The hadronic stress-energy tensor on the light front

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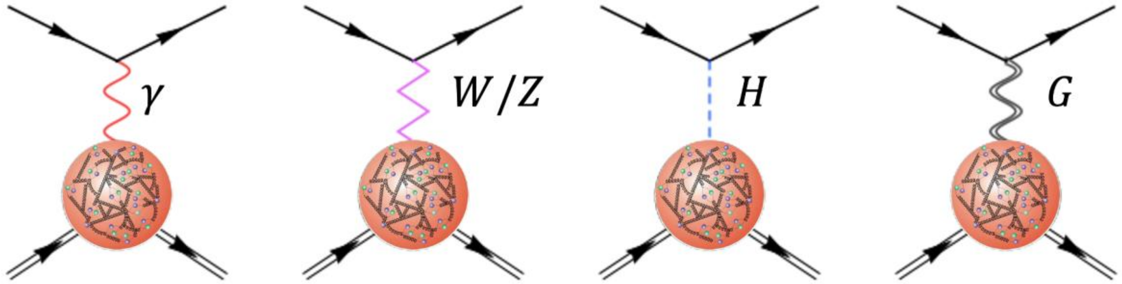
Stress-energy tensor



- Hadron matrix elements and gravitational form factors (GFFs):

[Kobzarev:1962wt, Pagels:1966zza]

$$\langle p', s'|\hat{T}_i^{\mu\nu}(0)|p, s\rangle =$$

$$\frac{1}{2M}\bar{u}_{s'}(p')\left[2P^\mu P^\nu A_i(q^2) + iP^{\{\mu}\sigma^{\nu\}\rho}q_\rho J_i(q^2) + \frac{1}{2}(q^\mu q^\nu - g^{\mu\nu}q^2)D_i(q^2) + 2g^{\mu\nu}\bar{c}_i(q^2)\right]u_s(p)$$

where $P = (p + p')/2$, $q = p' - p$.

[Polyakov:2018zvc, Cotogno:2019xcl, Lorce:2019sbq]

- Conservation laws constrain gravitational form factors except D

$$A(0) = 1, \quad J(0) = \frac{1}{2}, \quad \lim_{Q^2\to 0} Q^2 D(Q^2) = 0$$

em: $\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N'\|J^\mu_{\textbf{em}}\|N\rangle$	$\longrightarrow$	Q	$= 1.602176487(40) \times 10^{-19}\text{C}$
			μ	$= 2.792847356(23)\mu_N$
weak: PCAC	$\langle N'\|J^\mu_{\textbf{weak}}\|N\rangle$	$\longrightarrow$	g_A	$= 1.2694(28)$
			g_p	$= 8.06(55)$
gravity: $\partial_\mu T^{\mu\nu}_{\textbf{grav}} = 0$	$\langle N'\|T^{\mu\nu}_{\textbf{grav}}\|N\rangle$	$\longrightarrow$	m	$= 938.272013(23)\,\text{MeV}/c^2$
			J	$= \frac{1}{2}$
			D	$= ?$

Mechanical properties of hadrons

$D(q^2)$ is related to the pressure and shear forces inside hadrons

[Polyakov:2018zvc]

$$T^{ij}(\boldsymbol{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)s(r) + \delta^{ij}p(r)$$

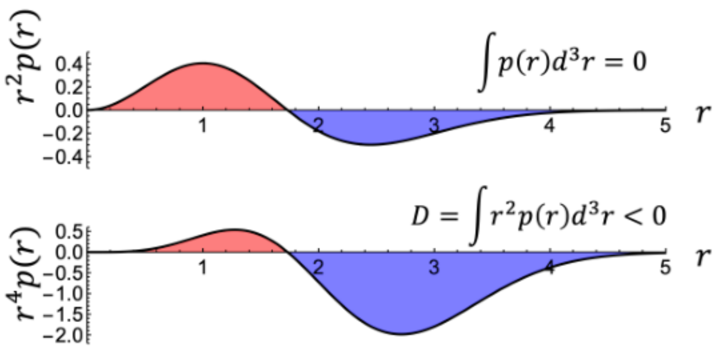
$$\Downarrow$$

$$p(r) = \frac{1}{6M}\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}r^2\frac{\mathrm{d}}{\mathrm{d}r}\tilde{D}(r), \qquad s(r) = -\frac{1}{4M}r\frac{\mathrm{d}}{\mathrm{d}r}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\tilde{D}(r)$$

Hadron stability conditions:

[Perevalova:2016dln]

- Force equilibrium (von Laue condition): $\int d^3 r p(r) = 0$
- Stability conjecture: $D(0) = \int \mathrm{d}^3 r r^2 p(r) < 0$



Light-front wave functions

$$H_{\mathrm{LF}}|\psi\rangle = M^2|\psi\rangle$$

$$|\psi_h(P,j,\lambda)\rangle = \sum_{n=1}^{\infty}\int [\mathrm{d}x_i\mathrm{d}^2k_{i\perp}]_n \psi_{h/n}(\{\vec{k}_{i\perp},x_i,\lambda_i\}_n)\big|\{\vec{p}_{i\perp},p_i^+,\lambda_i\}_n\big\rangle$$

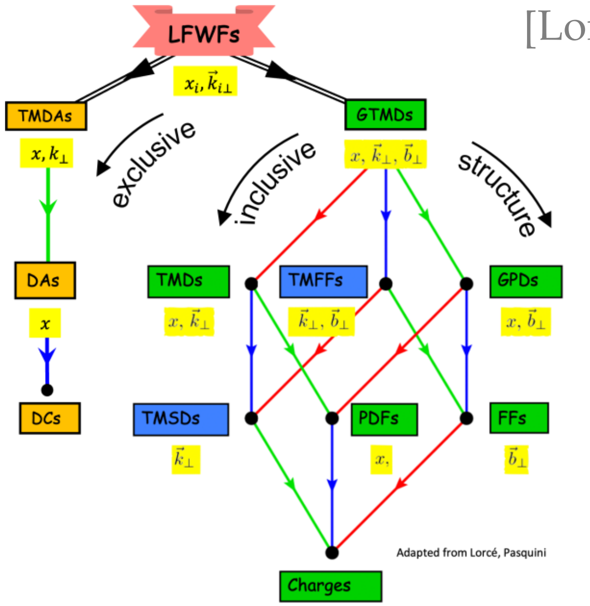
Light-front wave functions (LFWFs) are boost invariant and only depend on relative variables

$$x_i \equiv p_i^+/P^+, \quad \vec{k}_{i\perp} \equiv \vec{p}_{i\perp} - x_i\vec{P}_\perp \Longrightarrow \sum_i x_i = 0, \sum_i \vec{k}_{i\perp} = 0$$

Light-front wave functions provide intrinsic information of the structure of hadrons

- Overlap of LFWFs: Structure functions (e.g. PDFs), form factors
- Integrating out LFWFs: light-cone distributions (e.g. DAs)

[Lorce:2011kd]



Light-front wave function representation

Diagonal representation for charge form factor and GFF $A(q^2)$

[Drell:1969km, West:1970av, Brodsky:1980zm]

$$F_1(q_\perp^2) = \sum_j \int [\mathrm{d}x_i \mathrm{d}^2\boldsymbol{r}_{i\perp}]_n \psi_n^*(\{x_i, \boldsymbol{r}_{i\perp}\})\psi_n(\{x_i, \boldsymbol{r}_{i\perp}\}) e_j e^{i\boldsymbol{r}_{j\perp}\cdot\boldsymbol{q}_\perp}$$

