Heavy Meson Profiles from First-Principle: Challenges, Advances, and Implementations

Qi-An Zhang

Beihang University (BUAA)

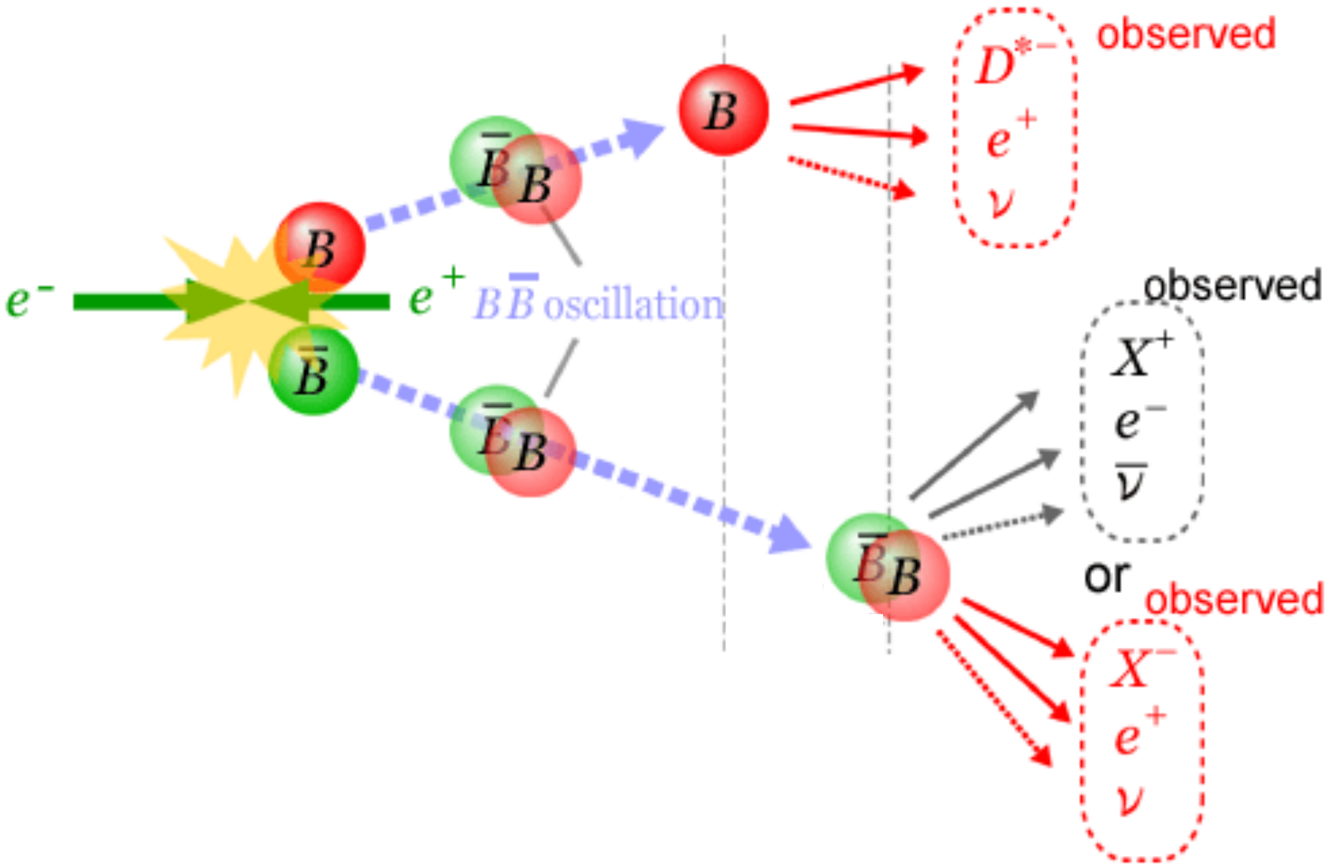
Apr. 21, 2025 @ Nanjing Normal University

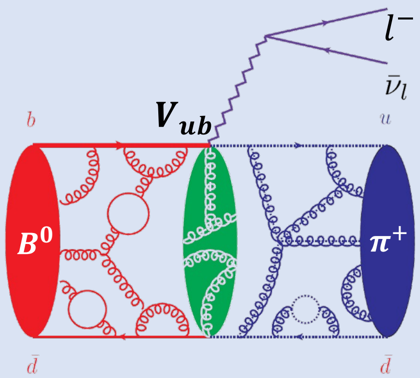
Outline

- Motivation
- Challenges in profiling the heavy mesons
- Sequential Effective Theory (SET)
- Implementing SET: Heavy Meson LCDAs & Shape Functions
- Improving SET: Power Corrections
- Extending SET: Heavy Quark Mass RGE
- Summary and Outlook

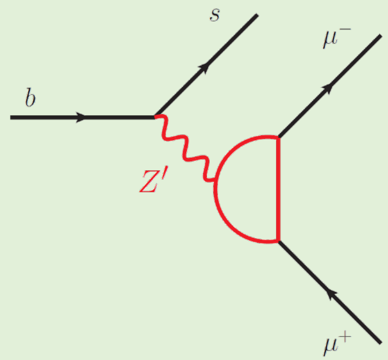
Motivation

- Rich physics in heavy meson decays:

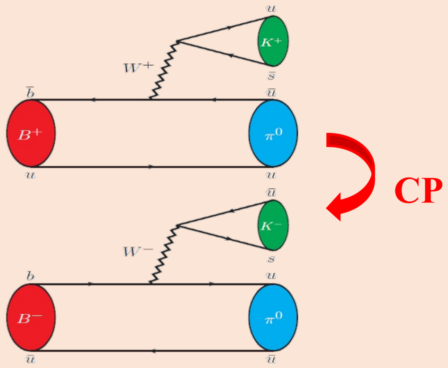




Precisely testing standard model



Indirect search for new physics



Study on CP violation

$|V_{xb}|$ Puzzle

- $|V_{xb}|$ can be measured in semileptonic B decays with **inclusive** or **exclusive** processes (leptonic probe refers to larger uncertainty)

- Long-standing "**$|V_{xb}|$ puzzle**": discrepancy between inclusive and exclusive determinations

