# Scrutinizing semileptonic decays of charmed mesons to vector mesons via QCD LCSR

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# Contents

# I. Introduction

# $D \to V \ell \nu$ Decays


## Physical Information in the Decay Process

- Transition form factors
- CKM element $|V_{cq}|$
- Branching fraction

## Physical Significance

- Explore quark flavor dynamics and weak interactions
- Provide stringent tests of the Standard Model

# Experimental Measurements of $D \to V\ell\nu$

| Decay Mode | Collaboration | Year | Reference |
|---|---|---|---|
| $D^0 \to K^{*-}\mu^+\nu_\mu$ | BESIII | 2025 | PRL 134(2025)1,011803 |
| $D^0 \to \rho^- e^+\nu_e$ | BESIII | 2024 | PRD 110(2024)11,112018 |
| $D_s^+ \to K^{*0} e^+\nu_e$ | BESIII | 2019 | PRL 122 (2019) 6, 061801 |
| $D^+ \to \rho^0 e^+\nu_e$ | CLEO | 2013 | PRL 110(2013)13,131802 |
| $D^+ \to K^{*0}\mu^+\nu_\mu$ | FOCUS | 2006 | PLB 637(2006)32-38 |

# Status of theoretical studies

- **QCD based approaches:**
  - ▲ **light-cone sum rules**
    Y.-L. Wu, Int. J. Mod. Phys. A 21, 6125 (2006)
    H.-B. Fu, Phys. Rev. Res. 2, 043129 (2020)
  - ▲ **QCD sum rules**
    P. Ball, Phys. Rev. D 48, 3190 (1993)
  - ▲ **QCD factorization**
    T. Feldmann, JHEP 08, 105 (2017)
- **Quark model formulations:**
  - ▲ **Quark Model**
    D. Scora, Phys. Rev. D 52, 2783 (1995)
  - ▲ **Covariant Light-front QM**
    H.-Y. Cheng, Phys. Rev. D 69, 074025 (2004)
    R. C. Verma, J. Phys. G 39, 025005 (2012), 1103.2973.
  - ▲ **Covariant Confined QM**
    N. R. Soni, Phys. Rev. D 98,114031 (2018), 1810.11907
  - ▲ **Relativistic QM**
    R. N. Faustov,Phys. Rev. D 101, 013004 (2020), 1911.0820
- **Low-energy effective theories:**
  - ▲ **HChPT**
    S. Fajfer, Phys. Rev. D 72, 034029 (2005)
  - ▲ **Chiral Unitarity Approach**
    T. Sekihara, Phys. Rev. D 92, 054038 (2015)

# II. Transition FFs from LCSR

- **Definition of Form Factors(traditional):**

$$\langle D_{(s)}(p_{D_{(s)}})|\, j_2(0)\,|0\rangle = \frac{m^2_{D_{(s)}}}{m_c + m_{q_2}} f_{D_{(s)}}$$

$$\begin{aligned}\langle V(p,\eta^*)|\, j_{1\mu}(x)\,|D_{(s)}(p_{D_{(s)}})\rangle =& - i\eta^*_\mu (m_{D_{(s)}} + m_V) A_1(q^2) + i(p_{D_{(s)}} + p)_\mu (\eta^* q)\frac{A_2(q^2)}{m_{D_{(s)}} + m_V} \\ &+ i q_\mu (\eta^* q)\frac{2m_V}{q^2}\left(A_3(q^2) - A_0(q^2)\right) - \epsilon_{\mu\nu\rho\sigma}\eta^{*\nu} p^\rho_{D_{(s)}} p^\sigma \frac{2V(q^2)}{m_{D_{(s)}} + m_{K^*}}\end{aligned}$$

$$A_3(q^2) = \left[(m_D + m_V)A_1(q^2) - (m_D - m_V)A_2(q^2)\right]/2/m_V$$

$$j_{1\mu}(x) = \bar{q_1}(x)\gamma_\mu(1-\gamma_5)c(x) \quad , j_2(0) = \bar{c}(0) i\gamma_5 q_2(0)$$

- **Definition of Form Factors:**

$$\langle V(p,\eta^*)|j_{1\mu}(x)|D_{(s)}(p_{D_{(s)}})\rangle = p_{1\mu}\mathcal{V}_1(q^2) + p_{2\mu}\mathcal{V}_2(q^2) + p_{3\mu}\mathcal{V}_3(q^2) + p_{P\mu}\mathcal{V}_P(q^2),$$

$$p_{P\mu} = i(\eta^*\cdot q)q^\mu, \qquad p_{1\mu} = -2\varepsilon^{\mu\nu\rho\sigma}\eta^*_\nu p_\rho q_\sigma,$$

$$p_{2\mu} = i\left\{\left(m_D^2 - m_V^2\right)\eta^{*\mu} - (\eta^*\cdot q)\left(p+p_D\right)^\mu\right\}, \quad p_{3\mu} = i\left(\eta^*\cdot q\right)\left\{q^\mu - \frac{q^2}{m_D^2 - m_V^2}\left(p+p_D\right)^\mu\right\}.$$

- **Relations:**

$$\mathcal{V}_P(q^2) = -\frac{2m_V}{q^2}A_0(q^2), \qquad \mathcal{V}_1(q^2) = -\frac{V(q^2)}{m_D+m_V},$$

$$\mathcal{V}_2(q^2) = -\frac{A_1(q^2)}{m_D-m_V}, \qquad \mathcal{V}_3(q^2) = \frac{2m_V}{q^2}A_3(q^2).$$

# Form Factors Calculation in the LCSR Framework



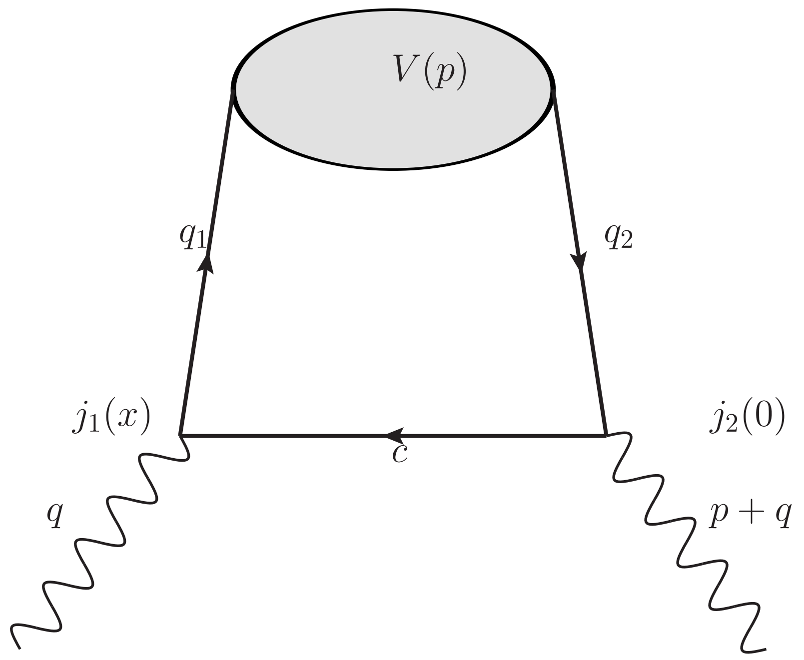
- **Correlation function：**

$$C[\Gamma_\mu] = i\int d^4x e^{iq\cdot x}\langle V(p,\eta^*)|T\{\Gamma_\mu(0),\Gamma_5(x)\}|0\rangle$$

