F_{UT,L}^{\sin(\phi_h - \phi_S)} = \frac{\alpha_s}{2\pi} \frac{P_{h\perp}}{M} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} \left[4T_F z \bar{z} u_{1gT}^h(x_B/z, \xi_h, P_{h\perp}) + 2C_F z u_{1T}^h(x_B/z, \xi_h, P_{h\perp}) \right].$$

Twist-4 if at
the tree level

$$F_{UU,T} = x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} \left[\mathcal{H}_g(z) u_{1g}(x_B/z, \xi_h, P_{h\perp}) + \mathcal{H}_q(z) u_1(x_B/z, \xi_h, P_{h\perp}) \right],$$

$$F_{UT,T}^{\sin(\phi_h - \phi_S)} = \frac{P_{h\perp}}{M} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} \left[\mathcal{H}_g(z) u_{1gT}^h(x_B/z, \xi_h, P_{h\perp}) + \mathcal{H}_q(z) u_{1T}^h(x_B/z, \xi_h, P_{h\perp}) \right],$$

$$F_{LL} = x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} \left[\Delta \mathcal{H}_g(z) l_{1gL}(x_B/z, \xi_h, P_{h\perp}) + \Delta \mathcal{H}_q(z) l_{1L}(x_B/z, \xi_h, P_{h\perp}) \right],$$

$$F_{LT}^{\cos(\phi_h - \phi_S)} = \frac{P_{h\perp}}{M} x_B \sum_{q,\bar{q}} e_q^2 \int_{x_B/\bar{\xi}_h}^1 \frac{dz}{z} \left[\Delta \mathcal{H}_g(z) l_{1gT}^h(x_B/z, \xi_h, P_{h\perp}) + \Delta \mathcal{H}_q(z) l_{1T}^h(x_B/z, \xi_h, P_{h\perp}) \right],$$

LO + NLO corrections

$$\mathcal{H}_q(z) = \delta(\bar{z}) + \frac{\alpha_s}{2\pi} \left\{ P_{qq}(z) \ln \frac{Q^2}{\mu^2} + C_F \left[2 \left(\frac{\ln \bar{z}}{\bar{z}} \right)_+ - \frac{3}{2} \left(\frac{1}{\bar{z}} \right)_+ - (1+z) \ln \bar{z} - \frac{1+z^2}{\bar{z}} \ln z + 3 - \left(\frac{\pi^2}{3} + \frac{9}{2} \right) \delta(\bar{z}) \right] \right\}, \quad \mathcal{H}_g(z) = \frac{\alpha_s}{2\pi} \left[P_{qg}(z) \ln \frac{Q^2 \bar{z}}{\mu^2 z} - T_F (1-2z)^2 \right],$$

$$\Delta \mathcal{H}_q(z) = \delta(\bar{z}) + \frac{\alpha_s}{2\pi} \left\{ \Delta P_{qq}(z) \ln \frac{Q^2}{\mu^2} + C_F \left[(1+z^2) \left(\frac{\ln \bar{z}}{\bar{z}} \right)_+ - \frac{3}{2} \left(\frac{1}{\bar{z}} \right)_+ - \frac{1+z^2}{\bar{z}} \ln z + 2 + z - \left(\frac{\pi^2}{3} + \frac{9}{2} \right) \delta(\bar{z}) \right] \right\}, \quad \Delta \mathcal{H}_g(z) = \frac{\alpha_s}{2\pi} \left[\Delta P_{qg}(z) \left(\ln \frac{Q^2 \bar{z}}{\mu^2 z} - 1 \right) + 2T_F \bar{z} \right].$$



One-loop contributions for TFR SIDIS

Structure functions	Twist	$\mathcal{O}(\alpha_S)$
$F_{UU,T}$	✓ (2)	✓ (α_S^0)
$F_{UU,L}$		✓ (α_S^1)
$F_{UU}^{\cos \phi_h}$	✓ (3)	
$F_{UU}^{\cos 2\phi_h}$		✓ (α_S^1)
$F_{LU}^{\sin \phi_h}$	✓ (3)	
$F_{UL}^{\sin \phi_h}$	✓ (3)	
$F_{UL}^{\sin 2\phi_h}$		✓ (α_S^1)
F_{LL}	✓ (2)	✓ (α_S^0)
$F_{LL}^{\cos \phi_h}$	✓ (3)	

