Fragmentation functions at NNLO and constraints on PDFs

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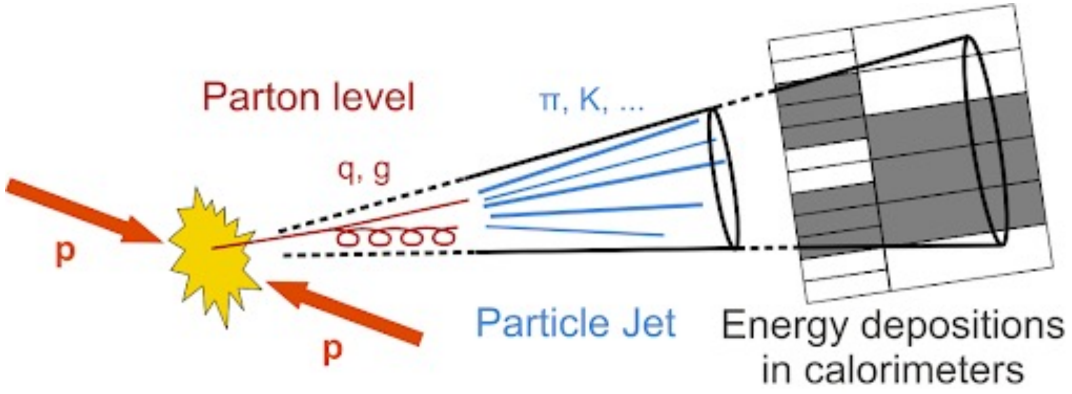
Fragmentation functions (FFs) as extension of the parton model

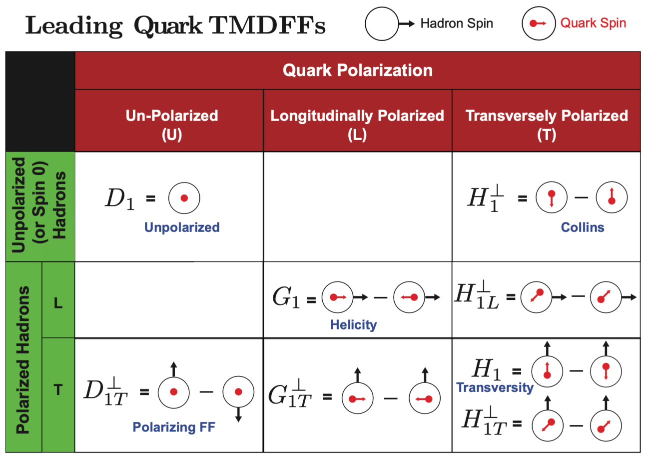
1969 — Feynman, '69
Parton model

1971 — Berman, Bjorken, Kogut, '71
FFs proposed

1977 — Field, Feynman, '77
First determination of FFs



$$D_{h/i}\left(z = \frac{p^+_{\mathrm{hadron}\,h}}{p^+_{\mathrm{parton}\,i}}\right)$$



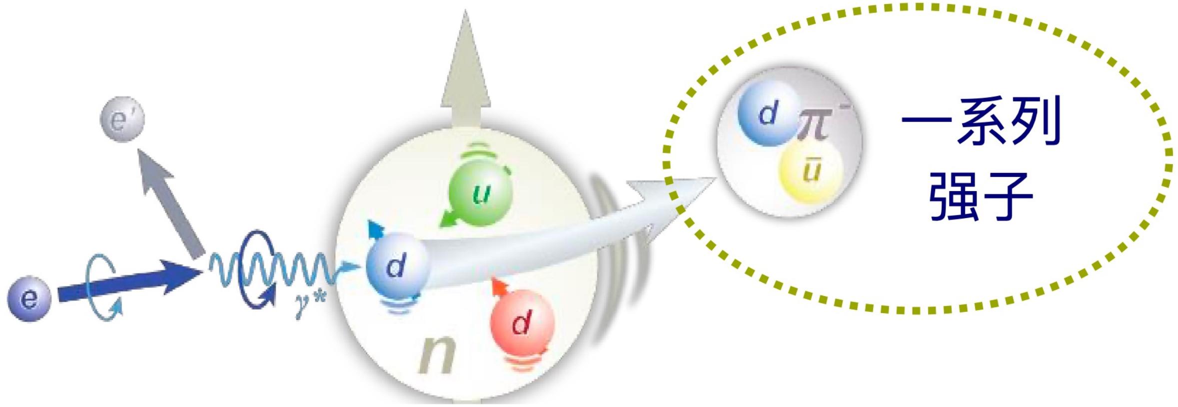
[2304.03302]

$D_i^h(z)$: number density of hadron h carrying a fraction of momentum z of parton i

Why FFs: key ingredients of QCD factorization framework

1969	1971	1977	1982	1989
Feynman, '69 Parton model	Berman, Bjorken, Kogut, '71 FFs proposed	Field, Feynman, '77 First determination of FFs	Collins, Soper, '82 FFs definition in QCD	Collins et al., '89 Factorization theorem

