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Towards High-Precision Lattice QCD Calculations of Heavy Meson LCDAs

Based on arXiv: 25xx.xxxxx & arXiv: 2403.17492,
& Phys.Rev.D 111 (2025) 3, 034503

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Outline

- Motivation
- Theoretical framework
    - Sequential effective theory
    - Discussion of renormalization
- Lattice simulation
- Summary and prospect

🤔 Motivation

Factorization

[Beneke, Buchalla, Neubert, Sachrajda, 1999]

$$\langle \pi(p')\pi(q)|Q_i|\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx T_i^{\mathrm{I}}(x)\phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{\mathrm{II}}(\xi,x,y)\phi_B(\xi)\phi_\pi(x)\phi_\pi(y)$$

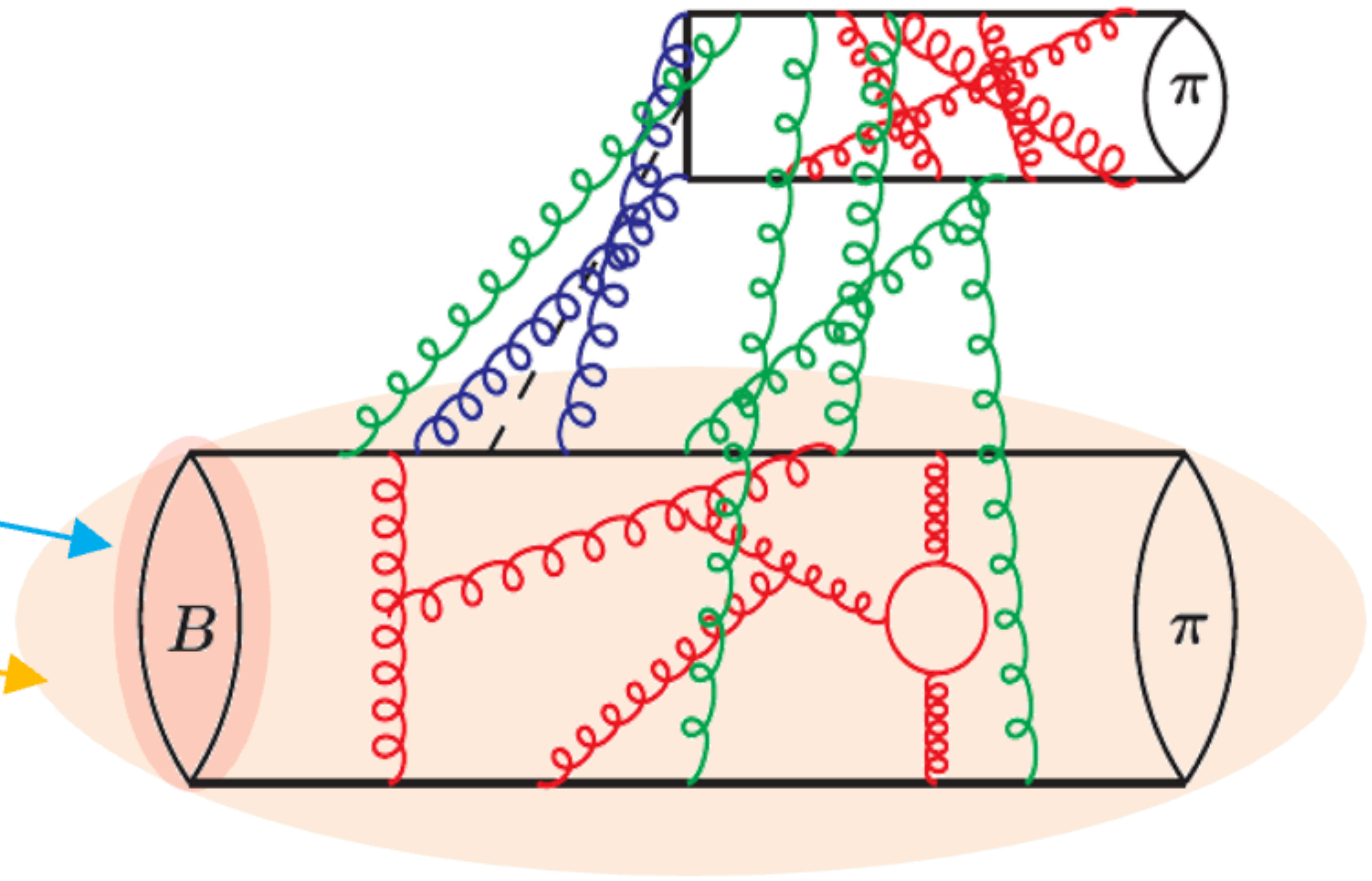
Form factor

Heavy meson LCDA

Hard kernel ⊗ LCDA

Hard kernel: Perturbative

Meson LCDAs: Nonperturbative



$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

Model dependence

$$f^0_{B\to\pi}(0) = 0.122\times\left[1\pm 0.07\Big|_{S_0^\pi}\pm 0.11\Big|_{\Lambda_q}\pm 0.02\Big|_{\lambda_E^2/\lambda_H^2}{}^{+0.05}_{-0.06}\Big|_{M^2}\pm 0.05\Big|_{2\lambda_E^2+\lambda_H^2}{}^{+0.06}_{-0.10}\Big|_{\mu_h}\pm 0.04\Big|_{\mu}{}^{+1.36}_{-0.56}\Big|_{\lambda_B}{}^{+0.25}_{-0.43}\Big|_{\sigma_1,\sigma_2}\right]$$

Inverse moment and log moment

[Gao, Lu, Shen, Wang, Wei, 2020; Cui, Huang, Shen, Wang, 2023]

🤔 Model dependence of heavy meson LCDAs

Models of leading twist heavy meson LCDAs

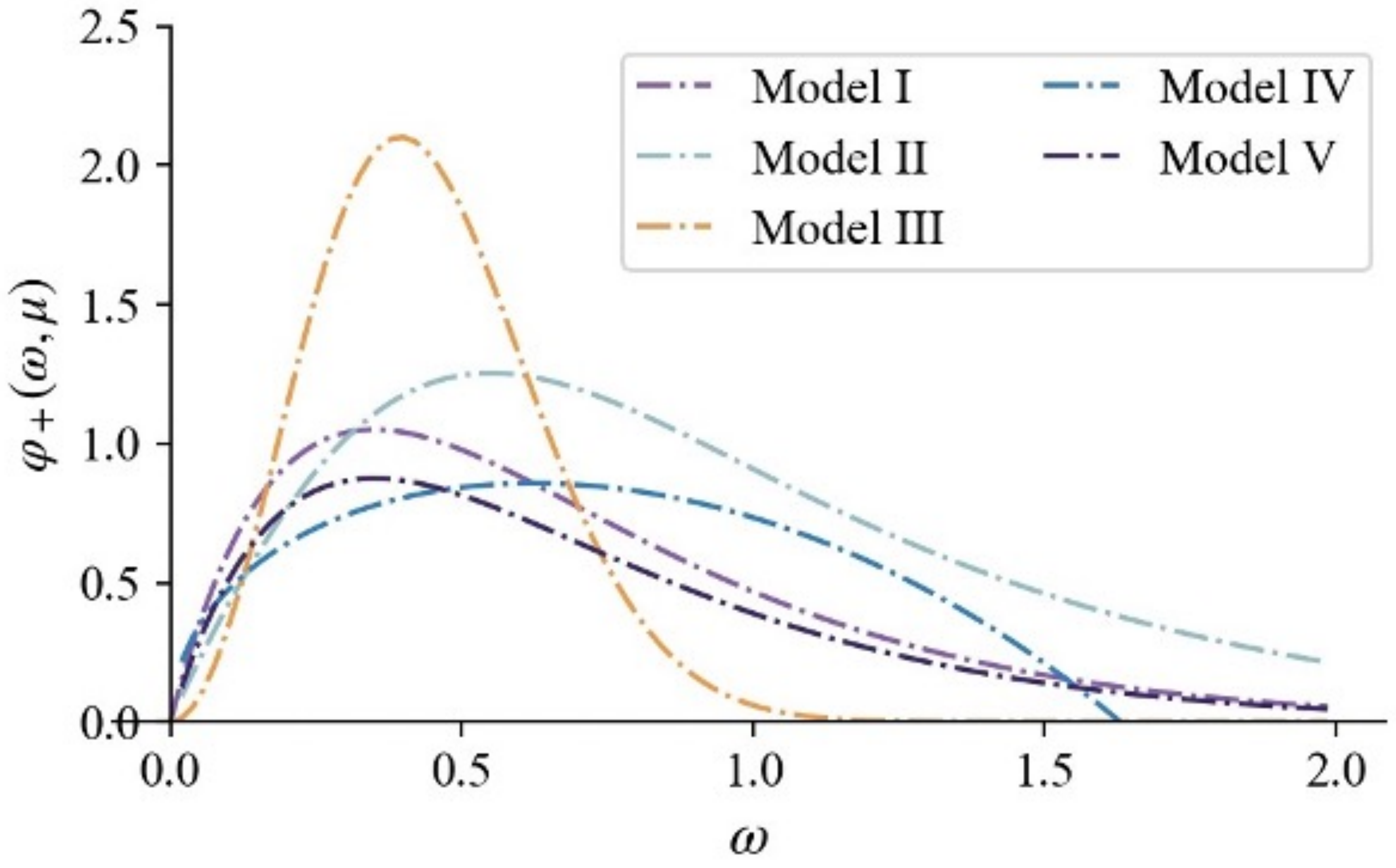
$$\varphi_{\rm I}^{+}(\omega,\mu_0) = \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0} \,,$$

