

Bottom quark hadronization in proton-proton collisions

**Based on MH & R. Rapp, PRL131, 012301 (2023)
Y. Dai & MH, PRC110, 034905 (2024)**

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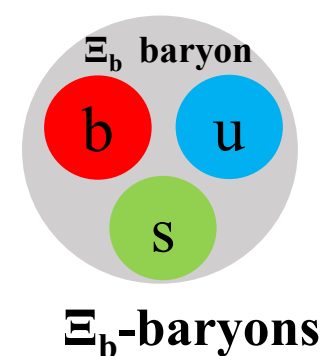
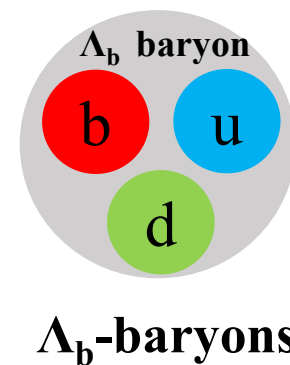
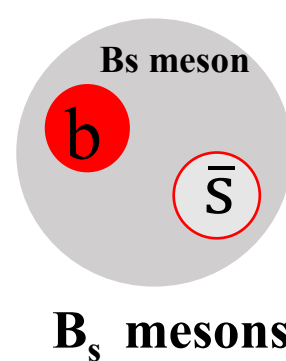
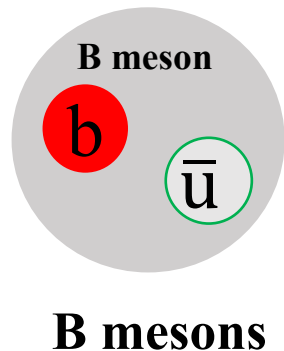
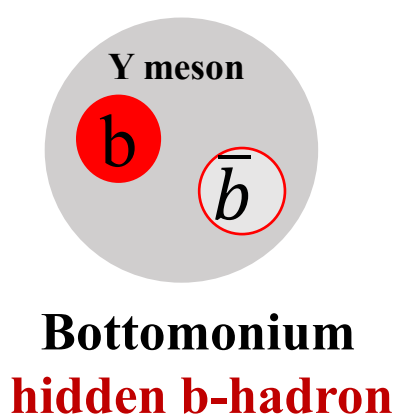


Heavy quarks & heavy hadrons

- Heavy quark $m_Q \gg \Lambda_{\text{QCD}}$
 $m_c \sim 1.5 \text{ GeV}$, $m_b \sim 4.5 \text{ GeV}$

质量→	2.4 MeV	1.27 GeV	171.2 GeV	0
电荷→	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0
自旋→	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
名字→	u 上夸克	c 粲夸克	t 顶夸克	γ 光子
夸克	4.8 MeV	104 MeV	4.2 GeV	0
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	d 下夸克	s 奇夸克	b 底夸克	g 胶子

- Bottomonium vs open bottom hadrons

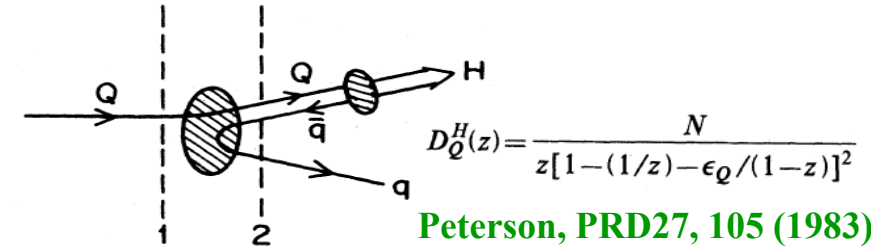
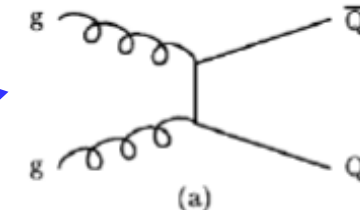


open b-hadrons

Heavy quark hadronization in pp collisions

- Heavy hadron production cross section: **factorization**

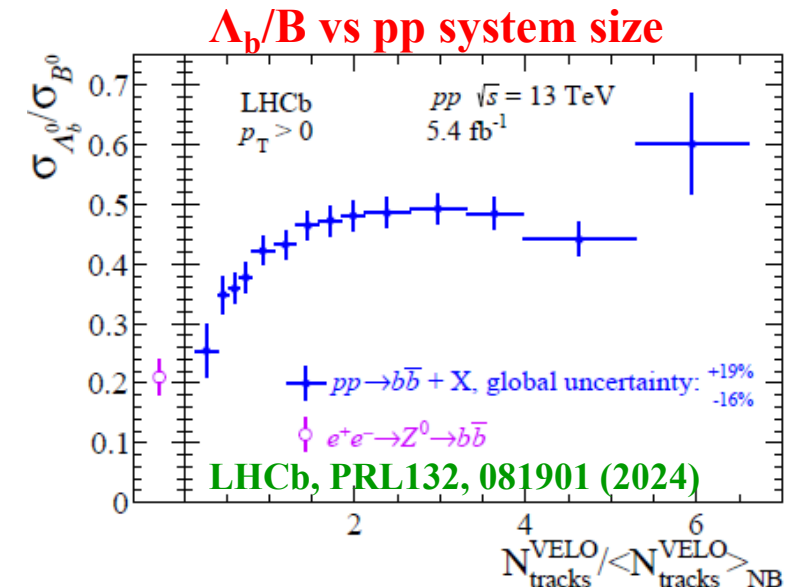
$$\frac{d\sigma^{H_c}}{dp_T^{H_c}}(p_T; \mu_F, \mu_R) = \underbrace{\text{PDF}(x_1, \mu_F) \cdot \text{PDF}(x_2, \mu_F)}_{\text{Parton distribution functions (PDFs)}} \otimes \underbrace{\frac{d\sigma^c}{dp_T^c}(x_1, x_2, \mu_R, \mu_F)}_{\text{Hard scattering cross section (pQCD)}} \otimes \underbrace{D_{c \rightarrow H_c}(z = p_{H_c}/p_c, \mu_F)}_{\text{Fragmentation function (hadronization)}}$$



- Hadronization: heavy quark conservation $f_u + f_d + f_s + f_{baryon} = 1$
- Heavy **hadro-chemistry**: universal? $\rightarrow$ non-universal!