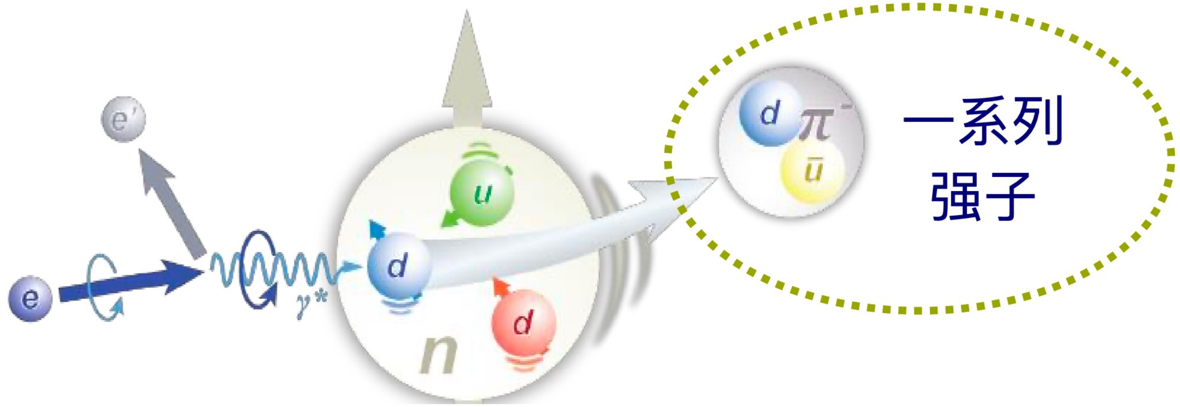
Example: e + N -> e + **h** + X (Semi-Inclusive DIS (SIDIS))



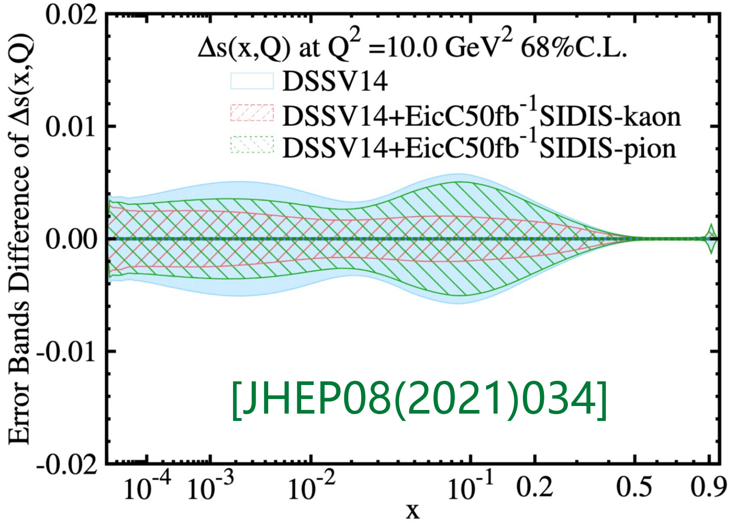
$$\sigma^h = f_{i/N} \otimes \hat{\sigma}_{j \leftarrow i} \otimes D_{h/j}$$

Why FFs: phenomenologial applications

Identified hadron helps discriminate initial parton



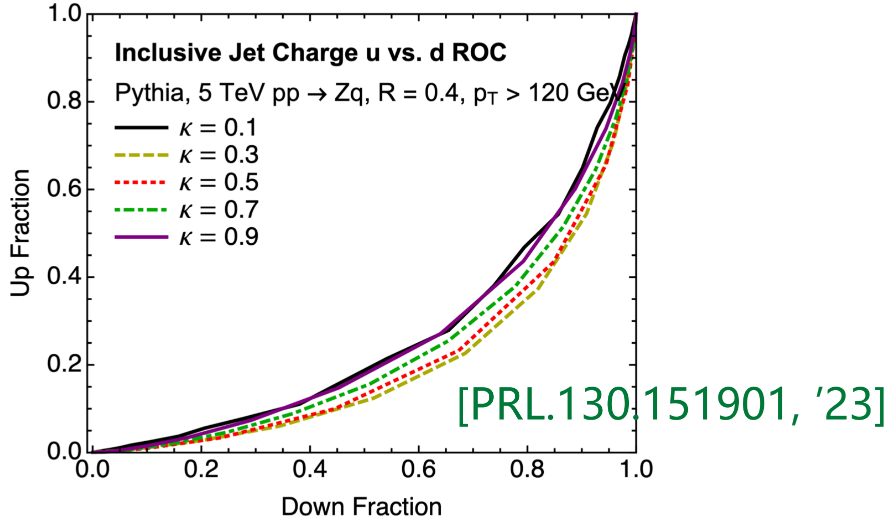
FFs are key inputs of pPDFs determination



The **jet charge**

$$Q_J = \frac{1}{(p_{\mathrm{T}\,J})^\kappa} \sum_{i \in Tracks} q_i \times (p_{\mathrm{T,i}})^\kappa$$

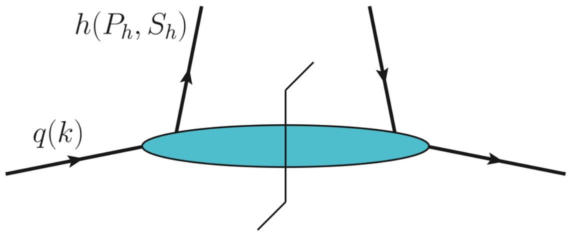
may be used for u/d quark jet separation