$$\varphi_{\rm II}^{+}(\omega,\mu_0) = \frac{4}{\pi\omega_0}\frac{k}{k^2+1}\left[\frac{1}{k^2+1} - \frac{2\left(\sigma_B^{(1)}-1\right)}{\pi^2}\ln k\right] \,,$$

$$\varphi_{\rm III}^{+}(\omega,\mu_0) = \frac{2\omega^2}{\omega_0\omega_1^2} e^{-(\omega/\omega_1)^2} \,,$$

$$\varphi_{\rm IV}^{+}(\omega,\mu_0) = \frac{\omega}{\omega_0\omega_2}\frac{\omega_2-\omega}{\sqrt{\omega\left(2\omega_2-\omega\right)}}\theta\left(\omega_2-\omega\right) \,,$$

$$\varphi_{\rm V}^{+}(\omega,\mu_0) = \frac{\Gamma(\beta)}{\Gamma(\alpha)}\frac{\omega}{\omega_0^2} e^{-\omega/\omega_0} U(\beta-\alpha, 3-\alpha, \omega/\omega_0) \,,$$



$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)} \,,$$

Model dependence

[Gao, Lu, Shen, Wang, Wei, 2020; Cui, Huang, Shen, Wang, 2023]

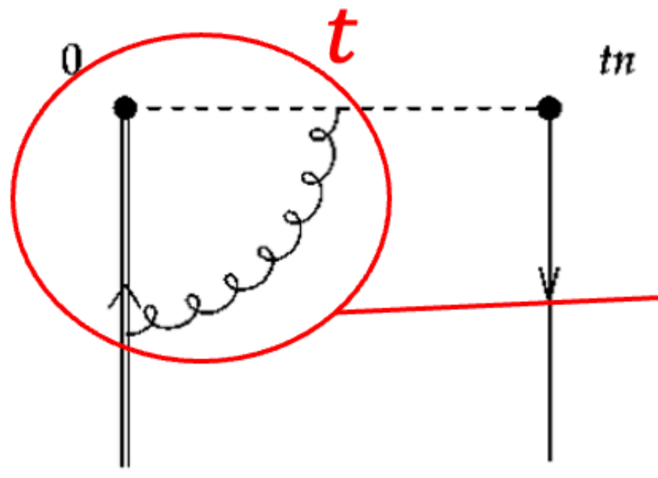
🤔 First-principle determination

Leading twist heavy meson LCDAs

$t \equiv z \cdot v$

$$\langle 0|O_v^P(t)|H(p_H)\rangle = i\tilde{f}_H m_H n_+ \cdot v \int_0^\infty d\omega e^{i\omega t n_+ \cdot v} \varphi^+(\omega;\mu),$$

[Braun, Ivanov, Korchemsky, 2004]



(a)

$$O_v^{P,\text{ren}}(t,\mu) = O_v^{P,\text{bare}}(t) + \frac{\alpha_s C_F}{4\pi}\left\{\left(\frac{4}{\hat{\epsilon}^2} + \frac{4}{\hat{\epsilon}}\ln(it\mu)\right) O_v^{P,\text{bare}}(t) - \frac{4}{\hat{\epsilon}}\int_0^1 du \frac{u}{1-u}\left[O_v^{P,\text{bare}}(ut) - O_v^{P,\text{bare}}(t)\right]\right\}$$

Diverge at t→0!

- Diverge at t→0 ⟺ No local limit
- Non-negative moments $\int dk\, k^n \varphi_+(k)$ for n=0,1,2,… are not related to OPE
- Cannot obtain φ_B^+ from lattice QCD through moments.

‣ How to solve the problem?

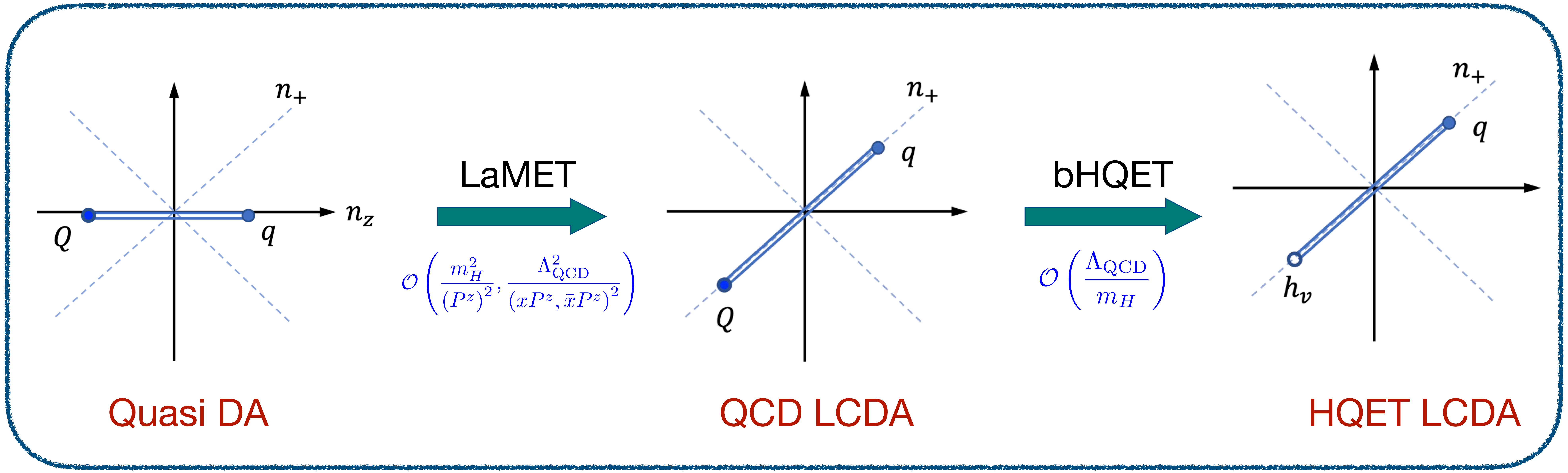
🤔 Off light-cone Wilson line, still heavy quark field h_v;

[Wang², Xu, Zhao, 2020; Xu, Zhang, 2022; Hu, Wang, Xu, Zhao, 2024]

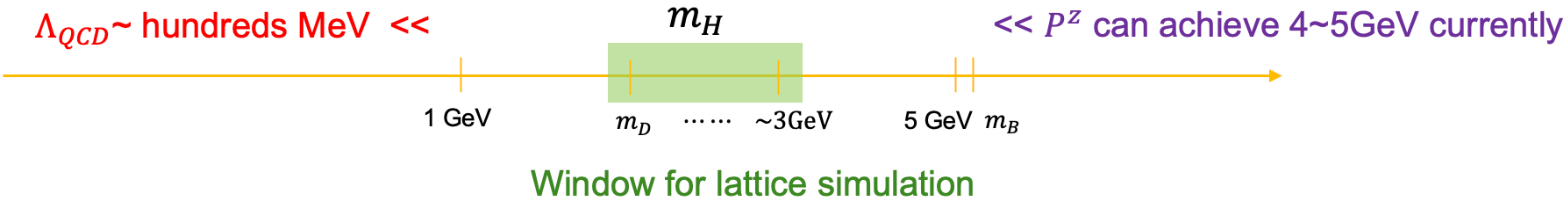
🤔 No h_v ⇒ QCD heavy quark.

