Baryonic B decays in PQCD

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OUTLINE

- RESEARCH MOTIVATION
- PQCD CALCULATIONS
- RESULTS AND DISCUSSIONS
- SUMMARY

MOTIVATION

- CPV is an intriguing topic of heavy flavor physics.

- Searching for CPV in baryonic B decays might mark a new milestone in the discovery of CPV.

- First evidence (4.0σ) for CPV in $B \to p\bar{p}K$ decays.
  PRL 113, 141801 (2014)

- It can be expected that CPV in baryonic B decays might be observed soon !!

MOTIVATION

- Baryonic B decays offer alternative robust ways to test the SM and search for new physics, complementing searches with mesonic B decays.

- At least two baryons with half-integer spin in the final state: more plentiful CPV observations in the angular distribution.

- Not only direct CPV but also mixing induced CPV.

- New phenomena in baryonic B decays：
  - Threshold enhancement
  - Multiplicity effects [BR(4-body)>BR(3-body)>BR(2-body)]
  - Angular correlation puzzle

  The Physics of the B Factories, Eur. Phys. J. C (2014) 74:3026.

- Status of baryonic B decays:

  Cheng(2005,2007,2009), Huang, Hsiao, Wang, and Sun (2022).

- **Many phenomenological methods have explored the baryonic B decays :**

  - QCD sum rule, Chernyak and Zhitnitsky (1990);
  - Pole model, Jarfi et al. (1990) , Cheng and Yang (2002);
  - Diquark model, Ball and Dosch (1991), Chang and Hou(2002);
  - $3P_0$ model, Cheng et al. , (2006,2009);
  - PQCD, He, Li ,Li, and Wang(2006);
  - Topological Diagrammatic Approach, Chua (2014,2015,2022);
  - Bag model+SU3, Geng, Liu, and Jin (2022);
  - 3*P*0 model and chiral selection rule , Geng, Liu, and Jin (2023);
  - SU(3) flavor symmetry, Hsiao(2023);
  - Final state interactions, Geng, Liu, Jin, and Yu (2024,2025).

- **However, baryonic B decays are more challenging for the calculations based on QCD-inspired approaches!**

  1) More complex dynamics

  Baryons are made of three quarks, one more quark requires one more gluon.

  2) Baryon LCDAs are not well determined

  A primary source of theoretical uncertainties.

  3) Factorization assumption may not work well

  The nonfactorizable internal W-emission is not necessarily color suppressed, while the factorizable W-exchange and W annihilation are expected to be helicity suppressed in baryonic B decays.

- **Dynamics of B-meson baryonic decays are not well understood.**

$$\mathcal{B}_c\bar{\mathcal{B}}'_c(\sim 10^{-3}) \gg \mathcal{B}_c\bar{\mathcal{B}}(\sim 10^{-5}) \gg \mathcal{B}_1\bar{\mathcal{B}}_2(\lesssim 10^{-7})$$

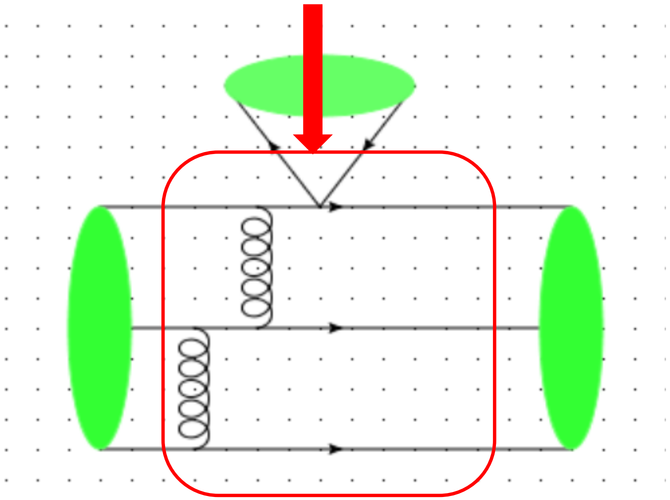
Advantages IN PQCD

- Keeping the parton transverse momenta to avoid the endpoint singularity.
- Large logarithmic corrections are organized to all orders by Sudakov resummation.
- PQCD successfully predict CPV in $B \to \pi\pi, K\pi$ decays. *[Keum, H.n.Li, Sanda, 2000; C.D.Lu, Ukai, M.Z.Yang, 2000].*
- Nonfactorizable diagrams including internal W-emission diagrams provide a significant source of strong phase.
- PQCD approach is powerful for predicting CPV in baryonic B decays.
- Many asymmetries in the angular distribution can be evaluated reliably in PQCD.