Structure functions	Twist	$\mathcal{O}(\alpha_S)$
$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	✓ (2)	✓ (α_S^0)
$F_{UT,L}^{\sin(\phi_h-\phi_S)}$		✓ (α_S^1)
$F_{UT}^{\sin(\phi_h+\phi_S)}$		✓ (α_S^1)
$F_{UT}^{\sin \phi_S}$	✓ (3)	
$F_{UT}^{\sin(2\phi_h-\phi_S)}$	✓ (3)	
$F_{UT}^{\sin(3\phi_h-\phi_S)}$		✓ (α_S^1)
$F_{LT}^{\cos \phi_S}$	✓ (3)	
$F_{LT}^{\cos(\phi_h-\phi_S)}$	✓ (2)	✓ (α_S^0)
$F_{LT}^{\cos(2\phi_h-\phi_S)}$	✓ (3)	

All 18 structure functions are non-zero up to twist-3 or $\mathcal{O}(\alpha_S^1)$ level.



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Connecting NEEC with fracture function

■ Energy-energy correlator (EEC)

EEC was proposed as an event-shape observable in e^+e^- annihilation in 1970's.

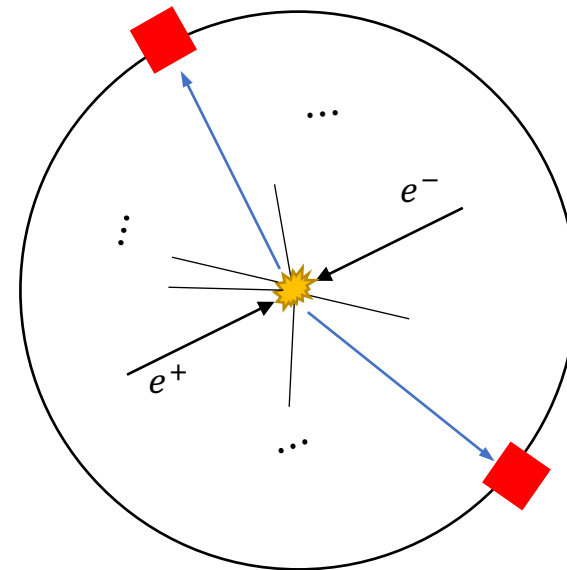
$$\text{EEC} \sim \sum_{a,b} d\sigma(e^+e^- \rightarrow h_a h_b X) \frac{E_a E_b}{s} \delta(\Omega_{ab} - \Omega)$$

Basham, Brown, Ellis and Love, PRL 41 (1978) 1585

Energy weighted cross section, infrared safe,
precise test of pQCD, easy to implement in
experiments

Applications to DIS process and others,

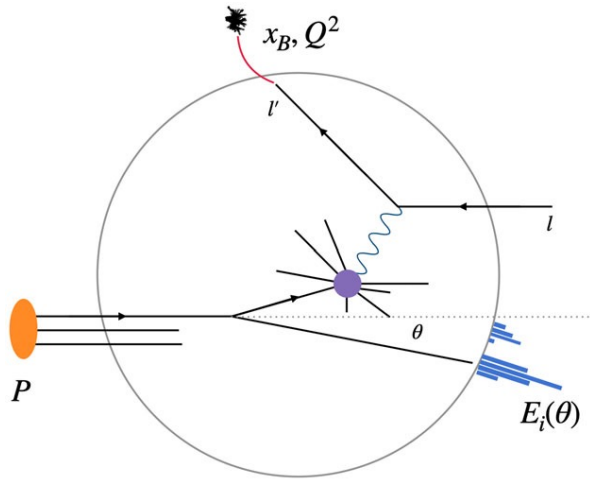
developing fast recently..... X.H. Liu's talk





Connecting NEEC with fracture function

■ Nucleon energy-energy correlator (NEEC)



X.H. Liu and H.X. Zhu, PRL 130 (2023) 091901

H.T. Cao, X.H. Liu and H.X. Zhu, PRD 107 (2023) 114008

$$\text{NEEC: } f_{EEC}(z, \theta) = \int \frac{dy^-}{4\pi} e^{-izP^+y^-} \langle P | \bar{\psi}(y^-) \frac{\gamma^+}{2} \hat{\mathcal{E}}(\theta) \psi(0) | P \rangle$$

$$\text{The energy flow operator: } \hat{\mathcal{E}}(\theta) |X\rangle \equiv \sum_{i \in X} \delta(\theta^2 - \theta_i^2) \frac{E_i}{E_P} |X\rangle$$

$$\Sigma(x_B, Q^2, \theta) \equiv \sum_i \int \frac{E_i}{E_P} d\sigma(x_B, Q^2, p_i) \delta(\theta - \theta_i)$$

$$\xrightarrow{\theta \ll 1} = \int \frac{dz}{z} \hat{\sigma}\left(\frac{x_B}{z}, Q^2, \mu\right) f_{EEC}(z, \theta, \mu)$$