*[**Han,** Wang, Zhang, et.al, 2403.17492; Han, Wang, Zhang, Zhang, 2408.13486; Deng, Wang, Wei, Zeng, 2409.00632]*

Sequential effective theory

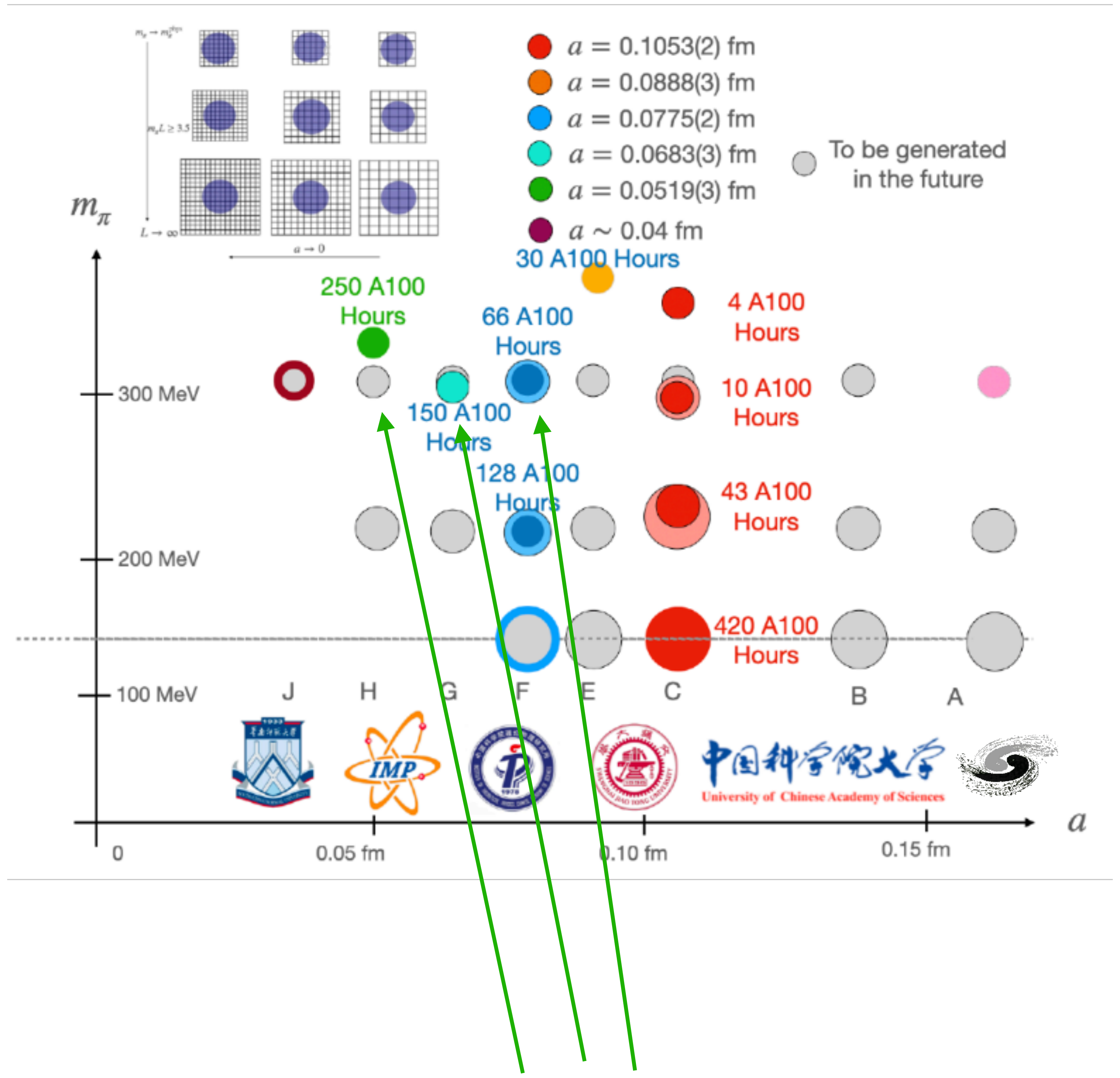


*[**Han**, Hua, Ji, Lü, et al., 2024; **Han**, Hua, Ji, Lü, et al., 2025]*



Lattice Setup

	Previous work	Towards precision calculation
Action	Tadpole improved Wilson clover fermion action	
Improvement	—	HYP smear for Wilson line; Multi-source enhancement
Lattice spacing	a=0.05187fm	a=0.0775fm, 0.06826fm, 0.05187fm
NPR	Simplified hybrid scheme	Hybrid scheme based on self-renormalization
Pz extrapolation	—	Infinite momentum extrapolation
Statistics	~5k	60k~100k
	arXiv: 2403.17492 Phys.Rev.D 111 (2025) 3, 034503	arXiv: 25xx.xxxxx



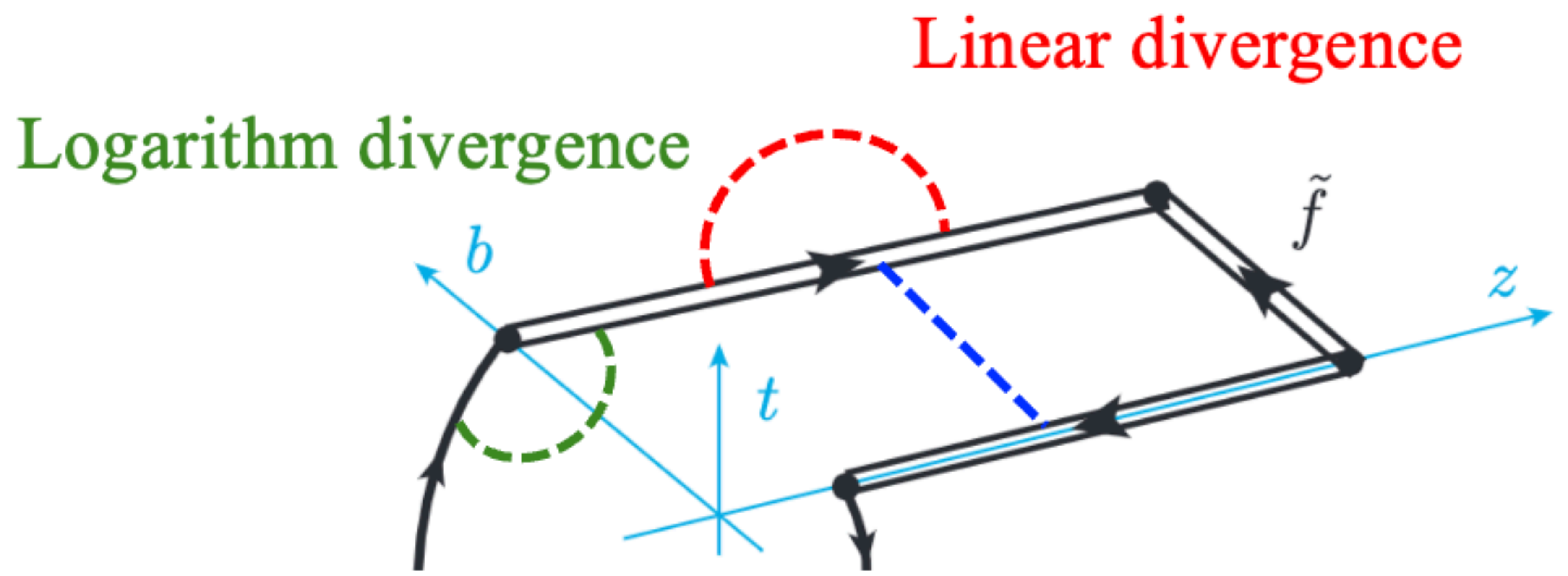
Ensembles we use

NPR of nonlocal operators

Linear divergence + ...

$$\left[\bar{\psi}(z)\,\Gamma W(z,0)\,\psi(0)\right]_B = \underline{e^{\delta m|z|}}\,\underline{Z(a)}\left[\bar{\psi}(z)\,\Gamma W(z,0)\,\psi(0)\right]_R$$