[Brodsky:2000ii]

$$A(q_\perp^2) = \sum_j \int [\mathrm{d}x_i \mathrm{d}^2\boldsymbol{r}_{i\perp}]_n \psi_n^*(\{x_i, \boldsymbol{r}_{i\perp}\})\psi_n(\{x_i, \boldsymbol{r}_{i\perp}\}) x_j e^{i\boldsymbol{r}_{j\perp}\cdot\boldsymbol{q}_\perp}$$

Number densities:

$$\rho_{\mathrm{ch}}(r_\perp) = \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} F_1(q_\perp^2) = \Big\langle \sum_j e_j \delta^{(2)}(\boldsymbol{r}_\perp - \boldsymbol{r}_{j\perp}) \Big\rangle$$

$$\mathcal{A}(r_\perp) = \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} A(q_\perp^2) = \Big\langle \sum_j x_j \delta^{(2)}(\boldsymbol{r}_\perp - \boldsymbol{r}_{j\perp}) \Big\rangle$$

where the quantum average is defined as

$$\langle \hat{O} \rangle = \int [\mathrm{d}x_i \mathrm{d}^2\boldsymbol{r}_{i\perp}]_n \psi_n^*(\{x_i, \boldsymbol{r}_{i\perp}\}) \hat{O} \psi_n(\{x_i, \boldsymbol{r}_{i\perp}\})$$

Light-front wave function representation for $D(q^2)$

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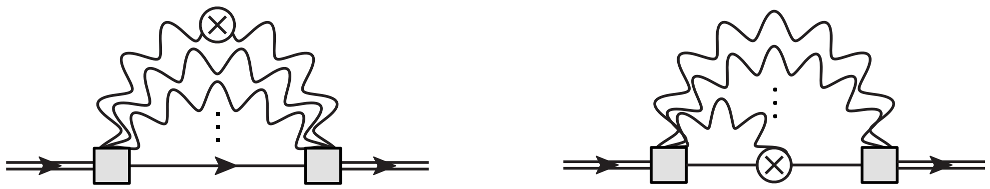
| Reviews

Forces inside hadrons: Pressure, surface tension, mechanical radius, and all that

Maxim V. Polyakov and Peter Schweitzer

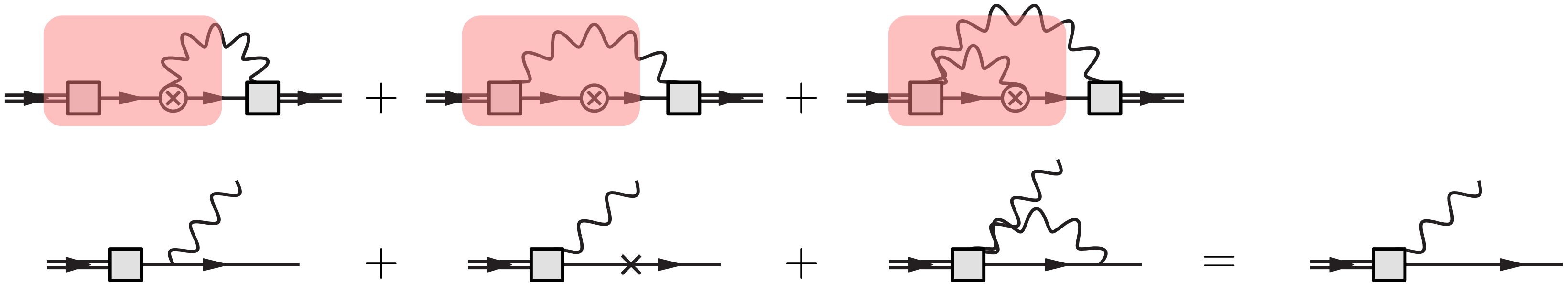
https://doi.org/10.1142/S0217751X18300259 | **Cited by:** 241 **(Source: Crossref)**

$\hat{T}_{++}$ of the EMT. Being related to the stress tensor $\hat{T}_{ij}$ the form factor $D(t)$ naturally "mixes" good and bad light-front components and is described in terms of transitions between different Fock state components in overlap representation. As a quantity intrinsically nondiagonal in a Fock space, it is difficult to study the D-term in approaches based on light-front wave functions. This is due to the rela-



diagonal non-diagonal

- We start from the scalar Yukawa theory to seek inspiration
- $D(q^2)$ contains the overlap between different Fock state components. However, the non-diagonal diagrams add up to a diagonal diagram



Covariant decomposition on the light-front

$$\langle p'|\,\hat{T}_i^{\alpha\beta}(0)\,|p\rangle = 2P^\alpha P^\beta A_i(q^2) + \frac{1}{2}(q^\alpha q^\beta - q^2 g^{\alpha\beta})D_i(q^2) + 2M^2 g^{\alpha\beta}\bar{c}_i(q^2) + \frac{M^4 \omega^\alpha \omega^\beta}{(\omega\cdot P)^2} S_{1i}(q^2) + (V^\alpha V^\beta + q^\alpha q^\beta) S_{2i}(q^2)$$

[Cao:2024rul]

where $P = (p + p')/2$, $q = p' - p$, $V^\alpha = \epsilon^{\alpha\beta\rho\sigma} P_\beta q_\rho \omega_\sigma/(\omega\cdot P)$. $\omega^\mu = (\omega^+, \omega^-, \boldsymbol{\omega}_\perp) = (0,2,0)$ is a null vector indicating the light-front direction.

- $S_{1,2}(q^2)$ are two spurious gravitational form factors which appear due to the violation of the full Lorentz symmetry
- Identify $T^{++}, T^{+i}, T^{12}, T^{+-}$ as good currents to extract GFFs

$$t_i^{++} = 2(P^+)^2 A_i(q_\perp^2), \quad t_i^{11} + t_i^{22} = -\frac{1}{2} q_\perp^2 D_i(q_\perp^2) - 4M^2 \bar{c}_i(q_\perp^2) + 2q_\perp^2 S_{2i}(q_\perp^2),$$

$$t_i^{12} = \frac{1}{2} q_\perp^1 q_\perp^2 D_i(q_\perp^2), \quad t_i^{--} = 2\left(\frac{M^2 + \frac{1}{4}q_\perp^2}{P^+}\right)^2 A_i(q_\perp^2) + \frac{4M^4}{(P^+)^2} S_{1i}(q_\perp^2)$$

$$t_i^{+-} = 2(M^2 + \frac{1}{4}q_\perp^2)A_i(q_\perp^2) + q_\perp^2 D_i(q_\perp^2) + 4M^2 \bar{c}_i(q_\perp^2)$$

$$t^{\alpha\beta} = \langle p'|\,\hat{T}^{\alpha\beta}(0)\,|p\rangle$$

Light-front wave function representation

[Cao:2023ohj, Cao:2024fto]

$$t^{12} = \frac{1}{2}\Big\langle \sum_j e^{-i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{j\perp}} \frac{i\overleftrightarrow{\nabla}_j^1\overleftrightarrow{\nabla}_j^2 - q^1q^2}{x_j}\Big\rangle$$

