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Semileptonic form factors of the open-charm mesons in $N_f = 4$ holographic QCD

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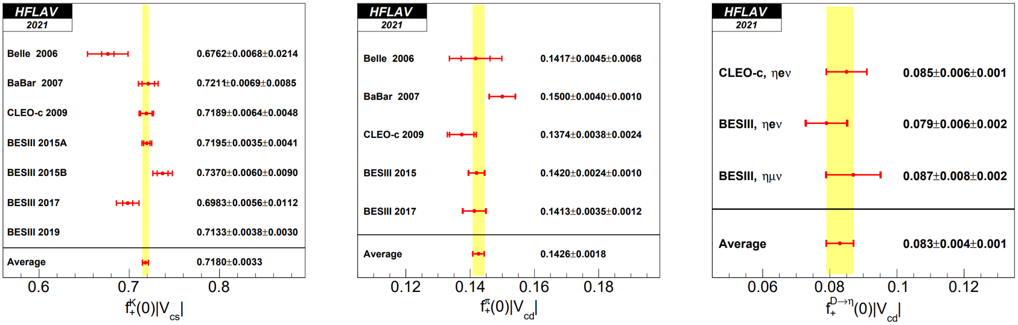
Introduction

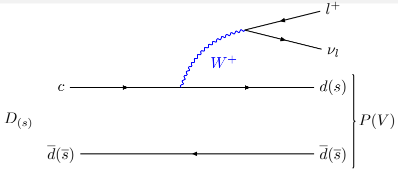
- Semileptonic decays involve the transition of a heavy meson (such as B or D) to a lighter meson via the exchange of a W boson.
- Understanding the form factors governing these transitions is essential for precision measurements of CKM matrix elements and testing the Standard Model.
- In particular, semileptonic $D_{(s)}$ meson decays offer a valuable avenue for directly determining the CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$.



(HFLAV Collaboration, PhysRevD.107.052008)

Introduction

- The Feynman diagram of the semileptonic decay process of $D_{(s)}$ to a pseudoscalar or a vector meson:



- The matrix elements within the SM is defined by (M. A. Ivanov et al., Front. Phys.(2019))

$$\mathcal{M}\left(D_{(s)} \to (P,V) l^+ \nu_l\right) = \frac{G_F}{\sqrt{2}} V_{cq}^* \left\langle (P,V) \right| \bar{q} \gamma^\mu (1-\gamma_5) c \left| D_{(s)} \right\rangle \bar{\nu}_l \gamma^\mu (1-\gamma_5) l$$

- The transition form factors are defined by (M. Wirbel et al., Z. Phys. C(1985))

$$\left\langle P\left(p_2\right)\left|V^\mu\right| D_{(s)}\left(p_1\right)\right\rangle = F_+\left(q^2\right)\left[P^\mu - \frac{M_1^2 - M_2^2}{q^2} q^\mu\right] + F_0\left(q^2\right) \frac{M_1^2 - M_2^2}{q^2} q^\mu$$

$$\left\langle V\left(p_2, \epsilon_2\right)\left|V^\mu - A^\mu\right| D_{(s)}\left(p_1\right)\right\rangle = -\left(M_1 + M_2\right) \epsilon_2^{*\mu} A_1\left(q^2\right) + \frac{\epsilon_2^* \cdot q}{M_1 + M_2} P^\mu A_2\left(q^2\right)$$

$$+ 2M_2 \frac{\epsilon_2^* \cdot q}{q^2} q^\mu \left[A_3\left(q^2\right) - A_0\left(q^2\right)\right] + \frac{2i\varepsilon_{\mu\nu\rho\sigma} \epsilon_2^{*\nu} p_1^\rho p_2^\sigma}{M_1 + M_2} V\left(q^2\right)$$

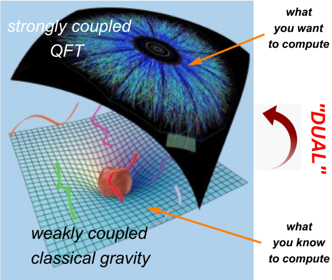
AdS/CFT Correspondence

- AdS/CFT Correspondence (Adv.Theor.Math.Phys.2,231 (1998))