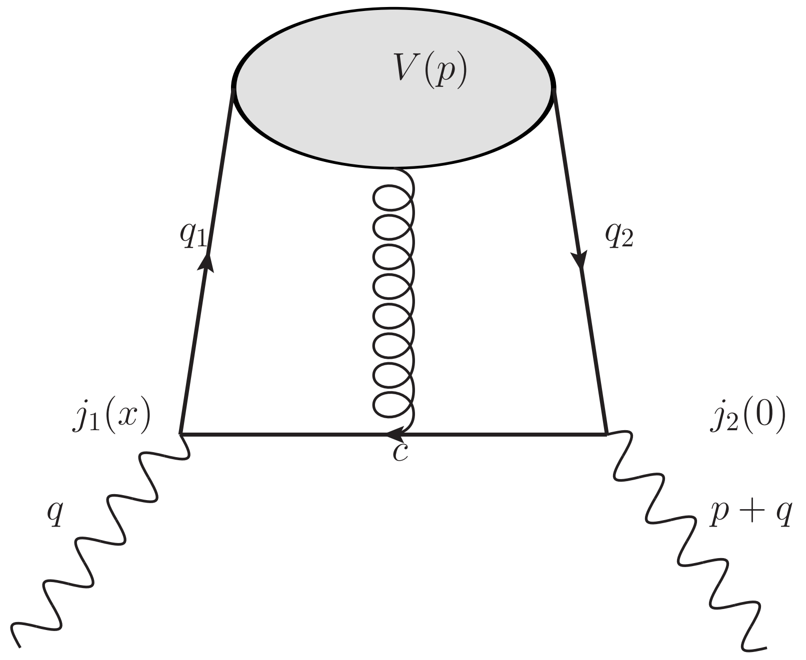
▲ $\Gamma_\mu = \bar{q_1}\gamma_\mu(1-\gamma_5)c$ ▲ $\Gamma_5 = \bar{c}i\gamma_5 q_2$

★ $C[\Gamma_\mu] = i\int d^4x e^{iq\cdot x}\,\langle V(p,\eta^*)|\,\bar{q_1}_\alpha(x)q_{2\beta}(0)\,|0\rangle\,[\gamma_\mu\,(1-\gamma_5)]_{\alpha\alpha'}[i\gamma_5]_{\beta'\beta}[iS_c(x,0,m_c)]_{\alpha'\beta'}$

$$\begin{aligned}S_c(x,0,m_c) \equiv& -i\,\langle 0|\,T\{c_i(x)\bar{c}_j(0)\}\,|0\rangle\\ =& \frac{-im_c^2}{4\pi^2}\left[\frac{K_1(m_c\sqrt{|x^2|})}{\sqrt{|x^2|}} + i\frac{\not{x}}{|x^2|}K_2(m_c\sqrt{|x^2|})\right]\delta_{ij}\\ &- \frac{igm_c}{16\pi^2}\int_0^1 du\left[G\cdot\sigma\,K_0(m_c\sqrt{|x^2|}) + i\frac{[\not{x}G\cdot\sigma - 4iux_\mu G^{\mu\nu}\gamma_\nu]K_1(m_c\sqrt{|x^2|})}{\sqrt{|x^2|}}\right]\delta_{ij} + \cdots\end{aligned}$$



two-particle



three-particle

$$C[\Gamma_\mu] = i\int d^4x e^{iq\cdot x} \langle V(p,\eta^*)|\, \bar{q_1}_\alpha(x) {q_2}_\beta(0)\,|0\rangle \gamma_\mu\,(1-\gamma_5)]_{\alpha\alpha'}[i\gamma_5]_{\beta'\beta}[iS_c(x,0,m_c)]_{\alpha'\beta'}$$

Fierz identity

$$\langle V(p,\eta^*)|\,\bar{q_1}_\alpha(x){q_2}_\beta(0)\,|0\rangle = \frac{1}{4}\Big\{(\mathbf{1})_{\beta\alpha}\langle V(p,\eta^*)|\,\bar{q_1}(x)q_2(0)\,|0\rangle + (\gamma_5)_{\beta\alpha}\langle V(p,\eta^*)|\,\bar{q_1}(x)\gamma_5 q_2(0)\,|0\rangle$$
$$+ (\gamma_\mu)_{\beta\alpha}\langle V(p,\eta^*)|\,\bar{q_1}(x)\gamma_\mu q_2(0)\,|0\rangle - (\gamma_\mu\gamma_5)_{\beta\alpha}\langle V(p,\eta^*)|\,\bar{q_1}(x)\gamma_\mu\gamma_5 q_2(0)\,|0\rangle$$
$$+ \frac{1}{2}(\sigma_{\mu\nu})_{\beta\alpha}\langle V(p,\eta^*)|\,\bar{q_1}(x)\sigma_{\mu\nu} q_2(0)\,|0\rangle\Big\}$$

# Form Factors Calculation in the LCSR Framework

Definition of Form Factors | Calculate Correlation Function | Borel Transformation

- **Two-Particle LCDAs:**

$$\langle V(p,\eta^*)|\bar{q}_1(x)\gamma_\mu q_2(0)|0\rangle = f_V^{\parallel} m_V \int_0^1 du e^{i\bar{u}p\cdot x}\Big\{\eta_\mu^*\Big[\phi_3^\perp(u)+\frac{m_V^2x^2}{16}\phi_5^\perp(u)\Big]$$
$$+p_\mu\frac{\eta^*\cdot x}{p\cdot x}\Big[\phi_2^{\parallel}(u)-\phi_3^\perp(u)+\frac{m_V^2x^2}{16}\Big(\phi_4^{\parallel}(u)-\phi_5^\perp(u)\Big)\Big]$$
$$-\frac{(\eta^*\cdot x)m_V^2}{2(p\cdot x)^2}x_\mu\Big[\psi_4^{\parallel}(u)-2\phi_3^\perp(u)+\phi_2^{\parallel}(u)\Big]\Big\},$$

$$\langle V(p,\eta^*)|\bar{q}_1(x)\sigma_{\mu\nu}q_2(0)|0\rangle = -if_V^\perp\int_0^1 du e^{i\bar{u}p\cdot x}\Big\{(\eta_\mu^*p_\nu-\eta_\nu^*p_\mu)\Big[\phi_2^\perp(u)+\frac{m_V^2x^2}{16}\phi_4^\perp(u)\Big]$$
$$+(p_\mu x_\nu-p_\nu x_\mu)\frac{(\eta^*\cdot x)m_V^2}{(p\cdot x)^2}\Big[\phi_3^{\parallel}(u)-\frac{1}{2}\phi_2^\perp(u)-\frac{1}{2}\psi_4^\perp(u)\Big]$$
$$+\frac{m_V^2}{2(p\cdot x)}(\eta_\mu^*x_\nu-\eta_\nu^*x_\mu)\Big[\psi_4^\perp(u)-\phi_2^\perp(u)\Big]\Big\},$$

$$\langle V(p,\eta^*)|\bar{q}_1(x)\gamma_\mu\gamma_5 q_2(0)|0\rangle = \frac{f_V^{\parallel}m_V\varepsilon_{\mu\nu\rho\sigma}\eta^{*\nu}p^\rho x^\sigma}{4}\int_0^1 du e^{i\bar{u}p\cdot x}\Big[\tilde{\psi}_3^\perp(u)+\frac{m_V^2x^2}{16}\tilde{\psi}_5^\perp(u)\Big],$$

$$\langle V(p,\eta^*)|\bar{q}_1(x)q_2(0)|0\rangle = -\frac{i}{2}f_V^\perp(\eta^*\cdot x)m_V^2\int_0^1 du e^{i\bar{u}p\cdot x}\tilde{\psi}_3^{\parallel}(u).$$

P. Ball,JHEP 08, 090 (2007), 0707.1201.
Bharucha,JHEP08(2016)098

OPE Valid in the Low-$q^2$ Region: $0\le q^2\le m_c^2-2m_c\chi, \quad \chi\sim 500MeV$