PQCD CALCULATIONS

- The decay amplitudes are factorized into the convolution of hard scattering kernels with the hadronic LCDAs

$$M \propto \psi_{\mathrm{B}} \otimes H \otimes \psi_{\overline{B_c}} \otimes \psi_{B_c}$$



- The hard amplitude involves **eight** external on shell quarks, four of which correspond to the four-fermion operators and four of which are the spectator quarks in the final states.
- The hard kernels start at α_s^2 in the PQCD approach.
- Hadronic LCDAs are the necessary inputs in PQCD calculations.

Heavy hadronic LCDAs:

- Heavy baryons LCDAs can be simplified in the heavy-quark symmetry limit.
  - *P.Ball, V.M.Braun, E.Gardi (2008)*
  - *A. Ali, C. Hambrock, and A. Y. Parkhomenko (2012)*
  - *G.Bell, T.Feldmann, Yu-Ming Wang, M.W.Y.Yip (2013)*
  - *A. Ali, C. Hambrock, A. Y. Parkhomenko, and W. Wang (2013)*
  - *V. M. Braun, S. E. Derkachov, and A. N. Manashov (2014)*
  - *Yu-Ming Wang, Yue-Long Shen (2016)*
- Based on heavy-quark symmetry, we can use the same LCDAs for the baryon containing the charm quark and the bottom quark.

$$\begin{aligned}\epsilon^{ijk}\langle 0|q^i_{1\alpha}(t_1)q^j_{2\beta}(t_2)c^k_\gamma(0)|\mathcal{B}_c\rangle &= \frac{f^{(1)}}{8}\Big[(\bar{\not n}\gamma_5 C)_{\alpha\beta}\phi_2(t_1,t_2)+(\not n\gamma_5 C)_{\alpha\beta}\phi_4(t_1,t_2)\Big]u_\gamma\\ &+\frac{f^{(2)}}{4}\Big[(\gamma_5 C)_{\alpha\beta}\phi^s_3(t_1,t_2)-\frac{i}{2}(\sigma_{\bar n n}\gamma_5 C)_{\alpha\beta}\phi^a_3(t_1,t_2)\Big]u_\gamma\end{aligned}$$

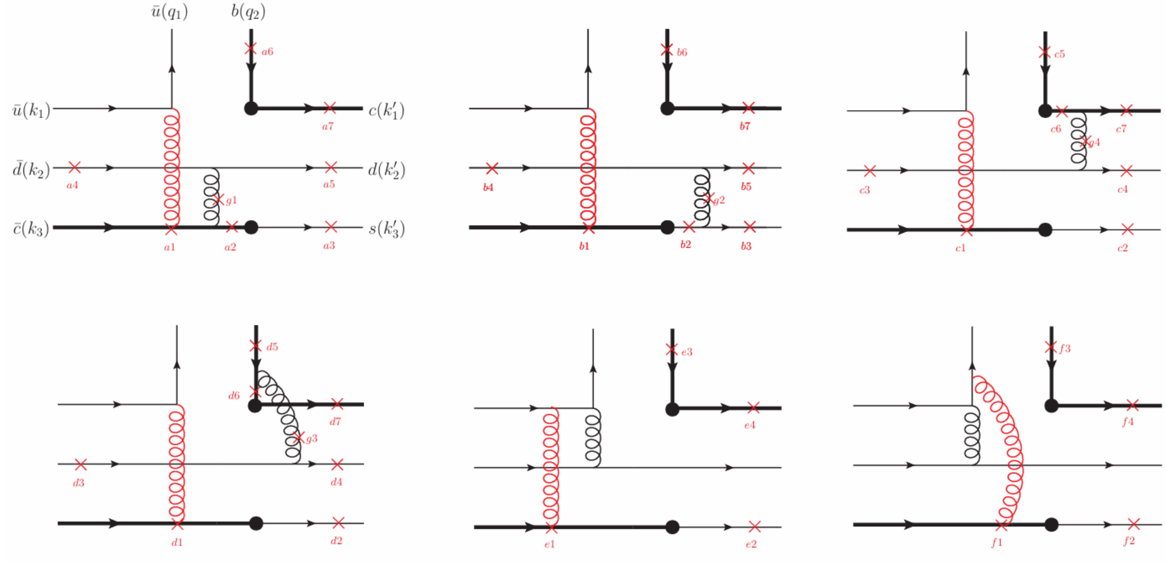
- B-meson LCDAs: [Phys. Rev. D 74 (2006) 014027]

$$\Phi_B=-\frac{i}{\sqrt{2N_c}}(\not q+M)\gamma_5\left(\phi_B^-+\frac{\not n_+}{\sqrt{2}}(\phi_B^--\phi_B^+)\right)$$