Log divergence

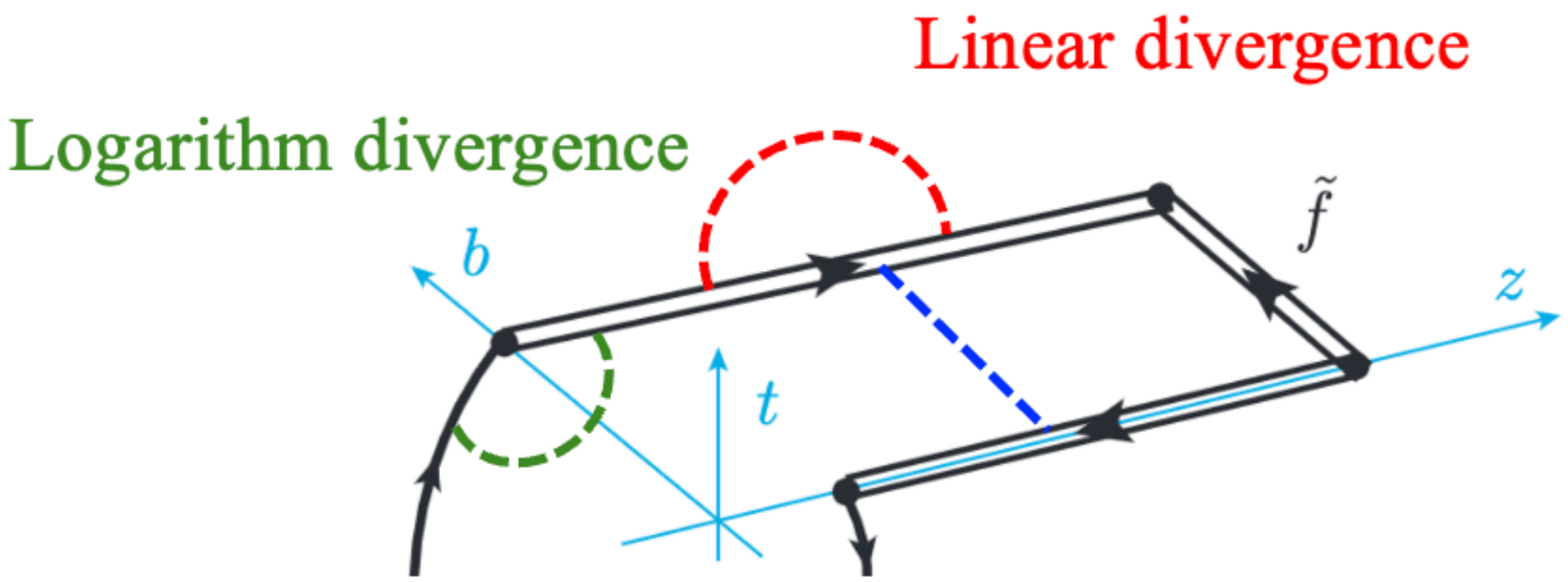


NPR of nonlocal operators

Linear divergence + …

$$\left[\bar{\psi}(z)\,\Gamma W(z,0)\,\psi(0)\right]_B = e^{\delta m|z|}Z(a)\left[\bar{\psi}(z)\,\Gamma W(z,0)\,\psi(0)\right]_R$$

Log divergence



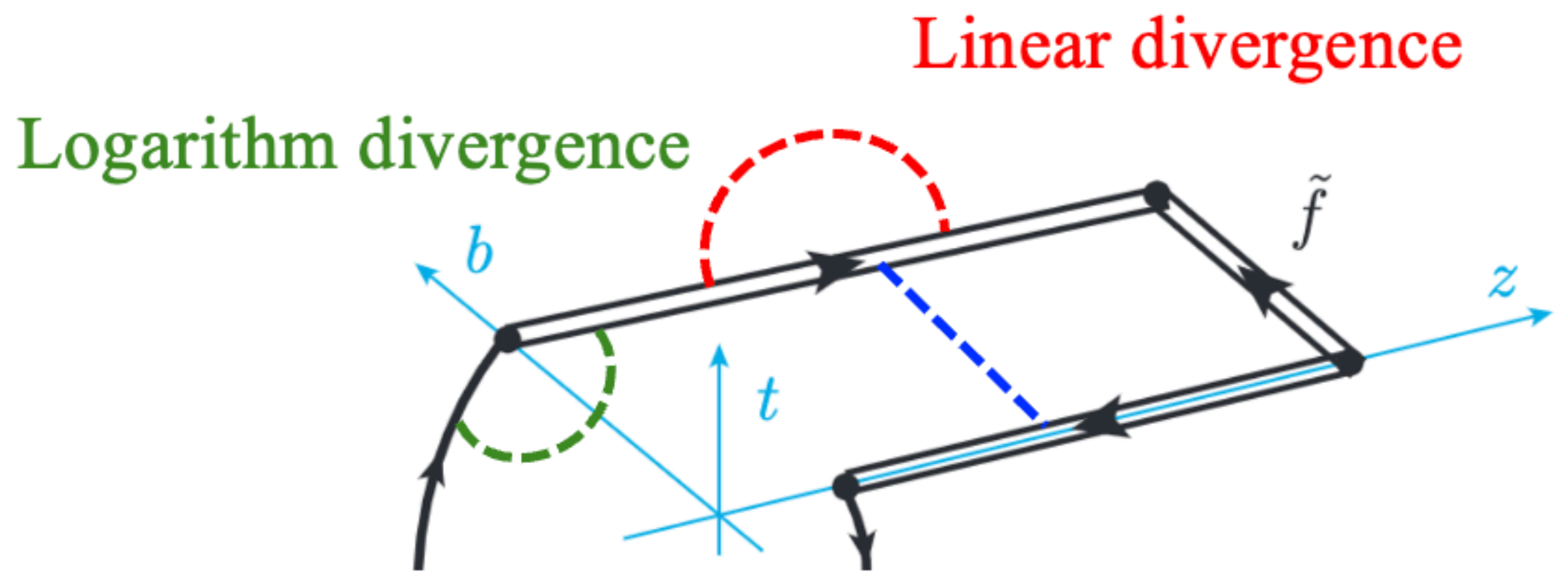
	RI/MOM scheme	Ratio scheme	Self-renormalization
Short distance	Local	Well	—
Large distance	Non-perturbative effects; IR logarithms of z	Invalid OPE	Cancel the linear divergence and discretization effect

NPR of nonlocal operators

Linear divergence + …

$$\left[\bar{\psi}(z)\,\Gamma W(z,0)\,\psi(0)\right]_B = e^{\delta m|z|} Z(a) \left[\bar{\psi}(z)\,\Gamma W(z,0)\,\psi(0)\right]_R$$

Log divergence



$$\langle O_\Gamma(z)\rangle = \Gamma\left(1 + \gamma g^2 \log(z^2/a^2) - m_{-1}\frac{z}{a} + \ldots\ldots\right)$$

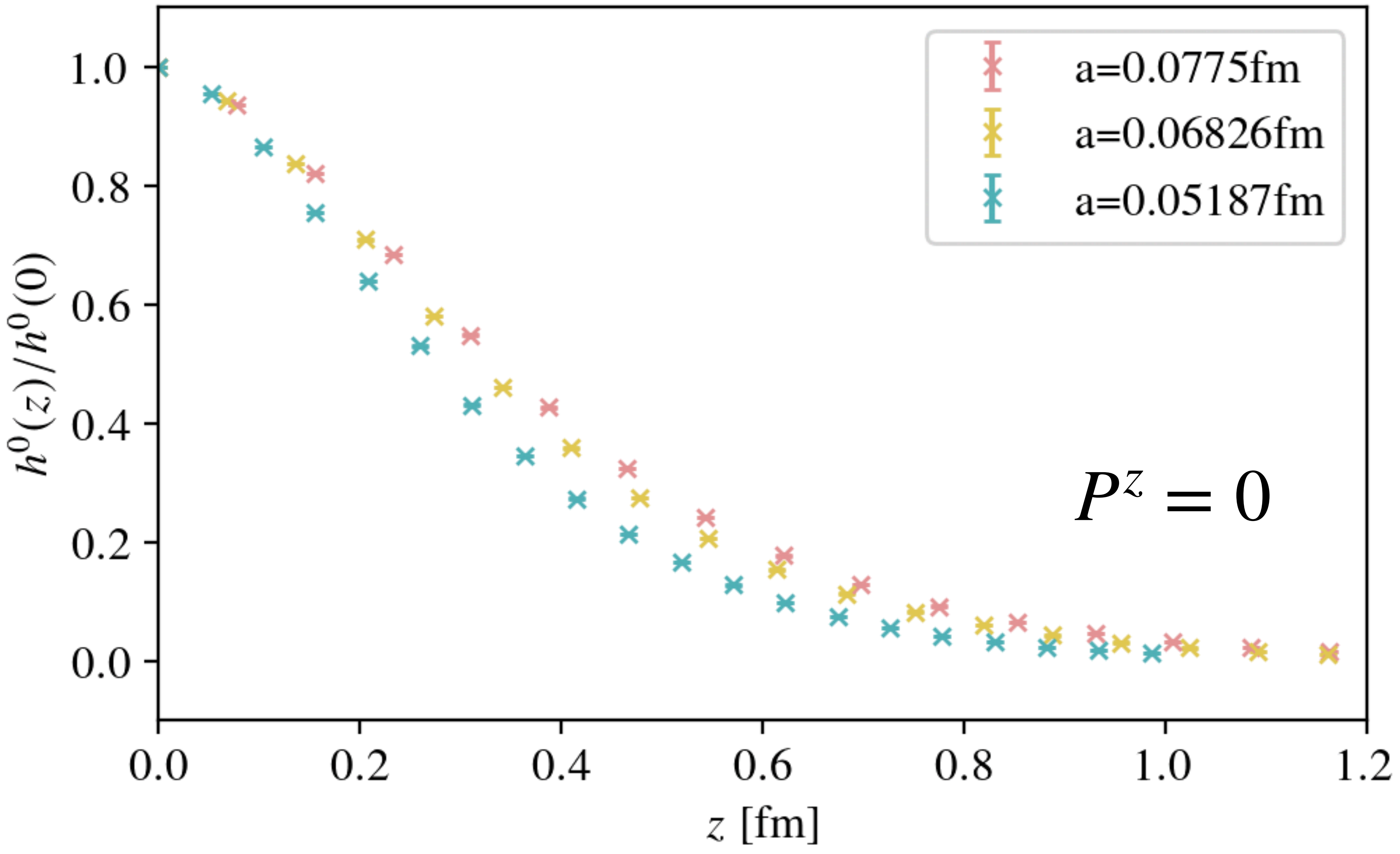
	⬅ v1	⬅ v2	➡ Current work
Data	Very limited	Limited	Relatively sufficient
Short distance	Ratio scheme	Ratio scheme	Ratio scheme
Large distance	Ratio scheme	RGR+LRR	Self renormalization

arXiv: 2403.17492 *Phys.Rev.D 111 (2025) 3, 034503*

Extraction of renormalization factor

[Hua, Su, Gui, et al. 2021]