Quantity		Z decays e^+e^-	Tevatron $p\text{-}p\bar{p}$
Mixing probability	$\bar{\chi}$	0.1259 ± 0.0042	0.147 ± 0.011
B^+ or B^0 fraction	$f_u = f_d$	0.407 ± 0.007	$\rightarrow 0.344 \pm 0.021$
B_s^0 fraction	f_s	0.101 ± 0.008	$\rightarrow 0.115 \pm 0.013$
b -baryon fraction	f_{baryon}	0.085 ± 0.011	$\rightarrow 0.198 \pm 0.046$

HFAG, EPJC81, 226 (2021)



Bottom hadro-chemistry in minimum bias pp collisions

Grand-canonical Ensemble Statistical Hadronization Model

Statistical Hadronization Model (SHM)

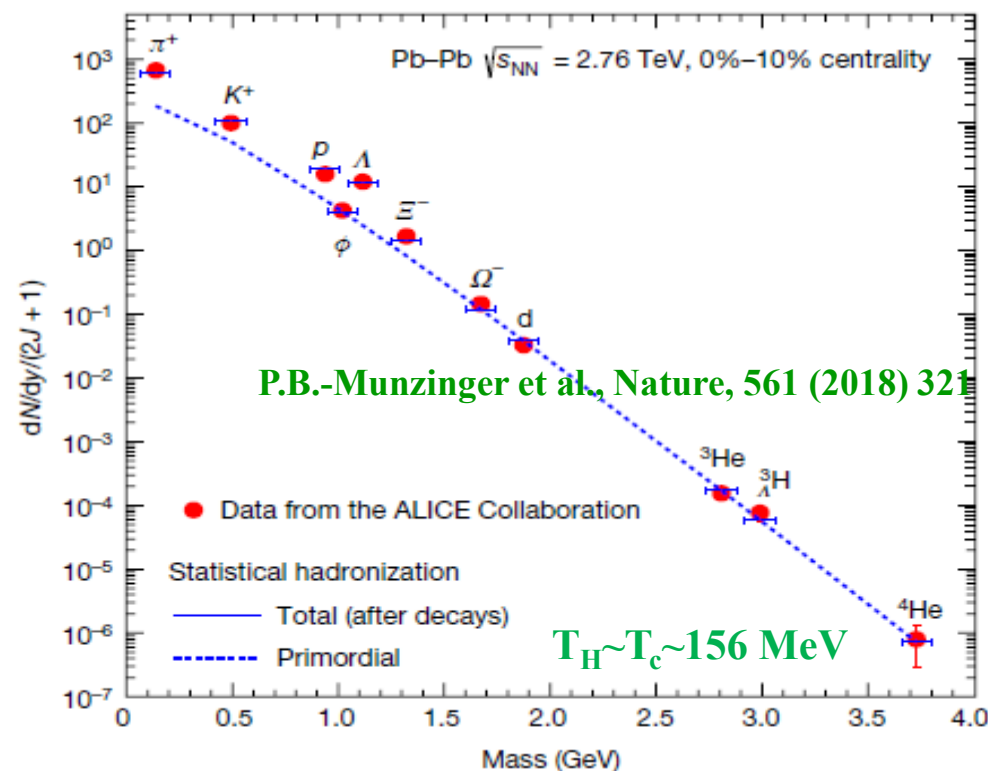
- QCD hadronic population from partons: born into equilibrium = **maximum entropy** state
- Hadron yields governed by partition function of a free hadron resonance gas (**HRG**)
- **Grand-canonical ensemble** SHM for light hadrons in PbPb

$$Z^{GC}(T, V, \mu_Q) = \text{Tr}[e^{-\beta(H - \sum_i \mu_{Q_i} Q_i)}]$$

$$\log Z^{GC}(T, V, \mu_Q) = \sum_i \log Z_i(T, V, \mu_{Q_i})$$

- **hadron yield**

$$\langle N_i \rangle = \frac{g_i V T}{2\pi^2} \sum_n \frac{(\pm 1)^{n+1}}{n} \lambda_i^n m_i^2 K_2\left(\frac{nm_i}{T}\right)$$



SHM for Bottom-hadrons in pp

- QCD hadronic population from partons: born into equilibrium = **maximum entropy** state

Khazeev & Satz, EPJC 52,187 (2007)

- Statistical Hadronization Model (SHM) for bottom-hadron production in pp

→ bottom quarks produced in early hard processes, bottom-hadron yields not in absolute equilibrium (unlike light hadrons)

→ **relative chemical equilibrium** achieved between different bottom-hadron species
primary production yields $N_i \propto$ statistical thermal densities

- Mass spectrum of bottom-hadrons: **PDG** incomplete? → to be augmented by **RQM**



Grand-canonical SHM for b-hadrons

- Grand-canonical ensemble → thermal density for primary b-hadrons

$$n_i^{\text{primary}} = \frac{d_i}{2\pi^2} \gamma_s^{N_s^i} m_i^2 T_H K_2\left(\frac{m_i}{T_H}\right) \begin{cases} \gamma_s=0.6 \text{ -- strangeness suppression factor} \\ T_H=170 \text{ MeV -- hadronization temperature} \end{cases}$$

- PDG: 5 B, 4 B_s,
5 Λ_b, 2 Σ_b, 4 Ξ_b, 1 Ω_b

- RQM: 25 B, 20 B_s, Ebert et al., PRD 84 (2011) 014025
30 Λ_b, 46 Σ_b, 75 Ξ_b, 42 Ω_b

Λ_b⁰

$$I(J^P) = 0(\frac{1}{2}^+)$$

$I(J^P)$ not yet measured; $0(\frac{1}{2}^+)$ is the quark model prediction.

Mass $m = 5619.60 \pm 0.17 \text{ MeV}$

$m_{\Lambda_b^0} - m_{B^0} = 339.2 \pm 1.4 \text{ MeV}$

$m_{\Lambda_b^0} - m_{B^+} = 339.72 \pm 0.28 \text{ MeV}$

Mean life $\tau = (1.471 \pm 0.009) \times 10^{-12} \text{ s}$

$c\tau = 441.0 \text{ } \mu\text{m}$

Λ_b(5912)⁰

$$J^P = \frac{1}{2}^-$$

Mass $m = 5912.20 \pm 0.21 \text{ MeV}$

Full width $\Gamma < 0.66 \text{ MeV}$, CL = 90%

Λ_b(5920)⁰

$$J^P = \frac{3}{2}^-$$

Mass $m = 5919.92 \pm 0.19 \text{ MeV}$ (S = 1.1)

Full width $\Gamma < 0.63 \text{ MeV}$, CL = 90%

Λ_b(6146)⁰

$$J^P = \frac{3}{2}^+$$

Mass $m = 6146.2 \pm 0.4 \text{ MeV}$

Full width $\Gamma = 2.9 \pm 1.3 \text{ MeV}$

Full width $\Gamma = 526.55 \pm 0.34 \text{ MeV}$

Λ_b(6152)⁰

$$J^P = \frac{5}{2}^+$$

Mass $m = 6152.5 \pm 0.4 \text{ MeV}$

Full width $\Gamma = 2.1 \pm 0.9 \text{ MeV}$

Full width $\Gamma = 532.89 \pm 0.28 \text{ MeV}$

Full width $\Gamma = 6.34 \pm 0.32 \text{ MeV}$

TABLE II. Masses of the Λ_Q (Q = c, b) heavy baryons (in MeV).

