基于 QCD 因子化方案 $B \to \pi\pi$ 中色八重态的贡献

杨冰冰

河南师范大学

Based on arXiv:2502.12461.

第七届全国重味物理和量子色动力学研讨会

南京，2025.04.21

目录

- 动机
- 色八重态矩阵元
  - $B \to \pi\pi$ 的振幅表达式 A_{ij}
  - 色八重态贡献
- 结果与讨论
  - 待定参数
  - 分支比
  - CP 破坏
- 总结

动机

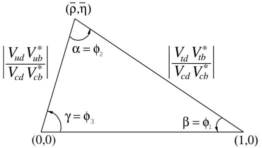


figure1: $(\bar{\rho}, \bar{\eta})$ 复平面上的幺正三角形

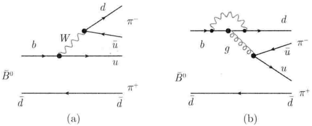


figure2: $\overline{B}^0 \to \pi^+\pi^-$ 过程的费曼图
(a) 树图; (b)QCD 企鹅图

CKM 混合矩阵 $\mathbf{V}_{\mathbf{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$

三个相角的约束条件: $\alpha + \beta + \gamma = 180°$

$$V_{ub} = |V_{ub}|e^{-i\gamma} \qquad V_{td} = |V_{td}|e^{-i\beta}$$

$$V_{ub}V_{ud}^* = |V_{ub}V_{ud}^*|e^{-i\gamma} \qquad V_{tb}V_{td}^* = |V_{tb}V_{td}^*|e^{+i\beta}$$

$$\begin{aligned} \mathcal{A} &= V_{ub}V_{ud}^*T - V_{tb}V_{td}^*P \\ &= |V_{ub}V_{ud}^*|e^{-i\gamma}T - |V_{tb}V_{td}^*|e^{+i\beta}P \\ &= |V_{ub}V_{ud}^*|e^{-i\gamma}T(1 + \frac{|V_{tb}V_{td}^*|}{|V_{ub}V_{ud}^*|}e^{-i\alpha}\frac{P}{T}) \end{aligned}$$

$B \to \pi\pi$ 衰变道可以提取 CKM 相角 α

动机

理论预测的 $\overline{B}^0 \to \pi^0\pi^0$ 的分支比与实验值相比小很多（$\pi\pi$ puzzle）

Experimental data[1]

mode	PDG	Belle	BaBar	Belle II
$10^6 \times \mathcal{B}(B^- \to \pi^-\pi^0)$	5.31±0.26	5.86±0.46	5.02±0.54	5.10±0.40
$10^6 \times \mathcal{B}(\overline{B}^0 \to \pi^0\pi^0)$	1.55±0.17	1.31±0.27	1.83±0.25	1.38±0.35
$10^6 \times \mathcal{B}(\overline{B}^0 \to \pi^+\pi^-)$	5.43±0.26	5.04±0.28	5.5 ±0.5	5.83±0.28
$\mathcal{A}_{CP}(B^- \to \pi^-\pi^0)$	−0.01 ±0.04	0.025±0.044	0.03 ±0.08	−0.081±0.055
$\mathcal{C}_{CP}(\overline{B}^0 \to \pi^0\pi^0)$	−0.25 ±0.20	−0.14 ±0.37	−0.43 ±0.26	0.14 ±0.47
$\mathcal{C}_{CP}(\overline{B}^0 \to \pi^+\pi^-)$	−0.314±0.030	−0.33 ±0.07	−0.25 ±0.08	
$\mathcal{S}_{CP}(\overline{B}^0 \to \pi^+\pi^-)$	−0.67 ±0.03	−0.64 ±0.09	−0.68 ±0.10	

Theoretical results

mode	QCDF		PQCD		
	+NLO[2]	+NNLO[3]	+NLO[4]	+NLO[5]	+NLOG[5]
$10^6 \times \mathcal{B}(B^- \to \pi^-\pi^0)$	5.1	5.82±1.42	$4.27^{+1.85}_{-1.47}$	3.35±1.10	4.45±1.43
$10^6 \times \mathcal{B}(\overline{B}^0 \to \pi^0\pi^0)$	0.7	0.63±0.65	$0.23^{+0.19}_{-0.15}$	0.29±0.11	0.61±0.21
$10^6 \times \mathcal{B}(\overline{B}^0 \to \pi^+\pi^-)$	5.2	5.70±1.35	$7.67^{+3.47}_{-2.64}$	6.19±2.12	5.39±1.88

[1] Phys. Rev. D 110, 030001 (2024). [2] Nucl. Phys. B 675, 333 (2003). [3] M. Beneke, T. Huber, X. Li, Nucl. Phys. B 832, 109 (2010).
[4] Y. Zhang, X. Liu, Y. Fan, S. Cheng, Z. Xiao, Phys. Rev. D 90, 014029 (2014). [5] X. Liu, H. Li, Z. Xiao, Phys. Rev. D 91, 114019 (2015).

