# Form factors in semileptonic decay $D \to K^* l\nu$ from lattice QCD

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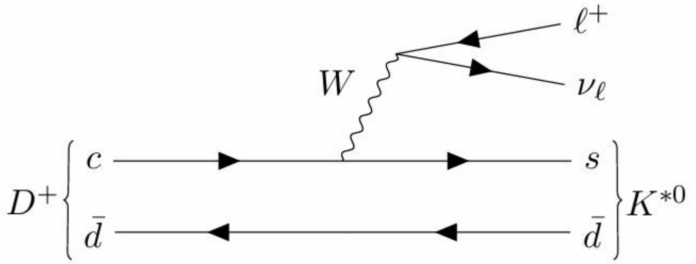
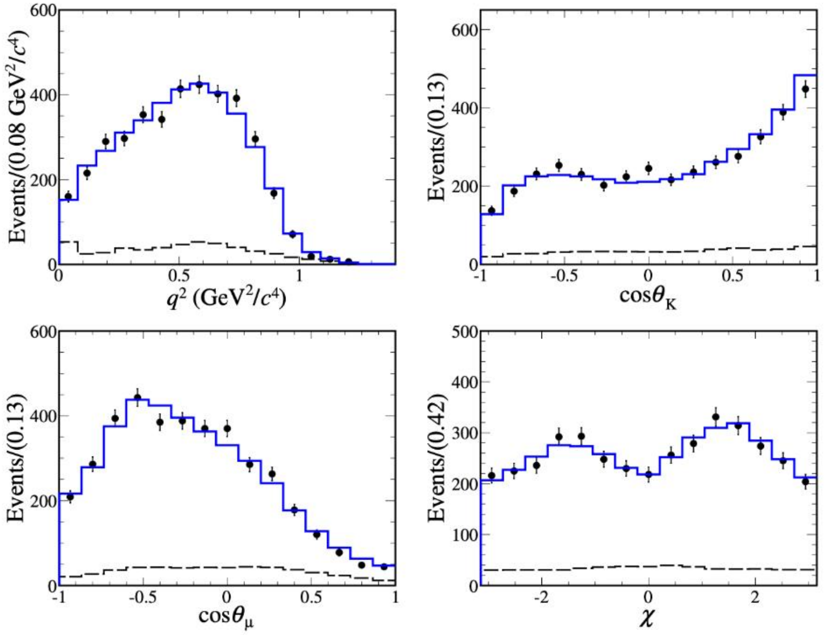
# Outline

- Motivations
- Lattice set up
- Method
- Preliminary results

# Motivations

- Semi-leptonic decays offer an ideal place to deeply understand hadronic transitions in the nonperturbative region of QCD and explore the weak and strong interactions in charm sector
- Vector meson decay makes this transition notoriously difficult to model due to theoretical complexity
- Combining with experimental data, the CKM matrix element can be extracted, and it helps to test unitarity of CKM matrix and searching for new physics beyond SM
- Calculating branching fractions helps to test $\mu - e$ lepton flavor universality




[BESIII, Phys. Rev. Lett. 134, 011803 (2025)]

SM parameter

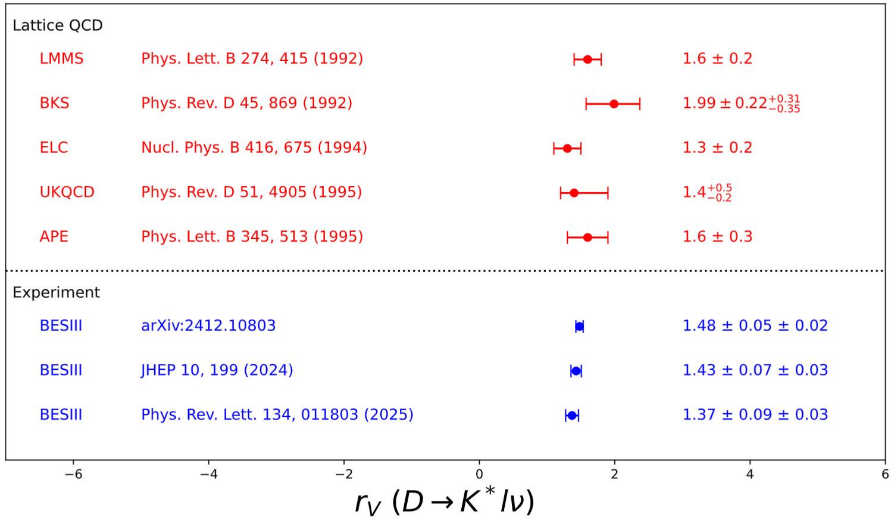
$$\boxed{\Gamma(D \to K^*\ell\nu)} = \frac{G_F^2 \boxed{|V_{cs}|^2}}{192\pi^3 m_D^3} \int_0^{q^2_{\max}} dq^2 q^2 \lambda(q^2)^{1/2} \times \boxed{\left(|H^+(q^2)|^2 + |H^-(q^2)|^2 + |H^0(q^2)|^2\right)}$$

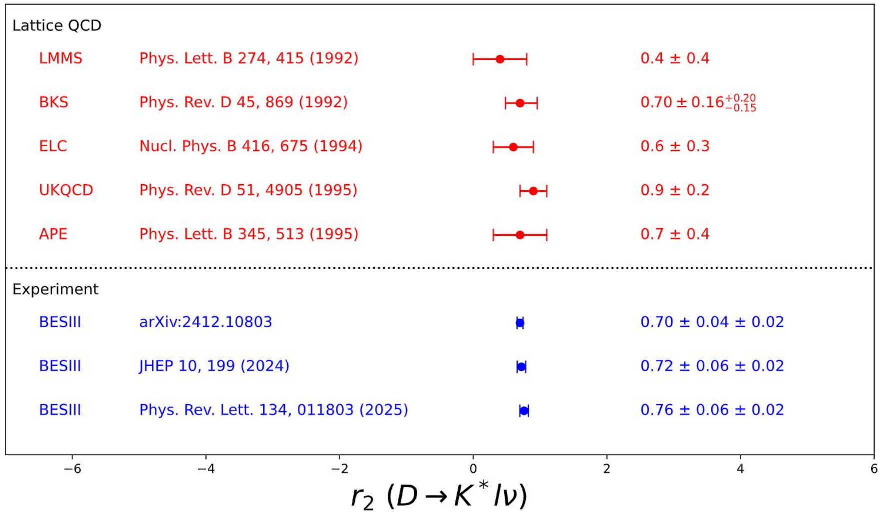
Experimental data

non-perturbative

# Motivations

- No un-quenched $D \to K^* l\nu$ lattice QCD results were reported. More BESIII high-precision measurements on charm meson semi-leptonic decays to vector mesons results are on the way. A full nonperturbative lattice calculation is important



