# Triangle Singularity in Isospin Breaking Process $J/\psi \to \Lambda\bar{\Lambda}\pi$

Jia-Jun Wu (UCAS)

Collaborators: Qi Huang, Zixuan Ma, Ronggang Ping, Jun He, Hongxia Huang

PRD 110 (2024) 3, 034018 arXiv: 2405.11127 [hep-ph]

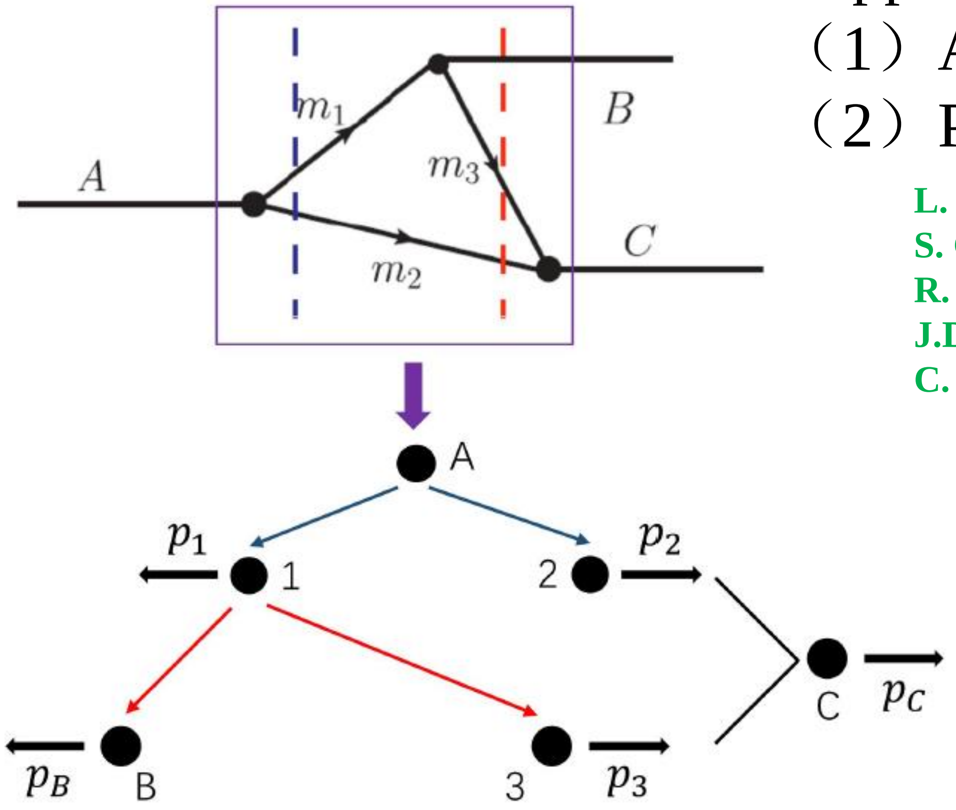
workshop Oct-18-2024 IHEP, Beijing

中国科学院大学
University of Chinese Academy of Sciences

# Content

- What is Triangle Singularity?
- TS + Iso-spin breaking
- TS in $J/\psi \to \bar{\Lambda}\Lambda\pi$
- Summary

# What is Triangle Singularity?

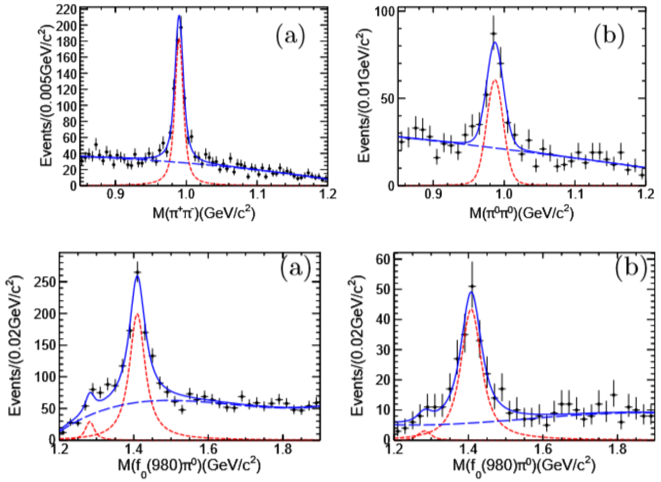


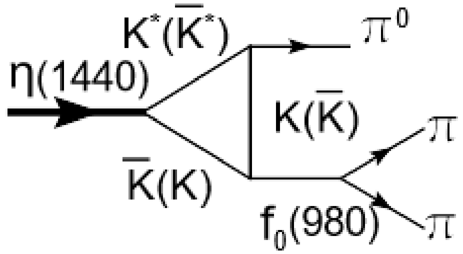

Happened Process

（1）All particles are on mass-shell;

（2）Particle 3 catch up particle 2.

**L. D. Landau, NP 13, 181 (1960)**
**S. Coleman, R.E. Norton, Nuovo Cim. 38, 438 (1965)**
**R. Karplus, C.M. Sommerfield, E.H. Wichmann, PR 111, 1187 (1958).**
**J.D. Bjorken, Ph.D. Thesis, Stanford University, Stanford, CA, USA, (1959).**
**C. Schmid, PR 154, 1363 (1967)**




**BESIII collaboration,**
**PRL 108, 182001(2012)**
**Wu, Liu, Zhao, Zou**
**PRL, 108, 081803 (2012)**

# Why Triangle Singularity interesting ?

Guo, Liu, Sakai PPNP 112, 103757 (2020)