Notice: Hadrons with $\theta \ll 1$ are almost **collinear with the target nucleon**.
They are produced in the **Target Fragmentation Region!**



Connecting NEEC with fracture function

■ Relation between NEEC and fracture functions

Matrix elements for fracture functions and NEECs:

$$\mathcal{M}_{ij,\text{FrF}}^q(x, \xi_h, \mathbf{P}_{h\perp}) = \int \frac{d\eta^-}{2\xi_h(2\pi)^4} e^{-ixP^+\eta^-} \sum_X \int \frac{d^3\mathbf{P}_X}{2E_X(2\pi)^3} \\ \times \langle PS | \bar{\psi}_j(\eta^-) \mathcal{L}_n^\dagger(\eta^-) | P_h X \rangle \langle X P_h | \mathcal{L}_n(0) \psi_i(0) | PS \rangle$$

$$\sum_X \int \frac{d^3\mathbf{P}_X}{2E_X(2\pi)^3} |P_h X\rangle \langle X P_h| = a_h^\dagger a_h$$

$$\mathcal{M}_{ij,\text{NEEC}}^q(x, \theta, \phi) = \int \frac{d\eta^-}{2\pi} e^{-ixP^+\eta^-} \langle PS | \bar{\psi}_j(\eta^-) \mathcal{L}_n^\dagger(\eta^-) \mathcal{E}(\theta, \phi) \mathcal{L}_n(0) \psi_i(0) | PS \rangle$$

$$\mathcal{E}(\theta, \phi) = \sum_h \int \frac{d^3\mathbf{P}_h}{2E_h(2\pi)^3} \frac{E_h}{E_N} \delta(\theta^2 - \theta_h^2) \delta(\phi - \phi_h) a_h^\dagger a_h$$



$$\mathcal{M}_{ij,\text{NEEC}}^q(x, \theta, \phi) = \sum_h \int_0^{1-x} d\xi_h \xi_h \frac{\mathbf{P}_{h\perp}^2}{2\theta^2} \mathcal{M}_{ij,\text{FrF}}^q(x, \xi_h, \mathbf{P}_{h\perp}) \Big|_{\mathbf{P}_{h\perp} = \frac{\xi_h \theta P^+}{\sqrt{2}} \mathbf{n}_t}$$

$$\mathbf{n}_t \equiv (\cos \phi, \sin \phi)$$

Fracture functions act as **generating functions** of NEECs.



Connecting NEEC with fracture function

■ Relation between NEEC and fracture functions

$$\mathcal{M}_{ij,\text{FrF}}^q(x, \xi_h, \mathbf{P}_{h\perp}) = \frac{(\gamma_\rho)_{ij}}{2N_c} \left[\bar{n}^\rho \left(u_1^q - \frac{P_{h\perp} \cdot \tilde{S}_\perp}{M} u_{1T}^{h,q} \right) \right] + \frac{(\gamma_5 \gamma_\rho)_{ij}}{2N_c} \left[\bar{n}^\rho \left(S_L l_{1L}^q - \frac{P_{h\perp} \cdot S_\perp}{M} l_{1T}^{h,q} \right) \right] + \dots$$

$$\mathcal{M}_{ij,\text{NEEC}}^q(x, \theta, \phi) = \frac{(\gamma_\rho)_{ij}}{2N_c} \left[\bar{n}^\rho \left(\frac{1}{2\pi} f_1^q - n_t \cdot \tilde{S}_\perp f_{1T}^{t,q} \right) \right] + \frac{(\gamma_5 \gamma_\rho)_{ij}}{2N_c} \left[\bar{n}^\rho \left(\frac{1}{2\pi} S_L g_{1L}^q - n_t \cdot S_\perp g_{1T}^{t,q} \right) \right] + \dots$$

➡

$$f_1^q(x, \theta) = 2\pi \oint u_1^q(x, \xi_h, \mathbf{P}_{h\perp}^2), \quad f_{1T}^{t,q}(x, \theta) = \oint \frac{|\mathbf{P}_{h\perp}|}{M} u_{1T}^{h,q}(x, \xi_h, \mathbf{P}_{h\perp}^2),$$

$$g_{1L}^q(x, \theta) = 2\pi \oint l_{1L}^q(x, \xi_h, \mathbf{P}_{h\perp}^2), \quad g_{1T}^{t,q}(x, \theta) = \oint \frac{|\mathbf{P}_{h\perp}|}{M} l_{1T}^{h,q}(x, \xi_h, \mathbf{P}_{h\perp}^2), \quad \dots$$

One to one correspondence.

$$\oint \mathcal{A} \equiv \sum_h \int_0^{1-x} d\xi_h \xi_h \frac{P_{h\perp}^2}{2\theta^2} \mathcal{A} \Big|_{|\mathbf{P}_{h\perp}| = \frac{\theta \xi_h P^+}{\sqrt{2}}}$$



Connecting NEEC with fracture function

Accessing NEEC properties through fracture functions.