  - From inclusive B decays

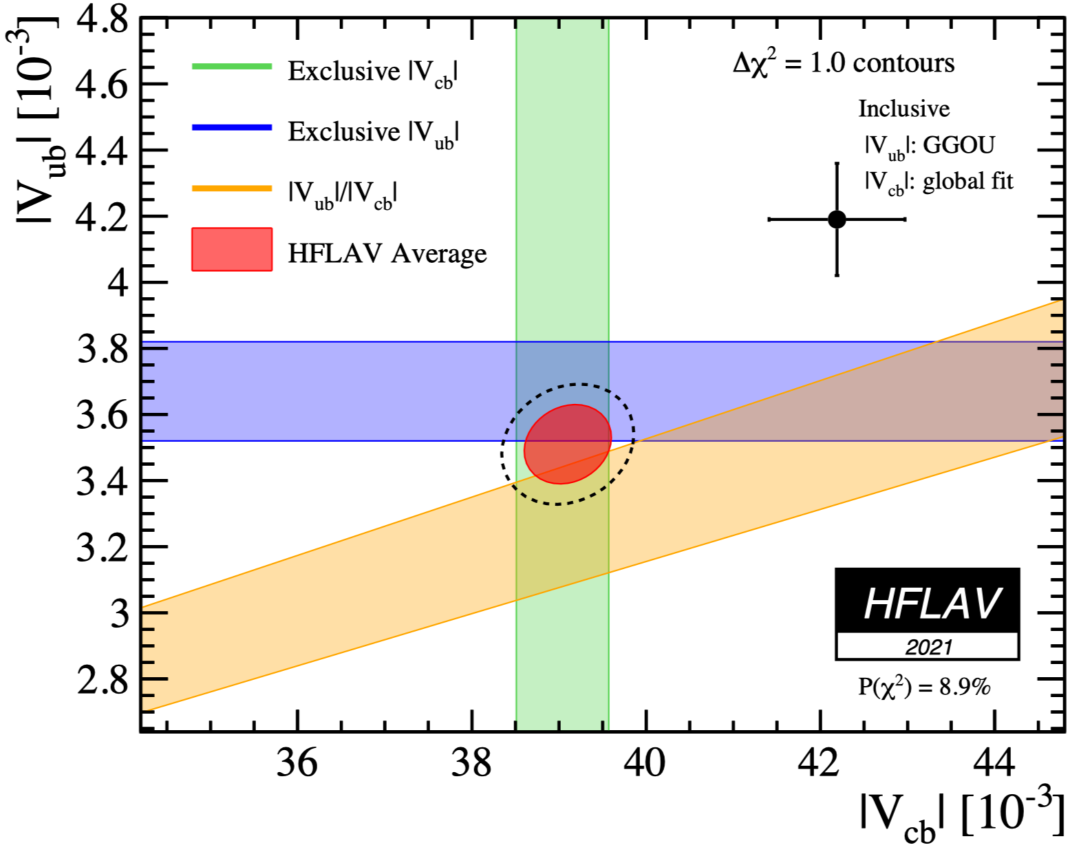
$$|V_{ub}^{\text{incl.}}| = (4.19 \pm 0.17) \times 10^{-3}$$
$$|V_{cb}^{\text{incl.}}| = (42.19 \pm 0.78) \times 10^{-3}$$

  - From exclusive B, B_s and Λ_b decays

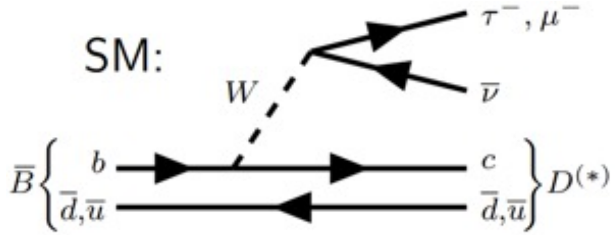
$$|V_{ub}^{\text{excl.}}| = (3.51 \pm 0.12) \times 10^{-3}$$
$$|V_{cb}^{\text{excl.}}| = (39.10 \pm 0.50) \times 10^{-3}$$

3.3σ discrepancy!

HFLAV, PRD 107, 052008 (2023)



$\mathcal{R}(D)$ and $\mathcal{R}(D^*)$



$$\mathcal{R}(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu_\ell)}$$

- **Precisely determined** both experimentally and theoretically.

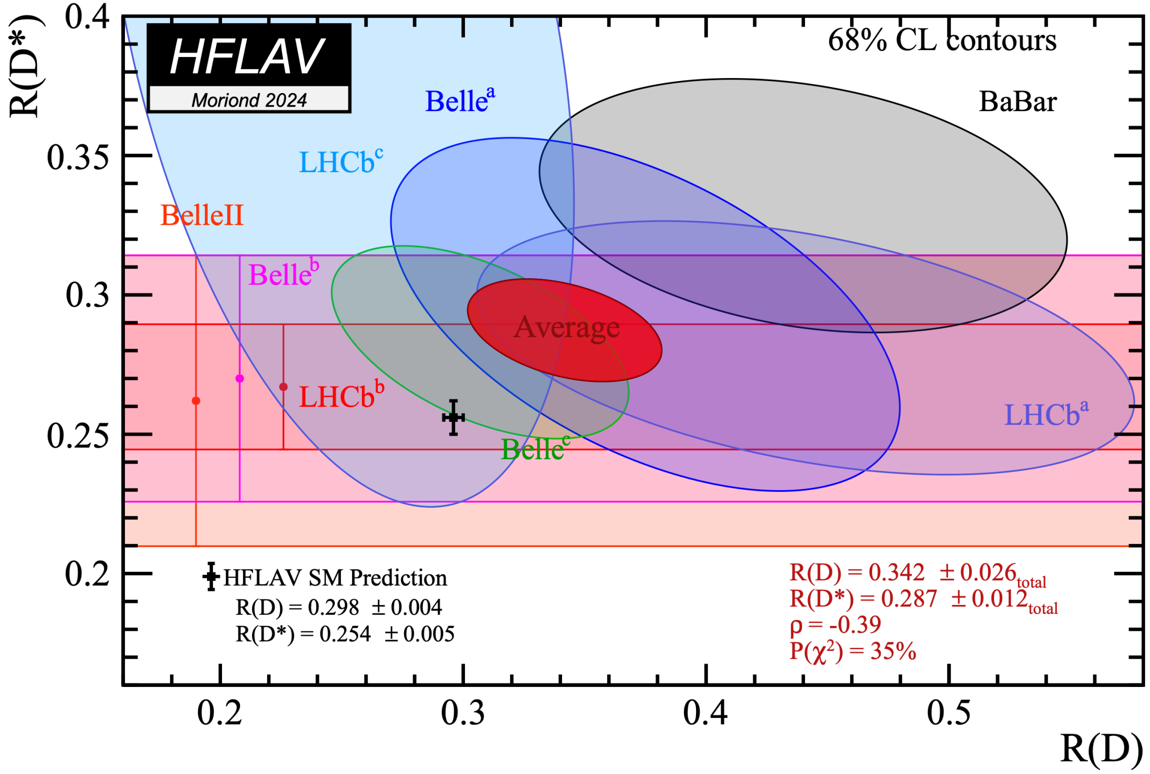
$$\mathcal{R}(D)_{\mathbf{exp}} = \mathbf{0.342 \pm 0.026}$$
$$\mathcal{R}(D^*)_{\mathbf{exp}} = \mathbf{0.287 \pm 0.012}$$

vs

$$\mathcal{R}(D)_{\mathbf{SM}} = \mathbf{0.298 \pm 0.004}$$
$$\mathcal{R}(D^*)_{\mathbf{SM}} = \mathbf{0.254 \pm 0.005}$$

- Current combined tension at the level of $\mathbf{3.31\sigma}$.