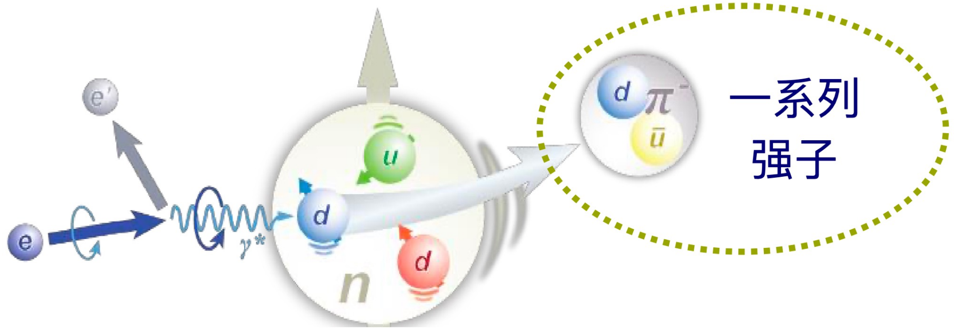
Extraction of FFs

❖ Field theory definiton of the collinear (integrated) quark FFs [Collins, Soper '82]

$$D_{h/q}(z) = \frac{z}{4}\sum_X \int \frac{d\xi^+}{2\pi} e^{iP_h^- \xi^+/z}\,\mathrm{Tr}\Big[\langle 0\,|\,\mathcal{W}(\infty^+,\xi^+)\ \psi_q(\xi^+,0^-,\vec{0}_T)\,|\,P_h, S_h; X\rangle \times \langle P_h, S_h; X\,|\,\bar{\psi}_q(0^+,0^-,\vec{0}_T)\ \mathcal{W}(0^+,\infty^+)\,|\,0\rangle\ \gamma^-\Big]$$



❖ Global analyses of **ee(SIA), ep(SIDIS) and pp** data based on **factorization formula.**



$$\frac{d^3\sigma_h}{dx\,dy\,dz_h} = f_{i/p}(x) \otimes \hat{\sigma}_{j\leftarrow i}(x,y,z) \otimes D_{h/j}(z_h) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

measurement　　input　　pQCD　　to be determined. Universal !

FFs can be extracted via global fit of ee, ep, pp data.

Some representative FFs

Berman, Bjorken, Kogut, '71
FFs proposed

Collins, Soper, '82
FFs definition in QCD

Chiappetta et al., '92
First FFs extraction at NLO (ee, pp)

TODO: Global NNLO or N3LO FFs analyses

1969 **1971** **1977** **1982** **1989** **1992** **2015** **high-precision era**

Feynman, '69
Parton model

Field, Feynman, '77
First determination of FFs

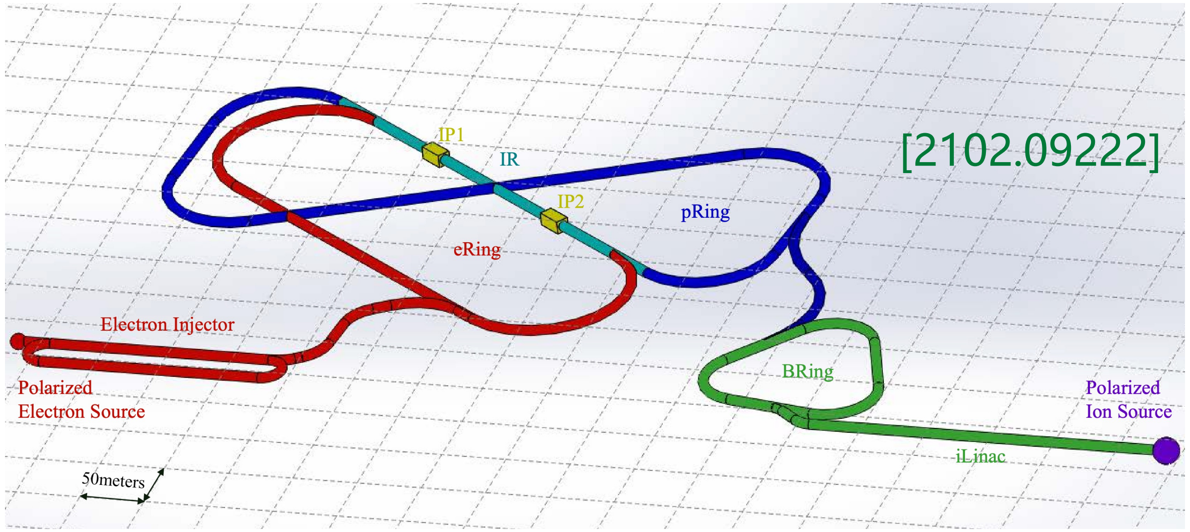
Collins et al., '89
Factorization theorem

Anderle et al., '15
First FFs extraction at NNLO (**ee only**)

collarboration	JAM	DSS+	NPC23	BDSSV	MAP
SIA (ee)	✓	✓	✓	✓	✓
SIDIS (ep)	✓	✓	✓	✓	✓
pp incl. hadron	✘	✓	✓	✘	✘
pp hadron in jet	✘	✘	✓	✘	✘
identified hadron	π, K	π, K	π, K, p	π	π, K
pQCD order	NLO	NLO	NLO	approx. NNLO	approx. NNLO
publication	2101.04664	2311.17768	2401.02781	2202.05060	2204.10331

NNLO or N3LO FFs will be needed for the forthcoming high-precision era.