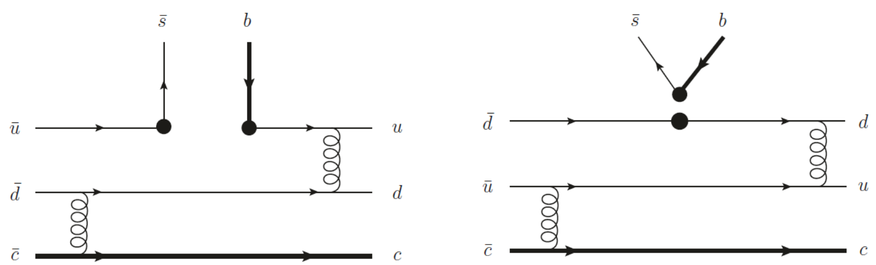
$\phi_B=\phi_B^-$ leading, $\overline{\phi_B}=\phi_B^--\phi_B^+$ subleading

FEYNMAN DIAGRAMS

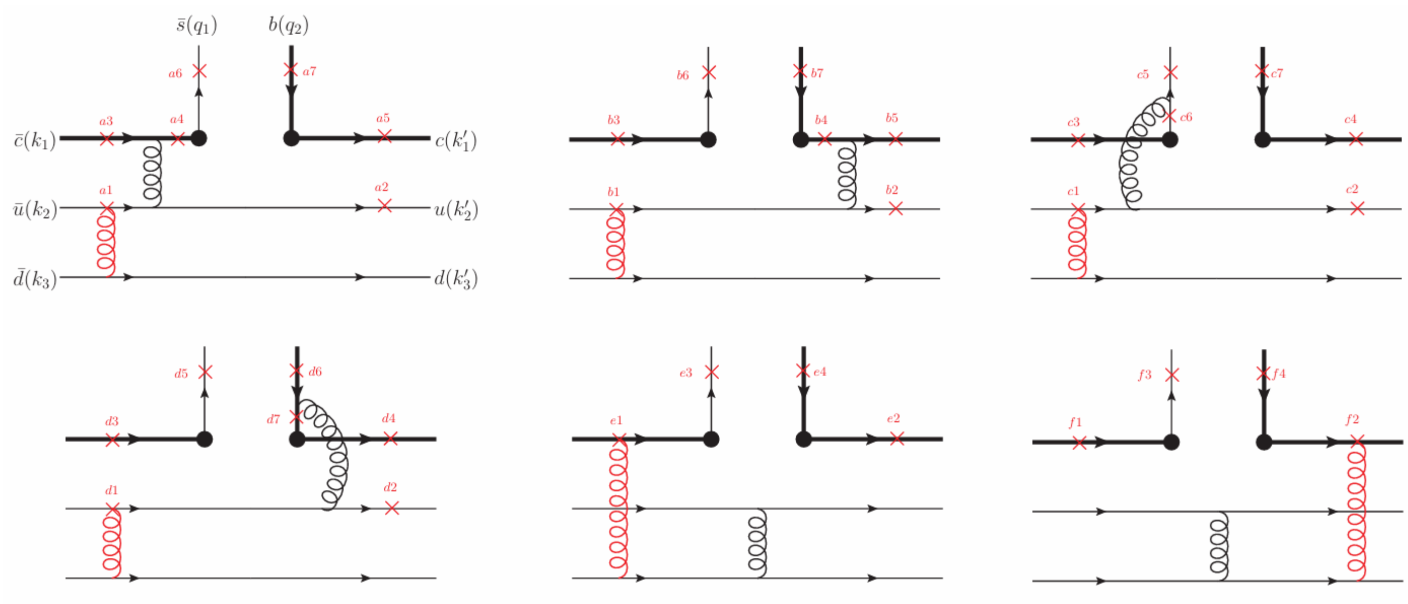
- Internal W-emission diagrams (C) for the decay $B^- \longrightarrow \Xi_c^0 \Lambda_c^-$ (40)



- Weak annihilation diagrams for $b \to u$ and penguin diagrams for $b \to s$ transition suffer highly suppression.



- W-exchange diagrams (E) for the decay $B_s \longrightarrow \Lambda_c^+ \Lambda_c^-$ (36)



- $B \longrightarrow \Lambda_c^+ \Lambda_c^-$ proceeds through both the topological W-emission and W-exchange diagrams.