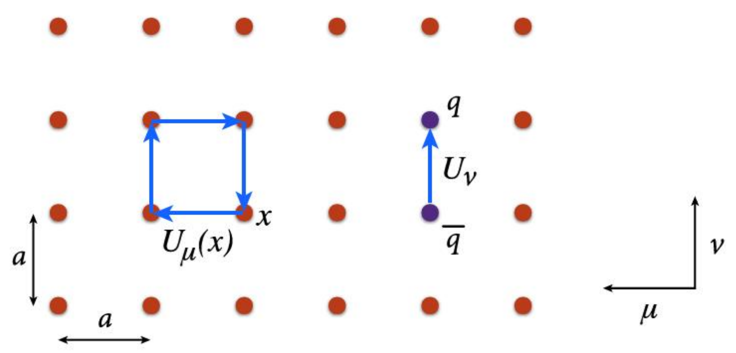

- By combining $D \to K^* l\nu$ and $D_s \to \phi l\nu$ LQCD results, the impact of SU(3) symmetry can be investigated

# Introduction to LQCD

- Path integral in discrete Euclidean space

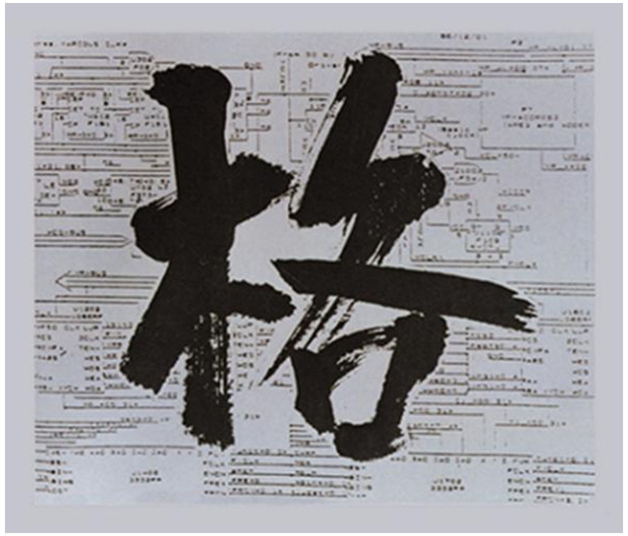
$$Z = \int [dU] \prod_f [dq_f][d\bar{q}_f] e^{-S_g[U] - \sum_f \bar{q}_f (D[U]+m_f) q_f}$$

$$Z = \int [dU] e^{-S_g[U]} \prod_f \det(D[U] + m_f)$$




- Expectation values of gauge-invariant operators, also known as “correlation functions”

$$\langle \mathcal{O}(U, q, \bar{q}) \rangle = (1/Z) \int [dU] \prod_f [dq_f][d\bar{q}_f] \mathcal{O}(U, q, \bar{q}) e^{-S_g[U] - \sum_f \bar{q}_f (D[U]+m_f) q_f}$$

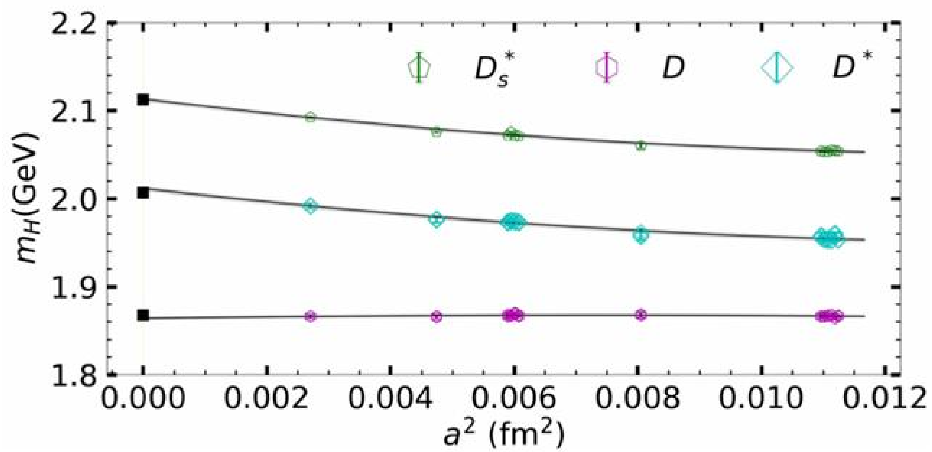



- Monte-Carlo method and data analysis

# Lattice set up

- (2+1)-flavor Wilson-clover gauge ensembles by CLQCD collaboration [CLQCD, PRD 111, 054504 (2025)]
- Charm quark mass $\tilde{m}_c^{\mathrm{b}}$ is tuned by physical $D_s$ mass
- C32P23, F32P30 with different $a$ and $\pi$ mass will be used in the future
- Computer resources: "SongShan" supercomputer at Zhengzhou University

| Ensemble | C24P29 |
|---|---|
| $a$ (fm) | 0.10524(05)(62) |
| $\tilde{m}_s^{\mathrm{b}}$ | $-0.2400$ |
| $\tilde{m}_l^{\mathrm{b}}$ | $-0.2770$ |
| $\tilde{m}_c^{\mathrm{b}}$ | 0.4159(07) |
| $L^3 \times T$ | $24^3 \times 72$ |
| $N_{\mathrm{cfg}} \times N_{\mathrm{src}}$ | $450 \times 72$ |
| $m_\pi$ (MeV) | 292.3(1.0) |
| $t$ | $2 - 17$ |
| $Z_V^s$ | 0.85184(06) |
| $Z_V^c$ | 1.57353(18) |
| $Z_A/Z_V$ | 1.07244(70) |



# Formulism of correlation function

- 2-point correlation function (2pt),

$$C^{(2)}(\vec{p}, t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle \mathcal{O}_h(\vec{x}, t)\mathcal{O}_h^\dagger(0)\rangle$$