$B \to \pi\pi$ 的振幅表达式 A_{ij}

$$\mathcal{A}_{+-} = \mathcal{A}_{\pi\pi}\{V_{ub}V_{ud}^{*}a_1 - V_{tb}V_{td}^{*}[a_4 + a_{10} + (a_6 + a_8)R]\}$$

$$\mathcal{A}_{-0} = \frac{\mathcal{A}_{\pi\pi}}{\sqrt{2}}\{V_{ub}V_{ud}^{*}(a_1 + a_2) - V_{tb}V_{td}^{*}\frac{3}{2}(-a_7 + a_8R + a_9 + a_{10})\}$$

$$\mathcal{A}_{00} = -\mathcal{A}_{\pi\pi}\{V_{ub}V_{ud}^{*}a_2 + V_{tb}V_{td}^{*}[a_4 - \frac{1}{2}a_{10} + \frac{3}{2}(a_7 - a_9) + (a_6 - \frac{1}{2}a_8)R]\}$$

其中，$\mathcal{A}_{\pi\pi} = i\frac{G_F}{2}(m_B^2 - m_\pi^2)F_0^{B\pi}f_\pi$，$R = \frac{2m_\pi^2}{\bar{m}_b(\bar{m}_u + \bar{m}_d)}$

NLO 系数 $a_i = C_i^{\text{NLO}} + \frac{1}{N}C_j^{\text{NLO}} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_j^{\text{LO}}V_i$　当 i 为奇数时，j=i+1；i 为偶数时，j=i-1。

$$a_1 = C_1^{\text{NLO}} + \frac{1}{N}C_2^{\text{NLO}} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_2^{\text{LO}}V_1$$

$$a_2 = C_2^{\text{NLO}} + \frac{1}{N}C_1^{\text{NLO}} + \frac{\alpha_s}{4\pi}\frac{C_F}{N}C_1^{\text{LO}}V_2$$

根据颜色 SU(3) 群生成元的代数关系 $T^a_{i,j}T^a_{k,l} = -\frac{1}{2N}\delta_{i,j}\delta_{k,l} + \frac{1}{2}\delta_{i,l}\delta_{k,j}$

一个四夸克算符可以表示为颜色单态和颜色八重态算符的和，Γ 代表任意 Dirac 流结构：

$$(\bar{q}_{1,\alpha}\Gamma_1 q_{2,\beta})(\bar{q}_{3,\beta}\Gamma_2 q_{4,\alpha}) = \frac{1}{N}(\bar{q}_{1,\alpha}\Gamma_1 q_{2,\alpha})(\bar{q}_{3,\beta}\Gamma_2 q_{4,\beta}) + 2(\bar{q}_1\Gamma_1 T^a q_2)(\bar{q}_3\Gamma_2 T^a q_4)$$

$$\begin{aligned}
&C_1\langle\pi^0\pi^0|O_1|\overline{B}^0\rangle \\
=&C_1\langle\pi^0\pi^0|(\overline{u}_\alpha b_\alpha)_{V-A}(\overline{d}_\beta u_\beta)_{V-A}|\overline{B}^0\rangle \\
=&C_1\langle\pi^0\pi^0|(\overline{u}_\alpha u_\beta)_{V-A}(\overline{d}_\beta b_\alpha)_{V-A}|\overline{B}^0\rangle \\
=&\frac{C_1}{N}\langle\pi^0\pi^0|(\overline{u}_\alpha u_\alpha)_{V-A}(\overline{d}_\beta b_\beta)_{V-A}|\overline{B}^0\rangle \\
&+2C_1\langle\pi^0\pi^0|(\overline{u}T^a u)_{V-A}(\overline{d}T^a b)_{V-A}|\overline{B}^0\rangle
\end{aligned}$$

参考 PQCD 的方法 [1,2]

[1] S. Lü and M. Z. Yang, Phys. Rev. D 107, 013004 (2023).
[2] R. X. Wang and M. Z. Yang, Phys. Rev. D 108, 013003 (2023).

$$\langle\pi^0\pi^0|(\overline{u}_\alpha u_\alpha)_{V-A}(\overline{d}_\beta b_\beta)_{V-A}|\overline{B}^0\rangle = -i(m_B^2 - m_\pi^2)F_0^{B\pi}f_\pi$$

类比色单态的强子矩阵元参数化色八重态矩阵元贡献:

$$\langle\pi^0\pi^0|(\overline{u}T^a u)_{V-A}(\overline{d}T^a b)_{V-A}|\overline{B}^0\rangle = -i(m_B^2 - m_\pi^2)F_0^{B\pi}f_\pi X_{LL}$$

同理,

$$\langle\pi^0\pi^0|(\overline{d}T^a d)_{V+A}(\overline{d}T^a b)_{V-A}|\overline{B}^0\rangle = -i(m_B^2 - m_\pi^2)F_0^{B\pi}f_\pi X_{RL}$$

$$-2\langle\pi^0\pi^0|(\overline{d}T^a d)_{S+P}(\overline{d}T^a b)_{S-P}|\overline{B}^0\rangle = +i(m_B^2 - m_\pi^2)F_0^{B\pi}f_\pi R X_{SP}$$