$I(J^P)$	Qd state	Q = c		Q = b	
		M	M ^{exp} [1]	M	M ^{exp} [1]
0($\frac{1}{2}^+$)	1S	2286	2286.46(14)	5620	5620.2(1.6)
0($\frac{1}{2}^+$)	2S	2769	2766.6(2.4)?	6089	
0($\frac{1}{2}^+$)	3S	3130	6455		
0($\frac{1}{2}^+$)	4S	3437	6756		
0($\frac{1}{2}^+$)	5S	3715	7015		
0($\frac{1}{2}^+$)	6S	3973	7256		
0($\frac{1}{2}^-$)	1P	2598	2595.4(6)	5930	
0($\frac{1}{2}^-$)	2P	2983	2939.3($\frac{1}{2}$)?	6326	
0($\frac{1}{2}^-$)	3P	3303	6645		
0($\frac{1}{2}^-$)	4P	3588	6917		
0($\frac{1}{2}^-$)	5P	3852	7157		
0($\frac{3}{2}^-$)	1P	2627	2628.1(6)	5942	
0($\frac{3}{2}^-$)	2P	3005	6333		
0($\frac{3}{2}^-$)	3P	3322	6651		
0($\frac{3}{2}^-$)	4P	3606	6922		
0($\frac{3}{2}^-$)	5P	3869	7171		
0($\frac{3}{2}^+$)	1D	2874	6190		
0($\frac{3}{2}^+$)	2D	3189	6526		
0($\frac{3}{2}^+$)	3D	3480	6811		
0($\frac{3}{2}^+$)	4D	3747	7060		
0($\frac{5}{2}^+$)	1D	2880	2881.53(35)	6196	
0($\frac{5}{2}^+$)	2D	3209	6531		
0($\frac{5}{2}^+$)	3D	3500	6814		

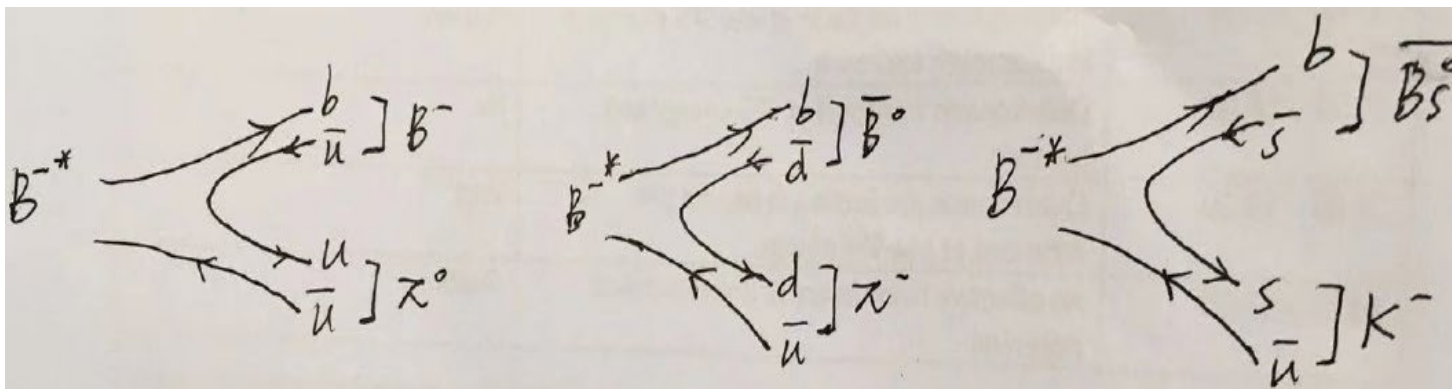
TABLE III. Masses of the Σ_Q (Q = c, b) heavy baryons (in MeV).

$I(J^P)$	Qd state	Q = c		Q = b	
		M	M ^{exp} [1]	M	M ^{exp} [1]
1($\frac{1}{2}^+$)	1S	2443	2453.76(18)	5808	5807.8(2.7)
1($\frac{1}{2}^+$)	2S	2901		6213	
1($\frac{1}{2}^+$)	3S	3271		6575	
1($\frac{1}{2}^+$)	4S	3581		6869	
1($\frac{1}{2}^+$)	5S	3861		7124	
1($\frac{3}{2}^+$)	1S	2519	2518.0(5)	5834	5829.0(3.4)
1($\frac{3}{2}^+$)	2S	2936	2939.3($\frac{1}{2}$)?	6226	
1($\frac{3}{2}^+$)	3S	3293		6583	
1($\frac{3}{2}^+$)	4S	3598		6876	
1($\frac{3}{2}^+$)	5S	3873		7129	
1($\frac{3}{2}^-$)	1P	2799	2802($\frac{4}{3}$)	6101	
1($\frac{3}{2}^-$)	2P	3172		6440	
1($\frac{3}{2}^-$)	3P	3488		6756	
1($\frac{3}{2}^-$)	4P	3770		7024	
1($\frac{3}{2}^-$)	1P	2713		6095	
1($\frac{3}{2}^-$)	2P	3125		6430	
1($\frac{3}{2}^-$)	3P	3455		6742	
1($\frac{3}{2}^-$)	4P	3743		7008	
1($\frac{3}{2}^-$)	1P	2798	2802($\frac{4}{3}$)	6096	
1($\frac{3}{2}^-$)	2P	3172		6430	
1($\frac{3}{2}^-$)	3P	3486		6742	
1($\frac{3}{2}^-$)	4P	3768		7009	
1($\frac{3}{2}^-$)	1P	2773	2766.6(2.4)?	6087	
1($\frac{3}{2}^-$)	2P	3151		6423	
1($\frac{3}{2}^-$)	3P	3469		6736	
1($\frac{3}{2}^-$)	4P	3753		7003	
1($\frac{3}{2}^-$)	1P	2789		6084	