Type IIB string theory on $AdS_5 \times S^5$ in low-energy approximation $\Longleftrightarrow$ $\mathcal{N}=4$ Super Yang-Mills theory on AdS boundary in the limit $\lambda = g_{YM}^2 N_c >> 1$

- GKP-Witten relation (hep-th/9802109, hepth/9802150):
  The partition function of a gauge theory with scale invariance (conformal invariance) is equivalent to the partition function of string theory on the AdS_5 spacetime.

$$Z_{CFT} = Z_{AdS_5}$$

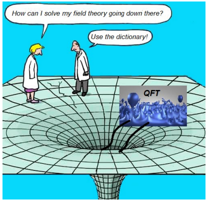


1908.02667v2 [hep-th]

AdS/CFT dictionary

- The parameters on the field theory side, i.e., g_{YM} and N_c, are mapped to the parameters g_s and l_s on the string theory side by

$$g_{YM}^2 = 2\pi g_s \quad \text{and} \quad 2\lambda = 2g_{YM}^2 N_c = L^4/l_s$$



- Operator/Field correspondence:

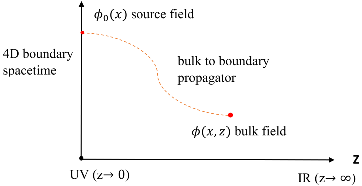
4D boundary operator $\mathcal{O}(x)$
local, gauge invariant, scaling dim. Δ

$\Longleftrightarrow$

5D bulk field with mass: $\Delta(\Delta - d) = m^2 L^2$
$\phi(x, z \to 0) \to z^{4-\Delta}\phi_0(x) + z^{\Delta} < \mathcal{O}(x) >$

$$\left\langle e^{i\int d^4x \phi_0(x)\mathcal{O}(x)} \right\rangle_{CFT} = e^{iS_{5D}[\phi(x,z)]}|_{\phi(x,z\to 0)\to\phi_0(x)}$$

$$\langle 0|\mathcal{T}\mathcal{O}(x_1)\ldots\mathcal{O}(x_n)|0\rangle = \left.\frac{(-i)^n \delta^n e^{iS_{AdS}}}{\delta\phi^0(x_1)\ldots\delta\phi^0(x_n)}\right|_{\phi^0=0}$$



$N_f = 4$ Soft Wall Model

- AdS metric : $ds^2 = g_{MN}dx^M dx^N = \frac{L}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right), 0 < z < \infty$
- Conformal invariance broken by a background dilaton field in the bulk: $\phi(z) = \mu^2 z^2$

- Operators/fields correspondence:

4D : $\mathcal{O}(x)$	5D : $\phi(x,z)$	p	Δ	$(M_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

- The 5D action (Y. Chen and M. Huang,prd(2022)):

$$S_M = -\int d^5x\sqrt{-g}e^{-\phi}\,\mathrm{Tr}\left\{\left(D^M X\right)^\dagger (D_M X) + M_5^2|X|^2 - \kappa|X|^4 + \frac{1}{4g_5^2}\left(L^{MN}L_{MN} + R^{MN}R_{MN}\right)\right\}$$

$M_5^2 = (\Delta - p)(\Delta + p - 4)$, $X = e^{i\pi}X_0 e^{i\pi}$, $X_0 = \frac{1}{2}\,\mathrm{diag}\left[v_u(z), v_d(z), v_s(z), v_c(z)\right]$.

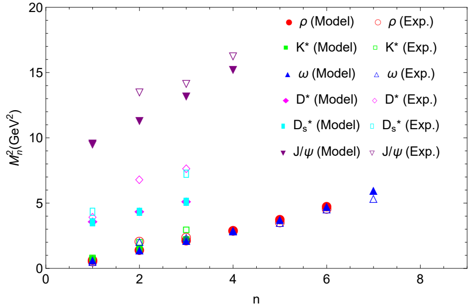
- VEV at the UV and IR regions

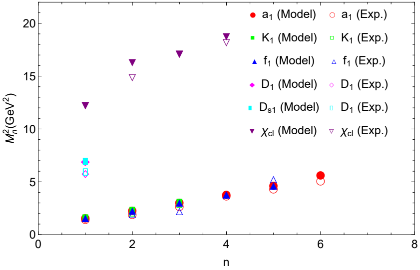
$$\begin{aligned} v(z\to 0) &= m_q z + \sigma z^3 + \mathcal{O}(z^5), \\ \Delta m^2 &\equiv \left(m^2_{A_n} - m^2_{V_n}\right)_{n\to\infty} = g_5^2\frac{L^2 v^2(z)}{z^2}(z\to\infty) \longrightarrow v(z\to\infty)\sim z. \end{aligned}$$