| Structures | Processes | Loops | I/F | Refs. |
|---|---|---|---|---|
| 2.1 GeV [141] | $\gamma p^+ \to N^*(2030) \to K^+\Lambda(1405)$ | $K^*\Sigma\pi$ | I | [142] |
| 2.1 GeV | $\pi^- p^+ \to K^0\Lambda(1405)$, $pp \to pK^+\Lambda(1405)$ | $K^*\Sigma\pi$ | I | [143] |
| 1.88 GeV | $\Lambda_c^+ \to \pi^+\pi^0\pi\Sigma$ | $\bar{K}^*N\bar{K}$ | I | [144, 145][a] |
| $N(1700)$ [10] | $N(1700) \to \pi\Delta$ | $\rho N\pi$ | I | [146] |
| $N(1875)$ [10] | $N(1875) \to \pi N(1535)$ | $\Sigma^*K\Lambda$ | I | [147] |
| $\Delta(1700)$ [148–150] | $\gamma p \to \Delta(1700) \to \pi N(1535) \to p\pi^0\eta$ | $\Delta\eta p$ | I | [151] |
| 2.2 GeV [152] | $\Lambda_c^+ \to \pi^0\phi p$ | $\Sigma^*K^*\Lambda$ | F | [153] |
| 1.66 GeV [154, 155] | $\Lambda_c^+ \to \pi^+K^-p$ | $a_0\Lambda\eta, \Sigma^*\eta\Lambda$ | F | [156] |
| $P_c(4450)$ [35] | $\Lambda_b^0 \to K^-J/\psi p$ | $\Lambda(1890)\chi_{c1}p$ | F | [157–160][b] |
| | | $N(1900)\chi_{c1}p$ | F | [159] |
| peaks relevant for $P_c$ | $\Lambda_b^0 \to K^-J/\psi p$ | $\bar{D}_{sJ}\Lambda_c^{(*)}\bar{D}^{(*)}$ | F | [36, 158] |

| Structures | Processes | Loops | I/F | Refs. |
|---|---|---|---|---|
| $\rho(1480)$ [78, 79] | $\pi^-p \to \phi\pi^0 n$ | $K^*\bar{K}K$ | I | [80, 81] |
| $\eta(1405/1475)$ [82–86] | $\eta(1405/1475) \to \pi f_0$ | $K^*\bar{K}K$ | I | [87–91][a,b] |
| $f_1(1420)$ [92] | $f_1(1285) \to \pi a_0/\pi f_0$ | $K^*\bar{K}K$ | I | [89, 93–95][b] |
| $a_1(1420)$ [96, 97] | $a_1(1260) \to f_0\pi \to 3\pi$ | $K^*\bar{K}K$ | I | [97–99] |
| 1.4 GeV [100] | $J/\psi \to \phi\pi^0\eta/\phi\pi^0\pi^0$ | $K^*\bar{K}K$ | I | [101][b] |
| 1.42 GeV | $B^- \to D^{*0}\pi^- f_0(a_0), \tau \to \nu_\tau\pi^- f_0(a_0)$ | $K^*\bar{K}K$ | I | [102, 103] |
| | $D_s^+ \to \pi^+\pi^0 f_0(a_0), \bar{B}_s^0 \to J/\psi\pi^0 f_0(a_0)$ | $K^*\bar{K}K$ | I | [104, 105] |
| $f_2(1810)$ [10] | $f_2(1640) \to \pi\pi\rho$ | $K^*\bar{K}^*K$ | I | [106] |
| 1.65 GeV | $\tau \to \nu_\tau\pi^- f_1(1285)$ | $K^*\bar{K}^*K$ | I | [107] |
| 1515 MeV | $J/\psi \to K^+K^- f_0\,(a_0)$ | $\phi\bar{K}K$ | I | [108] |
| 2.85 GeV, 3.0 GeV | $B^- \to K^-\pi^-D_{s0}^*/K^-\pi^-D_{s1}$ | $K^{*0}D^{(*)0}K^+$ | I | [109, 110] |
| 5.78 GeV | $B_c^+ \to \pi^0\pi^+B_s^0$ | $\bar{K}^{*0}B^+\bar{K}$ | F | [111] |
| [4.01, 4.02] GeV | $[\bar{D}^{*0}D^{*0}] \to \gamma X$ | $D^{*0}\bar{D}^{*0}D^0$ | I | [112] |
| 4015 MeV | $e^+e^- \to \gamma X$ | $D^{*0}\bar{D}^{*0}D^0$ | I | [113, 114] |
| 4015 MeV | $B \to KX\pi$, $pp/p\bar{p} \to X\pi$+anything | $D^{*0}\bar{D}^{*0}D^0$ | I | [115, 116] |
| $\Upsilon(11020)$ [117, 118] | $e^+e^- \to Z_b\pi$ | $B_1(5721)\bar{B}B^*$ | I | [119, 120] |
| 3.73 GeV | $X \to \pi^0\pi^+\pi^-$ | $D^{*0}\bar{D}^0D^0$ | F | [121] |
| [4.22, 4.24] GeV | $e^+e^- \to \gamma J/\psi\phi/\pi^0J/\psi\eta$ | $D_{s0(s1)}^*\bar{D}_s^{(*)}D_s^{(*)}$ | F | [122] |
| [4.08, 4.09] GeV | $e^+e^- \to \pi^0J/\psi\eta$ | $D_{s0(s1)}^*\bar{D}_s^{(*)}D_s^{(*)}$ | F | [122] |
| $Z_c(3900)$ [31, 32] | $e^+e^- \to J/\psi\pi^+\pi^-$ | $D_1\bar{D}D^*$ | F | [119, 123–127][c] |
| | | $D_0^*(2400)\bar{D}^*D$ | F | [128, 129] |
| $Z_c(4020, 4030)$ [33, 130] | $e^+e^- \to \pi^+\pi^-h_c(\psi')$ | $D_{1(2)}\bar{D}^{(*)}D^{(*)}$ | F | [125] |
| $X(4700)$ [131, 132] | $B^+ \to K^+J/\psi\phi$ | $K_1(1650)\psi'\phi$ | F | [133] |
| $Z_c(4430)$ [30, 134] | $\bar{B}^0 \to K^-\pi^+J/\psi$ | $\bar{K}^{*0}\psi(4260)\pi^+$ | F | [135] |
| $Z_c(4200)$ [136, 137] | $\bar{B}^0 \to K^-\pi^+\psi(2S)$ | $\bar{K}_2^*\psi(3770)\pi^+$ | F | [135] |
| | $\Lambda_b^0 \to p\pi^-J/\psi$ | $N^*\psi(3770)\pi^-$ | F | [135] |
| $X(4050)^\pm$ [138] | $\bar{B}^0 \to K^-\pi^+\chi_{c1}$ | $\bar{K}^{*0}X\pi^+$ | F | [139] |
| $X(4250)^\pm$ [138] | $\bar{B}^0 \to K^-\pi^+\chi_{c1}$ | $\bar{K}_2^*\psi(3770)\pi^+$ | F | [139] |
| $Z_b(10610)$ [34] | $e^+e^- \to \Upsilon(1S)\pi^+\pi^-$ | $B_J^*\bar{B}^*B$ | F | [128] |