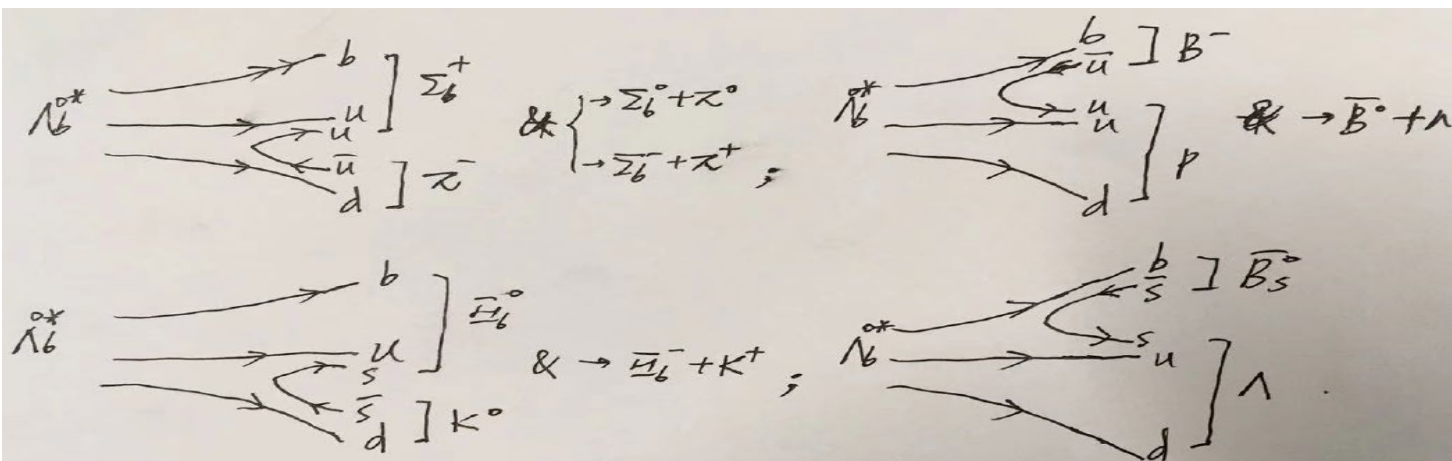


Strong decay of excited states: Branching ratios

- 3P_0 model: $A \rightarrow B + C$ via creating a $q\text{-}\bar{q}$ pair of $J^{PC}=0^{++}$
 → **Branching Ratio** $\propto$ # of possible diagrams once a decay channel opens up



$$\begin{aligned} \text{BR}(B^{*-} \rightarrow B^{-} + \pi^0) &= 1/(1+1+1/3) = 43\%; \\ \text{BR}(B^{*-} \rightarrow \bar{B}_s^0 + \pi^0) &= 1/3/(1+1+1/3) = 14\%; \\ \text{BR}(B_s^{*-} \rightarrow B^{-} + K) &= 1/(1+1+1/3) = 43\% \end{aligned}$$



$$\begin{aligned} \text{BR}(\Lambda_b^{0*} \rightarrow \Lambda_b^0 + 2\pi) &= 3/(3+2+2*1/3+1/3) = 54\% \\ \text{BR}(\Lambda_b^{0*} \rightarrow B^{-} + p) &= 1/(3+2+1/3+2*1/3) = 16\% \\ \text{BR}(\Lambda_b^{0*} \rightarrow \Xi_b + K) &= 2/3/(3+2+1/3+2*1/3) = 11\% \\ \text{BR}(\Lambda_b^{0*} \rightarrow \bar{B}_s^0 + \Lambda) &= 1/3/(3+2+1/3+2*1/3) = 6\% \end{aligned}$$

Ground-state b-hadron densities/ratios

- total density & production fractions of ground state b-hadrons @ $T_H=170$ MeV

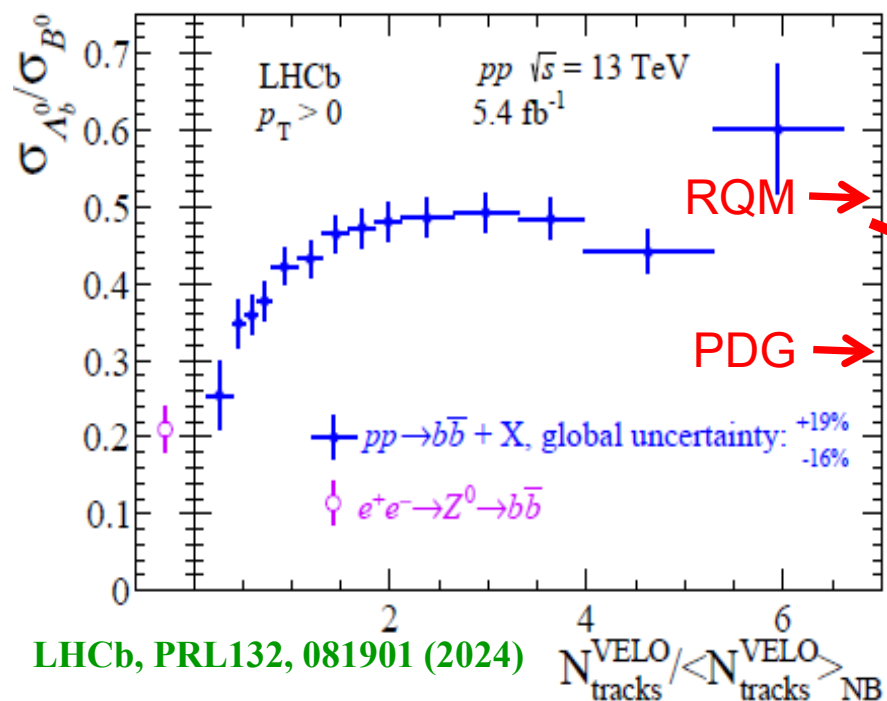
$$n_\alpha = n_\alpha^{\text{primary}} + \sum_i n_i^{\text{primary}} \cdot BR(i \rightarrow \alpha)$$

n_α ($\cdot 10^{-12} \text{ fm}^{-3}$)	B^-	$\bar{B}^0$	$\bar{B}_s^0$	Λ_b^0	$\Xi_b^{0,-}$	Ω_b^-
PDG	<u>1.0094</u>	1.0089	0.29308	<u>0.31591</u>	0.10097	0.002341
RQM	<u>1.2045</u>	1.2041	0.32513	<u>0.61702</u>	0.19548	0.0063204

$$f_u + f_d + f_s + f_{\Lambda_b^0} + f_{\Xi_b^{0,-}} + f_{\Omega_b^-} = 1$$

f_α	B^-	$\bar{B}^0$	$\bar{B}_s^0$	Λ_b^0	$\Xi_b^{0,-}$
PDG	0.3697	0.3695	0.1073	0.1157	0.03698
RQM	0.3391	<u>0.3389</u>	0.09152	<u>0.1737</u>	0.05503

