QCD Factorization for Radiative Leptonic B-Meson Decays

魏焰冰

华中师范大学

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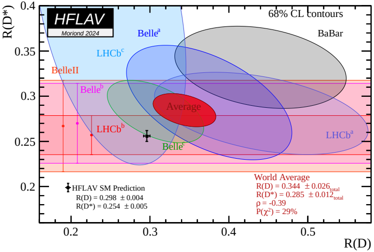
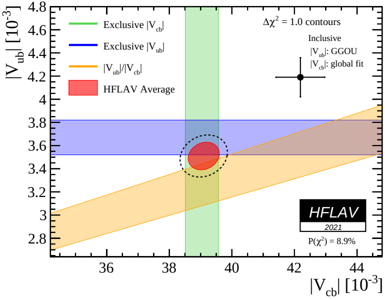
Beneke, König, Ji and **YBW**, Ongoing

B Physics



New physics beyond the SM

- Direct search: new particles
- Indirect search: flavour physics
  CPV, $R(D^{(*)})$, $|V_{ub}|$, $|V_{cb}|$, $\cdots$



- BaBar, Belle
- LHC, Belle-II
- HL-LHC

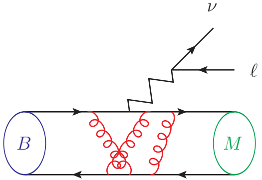
HQET LCDA of B meson

Light-cone distribution amplitudes (LCDA)

$$\langle H_v|\bar{h}_v(0)\not{n}_+\gamma^5[0,tn_+]q_s(tn_+)|0\rangle = -i\,\widetilde{f}_H(\mu)\,n_+\cdot v\int_0^\infty d\omega\, e^{i\omega t n_+\cdot v}\varphi_+(\omega)$$

Most important long-distance function in exclusive B decays

- Non-leptonic decays: $B\to\pi\pi$, $B\to\pi K\cdots$
- Semi-leptonic decays: $B\to D^{(*)}\ell\nu$, $B\to K^{(*)}\ell\nu\,\cdots$
- Radiative decay: $B\to\gamma\ell\nu$, $B\to\gamma\gamma\,\cdots$



Non-perturbative LCDA of hadrons

LCDA: non-perturbative physics

- Extract from the experiments: clean process
  - $B \to \gamma \ell \nu$ decay [Belle, 18']

$$F_{V/A,\,\mathrm{LP}} = \frac{Q_u m_B}{n \cdot p} \tilde{f}_B(\mu) \int_0^\infty \frac{d\omega}{\omega} T(\omega, \mu) \phi_B^+(\omega, \mu)$$

  - Rare decay $W^- \to B^- \gamma$ [Grossman, König and Neubert, 15']

- Calculate with LQCD: (LaMET [Ji, 13'])
  quasi DA $\to$ QCD LCDA [Han, et. al. 24']

QCD LCDA $\to$ HQET LCDA [Beneke, Finauri, Vos, and **YBW** 23'], [Deng, Wang, **YBW** and Zeng 24']

Decay amplitude of the $B^- \to \gamma\ell\bar{\nu}_\ell$ process

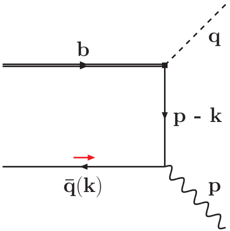
$$A(B^- \to \gamma\ell\bar{\nu}_\ell) = \frac{G_F V_{ub}}{\sqrt{2}} \langle (\ell\bar{\nu}_\ell)(q)\gamma(p)|\bar{\ell}\,\gamma^\nu(1-\gamma_5)\nu_\ell\,\bar{u}\gamma_\nu(1-\gamma_5)b|B^-(p_B)\rangle$$

Theoretically clean: factorization properties of exclusive B decays, extract important non-perturbative parameters of B meson.

[Korchemsky, Pirjol and Yan 99'; Descotes-Genon and Sachrajda 02'; Lunghi, Pirjol and Wyler 02';
Beneke, Rohrwild 11': NLL resummation and NLP correction

Braun and Khodjamirian 12'; Wang and Shen 18'; Beneke, Braun, Ji and **YBW** 18'; Galda and Neubert 20'; Shen, **YBW**, Zhao and Zhou 20'; Cui, Shen, Wang and **YBW** 23';$\cdots$]

$E_\gamma \sim \mathcal{O}(m_b)$, light-cone vectors n and $\bar{n}$: $n^2 = \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$.
Power-expansion parameter $\lambda = \mathcal{O}(\sqrt{\Lambda_{\mathrm{QCD}}/m_b})$.



