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Global Analysis of Fragmentation Functions to Light Neutral Hadrons

based on arXiv:2503.21311

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1. Introduction

1.1 Introduction to FFs

In its simplest form, fragmentation functions (FFs) describe number density of the identified hadron wrt the fraction of momentum of the initial parton it carries, as measured in single inclusive hadron production, e.g., from single-inclusive annihilation (SIA), semi-inclusive DIS (SIDIS), *pp* collisions

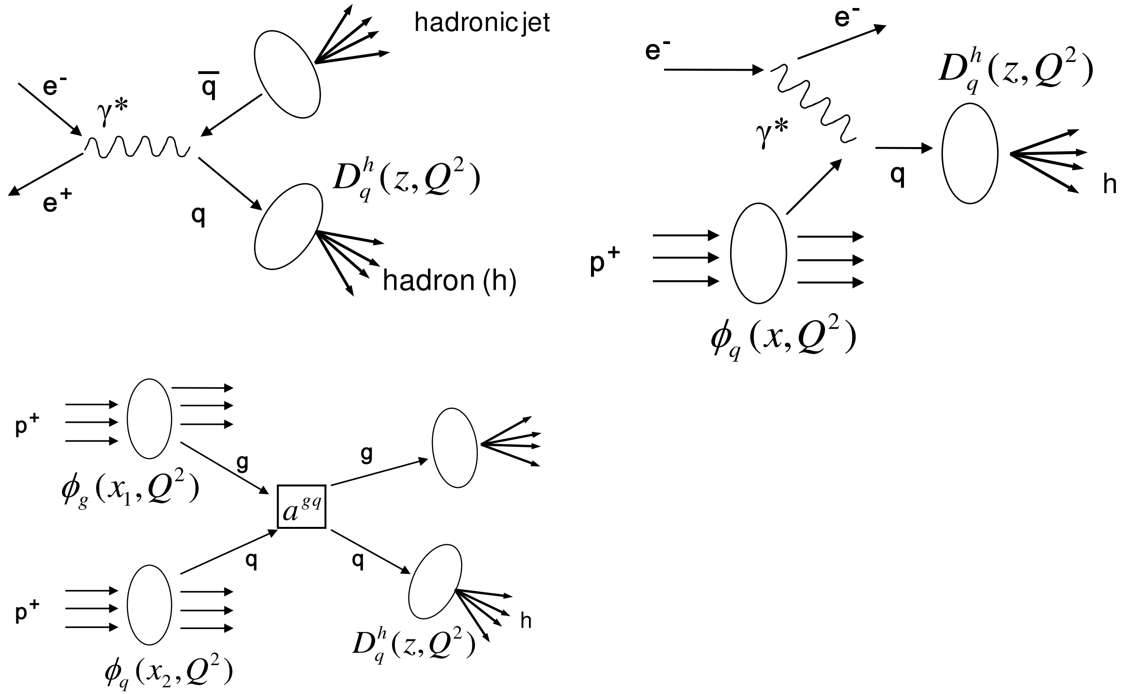
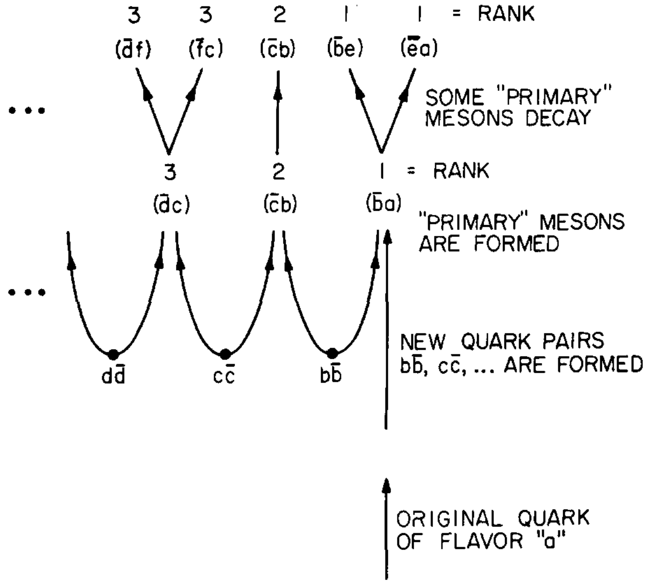


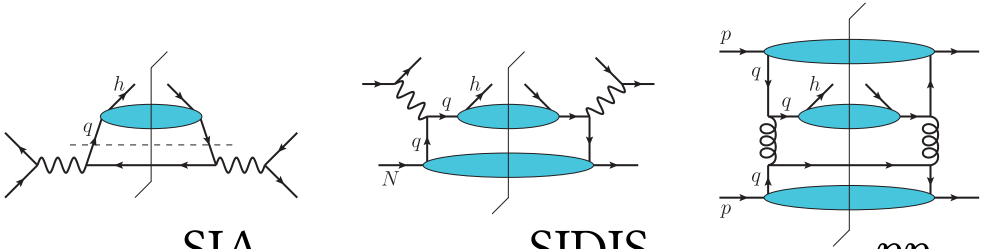
Illustration of hadron production in SIA, SIDIS and *pp* processes



Cascade decay ansatz (R.D. Field & R.P. Feynman)

1.2 QCD Factorization Theorem

- QCD factorization theorem enables the separation of the perturbatively calculable part of the cross section from the non-perturbative part which involes initial and final state hadrons. [J. C. Collins, D. E. Soper, G. Sterman]



(a) SIA (b) SIDIS (c) pp

$$\underset{\substack{\text{experimental}\\ \text{measurements}}}{\underset{\Downarrow}{\sigma^{pp\to hX}}} = f_{i/p}\otimes f_{j/p}\otimes \underset{\text{pQCD}}{\underset{\Downarrow}{\hat{\sigma}_{ij\to k}}} \otimes \underset{\substack{\text{global}\\ \text{extraction}}}{\underset{\Downarrow}{D_k^h}}$$

- Hard scattering processes are independent of hadrons involved. Coefficient functions are perturbatively calculable.
- PDFs/FFs are non-perturbative part that describe the inner structure of hadrons or parton-hadron transition. They are universal and can be fitted from data.
- Evolution of FFs w.r.t. scale is governed by DGLAP equation.

$$\frac{\partial}{\partial \ln Q^2} D_i^h(z,Q) = \sum_j P_{ji}(z,\alpha_s(Q))\otimes D_j^h(z,Q), i,j = q,\bar{q},g$$

- Operator definition:

$$D_1^{h/q}(z) = \frac{z}{4}\sum_X \int \frac{d\xi^+}{2\pi} e^{ik^-\xi^+}\ \mathrm{Tr}\Big(\big[\langle 0 \mid \mathcal{W}(\infty^+,\xi^+)\psi_q\big(\xi^+,0^-,\vec{0}_T\big) \mid P_h,S_h;X\rangle \\ \times \langle P_h,S_h;X \mid \overline{\psi}_q\big(0^+,0^-,\vec{0}_T\big)\mathcal{W}(0^+,\infty^+) \mid 0\rangle\gamma^-\big]\Big)$$

2. Overview of NPC FFs

2.1 Previous Work: NPC FFs for Charged Hadrons

LHAPDF 6.5.5

Main page | PDF sets | Class hierarchy | Examples | More...