→ Agree with Tevatron p-pbar



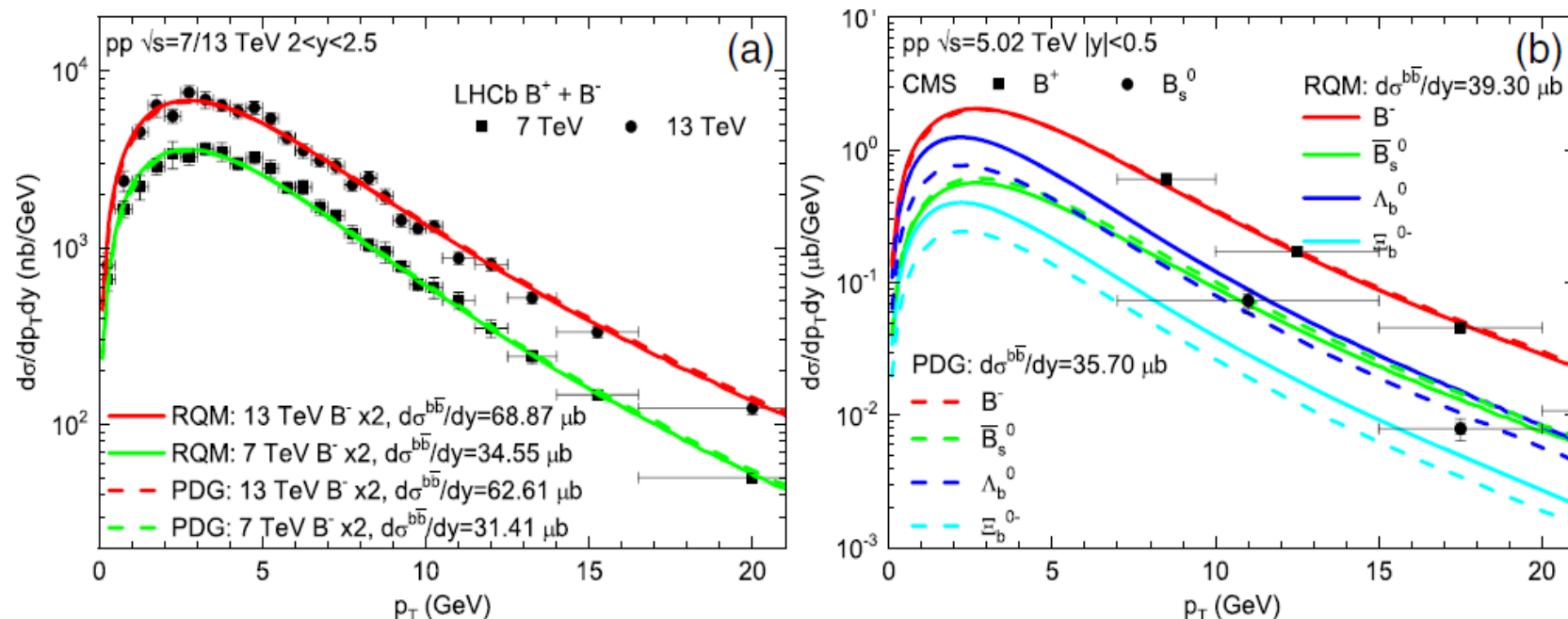
- production ratios at large $dN_{ch}/d\eta$:
RQM favored!

r_α	$\bar{B}^0/B^-$	$\bar{B}_s^0/B^-$	<u>Λ_b^0/B^-</u>	$\Xi_b^{0,-}/B^-$
PDG	0.9995	0.2904	<u>0.3129</u>	0.1000
RQM	0.9994	0.2699	<u>0.5122</u>	0.1623

Fragmentation & p_T -spectra

- FONLL b-quark p_t -spectrum + fragmentation into **all primary** states + decay simulations
 $\rightarrow$ ground-state b-hadrons p_T -spectra: $z = p_T/p_t$

$$D_{b \rightarrow H_b}(z) \propto z^\alpha (1-z), \quad \left\{ \begin{array}{l} \text{weight} \propto \text{primary density (relative chemical equilibrium)} \\ \alpha_B = 45, \alpha_{B_s} = 25, \alpha_{\text{baryon}} = 8 \text{ to tune the slope of spectra} \end{array} \right.$$



- Fitting meson spectra $\rightarrow$ **predicting** baryon & total $d\sigma^{b\bar{b}}/dy = 39.3 \mu\text{b}$ for 5.02 TeV mid-y based on SHM chemistry $\rightarrow$ **baseline for b-hadron production in Pb-Pb collisions**

**System size ($dN_{ch}/d\eta$) dependence of
bottom hadro-chemistry in pp collisions**

**Canonical Ensemble
Statistical Hadronization Model**

Canonical ensemble (CE) SHM

- **Canonical ensemble partition function**: **strict conservation** of quantum charges (electric charge, baryon-number, strangeness, charm-, bottom-number)

$$Z(\vec{Q}) = \int_0^{2\pi} \frac{d^5\phi}{(2\pi)^5} e^{i\vec{Q}\cdot\vec{\phi}} \exp\left[\sum_j \gamma_s^{N_{sj}} \gamma_c^{N_{cj}} \gamma_b^{N_{bj}} e^{-i\vec{q}_j\cdot\vec{\phi}} z_j\right] \quad \vec{Q} = (Q, N, S, C, B)$$

$$z_j = (2J_j + 1) \frac{V T_H}{2\pi^2} m_j^2 K_2\left(\frac{m_j}{T_H}\right)$$

correlation volume ~ system size

- Primary hadron yield: CE vs GCE

$$\langle N_j \rangle^{CE} = \gamma_s^{N_{sj}} \gamma_c^{N_{cj}} \gamma_b^{N_{bj}} z_j \frac{Z(\vec{Q} - \vec{q}_j)}{Z(\vec{Q})}$$

$$= \langle N_j \rangle^{GCE} \frac{Z(\vec{Q} - \vec{q}_j)}{Z(\vec{Q})}$$

chemical factor <1:
canonical suppression for
charged hadron with $\vec{q}_j \neq 0$

- E.g. exact baryon-number conservation requires: simultaneous creation of a pair of baryon and antibaryon → **energy-expensive** $\exp(-2m_N/T_H)$
→ **canonical suppression** for baryon production