Bare matrix element



$$\ln\tilde{h}^{\pi}_{B}(z, P_z = 0, 1/a) = \frac{kz}{a\ln[a\Lambda_{\rm QCD}]} + f(z)a^2 + \frac{3C_F}{b_0}\ln\left[\frac{\ln(1/a\Lambda_{\rm QCD})}{\ln(\mu/\Lambda_{\rm QCD})}\right] + \frac{1}{2}\ln\left[1 + \frac{d}{\ln[a\Lambda_{\rm QCD}]}\right]^2 + \begin{cases} \ln\left[C_{0,\rm NLO}(z,\mu)\right] + m_0 z & \text{if } z_0 \le z \le z_1 \\ g(z) & \text{if } z_1 < z \end{cases}, \quad (21)$$

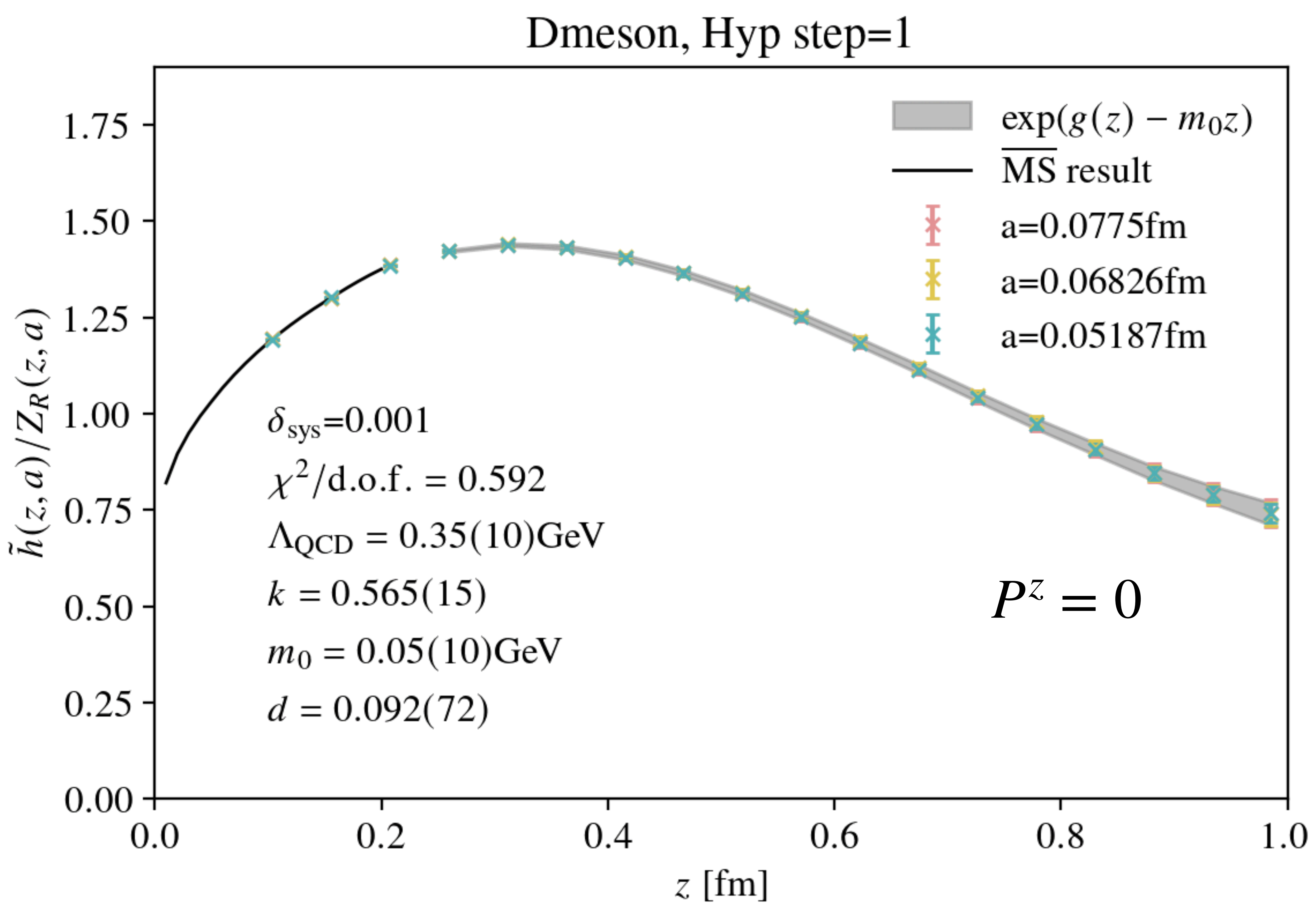
$$C_0^{\rm NLO}(z,\mu) = 1 + \frac{\alpha_s C_F}{2\pi}\left[\frac{3}{2}\log\left(\frac{1}{4}e^{2\gamma_E}z^2\mu^2\right) + A\right]$$

$$C_0^{\rm NLO+LRR}(z,\mu) = C_0^{\rm NLO}(z,\mu) + z\mu\left(C_{\rm PV}(z,\mu) - \sum_{i=0}^{k-1}\alpha_s^{i+1}(\mu) r_i\right)$$