1.It is a pure kinematic effect –> Model independent

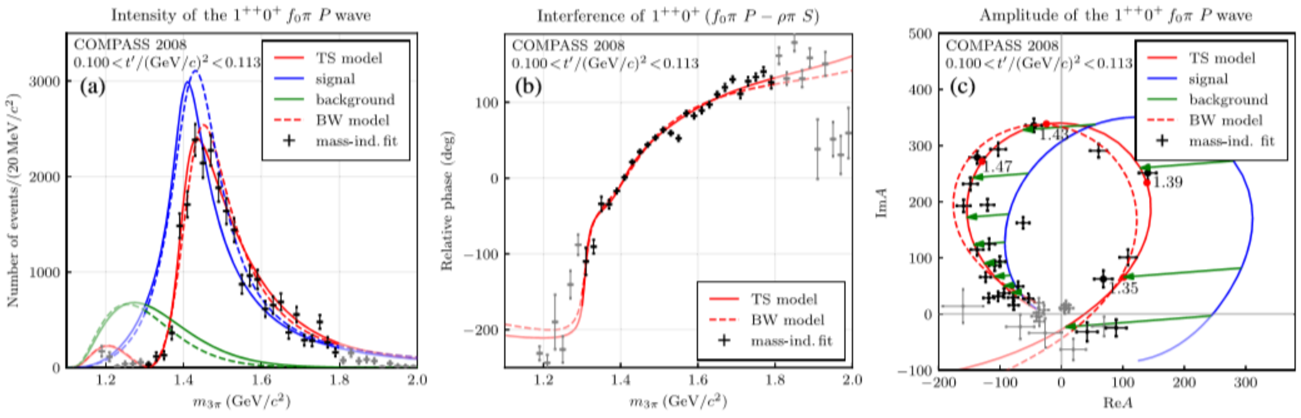
2.The effect of Loop –> Understand hadronic loop contribution

3.Provide a peak structure –> May mixing with resonance

4.Extract the nature of hadron –> Study the coupling at the energy point

5. ……

$$\pi p \to a_1(1420) \to f_0(980)\pi \to 3\pi$$



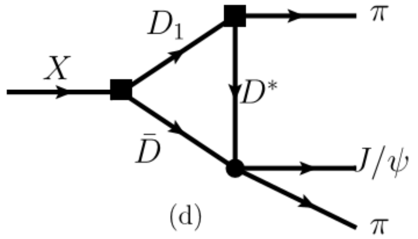

COMPASS Collaboration PRL 127, 082501 (2021)

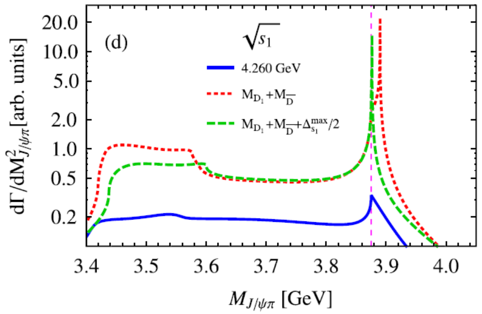
Mikhasenko, Ketzer, Sarantsev
PRD 91, 094015 (2015)

**But not confirm yet …**

# How to confirm Triangle Singularity?

## 1.Threshold

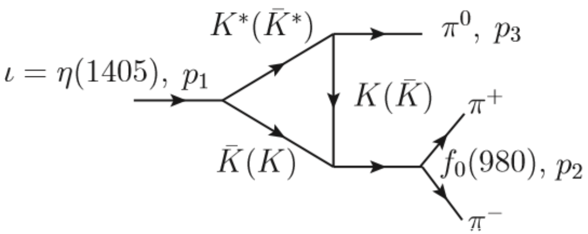


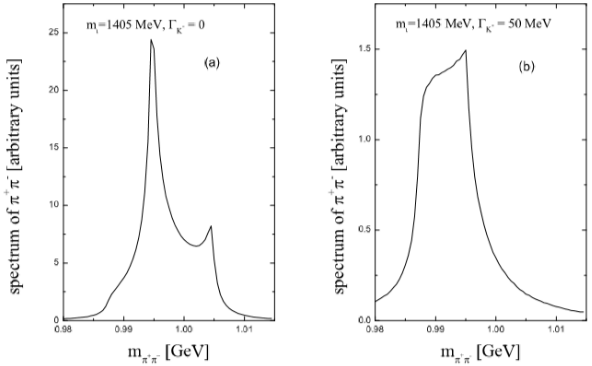




The corresponding values are listed in Table 1. The ATS peak will then stay close to the normal threshold, as illustrated in Fig. 4(d). In this sense, it would be difficult to distinguish the ATS peak from the pole structure in the invariant mass of the $J/\psi\pi$. We shall come back to the relevant issue later in this Section. It should be