# Form Factors Calculation in the LCSR Framework



- **Three-Particle LCDAs：**

$$\langle V(p,\epsilon^*)|\,\bar{s}(x)\sigma_{\alpha\beta}gG_{\mu\nu}(vx)d(0)\,|0\rangle = f_V^{\perp}m_V^2\frac{\epsilon^* x}{2px}[p_\alpha p_\mu g^{\perp}_{\beta\nu} - p_\beta p_\mu g^{\perp}_{\alpha\nu} - p_\alpha p_\nu g^{\perp}_{\beta\mu} + p_\beta p_\nu g^{\perp}_{\alpha\mu}]\mathcal{T}(v,px)$$
$$+ f_V^{\perp}m_V^2[p_\alpha \epsilon^*_{\perp\mu} g^{\perp}_{\beta\nu} - p_\beta \epsilon^*_{\perp\mu} g^{\perp}_{\alpha\nu} - p_\alpha \epsilon^*_{\perp\nu} g^{\perp}_{\beta\mu} + p_\beta \epsilon^*_{\perp\nu} g^{\perp}_{\alpha\mu}]T_1^{(4)}(v,px)$$
$$+ f_V^{\perp}m_V^2[p_\mu \epsilon^*_{\perp\alpha} g^{\perp}_{\beta\nu} - p_\mu \epsilon^*_{\perp\beta} g^{\perp}_{\alpha\nu} - p_\nu \epsilon^*_{\perp\alpha} g^{\perp}_{\beta\mu} + p_\nu \epsilon^*_{\perp\beta} g^{\perp}_{\alpha\mu}]T_2^{(4)}(v,px)$$
$$+ \frac{f_V^{\perp}m_V^2}{px}[p_\alpha p_\mu \epsilon^*_{\perp\beta}x_\nu - p_\beta p_\mu \epsilon^*_{\perp\alpha}x_\nu - p_\alpha p_\nu \epsilon^*_{\perp\beta}x_\mu + p_\beta p_\nu \epsilon^*_{\perp\alpha}x_\mu]T_3^{(4)}(v,px)$$
$$+ \frac{f_V^{\perp}m_V^2}{px}[p_\alpha p_\mu \epsilon^*_{\perp\nu}x_\beta - p_\beta p_\mu \epsilon^*_{\perp\nu}x_\alpha - p_\alpha p_\nu \epsilon^*_{\perp\mu}x_\beta + p_\beta p_\nu \epsilon^*_{\perp\mu}x_\alpha]T_4^{(4)}(v,px)$$

$$\langle V(p,\epsilon^*)|\,\bar{s}(x)gG_{\mu\nu}(vx)d(0)\,|0\rangle = -if_V^{\perp}m_V^2(\epsilon^*_{\perp\mu}p_\nu - \epsilon^*_{\perp\nu}p_\mu)S(v,px)$$

$$\langle V(p,\epsilon^*)|\,\bar{s}(x)ig\tilde{G}_{\mu\nu}(vx)\gamma_5 d(0)\,|0\rangle = if_V^{\parallel}m_V^2(\epsilon^*_{\perp\mu}p_\nu - \epsilon^*_{\perp\nu}p_\mu)\tilde{S}(v,px)$$

$$\langle V(p,\epsilon^*)|\,\bar{s}(x)g\tilde{G}_{\mu\nu}(vx)\gamma_\alpha\gamma_5 d(0)\,|0\rangle = f_V^{\parallel}m_V\Big[p_\alpha(p_\mu\epsilon^*_{\perp\nu} - p_\nu\epsilon^*_{\perp\mu})\mathcal{A}(v,px)$$
$$-m_V^2\frac{\epsilon^* x}{px}(p_\mu g^{\perp}_{\alpha\nu} - p_\nu g^{\perp}_{\alpha\mu})\Phi(v,px) - m_V^2\frac{\epsilon^* x}{(px)^2}p_\alpha(p_\mu x_\nu - p_\nu x_\mu)\Psi(v,px)\Big]$$

$$\langle V(p,\epsilon^*)|\,\bar{s}(x)gG_{\mu\nu}(vx)\gamma_\alpha d(0)\,|0\rangle = -if_V^{\parallel}m_V\Big[p_\alpha(p_\mu\epsilon^*_{\perp\nu} - p_\nu\epsilon^*_{\perp\mu})\mathcal{V}(v,px)$$
$$-m_V^2\frac{\epsilon^* x}{px}(p_\mu g^{\perp}_{\alpha\nu} - p_\nu g^{\perp}_{\alpha\mu})\tilde{\Phi}(v,px) - m_V^2\frac{\epsilon^* x}{(px)^2}p_\alpha(p_\mu x_\nu - p_\nu x_\mu)\tilde{\Psi}(v,px)\Big]$$

P. Ball Phys. Rev. D 58, 094016 (1998)

OPE Valid in the Low-$q^2$ Region: $0 \le q^2 \le m_c^2 - 2m_c\chi, \quad \chi \sim 500MeV$

# Form Factors Calculation in the LCSR Framework

Definition of Form Factors | Calculate Correlation Fnction | Borel Transformation

| LCDAs | $\phi_2^{\parallel}$ | $\phi_2^{\perp}$ | $\phi_3^{\parallel}$ | $\psi_3^{\parallel}$ | $\phi_3^{\perp}$ | $\psi_3^{\perp}$ | $\phi_4^{\parallel}$ | $\psi_4^{\parallel}$ | $\phi_4^{\perp}$ | $\psi_4^{\perp}$ | $\phi_5^{\perp}$ | $\psi_5^{\perp}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| twists | 2 | 2 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 5 | 5 |
| dirac spinors | $\gamma_\mu$ | $\sigma_{\mu\nu}$ | $\sigma_{\mu\nu}$ | $\mathbf{1}$ | $\gamma_\mu$ | $\gamma_\mu\gamma_5$ | $\gamma_\mu$ | $\gamma_\mu$ | $\sigma_{\mu\nu}$ | $\sigma_{\mu\nu}$ | $\gamma_\mu$ | $\gamma_\mu\gamma_5$ |

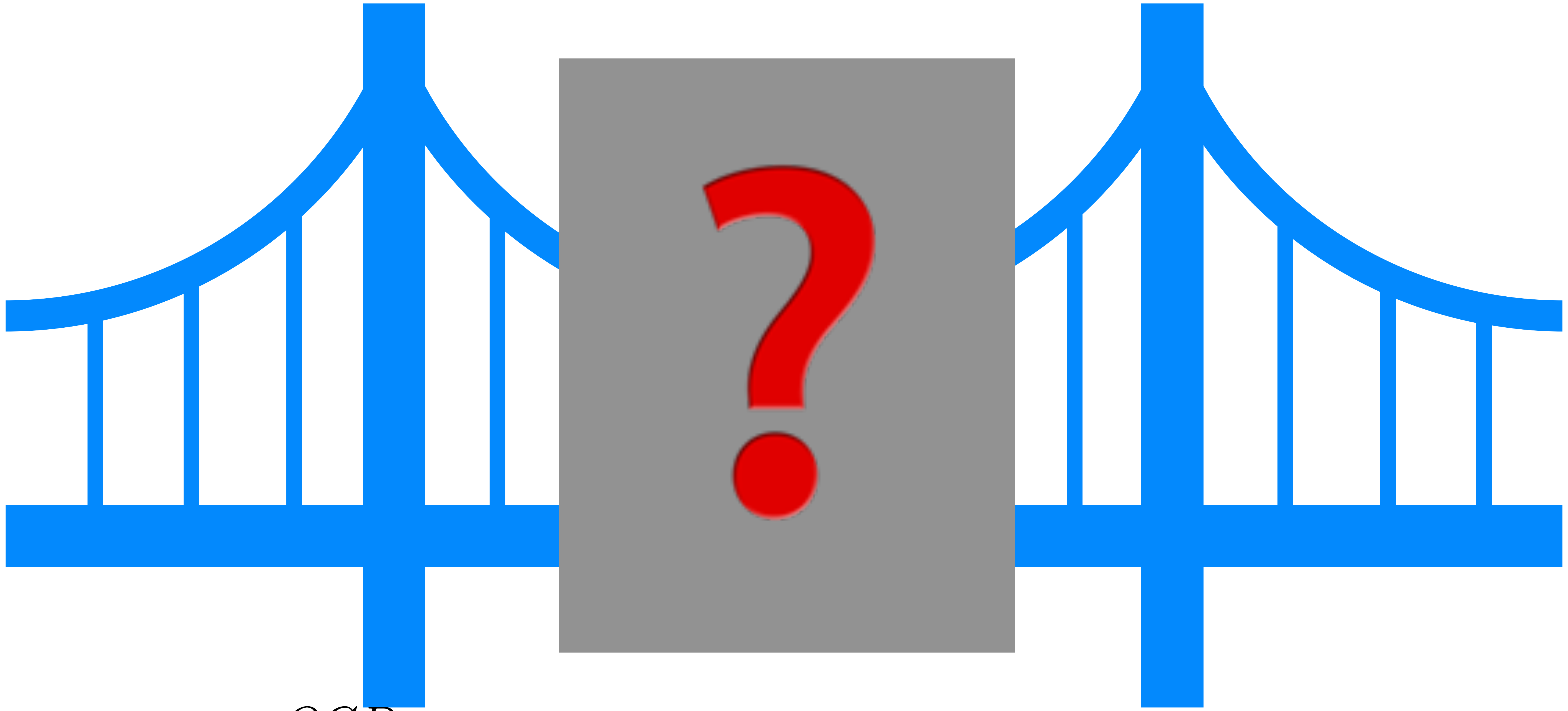
| LCDAs | $\mathcal{V}$ | $\mathcal{A}$ | $\mathcal{T}$ | $\mathcal{S}$ | $\tilde{\mathcal{S}}$ | $T_1^{(4)}$ | $T_2^{(4)}$ | $T_3^{(4)}$ | $T_4^{(4)}$ |
|---|---|---|---|---|---|---|---|---|---|
| twists | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 4 | 4 |
| dirac spinors | $\gamma_\mu$ | $\gamma_\mu\gamma_5$ | $\sigma_{\mu\nu}$ | $\mathbf{1}$ | $\gamma_5$ | $\sigma_{\mu\nu}$ | $\sigma_{\mu\nu}$ | $\sigma_{\mu\nu}$ | $\sigma_{\mu\nu}$ |