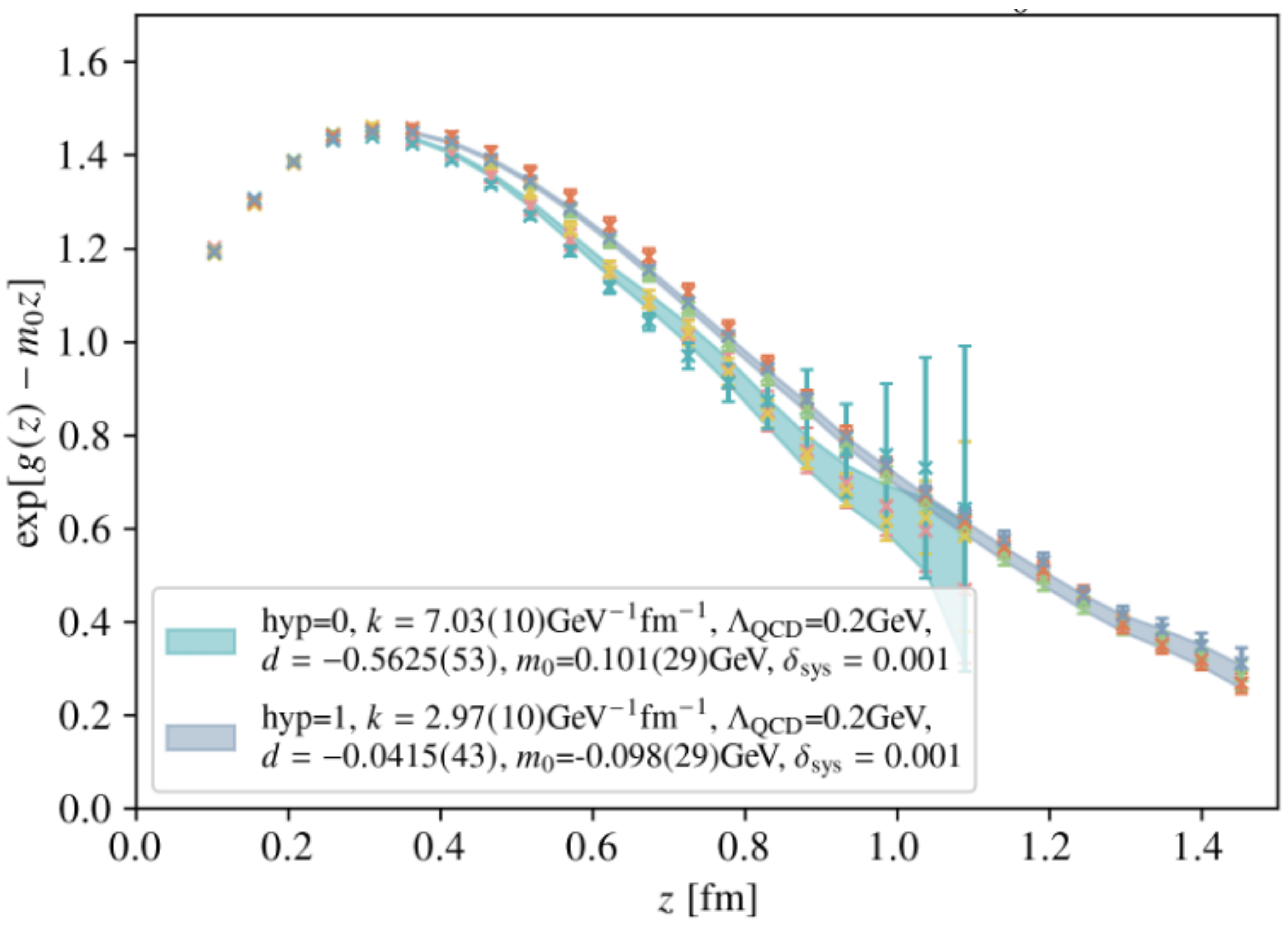
- Hypercubic smearing improved the SNR

Extraction of renormalization factor

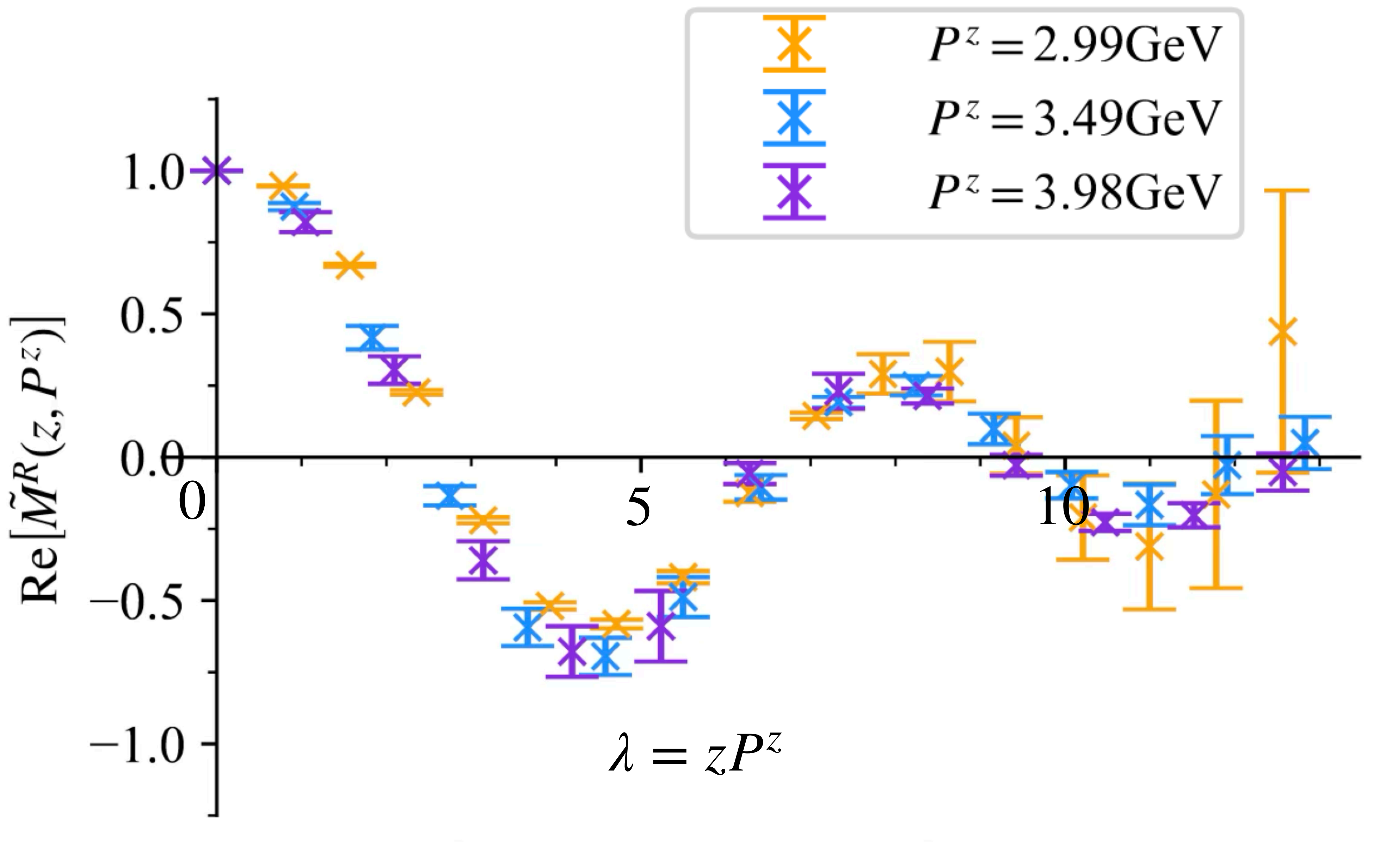
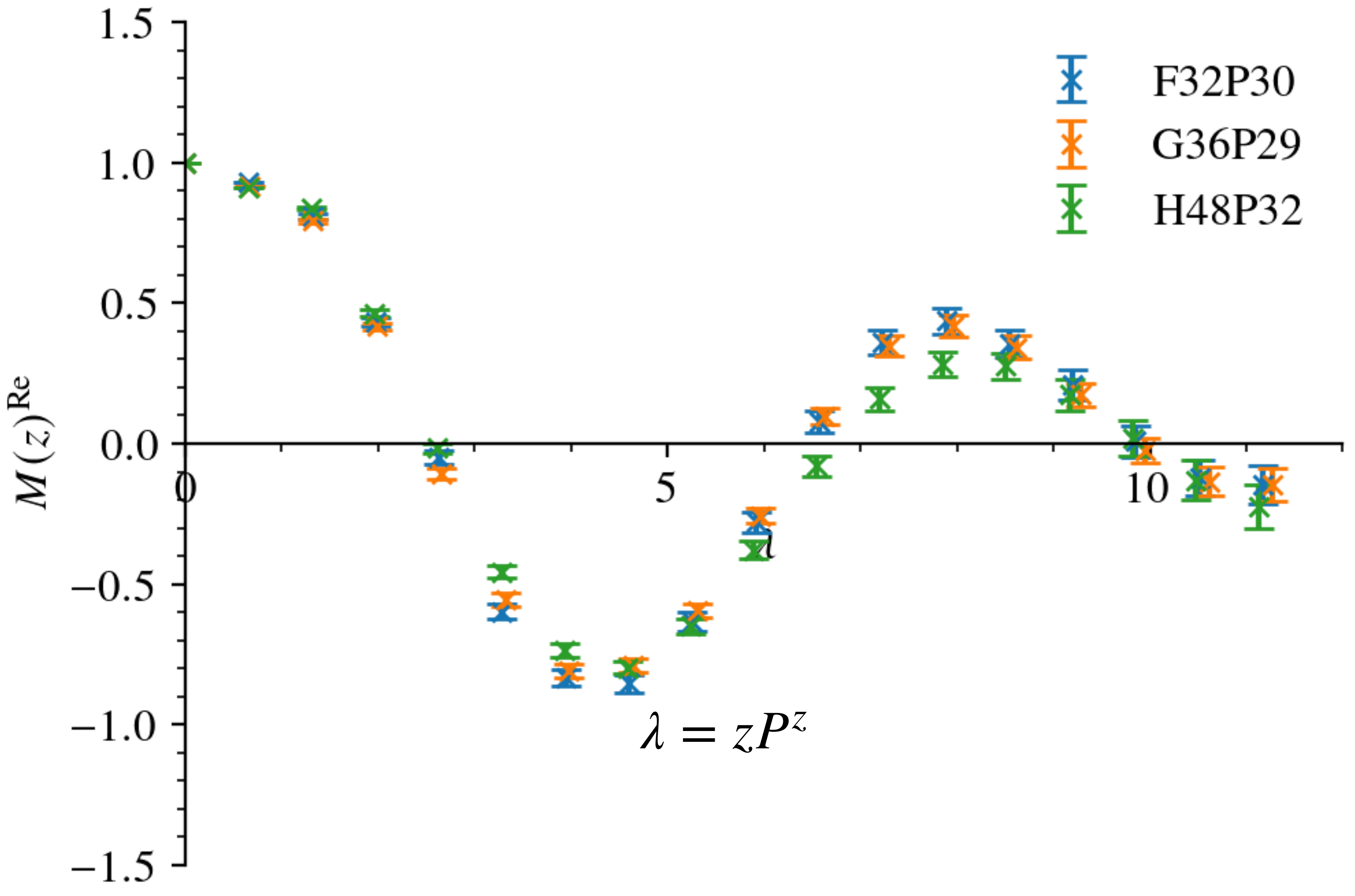
Renormalized matrix element



- Hypercubic smearing has only a minor impact on renormalization.