$$\int d^3x T^{+\mu}(x) = P^\mu$$

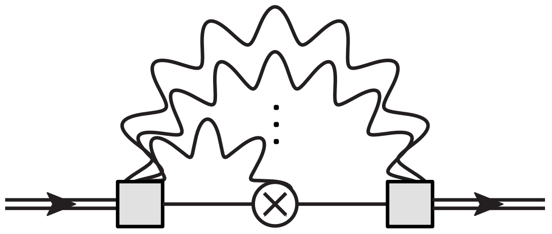
$$t^{+-} = 2\Big\langle \underbrace{\sum_j e^{i\boldsymbol{q}_\perp\cdot\boldsymbol{r}_{j\perp}} \frac{-\frac{1}{4}\overleftrightarrow{\nabla}_{j\perp}^2 + m_j^2 - \frac{1}{4}q_\perp^2}{x_j}}_{\text{kinetic part}} + \underbrace{V e^{i\boldsymbol{r}_{N\perp}\cdot\boldsymbol{q}_\perp}}_{\text{potential part}}\Big\rangle$$

$$P^- = \frac{P_\perp^2 + M^2}{P^+}$$

where $V = M^2 - \sum_j \frac{-\nabla_{j\perp}^2 + m_j^2}{x_j}$ in the scalar Yukawa model. The quantum average is defined as

$$\langle\hat{O}\rangle = \int [\mathrm{d}x_i\mathrm{d}^2\boldsymbol{r}_{i\perp}]_n \psi_n^*(\{x_i,\boldsymbol{r}_{i\perp}\})\hat{O}\psi_n(\{x_i,\boldsymbol{r}_{i\perp}\})$$

- Modify V in phenomenological models
- $e^{i\boldsymbol{r}_{N\perp}\cdot\boldsymbol{q}_\perp} \xrightarrow{\text{F.T.}} \delta^{(2)}(\boldsymbol{r}_\perp - \boldsymbol{r}_{N\perp})$ indicates the location of interaction



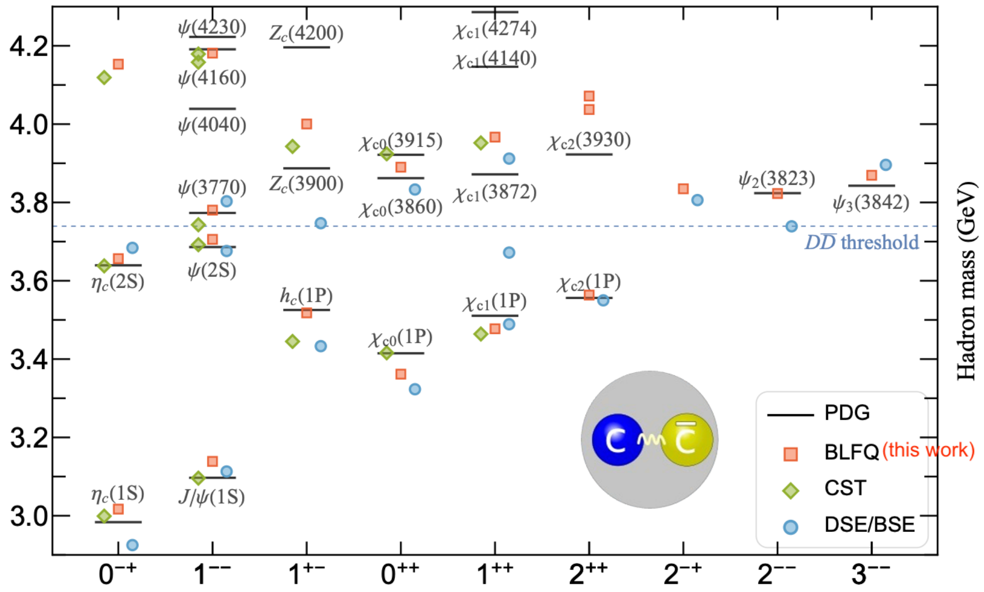
Charmonium: hydrogen atom of QCD

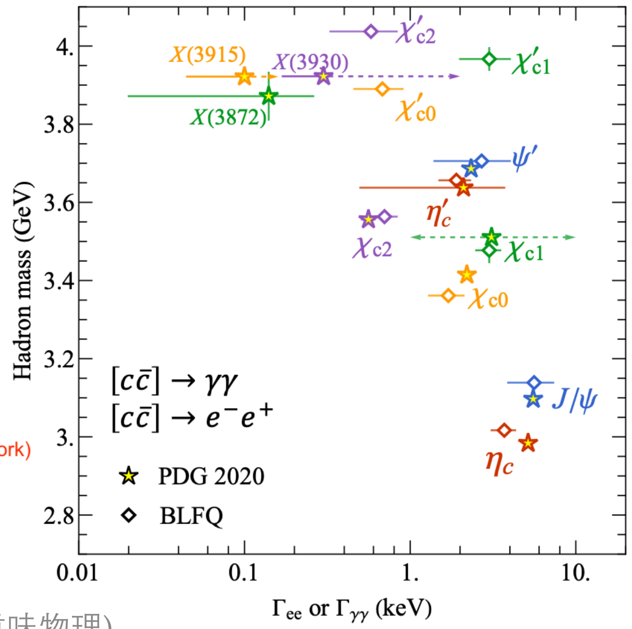
Effective Hamiltonian in the $q\bar{q}$ Fock sector [Li:2015zda, Li:2017mlw, Li:2021ejv]

kinetic energy, confinement, one gluon exchange

$$H_{\text{eff}} = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\partial_x(x(1-x)\partial_x) - \frac{C_F 4\pi\alpha_s(Q^2)}{Q^2}\bar{u}_{s'}(k')\gamma_\mu u_s(k)\bar{v}(\bar{k})\gamma^\mu v_{\bar{s}'}(\bar{k}')$$

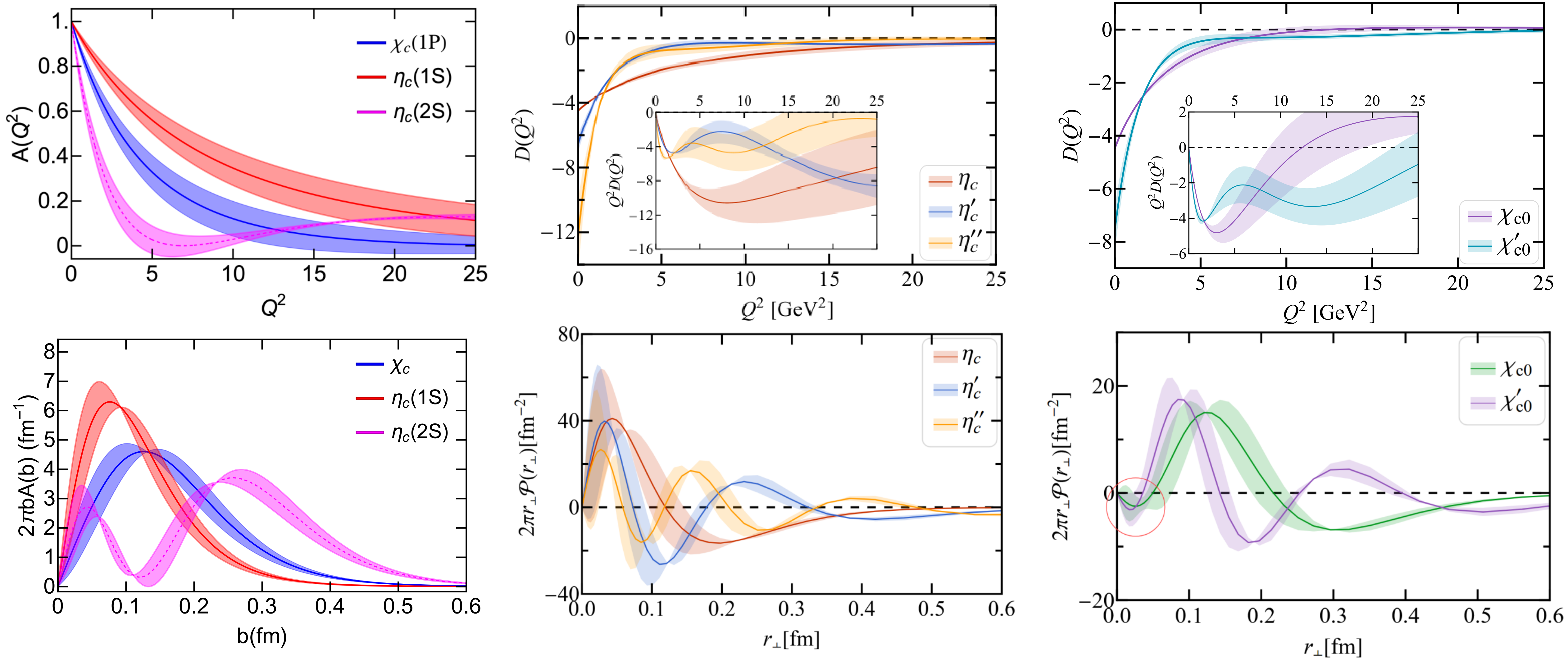
Two parameters (m_q, κ) are fixed by fitting the charmonium mass spectrum