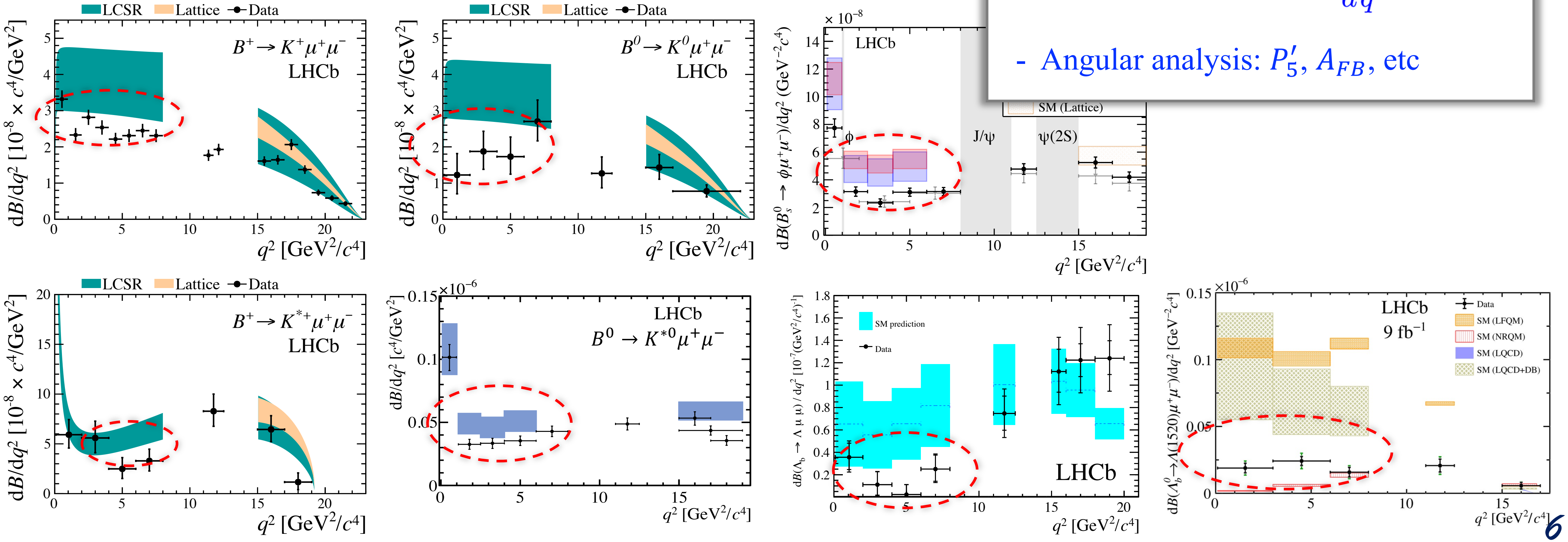
HFLAV, preliminary average for Moriond 2024



Flavor-changing neutral current (FCNC) processes: $b \to s\ell\ell$ rare decays

- Small SM amplitude $\Rightarrow$ Excellent place to search for NP
- Current experimental results show significant deviations from theoretical predictions:

- Lepton Flavor Universality: $\dfrac{\mathcal{B}(H_b \to F\mu^+\mu^-)}{\mathcal{B}(H_b \to \mathrm{F}e^+e^-)}$
- Differential BFs: $\dfrac{d\Gamma(H_b - F\ell\ell)}{dq^2}$
- Angular analysis: P_5', A_{FB}, etc



Theoretical Tools: Factorization Theorem

- Nonleptonic decays, such as $B^0/\bar{B}^0 \to \pi^+\pi^-$, are important to explore the CP violation

$$\langle \pi^+\pi^-|Q_i|\bar{B}^0\rangle = f^0_{B\to\pi}(m_\pi^2)\int_0^1 dx T_i^I(x,\mu)\phi_\pi(x,\mu) + \int d\omega dx dy T_i^{II}(\xi,x,y,\mu)\varphi_+^B(\omega,\mu)\phi_\pi(x,\mu)\phi_\pi(y,\mu)$$

Light-cone distribution amplitudes (LCDAs)

- FCNC processes, such as $B^0 \to K^*\ell\ell$, are sensitive to new physics:

$$\langle K_a^*\ell^+\ell^-|\mathcal{H}_{\rm eff}|B\rangle = T_a^I(q^2)\xi_a(q^2) + \sum_{\pm}\int_0^\infty \frac{d\omega}{\omega}\varphi_\pm^B(\omega,\mu)\int_0^1 du\phi_{K^*}(u,\mu)T^{II}_{a,\pm}(\omega,u,q^2)$$

- Semileptonic $B \to M\ell\bar{\nu}$ decays, $M = \pi, D, \ldots$, contribute to the determination of $|V_{ub}^{\rm excl}|$, $\mathcal{R}(D^{(*)})$:

$$F_i^{B\to M}(q^2) = C_i^{(A0)}(q^2)\xi_a(q^2) + \int_0^\infty \frac{d\omega}{\omega}\int_0^1 dv T_i(\ln\omega, v, q^2)\varphi_\pm^B(\omega,\mu)\phi_M(v,\mu)$$

- Inclusive decays, such as $\bar{B} \to X_s\gamma$ or $\bar{B} \to X_u\ell\bar{\nu}$, will contribute to $|V_{ub}^{\rm incl}|$:

Shape function

$$\frac{d\Gamma^{\bar{B}\to X_s\gamma}}{dx} = \Gamma(b\to s\gamma)\frac{M}{v^+}\int d\ell_+ S(\ell_+)J(P_+ - \ell_+)H(P_-), \qquad P_+ = \frac{M}{\sqrt{2}}(1-x)$$

Crucial Nonperturbative Parameters

- Light-cone distribution amplitudes (LCDAs) of light meson, contribute to the exclusive processes and describe the partonic structures of final-state light meson:

$$\phi(x,\mu) = \frac{1}{i f_H}\int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \langle H(P_H)\,|\,\bar{q}(\xi^-)\not{n}_+\gamma_5\, W_c(\xi^-,0)q(0)\,|\,0\rangle$$

- LCDAs of heavy meson with QCD fields (QCD LCDAs), contribute to the heavy meson final state of exclusive processes, such as $e^+e^- \to \boldsymbol{B\bar{B}}$, $B \to \boldsymbol{D}\ell\nu$:

$$\phi(x,\mu) = \frac{1}{i f_{H_Q}}\int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \langle H_Q(P_{H_Q})\,|\,\bar{q}(\xi^-)\not{n}_+\gamma_5\, W_c(\xi^-,0)Q(0)\,|\,0\rangle$$

- LCDAs of heavy meson with HQET fields (HQET LCDAs), contribute to the heavy meson initial state of exclusive processes, such as $\boldsymbol{B} \to D\ell\nu$, $\boldsymbol{B} \to K^*\ell\ell$:

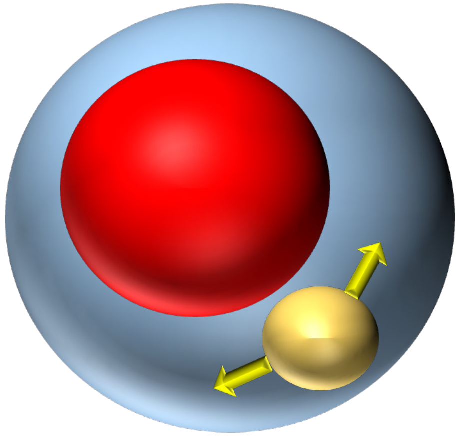
$$\varphi^+(\omega,\mu) = \frac{1}{i\,\tilde{f}_{H_Q} m_{H_Q} n_+\cdot v}\int \frac{dt}{2\pi} e^{-i\omega t n_+\cdot v} \langle 0\,|\,\bar{q}(tn_+)\not{n}_+\gamma_5\, W_c(tn_+,0)h_v(0)|\,H_Q(v)\rangle$$

- Shape functions (SFs) of heavy meson with HQET fields, contribute to the heavy meson initial state of inclusive processes, such as $\boldsymbol{\bar{B}} \to X_s\gamma$, $\boldsymbol{\bar{B}} \to X_u\ell\bar{\nu}$:

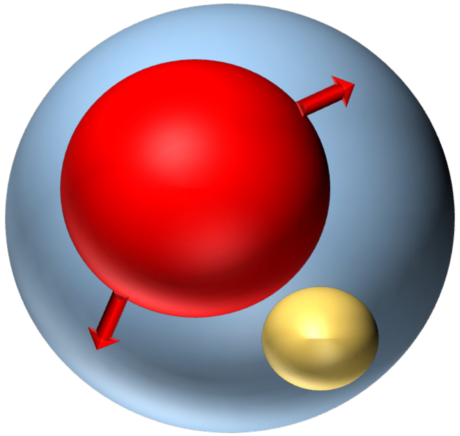
$$S(\omega,\mu) = \frac{1}{2m_{H_Q}}\int \frac{dt}{2\pi} e^{-i\omega t n_+\cdot v} \langle H_Q(v)|\bar{h}_v(tn_+,0)W_c(tn_+,0)h_v(0)|\,H_Q(v)\rangle$$

Heavy Meson Profiles

- A heavy flavor meson consists of a pair of heavy and light quarks.