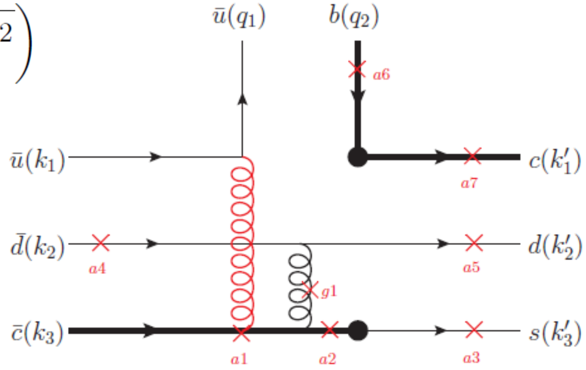
KINEMATICS:

- In the rest frame of ***B*** in the light-cone coordinates

$$q = \frac{M}{\sqrt{2}}(1, 1, \mathbf{0}_T) \qquad p = \frac{M}{\sqrt{2}}(f^+, f^-, \mathbf{0}_T), \quad p' = \frac{M}{\sqrt{2}}(1-f^+, 1-f^-, \mathbf{0}_T).$$

$$f^{\pm} = \frac{1}{2}\left(1 - r^2 + \bar{r}^2 \pm \sqrt{(1 - r^2 + \bar{r}^2)^2 - 4\bar{r}^2}\right)$$

- The b and c quarks are considered to be massive, while the masses of all light quarks are neglected.



$$q_1 = \left(0, \frac{M}{\sqrt{2}}y, \mathbf{q}_T\right), \quad q_2 = q - q_1,$$

$$k_1 = \left(\frac{M}{\sqrt{2}}f^+ x_1, 0, \mathbf{k}_{1T}\right), \quad k_2 = \left(\frac{M}{\sqrt{2}}f^+ x_2, 0, \mathbf{k}_{2T}\right), \quad k_3 = p - k_1 - k_2,$$

$$k_1' = p' - k_1' - k_2', \quad k_2' = \left(0, \frac{M}{\sqrt{2}}(1-f^-)x_2', \mathbf{k}_{2T}'\right), \quad k_3' = \left(0, \frac{M}{\sqrt{2}}(1-f^-)x_3', \mathbf{k}_{3T}'\right),$$

- The conservation laws $x_1 + x_2 + x_3 = 1, \quad \mathbf{k}_{1T} + \mathbf{k}_{2T} + \mathbf{k}_{3T} = 0,$

NUMERICAL RESULTS

Invariant amplitudes

$$\mathcal{M} = \langle \mathcal{B}_c \bar{\mathcal{B}}_c | \mathcal{H}_{eff} | B \rangle = \bar{u}[H_S + H_P \gamma_5] v$$

Mode	Type	Amplitude	ϕ_B	$\bar{\phi}_B$	$\phi_B + \bar{\phi}_B$
		H_S	$1.2 \times 10^{-7} + i8.3 \times 10^{-9}$	$2.0 \times 10^{-8} + i3.2 \times 10^{-8}$	$1.4 \times 10^{-7} + i4.0 \times 10^{-8}$
$B^- \to \Xi_c^0 \bar{\Lambda}_c^-$	C	H_P	$-7.8 \times 10^{-9} + i4.9 \times 10^{-8}$	$-1.0 \times 10^{-8} + i1.5 \times 10^{-8}$	$-1.8 \times 10^{-8} + i6.4 \times 10^{-8}$
		$\|\mathcal{M}\|$(GeV)	3.7×10^{-7}	1.3×10^{-7}	4.8×10^{-7}
		H_S	$4.8 \times 10^{-9} - i1.1 \times 10^{-8}$	$5.0 \times 10^{-9} + i8.6 \times 10^{-9}$	$9.8 \times 10^{-9} - i2.4 \times 10^{-9}$
$\bar{B}_s^0 \to \Lambda_c^+ \bar{\Lambda}_c^-$	E	H_P	$-9.6 \times 10^{-10} + i1.9 \times 10^{-8}$	$5.8 \times 10^{-9} - i3.0 \times 10^{-9}$	$4.8 \times 10^{-9} + i1.6 \times 10^{-8}$
		$\|\mathcal{M}\|$(GeV)	1.1×10^{-7}	4.5×10^{-8}	9.5×10^{-8}
		H_S	$7.8 \times 10^{-9} + i6.1 \times 10^{-9}$	$1.0 \times 10^{-10} + i1.7 \times 10^{-9}$	$7.9 \times 10^{-9} + i7.8 \times 10^{-9}$
$\bar{B}^0 \to \Lambda_c^+ \bar{\Lambda}_c^-$	$C + E$	H_P	$-2.5 \times 10^{-9} + i1.5 \times 10^{-9}$	$-2.8 \times 10^{-9} + i2.3 \times 10^{-9}$	$-5.3 \times 10^{-9} + i3.8 \times 10^{-9}$
		$\|\mathcal{M}\|$(GeV)	3.0×10^{-8}	2.0×10^{-8}	4.5×10^{-8}

- The subleading contributions can reach as much as (30−70)% of leading ones.
- The interference patterns for C and E amplitudes differ, with the former being constructive and the latter destructive.
- The inclusion of subleading correction can obviously enhance or reduce the total amplitudes.