Canonical suppression: chemical factors

CF	$V_C=5 \text{ fm}^3$	10	20	30	50	100	200
$\bar{B}^0$	0.0097194	0.023927	0.058660	0.094845	0.16493	0.32591	0.56988
B^-	0.0078259	0.021863	0.056893	0.093168	0.16331	0.32438	0.56858
$\bar{B}_s^0$	0.0039920	0.013624	0.045935	0.082725	0.15364	0.31546	0.56101
Λ_b^0	0.0049325	0.014844	0.047305	0.084415	0.15574	0.31768	0.56300
Ξ_b^{0-}	0.0021863	0.0089128	0.037336	0.073498	0.14477	0.30720	0.55402
Ω_b^-	0.0004649	0.0030092	0.019475	0.047296	0.11221	0.27231	0.52265
$\bar{B}_s^0/\bar{B}^0$	0.41072	0.56939	0.78307	0.87221	0.93155	0.96793	0.98443
$\Lambda_b^0/\bar{B}^0$	0.50749	0.62039	0.80643	0.89003	0.94427	0.97474	0.98793
$\Xi_b^{0-}/\bar{B}^0$	0.22494	0.37250	0.63648	0.77493	0.87776	0.94259	0.97217

At a small volume/system size,

- CF of $\mathbf{B}_s$ & $\mathbf{\Lambda}_b < \mathbf{B}$, canonical strangeness & baryon suppression
- CF of $\mathbf{\Omega}_b < \mathbf{\Xi}_b < \mathbf{\Lambda}_b$, increasing strangeness content despite common baryon

Canonical suppression: chemical factors

CF	$V_C=5 \text{ fm}^3$	10	20	30	50	100	200
$\bar{B}^0$	0.0097194	0.023927	0.058660	0.094845	0.16493	0.32591	0.56988
B^-	0.0078259	0.021863	0.056893	0.093168	0.16331	0.32438	0.56858
$\bar{B}_s^0$	0.0039920	0.013624	0.045935	0.082725	0.15364	0.31546	0.56101
Λ_b^0	0.0049325	0.014844	0.047305	0.084415	0.15574	0.31768	0.56300
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$\Xi_b^{0-}/\bar{B}^0$	0.22494	0.37250	0.63648	0.77493	0.87776	0.94259	0.97217

As volume/system size increases,

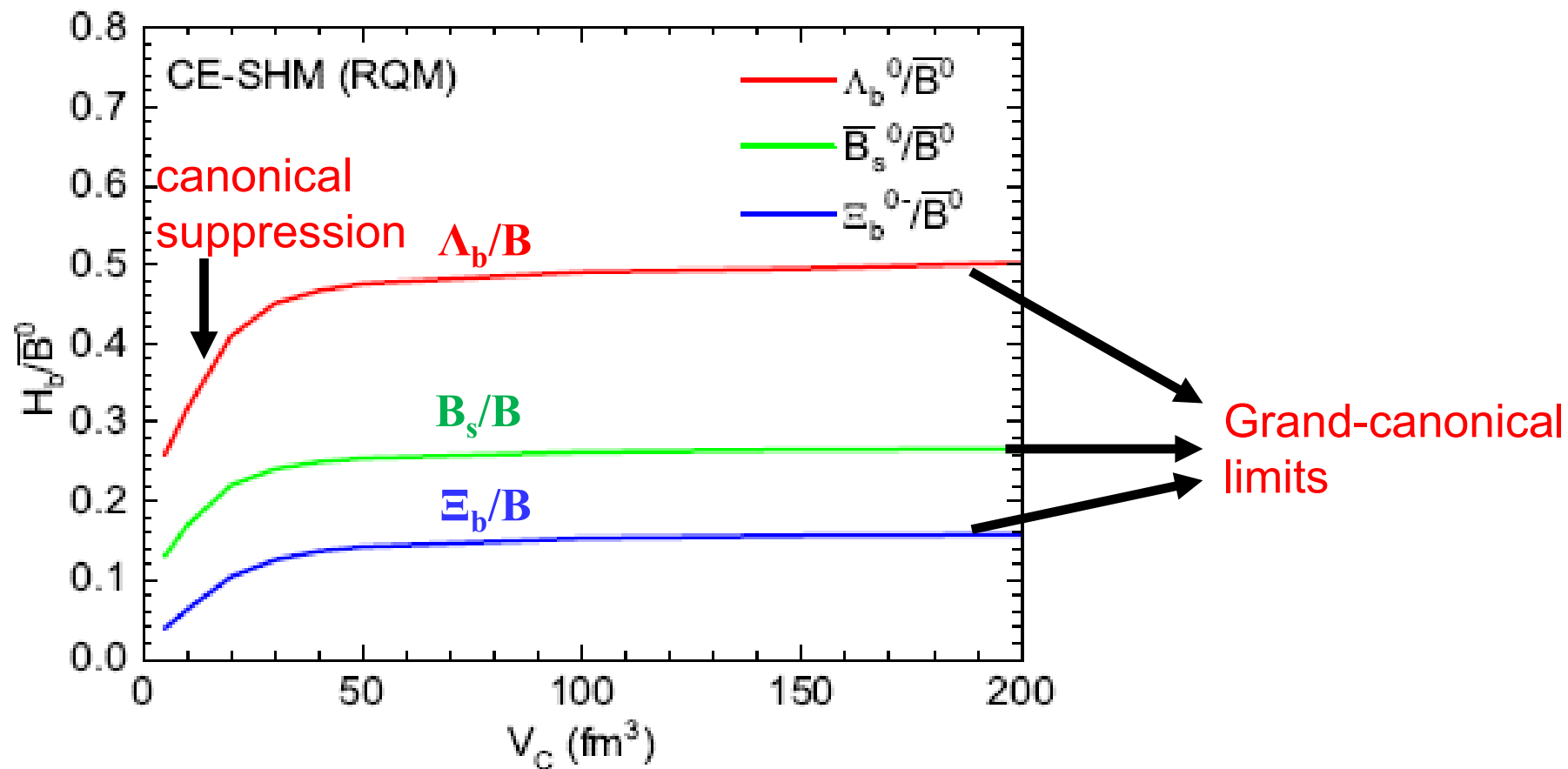
- canonical strangeness & baryon suppression attenuates
- same residual CF at large V: common canonical bottom number suppression