Charmonium gravitational form factors

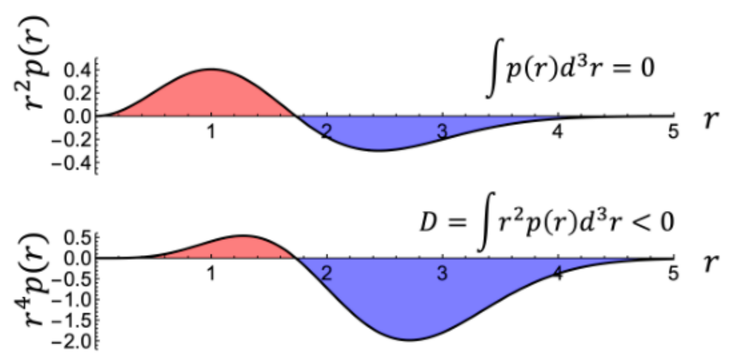
[Xu:2024hfx, Hu:2024edc]

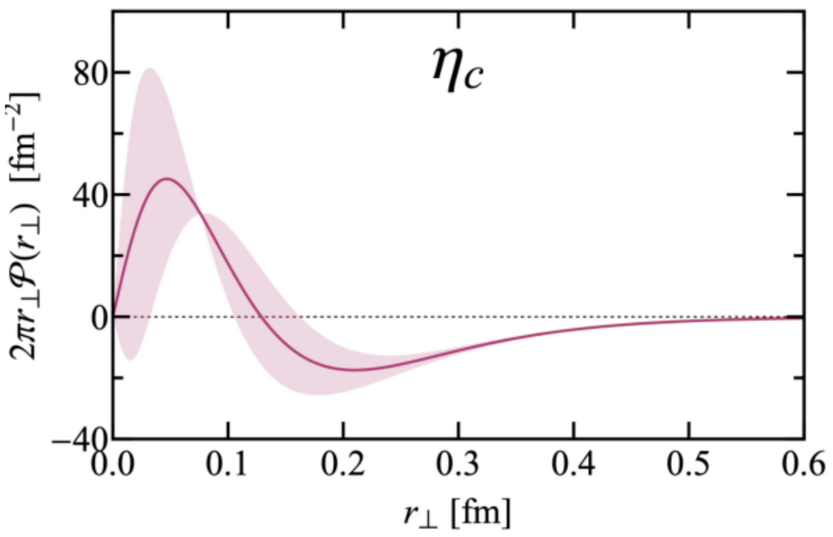


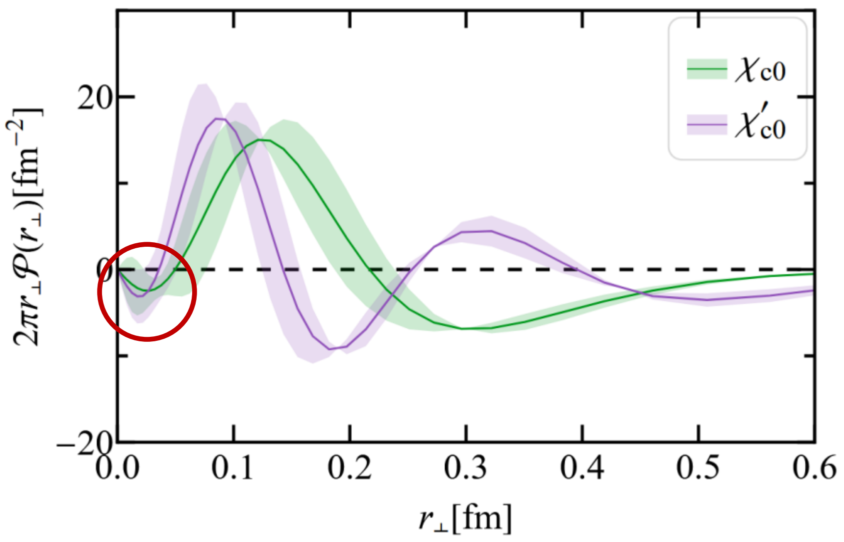
Mechanical stability

$$D = \int \mathrm{d}^3 r r^2 p(r) < 0$$

- A mechanically stable system should have a repulsive core and an attractive edge
- η_c has a repulsive core but χ_{c0} has an attractive core





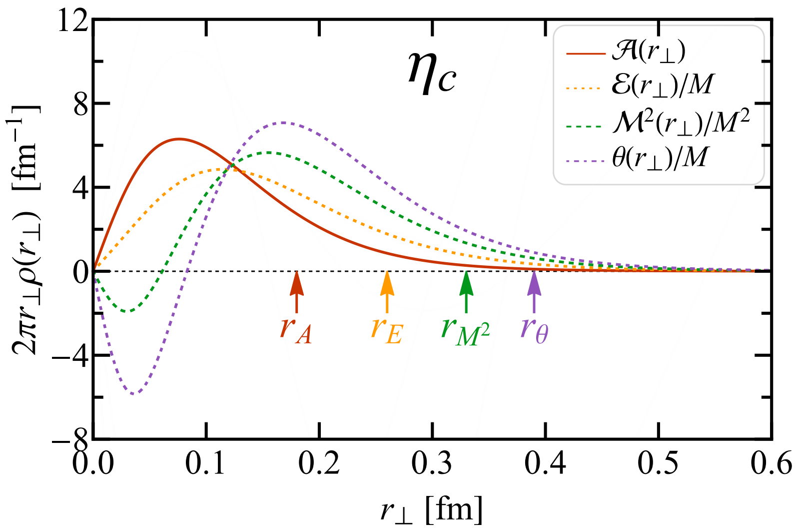


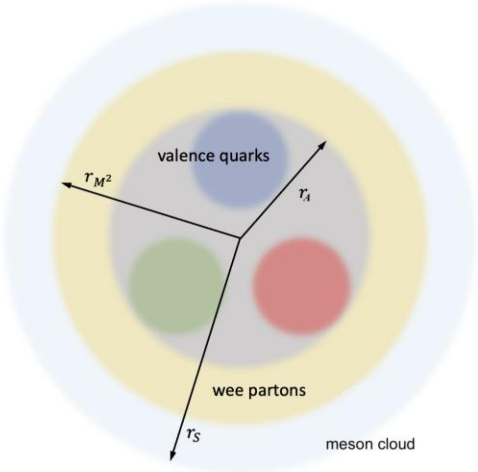
Physical densities

- Momentum density $\mathcal{A}(r_\perp)$, energy density $\mathcal{E}(r_\perp)$, invariant mass squared density $\mathcal{M}^2(r_\perp)$ and the trace scalar density $\theta(r_\perp) = \mathcal{T}^\alpha_\alpha(r_\perp) = \mathcal{E}(r_\perp) - 3\mathcal{P}(r_\perp)$
- The negative D suggests a chain of inequalities about different radii