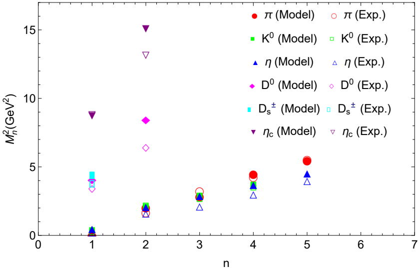
Meson spectra

$\mu = 0.43$	$\kappa = 30$
$m_u = 0.0032$	$\sigma_u = (0.2962)^3$
$m_s = 0.1423$	$\sigma_s = (0.2598)^3$
$m_c = 1.5971$	$\sigma_c = (0.302)^3$

The values of the free parameters with the unit of GeV.







Three-point functions

- To obtain the transition form factors from the holographic model, we need to expand the action up to the third order in the fields.

$$S^{(3)} = -\int d^5x \Big\{ \eta^{MN} \frac{e^{-\phi(z)}}{z^3} \big(2\left(A^a_M - \partial_M \pi^a\right) V^b_N \pi^c g^{abc} + V^a_M \left(\partial_N \left(\pi^b \pi^c\right) - 2A^b_M \pi^c\right) h^{abc}$$

$$- V^a_M V^b_N \pi^c k^{abc}\big) + \frac{e^{-\phi(z)}}{2g_5^2 z} \eta^{MP} \eta^{NQ} \left(V^a_{MN} V^b_P V^c_Q + V^a_{MN} A^b_P A^c_Q + A^a_{MN} V^b_P A^c_Q + A^a_{MN} A^b_P V^c_Q\right) f^{bca} \Big\}$$

$$g^{abc} = iTr\left(\{t^a, X_0\}[t^b, \{t^c, X_0\}]\right),$$

$$h^{abc} = iTr\left([t^a, X_0]\{t^b, \{t^c, X_0\}\}\right),$$

$$k^{abc} = -2Tr\left([t^a, X_0][t^b, \{t^c, X_0\}]\right).$$

$$V^a_{\mu\perp}(q,z) = V^{0a}_{\mu\perp}(q)\mathcal{V}^a(q^2,z),$$

$$A^a_{\mu\perp}(q,z) = A^{0a}_{\mu\perp}(q)\mathcal{A}^a(q^2,z),$$

$$\phi^a(q,z) = \phi^a\left(q^2,z\right) \frac{iq^\alpha}{q^2} A^{0a}_{\|\alpha}(q),$$

$$\pi^a(q,z) = \pi^a\left(q^2,z\right) \frac{iq^\alpha}{q^2} A^{0a}_{\|\alpha}(q),$$

$$\langle 0|\mathcal{T} J^{\mu a}_{A\|}(x) J^{\nu b}_{V\perp}(0) J^{\alpha c}_{A\|}(w)|0\rangle = \frac{i\delta S_{V\pi\pi}}{i^3 \delta A^{0a}_{\|\mu}(x) \delta V^{0b}_{\perp\nu}(y) \delta A^{0c}_{\|\alpha}(w)}$$

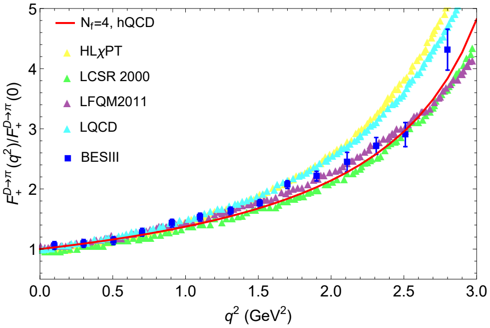
$$\langle 0|\mathcal{T} J^{\mu a}_{V\perp}(x) J^{\nu b}_{V\perp}(0) J^{\alpha c}_{A\|}(w)|0\rangle = \frac{i\delta S_{VV\pi}}{i^3 \delta V^{0a}_{\perp\mu}(x) \delta V^{0b}_{\perp\nu}(y) \delta A^{0c}_{\|\alpha}(w)}$$