LCDAs describe the momentum distribution amplitude of the light quark.



Shape function characterizes the momentum distribution function of the heavy quark.

- **Together, they provide the most essential information about the profile of heavy mesons.**

LCDAs of Light Mesons Have Been Extensively Studied

- Asymptotic form

  Chernyak, Zhitnitsky, 1977; Lepage, Brodsky, 1979; ……

- QCD Sum rules

  Chernyak, Zhitnitsky, 1982; Braun, Filyanov, 1989; ……

- Dyson-Schwinger Equation

  Chang, Cloet, et.al, 2013; Gao, Chang, et.al, 2014; ……

- Global Fits

  Cheng, et.al, 2020; Hua, Li, Lu, Wang, Xing, 2021; ……

- Models

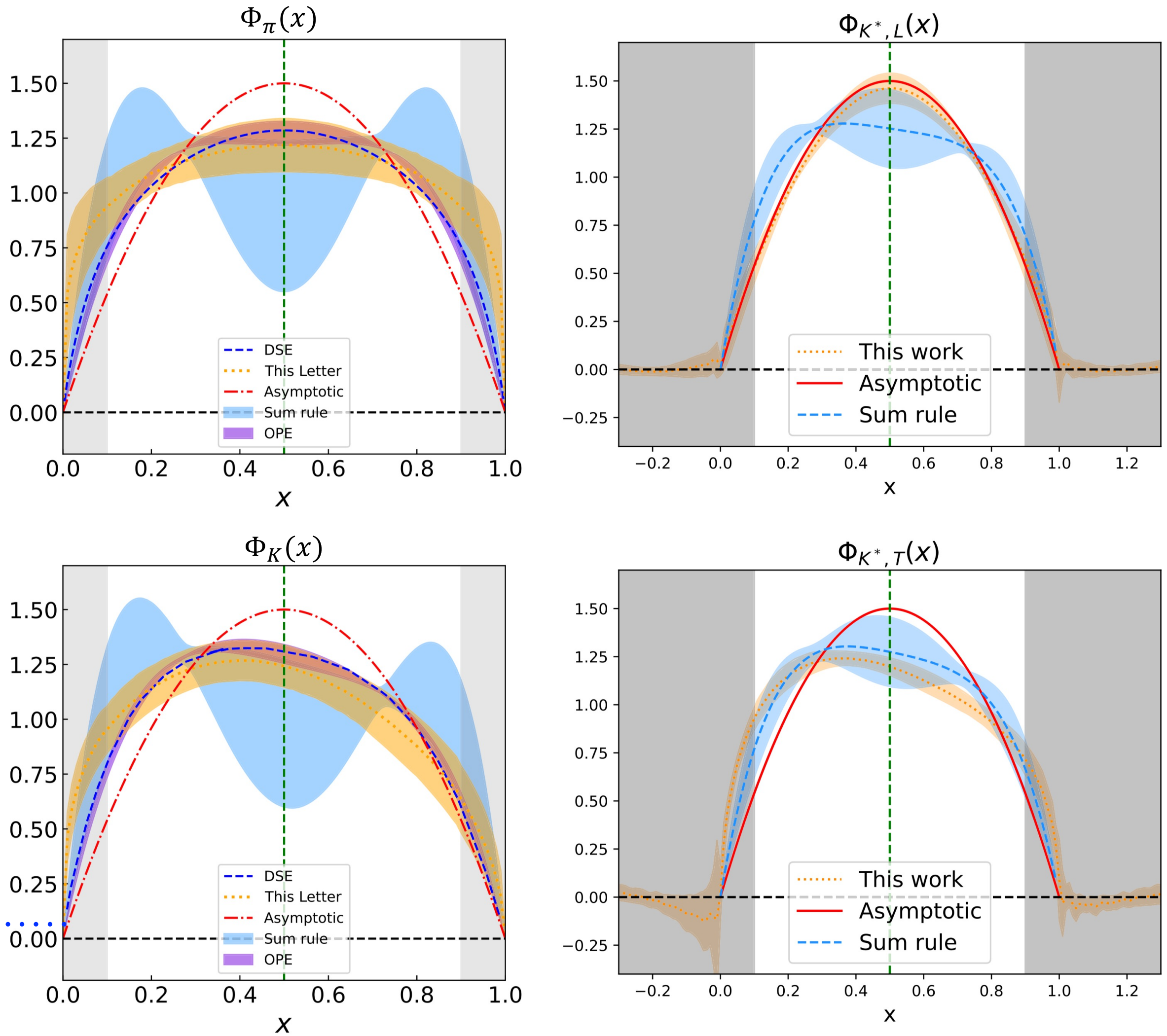
  Arriola, Broniowski, 2002; Zhong, Zhu, et.al, 2021; ……

- Lattice QCD with OPE

  Braun, Bruns, et al., 2016; RQCD collaboration, 2019, 2020; ……

- Lattice QCD with LaMET

  LP3, 2019; LPC, 2021, 2022; ……



PRL127, 062002 (2021); PRL129, 132001 (2022)

When Extending to Heavy Mesons……

- **Limited understanding** of the nonperturbative heavy meson LCDAs and SFs:

  - Only models for heavy meson LCDAs:

    Grozin, Neubert, 1997; Braun, Ivanov, Korchemsky, 2004; Beneke, Braun, Ji, Wei, 2018; ……

  - Only models for heavy meson SFs:

    Korchemsky, Sterman, 1994; Bauer, Luke, Mannel, 2001; Neubert, 2005; Lee, Ligeti, Stewart, Tackmann, 2006; ……

  - Relations between LCDAs and SFs?

    Yaouanc, Oliver, Raynal, 2008

- Uncertainties from heavy meson LCDAs **dominate** the errors in theoretical calculation.