假设 $X_{LL} = X_{RL} = X_{SP} = X = |X|e^{i\delta}$, 则色八重态的振幅为 $\mathcal{C}_{ij} = 2X\mathcal{A}_{ij}$ $(a \to C)$

待定参数: $F_0^{B\pi}$、$|X|$、δ χ^2 函数: $\chi^2(\theta) = \sum_i \frac{(y_i-\mu(x_i;\theta))^2}{\sigma_i^2}$

待定参数

the fit results

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2\,m_b$
PDG			
χ^2/n_{dof}	$120.5/4$	$170.1/4$	$205.2/4$
$\lvert X\rvert$	0.31 ± 0.02	0.40 ± 0.02	0.42 ± 0.02
δ	$(-61.5 \pm 5.5)^\circ$	$(75.6 \pm 4.3)^\circ$	$(83.5 \pm 4.0)^\circ$
$F_0^{B\pi}$	0.218 ± 0.004	0.218 ± 0.005	0.220 ± 0.005
$\rho_{\lvert X\rvert,F_0}$	-0.58	-0.51	-0.47
Belle			
χ^2/n_{dof}	$22.4/4$	$28.2/4$	$31.0/4$
$\lvert X\rvert$	0.28 ± 0.03	0.36 ± 0.04	0.37 ± 0.04
δ	$(-42.7 \pm 13.0)^\circ$	$(59.3 \pm 10.3)^\circ$	$(67.1 \pm 9.4)^\circ$
$F_0^{B\pi}$	0.220 ± 0.006	0.217 ± 0.006	0.216 ± 0.005
$\rho_{\lvert X\rvert,F_0}$	-0.50	-0.47	-0.38
BaBar			
χ^2/n_{dof}	$10.8/4$	$15.9/4$	$21.6/4$
$\lvert X\rvert$	0.33 ± 0.03	0.36 ± 0.03	0.37 ± 0.04
δ	$(-70.6 \pm 9.0)^\circ$	$(-79.7 \pm 8.8)^\circ$	$(-87.4 \pm 8.5)^\circ$
$F_0^{B\pi}$	0.216 ± 0.009	0.217 ± 0.009	0.221 ± 0.009
$\rho_{\lvert X\rvert,F_0}$	-0.63	-0.60	-0.54

[arXiv:2502.12461]

- 随能标 μ 变化
- 在 $\mu = m_b/2$ 时 χ^2 最小
- PDG 的最小 χ^2 更大
- $|X| \sim 0.3/0.4$
- $F_0^{B\pi} \sim 0.22/0.23$
- $\rho_{|X|,F_0} < 0$

格点计算结果 $F_0^{B\pi}$=0.183(92)[1]
光锥求和规则结果 $F_0^{B\pi}$=0.19(5)[2]

[1] JLQCD Collaboration, Phys. Rev. D106, 054502 (2022).   [2] B. Cui, Y. Huang, Y. Shen, C. Wang, Y. Wang, JHEP 03, 140 (2023).

Our result: $|X| \sim 0.3/0.4$

PQCD: $|X| \sim 0.25$, $F_0^{B\pi}$=0.27[1]

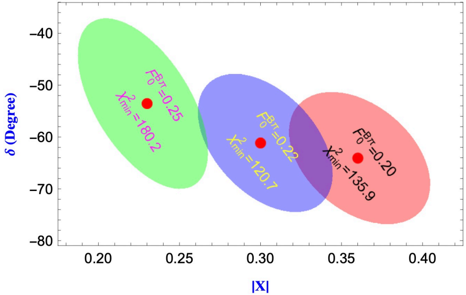


FIG. 1: The distribution of the fit parameter X obtained from the PDG data with different form factor $F_0^{B\pi}$ at the scale $\mu = m_b/2$, where the dots correspond to the optimal values of X, and the ellipses correspond to the errors of X.

[1] S. Lü and M. Z. Yang, Phys. Rev. D 107, 013004 (2023).