2070000	NPC23_PIp_nlo	(tarball) (info file)	127	1
2070200	NPC23_KAp_nlo	(tarball) (info file)	127	1
2070400	NPC23_PRp_nlo	(tarball) (info file)	127	1
2070600	NPC23_PIm_nlo	(tarball) (info file)	127	1
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2071000	NPC23_PRm_nlo	(tarball) (info file)	127	1
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2071400	NPC23_KAsum_nlo	(tarball) (info file)	127	1
2071600	NPC23_PRsum_nlo	(tarball) (info file)	127	1
2071800	NPC23_CHHAp_nlo	(tarball) (info file)	127	1
2072000	NPC23_CHHAm_nlo	(tarball) (info file)	127	1
2072200	NPC23_CHHAsum_nlo	(tarball) (info file)	127	1

Figure 1: From https://lhapdf.hepforge.org/pdfsets.html

- First time including jet fragmentation data.
- Joint determination of FFs to charged pion, kaon, proton at NLO including estimation of uncertainties with Hessian sets
- Strong selection criteria to ensure validity of leading twist factorization ($E_h/p_{T,h} > 4$ GeV, $z > 0.01$)
- We arrive at a best-fit of the charged pion, kaon and proton FFs ($\chi^2/N_{pt} = 0.90$ for 1370 points)together with 126 Hessian error FFs

2.1 Previous Work: NPC FFs for Charged Hadrons

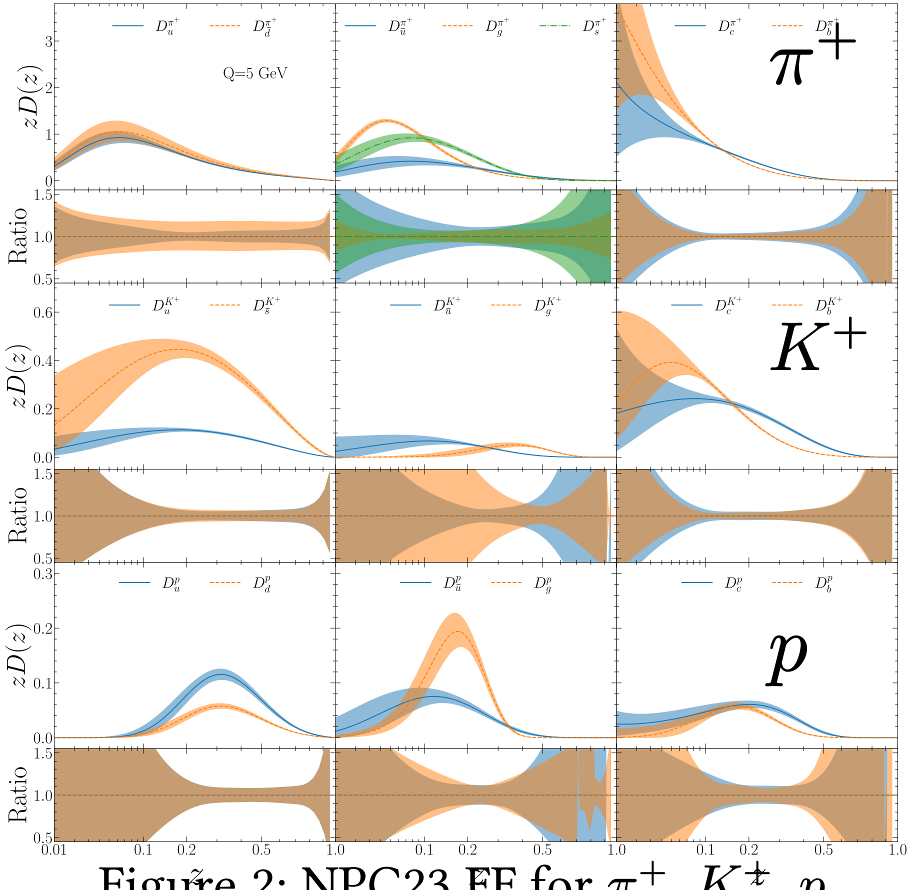


Figure 2: NPC23 FF for π^+, K^+, p
Phys.Rev.D 110 (2024) 11, 114019

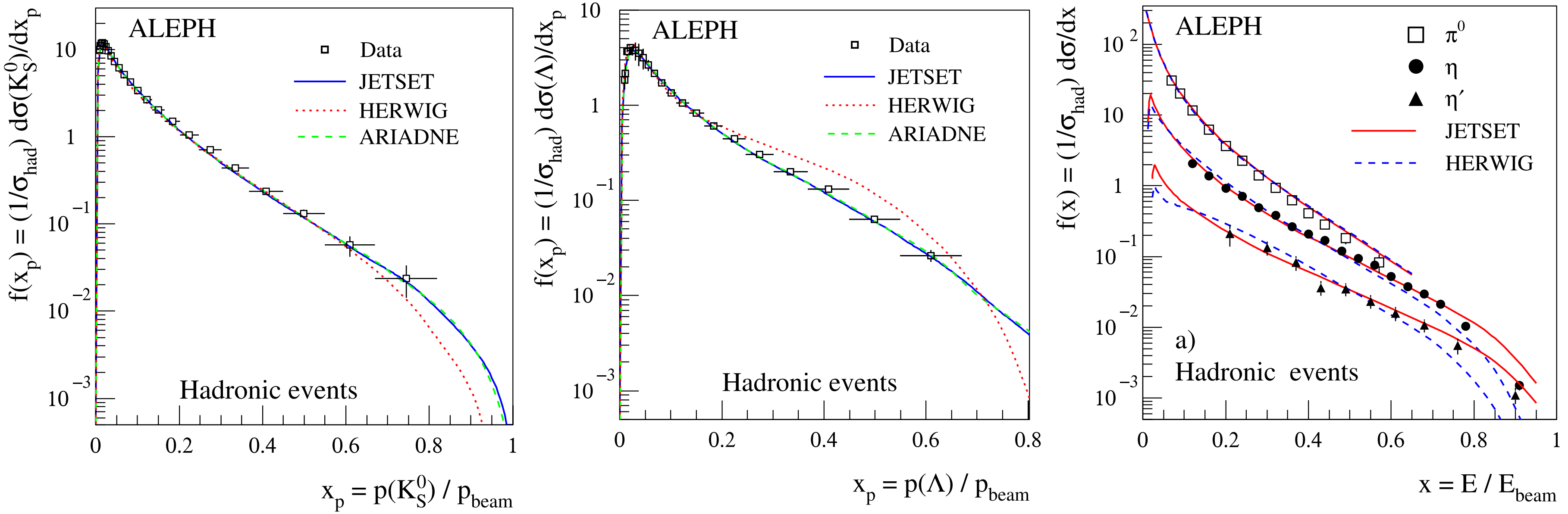
- High precision determination of FFs for charged hadrons
- 4%, 4% and 7% uncertainty for u-quark to π^+, K^+, p at $z = 0.3$, respectively
- 3%, 4% and 8% uncertainty for gluon to pion at $z = 0.05, 0.1, 0.3$, respectively, mainly due to the jet fragmentation data from LHC
- Heavy quark fragmentation are well constrained in $z = 0.1 \sim 0.5$

3. NPC FFs for Neutral Hadrons

3.1 Neutral Light Hadrons

Particle	Quark Composition	Mass (MeV/c^2)	Lifetime (s)	Spin
K_S^0	$\frac{1}{\sqrt{2}}\left(\lvert d\bar{s}\rangle - \lvert s\bar{d}\rangle\right)$	497.611 ± 0.013	0.8954×10^{-10}	0
Λ	$\lvert uds\rangle$	1115.683 ± 0.006	2.632×10^{-10}	1/2
η	$\frac{1}{\sqrt{6}}\left(\lvert u\bar{u}\rangle - \lvert d\bar{d}\rangle - 2\lvert s\bar{s}\rangle\right)$	547.862 ± 0.017	5.0×10^{-19}	0
π^0	$\frac{1}{\sqrt{2}}\left(\lvert u\bar{u}\rangle - \lvert d\bar{d}\rangle\right)$	134.977 ± 0.0005	8.52×10^{-17}	0