$$r_A < r_E < r_{M^2} < r_\theta$$

$$r_A^2 = -6A'(0), \quad r_E^2 = r_A^2 - \frac{3}{2}\lambda_C^2(1+D), \quad r_{M^2}^2 = r_A^2 - \frac{3}{2}\lambda_C^2(1+2D), \quad r_\theta^2 = r_A^2 - \frac{3}{2}\lambda_C^2(1+3D)$$





$$\lambda_C = \frac{1}{M}$$

$$p_i^- = \frac{p_{i\perp}^2 + m_i^2}{x_i p^+}$$

Summary

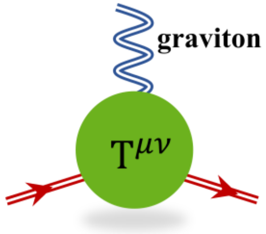
- We obtain a non-perturbative light-front wave function representation to evaluate the gravitational form factors
- We apply the light-front wave function representation to charmonium solved from an effective Hamiltonian
- The extracted densities provide novel insights into the hadron structures

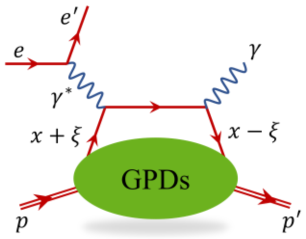
Based on:
Cao, Li, Vary, PRD 108, 056026 (2023)
Xu, Cao, Hu, Li, Zhao, Vary, PRD 109, 114024 (2024)
Cao, Xu, Li, Chen, Zhao, Karmanov, Vary, JHEP (2024) 95
Cao, Li, Vary, PRD 110, 076025 (2024)
Hu, Cao, Xu, Li, Zhao, Vary, arXiv:2408.09689

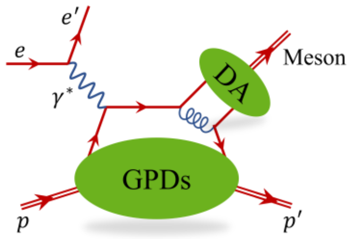
Thank you!

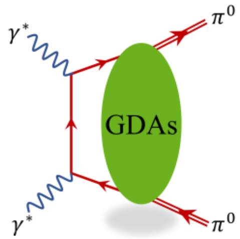
Backup Slides

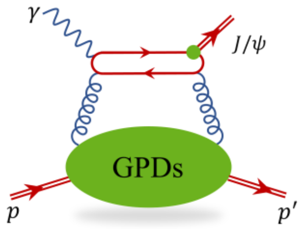
How to access gravitational form factors











- Deeply virtual Compton scattering
- Deeply virtual meson production
- Two-photon pair production
- J/ψ threshold photoproduction

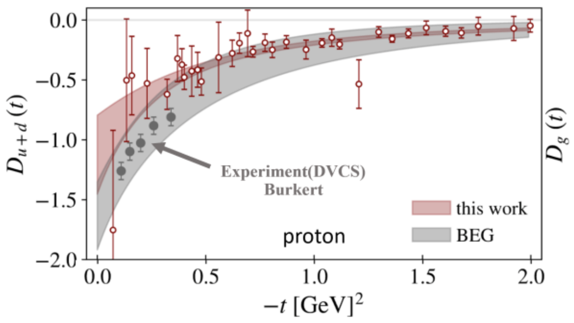
[Kumano:2017lhr, Duran:2022xag, Burkert:2023wzr]

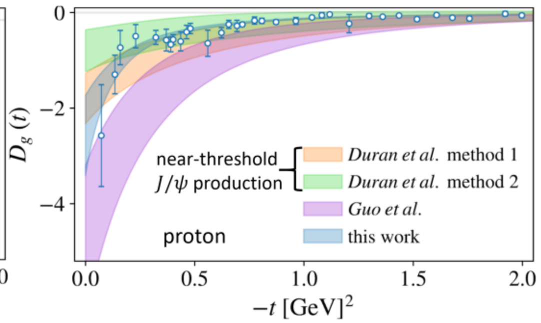
Ji's sum rule:

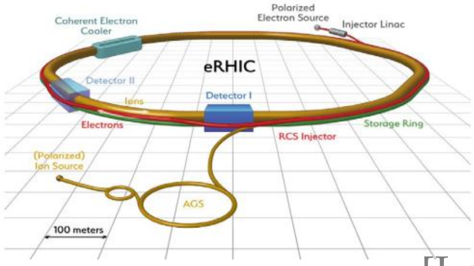
$$\int_{-1}^{1} dx x H^{q,g}(x,\xi,t) = A^{q,g}(t) + \xi^2 D^{q,g}(t), \quad \int_{-1}^{1} dx x E^{q,g}(x,\xi,t) = B^{q,g}(t) - \xi^2 D^{q,g}(t)$$

[Ji:1996nm]

Here, $H^{q,g}$ and $E^{q,g}$ are generalized parton distributions



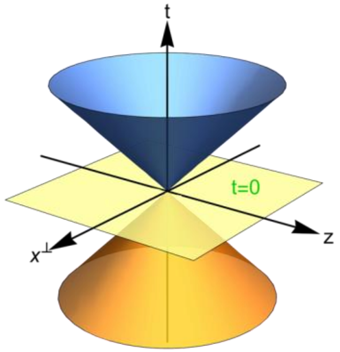
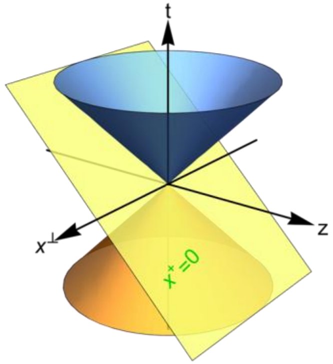




[Lattice'23: Hackett:2023nkr]

Light-front quantization

[Dirac:1949cp]

	equal time quantization	light-front quantization
		
■ Time:	$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
■ Hamiltonian:	$H \equiv P^0$	$H \equiv P^- = P^0 - P^3$
■ Dispersion relation:	$p^0 = \sqrt{\vec{p}^2 + m^2}$	$p^- = (\vec{p}_\perp^2 + m^2)/p^+$

light-front coordinates

$$x^\pm = x^0 \pm x^3$$

$$\vec{x}^\perp = (x^1, x^2)$$

Light-front quantization is a Hamiltonian method of the quantum field theory

[Brodsky:1997de]

$$(P^+ \hat{P}^- - \vec{P}_\perp^2) \left|\psi_H\right\rangle = M_h^2 \left|\psi_h\right\rangle$$