Ground-state b-hadron densities with feeddowns

$n_\alpha (\cdot 10^{-5} \text{ fm}^{-3})$	$V_C=5 \text{ fm}^3$	10	20	30	50	100	200	GCE
$\bar{B}^0$	1.1220	2.7920	6.9508	11.313	19.759	39.148	68.534	120.41
B^-	0.96934	2.6261	6.8105	11.181	19.635	39.038	68.452	120.45
$\bar{B}_s^0$	0.14641	0.47267	1.5299	2.7242	5.0273	10.285	18.263	32.513
Λ_b^0	0.29886	0.90201	2.8845	5.1551	9.5210	19.435	34.453	61.702
Ξ_b^{0-}	0.043883	0.17479	0.72393	1.4247	2.8132	5.9882	10.818	19.548
Ω_b^-	0.00028060	0.0018164	0.011755	0.028549	0.067730	0.16437	0.31548	0.63204
$\bar{B}_s^0/\bar{B}^0$	0.13049	0.16929	0.22010	0.24080	0.25443	0.26273	0.26648	0.27002
$\Lambda_b^0/\bar{B}^0$	0.26635	0.32307	0.41499	0.45568	0.48186	0.49644	0.50271	0.51243
$\Xi_b^{0-}/\bar{B}^0$	0.039110	0.062602	0.10415	0.12594	0.14238	0.15296	0.15785	0.16235

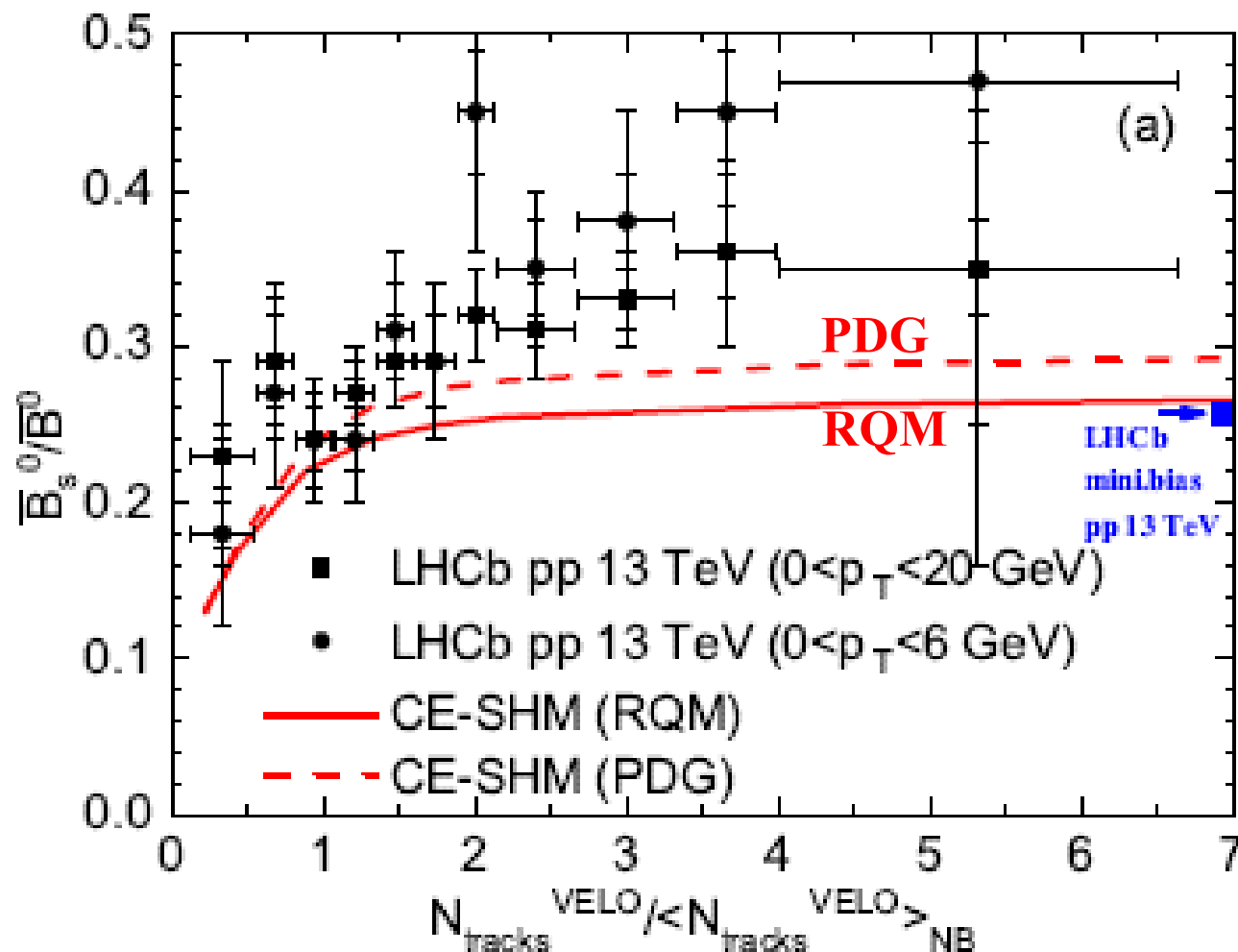
- As volume/system size reduces, $\mathbf{B}_s/\mathbf{B}$, $\mathbf{\Lambda}_b/\mathbf{B}$ suppressed by a factor 2; $\mathbf{\Xi}_b/\mathbf{B}$ suppression stronger, two-fold role of baryon + strangeness
- All ratios tend to the corresponding GCE-SHM values at large system size

Ground-state b-hadron ratios vs Volume



- As volume/system size reduces, B_s/B , Λ_b/B suppressed by a factor 2; Ξ_b/B suppression stronger, two-fold role of baryon + strangeness
- All ratios tend to the corresponding GCE-SHM values at large system size