$$\phi_2^{\perp}(u) = 6u(1-u)\left(1 + a_1^{\perp} C_1^{3/2}(\xi) + a_2^{\perp} C_2^{3/2}(\xi)\right),$$
$$\phi_2^{\parallel}(u) = 6u(1-u)\left(1 + a_1^{\parallel} C_1^{3/2}(\xi) + a_2^{\parallel} C_2^{3/2}(\xi)\right),$$
$$\vdots$$

# Form Factors Calculation in the LCSR Framework



**Quark Level**



**Hadron Level**

$$\Pi_{\mu}^{QCD}(p,q) = \frac{1}{\pi}\int_{(m_c+m_{q_2})^2}^{s_0} ds \frac{\mathrm{Im}\,\Pi_{\mu}^{QCD}}{s-(p+q)^2} + \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\,\Pi_{\mu}^{QCD}}{s-(p+q)^2}$$

$$\Pi_{\mu}^{hardon}(p,q) = \frac{\langle V(p,\eta^*)|\, j_{1\mu}(x)\,|D_{(s)}(p+q)\rangle \langle D_{(s)}(p+q)|\, j_2(0)\,|0\rangle}{m_{D_{(s)}}^2-(p+q)^2} + \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\rho^h(s,q^2)}{s-(p+q)^2}$$

hadron–quark duality

## Form Factor densities:

**Two particle:**

$$R_{\mathcal{V}_1} = \int_0^1 du[-\frac{f_V^\perp \phi_2^\perp(u)}{2\Delta} + \frac{m_V^2 f_V^\perp(2m_c^2+\Delta)}{8\Delta^3}\phi_4^\perp(u) - \frac{m_c f_V^\parallel m_V \tilde{\psi}_3^\perp(u)}{4\Delta^2} + \frac{3m_c^3 f_V^\parallel m_V^3 \tilde{\psi}_5^\perp(u)}{8\Delta^4}]$$

$$\vdots$$

$$\triangle \equiv m_c^2 - (q+up)^2$$

**Three particle:**

$$R_{\mathcal{V}_1} = -f_V^\perp m_V^2 \int_0^1 du \int_0^u d\alpha_1 \int_0^{\bar{u}} \frac{d\alpha_2}{1-\alpha_1-\alpha_2} \left[\frac{1}{2\Box^2(k)}\left(S(\alpha_i)-(1-2v)\tilde{S}(\alpha_i)\right) + \frac{1}{\Box^2(k)}\left(T_1^{(4)}(\alpha_i)-T_2^{(4)}(\alpha_i)+T_3^{(4)}(\alpha_i)-T_4^{(4)}(\alpha_i)\right) + \frac{8v}{\alpha_1+v\alpha_3}\left(\frac{1}{\Box^2(k)}+\frac{m_c^2-q^2+(\alpha_1+v\alpha_3)^2p^2}{\Box^3(k)}\right)\left(-T_3^{(4)}(\bar{\alpha}_1,\alpha_3)+\frac{1}{2}T_4^{(4)}(\bar{\alpha}_1,\alpha_3)\right)\right]$$

$$\vdots$$

$$\Box \equiv [q+(\alpha_1+v\alpha_3)\,p]^2 - m_c^2$$

$$\Pi_\mu^{QCD}(p,q) = \frac{1}{\pi}\int_{(m_c+m_{q_2})^2}^{s_0} ds \frac{\mathrm{Im}\,\Pi_\mu^{QCD}}{s-(p+q)^2} + \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\,\Pi_\mu^{QCD}}{s-(p+q)^2}$$

$$\Pi_\mu^{hardon}(p,q) = \frac{\langle V(p,\eta^*)|\, j_{1\mu}(x)\,|D_{(s)}(p+q)\rangle \langle D_{(s)}(p+q)|\, j_2(0)\,|0\rangle}{m_{D_{(s)}}^2-(p+q)^2} + \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\rho^h(s,q^2)}{s-(p+q)^2}$$

hadron–quark duality

## Limitations of Quark Hadron Duality:

1. Duality is approximate, especially near $s_0$
2. Excited-state structures are not fully canceled

## Borel Transformation:

- $$X_n = \int_{u_0}^{1} du \frac{F(u)}{\Delta^n}, \quad X_1 = \int_{u_0}^{1} du \frac{F(u)}{\Delta} = \int_{u_0}^{1} \frac{du}{u} \frac{F(u)}{s(u,q^2) - p^2_{D_{(s)}}}$$

▲ $$\Delta = m_c^2 - (q+up)^2, \quad s(u,q^2) = \bar{u} m_V^2 + \frac{1}{u}(m_c^2 - \bar{u} q^2)$$

- $$\hat{B}[X_1] = \int_{u_0}^{1} du \frac{F(u) e^{-\frac{s(u,q^2)}{M^2}}}{u}, \quad u_0 \equiv \frac{-(s_0 - q^2 - m_V^2) + \sqrt{(s_0 - q^2 - m_V^2)^2 + 4m_V^2(m_c^2 - q^2)}}{2m_V^2}$$

- $$\hat{B}[X_n] = \frac{1}{\Gamma[n]} \left( -\frac{d}{dm_c^2} \right)^{(n-1)} \hat{B}[X_1]$$

- $$F_i(q^2) = \kappa_{F_i} \frac{e^{m^2_{D_{(s)}}/M^2}}{m^2_{D_{(s)}} f_{D_{(s)}}} \hat{B}\left[\mathcal{V}_i\right].$$