Renormalized matrix elements

⬅ Previous work	➡ Towards precision calculation
	

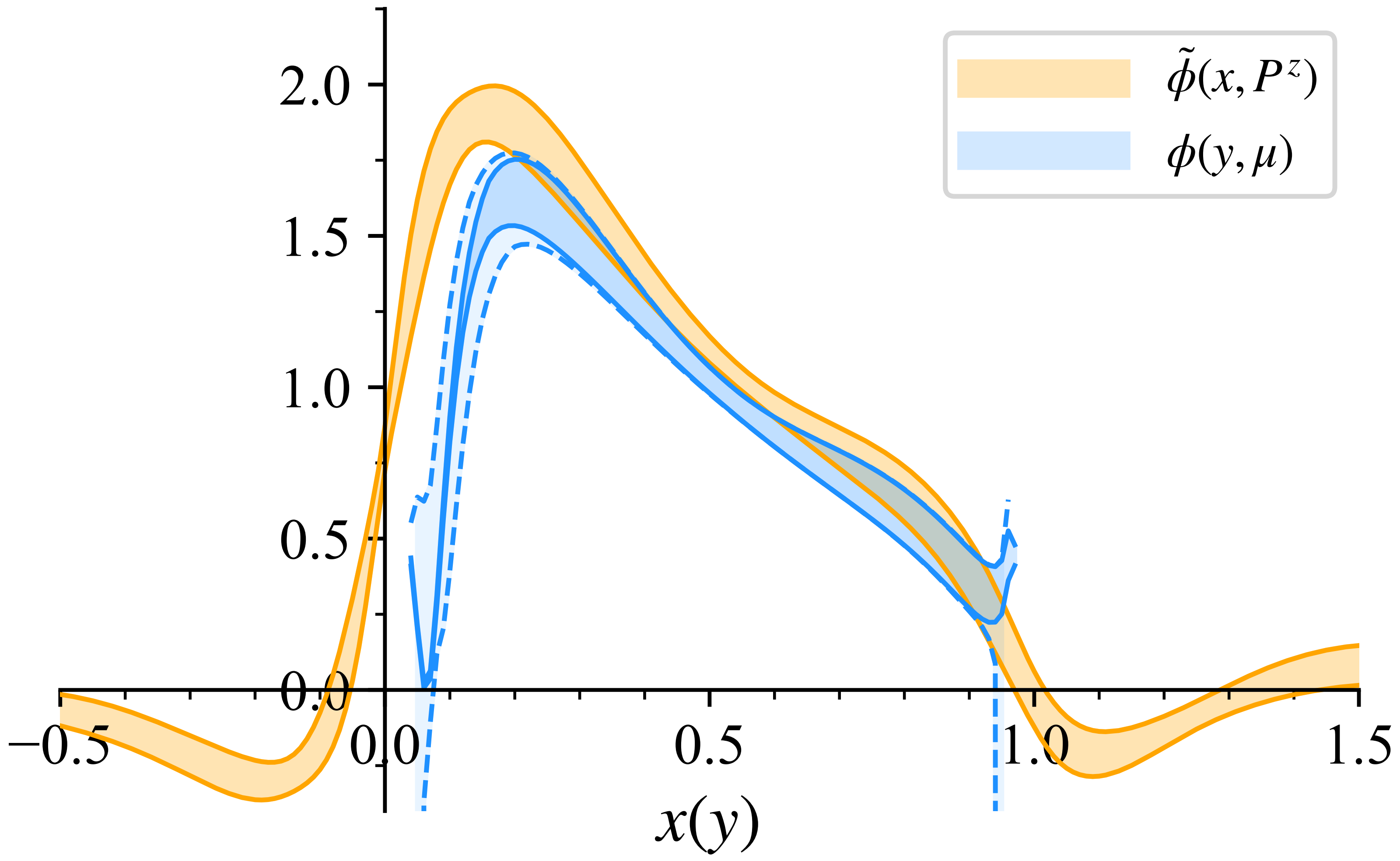
- Obtain a more precise results at large λ
- Multi lattice spacings to perform the continuum extrapolation

Matching for previous results

- LaMET matching

$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

[Liu, Wang, Xu, Zhang, Zhao, 2019]

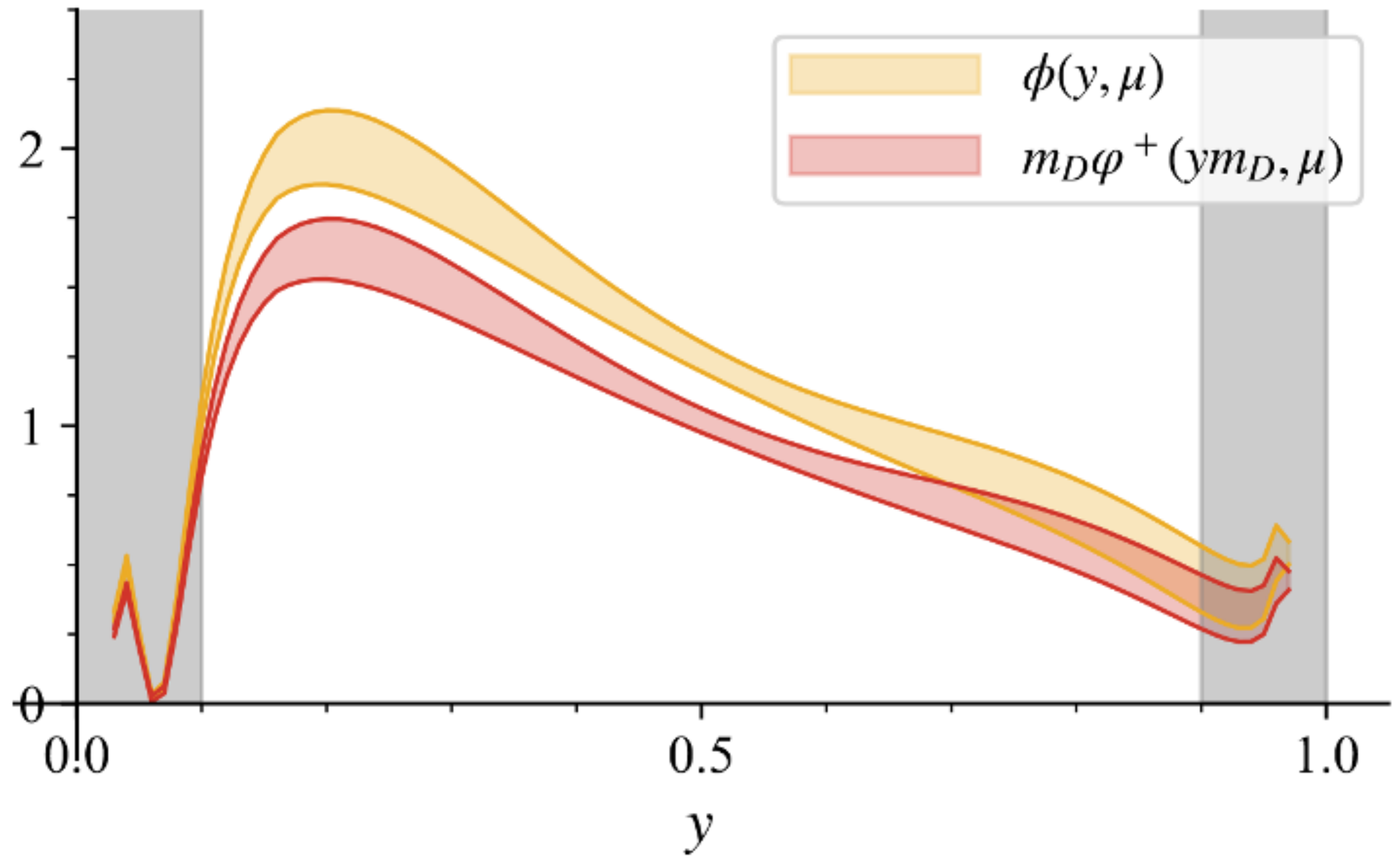


Quasi DA & QCD LCDA

- bHQET matching

$$\varphi^+_{\rm peak}(\omega, m_H) = \frac{f_H}{\tilde{f}_H} \frac{1}{\mathcal{J}_{\rm peak}} \phi(y, \mu; m_H) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_H}\right)$$

[Beneke, Finauri, Vos, Wei, 2023; Ishaq, Jia, et al. 2019]



QCD LCDA & HQET LCDA ($\omega \sim \Lambda_{\rm QCD}$)