Eur. Phys. J. C 16, 613–634 (2000)



3.2 Datasets

We include a comprehensive set of data: SIA below Z-pole energy (9.46 GeV $\sim$ 58 GeV), SIA at Z-pole inclusive, jet fragmentation and tagged data. pp data and SIDIS data (K_S^0, Λ only). Jet fragmentation and SIDIS data are incorporated for the first time.

collaboration	year	$\sqrt{s}$[GeV]	final state	observable	N_{pt}
TASSO	1985	14, 22 & 34	K_S^0	$\frac{s}{\beta}\frac{d\sigma}{dx_h}$	9, 6 & 13
TASSO	1990	14.8, 21.5, 34.5, 35 & 42.6	K_S^0	$\frac{s}{\beta}\frac{d\sigma}{dx_h}$	9, 6, 13, 13 & 13
TPC	1984	29	K_S^0	$\frac{1}{\beta\sigma_h}\frac{d\sigma}{dx_h}$	8
MARK II	1985	29	K_S^0	$\frac{1}{\beta\sigma_h}\frac{d\sigma}{dx_h}$	17
HRS	1987	29	K_S^0	$\frac{s}{\beta}\frac{d\sigma}{dx_h}$	12
CELLO	1990	35	K_S^0	$\frac{1}{\beta\sigma_h}\frac{d\sigma}{dx_h}$	9
TOPAZ	1995	58	K_S^0	$\frac{1}{\sigma_h}\frac{d\sigma}{d\xi}$	4
OPAL	1991	91.2	K_S^0	$\frac{1}{\beta\sigma_h}\frac{d\sigma}{dx_h}$	7
OPAL	1995	91.2	K_S^0	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_h}$	16
OPAL	2000	91.2	K_S^0	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_h}$	16
ALEPH	1998	91.2	K_S^0	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	16
ALEPH	2000	91.2	K_S^0	$\frac{1}{\sigma_h}\frac{d\sigma}{d\xi}$	14
ALEPH	2000	91.2	K_S^0 (jet)	$\frac{1}{\sigma_{\text{jet}}}\frac{d\sigma}{d\xi}$	12, 13 & 11
DELPHI	1995	91.2	K_S^0	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	13
SLD	1999	91.2	K_S^0	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	9
SLD (tagged)	1999	91.2	K_S^0 (tagged)	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	9 & 9
ZEUS	2012	318	K_S^0	$\frac{1}{N_{\text{DIS}}}\frac{dN_{K_S^0}}{dz_p}$	5, 5 & 2
ALICE	2021	13000 & 7000	K_S^0	$\frac{dN_{K_S^0}^{13\,\text{TeV}}}{dp_T} / \frac{dN_{K_S^0}^{7\,\text{TeV}}}{dp_T}$	10
ALICE	2021	13000	K_S^0 & $\pi^\pm$	$\frac{dN_{K_S^0}}{dp_T} / \frac{dN_{\pi^\pm}}{dp_T}$	15

Table 2: Data table for K_S^0

collaboration	year	$\sqrt{s}$[GeV]	final state	observable	N_{pt}
TASSO	1985	14, 22 & 34	Λ	$\frac{s}{\beta}\frac{d\sigma}{dx_h}$	3, 4 & 7
SLAC	1985	29	Λ	$\frac{s}{\beta}\frac{d\sigma}{dx_h}$	15
HRS	1987	29	Λ	$\frac{s}{\beta}\frac{d\sigma}{dx_h}$	8
CELLO	1990	35	Λ	$\frac{1}{\beta\sigma_h}\frac{d\sigma}{dx_h}$	7
DELPHI	1993	91.2	Λ	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	7
ALEPH	1994	91.2	Λ	$\frac{1}{\sigma_h}\frac{d\sigma}{d\xi}$	14
ALEPH	1998	91.2	Λ	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	16
ALEPH	2000	91.2	Λ(jet)	$\frac{1}{\sigma_{\text{jet}}}\frac{d\sigma}{d\xi}$	13, 12 & 9
OPAL	1997	91.2	Λ	$\frac{1}{\beta\sigma_h}\frac{d\sigma}{dx_h}$	12
SLD	1999	91.2	Λ	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	9
SLD (tagged)	1999	91.2	Λ(tagged)	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	4 & 4
ZEUS	2012	318	Λ	$\frac{1}{N_{\text{DIS}}}\frac{dN_\Lambda}{dz_p}$	5, 3 & 1
CMS	2011	900	Λ & K_S^0	$\frac{dN_\Lambda}{dp_T} / \frac{dN_{K_S^0}}{dp_T}$	4
ALICE	2021	13000 & 7000	Λ	$\frac{dN_\Lambda^{13\,\text{TeV}}}{dp_T} / \frac{dN_\Lambda^{7\,\text{TeV}}}{dp_T}$	7

Table 3: Data table for Λ

3.2 Datasets

collaboration	year	$\sqrt{s}$[GeV]	final state	observable	N_{pt}
ARGUS	1990	9.46	η	$\frac{s}{\beta}\frac{d\sigma}{dx_h}$	6
HRS	1988	29	η	$\frac{s}{\beta}\frac{d\sigma}{dx_h}$	13
JADE	1985	34.4	η	$\frac{s}{\beta}\frac{d\sigma}{x_h}$	2
JADE	1990	35	η	$\frac{s}{\beta}\frac{d\sigma}{dx_h}$	3
CELLO	1990	35	η	$\frac{s}{\beta}\frac{d\sigma}{dx_h}$	5
L3	1992	91.2	η	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	4
L3	1994	91.2	η	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_h}$	10
ALEPH	1992	91.2	η	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_h}$	8
ALEPH	2000	91.2	η	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_h}$	18
ALEPH	2000	91.2	η(jet)	$\frac{1}{\sigma_{\text{jet}}}\frac{d\sigma}{dx_h}$	7, 6 & 4
ALEPH	2002	91.2	η	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	5
OPAL	1998	91.2	η	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_h}$	11
PHENIX	2011	200	η & π^0	$\frac{Ed^3\sigma_\eta}{dp^3}/\frac{Ed^3\sigma_{\pi^0}}{dp^3}$	14
ALICE	1207	2760	η & π^0	$\frac{Ed^3\sigma_\eta}{dp^3}/\frac{Ed^3\sigma_{\pi^0}}{dp^3}$	6
ALICE	1202	7000	η & π^0	$\frac{Ed^3\sigma_\eta}{dp^3}/\frac{Ed^3\sigma_{\pi^0}}{dp^3}$	4
ALICE	1208	8000	η & π^0	$\frac{Ed^3\sigma_\eta}{dp^3}/\frac{Ed^3\sigma_{\pi^0}}{dp^3}$	13
ALICE	2204	13000	η & π^0	$\frac{Ed^3\sigma_\eta}{dp^3}/\frac{Ed^3\sigma_{\pi^0}}{dp^3}$	14