**X. H. Liu, M. Oka and Q. Zhao,**
**PLB 753, 297-302 (2016)**

## 2.Width of the internal particle of the loop



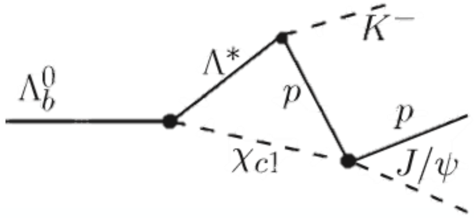



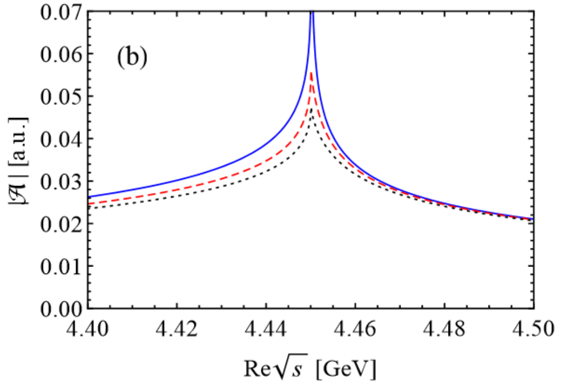

**Achasov, Kozhevnikov, Sobolev IM, Shestakov PRD 92 (2015) 3, 036003**

## 3.Unknow vertex

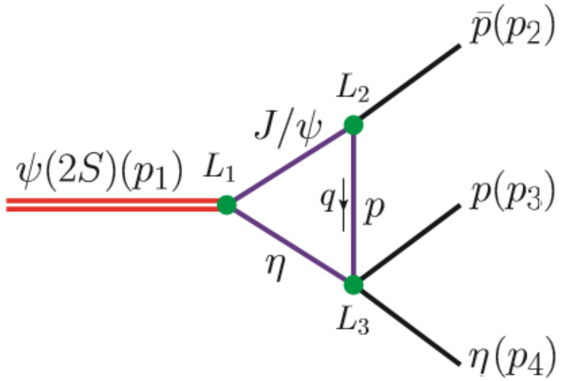
**Guo, Meißner, Wang, Yang, PRD92, 071502 (2015)**

$$\Lambda_b \to (c\bar{c})\Lambda^*$$
$$\to (c\bar{c})\Lambda K^-$$
$$\to J/\psi p K^-$$



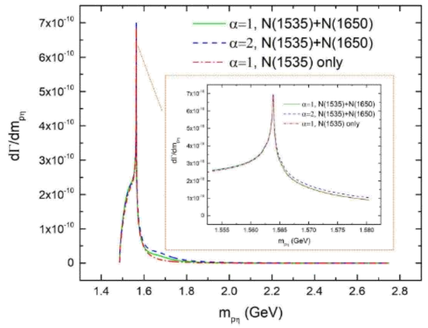
# A Proposal ----- Problem




Qi Huang, Chaowei Sheng, Jia-Jun Wu PRD 103 (2021) 1, 016014

**1.Threshold**

Far away from pη threshold. Singularity point is 1.563 GeV of p η invariant mass.

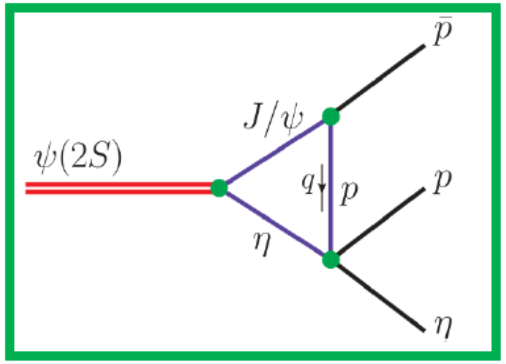


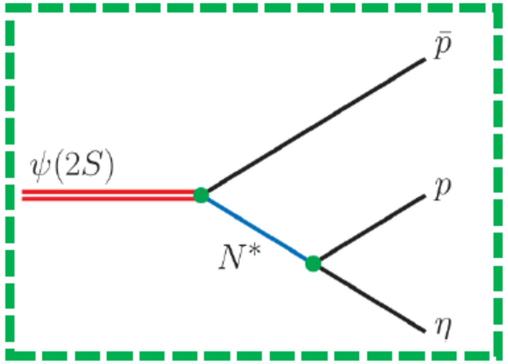

**2.Width of the internal particle of the loop**

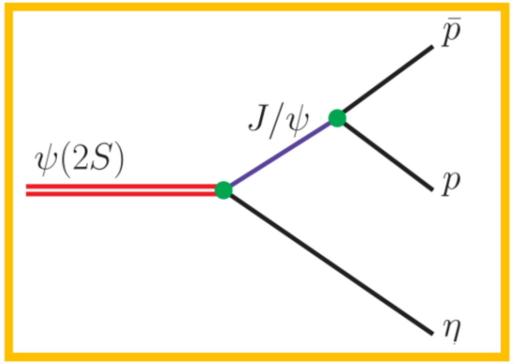
All narrow internal particles, J/psi, p, η