- 3-point correlation function (3pt),

$$\begin{aligned} C_{\mu\nu} &= \langle \mathcal{O}_{K_\nu^*}(t) J_\mu^W(0) \mathcal{O}_D^\dagger(-t_s)\rangle \\ &= \langle \bar{u}(t)\gamma_\nu s(t)\bar{s}(0)\gamma_\mu(1-\gamma_5)c(0)\bar{c}(-t_s)\gamma_5 u(-t_s)\rangle \\ &= \langle \mathrm{Tr}[\gamma_5\gamma_5 S_{-u}^\dagger(t, -t_s)\gamma_5\gamma_\nu S_s(t, 0)\gamma_\mu(1-\gamma_5)S_c(0, -t_s)]\rangle \end{aligned}$$

# Method

- Correlation functions $\longrightarrow$ Scalar functions $\longrightarrow$ Form factors

$\langle K^*_\sigma(\vec{p})|J^W_\mu(0)|D(p')\rangle$ $\qquad \tilde{\mathcal{I}}_j \qquad$ $V, A_0, A_1, A_2$

- The parameterization for $P \to V$ semileptonic matrix element

$$\langle K^*_\sigma(\vec{p})|J^W_\mu(0)|D(p')\rangle = \frac{F_0(q^2)}{Mm}\epsilon_{\mu\sigma\alpha\beta}p'^\alpha p^\beta + F_1(q^2)\delta_{\mu\sigma} + \frac{F_2(q^2)}{Mm}p_\mu p'_\sigma + \frac{F_3(q^2)}{M^2}p'_\mu p'_\sigma,$$

$$\langle K^*(\varepsilon,\vec{p})|J^W_\mu(0)|D(p')\rangle = \varepsilon^*_\nu\epsilon_{\mu\nu\alpha\beta}p'_\alpha p_\beta\frac{2V(q^2)}{m+M} + (M+m)\varepsilon^*_\mu A_1 + \frac{\varepsilon^*\cdot q}{M+m}(p+p')_\mu A_2 - 2m\frac{\varepsilon^*\cdot q}{Q^2}q_\mu(A_0 - A_3)$$

- Relationship with the form factor

$$V = \frac{(m+M)}{2mM}F_0,$$
$$A_1 = \frac{F_1}{M+m},$$
$$A_2 = \frac{M+m}{2mM^2}(MF_2 + mF_3),$$
$$A_0 - A_3 = Q^2\left(\frac{F_2}{4m^2M} - \frac{F_3}{4mM^2}\right).$$

$A_3$ is not an independent form factor

$$A_3(q^2) = \frac{M+m}{2m}A_1(q^2) - \frac{M-m}{2m}A_2(q^2)$$

$A_0$ is then

$$A_0(q^2) = \frac{F_1}{2m} + \frac{m^2 - M^2 + Q^2}{4m^2M}F_2 + \frac{m^2 - M^2 - Q^2}{4mM^2}F_3$$

$A_0(0) = A_3(0)$ is automatically perserved

# Method

- A similar scalar function scheme has been used for high-precision calculation

  - $\Gamma(\eta_c \to 2\gamma) = 6.67(16)(6)\ \mathrm{keV}$

    [Y. M et al, Science Bulletin 68, 1880 (2023)]

  - $\Gamma(D_s^* \to \gamma D_s) = 0.0549(54)\ \mathrm{keV}$

    [Y. M et al, PRD109, 074511 (2024)]

  - $\mathrm{Br}(J/\psi \to D e \nu_e) = 1.21(11) \times 10^{-11}$
    $\mathrm{Br}(J/\psi \to D \mu \nu_\mu) = 1.18(11) \times 10^{-11}$
    $\mathrm{Br}(J/\psi \to D_s e \nu_e) = 1.90(8) \times 10^{-10}$
    $\mathrm{Br}(J/\psi \to D_s \mu \nu_\mu) = 1.84(8) \times 10^{-10}$

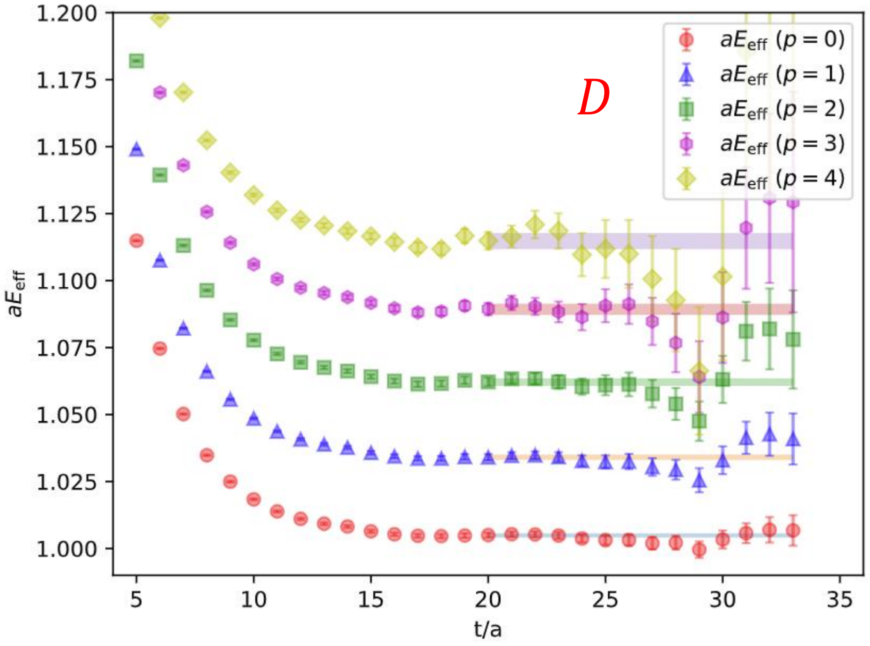
    [Y. M et al, PRD110, 074510 (2024)]