分支比

Branching ratios

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2\, m_b$	data
		PDG		
$10^6 \times \mathcal{B}(\pi^-\pi^0)$	$5.35^{+0.55}_{-0.52}$	$5.29^{+0.55}_{-0.52}$	$5.20^{+0.54}_{-0.51}$	$5.31{\pm}0.26$
$10^6 \times \mathcal{B}(\pi^0\pi^0)$	$1.62^{+0.29}_{-0.26}$	$1.64^{+0.28}_{-0.25}$	$1.70^{+0.28}_{-0.25}$	$1.55{\pm}0.17$
$10^6 \times \mathcal{B}(\pi^+\pi^-)$	$5.35^{+0.42}_{-0.41}$	$5.38^{+0.35}_{-0.34}$	$5.42^{+0.31}_{-0.30}$	$5.43{\pm}0.26$
		Belle		
$10^6 \times \mathcal{B}(\pi^-\pi^0)$	$5.88^{+0.95}_{-0.93}$	$5.89^{+0.98}_{-0.94}$	$5.82^{+0.97}_{-0.92}$	$5.86{\pm}0.46$
$10^6 \times \mathcal{B}(\pi^0\pi^0)$	$1.35^{+0.43}_{-0.36}$	$1.34^{+0.43}_{-0.35}$	$1.39^{+0.41}_{-0.35}$	$1.31{\pm}0.27$
$10^6 \times \mathcal{B}(\pi^+\pi^-)$	$5.02^{+0.62}_{-0.55}$	$5.02^{+0.51}_{-0.48}$	$5.04^{+0.40}_{-0.39}$	$5.04{\pm}0.28$
		BaBar		
$10^6 \times \mathcal{B}(\pi^-\pi^0)$	$5.04^{+1.07}_{-0.94}$	$5.01^{+1.09}_{-0.96}$	$4.98^{+1.09}_{-0.98}$	$5.02{\pm}0.54$
$10^6 \times \mathcal{B}(\pi^0\pi^0)$	$1.85^{+0.49}_{-0.41}$	$1.82^{+0.49}_{-0.41}$	$1.81^{+0.48}_{-0.41}$	$1.83{\pm}0.25$
$10^6 \times \mathcal{B}(\pi^+\pi^-)$	$5.46^{+0.77}_{-0.74}$	$5.52^{+0.69}_{-0.64}$	$5.55^{+0.61}_{-0.57}$	$5.5{\pm}0.5$
		Belle II		
$10^6 \times \mathcal{B}(\pi^-\pi^0)$	$5.11^{+0.62}_{-0.60}$	$5.10^{+0.61}_{-0.60}$	$5.10^{+0.61}_{-0.60}$	$5.10{\pm}0.40$
$10^6 \times \mathcal{B}(\pi^0\pi^0)$	$1.42^{+0.44}_{-0.37}$	$1.40^{+0.44}_{-0.37}$	$1.39^{+0.45}_{-0.37}$	$1.38{\pm}0.35$
$10^6 \times \mathcal{B}(\pi^+\pi^-)$	$5.82^{+0.43}_{-0.42}$	$5.82^{+0.42}_{-0.40}$	$5.83^{+0.38}_{-0.36}$	$5.83{\pm}0.28$

$\mathcal{A}_{+-} \sim a_1$
$\mathcal{A}_{00} \sim a_2$
$\mathcal{A}_{-0} \sim a_1 + a_2$

拟合得到的分支比在误差范围内都与实验值符合很好。

CP asymmetries

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2\,m_b$	data
PDG				
$\mathcal{A}_{CP}(\pi^-\pi^0)$	$-0.0013^{+0.0003}_{-0.0002}$	-0.0024 ± 0.0001	0.0009 ± 0.0003	-0.01 ± 0.04
$\mathcal{C}_{CP}(\pi^0\pi^0)$	$-0.504^{+0.043}_{-0.042}$	$0.418^{+0.035}_{-0.034}$	$0.314^{+0.024}_{-0.022}$	-0.25 ± 0.20
$\mathcal{S}_{CP}(\pi^0\pi^0)$	$0.049^{+0.059}_{-0.054}$	$-0.053^{+0.045}_{-0.041}$	$-0.108^{+0.029}_{-0.027}$	
$\mathcal{C}_{CP}(\pi^+\pi^-)$	$-0.023^{+0.002}_{-0.001}$	-0.012 ± 0.001	-0.025 ± 0.001	-0.314 ± 0.030
$\mathcal{S}_{CP}(\pi^+\pi^-)$	$-0.522^{+0.001}_{-0.002}$	-0.443 ± 0.001	-0.365 ± 0.002	-0.67 ± 0.03
Belle				
$\mathcal{A}_{CP}(\pi^-\pi^0)$	$-0.0018^{+0.0005}_{-0.0004}$	-0.0024 ± 0.0002	0.0001 ± 0.0005	0.025 ± 0.044
$\mathcal{C}_{CP}(\pi^0\pi^0)$	$-0.459^{+0.113}_{-0.100}$	$0.396^{+0.082}_{-0.081}$	$0.307^{+0.061}_{-0.054}$	-0.14 ± 0.37
$\mathcal{S}_{CP}(\pi^0\pi^0)$	$0.218^{+0.144}_{-0.129}$	$0.095^{+0.117}_{-0.104}$	$-0.002^{+0.076}_{-0.068}$	
$\mathcal{C}_{CP}(\pi^+\pi^-)$	$-0.020^{+0.004}_{-0.003}$	$-0.010^{+0.002}_{-0.001}$	$-0.022^{+0.004}_{-0.003}$	-0.33 ± 0.07
$\mathcal{S}_{CP}(\pi^+\pi^-)$	$-0.526^{+0.003}_{-0.004}$	$-0.441^{+0.002}_{-0.001}$	$-0.359^{+0.004}_{-0.003}$	-0.64 ± 0.09