We have momenta:

- hard p_b: $(n \cdot p_b, \bar{n} \cdot p_b, p_{b\perp}) \sim m_b(1,1,1)$
- soft k: $m_b(\lambda^2, \lambda^2, \lambda^2)$
- collinear p: $m_b(1, \lambda^4, \lambda^2)$
- hard-collinear $p - k$: $m_b(1, \lambda^2, \lambda)$

Two perturbative scales: hard scale $\mathcal{O}(m_b)$, hard-collinear scale $\mathcal{O}(\lambda m_b)$

$$F_{V/A,\,\mathrm{LP}} = \frac{Q_u m_B}{n \cdot p} \tilde{f}_B(\mu)\, C(n \cdot p, m_b, \mu) \int_0^\infty \frac{d\omega}{\omega} J(n \cdot p\, \omega, \mu) \phi_B^+(\omega, \mu)$$

$B^- \to \gamma \ell \bar{\nu}_\ell$ form factors at LP

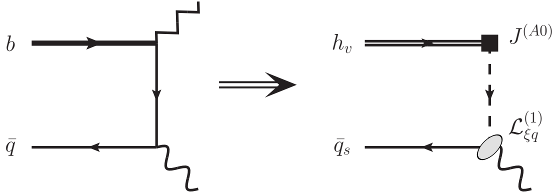
- Operators in $\mathrm{SCET_I}$ scale as λ^4

$$\mathrm{T}\{\bar{\xi}_{hc}(1+\gamma_5)\gamma_\nu h_v,\ \mathcal{L}^{(1)}_{\xi q}\}$$

Translating $A^{\mathrm{em}}_{\perp hc}$ to $A^{\mathrm{em}}_{\perp c}$ will generate an additional λ factor.

- Factorizable $\mathrm{SCET_{II}}$ operator scale as λ^6

$$O_{\mathrm{LP}} = \bar{q}_s \frac{\bar{\not{n}}}{2} \not{A}^{\mathrm{em}}_{\perp c} \frac{1}{i\bar{n}\cdot\overleftarrow{\partial}} \gamma_\nu (1-\gamma_5) h_v$$

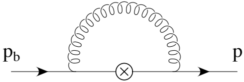


QCD to SCET$_\mathrm{I}$: hard function

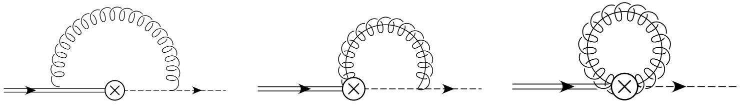
From QCD to SCET$_\mathrm{I}$

$$\bar{u}(1+\gamma_5)\gamma_\nu b \Rightarrow \int d\hat{s}\, \tilde{C}(\hat{s}) \, [(\bar{\xi}_{hc} W_{hc})(sn)(1+\gamma_5)\gamma_\nu h_v]$$

QCD diagram at NLO



SCET$_\mathrm{I}$ diagrams at NLO



C at NNLO

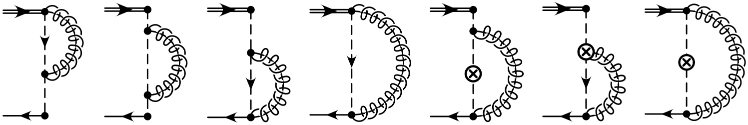
[Bonciani and Ferroglia 08'; Asatrian, Greub and Pecjak 08'; Beneke, Huber and Li 08'; Bell 08']

SCET$_\mathrm{I}$ to SCET$_\mathrm{II}$: jet function

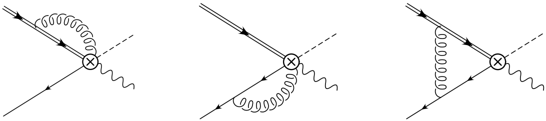
From SCET$_\mathrm{I}$ to SCET$_\mathrm{II}$

$$\mathrm{T}\{(\bar{\xi}_{hc} W_{hc})(1+\gamma_5)\gamma_\nu h_v,\, \mathcal{L}^{(1)}_{\xi q}\} \Rightarrow \int \frac{d\omega}{\omega} J(\omega)[(\bar{q}_s Y_s)(t\bar{n})\frac{\not{\bar{n}}}{2} \not{A}^{\mathrm{em}}_{\perp c}\gamma_\nu(1-\gamma_5) Y_s^\dagger h_v]_{\mathrm{FT}}$$

SCET$_\mathrm{I}$ diagrams at NLO



SCET$_\mathrm{II}$ diagrams at NLO



Jet function at NNLO: same as $H \to \gamma\gamma$ [Liu and Neubert 20']