The needs of high-precision FFs extractions



❖ The Electron-Ion Collider (EIC)

- expected to start operation in the early 2030s
- unprecedented access to the spatial and spin structure of the proton
- **FFs** as keys ingredients of SIDIS at the EIC

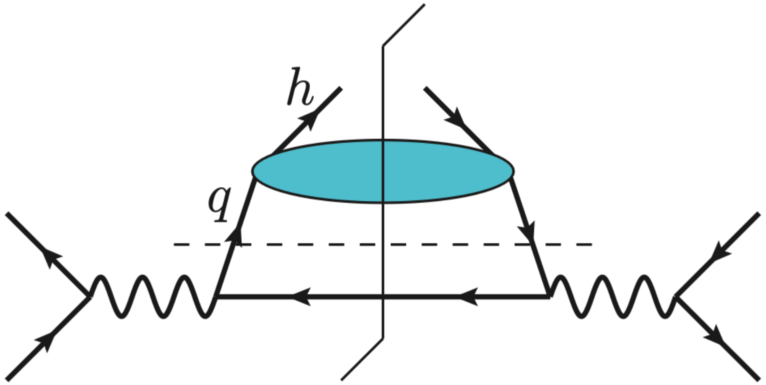
❖ Efforts from China

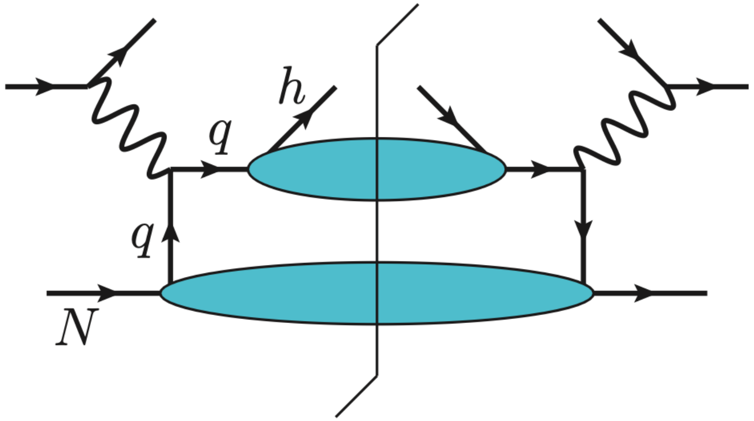
- ep collisions: EicC
- ee collisions: BESIII measurements
- ee collisions: CEPC

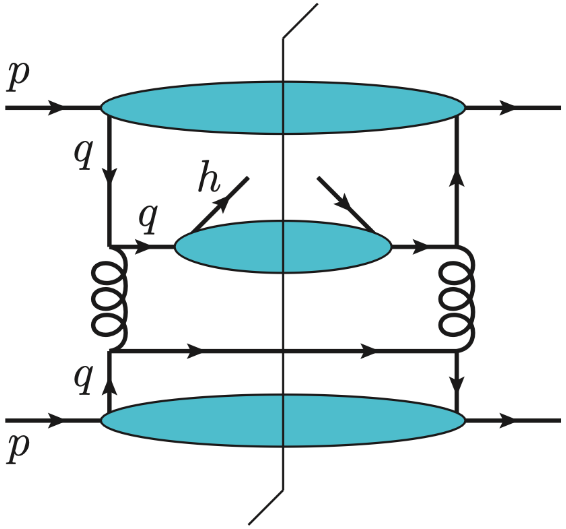
See Wenbiao Yan's talk

High-precision (NNLO, N3LO) FFs as key output & input

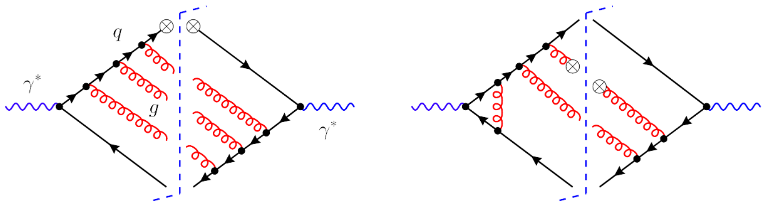
Progress on the theoretical side







- SIA(e^+e^-) at N3LO



(a) RRR (b) VRR

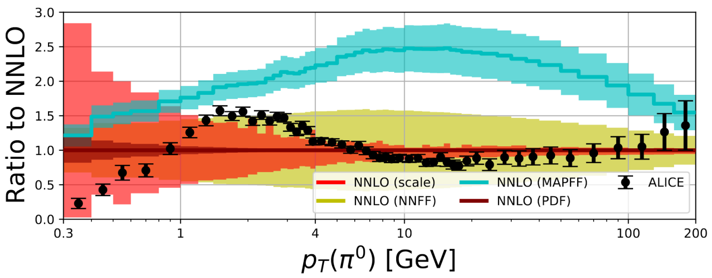
[He, Xing, Yang, Zhu, 2503.20441]

See HuaXing Zhu's talk

- SIDIS(ep) at NNLO

[PRL.132.251901, 2024
PRL.132.251902, 2024
PRL.133.211904, 2024
PRL.133.211905, 2024
2504.05376]

- pp at NNLO



[Czakon et al. 2503.11489]

Ingredients ready for a global analysis of FFs at NNLO

NNLO FFs analysis: parameterization

[Gao, **XS**, Xing, Zhao, Zhou, 2502.17837]

We performed the **first** global analyses of FFs to $\pi^{\pm}, K^{\pm}$ at NNLO in QCD

- Light flavor FFs are parameterized at $Q_0 = 1.4$ GeV

$$zD_i^h(z, Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h} \exp\left(\sum_{n=0}^{m} a_{i,n}^h z^{n/2}\right)$$

- Heavy quark FFs are freezed below threshold.