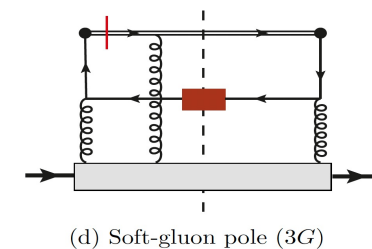
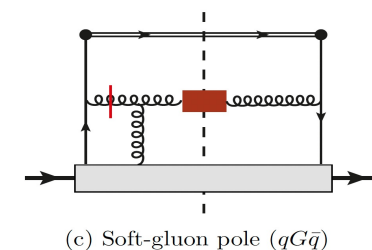
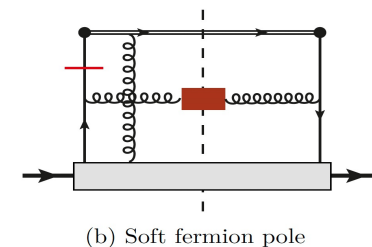
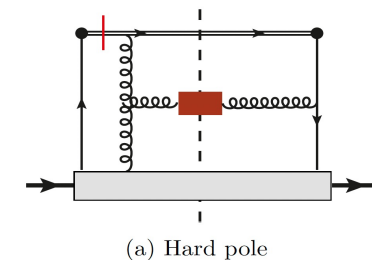
Example:

Matching of Sivers-type fracture function $u_{1T}^{h,q}(x, \xi_h, P_{h\perp})$ at $\Lambda_{QCD} \ll P_{h\perp} \ll Q$



Matching of Sivers-type NEEC $f_{1T}^{t,q}(x, \theta)$ at $\Lambda_{QCD} \ll \theta Q \ll Q$

Diagram examples



$$f_{1T}^{t,q}(x, \theta) \Big|_{\text{HP}} = \frac{\alpha_s N_c}{2(2\pi)^2 \theta^3 E_N} \int_x^1 \frac{dy}{y} \left[T_{\Delta}(y, x) - (1 + 2x/(y-x)) T_F(y, x) \right],$$

$$f_{1T}^{t,q}(x, \theta) \Big|_{\text{SFP}} = \frac{\alpha_s}{2(2\pi)^2 N_c \theta^3 E_N} \int_x^1 dy \frac{x}{y^3} \left[(2x-y) T_F(y, 0) - y T_{\Delta}(y, 0) \right],$$

$$f_{1T}^{t,q}(x, \theta) \Big|_{\text{SGP}, qG\bar{q}} = \frac{\alpha_s N_c}{2(2\pi)^2 \theta^3 E_N} \int_x^1 \frac{dy}{y^3} \left[\frac{1}{y-x} (y^3 + 3x^2 y - 2x^3) T_F(y, y) - y(y^2 + x^2) \frac{dT_F(y, y)}{dy} \right],$$

$$f_{1T}^{t,q}(x, \theta) \Big|_{\text{SGP}, 3G} = \frac{\alpha_s}{2\pi \theta^3 E_N} \int_x^1 dy \frac{y-x}{y^5} \left\{ 2(4x^2 - 3xy + y^2) [N(y, y) - O(y, y)] \right. \\ \left. - 2(8x^2 - 5xy + y^2) [N(y, 0) + O(y, 0)] + y(2x-y)^2 \frac{d}{dy} [N(y, 0) + O(y, 0)] \right. \\ \left. - y(2x^2 + y^2 - 2xy) \frac{d}{dy} [N(y, y) - O(y, y)] \right\}.$$



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Summary

- SIDIS in the TFR is factorized with fracture functions.
- TFR SIDIS is calculated up to twist-3 at the tree level. Structure functions and azimuthal asymmetries are obtained using the gauge-invariant fracture functions.
- By adding one-loop contributions at twist-2, all 18 structure functions for TFR SIDIS are non-zero.
- Connection between NEEC and fracture functions is established, which will facilitate the study of NEEC through fracture functions.

谢谢!