$B^- \to \gamma\ell\bar{\nu}_\ell$ form factors at NNLO

$$F_{V/A,\,\mathrm{LP}} = \frac{Q_u m_B}{n\cdot p}\tilde{f}_B(\mu)\, C(n\cdot p, m_b, \mu)\int_0^\infty \frac{d\omega}{\omega} J(n\cdot p\,\omega, \mu)\phi_B^+(\omega,\mu)$$

Factorization scale $\mu \to$ hard-collinear scale

Large Logs

- $\ln(m_b/\mu)$, $\ln(n\cdot p/\mu)$ from C and $\tilde{f}_B$
- $\ln(\Lambda_{\mathrm{QCD}}/\mu)$ from ϕ_B^+ (two-loop γ [Braun, Ji and Manashov 19'])

Numerical result [Galda and Neubert 20']

$$I_{\mathrm{NLO}} = \frac{1}{\lambda_B}\left[0.731 + 0.035\,\sigma_2 - 0.003\,\sigma_3 + \ldots\right],$$

$$I_{\mathrm{NNLO}} = \frac{1}{\lambda_B}\left[0.664\,^{+0.026}_{-0.040} + 4.36\,^{+0.17}_{-0.46}\cdot 10^{-2}\,\sigma_2 + 0.35^{+2.97}_{-1.99}\cdot 10^{-3}\,\sigma_3 + \ldots\right],$$

NNLO result is about 10% smaller than the NLO result.

Subleading power corrections

- New phenomena will appear

$$F_{V/A} = \underbrace{\frac{Q_u\, m_B}{2E_\gamma} R(E_\gamma)}_{\text{LP}} + \left[\xi(E_\gamma) \pm \left(\frac{Q_b\, m_B f_B}{2E_\gamma m_b} + \frac{Q_u\, m_B f_B}{(2E_\gamma)^2}\right)\right]$$

  - Local symmetry breaking terms [Beneke and Rohrwild 11']
  - End-point singularity: [Liu and Neubert 19'; Beneke et. al. 20']
  - Renormalon ambiguity
  - Generalized LCDA [Qin, Shen Wang and Wang 22']
- Power corrections are numerically important in B decays

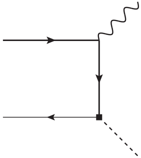
$$\Lambda_{\text{QCD}}/m_b \quad \sim \quad \alpha_s(\mu)/\pi$$

$$\text{NLP@LO} \sim \text{LP@NLO}, \qquad \text{NNLP@LO} \sim \text{NLP@NLO} \sim \text{LP@NNLO}$$

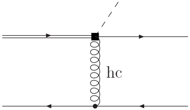
Subleading power corrections

NLP SCET$_{\mathrm{II}}$ operators scale as λ^8 [Beneke, König Ji and **YBW** Ongoing]

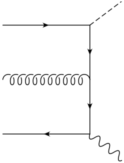
- Power suppressed operators



  - Local SCET$_{\mathrm{II}}$ operator $\bar{q}_s A^{\mathrm{em}}_{\perp c} h_v$

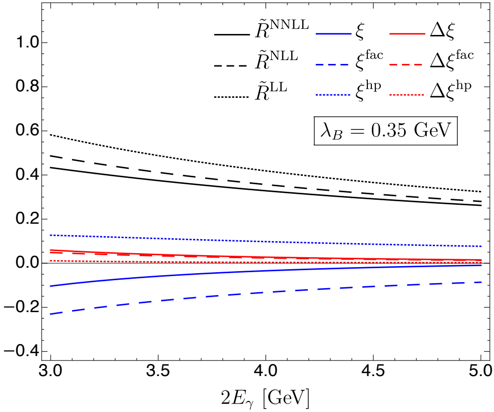


  - SCET$_{\mathrm{I}}$ operators $\bar{\xi}_{hc} A_{\perp hc} h_v$:
    hadronic structure of the photon



- LP operator with power suppressed Lagrangian

NLP numerical results



[Cui, Shen, Wang and **YBW** 23']

$$F_{V/A} = \tilde{R} + \xi \pm \Delta\xi$$

$\tilde{R}$: LP

ξ: symmetry conserving

$\Delta\xi$: symmetry breaking

$\xi^{\rm hp}$, $\Delta\xi^{\rm hp}$:
hadronic photon

NLP contributions bring about $\mathcal{O}(20\%)$ correction

Summary

※ $B^- \to \gamma \ell \bar{\nu}_\ell$ form factors F_V and F_A at LP

- Hrad function: QCD to $\mathrm{SCET_I}$
- Jet function: $\mathrm{SCET_I}$ to $\mathrm{SCET_{II}}$

※ Subleading power corrections: numerical results

Thank you!