- Magnitude of amplitude |M| (GeV) from various twist combinations of the baryon and antibaryon LCDAs.

	Twist-2	Twist-3	Twist-4
$B^- \to \Xi_c^0 \bar{\Lambda}_c^-$			
Twist-2	3.5×10^{-8}	1.7×10^{-7}	9.6×10^{-8}
Twist-3	1.4×10^{-7}	1.9×10^{-7}	1.4×10^{-7}
Twist-4	1.1×10^{-7}	2.0×10^{-7}	1.6×10^{-7}
$\bar{B}_s^0 \to \Lambda_c^+ \bar{\Lambda}_c^-$			
Twist-2	3.2×10^{-9}	0	1.5×10^{-7}
Twist-3	0	1.5×10^{-7}	0
Twist-4	5.8×10^{-8}	0	1.5×10^{-8}
$\bar{B}^0 \to \Lambda_c^+ \bar{\Lambda}_c^-$			
Twist-2	5.0×10^{-9}	2.6×10^{-8}	4.1×10^{-8}
Twist-3	2.1×10^{-8}	5.0×10^{-8}	1.5×10^{-8}
Twist-4	2.4×10^{-8}	3.0×10^{-8}	2.4×10^{-8}

- Higher-twist baryon LCDAs give significant contributions to the decay amplitudes due to the endpoint enhancement behaviors caused by the higher-twist baryon LCDAs. [Eur. Phys. J. C 82 (2022) 686]

- The contributions of the twist-4-twist-4 combination are less than the dominant twist-3-twist-3 combination, indicating the reliability of twist expansion of the baryon LCDAs.

Branching ratios

$$\mathcal{B} = \frac{P_c \tau_B}{8\pi M^2}|\mathcal{M}|^2 = \frac{P_c \tau_B}{8\pi M^2}(|H_S|^2 Q_+ + |H_P|^2 Q_-). \qquad Q_\pm = M^2 - (m \pm \bar{m})^2$$

Mode	PQCD	SU(3)	Data
$B^- \to \Xi_c^0 \overline{\Lambda_c^-}$	$9.5^{+3.0+2.6+1.7+1.2}_{-2.3-3.5-1.4-1.1} \times 10^{-4}$	$7.8^{+2.3}_{-2.0} \times 10^{-4}$	$(9.5 \pm 2.3) \times 10^{-4}$
$\overline{B^0} \to \Xi_c^+ \overline{\Lambda_c^-}$	$8.8^{+2.7+2.6+1.5+1.1}_{-2.1-3.1-1.2-1.0} \times 10^{-4}$	$7.2^{+2.1}_{-1.9} \times 10^{-4}$	$(12 \pm 8) \times 10^{-4}$
$\bar{B}_s^0 \to \Lambda_c^+ \overline{\Lambda_c^-}$	$4.0^{+0.7+0.2+0.9+1.0}_{-0.3-0.1-0.7-0.8} \times 10^{-5}$	$8.1^{+1.7}_{-1.5} \times 10^{-5}$	$< 9.9 \times 10^{-5}$
$\overline{B^0} \to \Lambda_c^+ \overline{\Lambda_c^-}$	$8.8^{+4.4+3.5+1.1+1.0}_{-2.8-3.6-0.9-0.6} \times 10^{-6}$	$2.1^{+1.0}_{-0.8} \times 10^{-5}$	$< 1.6 \times 10^{-5}$

- Theoretical uncertainties: B meson LCDAs, charmed baryon LCDAs, the scale dependence, and the Sudakov resummation.
- The branching ratios suffer large theoretical uncertainties from the nonperturbative hadronic parameters.
- The PQCD predictions for the first two modes agree with the SU(3) and PDG data, while those of the last two modes reach half of the measured upper limits.

How to understand the measurements of $BR(B \longrightarrow \Lambda_c^+\Lambda_c^-)$ and $BR(B^- \longrightarrow \Xi_c^0\Lambda_c^-)$?

- In SU(3) limit and without the E amplitude, one has,

  $\mathrm{BR}(B \longrightarrow \Lambda_c^+\Lambda_c^-) \approx (V_{cd}/V_{cs})^2\ \tau_{B^0}/\tau_{B^-} BR(B^- \longrightarrow \Xi_c^0\Lambda_c^-) = (4.7 \pm 1.1) \times 10^{-5}$,
- Deviation from the experimental upper limit of 1.6×10^{-5} by around 3σ.
- By considering the E and assuming $Arg\left(\frac{E}{C}\right) = \pi$, the SU(3) approach gives

$$\mathrm{BR}(B \longrightarrow \Lambda_c^+\Lambda_c^-) = 2.1^{+1.0}_{-0.8} \times 10^{-5}$$

- In our calculations, the SU(3) breaking effect is taken into account.