Scalar Yukawa model

$$\mathcal{L} = \partial_\mu N^\dagger \partial^\mu N - m^2 N^\dagger N + \frac{1}{2}\partial_\mu \pi \partial^\mu \pi - \frac{1}{2}\mu^2 \pi^2 + g_0 N^\dagger N \pi + \delta m^2 N^\dagger N$$

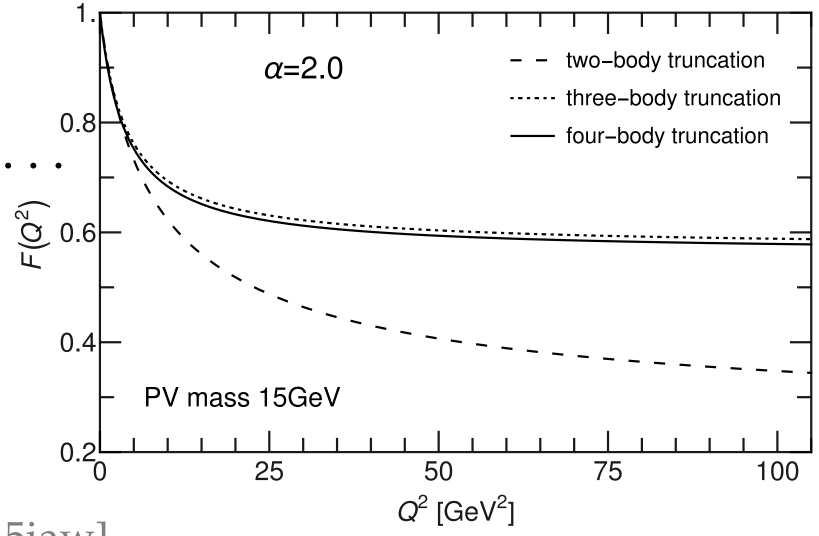
$$\Downarrow$$

$$\hat{T}^{\mu\nu} = \partial^{\{\mu} N^\dagger \partial^{\nu\}} N - g^{\mu\nu}\left[\partial_\sigma N^\dagger \partial^\sigma N - (m^2 - \delta m^2) N^\dagger N\right] - g^{\mu\nu} g_0 N^\dagger N \pi + \partial^\mu \pi \partial^\nu \pi - \frac{1}{2} g^{\mu\nu}\left(\partial^\rho \pi \partial_\rho \pi - \mu_0^2 \pi^2\right)$$

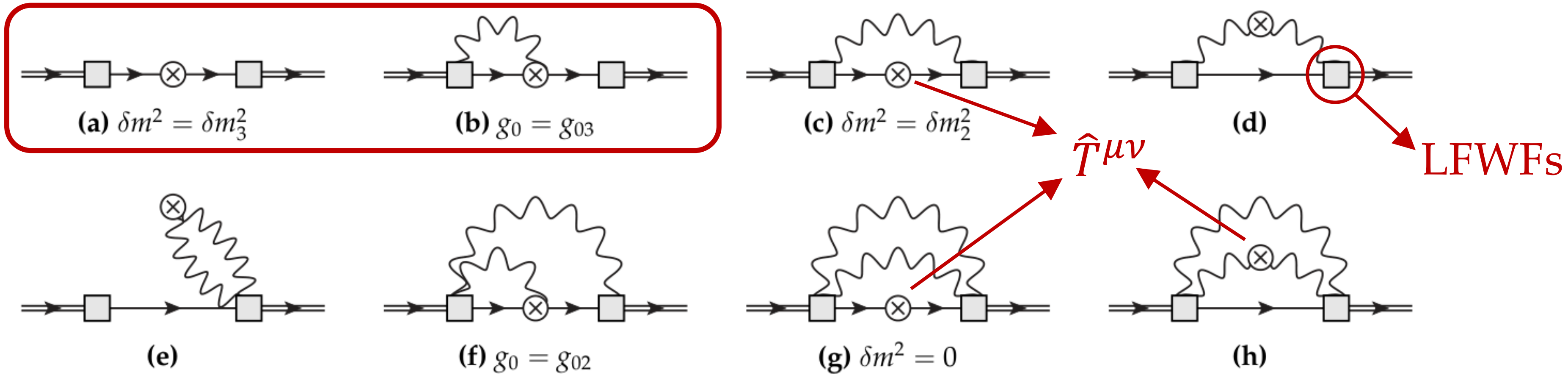
where $m = 0.94\text{GeV}$, $\mu = 0.14\text{GeV}$. g_0 and δm^2 are bare parameters.

- N: mock nucleon, π: mock pion [Gross:2001ha]
- Quenched approximation: to avoid vacuum instability
- Fock sector expansion: $|p\rangle = |N\rangle + |N\pi\rangle + |N\pi\pi\rangle + |N\pi\pi\pi\rangle \cdots$
- Solved up to $|N\pi\pi\pi\rangle$ sector at non-perturbative couplings
- Fock sector dependent renormalization [Karmanov:2008br]
- Fock sector expansion converged up to $|N\pi\pi\rangle$ sector [Li:2015iaw]

$$\alpha = g^2/(16\pi m^2), \quad Q^2 = -q^2$$



Stress-energy tensor renormalization



- Light-front wave functions (LFWFs) & sector dependent counterterms from [Li, Karmanov & Vary:2015iaw,2016yzu]
- Light-front graphical rules extended to non-perturbative regime using LFWFs [Carbonell:1998rj]
- All divergences cancel out with sector dependent counterterms, e.g. $(a) + (b)$:

$$t_a^{\alpha\beta} = Z[2P^\alpha P^\beta + (\frac{1}{2}q^2 - \delta m_3^2)g^{\alpha\beta} - \frac{1}{2}q^\alpha q^\beta]$$

$$t_b^{\alpha\beta} = -\sqrt{Z}g^{\alpha\beta}\int \frac{dx}{2x(1-x)}\int \frac{d^2k_\perp}{(2\pi)^3} g_{03}\psi_2(x,k_\perp) = g^{\alpha\beta}Z\delta m_3^2$$

$$t^{\alpha\beta} = \langle p'|\,\hat{T}^{\alpha\beta}(0)\,|p\rangle$$

Energy and momentum densities

2D transverse densities on the light-front: [Xu:2024hfx, Freese:2021czn]

$$\mathcal{T}^{\alpha\beta}(\vec{r}_\perp; P) = \frac{1}{2P^+}\int \frac{d^2q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} \langle P + \frac{1}{2}q|\,\hat{T}^{\alpha\beta}(0)\,|P - \frac{1}{2}q\rangle$$