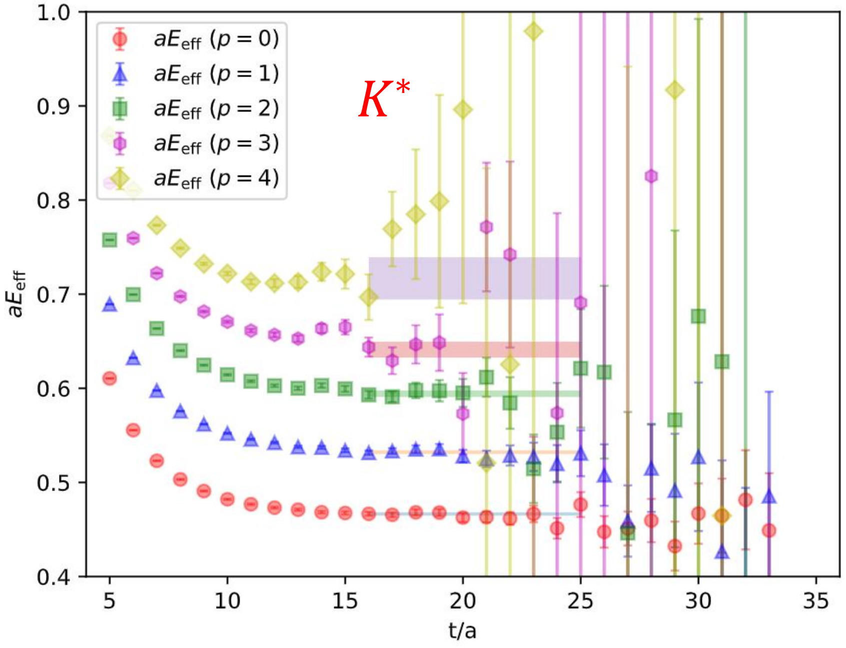
# Fitting of 2-point correlation function

- Least $\chi^2$ fitting considering covariance matrix between configurations and time

$$C^{(2)}(\vec{p},t) = \frac{Z_h^2}{2E_h}\left(e^{-E_h t} + e^{-E_h(T-t)}\right) \qquad C^{(2)}(\vec{p},t) = \left(-1 - \frac{|\vec{p}|^2}{3m_h^2}\right)\frac{Z_h^2}{2E_h}\left[e^{-E_h t} + e^{-E_h(T-t)}\right]$$

- There should be a plateau when meson ground states are dominant. Left for $D$ and right for $K^*$

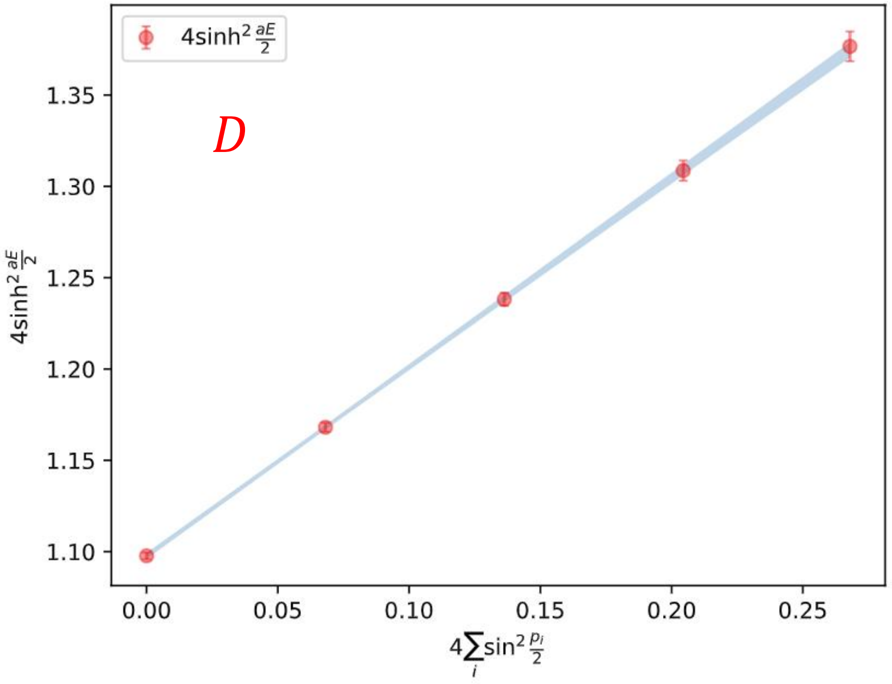


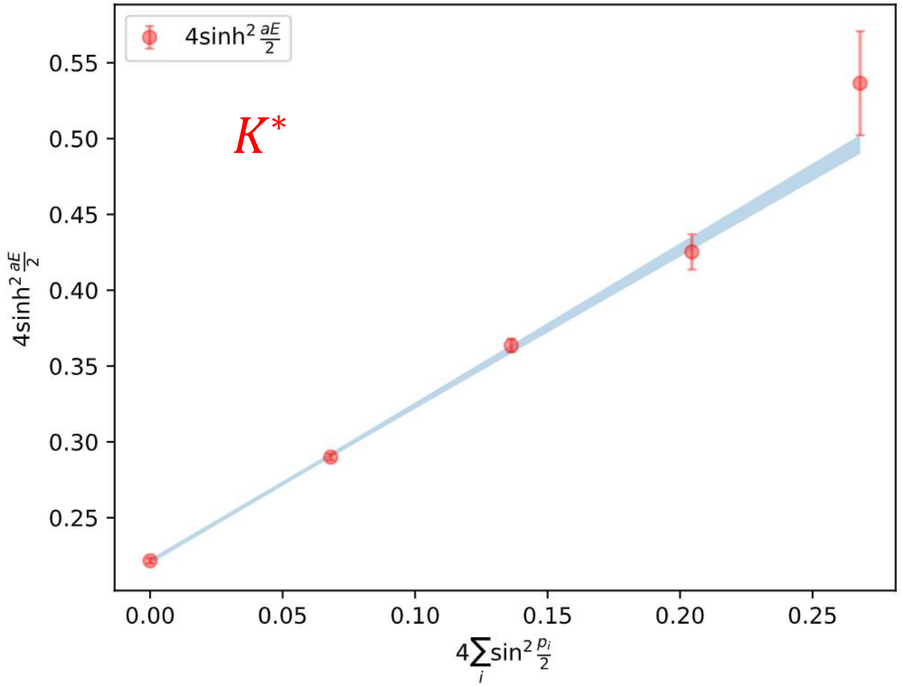



# Dispersion relation

- We checked the dispersion relation of $D$ meson and $K^*$ meson at four different momenta and use a discrete dispersion relation as the fitting function. $\mathcal{Z}_{\mathrm{latt}}$ for $D$ and $K^*$ is $1.034(19)$ and $1.027(27)$. Left for $D$ and right for $K^*$

$$4\sinh^2\frac{E_h}{2} = 4\sinh^2\frac{m_h}{2} + \mathcal{Z}^h_{\mathrm{latt}}\cdot 4\sum_i \sin^2\frac{p_i}{2}$$