  - *e.g.*: $B \to \pi,\ K^*$ form factors from LCSRs:

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

$$f^+_{B\to\pi}(0) = 0.122 \times \Big[1 \pm 0.07\Big|_{S_0^\pi} \pm 0.11\Big|_{\Lambda_q} \pm 0.02\Big|_{\lambda_E^2/\lambda_H^2}{}^{+0.05}_{-0.06}\Big|_{M^2} \pm 0.05\Big|_{2\lambda_E^2+\lambda_H^2}$$
$${}^{+0.06}_{-0.10}\Big|_{\mu_h} \pm 0.04\Big|_{\mu}{}^{+1.36}_{-0.56}\Big|_{\lambda_B}{}^{+0.25}_{-0.43}\Big|_{\sigma_1,\sigma_2}\Big].$$

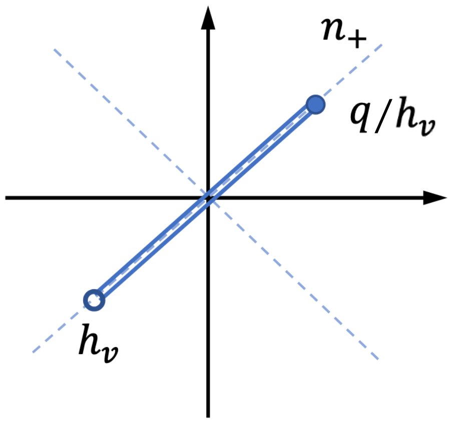
Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035

Cui, Huang, Shen, Wang, JHEP 03 (2023) 140

Challenges in profiling the heavy mesons from first-principles

- Light-like correlators containing HQET fields:

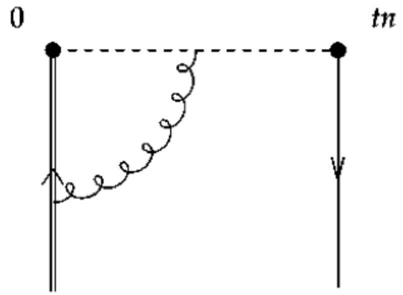
$$\langle 0|\bar{q}\, W_c\, h_v|H_Q\rangle, \qquad \langle H_Q|\bar{h}_v\, W_c\, h_v|H_Q\rangle$$



Challenge 1: **light-like correlators**

- **OPE**: Expansion into **local** operators matrix elements ⇒ QCD sum rule, Lattice QCD
- **LaMET**: From equal-time correlation functions to light-cone variables ⇒ Lattice QCD

Challenge 2: **Cusp divergence**



$$O_v^{\rm ren}(t,\mu) = \frac{4}{\hat{\epsilon}} \ln(it\mu)\, O_v^{\rm bare}(t) + \ldots \xrightarrow{t\to 0} \log 0! \quad \Rightarrow \quad \text{NO LOCAL LIMIT!}$$

OPE Breakdown……

…… the remaining issues can only be addressed by LaMET

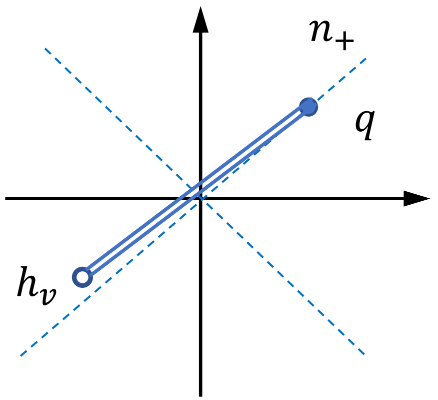
LaMET: matching from **equal-time correlators** of highly boosted hadrons to **light-cone observables**.

🤔 **Equal-time correlator?**

An intuitive approach: adopt **off light-cone Wilson line** to avoid cusp divergence

$$\langle H_Q(P_{H_Q})\,|\,\bar{q}(z)\not{n}_z\gamma_5\, W_c(z,0)h_v(0)\,|\,0\rangle \qquad \langle H_Q(v)\,|\,\bar{h}_v(z,0)W_c(z,0)h_v(0)\,|\,H_Q(v)\rangle$$

Wang, Wang, Xu, Zhao, PRD 102, 011502 (2020);
Xu, Zhang, PRD 106, 114019 (2022);
Hu, Xu, Zhao, EPJC 84, 502 (2024); ……



Need to realize the **boosted HQET fields on lattice**.

Boosted HQET on Lattice

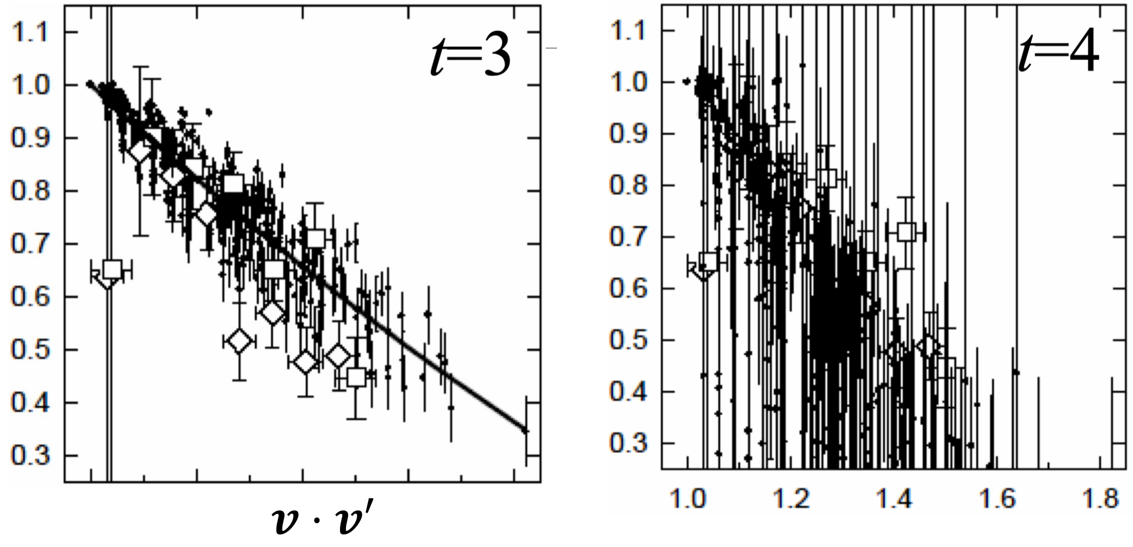
- HQET: $\mathcal{L}_{\mathrm{HQET}} = \bar{h}_v(x)\, i v\cdot D\, h_v(x)$ $\xrightarrow{\text{Discretization}}$ $\bar{h}_v(x)\,(-v_0 D_0 + i\vec{v}\cdot\vec{D})\, h_v(x)$
- The QCD field is recovered by: $\psi(x) = e^{-imv\cdot x}\dfrac{1+\not{v}}{2} h_v(x)$
- Evolution equation of HQET propagator:

$$G(x+\hat{t}) = U_4^\dagger(x)\,(1 + i\vec{v}\cdot\vec{D})G(x)$$

Significant signal-to-noise (StN) problem

e.g. Isgur-Wise function:

$\xi(v\cdot v')$



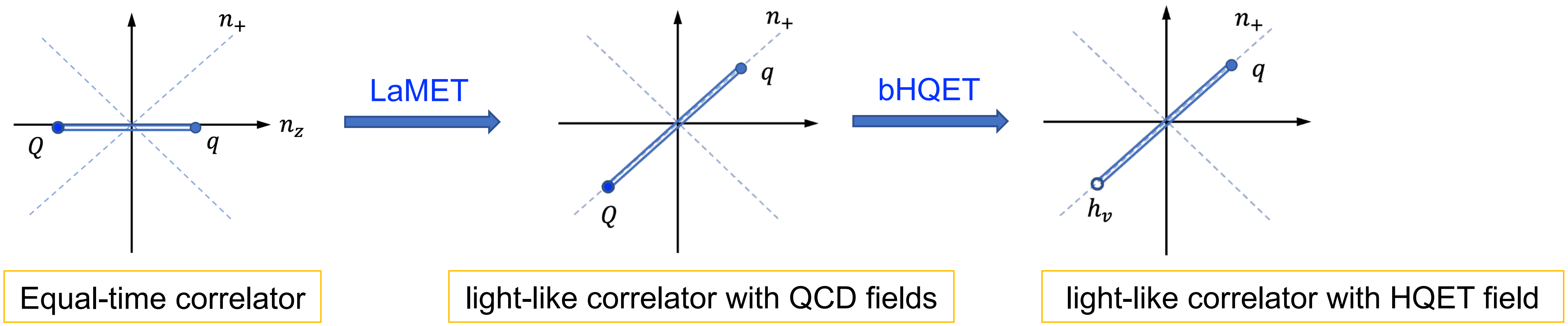
Mandula, Ogilvie, PRD 45, 2183-2187 (1992); NPB 34, 480-482 (1994)

Sequential Effective Theory (SET)

🤔 **Equal-time correlator + QCD fields?**

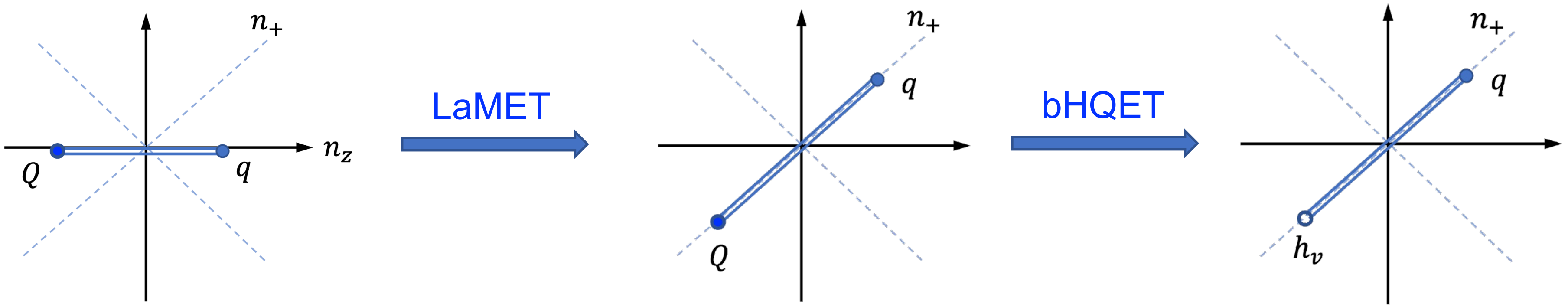
- Equal-time correlator ⇒ light-like correlator: **LaMET**
- QCD field ⇒ moving HQET field: **boosted HQET**

⇒ **A two-step factorization to combine LaMET and bHQET.**



arXiv:2403.17492 [hep-ph]; PRD111, 034503, (2025)

Sequential Effective Theory (SET)



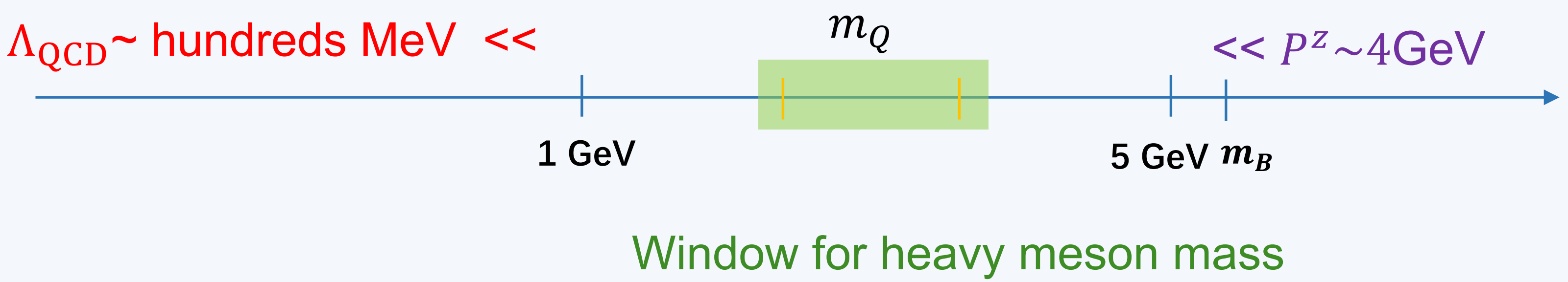
Equal-time correlator | light-like correlator with QCD fields | light-like correlator with HQET field

- **3 scales** in the equal-time correlator: $\boldsymbol{\Lambda_{\rm QCD}, m_Q, P^z}$
- Effective theories:
  - LaMET: $\Lambda_{\rm QCD}, m_{\rm Q} \ll P^z$ and integrate out P^z
  - bHQET: $\Lambda_{\rm QCD} \ll m_{\rm Q}$ and integrate out $m_{\rm Q}$

$\Rightarrow$ Introduce a hierarchy $\boldsymbol{\Lambda_{\rm QCD} \ll m_Q \ll P^z}$

SET on lattice

- **Lattice feasibility** at this stage:



Only valid for *D* mesons, rather than *B* mesons?

✓ Heavy quark flavor symmetry ensures that the HQET measurement is independent of heavy quark mass;

✓ m_Q (m_c or m_b) only contributes to the power corrections.

Implementing SET: Heavy Meson LCDAs

Quasi DA —LaMET→ **QCD LCDA** —bHQET→ **HQET LCDA**

- Step I: matching in LaMET

$$\tilde{\phi}(x, P^z) = \int_0^1 dy C\left(x, y, \frac{\mu}{P^z}\right)\phi(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

matching kernel @ NLO:

$$C\left(x, y, \frac{\mu}{P^z}\right) = \delta(x-y) + C_B^{(1)}\left(x, y, \frac{\mu}{P^z}\right) - C_{CT}^{(1)}(x, y) + \mathcal{O}(\alpha_s^2),$$

$$C_B^{(1)}\left(x, y, \frac{\mu}{P^z}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases} [H_1(x,y)]_+ & x<0<y \\ [H_2(x,y,P^z/\mu)]_+ & 0<x<y \\ \left[H_2\left(1-x, 1-y, \frac{P^z}{\mu}\right)\right]_+ & y<x<1 \\ [H_1(1-x, 1-y)]_+ & y<1<x \end{cases},$$

$$C_{CT}^{(1)} = -\frac{3\alpha_s C_F}{4\pi}\left[\frac{2\mathrm{Si}[(x-y)z_s P^z]}{\pi(x-y)}\right]_+,$$

- Step II: matching in bHQET

$$\varphi^+(\omega, \mu) = \begin{cases} \varphi^+_{\rm peak}(\omega, \mu), & \omega \sim \Lambda_{\rm QCD} \\ \varphi^+_{\rm tail}(\omega, \mu). & \omega \sim m_H \end{cases}$$

with:

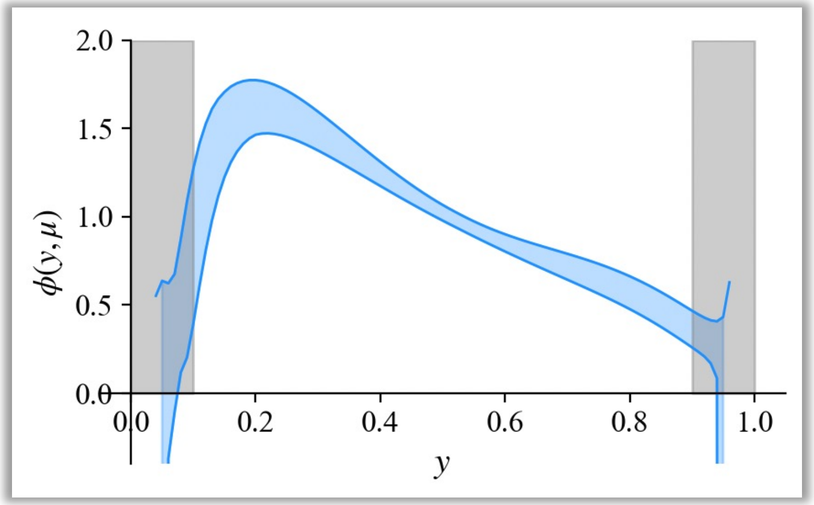
$$\varphi^+_{\rm peak}(\omega, \mu) = \frac{1}{m_H}\frac{f_H}{\tilde{f}_H}\frac{1}{\mathcal{J}_{\rm peak}}\phi(y, \mu; m_H)$$

$$\varphi^+_{\rm tail}(\omega, \mu) = \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right],$$

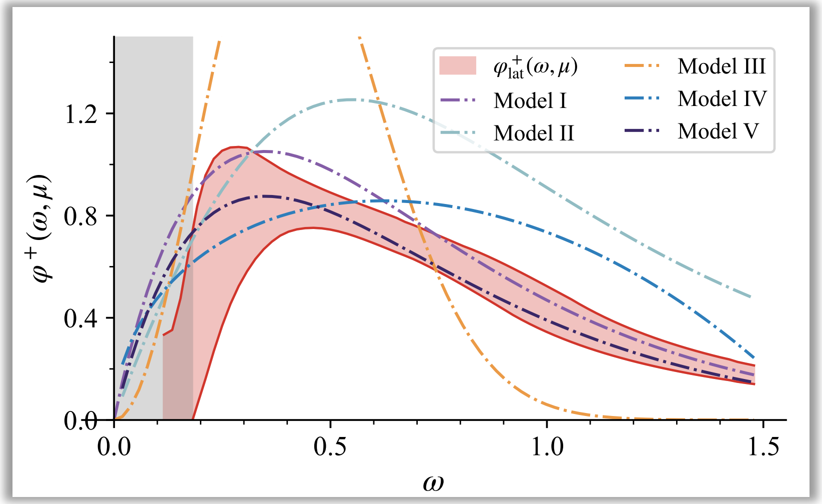
Liu, Wang, Xu, **QAZ**, Zhao, PRD 99, 094036 (2019)
Han, **QAZ**, *et.al.*, PRD111, 034503, (2025)
Ishaq, Jia, Xiong, Yang, PRL125, 132001 (2020)
Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)
Lee, Neubert, PRD 72, 094028 (2005)

Implementing SET: Heavy Meson LCDAs

D meson QCD LCDA



HQET LCDA φ^+



More details see Xue-Ying Han's talk

⇐ Numerical results for heavy meson LCDAs

⇓ Inverse and inverse-logarithmic moments

	λ_B (GeV)	$\sigma_B^{(1)}$	
This work	0.376(63)	1.66(13)	PRD111, 034503, (2025)
Ref.[16]	> 0.24		PRD98, 112016, (2018)
Ref.[54]	0.383(153)		JHEP 10, 043 (2020)
Ref.[7]	0.48(11)	1.6(2)	PRD72, 094028 (2005)
Ref.[12]	0.46(11)	1.4(4)	PRD69, 034014 (2004)
Ref.[5]	0.35(15)		PRD55, 272290 (1997)
Ref. [14]	$0.343^{+0.064}_{-0.079}$		PRD101, 074035 (2020)
Ref.[56]	0.338(68)		PLB848, 138345 (2024)

Implementing SET: Heavy Meson LCDAs

More details see Xue-Ying Han's talk

	⬅ Previous work	➡ Towards precision calculation
Action	Tadpole improved Wilson clover fermion action	
Improvement	—	HYP smear for Wilson line; Multi-source enhancement
Lattice spacing	a=0.05187fm	a=0.0775fm, 0.06826fm, 0.05187fm
NPR	Simplified hybrid scheme	Hybrid scheme based on self-renormalization
Pz extrapolation	—	Infinite momentum extrapolation
Statistics	~5k	60k~100k

Han, **QAZ**, *et.al.*, arXiv:2403.17492 [hep-ph]; PRD111, 034503, (2025)

Implementing SET: Heavy Meson Shape Functions

- Connections between SFs with HQET and QCD fields:

$$S^{\rm QCD}(x,\mu) = \begin{cases} Z_{\rm peak}(x,\omega,\mu) \otimes S^{\rm HQET}(\omega,\mu)\,, & x\sim 1 \\ Z_{\rm tail}(x,\mu)\,, & x\sim 0 \end{cases}$$

More details see Ji Xu's talk

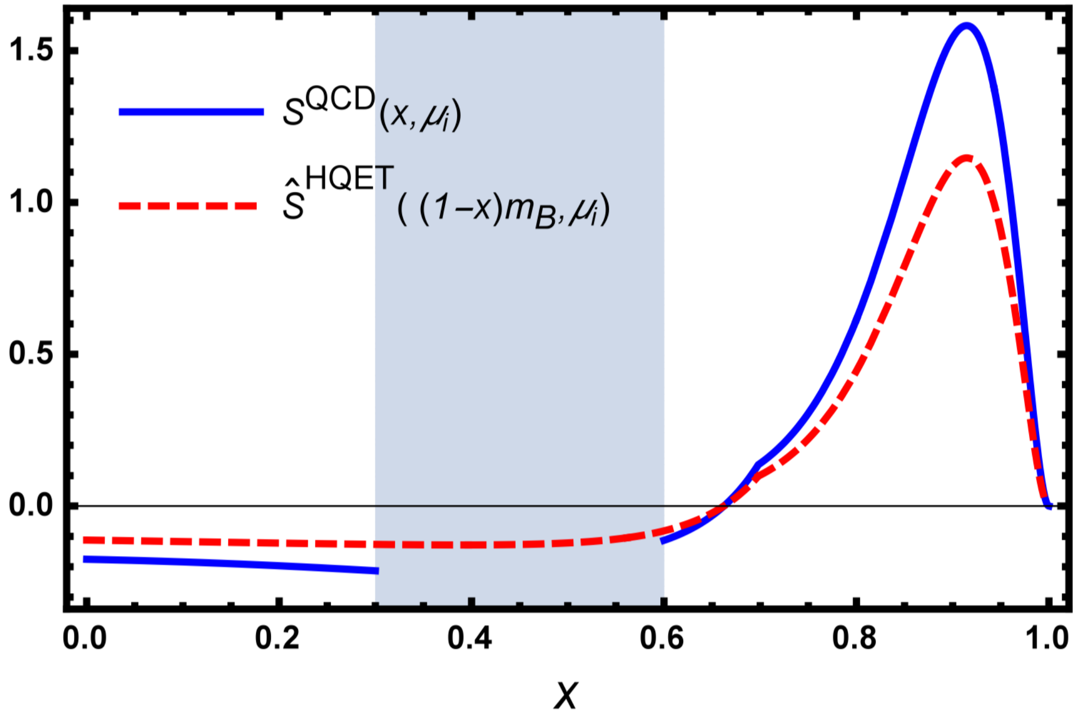
with NLO matching kernel in peak region:

$$Z^{(1)}_{\rm peak}(x,\omega,\mu) = \left(\frac{1}{2}\ln^2\frac{\mu^2}{m_b^2} - \frac{3}{2}\ln\frac{\mu^2}{m_b^2} + \frac{\pi^2}{12} - 2\right) \times\delta(xm_Bv^+ - m_bv^+ - \omega v^+)\,.$$

and tail region

$$Z^{(1)}_{\rm tail}(x,\mu) = \frac{1}{m_bv^+}\frac{1+x^2}{1-x}\left[-1+\ln\frac{\mu^2}{(1-x)^2m_b^2}\right].$$