Table 4: Data table for η

collaboration	year	$\sqrt{s}$[GeV]	final state	observable	N_{pt}
ARGUS	1990	9.46	π^0	$\frac{s}{\beta}\frac{d\sigma}{dx_h}$	14
JADE	1985	34.4	π^0	$\frac{s}{\beta}\frac{d\sigma}{dx_h}$	10
JADE	1990	35	π^0	$\frac{s}{\beta}\frac{d\sigma}{dx_h}$	9
JADE	1990	44	π^0	$\frac{s}{\beta}\frac{d\sigma}{dx_h}$	6
CELLO	1990	35	π^0	$\frac{s}{\beta}\frac{d\sigma}{dx_h}$	15
TASSO	1989	44	π^0	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	6
ALEPH	1997	91.2	π^0	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	20
ALEPH	2000	91.2	π^0(jet)	$\frac{1}{\sigma_{\text{jet}}}\frac{d\sigma}{dx_h}$	8, 8 & 6
DELPHI	1996	91.2	π^0	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	17
DELPHI	1996	91.2	π^0(b-tagged)	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	15
L3	1994	91.2	π^0	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$	12
OPAL	1998	91.2	π^0	$\frac{1}{\sigma_h}\frac{d\sigma}{dx_h}$	10
STAR	2010	200	π^0	$\frac{Ed^3\sigma}{dp^3}$	8
PHENIX	2007	200	π^0	$\frac{Ed^3\sigma}{dp^3}$	17
PHENIX	2016	510	π^0	$\frac{Ed^3\sigma}{dp^3}$	22
ALICE	2017	2760	π^0	$\frac{Ed^3\sigma}{dp^3}$	16
ALICE	2012	7000	π^0	$\frac{Ed^3\sigma}{dp^3}$	13
ALICE	2018	8000	π^0	$\frac{Ed^3\sigma}{dp^3}$	24

Table 5: Data table for π^0

3.3 Parametrization

$$zD_i^h(z,Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h}\exp\left(\sum_{n=0}^{m}\alpha_{i,n}^h z^{n/2}\right)$$

parton-to-K_S^0	favored	a_0	α	β	a_1
$u=\bar{u}$	✘	✓	✓	✓	✓
$d=\bar{d}$	✓	✓	$=\alpha_u$	✓	✓
$s=\bar{s}$	✓	✓	$=\alpha_u$	$=\beta_d$	$=a_{1,d}$
$c=\bar{c}$	✘	✓	✓	✓	✓
$b=\bar{b}$	✘	✓	✓	✓	✓
g	✘	✓	✓	✓	✓
parton-to-Λ	**favored**	**a_0**	**α**	**β**	**a_1**
$u=\bar{u}=d=\bar{d}$	✓	✓	✓	✓	✓
$s=\bar{s}$	✓	✓	✓	✓	✓
$c=\bar{c}$	✘	✓	✓	✓	✘
$b=\bar{b}$	✘	✓	✓	✓	✘
g	✘	✓	✓	✓	✓
parton-to-η	**favored**	**a_0**	**α**	**β**	**a_1**
$u=\bar{u}=d=\bar{d}$	✓	✓	✓	✓	✓
$s=\bar{s}$	✓	✓	$=\alpha_u$	$=\beta_u$	$=a_{1,u}$
$c=\bar{c}=b=\bar{b}$	✘	✓	✓	✓	✘
g	✘	✓	✓	✓	✓

Table 6: Parameterization for K_S^0, Λ, η

- π^0 FFs are constructed from $\pi^\pm$ FFs by

$$D_q^{\pi^0} = \frac{1}{4}\left(D_q^{\pi^+} + D_q^{\pi^-} + D_{q'}^{\pi^+} + D_{q'}^{\pi^-}\right), q(q') = u(d) \text{ or } d(u)$$

$$D_i^{\pi^0} = \frac{1}{2}\left(D_i^{\pi^+} + D_i^{\pi^-}\right), i = g, s, c, b$$

  and are tested by π^0 production datasets.

- K_S^0, Λ, η fits are performed independently.
- Theoretical calculations are performed using the FMNLO framework at NLO accuracy. Uncertainties are estimated using Hessian methods [hep-ph/0101032] and various combinations of fragmentation and factorization scales.
- For pp process $p_{T,h} > 4\text{GeV}$, and for SIA and SIDIS process $z > 0.05$

3.4 Fit Quality

K_S^0

collaboration	year	$\sqrt{s}$[GeV]	χ^2	N_{pt}	χ^2/N_{pt}
TASSO	1985	14	5.65	9	0.63
TASSO	1985	22	5.87	6	0.98
TASSO	1985	34	16.03	13	1.23
TASSO	1990	14.8	12.56	9	1.40
TASSO	1990	21.5	3.78	6	0.63
TASSO	1990	34.5	17.51	13	1.35
TASSO	1990	35	14.76	13	1.14
TASSO	1990	42.6	33.60	13	2.58
TPC	1984	29	2.75	8	0.34
MARK II	1985	29	12.65	17	0.74
HRS	1987	29	33.16	12	2.76
CELLO	1990	35	2.71	9	0.30
TOPAZ	1995	58	0.29	4	0.07
OPAL	1991	91.2	7.75	7	1.11
OPAL	1995	91.2	13.63	16	0.85
OPAL	2000	91.2	8.62	16	0.54
ALEPH	1998	91.2	6.39	16	0.40
ALEPH	2000	91.2	12.72	14	0.91
ALEPH jet 1	2000	91.2	14.91	12	1.24
ALEPH jet 2	2000	91.2	8.21	13	0.63
ALEPH jet 3	2000	91.2	8.55	11	0.78
DELPHI	1995	91.2	7.55	13	0.58
SLD	1999	91.2	7.39	9	0.82
SLD c-tagged	1999	91.2	17.44	9	1.94
SLD b-tagged	1999	91.2	11.12	9	1.24
SIA sum			285.60	277	1.03
ZEUS $Q^2 \in 160, 640\,\mathrm{GeV}^2$	2012	318	4.41	5	0.88
ZEUS $Q^2 \in 640, 2560\,\mathrm{GeV}^2$	2012	318	3.26	5	0.65
ZEUS $Q^2 \in 2560, 10240\,\mathrm{GeV}^2$	2012	318	2.74	2	1.37
SIDIS sum			10.41	12	0.87
ALICE $N_{K_S^0}^{13\,\mathrm{TeV}}/N_{K_S^0}^{7\,\mathrm{TeV}}$	2021	13000 & 7000	2.88	10	0.29
ALICE $N_{K_S^0}/N_{\pi^\pm}$	2021	13000	5.79	15	0.39
pp sum			8.67	25	0.35
total sum			**304.68**	**314**	**0.97**

Λ

collaboration	year	$\sqrt{s}$ [GeV]	χ^2	N_{pt}	χ^2/N_{pt}
TASSO	1985	14	1.45	3	0.48
TASSO	1985	22	2.93	4	0.73
TASSO	1985	34	3.95	7	0.56
SLAC	1985	29	19.16	15	1.28
HRS	1987	29	5.57	8	0.70
CELLO	1990	35	3.49	7	0.50
DELPHI	1993	91.2	9.58	7	1.37
ALEPH	1994	91.2	7.40	14	0.53
ALEPH	1998	91.2	4.43	16	0.28
ALEPH jet 1	2000	91.2	16.50	13	1.27
ALEPH jet 2	2000	91.2	3.08	12	0.26
ALEPH jet 3	2000	91.2	3.72	9	0.41
OPAL	1997	91.2	5.83	12	0.48
SLD	1999	91.2	8.08	9	0.90
SLD c-tagged	1999	91.2	13.96	4	3.49
SLD b-tagged	1999	91.2	0.75	4	0.19
SIA sum			109.88	144	0.76
ZEUS $Q^2 \in (160, 640)\mathrm{GeV}^2$	2012	318	14.44	5	2.89
ZEUS $Q^2 \in (640, 2560)\mathrm{GeV}^2$	2012	318	1.26	3	0.42
ZEUS $Q^2 \in (2560, 10240)\mathrm{GeV}^2$	2012	318	0.01	1	0.01
SIDIS sum			15.70	9	1.74
CMS $N_\Lambda/N_{K_S^0}$	2011	900	2.99	4	0.75
ALICE $N_\Lambda^{13\mathrm{TeV}}/N_\Lambda^{7\mathrm{TeV}}$	2021	13000 & 7000	1.59	7	0.23
pp sum			4.58	11	0.42
total sum			**130.16**	**164**	**0.79**