Amplitude	C	E	$\lvert\frac{E}{C}\rvert$	$\mathrm{Arg}(E/C)$
H_S	$1.0\times10^{-8}+i7.5\times10^{-9}$	$-2.3\times10^{-9}+i3.0\times10^{-10}$	0.18	2.37
H_P	$-4.1\times10^{-9}+i7.4\times10^{-9}$	$-1.1\times10^{-9}-i3.7\times10^{-9}$	0.44	2.34

- PQCD predictions:

  $\mathrm{BR}(B \longrightarrow \Lambda_c^+\Lambda_c^-) = 4.5 \times 10^{-5}$ without SU(3) breaking and without E

  $\mathrm{BR}(B \longrightarrow \Lambda_c^+\Lambda_c^-) = 1.4 \times 10^{-5}$ with SU(3) breaking (m_{B_c}, f_{B_c}, LCDAs)

  $\mathrm{BR}(B \longrightarrow \Lambda_c^+\Lambda_c^-) = 8.8 \times 10^{-6}$ with SU(3) breaking and E
- The significant SU(3) breaking effect can also explain why our value is lower than the SU(3) one by a factor 2.
- A large SU(3) breaking effect is also found by Geng et al.

$$\frac{\Gamma(B^0 \to \Xi_c^+\overline{\Xi}_c^-)}{\Gamma(B_s^0 \to \Lambda_c^+\overline{\Lambda}_c^-)} = 1.4\%$$

5.3% in the exact SU(3)! PRD110, 113008 (2024)

Asymmetry parameters

$$\alpha = \frac{|H_+|^2 - |H_-|^2}{|H_+|^2 + |H_-|^2}, \quad \beta = \frac{2Re(H_+ H_-^*)}{|H_+|^2 + |H_-|^2}, \quad \gamma = \frac{2Im(H_+ H_-^*)}{|H_+|^2 + |H_-|^2}$$

$$H_\pm = \frac{1}{\sqrt{2}}(\sqrt{Q_+}H_S \mp \sqrt{Q_-}H_P)$$

Mode	α	β	γ
$B^- \to \Xi_c^0 \bar{\Lambda}_c^-$	$-0.01^{+0.10+0.12+0.05+0.01}_{-0.10-0.29-0.14-0.01}$	$-0.99^{+0.01+0.09+0.00+0.00}_{-0.00-0.01-0.00-0.00}$	$-0.07^{+0.07+0.38+0.04+0.07}_{-0.06-0.13-0.05-0.08}$
$\bar{B}_s^0 \to \Lambda_c^+ \bar{\Lambda}_c^-$	$-0.03^{+0.05+0.03+0.05+0.01}_{-0.04-0.04-0.03-0.00}$	$-0.57^{+0.02+0.02+0.00+0.05}_{-0.03-0.02-0.02-0.05}$	$\boxed{-0.82^{+0.03+0.02+0.01+0.04}_{-0.01-0.01-0.00-0.03}}$
$\bar{B}^0 \to \Lambda_c^+ \bar{\Lambda}_c^-$	$0.17^{+0.08+0.08+0.03+0.02}_{-0.08-0.05-0.18-0.01}$	$-0.97^{+0.04+0.06+0.02+0.02}_{-0.03-0.00-0.02-0.01}$	$-0.15^{+0.17+0.54+0.14+0.09}_{-0.14-0.16-0.11-0.11}$

- The most important source of the theoretical errors is the charmed baryon LCDAs.
- Any significant reduction of the error requires more accurate information on the charmed baryon LCDAs.
- FSIs gives $\gamma(\bar{B}_s^0 \to \Lambda_c^+ \overline{\Lambda_c^-}) > 0.8$ by considering the LD contributions. Future experiments will tell us whether this process is dominated by the SD or LD contributions.

SUMMARY

- The QCD dynamics of baryonic B decay processes are more complicated than those of mesonic ones and poorly understood theoretically.
- We have made a first step to calculate the two-body doubly charmed baryonic B decays in PQCD.
- Some higher-power corrections arise from the nonperturbative hadronic LCDAs are taken into account in our numerical analysis.
- Branching ratios and asymmetry parameters are obtained and compared with other predictions and data.
- The accuracy of the theoretical predictions can be systematically improved once the charmed baryon LCDAs are available in the future.
- The SU(3) breaking effects are crucial in explaining the measured branching ratios of $\mathrm{BR}(B \longrightarrow \Lambda_c^+ \Lambda_c^-)$ and $BR(B^- \longrightarrow \Xi_c^0 \Lambda_c^-)$.
- PQCD is a powerful tool to analyze the baryonic B decays. The applications of the PQCD formalism extension to other baryonic B decays are in progress. We will focus on the CPV in these decays.