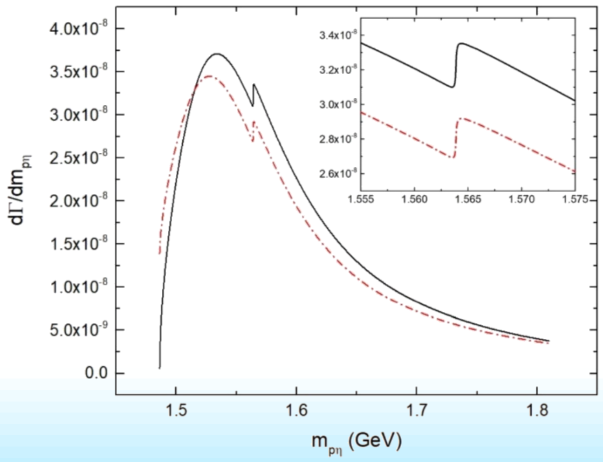
**3.Unknow vertex**

All vertices are constrained from experimental data

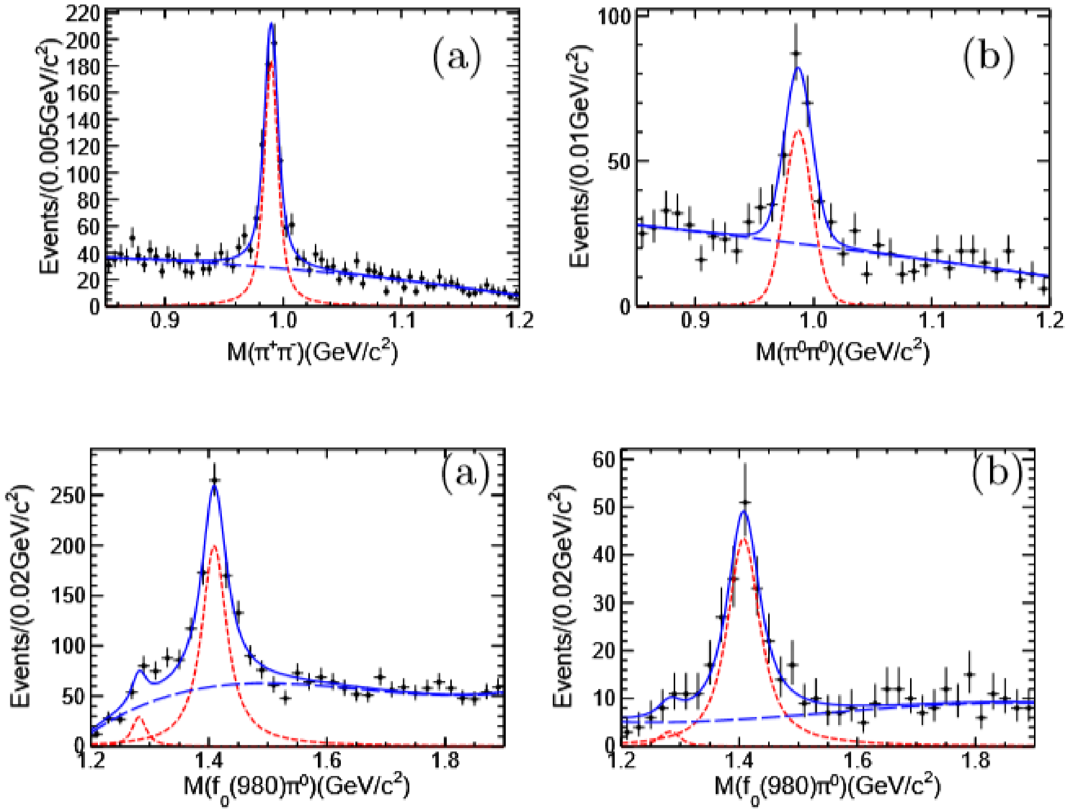


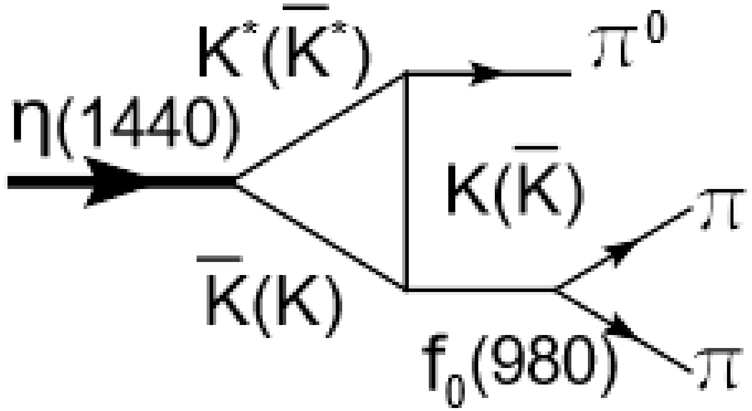



# TS in Iso-spin breaking process



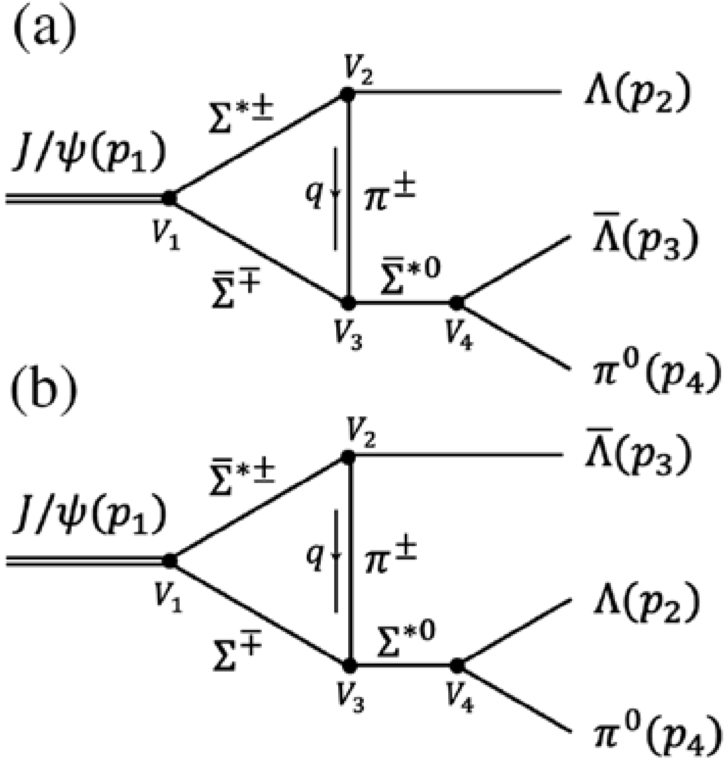

**IDEA: Searching TS happen in the Iso-spin breaking process**

$$|M_+ - M_0|^2 \sim d\Gamma$$

No direct tree diagram obeying the isospin conservation

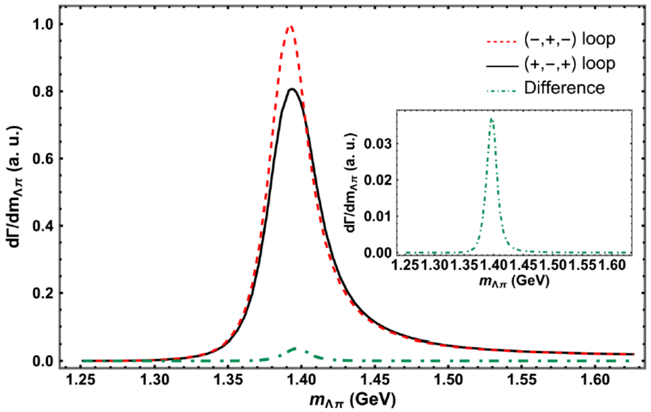
**BESIII collaboration, PRL 108, 182001(2012)** **Wu, Liu, Zhao, Zou, PRL, 108, 081803 (2012)**