Momentum ($\mu = +, 1, 2$) and energy ($\mu = -$) densities:

$$\mathcal{P}^\mu(r_\perp) \equiv \mathcal{T}^{+\mu}(r_\perp; P) = P^\mu \mathcal{A}(r_\perp),$$

$$\mathcal{P}^-(r_\perp) \equiv \mathcal{T}^{+-}(r_\perp; P) = \frac{P_\perp^2 \mathcal{A}(r_\perp) + \mathcal{M}^2(r_\perp)}{P^+}$$

$$\int d^3x T^{+\mu}(x) = P^\mu$$

Where (for spin-0 particles):

$$\mathcal{A}(r_\perp) = \int \frac{d^2q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} A(q_\perp^2),$$

$$\mathcal{M}^2(r_\perp) = \int \frac{d^2q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} [(M^2 + \frac{1}{4}q_\perp^2)A(q_\perp^2) + \frac{1}{2}q_\perp^2 D(q_\perp^2)]$$

$$P^- = \frac{P_\perp^2 + M^2}{P^+}$$

- $\mathcal{A}(r_\perp)$ can be interpreted as the momentum density
- $\mathcal{M}^2(r_\perp)$ can be interpreted as the invariant mass squared density

Hadron as a relativistic medium

[Li:2024vgv]

- The quantum expectation value of the stress-energy tensor:

$$\langle\Psi|\hat{T}^{\alpha\beta}(x)|\Psi\rangle = \langle\mathcal{E}\mathcal{U}^{\alpha}\mathcal{U}^{\beta} - \mathcal{P}\Delta^{\alpha\beta} + \Pi^{\alpha\beta} - g^{\alpha\beta}\Lambda\rangle_{\Psi}$$

where $\mathcal{U}^{\alpha}$ is hadronic 4-velocity ($\mathcal{U}^{\alpha}\mathcal{U}_{\alpha} = 1$), $\Delta^{\alpha\beta} = g^{\alpha\beta} - \mathcal{U}^{\alpha}\mathcal{U}^{\beta}$

- Physical densities:

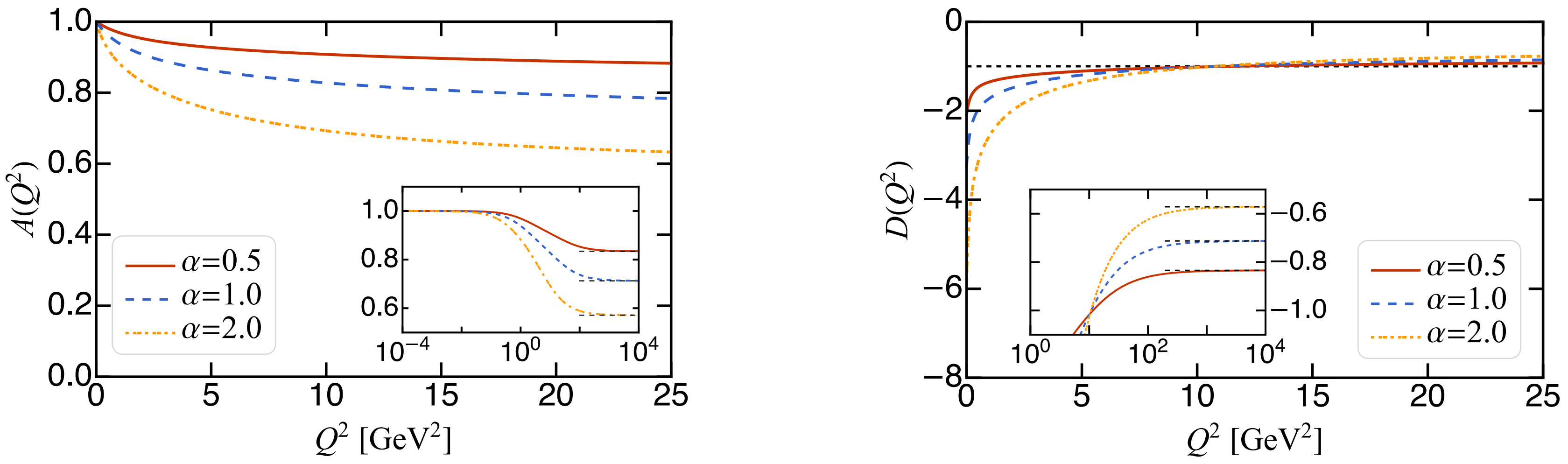
$$\text{Energy density: } \mathcal{E}(x) = M\int\frac{\mathrm{d}^3q}{(2\pi)^3}e^{iq\cdot x}\left\{A(q^2) - \frac{q^2}{4M^2}[A(q^2)+D(q^2)]\right\}$$

$$\text{Pressure: } \mathcal{P}(x) = \frac{1}{6M}\int\frac{d^3q}{(2\pi)^3}e^{iq\cdot x}q^2 D(q^2)$$

$$\text{Shear tensor: } \Pi^{\alpha\beta}(x) = \frac{1}{4M}\int\frac{d^3q}{(2\pi)^3}e^{iq\cdot x}(q^{\alpha}q^{\beta} - \frac{q^2}{3}\Delta^{\alpha\beta})D(q^2)$$

$$\text{Cosmological constant: } \Lambda = -M\int\frac{d^3q}{(2\pi)^3}e^{iq\cdot x}\bar{c}(q^2)$$

Strongly-coupled scalar nucleon



[Cao:2023ohj]

$$\alpha = \frac{g^2}{16\pi m^2}$$

- For small coupling, $D(Q^2)$ is close to -1, the free scalar particle's result
- In the forward limit ($Q^2 = 0$),

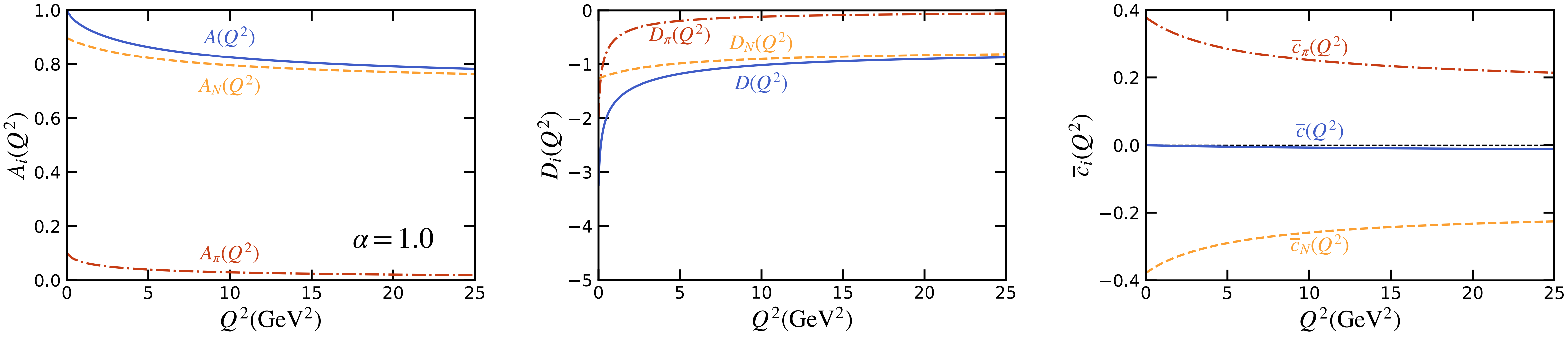
$$\lim_{Q^2=0} A(Q^2) = 1, \qquad \lim_{Q^2\to 0} Q^2 D(Q^2) = 0$$