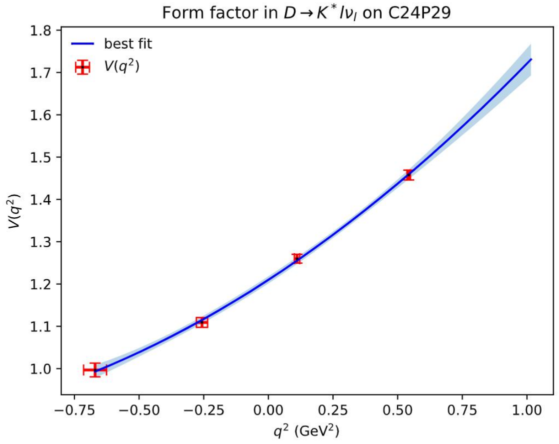


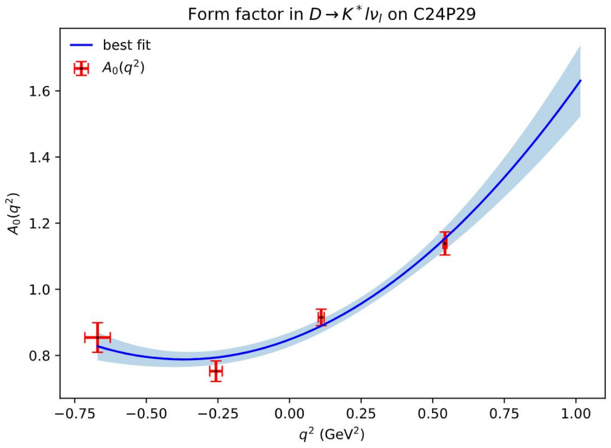



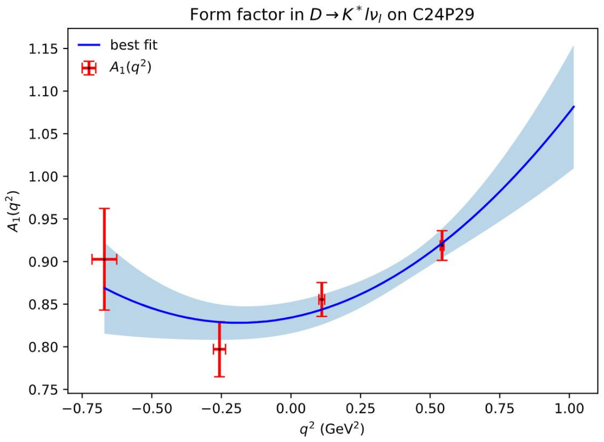
# Preliminary form factor results

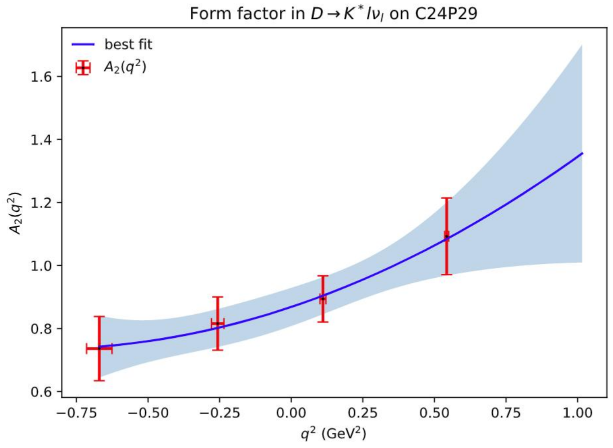
- The results have been multiplied by the renormalization constant
- A $q^2$ expansion by a polynomial form $F(q^2) = a_0 + a_1 \cdot q^2 + a_2 \cdot q^4$
- $A_0(0) - A_3(0) = 0.0003(51)$, which meets expectations

| | This work |
|---|---|
| $V(0)$ | 1.205(13) |
| $A_1(0)$ | 0.834(18) |
| $A_2(0)$ | 0.868(61) |
| $A_0(0)$ | 0.848(21) |