B_s^0/B vs $dN_{ch}/d\eta$



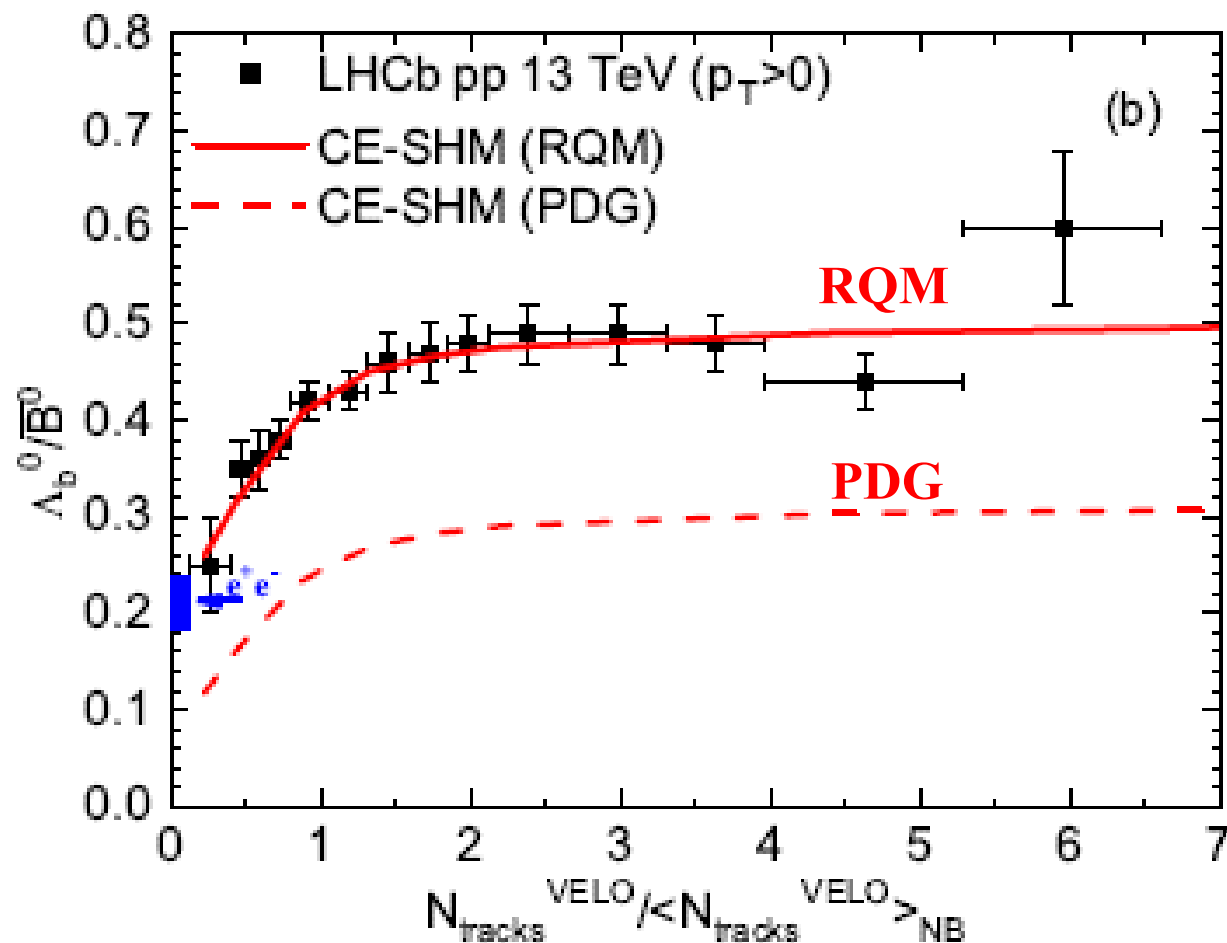
$$V_C / \langle V_C \rangle = N_{\text{tracks}}^{\text{VELO}} / \langle N_{\text{tracks}}^{\text{VELO}} \rangle_{\text{NB}}$$

$$\langle V_C \rangle = 22.6 \text{ fm}^3$$

Data taken from LHCb Collab.,
PRL131 (2023) 061901

- B_s^0/B vs $dN_{ch}/d\eta$ increasing from small multiplicity to saturation at large size
- RQM a bit smaller than PDG

Λ_b^0/B vs $dN_{ch}/d\eta$



$$V_C / \langle V_C \rangle = N_{tracks}^{VELO} / \langle N_{tracks}^{VELO} \rangle_{NB}$$

$$\langle V_C \rangle = 22.6 \text{ fm}^3$$

Data taken from LHCb Collab.,
PRL132 (2024) 081901

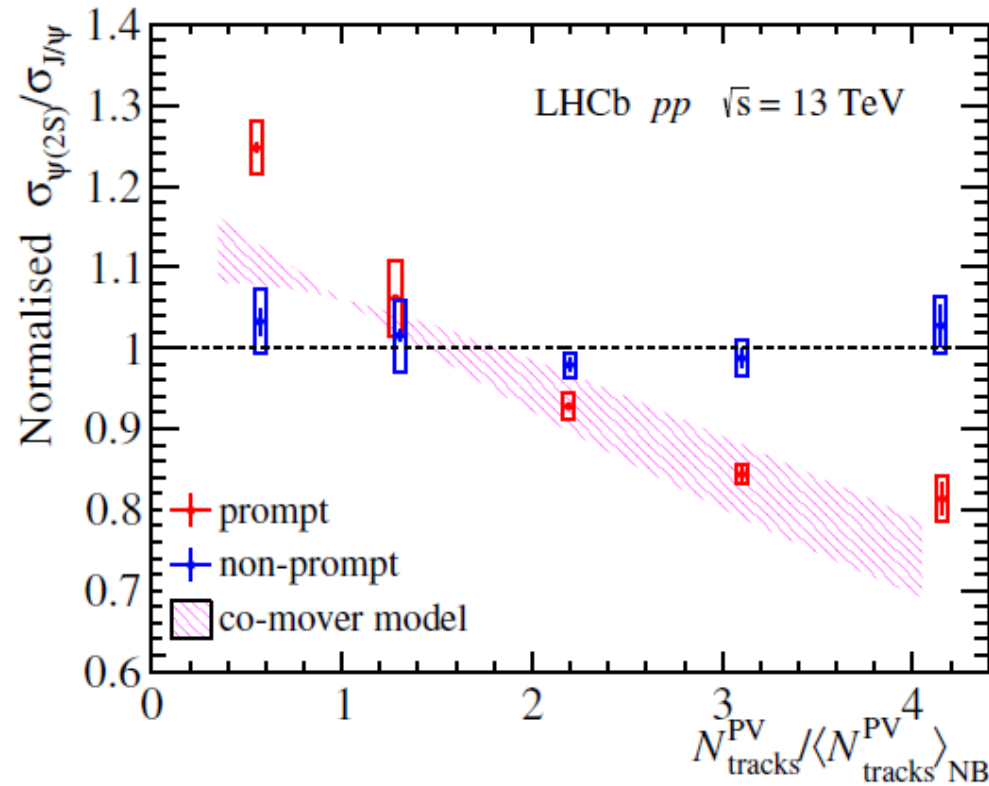
- Λ_b^0/B vs $dN_{ch}/d\eta$ increasing from e^+e^- value with small multiplicity to saturation/GCE limit at large size
- RQM strongly favored by data

Summary: b-quark hadronization

- Statistical hadronization of b-quarks in high-energy pp collisions:
grand-canonical → **canonical ensemble**
→ bottom hadro-chemistry well described in mini.bias & system-size scan
- role of **many “missing” heavy-baryons** highlighted
→ awaiting discovery!
- Heavy quark hadronization: **non-universal**, but environment dependent!



Back-up: $\psi(2S)/J/\psi$ vs $dN_{ch}/d\eta$ in pp collisions



LHCb collab.
JHEP05(2024)243