3.4 Fit Quality

collaboration	year	$\sqrt{s}$[GeV]	χ^2	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$
ARGUS	1990	9.46	5.94	6	0.99
HRS	1988	29	18.07	13	1.39
JADE	1985	34.4	2.29	2	1.14
JADE	1990	35	3.29	3	1.09
CELLO	1990	35	3.47	5	0.69
L3	1992	91.2	5.83	4	1.46
L3	1994	91.2	10.46	10	1.05
ALEPH	1992	91.2	1.48	8	0.18
ALEPH	2000	91.2	18.39	18	1.02
ALEPH jet 1	2000	91.2	11.26	7	1.61
ALEPH jet 2	2000	91.2	1.95	6	0.33
ALEPH jet 3	2000	91.2	10.49	4	2.62
ALEPH	2002	91.2	17.18	5	3.44
OPAL	1998	91.2	7.12	11	0.65
SIA sum			117.20	102	1.15
PHENIX	2011	200	7.61	14	0.51
ALICE	2017	2760	5.37	6	0.90
ALICE	2012	7000	1.26	4	0.32
ALICE	2018	8000	12.64	13	0.97
ALICE	2024	13000	7.11	13	0.51
pp sum			34.00	50	0.68
total sum			**151.21**	**152**	**0.99**

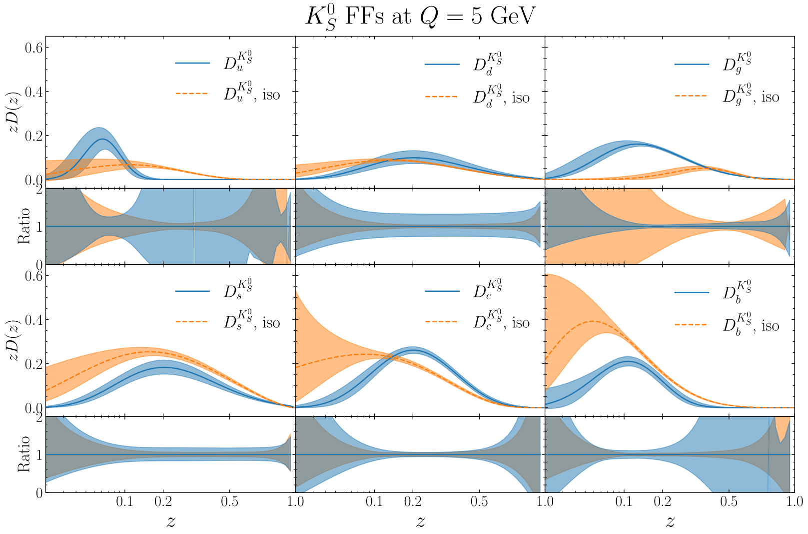
η

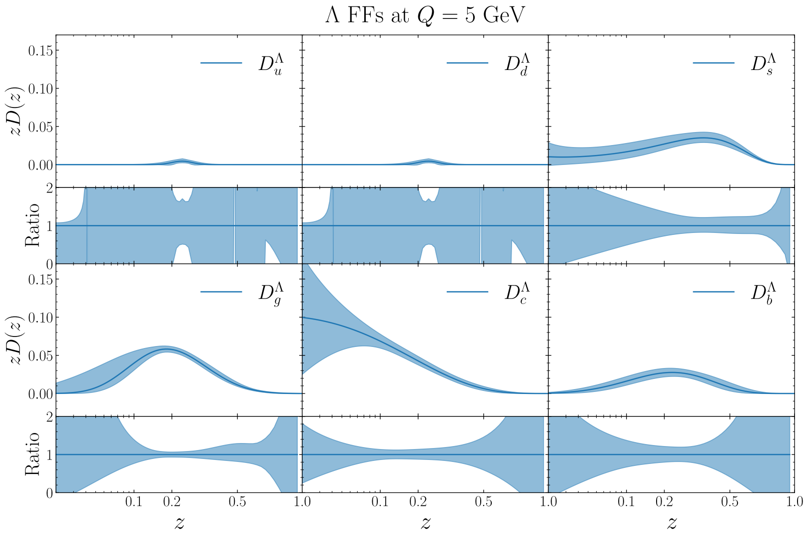
collaboration	year	$\sqrt{s}$[GeV]	χ^2	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$
ARGUS	1990	9.46	20.12	14	1.44
JADE	1985	34.4	7.92	10	0.79
JADE	1990	35	8.66	9	0.96
JADE	1990	44	7.30	6	1.22
CELLO	1990	35	31.31	15	2.09
TASSO	1989	44	1.66	6	0.28
ALEPH	1997	91.2	14.85	20	0.74
ALEPH jet 1	2000	91.2	2.43	8	0.30
ALEPH jet 2	2000	91.2	11.69	8	1.46
ALEPH jet 3	2000	91.2	34.65	6	5.77
DELPHI	1996	91.2	4.86	17	0.29
DELPHI *b*-tagged	1996	91.2	9.12	15	0.61
L3	1994	91.2	27.34	12	2.28
OPAL	1998	91.2	2.73	10	0.27
SIA sum			184.64	156	1.18
STAR	2009	200	6.09	8	0.76
PHENIX	2007	200	45.76	17	2.69
PHENIX	2016	510	66.95	22	3.04
ALICE	2017	2760	13.98	16	0.87
ALICE	2012	7000	29.27	13	2.25
ALICE	2017	8000	46.42	24	1.93
pp sum			208.47	100	2.08
total sum			**393.11**	**256**	**1.54**

π^0

- We arrive at $\chi^2/N_{pt} = 0.97, 0.79, 0.99$ for K_S^0, Λ, η respectively, indicating a successful fit.
- For each category, χ^2/N_{pt} is around 1 as well.
- π^0 data is tested against constructed FFs from $\pi^\pm$ FFs. The deviation is primarily from the few subsets with large discrepancies.