# TS in $J/\psi \to \bar{\Lambda}\Lambda\pi$

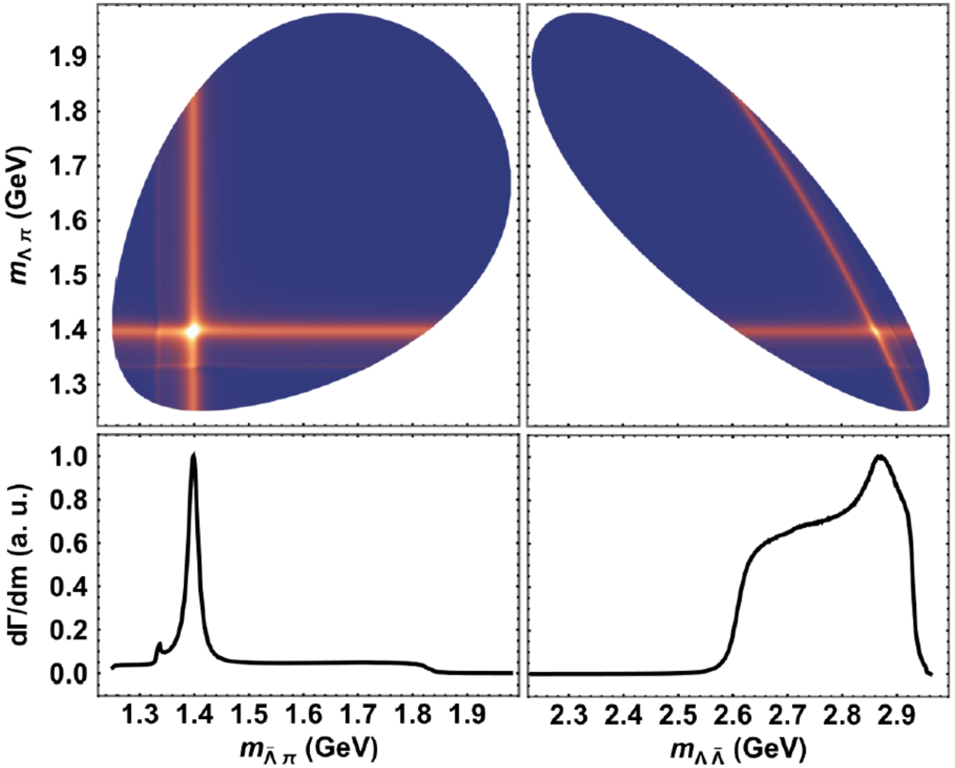



$\Sigma^{*\pm} = \Sigma^*(1620)\frac{1}{2}^-$, $\Sigma^*(1670)\frac{3}{2}^-$

$\Sigma^{*0} = \Sigma^*(1385)\frac{3}{2}^+$, $\Sigma^*(1380)\frac{1}{2}^-$




The mass difference considering here is:
$\Sigma^- = 1197.45$ MeV,
$\Sigma^+ = 1189.37$ MeV,
and the positions of triangle singularities are 1409.09 MeV and 1400.73 MeV




Two questions:
1. Other choices 2. Backgrounds

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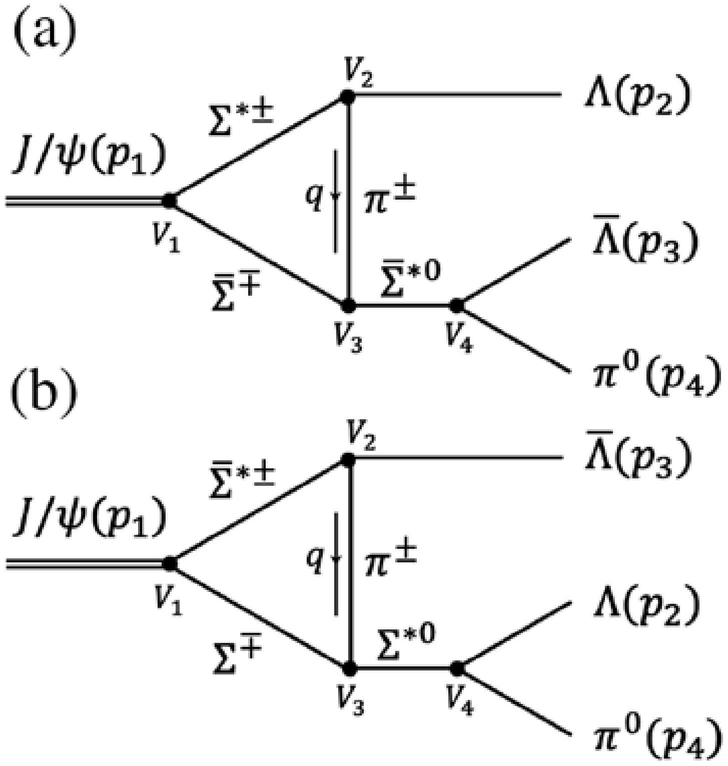
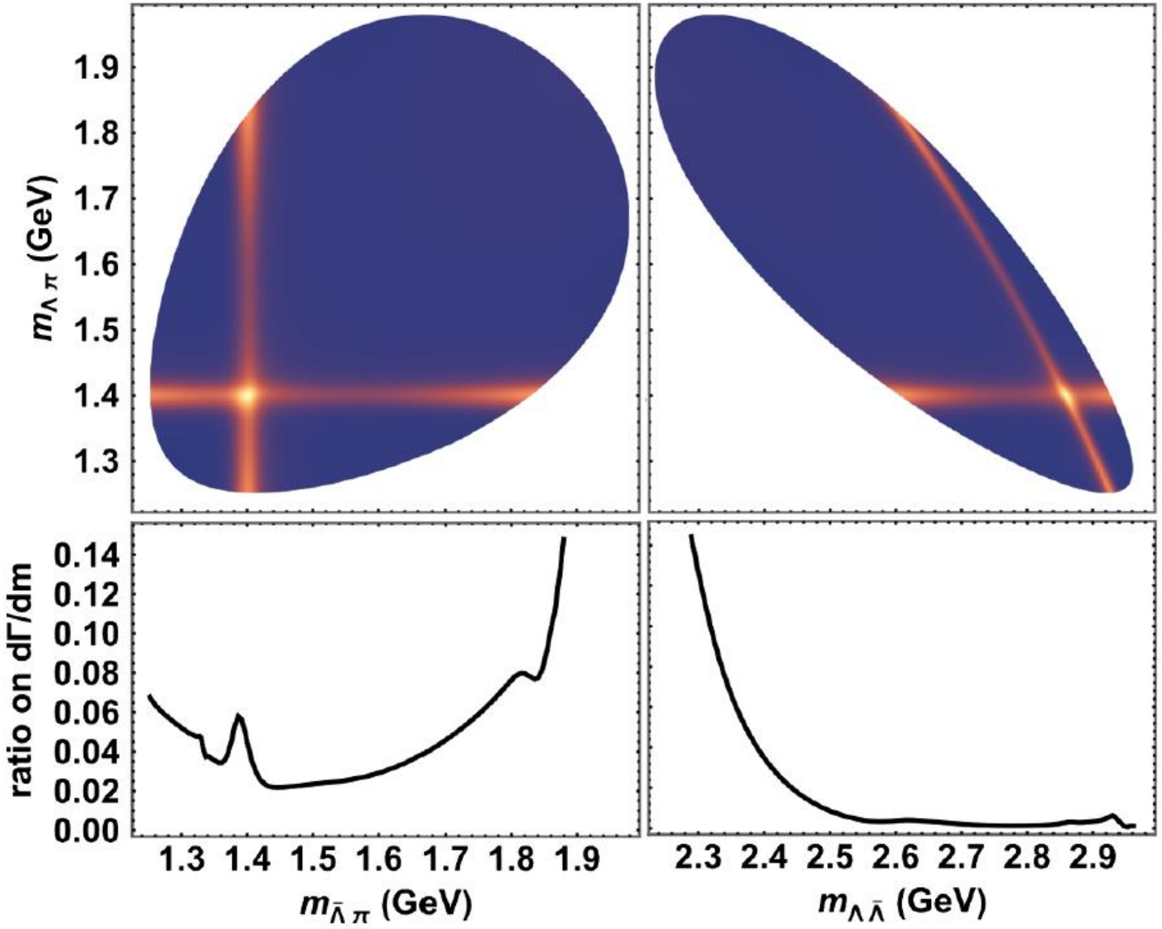
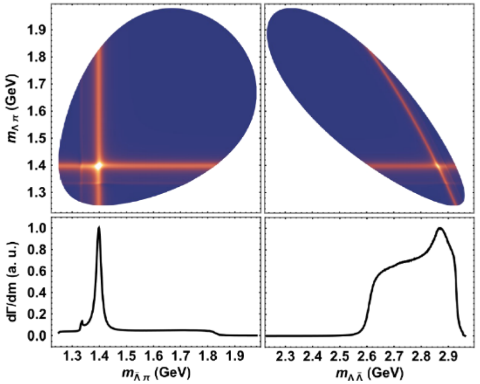
# TS in $J/\psi \to \bar{\Lambda}\Lambda\pi$