- 拟合值 $|\mathcal{A}_{CP}(\pi^-\pi^0)|<1\%$
- $\mathcal{C}_{CP}(\pi^0\pi^0)$ 对色八重态贡献敏感
- $|\mathcal{A}_{CP}(\pi^-\pi^0)|<|\mathcal{C}_{CP}(\pi^-\pi^+)|<|\mathcal{C}_{CP}(\pi^0\pi^0)|$

振幅关系 $\mathcal{A}_{+-}\sim a_1 \qquad \mathcal{A}_{00}\sim a_2$

$$\mathcal{A}_{-0} = \frac{\mathcal{A}_{\pi\pi}}{\sqrt{2}}\{V_{ub}V_{ud}^*(a_1+a_2) - V_{tb}V_{td}^*\frac{3}{2}(-a_7+a_8R+a_9+a_{10})\}$$

总结

- 我们将色八重态矩阵元的贡献考虑在内，基于 QCD 因子化方案在领头阶近似下重新研究了 $B \to \pi\pi$ 衰变。类比色单态矩阵元对色八重态矩阵元进行参数化，采用最小 χ^2 方法进行拟合。
- 色八重态贡献相对于色单态贡献较小，但是不能被忽略。拟合得到的形状因子 $F_0^{B\pi}$ ≈0.22，$F_0^{B\pi}$ 与 X 之间有紧密的关联性。
- 色八重态矩阵元的引入有力加强了 $\overline{B}^0 \to \pi^0\pi^0$ 的分支比，并且 $B \to \pi\pi$ 过程所有分支比在误差范围内都与实验值符合很好。
- 拟合得到的 CP 破坏结果存在与当前实验值的不一致还需要更多理论和实验的努力！

谢谢！

威尔森系数

TABLE IV: Wilson coefficients C_i with the naive dimensional regularization scheme.

μ	$m_b/2$		m_b		$2\,m_b$	
	LO	NLO	LO	NLO	LO	NLO
C_1	1.168	1.128	1.110	1.076	1.070	1.041
C_2	−0.338	−0.269	−0.237	−0.173	−0.160	−0.100
C_3	0.019	0.020	0.012	0.014	0.007	0.009
C_4	−0.046	−0.048	−0.032	−0.034	−0.022	−0.024
C_5	0.010	0.010	0.008	0.008	0.006	0.006
C_6	−0.057	−0.060	−0.037	−0.039	−0.023	−0.025
$C_7/\alpha_{\rm em}$	−0.103	−0.012	−0.096	0.004	−0.080	0.027
$C_8/\alpha_{\rm em}$	0.023	0.080	0.014	0.052	0.009	0.034
$C_9/\alpha_{\rm em}$	−0.095	−1.372	−0.090	−1.297	−0.076	−1.234
$C_{10}/\alpha_{\rm em}$	−0.025	0.360	−0.018	0.249	−0.013	0.166

待定参数

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2\,m_b$
	PDG		
$\chi^2/n_{\rm dof}$	$120.5/4$	$170.1/4$	$205.2/4$
$\lvert X\rvert$	0.31 ± 0.02	0.40 ± 0.02	0.42 ± 0.02
δ	$(-61.5 \pm 5.5)^\circ$	$(75.6 \pm 4.3)^\circ$	$(83.5 \pm 4.0)^\circ$
$F_0^{B\pi}$	0.218 ± 0.004	0.218 ± 0.005	0.220 ± 0.005
$\rho_{\lvert X\rvert,\delta}$	-0.44	0.47	0.41
$\rho_{\lvert X\rvert,F_0}$	-0.58	-0.51	-0.47
ρ_{δ,F_0}	0.14	0.08	0.22
	Belle		
$\chi^2/n_{\rm dof}$	$22.4/4$	$28.2/4$	$31.0/4$
$\lvert X\rvert$	0.28 ± 0.03	0.36 ± 0.04	0.37 ± 0.04
δ	$(-42.7 \pm 13.0)^\circ$	$(59.3 \pm 10.3)^\circ$	$(67.1 \pm 9.4)^\circ$
$F_0^{B\pi}$	0.220 ± 0.006	0.217 ± 0.006	0.216 ± 0.005
$\rho_{\lvert X\rvert,\delta}$	-0.63	0.74	0.68
$\rho_{\lvert X\rvert,F_0}$	-0.50	-0.47	-0.38
ρ_{δ,F_0}	0.49	-0.28	-0.05