- Number of free parameters suppressed by charge/isospin symmetries:

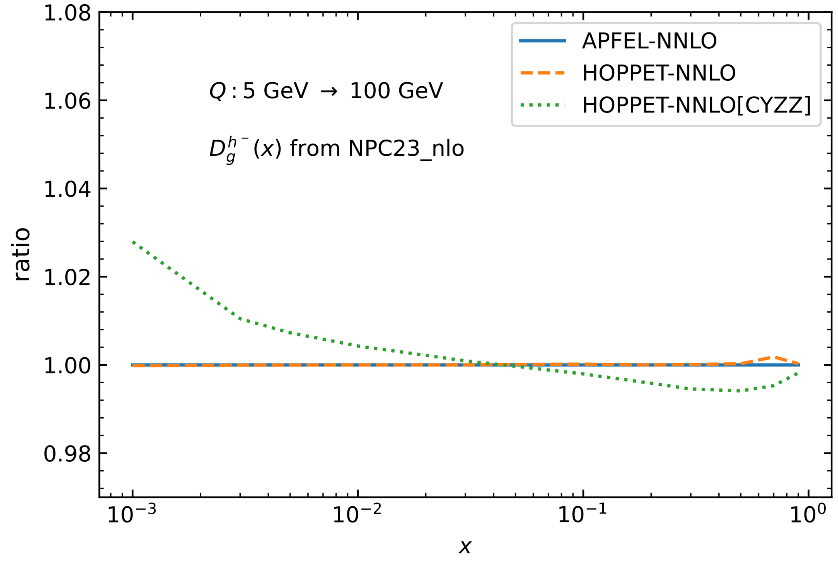
$$D_{u\to\pi^+}(z,Q) = D_{\bar{u}\to\pi^-}(z,Q)$$

$$D_{u\to\pi^+}(z,Q_0) = D_{\bar{d}\to\pi^+}(z,Q_0)$$

- NNLO (3-loop) DGLAP evolution from [Mitov, Moch, Vogt, Almasy]

- $+P_{qg}^{T,(2)}$ correction from [Chen, Yang, Zhu, Zhu, '20]

- implemented in HOPPET, with benchmark against APFEL



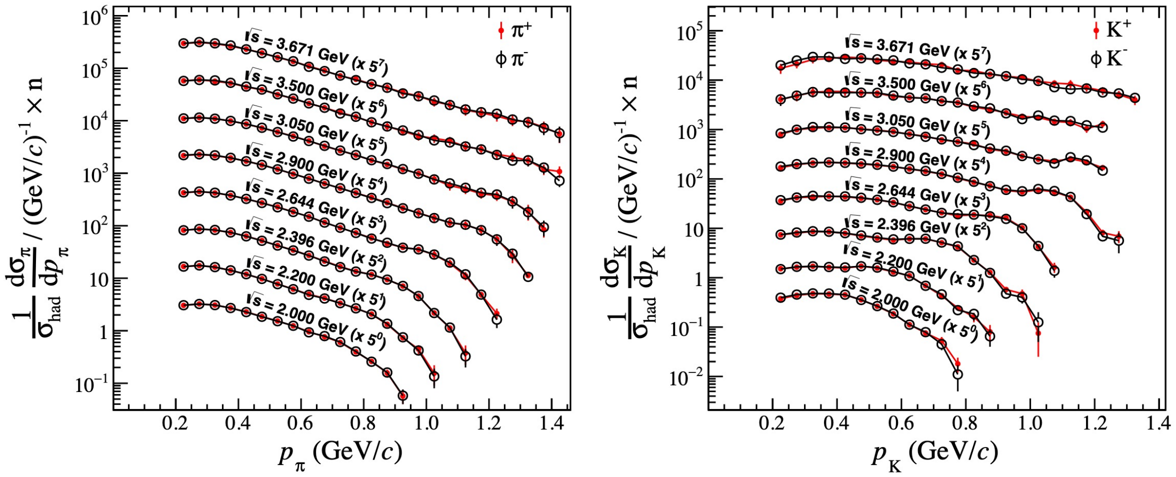
NNLO FFs analysis: overview

❖ Data

- Select SIA and SIDIS data with $Q > 3(2)\ GeV, z > 0.01, E_h > E_{h,min}$(0.8 GeV by default)
- New BESIII measurement included

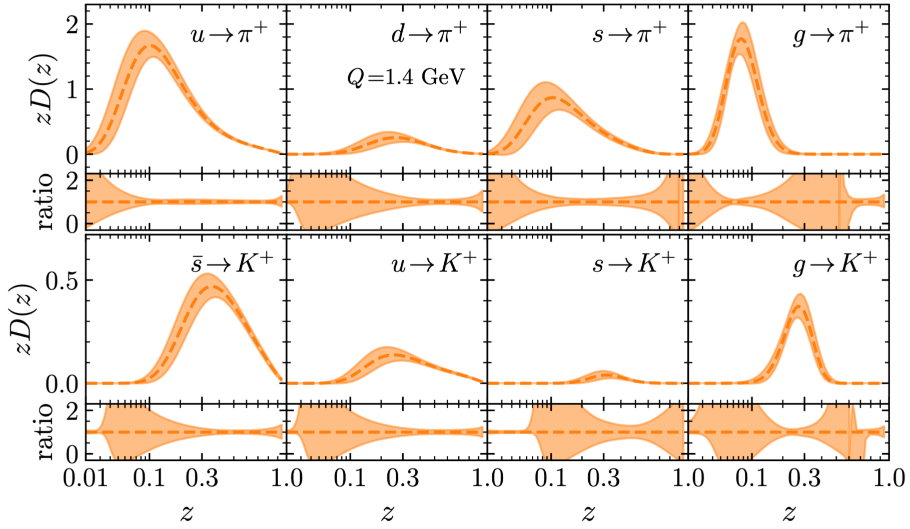
  See Wenbiao Yan's talk

[BESIII, 2502.16084]



❖ Theory

- SIDIS(ep): coefficient functions at NNLO from [Bonino+] and [Goyal+]
- SIA(ee): NNLO + hadron mass corrections
- include scale variations into the covarinace matrix