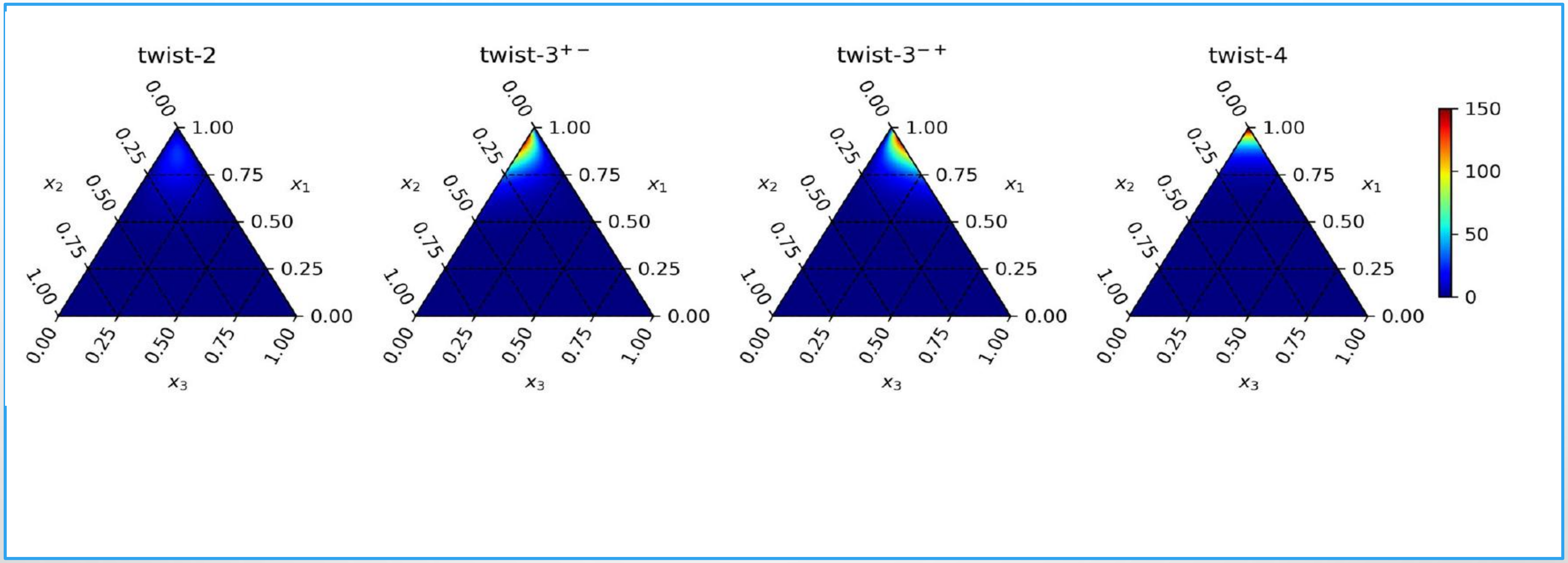
Thank you for attention!

Backup Slides

Baryon		ϵ_0	ϵ_1	ϵ_2	a_1	a_2
Λ_c	ϕ_2	$0.201^{+0.143}_{-0.059}$	0	$0.551^{+\infty}_{-0.356}$	0	$0.391^{+0.279}_{-0.279}$
	ϕ_3^s	$0.232^{+0.047}_{-0.056}$	0	$0.055^{+0.010}_{-0.020}$	0	$-0.161^{+0.108}_{-0.207}$
	ϕ_4	$0.352^{+0.067}_{-0.083}$	0	$0.262^{+0.116}_{-0.132}$	0	$-0.541^{+0.173}_{-0.090}$
Ξ_c	ϕ_2	$0.228^{+0.068}_{-0.061}$	$0.429^{+0.654}_{-0.281}$	$0.449^{+\infty}_{-0.473}$	$0.057^{+0.055}_{-0.034}$	$0.449^{+0.236}_{-0.380}$
	ϕ_3^s	$0.258^{+0.031}_{-0.038}$	$0.750^{+0.308}_{-0.093}$	$0.520^{+0.229}_{-0.060}$	$0.339^{+0.261}_{-0.160}$	$5.244^{+0.696}_{-1.132}$
	ϕ_4	$0.378^{+0.065}_{-0.080}$	$2.291^{+\infty}_{-0.842}$	$0.286^{+0.130}_{-0.150}$	$0.039^{+0.030}_{-0.018}$	$-0.090^{+0.037}_{-0.021}$