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2\,m_b$
	BaBar		
$\chi^2/n_{\rm dof}$	$10.8/4$	$15.9/4$	$21.6/4$
$\lvert X\rvert$	0.33 ± 0.03	0.36 ± 0.03	0.37 ± 0.04
δ	$(-70.6 \pm 9.0)^\circ$	$(-79.7 \pm 8.8)^\circ$	$(-87.4 \pm 8.5)^\circ$
$F_0^{B\pi}$	0.216 ± 0.009	0.217 ± 0.009	0.221 ± 0.009
$\rho_{\lvert X\rvert,\delta}$	-0.27	-0.27	-0.27
$\rho_{\lvert X\rvert,F_0}$	-0.63	-0.60	-0.54
ρ_{δ,F_0}	0.15	-0.05	-0.22
	Belle II		
$\chi^2/n_{\rm dof}$	$2.8/2$	$2.5/2$	$2.4/2$
$\lvert X\rvert$	0.36 ± 0.05	0.36 ± 0.05	0.37 ± 0.05
δ	$(72.2 \pm 6.9)^\circ$	$(78.1 \pm 7.0)^\circ$	$(83.9 \pm 7.0)^\circ$
$F_0^{B\pi}$	0.224 ± 0.006	0.225 ± 0.005	0.227 ± 0.005
$\rho_{\lvert X\rvert,\delta}$	0.69	0.67	0.66
$\rho_{\lvert X\rvert,F_0}$	-0.57	-0.49	-0.38
ρ_{δ,F_0}	-0.48	-0.28	-0.05

- 随能标 μ 变化
- PDG 的 χ^2 更大
- $\lvert X\rvert \sim 0.3/0.4$
- $F_0^{B\pi} \sim 0.22/0.23$
- $\rho_{\lvert X\rvert,F_0} < 0$

CP 破坏 (continue.)

BaBar

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2\, m_b$	data
$\mathcal{A}_{CP}(\pi^- \pi^0)$	-0.0009 ± 0.0004	-0.0024 ± 0.0002	-0.0049 ± 0.0005	0.03 ± 0.08
$\mathcal{C}_{CP}(\pi^0 \pi^0)$	$-0.493^{+0.055}_{-0.051}$	$-0.404^{+0.044}_{-0.043}$	$-0.308^{+0.033}_{-0.034}$	-0.43 ± 0.26
$\mathcal{S}_{CP}(\pi^0 \pi^0)$	$-0.035^{+0.087}_{-0.076}$	$-0.092^{+0.068}_{-0.060}$	$-0.123^{+0.050}_{-0.044}$	
$\mathcal{C}_{CP}(\pi^+ \pi^-)$	-0.024 ± 0.002	0.005 ± 0.001	0.020 ± 0.002	-0.25 ± 0.08
$\mathcal{S}_{CP}(\pi^+ \pi^-)$	-0.519 ± 0.003	-0.444 ± 0.001	-0.367 ± 0.003	-0.68 ± 0.10

Belle II

	$\mu = m_b/2$	$\mu = m_b$	$\mu = 2\, m_b$	data
$\mathcal{A}_{CP}(\pi^- \pi^0)$	$-0.0052^{+0.0003}_{-0.0004}$	-0.0026 ± 0.0002	$0.0006^{+0.0006}_{-0.0005}$	-0.081 ± 0.055
$\mathcal{C}_{CP}(\pi^0 \pi^0)$	$0.572^{+0.079}_{-0.065}$	$0.469^{+0.070}_{-0.055}$	$0.358^{+0.057}_{-0.044}$	0.14 ± 0.47
$\mathcal{S}_{CP}(\pi^0 \pi^0)$	$0.011^{+0.124}_{-0.097}$	$-0.057^{+0.095}_{-0.074}$	$-0.097^{+0.070}_{-0.054}$	
$\mathcal{C}_{CP}(\pi^+ \pi^-)$	0.012 ± 0.003	-0.011 ± 0.001	-0.022 ± 0.003	
$\mathcal{S}_{CP}(\pi^+ \pi^-)$	-0.522 ± 0.002	-0.444 ± 0.001	$-0.365^{+0.002}_{-0.003}$	

PQCD 结果

TABLE I. Branching ratios and direct CP violation ($\delta_8 = \delta_8^{SP}$, $l^2 = \frac{m_b^2}{4}$, $m_c = 1.3$ GeV), where NLO is the hard contribution up to next-to-leading order in QCD, "$+\xi_{B\pi}$" contribution of NLO + the contribution of the soft transition form factor $\xi_{B\pi}$, "$+T_8$" contribution of NLO + color-octet matrix element, "$+\xi_{\pi\pi}$" contribution of NLO + contribution of soft production form factor of $\pi\pi$, "$+\xi_{B\pi} + T_8 + \xi_{\pi\pi}$" total contribution of NLO + $\xi_{B\pi} + T_8 + \xi_{\pi\pi}$, for which the first uncertainty comes from the constraint of experimental data, the second is the quadratic combination of uncertainties from the variation of input parameters in B and pion wave functions. The last column is the experimental data from PDG [8].