$$F_i = \{V, A_1, A_3, A_0\}$$

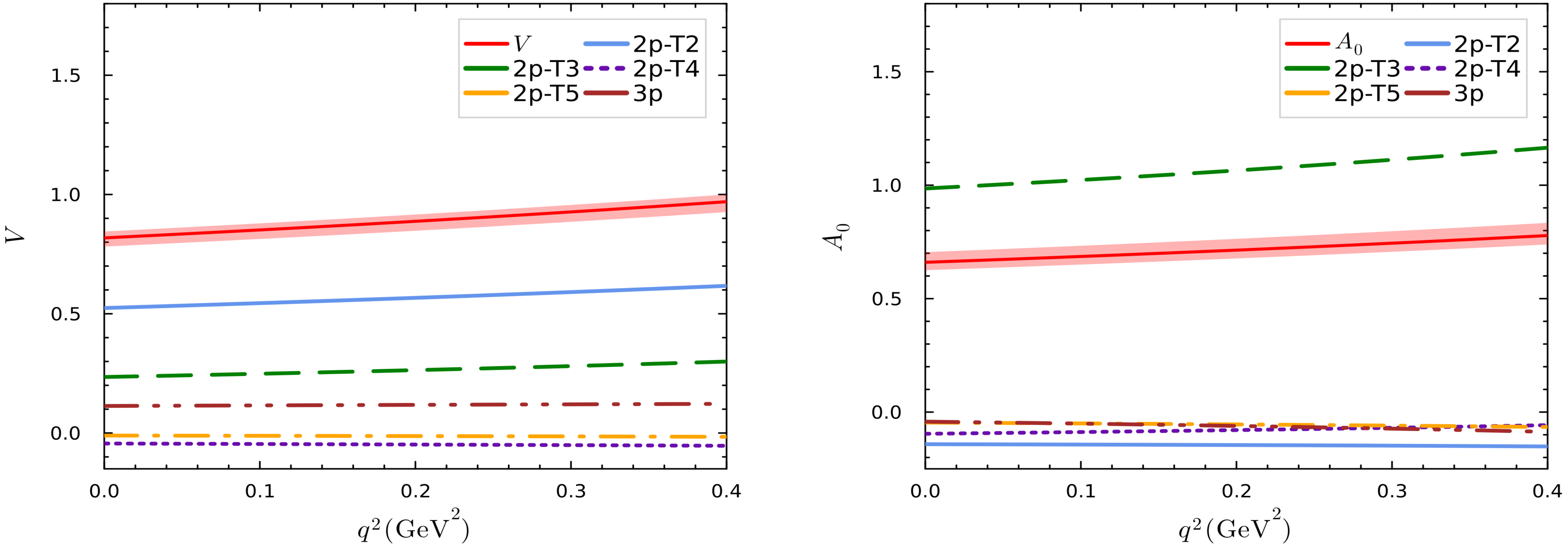
$$\mathcal{V}_i = \{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_P\}$$

$$\kappa_V = -\left(m_{D_{(s)}} + m_V\right), \quad \kappa_{A_1} = -\left(m_{D_{(s)}} - m_V\right), \quad \kappa_{A_3} = \frac{q^2}{2m_V}, \quad \kappa_{A_0} = -\frac{q^2}{2m_V}.$$

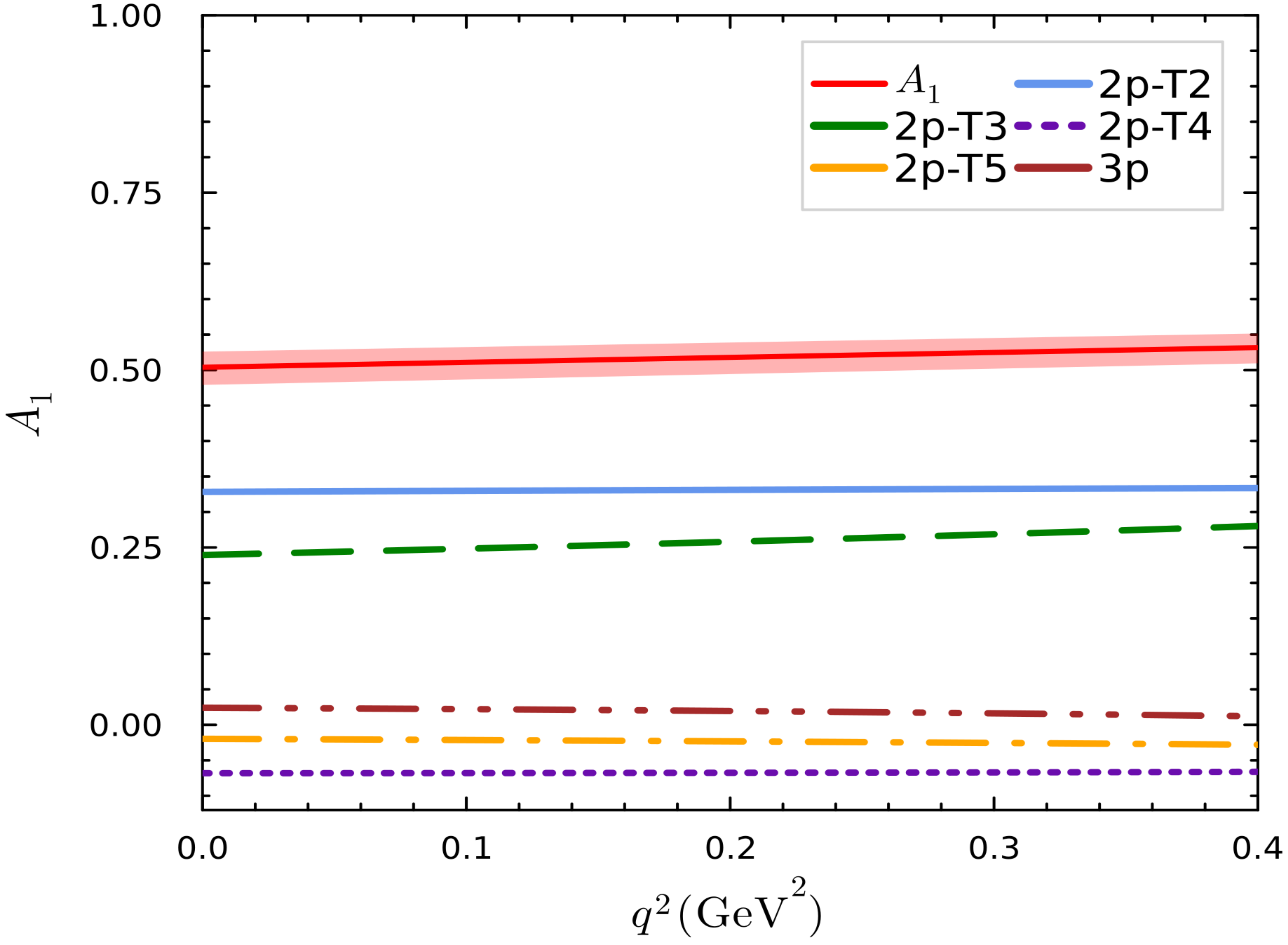
# III. Numerical Results

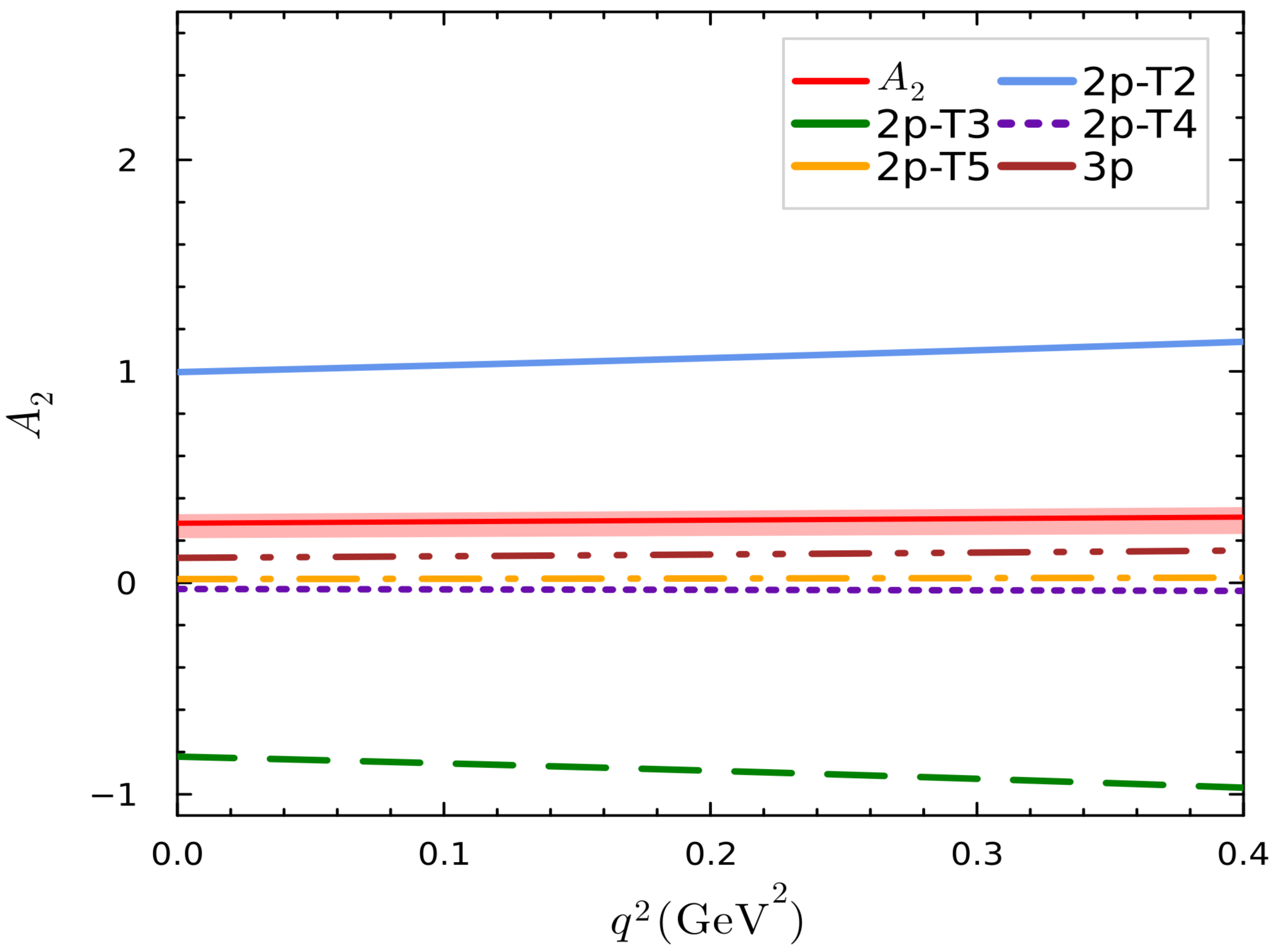
# Form Factors Calculation in the LCSR Framework