Illustration of the matching based on modelized QCD SF



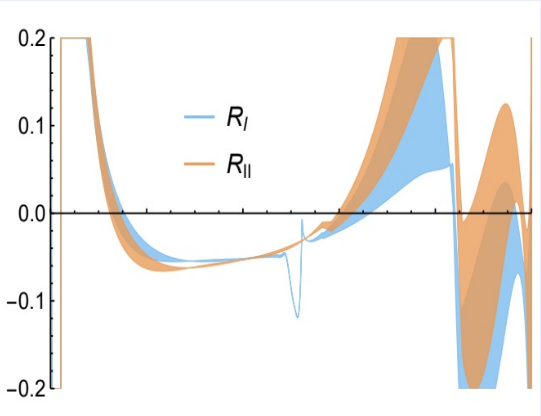
- QCD SFs are refer to the PDF of *B* meson.

Wang, Xu, **QAZ**, Zhao, arXiv:25xx.xxxx

Improving SET: Power Corrections

3 scales in SET introducing the following powers:

- Heavy hadron mass correction in LaMET: $M^2/(P^z)^2$:



- Smaller than 20% in most region;
- Smaller than 10% in the region we perform bHQET matching.

Han, Wang, Zhang, Zhang, *PRD* 110, 094038 (2024)

- Power correction in LaMET: $\Lambda_{\mathrm{QCD}}^2/(xP^z)^2$:
  - Significant at end-point region ($x \to 0, 1$) of the QCD LCDA;
  - Can be improved by considering the renormalon resummation, …

Su, Holligan, Ji, Yao, Zhang, Zhang, NPB 991, 116201 (2023)

- Heavy quark mass correction in bHQET: $\Lambda_{\mathrm{QCD}}/m_Q$:
  - A possible solution proposed in [Deng, Wang, Wei, Zeng, PRD110, 114006, (2024)]
  - HQET LCDA shows degeneracy in the Dirac structures due to heavy quark spin symmetry;
  - This power correction can be estimated from pseudoscalar and vector meson HQET LCDA.

Extending SET: Heavy Quark Mass RGE

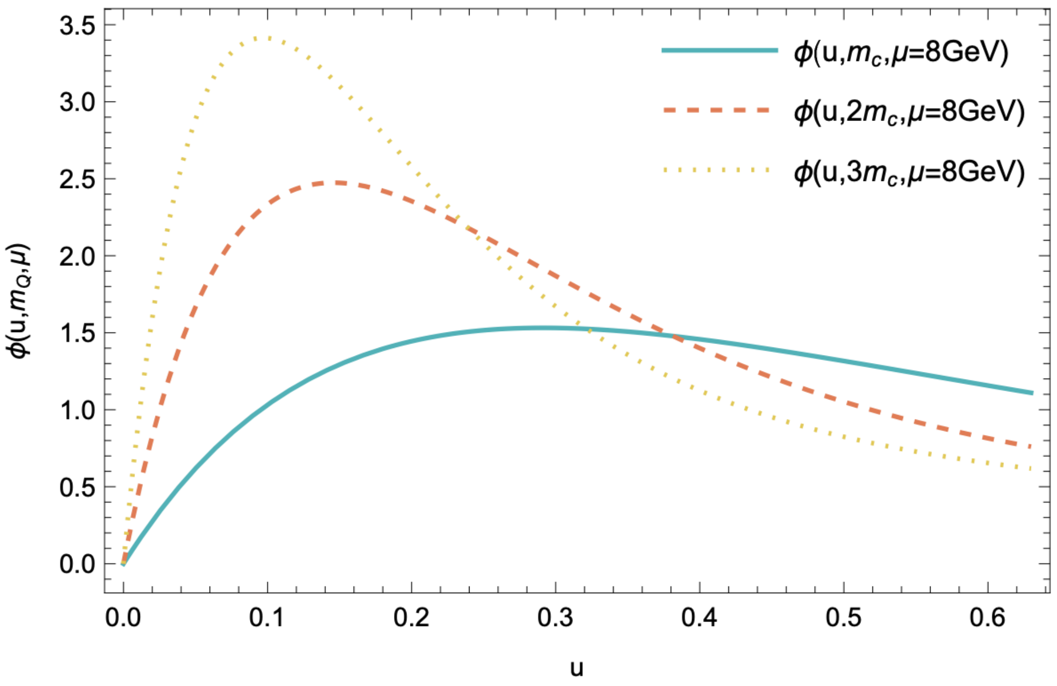
- As a key scale in HQET, m_Q reflects the intrinsic dynamics in heavy hadrons.
- Theoretically, m_Q is either $\boldsymbol{m_c}$ **or** $\boldsymbol{m_b}$. While lattice QCD enables simulations at arbitrary m_Q, this motivates systematic studies of $\boldsymbol{m_Q}$ **evolutions** in heavy hadronic systems.
- The $\boldsymbol{m_Q}$**-RGE** of heavy meson LCDAs:

Wang, Xu, **QAZ**, Zhao, arXiv:2411.07101 [hep-ph]

$$m_Q \frac{\partial}{\partial m_Q}\phi(u, m_Q; \mu) - u\frac{\partial}{\partial u}\phi(u, m_Q; \mu) - (1 + \gamma(m_Q, \mu))\phi(u, m_Q; \mu) = 0,$$

and its solution

$$\phi(u, m_Q; \mu) \approx \exp\left[\frac{2C_F}{\beta_0}\ln\frac{\alpha_s(m_Q)}{\alpha_s(m_{Q_0})} - \frac{4\pi C_F}{\beta_0^2}\left(\frac{1}{\alpha_s(m_{Q_0})}\ln\frac{\alpha_s(\mu)}{\alpha_s(m_{Q_0})e} - \frac{1}{\alpha_s(m_Q)}\ln\frac{\alpha_s(\mu)}{\alpha_s(m_Q)e}\right)\right]\frac{m_Q}{m_{Q_0}}\phi_0\left(u\frac{m_Q}{m_{Q_0}}\right).$$



Summary and Outlook

- We propose a sequential effective theory (SET) that bridges lattice calculable Euclidean correlators to the parton distribution profiles of heavy mesons.
- The factorization formula for heavy meson LCDAs within SET has been established. Preliminary lattice QCD results are now available, awaiting for more systematic lattice QCD investigations.
- The theoretical framework for heavy meson shape functions has also been proposed, expecting further validation in lattice QCD calculations.
- Power corrections within the SET remains under active discussions and investigations. Future improvements are anticipated to achieve precisely profiling the heavy meson.

Thanks for your attention!