3.5 Delivered FFs



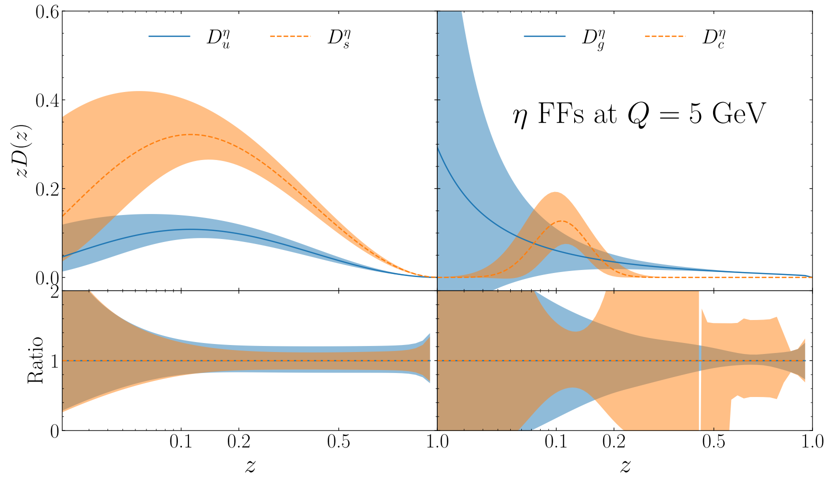


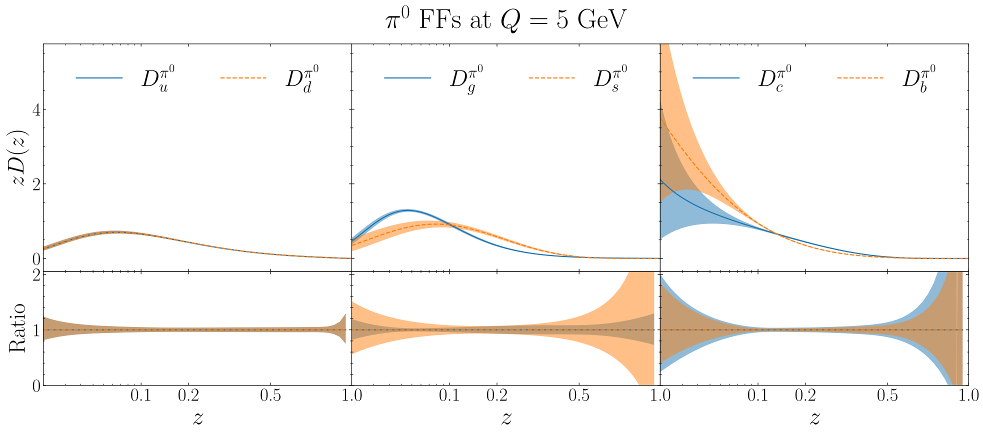
- K_S^0 FFs are generally well constrained in $z = 0.1 \sim 0.5$, In general, the fitted and constructed FFs of K_S^0 do not coincide with each other. But s-quark fragmentation is more favored in both cases. Due to the lack of data, flavor separation bwtween d, s is made by theoretical assumptions.
- In Λ FFs strange quark is clearly more favored compared to u and d quarks, despite the fact that they are all constituent quarks.

$$D_q^{K_S^0}, \text{iso} = \frac{1}{2}\left(D_{q'}^{K^+} + D_{q'}^{K^-}\right), q(q') = u(d) \text{ or } d(u)$$

$$D_i^{K_S^0}, \text{iso} = \frac{1}{2}\left(D_i^{K^+} + D_i^{K^-}\right), i = g, s, c, b$$

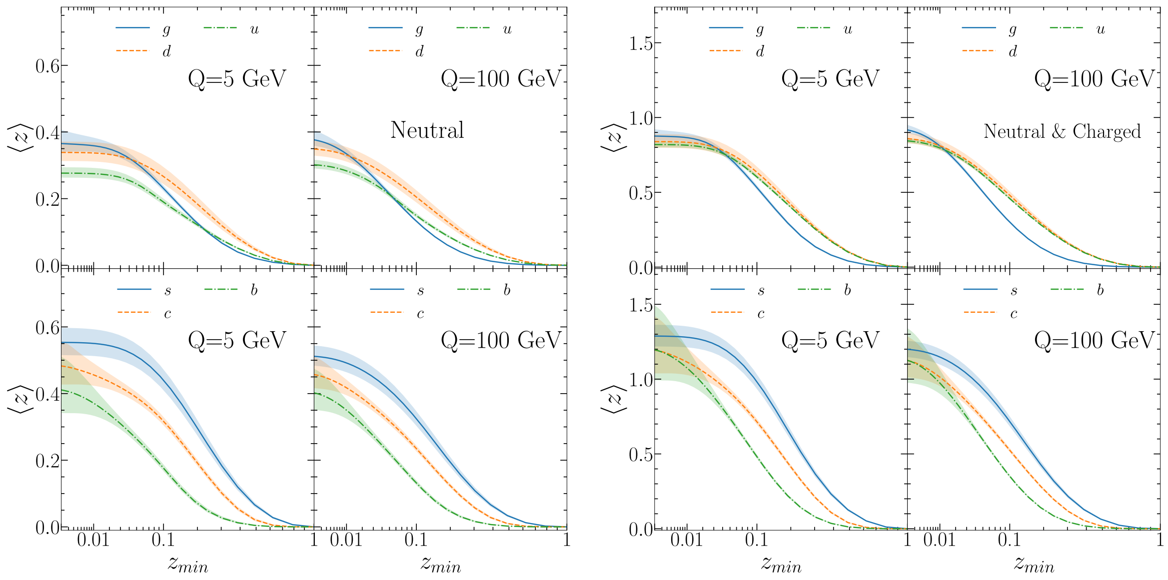
3.5 Delivered FFs





- In η FFs, s-quark fragmentaion is more favored than u, d, consistent with composition. FFs from heavy quarks are much less than those of the favored quarks, except at $z \sim 0.1$. FFs from gluons remain poorly constrained in the low z region.
- π^0 FFs are well-constrained due to the comprehensive datasets from charged hadron production data. In our formalism $D_u^{\pi^0} = D_{\bar{u}}^{\pi^0} = D_d^{\pi^0} = D_{\bar{d}}^{\pi^0}$, the FFs for remaining partons are essentially the same as those of $\pi^\pm$