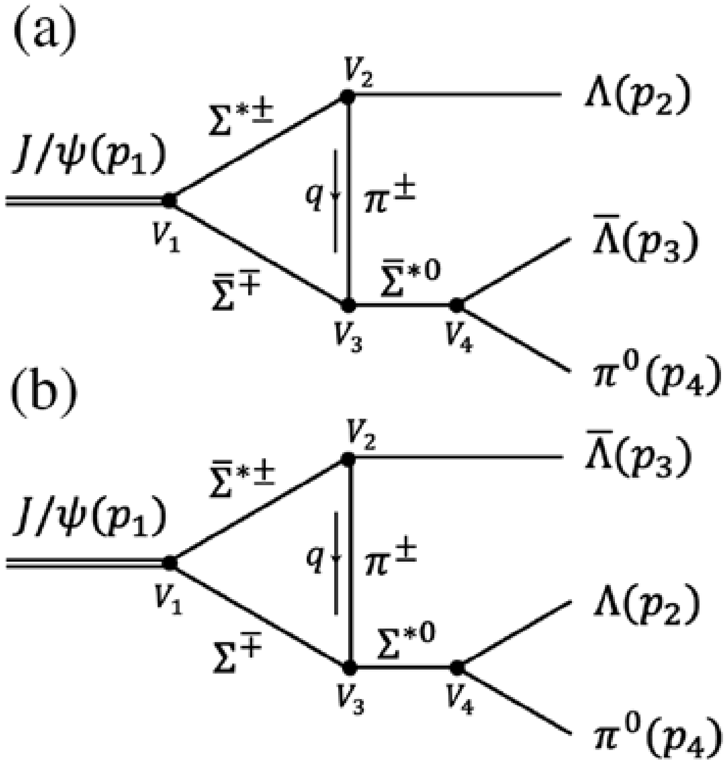

$\Sigma^{*\pm} = \Sigma^*(1620)\frac{1}{2}^-, \Sigma^*(1670)\frac{3}{2}^-$

$\Sigma^{*0} = \Sigma^*(1385)\frac{3}{2}^+, \Sigma^*(1380)\frac{1}{2}^-$

The particle wave between exchange $\pi^{\mp}$ and $\Sigma^{\pm}$ prefers the **S**-wave.