*[**Han**, Hua, Ji, Lü, et al., 2024; **Han**, Hua, Ji, Lü, et al., 2025]*

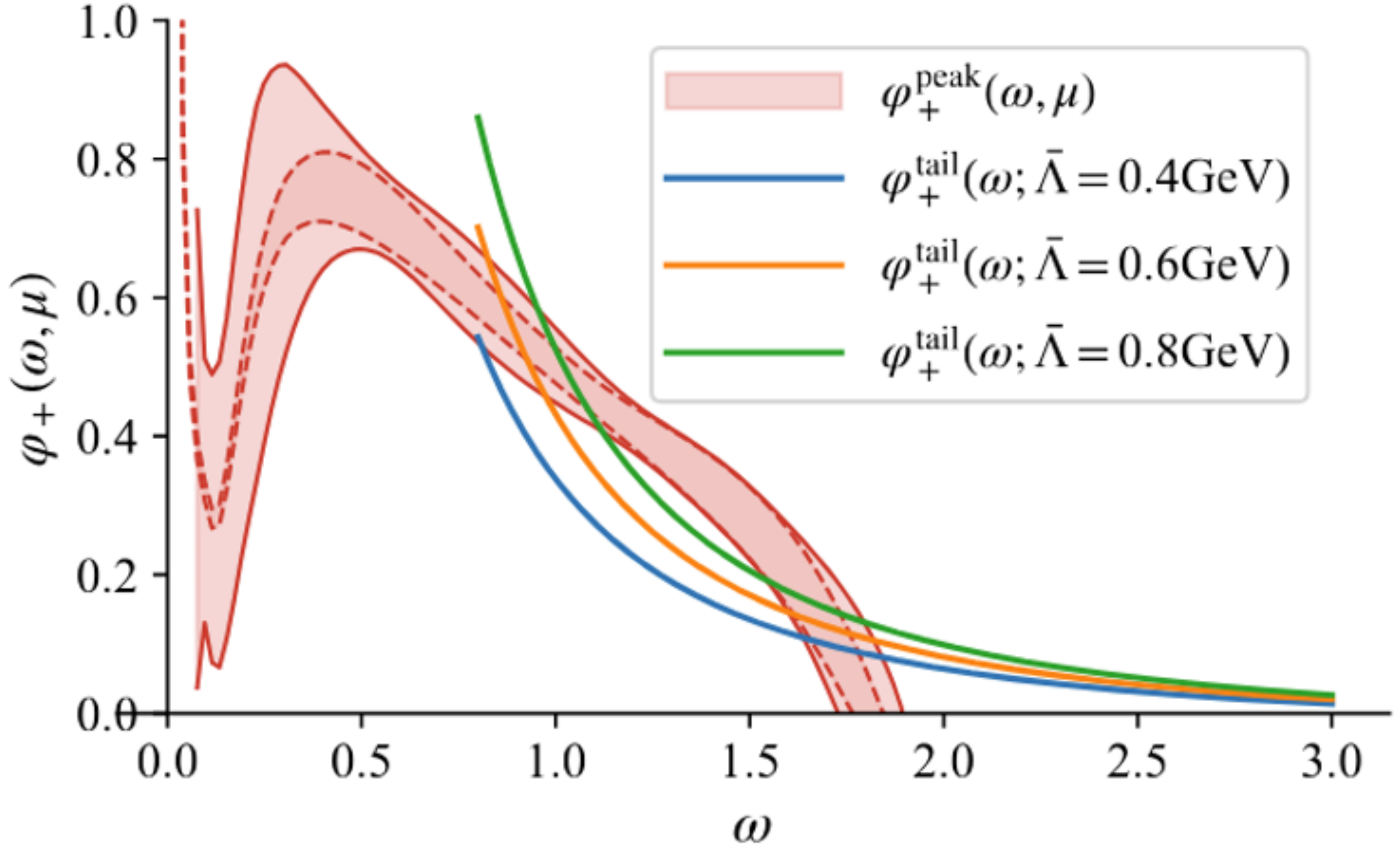
Matching for previous results

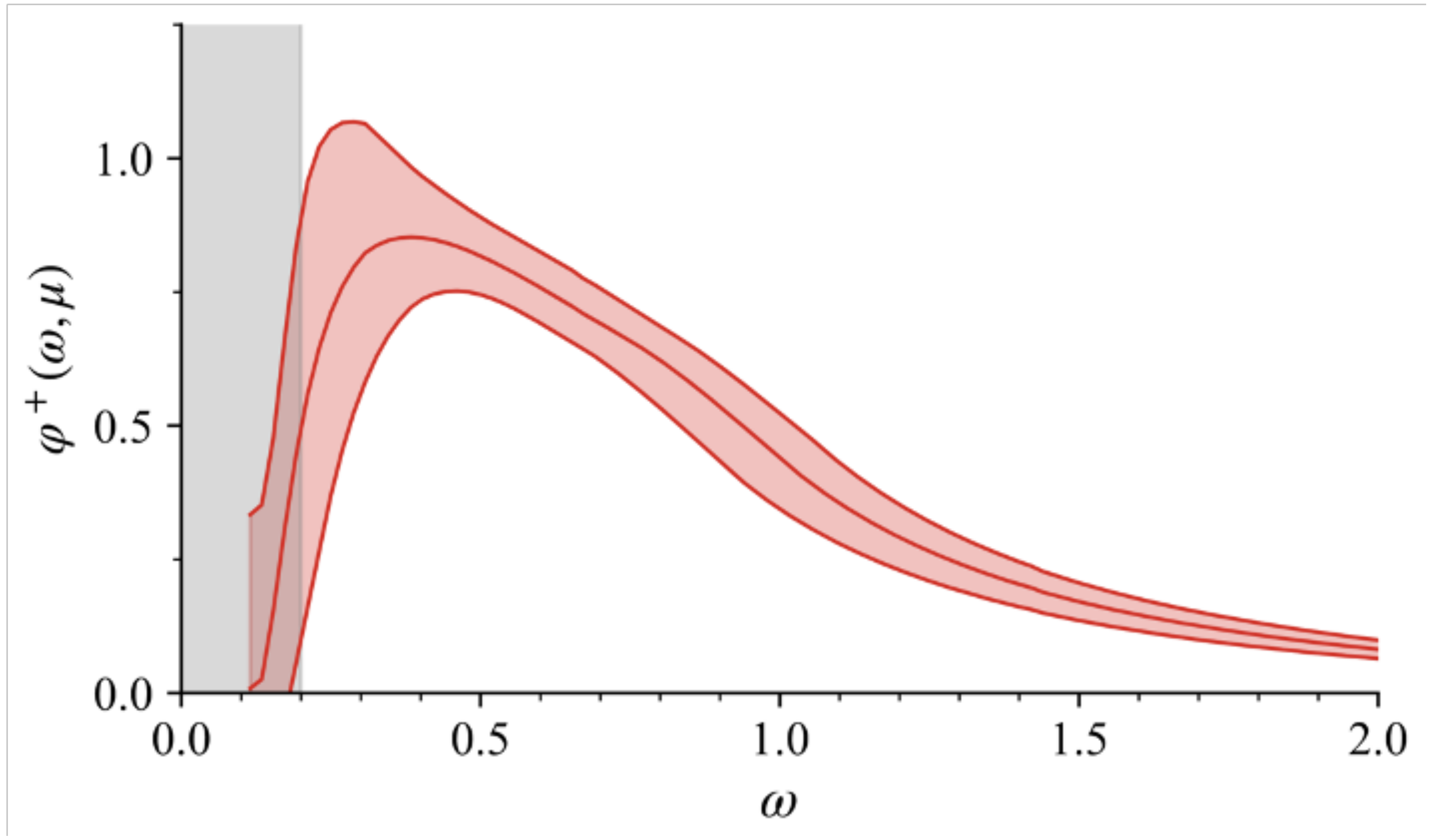
- Combine with $\omega \gg \Lambda_{\rm QCD}$ region

$$\varphi^+_{\rm tail} = \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right]$$

[Lee, Neubert, 2005]

$$\varphi^+(\omega,\mu) = \varphi^+_{\rm peak}(\omega,\mu)\theta(\omega_b - \omega) + \varphi^+_{\rm tail}(\omega,\mu)\theta(\omega - \omega_b)$$





HQET LCDA

Inverse moments

- λ_B and $\sigma_B^{(1)}$ at $\mu = 1\text{GeV}$

		λ_B (GeV)	$\sigma_B^{(1)}$
Our results	*N=1*	0.389(35)	1.63(8)
	N=2	0.393(37)	1.62(7)
	N=3	0.381(59)	1.63(12)
Experiment	*Belle 2018*	> 0.24	
Other theoretical approach	*Khodjamirian, Mandal, Mannel, 2020*	0.383(153)	
	Gao, Lu, Shen, Wang, Wei, 2020	$0.343^{+0.064}_{-0.079}$	
	Lee, Neubert, 2005	0.48(11)	1.6(2)
	Braun, Ivanov, Korchemsky, 2004	0.46(11)	1.4(4)
	Grozin, Neubert, 1997	0.35(15)	
	Mandal, Nandi, Ray, 2024	0.338(68)	

🚀 Summary and prospect

✅ What we achieved:

- First-principle strategy for heavy meson LCDA
- Hybrid renormalization:
  - Short-distance: ratio scheme (robust and clean)
  - Long-distance: self-renormalization (non-perturbative, removes linear divergence & discretization effects)
- Better StN ratio at large z via smearing & statistics

🌱 What's next:

- Continuum extrapolation & large P^z extrapolation;
- A more reliable extraction of HQET LCDA
- Extract λ_B, $\sigma_B^{(1)}$
- Apply to higher-twist & B meson.

Thank you!