3.6 Applications: Momentum Sum Rule



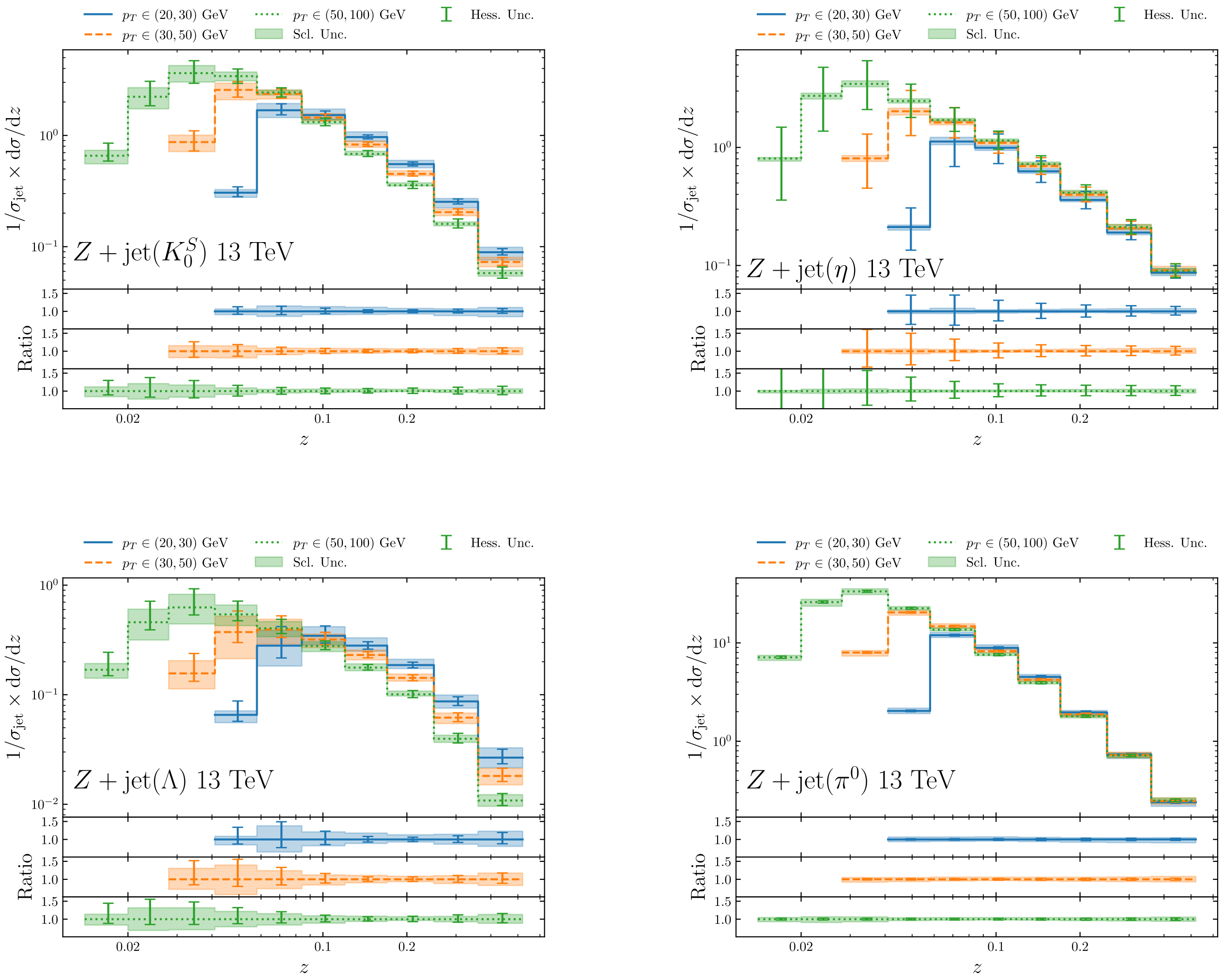
hadron	g ($z_{\rm min}=0.01$)	d ($z_{\rm min}=0.01$)	u ($z_{\rm min}=0.01$)	s ($z_{\rm min}=0.01$)	c ($z_{\rm min}=0.01$)	b ($z_{\rm min}=0.01$)
neutral	$0.3587^{+0.0243}_{-0.0184}$	$0.3370^{+0.0322}_{-0.0256}$	$0.2749^{+0.0173}_{-0.0133}$	$0.5504^{+0.0424}_{-0.0377}$	$0.4561^{+0.0350}_{-0.0325}$	$0.3711^{+0.0465}_{-0.0376}$
charged	$0.5068^{+0.0145}_{-0.0133}$	$0.5066^{+0.0420}_{-0.0367}$	$0.5306^{+0.0150}_{-0.0135}$	$0.7293^{+0.0440}_{-0.0416}$	$0.6581^{+0.0655}_{-0.0572}$	$0.7033^{+0.0903}_{-0.0723}$
total	$0.8655^{+0.0306}_{-0.0251}$	$0.8436^{+0.0591}_{-0.0500}$	$0.8056^{+0.0214}_{-0.0171}$	$1.2797^{+0.0728}_{-0.0678}$	$1.1142^{+0.0974}_{-0.0868}$	$1.0744^{+0.1350}_{-0.1080}$

Figure 3: Momentum fraction carried by each parton from hadrons

Sum rule: $\sum_h \int_0^1 dzzD_i^h(z) = 1$

- Momentum fraction carried by parton i from hadron h with cutoff $z_{\rm min}$ is defined as:
$$\langle z\rangle_i^h = \int_{z_{\rm min}}^1 dzzD_i^h(z)$$
- Definition of FFs will lead to violation of sum rule. [J. Collins and T. Rogers, (2023), arXiv:2309.03346]
- First data-driven test is reported in NPC23 charged fit and rescaled by PYTHIA8. [Gao, **C.Liu**, Shen, Xing, Zhao, PRL, 2024]
- It is tested again with direct fit results of neutral hadrons.

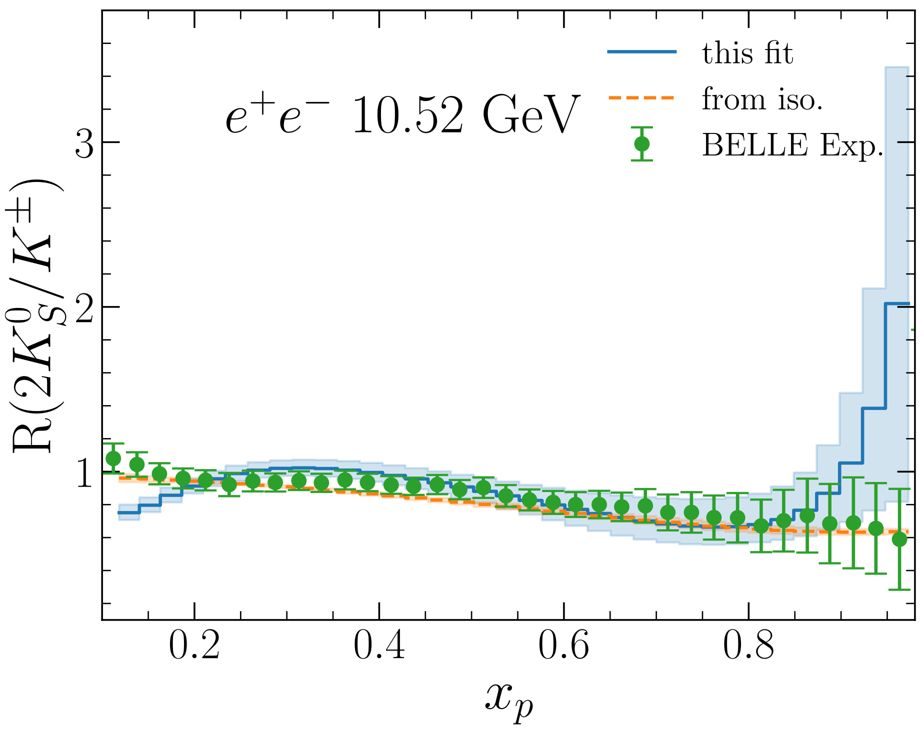
3.7 Applications: Predictions on LHCb Hadron Production

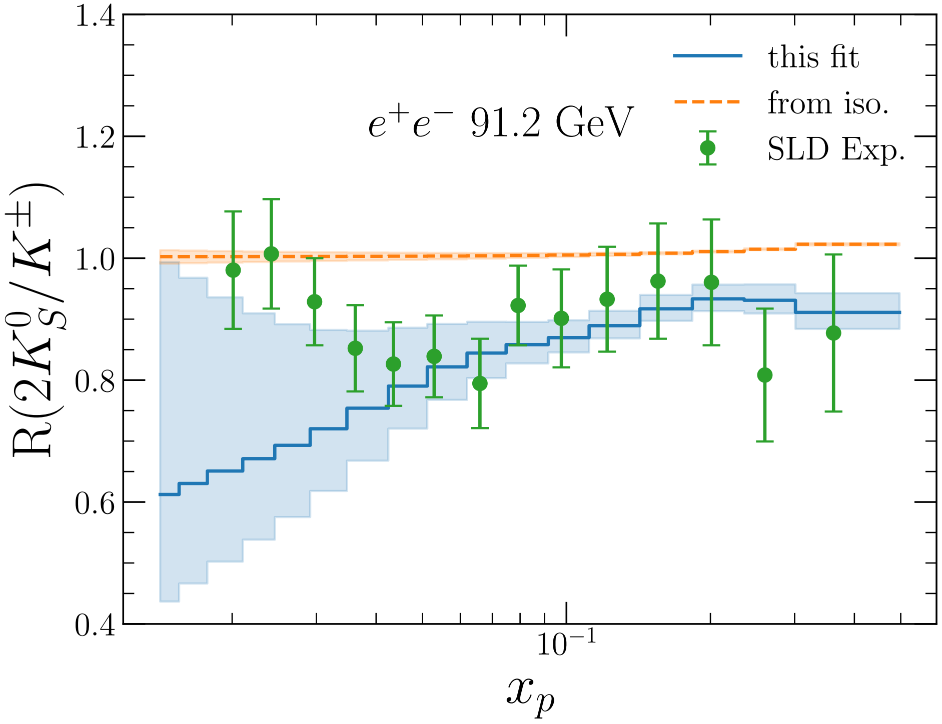


- Predictions of hadron in jet production on LHCb $Z +$ jet process at 13 TeV for 3 different jet p_T regions and 4 neutral hadrons are presented.
- Scale uncertainties, Hessian uncertainties and their relative uncertainties are shown.
- Kinematic bins are taken from previous LHCb measurements.