		η_1	η_2	η_3	b_2	b_3
Λ_c	ϕ_3^a	$0.324^{+0.054}_{-0.026}$	0	$0.633^{+0.0??}_{-0.0??}$	0	$-0.240^{+0.240}_{-0.147}$
Ξ_c	ϕ_3^a	$0.218^{+0.043}_{-0.047}$	$0.877^{+0.820}_{-0.152}$	$0.049^{+0.005}_{-0.012}$	$0.037^{+0.032}_{-0.019}$	$-0.027^{+0.016}_{-0.027}$

$$
\begin{aligned}
\phi_2(x_2,x_3) &= x_2 x_3 m^4 \sum_{n=0}^{2} \frac{a_n^{(2)}}{{\varepsilon_n^{(2)}}^4} C_n^{3/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-\frac{(x_2+x_3)m}{\varepsilon_n^{(2)}}}, \\
\phi_3^s(x_2,x_3) &= (x_2+x_3) m^3 \sum_{n=0}^{2} \frac{a_n^{(3)}}{{\varepsilon_n^{(3)}}^3} C_n^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-\frac{(x_2+x_3)m}{\varepsilon_n^{(3)}}} \\
\phi_3^a(x_2,x_3) &= (x_2+x_3) m^3 \sum_{n=0}^{3} \frac{b_n^{(3)}}{{\eta_n^{(3)}}^3} C_n^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-\frac{(x_2+x_3)m}{\eta_n^{(3)}}}, \\
\phi_4(x_2,x_3) &= m^2 \sum_{n=0}^{2} \frac{a_n^{(4)}}{{\varepsilon_n^{(4)}}^2} C_n^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-\frac{(x_2+x_3)m}{\varepsilon_n^{(4)}}},
\end{aligned}
$$



$4.5 \times 10^{-5} (In\ SU(3)\ limit)$
$\rightarrow 3.2 \times 10^{-5} (breaking\ from\ mass)$
$\rightarrow 2.1 \times 10^{-5} (from\ mass\ and\ decay\ constant)$
$\rightarrow 1.4 \times 10^{-5} (from\ mass\ and\ decay\ constant\ and\ LCDAs)$

$$\Phi_B = -\frac{i}{\sqrt{2N_c}}(\not{q} + M)\gamma_5\left(\phi_B^- + \frac{\not{n}_+}{\sqrt{2}}(\phi_B^- - \phi_B^+)\right),$$

$$\phi_B^-(y, b_q) = N_B y^2(1-y)^2 \exp\left[-\frac{y^2 M^2}{2\omega_b^2} - \frac{\omega_b^2 b_q^2}{2}\right]$$

$$\begin{aligned}\phi_B^-(x,b) \;=\; & N\,\frac{2\,\omega_B^4}{m_B^4}\exp\Big(-\frac{1}{2}\omega_B^2\,b^2\Big)\left\{\sqrt{\pi}\,\frac{m_B}{\sqrt{2}\,\omega_B}\mathrm{Erf}\Big(\frac{m_B}{\sqrt{2}\,\omega_B},\frac{x\,m_B}{\sqrt{2}\,\omega_B}\Big)\right.\\ & \left.+\Big[1+\Big(\frac{m_B\,\bar{x}}{\sqrt{2}\,\omega_B}\Big)^2\Big]\exp\Big[-\Big(\frac{x\,m_B}{\sqrt{2}\,\omega_B}\Big)^2\Big]-\exp\Big(-\frac{m_B^2}{2\,\omega_B^2}\Big)\right\}.\end{aligned}$$

$$\begin{aligned}S_B \;&=\; s(q_1^-, b_q) + \frac{5}{3}\int_{1/b_q}^{t} d\bar{\mu}\frac{\gamma(\alpha_s(\bar{\mu}))}{\bar{\mu}}\\ S_{\mathcal{B}_c} \;&=\; s_c(k_1^{'-}, cw') + \sum_{l=2}^{3} s(k_l^{'-}, cw') + \frac{8}{3}\int_{cw'}^{t} d\bar{\mu}\frac{\gamma(\alpha_s(\bar{\mu}))}{\bar{\mu}}\\ S_{\bar{\mathcal{B}}_c} \;&=\; s_c(k_1^+, cw) + \sum_{l=2}^{3} s(k_l^+, cw) + \frac{8}{3}\int_{cw}^{t} d\bar{\mu}\frac{\gamma(\alpha_s(\bar{\mu}))}{\bar{\mu}}\end{aligned}$$