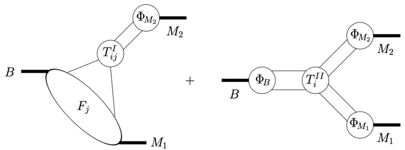
Mode	NLO	$+\xi_{B\pi}$	$+\xi_{\pi\pi}$	$+T_8$	$+\xi_{B\pi} + \xi_{\pi\pi} + T_8$	Data [8]
$\mathrm{B}(B^0 \to \pi^+\pi^-) \times 10^{-6}$	4.95	7.48	3.32	4.37	$5.14 \pm 0.61^{+0.34}_{-0.37}$	5.12 ± 0.19
$\mathrm{B}(B^+ \to \pi^+\pi^0) \times 10^{-6}$	3.27	4.40	3.27	4.23	$5.72 \pm 0.44^{+0.29}_{-0.37}$	5.5 ± 0.4
$\mathrm{B}(B^0 \to \pi^0\pi^0) \times 10^{-6}$	0.13	0.14	0.22	0.67	$1.50 \pm 0.24^{+0.18}_{-0.19}$	1.59 ± 0.26
$A_{CP}(B^0 \to \pi^+\pi^-)$	0.17	0.11	0.44	0.22	$0.33 \pm 0.04^{+0.04}_{-0.03}$	0.32 ± 0.04
$A_{CP}(B^+ \to \pi^+\pi^0)$	−0.0007	−0.0007	−0.0007	0.0053	$0.0054 \pm 0.0004^{+0.0001}_{-0.0001}$	0.03 ± 0.04
$A_{CP}(B^0 \to \pi^0\pi^0)$	0.27	0.48	−0.16	0.53	$0.23 \pm 0.07^{+0.07}_{-0.05}$	0.33 ± 0.22

QCD 因子化方案

- 1999 年 M. Beneke 等人首次提出一种计算强子矩阵元的方法——QCD 因子化方法 [1]。
- 在 QCD 因子化方案下，非微扰效应包含在普适的介子光锥分布振幅和形状因子中。
- 在重夸克近似下，$B \to M_1 M_2$ 过程的强子矩阵元公式表示为

(1) 当 M_1 和 M_2 都是轻介子

$$\langle M_1 M_2 | O_i | B \rangle = \sum_j F_j^{B\to M_1} \int_0^1 dx\ T_{ij}^I(x) \Phi_{M_2}(x) + (M_1 \leftrightarrow M_2) + \int_0^1 d\xi \int_0^1 dx \int_0^1 dy\ T_i^{II}(\xi, x, y) \Phi_B(\xi) \Phi_{M_1}(x) \Phi_{M_2}(y)$$



(2) 当 M_1 是重介子，M_2 是轻介子

$$\langle M_1 M_2 | O_i | B \rangle = \sum_j F_j^{B\to M_1} \int_0^1 dx\ T_{ij}^I(x) \Phi_{M_2}(x)$$

- QCD 因子化方案在 B 介子两体非轻弱衰变已经得到广泛应用 [1−9]，但也存在一些问题。

[1]Phys. Rev. Lett. 83, 1914 (1999). [2]Nucl. Phys. B 591, 313 (2000). [3]Nucl. Phys. B 606, 245 (2001). [4]Phys. Lett. B 488, 46 (2000). [5]Phys. Lett. B 509, 263 (2001). [6]Phys. Rev. D 64, 014036 (2001). [7]Nucl. Phys. B 675, 333 (2003). [8]Nucl. Phys. B 832, 109 (2010). [9]Phys. Rev. D 90, 054019 (2014).

QCDF 方案下 $B \to \pi\pi$ 的振幅表达式 (continue.)

NLO 系数 $a_i = C_i^{\text{NLO}} + \frac{1}{N} C_j^{\text{NLO}} + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_j^{\text{LO}} V_i$ 当 i 为奇数时，j=i+1；i 为偶数时，j=i-1。

顶角修正的贡献

$$V_i = \begin{cases} 12\ln\frac{m_b}{\mu} - 18 + \int_0^1 dx\, g(x)\ \Phi_M(x) & \text{if } i = 1,2,3,4,9,10 \\ -[12\ln\frac{m_b}{\mu} - 6\int_0^1 dx\, g(\bar{x})\ \Phi_M(x)] & \text{if } i = 5,7 \\ -6 & \text{if } i = 6,8 \end{cases}$$

$$g(x) = 3(\frac{1-2x}{1-x}\ln x - i\pi) + [2\text{Li}_2(x) - \ln^2 x + \frac{2\ln x}{1-x} - (3+2i\pi)\ln x - (x \leftrightarrow \bar{x})]$$

对于 $B \to \pi\pi$，twist-2 光锥分布振幅 $\Phi_\pi(x) = 6x(1-x)[1 + \sum_{n=1}^{\infty} a_n^\pi C_n^{(3/2)}(2x-1)]$。

故 $\int_0^1 dx\, g(x)\ \Phi_\pi(x)$ 可以表示为 $-\frac{1}{2} - i\,3\pi - \frac{21}{20}a_2^\pi - \frac{12}{35}a_4^\pi$

参考文献

[1] M. Beneke, G. Buchalla, M. Neubert, C. Sachrajda, QCD factorization for $B \to \pi\pi$ decays: strong phases and CP violation in the heavy quark limit, Phys. Rev. Lett. 83, 1914 (1999).

[2] M. Beneke, G. Buchalla, M. Neubert, C. Sachrajda, QCD factorization for exclusive nonleptonic B meson decays: General arguments and the case of heavy light final states, Nucl. Phys. B 591, 313 (2000).