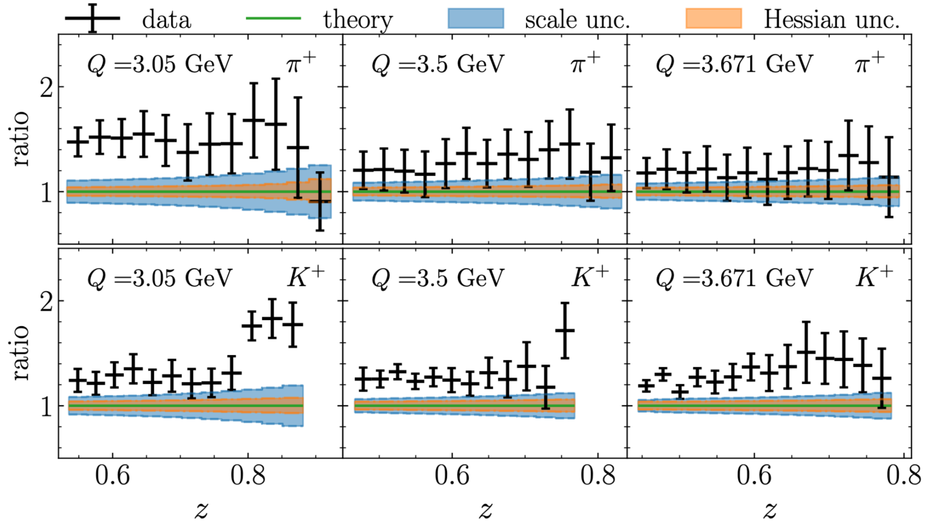
LHAgrids of our FFs will be published later

NNLO FFs analysis: fit quality

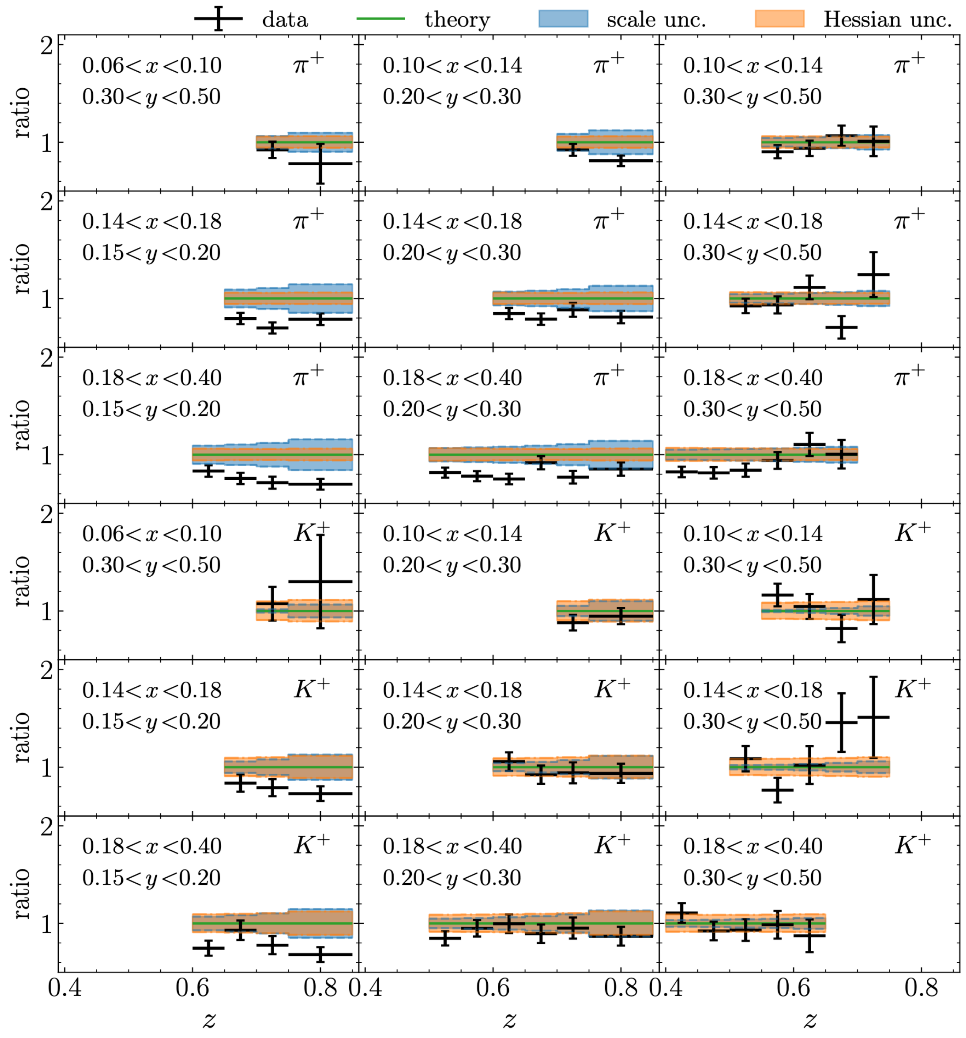
Quality of NNLO fit of SIA and SIDIS data

$E_{h,\min}$[GeV]	BESIII N_{pt}	BESIII χ^2/N_{pt}	COMPASS N_{pt}	COMPASS χ^2/N_{pt}	B-factories N_{pt}	B-factories χ^2/N_{pt}	HE-SIA N_{pt}	HE-SIA χ^2/N_{pt}	global N_{pt}	global χ^2	global χ^2/N_{pt}
0.5	242	1.26	358	1.65	233	1.06	426	1.19	1259	1650.2	1.31
0.6	212	1.21	290	1.59	228	0.92	423	0.97	1153	1338.8	1.16
0.7	182	1.11	214	1.47	223	0.61	413	0.84	1032	997.2	0.97
0.8	152	0.98	142	1.30	218	0.53	407	0.82	919	781.8	0.85
0.9	122	1.05	94	1.29	213	0.52	407	0.80	836	687.1	0.82
1.0	98	1.14	54	0.97	209	0.49	403	0.80	764	587.2	0.77