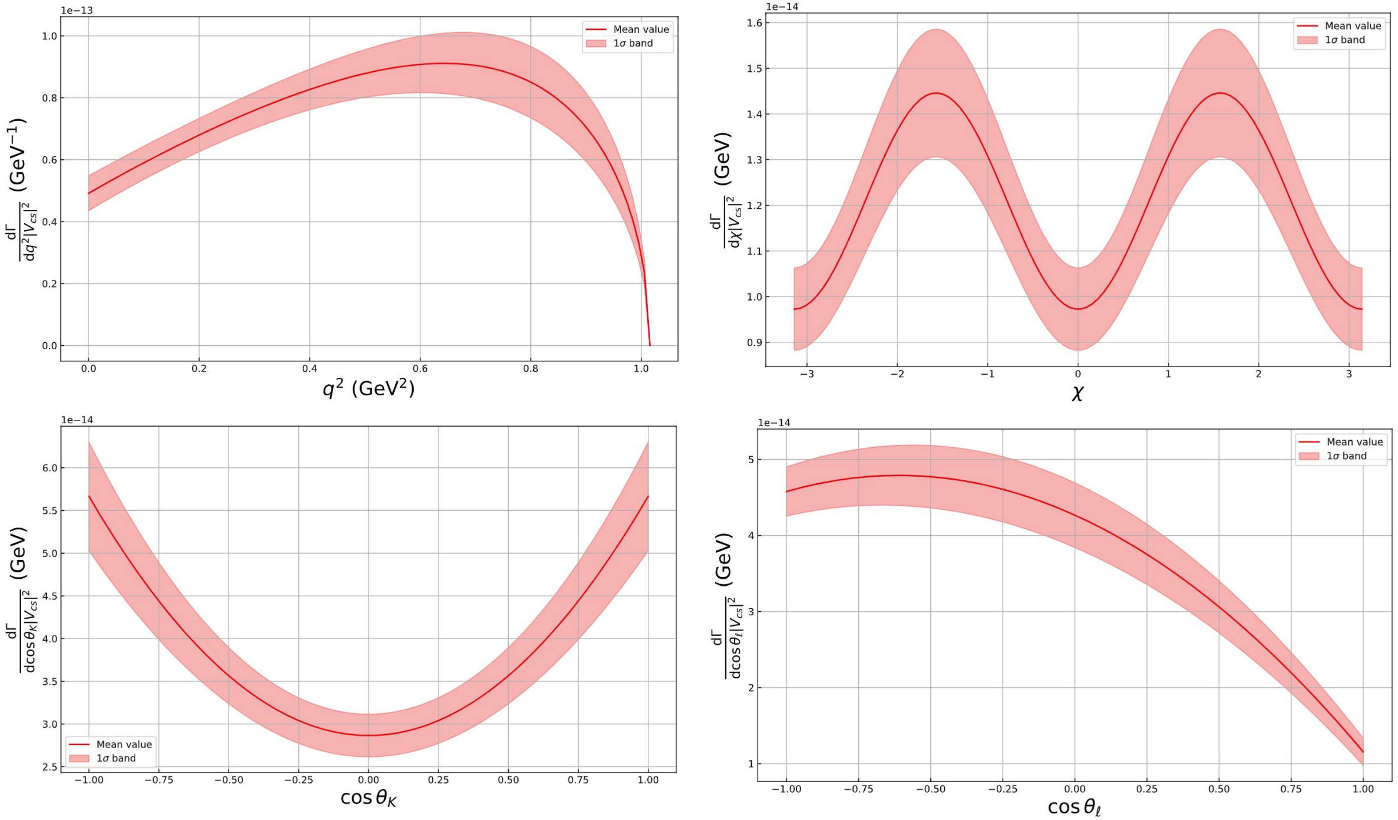



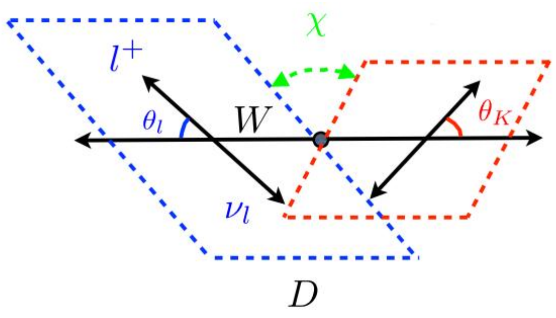



# Preliminary decay width results

- Differential decay width, where the lepton mass is neglected [Rev.Mod.Phys 67,893(1995)]




$$
\begin{aligned}
&\frac{d\Gamma(D\to K^*\ell\nu)}{dq^2 d\cos\theta_K d\cos\theta_\ell d\chi |V_{cs}|^2} = \frac{3}{8\,(4\pi)^4} G_F^2 \frac{p_{K^*} q^2}{M_D^2} \\
&\times \{(1+\cos\theta_\ell)^2 \sin^2\theta_K |H_+(q^2)|^2 \\
&+ (1-\cos\theta_\ell)^2 \sin^2\theta_K |H_-(q^2)|^2 \\
&+ 4\sin^2\theta_\ell \cos^2\theta_K |H_0(q^2)|^2 \\
&+ 4\sin\theta_\ell(1+\cos\theta_\ell)\sin\theta_K\cos\theta_K\cos\chi H_+(q^2)H_0(q^2) \\
&- 4\sin\theta_\ell(1-\cos\theta_\ell)\sin\theta_K\cos\theta_K\cos\chi H_-(q^2)H_0(q^2) \\
&-2\sin^2\theta_\ell \sin^2\theta_K \cos 2\chi H_+(q^2)H_-(q^2)\}
\end{aligned}
$$