$$\langle 0|\mathcal{T} J^{\mu a}_{V\perp}(x) J^{\nu b}_{A\perp}(0) J^{\alpha c}_{A\|}(w)|0\rangle = \frac{i\delta S_{VA\pi}}{i^3 \delta V^{0a}_{\perp\mu}(x) \delta A^{0b}_{\perp\nu}(y) \delta A^{0c}_{\|\alpha}(w)}$$

$D \to \pi l^+ \nu_l$

$$F_+(q^2) = \int dz \frac{e^{-\phi(z)}}{z} \left(f^{abc} \partial_z \phi^a \mathcal{V}^b(q^2, z) \partial_z \phi^c - \frac{2g_5^2}{z^2} (\pi^a - \phi^a) \mathcal{V}^b(q^2, z)(\pi^c - \phi^c)(g^{abc} - h^{bac}) \right)$$

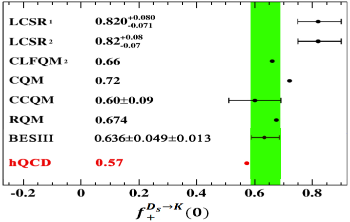
FFs	hQCD	LCSR2000	HLχPT	LFQM2011	LQCD	Exp.
$f_+^{D\to\pi}(0)$	0.58	0.65	0.61	0.66	0.64	0.6372±0.008

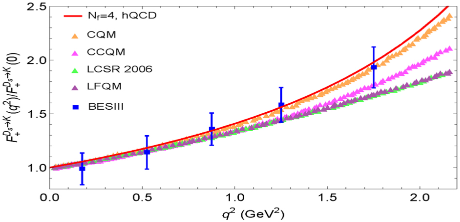


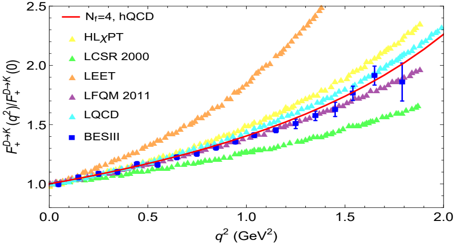
- HLχPT: S. Fajfer and J. F. Kamenik, prd(2005)
- LCSR: A. Khodjamirian et al., prd(2000)
- LFQM: R. C. Verma, J. Phys. G (2012)
- LQCD: C. Aubin et al, prl(2005)
- BESIII: M. Ablikim et al., prd(2015)

$D_{(s)} \to K l^+ \nu_l$

$$F_+(q^2) = \int dz \frac{e^{-\phi(z)}}{z} \left(f^{abc} \partial_z \phi^a \mathcal{V}^b(q^2, z) \partial_z \phi^c - \frac{2g_5^2}{z^2} (\pi^a - \phi^a) \mathcal{V}^b(q^2, z)(\pi^c - \phi^c)(g^{abc} - h^{bac}) \right)$$





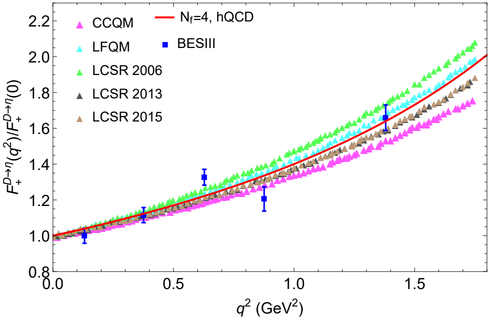


HLχPT: S. Fajfer and J. F. Kamenik, prd(2005) ; LCSR: A. Khodjamirian et al., prd(2000) ; LEET: J. Charles et al., prd(1999) ; LFQM: R. C. Verma, J. Phys. G (2012) ; LQCD: C. Aubin et al, prl(2005) ; BESIII: M. Ablikim et al., prd(2015) ; CQM: D. Melikhov and B. Stech, prd(2000) ; CCQM: N. R. Son et al., prd(2018) ; LCSR: Y. L. Wu, et al., IJMPA(2006) ; BESIII: M. Ablikim et al., prl(2019)