[3] M. Beneke, G. Buchalla, M. Neubert, C. Sachrajda, QCD factorization in $B \to \pi K, \pi\pi$ decays and extraction of Wolfenstein parameters, Nucl. Phys. B 606, 245 (2001).

[4] D. Du, D. Yang, G. Zhu, Analysis of the decays $B \to \pi\pi$ and πK with QCD factorization in the heavy quark limit, Phys. Lett. B 488, 46 (2000).

[5] D. Du, D. Yang, G. Zhu, Infrared divergence and twist-3 distribution amplitudes in QCD factorization for $B \to PP$, Phys. Lett. B 509, 263 (2001).

[6] D. Du, D. Yang, G. Zhu, QCD factorization for $B \to PP$, Phys. Rev. D 64, 014036 (2001).

[7] M. Beneke, M. Neubert, QCD factorization for $B \to PP$ and $B \to PV$ decays, Nucl. Phys. B 675, 333 (2003).

[8] M. Beneke, T. Huber, X. Li, NNLO vertex corrections to non-leptonic B decays: tree ampli- tudes, Nucl. Phys. B 832, 109 (2010).

[9] Q. Chang, J. Sun, Y. Yang, X. Li, Spectator scattering and annihilation contributions as a solution to the πK and $\pi\pi$ puzzles within QCD factorization approach, Phys. Rev. D 90, 054019 (2014).

[10] S. Navas, C. Amsler, T. Gutsche et al. (Particle Data Group), Review of particle physics, Phys. Rev. D 110, 030001 (2024).

[11] Y. Duh, T. Wu, P. Chang et al. (Belle Collaboration), Measurements of branching fractions and direct CP asymmetries for $B \to K\pi$, $B \to \pi\pi$ and $B \to KK$ decays, Phys. Rev. D 87, 031103 (2013).

[12] T. Julius, M. Sevior, G. Mohanty et al. (Belle Collaboration), Measurement of the branching fraction and CP asymmetry in $B^0 \to \pi^0\pi^0$ decays, and an improved constraint on ϕ_2, Phys. Rev. D 96, 032007 (2017).

[13] B. Aubert, M. Bona, D. Boutigny et al. (BaBar Collaboration), Study of $B^0 \to \pi^0\pi^0$, $B^\pm \to \pi^\pm\pi^0$, and $B^\pm \to K^\pm\pi^0$ decays, and isospin analysis of $B \to \pi\pi$ decays, Phys. Rev. D 76, 091102 (2007).

[14] J. Lees, V. Poireau, V. Tisserand et al. (BaBar Collaboration), Measurement of CP asymmetries and branching fractions in charmless two-body B-meson decays to pions and kaons, Phys. Rev. D 87, 052009 (2013).

[15] B. Aubert, R. Barate, M. Bona et al. (BaBar Collaboration), Improved measurements of the branching fractions for $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$, and a search for $B^0 \to K^+K^-$, Phys. Rev. D 75, 012008 (2007).

[16] I. Adachi, L. Aggarwal, H. Ahmed et al. (Belle II Collaboration), Measurement of branching fractions and direct CP asymmetries for $B \to K\pi$ and $B \to \pi\pi$ decays at Belle II, Phys. Rev. D 109, 012001 (2024).

[17] F. Abudinén, I. Adachi, K. Adamczyk et al. (Belle II Collaboration), Measurement of the branching fraction and CP asymmetry of $B^0 \to \pi^0\pi^0$ decays using 198×10^6 $B\bar{B}$ pairs in Belle II data, Phys. Rev. D 107, 112009 (2023).

[18] Y. Zhang, X. Liu, Y. Fan, S. Cheng, Z. Xiao, $B \to \pi\pi$ decays and effects of the next-to-leading order contributions, Phys. Rev. D 90, 014029 (2014).

[19] X. Liu, H. Li, Z. Xiao, Transverse-momentum-dependent wave functions with Glauber gluons in $B \to \pi\pi$, $\rho\rho$ decays, Phys. Rev. D 91, 114019 (2015).

[20] S. Lü, M. Yang, Possible solution of the puzzle for the branching ratio and CP violation in $B \to \pi\pi$ decays with a modified perturbative QCD approach, Phys. Rev. D 107, 013004 (2023).

[21] R. Wang, M. Yang, Branching ratio and CP violation of $B \to K\pi$ decays in a modified perturbative QCD approach, Phys. Rev. D 108, 013003 (2023).

[22] P. Ball, R. Zwicky, New results on $B \to \pi, K, \eta$ decay form factors from light-cone sum rules, Phys. Rev. D 71, 014015 (2005).

[23] B. Colquhoun, S. Hashimoto, T. Kaneko, J. Koponen (JLQCD Collaboration), Form factors of $B \to \pi\ell\nu$ and a determination of $|V_{ub}|$ with Möbius domain-wall fermions, Phys. Rev. D106, 054502 (2022).

[24] B. Cui, Y. Huang, Y. Shen, C. Wang, Y. Wang, Precision calculations of $B_{d,s} \to \pi, K$ decay form factors in soft-collinear effective theory, JHEP 03, 140 (2023).