Theory v.s. data for NNLO fit (BESIII)



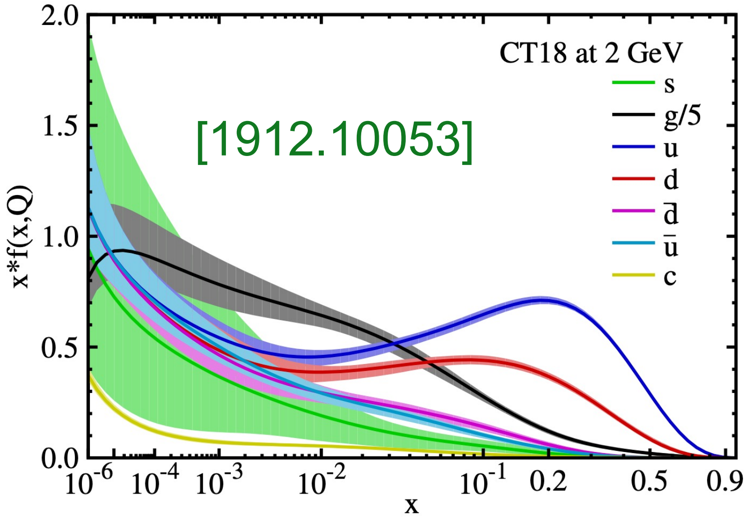
Theory v.s. data for NNLO fit (COMPASS06)



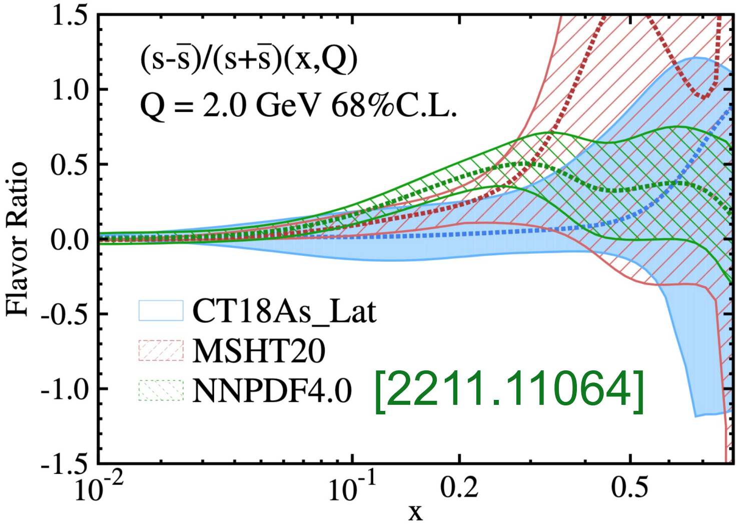
Satisfactory agreements found for both SIA and SIDIS data with Q~3 GeV

Applications: constraining SOTA proton PDFs at NNLO

We may for example constrain the **strange** quark PDFs with **SIDIS** data.

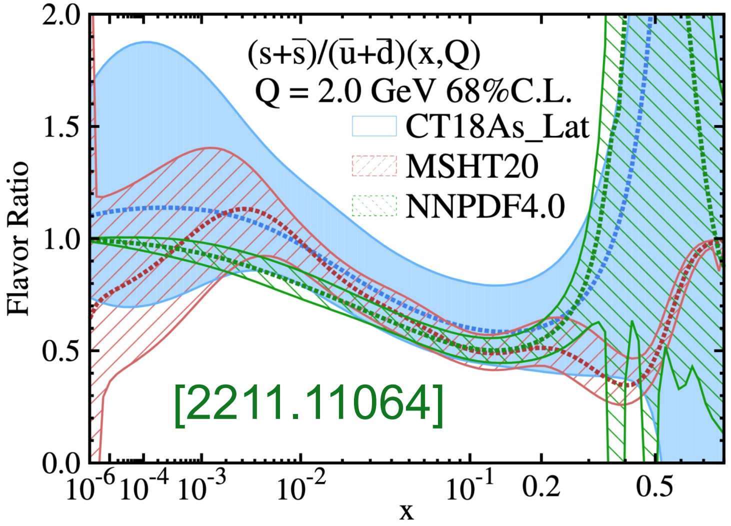


$$r_a = \frac{s-\bar{s}}{s+\bar{s}}$$



- CT18, ABMP16 and ATLAS21 assume zero strangeness asymmetry ($s-\bar{s}$) at initial scale
- MSHT20 and NNPDF4.0 show preference of a positive strangeness asymmetry from inclusive data