- For large Q^2,

$$\lim_{Q^2\to\infty} A(Q^2) = Z, \qquad \lim_{Q^2\to\infty} D(Q^2) = -Z$$

revealing a pointlike core, consistent with the physical picture of the model

Dissecting the strongly-coupled scalar nucleon

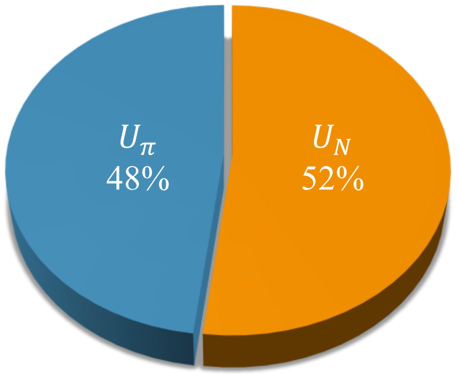


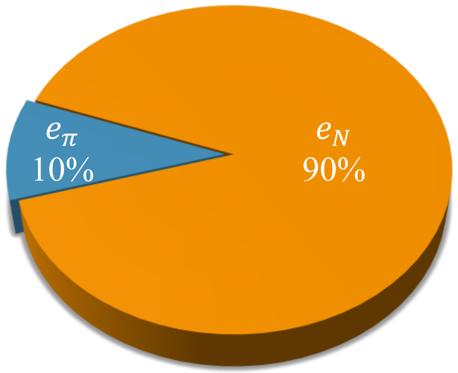
- A nonvanishing but small $\bar{c}(q^2)$ because of Fock space truncation $\sum_i \bar{c}_i(q^2) \neq 0$ [Cao:2024fto]
- Mass decomposition: [Lorce:2017xzd]

$$e_i = \int d^2 r_\perp \mathcal{E}(r_\perp) = A_i(0)$$

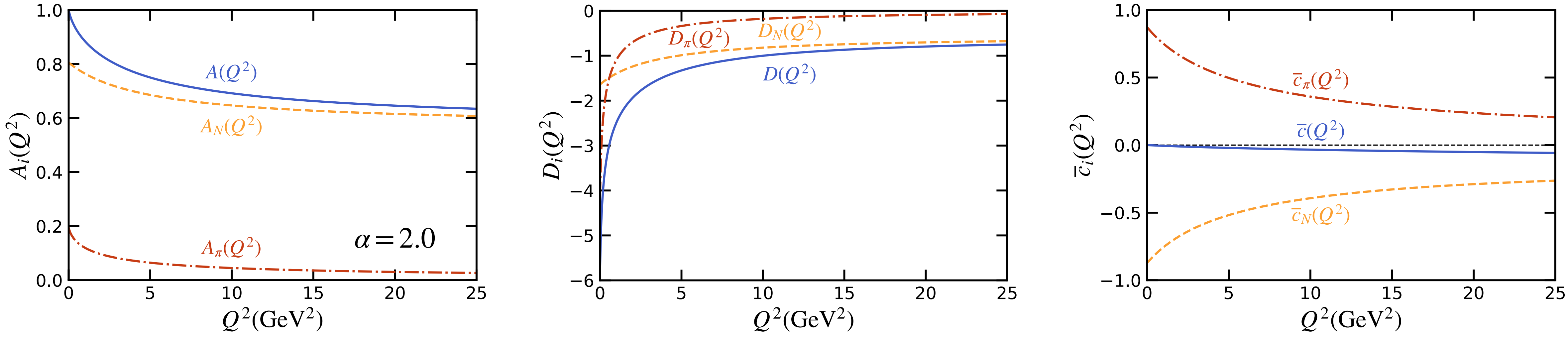
$$\lambda_i = \int d^2 r_\perp \Lambda_i(r_\perp) = \bar{c}_i(0)$$

$$U_i = e_i + \lambda_i$$





Dissecting the strongly-coupled scalar nucleon



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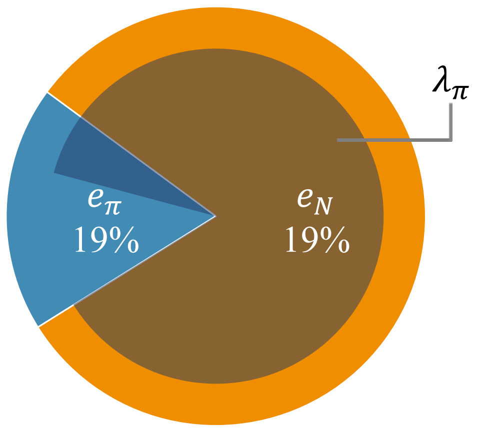
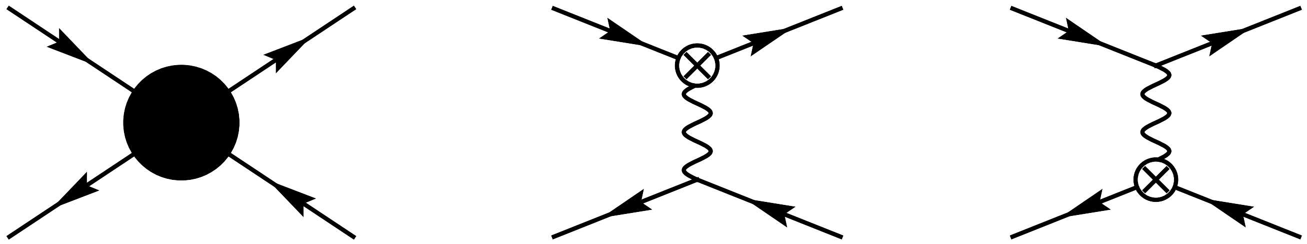


Impulse ansatz

- From the effective Hamiltonian, we can't give the exact stress-energy operator directly
- We adopt impulse ansatz for the interaction term in T^{+-}

$$t^{+-}_{\text{int}} = \frac{1}{2}\sum_{s,\bar{s}}\int \frac{\mathrm{d}x}{4\pi x(1-x)}\int \mathrm{d}^2 r_\perp \psi^*_{s\bar{s}}(x,\vec{r}_\perp)\left[e^{i\vec{q}_\perp\cdot\vec{r}_{1\perp}} + e^{i\vec{q}_\perp\cdot\vec{r}_{2\perp}}\right] v(x,\vec{r}_\perp,-i\nabla_\perp)\psi_{s\bar{s}}(x,\vec{r}_\perp)$$

where $v(x,\vec{r}_\perp,-i\nabla_\perp) = M^2 - \frac{-\nabla^2_\perp + m^2_q}{x} - \frac{-\nabla^2_\perp + m^2_{\bar{q}}}{1-x}$



Energy density and invariant mass squared density

$$\mathcal{E}\left(r_{\perp}\right)=M\int\frac{\mathrm{d}^{2}q_{\perp}}{(2\pi)^{2}}e^{-i\vec{q}_{\perp}\cdot\vec{r}_{\perp}}\left\{\left(1+\frac{q_{\perp}^{2}}{4M^{2}}\right)A\left(q_{\perp}^{2}\right)+\frac{q_{\perp}^{2}}{4M^{2}}D\left(q_{\perp}^{2}\right)\right\},$$

$$\mathcal{M}^{2}\left(r_{\perp}\right)=M^{2}\int\frac{\mathrm{d}^{2}q_{\perp}}{(2\pi)^{2}}e^{-i\vec{q}_{\perp}\cdot\vec{r}_{\perp}}\left\{\left(1+\frac{q_{\perp}^{2}}{4M^{2}}\right)A\left(q_{\perp}^{2}\right)+\frac{q_{\perp}^{2}}{2M^{2}}D\left(q_{\perp}^{2}\right)\right\}$$

- Energy density is positive
- Invariant mass squared density becomes negative at small $r_{\perp}$: tachyonic core?