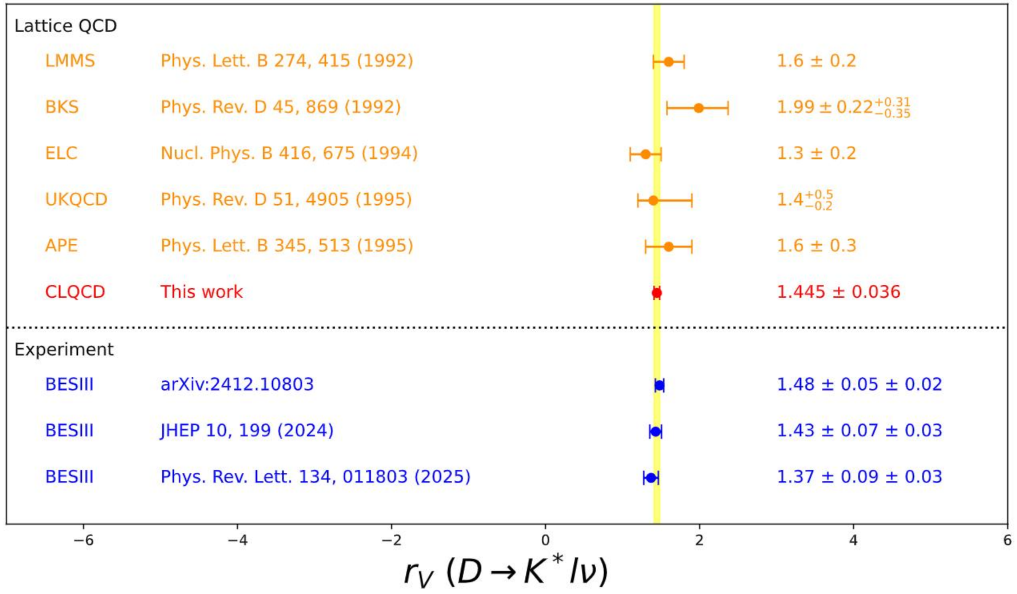
# Preliminary results

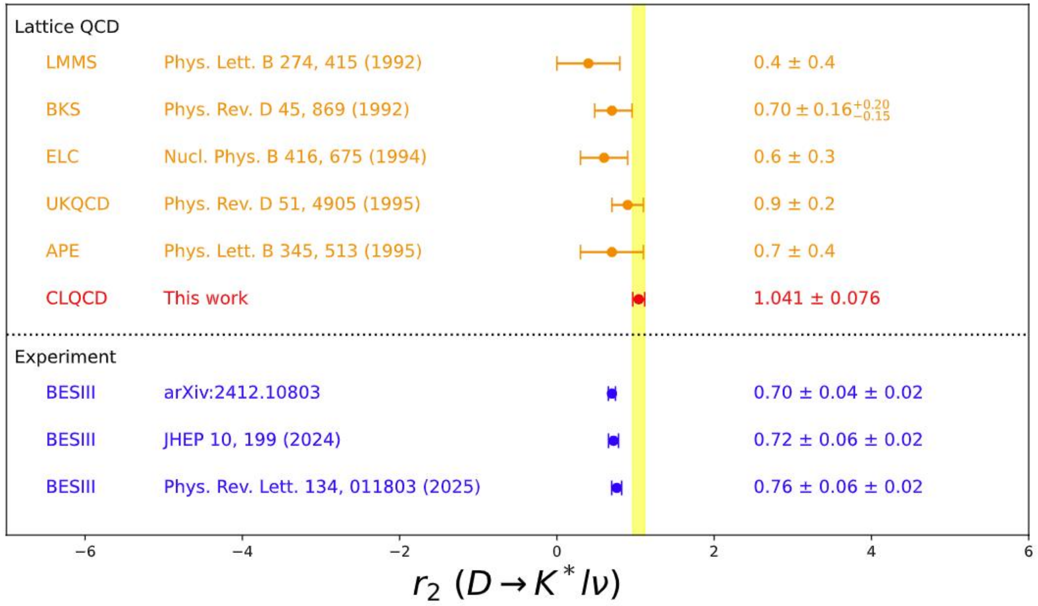
- Summary of preliminary results

$\pi$ mass $292.3(1.0)$ MeV
lattice spacing $0.10524(05)(62)$ fm

| | This work | EPJC (2022) | BESIII (2025) |
|---|---|---|---|
| $r_V \equiv V(0)/A_1(0)$ | 1.445(36) | 1.50 | 1.37(9)(3) |
| $r_2 \equiv A_2(0)/A_1(0)$ | 1.041(76) | 0.68 | 0.76(6)(2) |
| $\mathcal{B}(D \to K^*\ell\nu) \times 10^{-2}$ | 4.63(44) | 2.12(3) | 2.073(39)(32) |

[H. Y. Xing *et. al*, Eur.Phys.J.C 82 (2022) 10, 889]



# Summary and Outlook

## Summary

- Preliminary results for 2-point correlation functions of $D$ meson and $K^*$ meson
- Dispersion relation of $D$ meson and $K^*$ meson
- Preliminary results for form factors and decay width on one lattice set with four different $q^2$

## Outlook

- Consider $K\pi$ scattering contribution
- More statistics for decreasing statistical error
- Extrapolation/interpolation of results to the physical point/continuum limit
- Preliminary $D_s \to \phi$ form factors are ongoing

Thank you for your attention!

# Back up (Formulae)

- We start from the hadronic function

$$H_{\mu\nu}(x) = \langle K^*_\nu(\vec{x}, t)\, J^W_\mu(0)\,|V(p')\rangle$$
$$= \sum_p \frac{1}{2EV} e^{-Et+i\vec{p}\cdot\vec{x}} \left(-\delta_{\nu\sigma} - \frac{p_\nu p_\sigma}{m^2}\right) \langle 0|K^*(0)\,|K^*(\vec{p})\rangle \langle K^*_\sigma(\vec{p})\,|J^W_\mu(0)\,|D(p')\rangle$$

- Consider the parameterizations [Z. Phys. C 46, 93 (1990)]

$$\langle 0|\phi_h(0)\,|\phi_h(\vec{p})\rangle = Z_h$$

$$\langle K^*_\sigma(\vec{p})\,|J^W_\mu(0)\,|D(p')\rangle = \frac{F_0(q^2)}{Mm}\epsilon_{\mu\sigma\alpha\beta}p'^\alpha p^\beta + F_1(q^2)\,\delta_{\mu\sigma} + \frac{F_2(q^2)}{Mm}p_\mu p'_\sigma + \frac{F_3(q^2)}{M^2}p'_\mu p'_\sigma.$$

- It is easy to obtain

$$H_{\mu\nu}(\vec{x}, t) = \frac{2M}{Z_D} e^{Mt_s} C_{\mu\nu}(\vec{x}, t; t_s)$$

# Back up (Formulae)

- A spatial Fourier transform of $H_{\mu\nu} \equiv V_{\mu\nu} - A_{\mu\nu}$, $\widetilde{V}_{\mu\nu}$ and $\widetilde{A}_{\mu\nu}$ are

$$\tilde{V}_{\mu\nu} = -\frac{F_0(q^2)}{Mm}\epsilon_{\mu\nu\alpha\beta}p'^{\alpha}p^{\beta} \times \frac{Z_V e^{-Et}}{2E},$$

$$\tilde{A}_{\mu\nu} = \left[-F_1(q^2)\,\delta_{\mu\sigma} - \frac{F_2(q^2)}{Mm}p_\mu p'_\sigma - \frac{F_3(q^2)}{M^2}p'_\mu p'_\sigma\right]\left(-\delta_{\nu\sigma} - \frac{p_\nu p_\sigma}{m^2}\right) \times \frac{Z_V e^{-Et}}{2E}$$

- Construct the scalar functions

$$\mathcal{I}_0 = \frac{1}{M|\vec{p}|^2}\epsilon_{\mu\nu\alpha'\beta'}p'_{\alpha'}p_{\beta'}\int \mathrm{d}^3\vec{x}e^{-i\vec{p}\cdot\vec{x}}V_{\mu\nu}(\vec{x},t)$$

$$\mathcal{I}_1 = \delta_{\mu\nu}\int \mathrm{d}^3\vec{x}e^{-i\vec{p}\cdot\vec{x}}A_{\mu\nu}(\vec{x},t)$$

$$\mathcal{I}_2 = \frac{E}{M}\frac{p_\mu p'_\nu}{|\vec{p}|^2}\int \mathrm{d}^3\vec{x}e^{-i\vec{p}\cdot\vec{x}}A_{\mu\nu}(\vec{x},t)$$

$$\mathcal{I}_3 = \frac{p'_\mu p'_\nu}{|\vec{p}|^2}\int \mathrm{d}^3\vec{x}e^{-i\vec{p}\cdot\vec{x}}A_{\mu\nu}(\vec{x},t)$$

$$\tilde{\mathcal{I}}_j = \mathcal{I}_j \times \frac{2Ee^{Et}}{Z_{K^*}}$$

$$F_0(q^2) = \frac{m}{2}\tilde{\mathcal{I}}_0,$$

$$F_1(q^2) = \frac{1}{2}\tilde{\mathcal{I}}_1 + \frac{1}{2}\tilde{\mathcal{I}}_2 - \frac{m^2}{2M^2}\tilde{\mathcal{I}}_3,$$

$$F_2(q^2) = \frac{mE}{2(E^2-m^2)}\tilde{\mathcal{I}}_1 + \frac{mE^2+2m^3}{2(E^3-Em^2)}\tilde{\mathcal{I}}_2 - \frac{3Em^3}{2M^2(E^2-m^2)}\tilde{\mathcal{I}}_3,$$

$$F_3(q^2) = -\frac{m^2}{2(E^2-m^2)}\tilde{\mathcal{I}}_1 - \frac{3m^2}{2(E^2-m^2)}\tilde{\mathcal{I}}_2 + \frac{3m^4}{2M^2(E^2-m^2)}\tilde{\mathcal{I}}_3.$$