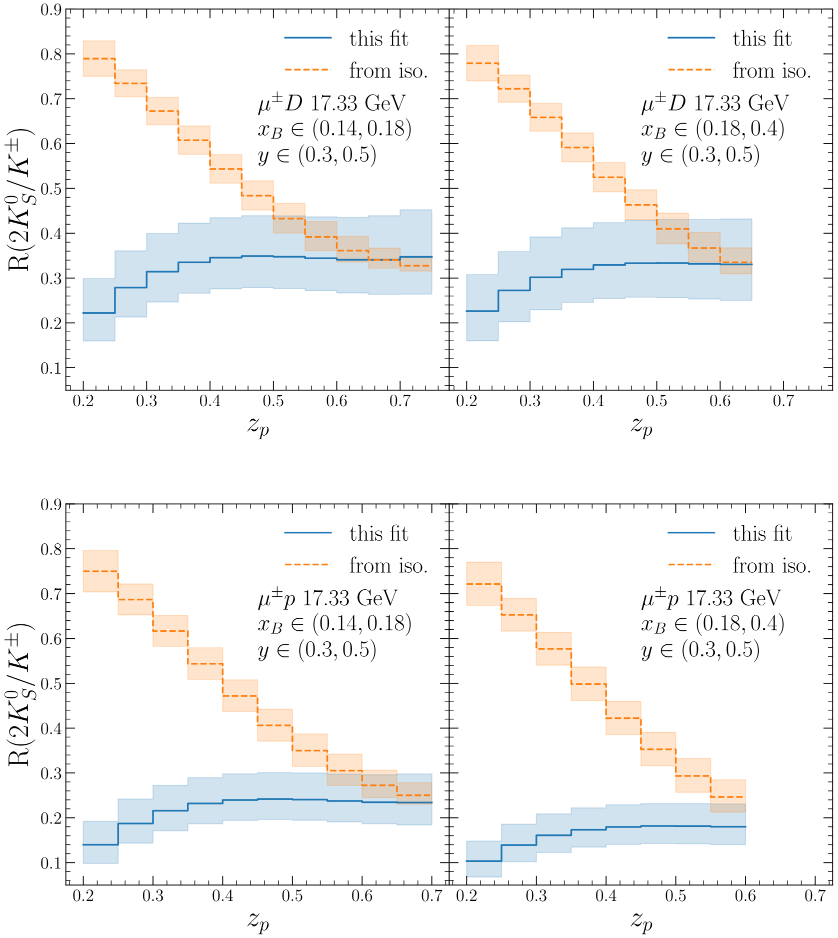
3.8 Applications: Production Ratio of K_S^0 to $K^\pm$

- In $\sqrt{s} = 10.52$GeV K_S^0 prediction of FFs from iso-spin symmetry is better aligned with experiment. Prediction of FFs from fit aligns in intermediate region.
- In $\sqrt{s} = 91.2$GeV, FFs from iso-spin symmetry predict neary unity. FFs from fit give better prediction.



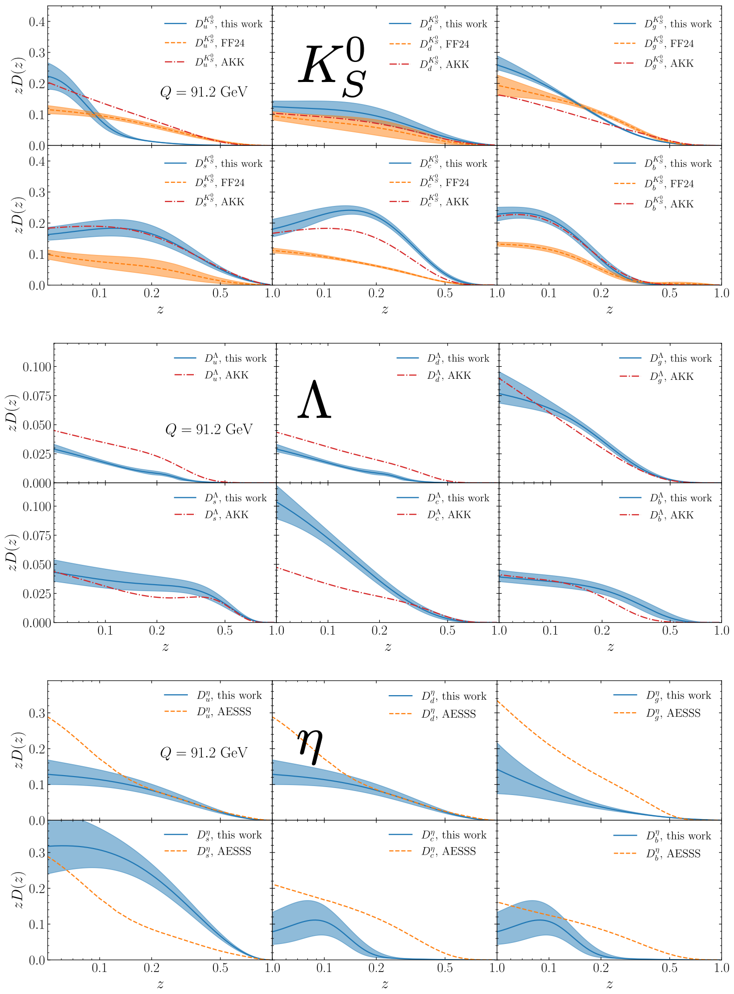


3.8 Applications: Production Ratio of K_S^0 to $K^{\pm}$



- Predictions of $R\left(2K_S^0/K^{\pm}\right)$ is presented following COMPASS kinematic bins
- The two curves diverge in the low z region. SIDIS process has the advantage of flavor separation
- Future measurements could help better contrain the K_S^0 FFs and test isospin symmetry

3.9 Comparison with Other Groups



- FF24 provides Monte Carlo uncertainty, AKK & AESSS uncertainties unavailable.
- K_S^0: NPC23 and AKK are close in d, s, b, NPC23 deviates from FF24 except d after considering uncertainty.
- Λ: NPC23 and AKK FFs from g, s, b agree in low an high z region, c agree in large z region,AKK is greater in u, d fragmentation
- η: AESS and NPC FFs from u, d agree in intermidiate and high z region, but differ significantly for other partons.

4. Summary

4.1 Summary

- In this talk, the previous work of NPC for charged hadrons is introduced.
- Then we present the FFs for $K_S^0, \Lambda, \eta, \pi^0$ in the context of global QCD analysis.
- Data from SIA, SIDIS and pp collisions is incorporated in the analysis. Data from SIA hadron-in-jet production and SIDIS process is included for the first time, which gives better gluon fragmentation constraints and flavor separation.
- With the comprehensive species of FFs extracted within the NPC framework, a test on the momentum sum rule with the light-flavor charged and neutral hadrons is performed.
- The extracted K_S^0 fragmentation functions, together with the K_S^0 FFs constructed from $K^\pm$ FFs via isospin symmetry, are used to test isospin symmetry in kaon fragmentation.
- Comparison of light neutral hadron FFs with other groups is also presented.