$D^0 \to \rho^-$ Form Factor



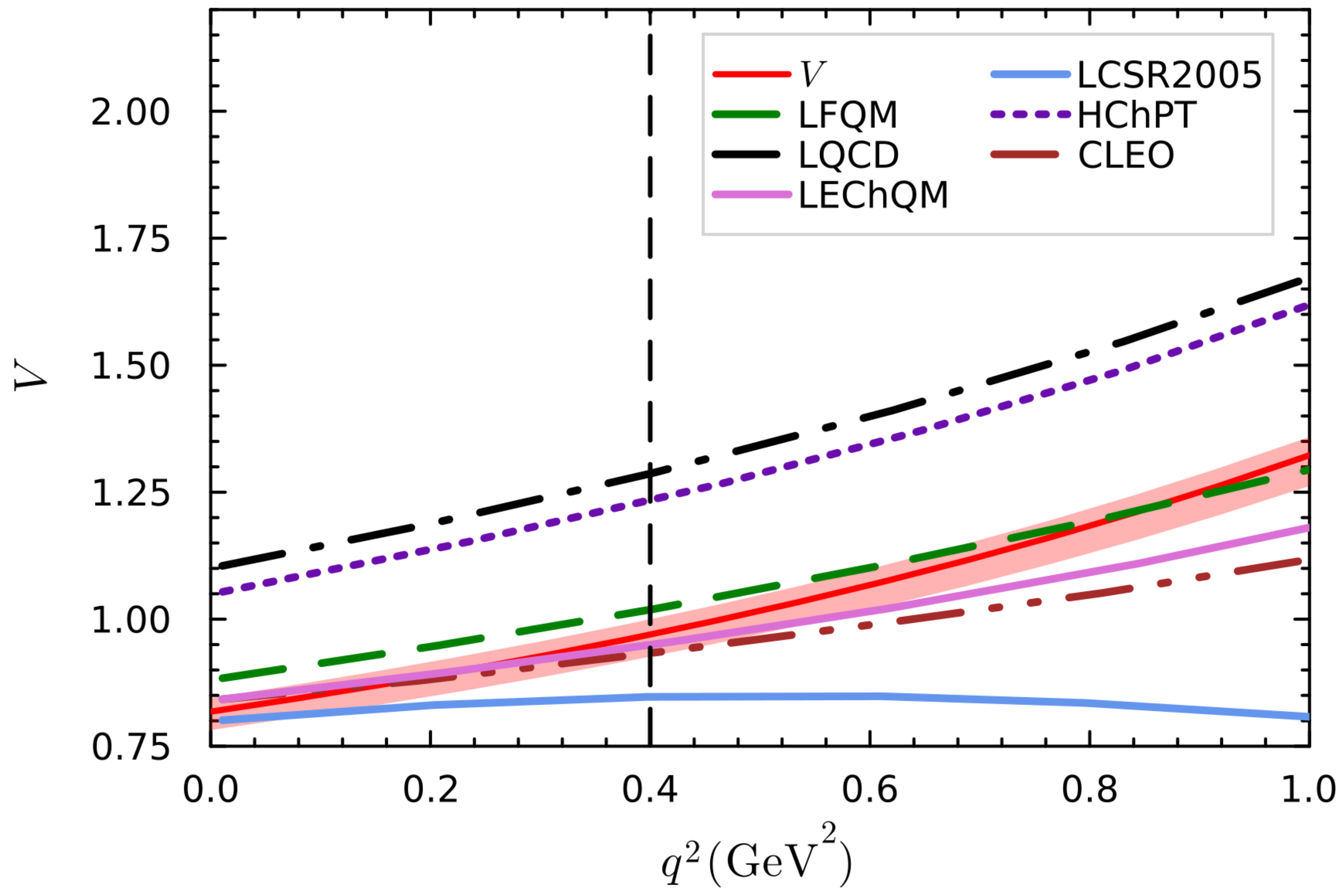
$D^0 \to \rho^-$ Form Factor

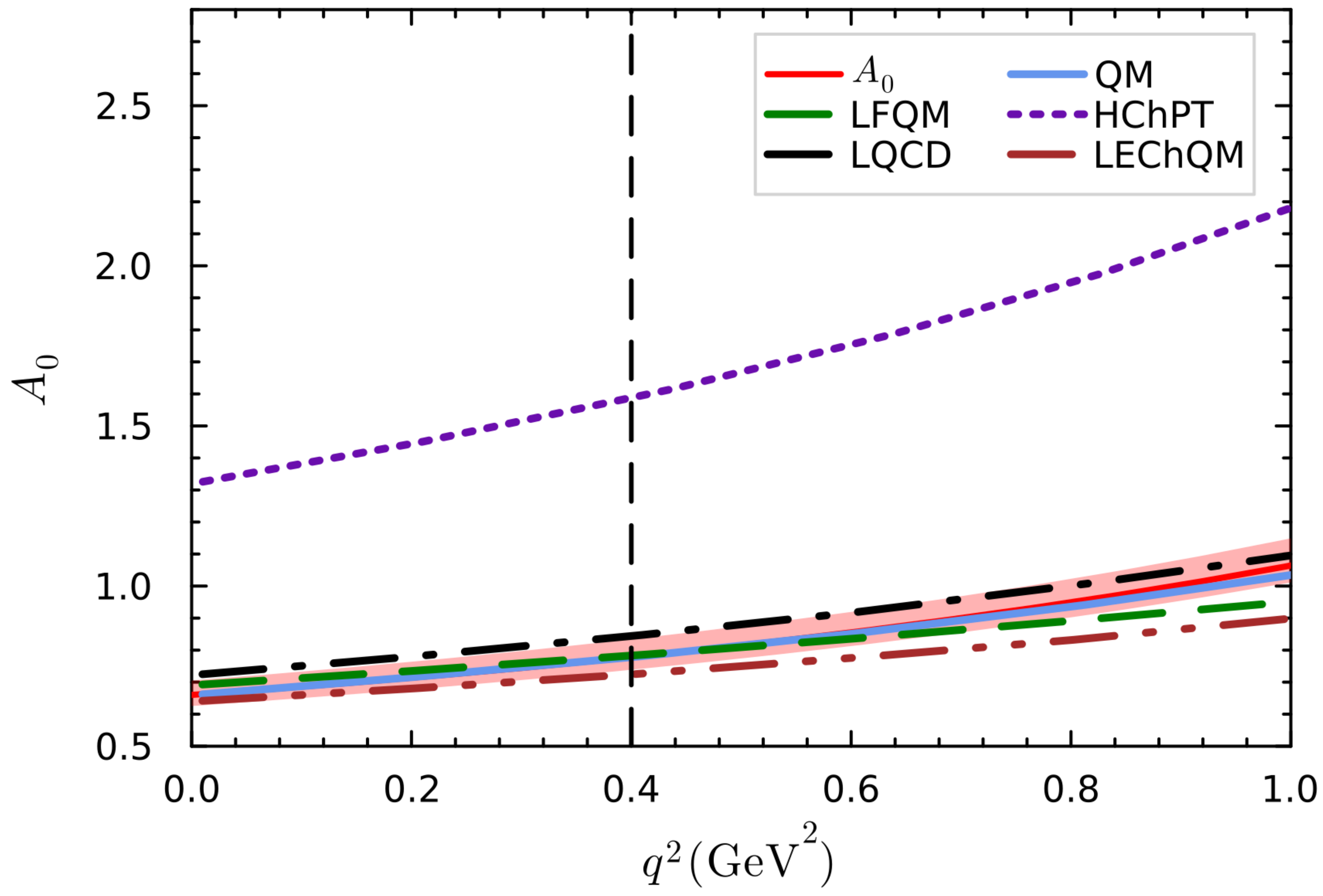


# Form Factors Calculation in the LCSR Framework

## $D^0 \to \rho^-$ Form Factor




**(BCL) parametrization** $F_i(q^2) = P_i(q^2) \sum_k \alpha_k^i \left[z(q^2) - z(0)\right]^k$. $P_i(q^2) = \frac{1}{1 - q^2/m_{R,i}^2}$, $z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$,

$t_\pm = (m_D \pm m_V)^2, t_0 = t_+(1 - \sqrt{1 - t_-/t_+})$

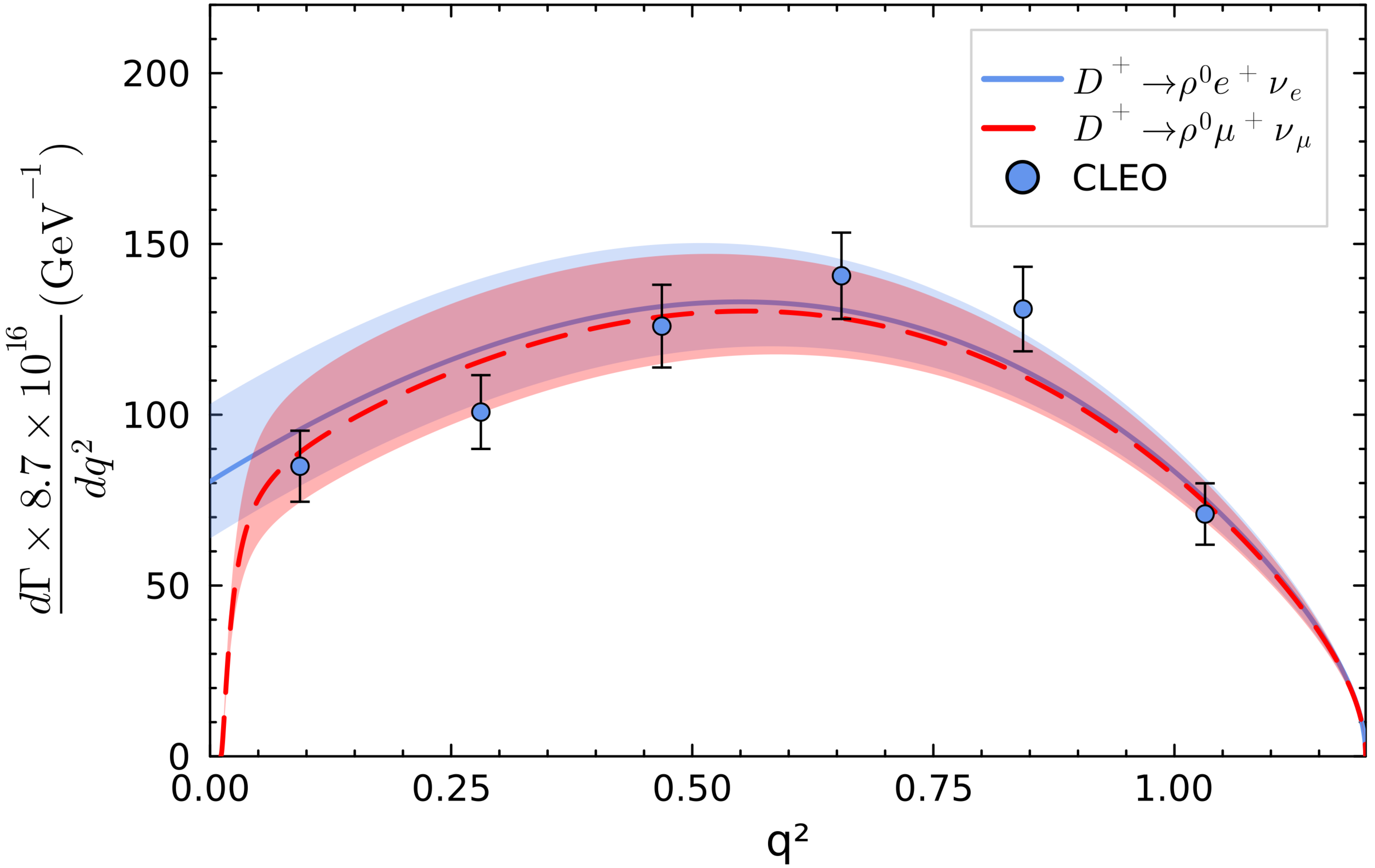
C. Bourrely, Phys. Rev. D 79, 013008 (2009)