# Back up (Formulae)

- The traditional parameterization for $P \to V$ semileptonic decay [Rev.Mod.Phys 67,893(1995)]

$$\langle K^*(\varepsilon,\vec{p})|J_\mu^W(0)|D(p')\rangle = \varepsilon_\nu^* \epsilon_{\mu\nu\alpha\beta} p'_\alpha p_\beta \frac{2V(q^2)}{m+M} + (M+m)\varepsilon_\mu^* A_1 + \frac{\varepsilon^*\cdot q}{M+m}(p+p')_\mu A_2 - 2m\frac{\varepsilon^*\cdot q}{Q^2} q_\mu (A_0 - A_3)$$

- Relationship with the form factor

$$V = \frac{(m+M)}{2mM} F_0,$$
$$A_1 = \frac{F_1}{M+m},$$
$$A_2 = \frac{M+m}{2mM^2}(MF_2 + mF_3),$$
$$A_0 - A_3 = Q^2\left(\frac{F_2}{4m^2M} - \frac{F_3}{4mM^2}\right).$$

$A_3$ is not an independent form factor

$$A_3(q^2) = \frac{M+m}{2m} A_1(q^2) - \frac{M-m}{2m} A_2(q^2)$$

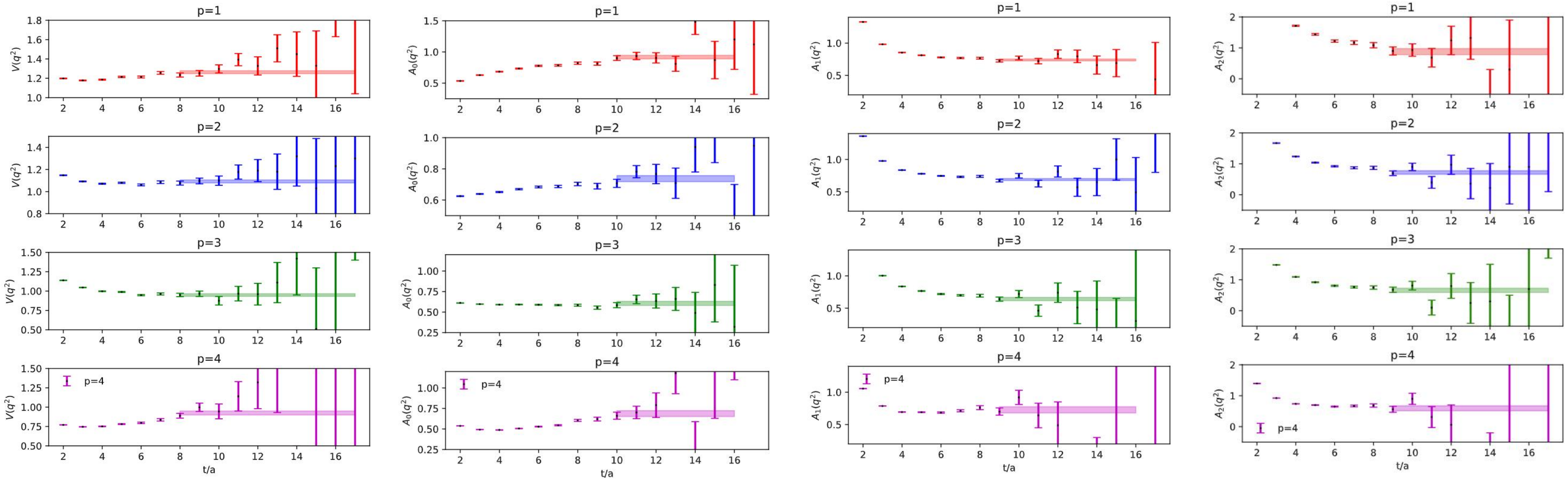
$A_0$ is then

$$A_0(q^2) = \frac{F_1}{2m} + \frac{m^2 - M^2 + Q^2}{4m^2M} F_2 + \frac{m^2 - M^2 - Q^2}{4mM^2} F_3$$

$A_0(0) = A_3(0)$ is automatically perserved

# Back up (Form factors fitting)

- Fit the form factor plateaus using constants at suitable time region



# Back up (Form factors $H_{\pm}$ and $H_0$)

$$H_{\pm}\left(q^2\right) = (M+m)\, A_1\left(q^2\right) \mp \frac{2Mp_{K^*}}{M+m} V\left(q^2\right),$$
$$H_0\left(q^2\right) = \frac{1}{2M\sqrt{q^2}} \times \left[\left(M^2 - m^2 - q^2\right)(M+m)\, A_1\left(q^2\right) - 4\frac{M^2 p_{K^*}^2}{M+m} A_2\left(q^2\right)\right].$$

- If do not neglect the lepton mass, the decay width should include term [Z. Phys. C 46, 93 (1990)]

$$\frac{3}{8(4\pi)^4} G_F^2 \frac{p_{K^*} m_\ell^2}{M_D^2} \times \{\sin^2\theta_K \sin^2\theta_\ell |H_+(q^2)|^2 + \sin^2\theta_K \sin^2\theta_\ell |H_-(q^2)|^2 + 4\cos^2\theta_K \cos^2\theta_\ell |H_0(q^2)|^2$$
$$+ 4\cos^2\theta_K |H_t(q^2)|^2 + \sin^2\theta_K \sin^2\theta_e \cos^2 2\chi H_+(q^2) H_-(q^2) + \sin 2\theta_K \sin 2\theta_\ell \cos 2\chi H_+(q^2) H_0(q^2)$$
$$+ \sin 2\theta_K \sin 2\theta_\ell \cos 2\chi H_-(q^2) H_0(q^2) + 2\sin 2\theta_K \sin\theta_\ell \cos\chi H_+(q^2) H_t(q^2) + 2\sin 2\theta_K \sin\theta_\ell \cos\chi H_-(q^2) H_t(q^2)$$
$$+ 8\cos^2\theta_K \cos\theta_\ell H_0(q^2) H_t(q^2)\},$$

where

$$H_t\left(q^2\right) = \frac{2Mp_{K^*}}{\sqrt{q^2}} A_0\left(q^2\right).$$