$D \to \eta$

$$F_+(q^2) = \int dz \frac{e^{-\phi(z)}}{z} \left(f^{abc} \partial_z \phi^a \mathcal{V}^b(q^2, z) \partial_z \phi^c - \frac{2g_5^2}{z^2} (\pi^a - \phi^a) \mathcal{V}^b(q^2, z)(\pi^c - \phi^c)(g^{abc} - h^{bac}) \right)$$

FFs	hQCD	LCSR2006	LCSR2013	LCSR2015	LFQM	CCQM	Exp.
$f_+^{D\to\eta}(0)$	0.31	0.556	0.552	0.429	0.71	0.67	0.39

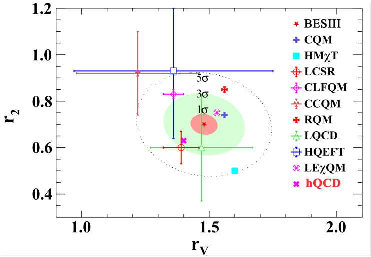


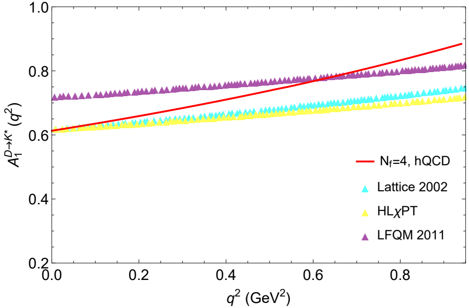
- CCQM: N. R. Son et al., prd(2018)
- LFQM: R. C. Verma, J. Phys. G (2012)
- LCSR2006: Y. L. Wu, et al., IJMPA(2006)
- LCSR2013: N. Offen et al., prd(2013)
- LCSR2015: G. Duplancic and B. Melic, JHEP(2015)
- BESIII: M. Ablikim et al., prl(2020)

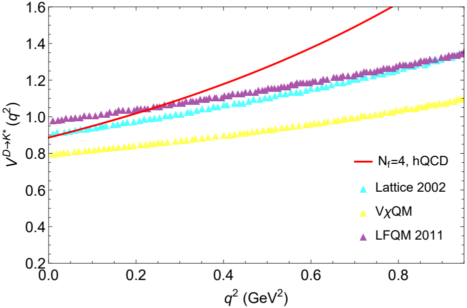
$D \to K^*$

$$V(q^2) = \frac{(M_1+M_2)g_5^2}{2}\int dz \frac{e^{-\phi(z)}}{z^3} k^{abc} V^a(z)\mathcal{V}^b(q^2,z)\pi^c(z)$$

$$A_1(q^2) = \int dz \frac{e^{-\phi(z)}}{z}\left(\frac{M_1^2+M_2^2-q^2}{2(M_1+M_2)}\right) f^{bac}\mathcal{A}^a(q^2,z)V^b(z)\phi^c(z)$$
$$-\int dz \frac{e^{-\phi(z)}}{z^3}\frac{2g_5^2}{(M_1+M_2)}\mathcal{A}^a(q^2,z)V^b(z)\pi^c(z)(g^{abc}-h^{bac})$$







Summary

- We investigate the semileptonic form factors of $D_{(s)}$ mesons from a modified soft-wall 4-flavor holographic model.
- The model successfully reproduces the masses of the vector mesons, ρ, K^*, ω, D^*, D_s^*, and J/ψ, axial vector mesons, a_1, K_1, f_1, D_1, D_{s1}, and χ_{c1}, and pseudoscalar mesons, π, K, η, D, D_s, and η_c.
- The result of the form factor for $D^+ \to \pi l^+ \nu_l$, $f_+(q^2)$ shows excellent agreement with the experimental data, and it is comparable with lattice QCD and other theoretical approaches.
- The normalized form factor $f_+(q^2)$ of the $D_{(s)}$-to-kaon is well consistent with the experimental and lattice data.
- Similarly the normalized $f_+(q^2)$ for the $D \to \eta$ is compatible with data.
- Finally, we predicted the vector form factors $V(q^2)$ and $A_1(q^2)$ for the decays $D \to K^*$.

谢谢大家!

Thanks for your attention!