# Form Factors Calculation in the LCSR Framework

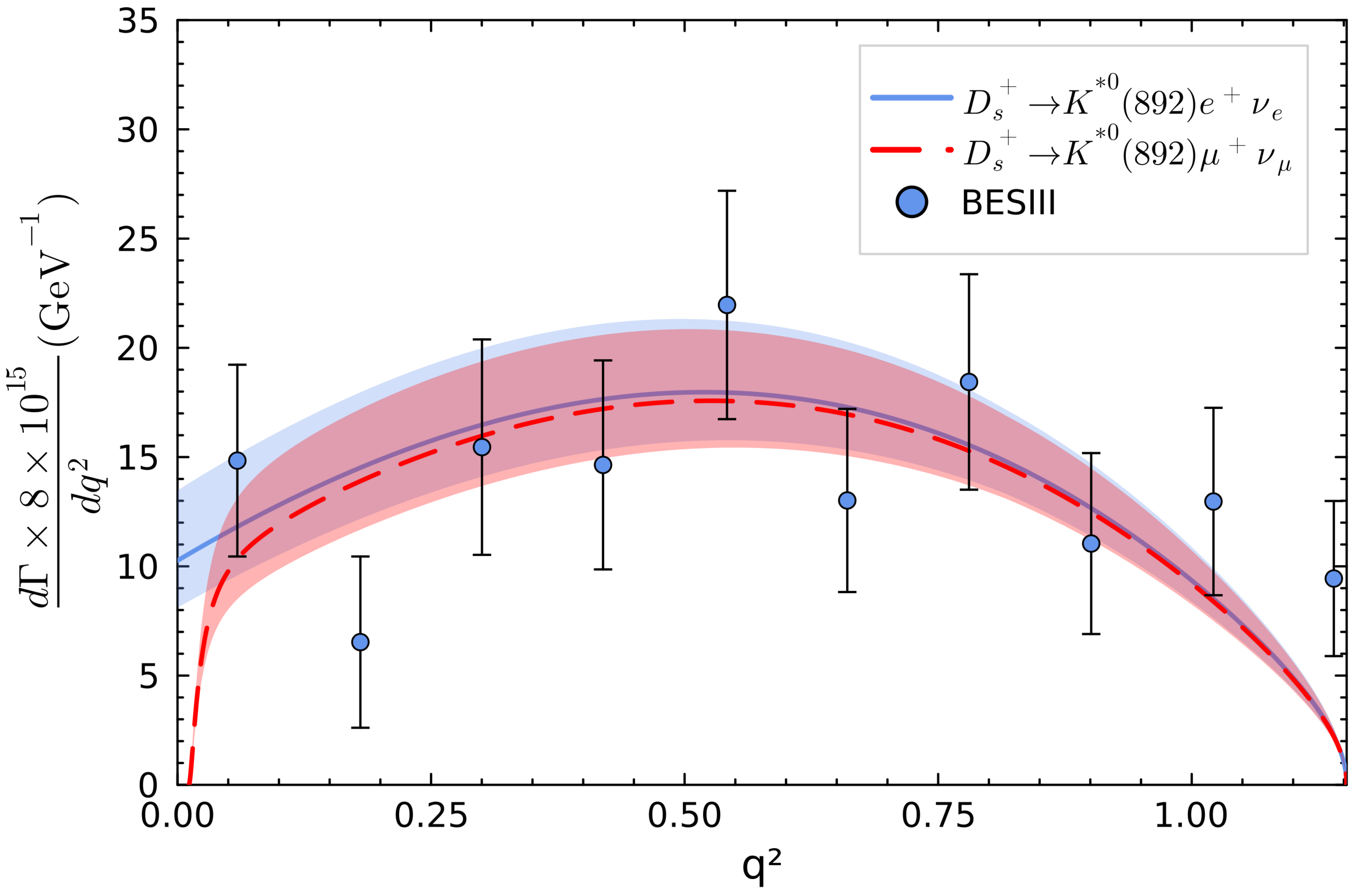
$D^0 \to \rho^-$ Form Factor

# Form Factors Calculation in the LCSR Framework

**Differential Decay Rate:**



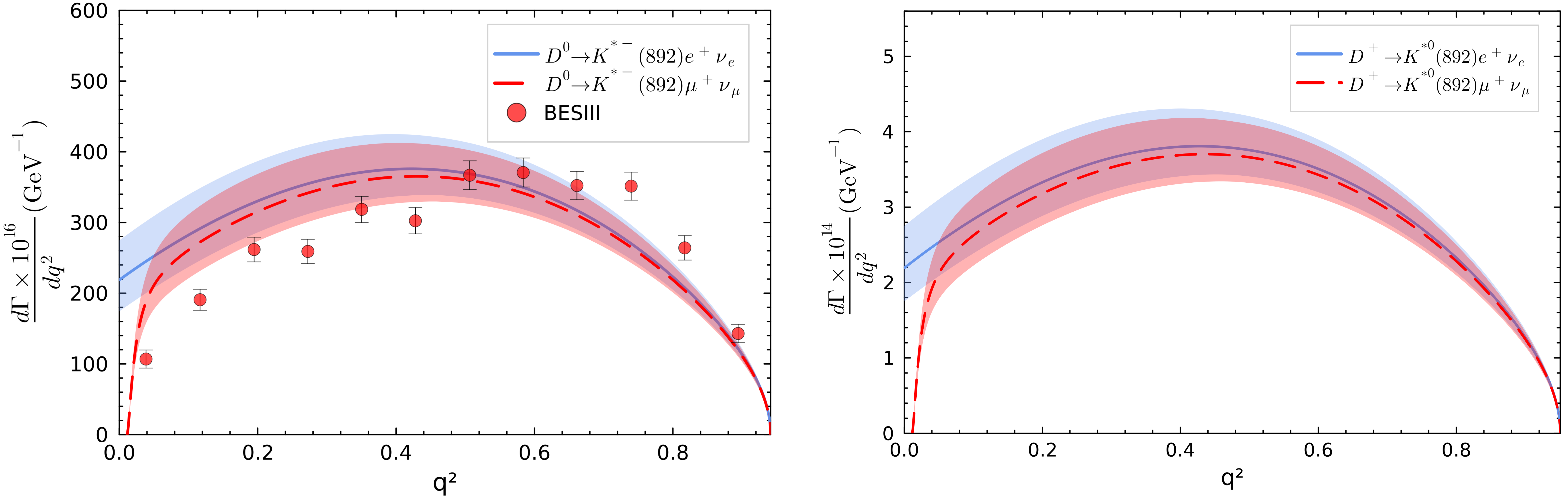

S. Dobbs, PRL 110(2013)13,131802




M. Ablikim, PRL 122 (2019) 6, 061801

# Form Factors Calculation in the LCSR Framework

## Differential Decay Rate:

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**Form Factors ratios:**

| transitions | $D \to \rho$ | | $D_s \to K^*$ | | $D \to K^*$ | |
|---|---|---|---|---|---|---|
| ratios | $r_V$ | $r_2$ | $r_V$ | $r_2$ | $r_V$ | $r_2$ |
| this work | $1.64^{+0.11}_{-0.17}$ | $0.56^{+0.11}_{-0.16}$ | $1.95^{+0.28}_{-0.16}$ | $0.50^{+0.13}_{-0.20}$ | $1.78 \pm 0.13$ | $0.68^{+0.11}_{-0.09}$ |
| LCSR2005 | $1.34^{+0.16}_{-0.13}$ | $0.62 \pm 0.08$ | $1.31^{+0.19}_{-0.16}$ | $0.53^{+0.09}_{-0.16}$ | $1.39^{+0.09}_{-0.10}$ | $0.60^{+0.09}_{-0.08}$ |
| QM | 1.53 | 0.83 | 1.82 | 0.74 | 1.56 | 0.74 |
| LFQM | 1.47 | 0.78 | 1.55 | 0.82 | 1.36 | 0.82 |
| LEChQM | 1.47 | 0.84 | 1.53 | 0.91 | 1.56 | 0.89 |
| HChPT | 1.72 | 0.51 | 1.93 | 0.55 | 1.60 | 0.50 |
| LQCD | 1.69 | 0.85 | — | — | $1.47^{+0.14}_{-0.13}$ | $0.60 \pm 0.07$ |
| CLEO | $1.48 \pm 0.15$ | $0.83 \pm 0.11$ | — | — | — | — |
| BESIII | $1.55 \pm 0.09$ | $0.82 \pm 0.06$ | $1.67 \pm 0.38$ | $0.77 \pm 0.29$ | $1.48 \pm 0.06$ | $0.70 \pm 0.05$ |

## Branching fraction:

| channels | $D^+ \to \rho^0 \ell^+ \nu_\ell$ | $D_s^+ \to K^{*0} \ell^+ \nu_\ell$ | $D^0 \to K^{*-} \ell^+ \nu_\ell$ | $D^+ \to \bar{K}^{*0} \ell^+ \nu_\ell$ |
|---|---|---|---|---|
| this work | $2.30^{+0.32}_{-0.25}$ | $1.55^{+0.30}_{-0.20}$ | $17.6^{+2.4}_{-1.9}$ | $45.2^{+6.2}_{-5.0}$ |
| | $2.20^{+0.30}_{-0.23}$ | $1.48^{+0.29}_{-0.19}$ | $16.6^{+2.2}_{-1.8}$ | $42.7^{+5.7}_{-4.5}$ |
| LCSR2005 | $2.29^{+0.23}_{-0.16}$ | $2.33^{+0.29}_{-0.30}$ | $21.2 \pm 0.9$ | $53.7^{+2.4}_{-2.3}$ |
| | $2.20^{+0.21}_{-0.16}$ | $2.24^{+0.27}_{-0.29}$ | $20.1 \pm 0.9$ | $51.0^{+2.3}_{-2.1}$ |
| CLFQM | 2.32 | 1.90 | — | 73.2 |
| | 2.22 | 1.82 | | 69.3 |
| HChPT | 2.50 | 2.20 | 22.0 | 56.0 |
| PDG | $1.90 \pm 0.10$ | $2.15 \pm 0.28$ | $21.5 \pm 1.60$ | $54.0 \pm 1.00$ |
| | $2.40 \pm 0.40$ | — | $18.9 \pm 2.40$ | $52.7 \pm 1.50$ |

# IIII. Summary and Outlook

# Summary and Outlook

## Summary

- Higher-Twist Corrections from Both Two- and Three-Particle LCDAs
- In good agreement with the experimental data for $D \to \rho$ and $D_s \to K^*$

## Outlook

- Improvement of DAs
- More Experimental and Lattice QCD Inputs to study the FFs in the whole kinematical region