$$r_s = \frac{s+\bar{s}}{\bar{u}+\bar{d}}$$



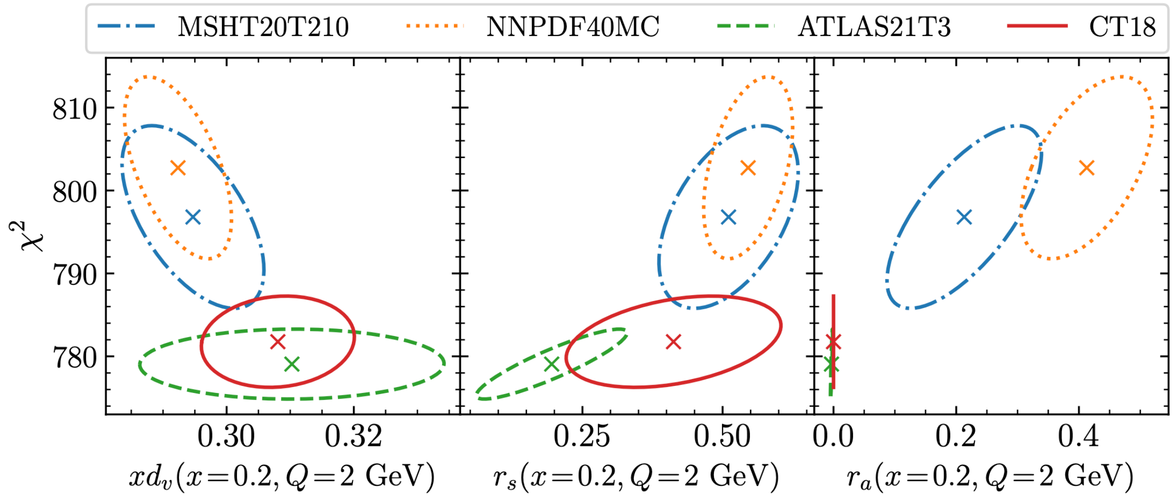
Applications: constraining SOTA proton PDFs at NNLO

We may for example constrain the **strange** quark PDFs with **SIDIS** data.

LO cross section difference of $K^{\pm}$ with iso-scalar target

$$\frac{d\sigma^{K^+}}{dxdydz} - \frac{d\sigma^{K^-}}{dxdydz}$$
$$\sim 2\Big(u_v(x) + d_v(x)\Big)\Big(D_u^{K^+}(z) - D_{\bar{u}}^{K^+}(z)\Big)$$
$$+\Big(s(x) - \bar{s}(x)\Big)\Big(D_s^{K^+}(z) - D_{\bar{s}}^{K^+}(z)\Big) + \cdots$$

- SIDIS measurement by the COMPASS Collaboration can be sensitive to strangeness asymmetry



$$r_s = \frac{s+\bar{s}}{\bar{u}+\bar{d}} \qquad r_a = \frac{s-\bar{s}}{s+\bar{s}}$$

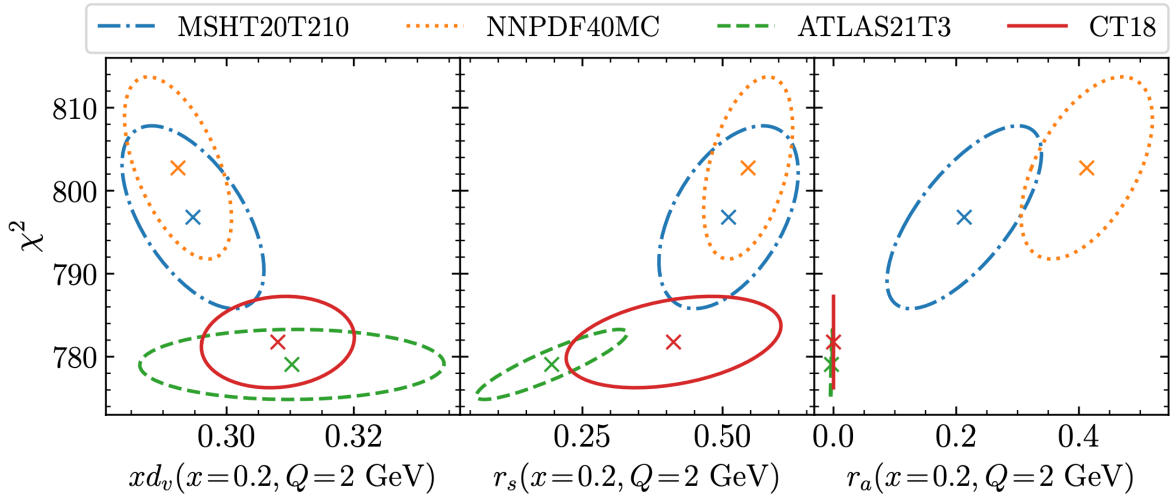
- The chi2 of best fit varies by 40 units with different choices of PDF sets
- A reduced strangeness asymmetry is preferred by SIDIS data

Applications: constraining SOTA proton PDFs at NNLO

❖ Modified PDF set with the inclusion of SIDIS data

- Reweighting of NNPDF4.0: applying a standard Gaussian weight $e^{-\chi^2/2}$
- Profiling of MSHT20 PDF

[see e.g. Phys.Rept.742(2018)1]



PDF variables before/after reweighting/profiling

	$xd_v(x=0.2,\,Q=2\text{GeV})$	$r_s(x=0.2,\,Q=2\text{GeV})$	$r_a(x=0.2,\,Q=2\text{GeV})$
NNPDF4.0	0.2924 ± 0.0084	0.547 ± 0.079	0.408 ± 0.107
NNPDF4.0(reweighting)	0.3021 ± 0.0069	0.438 ± 0.066	0.281 ± 0.086
MSHT20	0.295 ± 0.011	0.511 ± 0.124	0.213 ± 0.126
MSHT20(profiling)	0.298 ± 0.011	0.481 ± 0.121	0.167 ± 0.136

Modified PDF sets generated, taking into account SIDIS data

Summary

- First global analysis of FFs at full NNLO in QCD is performed.

- Recent BESIII measurements + SIDIS data allows a test of collinear factorization at low Q.

- Impacts of fragmentation data on the NNLO PDFs are studied.

Thank you for your attention!