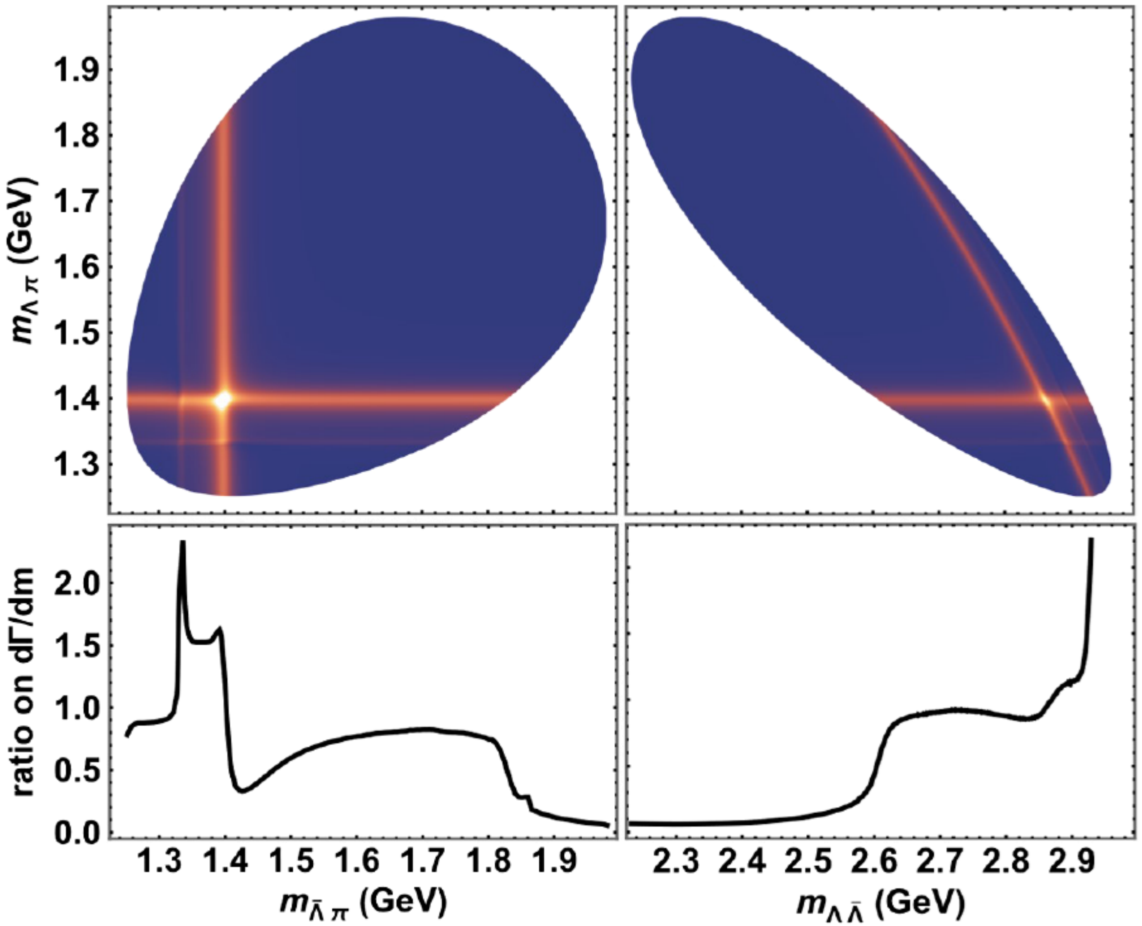
Two questions:
1. Other choices
2. Backgrounds

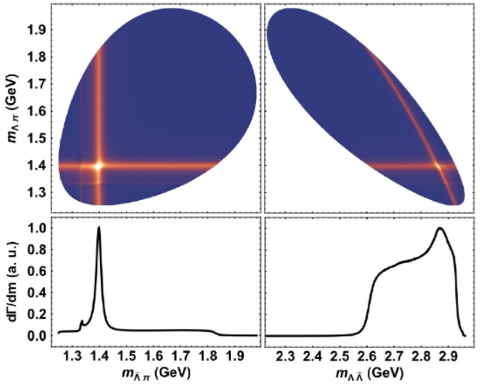
# TS in $J/\psi \to \bar{\Lambda}\Lambda\pi$




$$\Sigma^{*\pm} = \Sigma^*(1620)\frac{1}{2}^- , \Sigma^*(1670)\frac{3}{2}^-$$

$$\Sigma^{*0} = \Sigma^*(1385)\frac{3}{2}^+ , \Sigma^*(1380)\frac{1}{2}^-$$



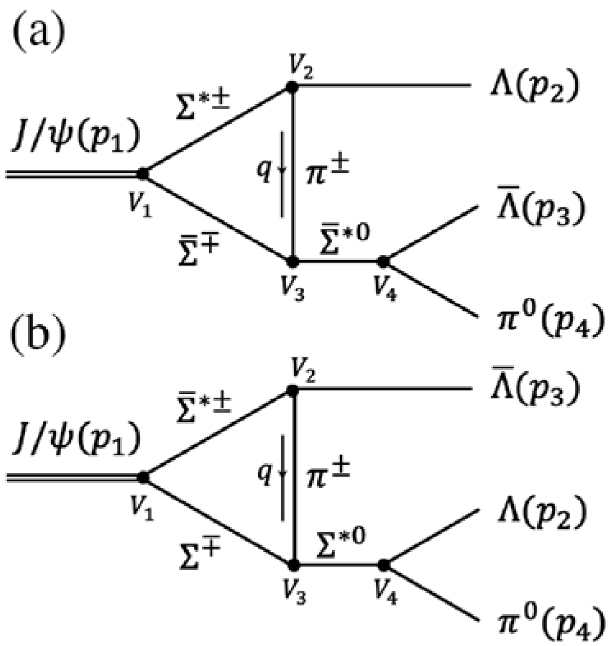



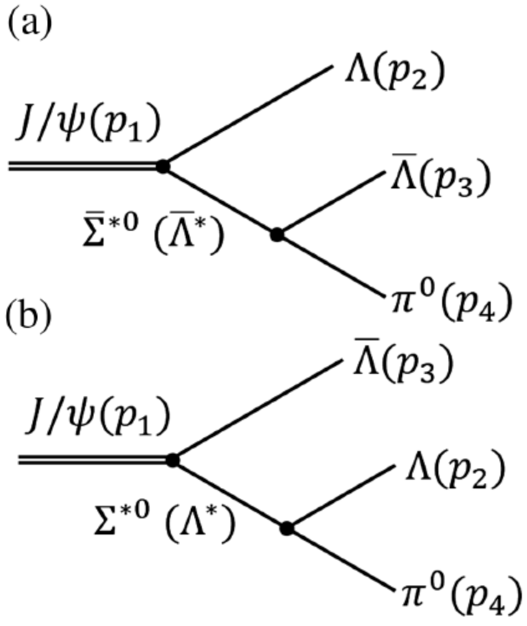

**It may provide a investigation into the existence of the two $\Sigma\left(\frac{1}{2}^-\right)$ states around 1.4 GeV and 1.6 GeV.**

# TS in $J/\psi \to \bar{\Lambda}\Lambda\pi$

Two questions:

1. Other choices
2. Backgrounds




$J/\psi \to \Sigma^{*-}(1385)\bar{\Sigma}^{+}$ $\quad Br = (3.1 \pm 0.5) \times 10^{-4}$

$J/\psi \to \Sigma^{*0}(1385)\bar{\Lambda}$ $\quad Br < (4.1 \pm 0.5) \times 10^{-6}$

So the isospin ratio is about 1%

$$\Sigma^{*\pm} = \Sigma^*(1620)\frac{1}{2}^{-}, \Sigma^*(1670)\frac{3}{2}^{-}$$

$$\Sigma^{*0} = \Sigma^*(1385)\frac{3}{2}^{+}, \Sigma^*(1380)\frac{1}{2}^{-}$$

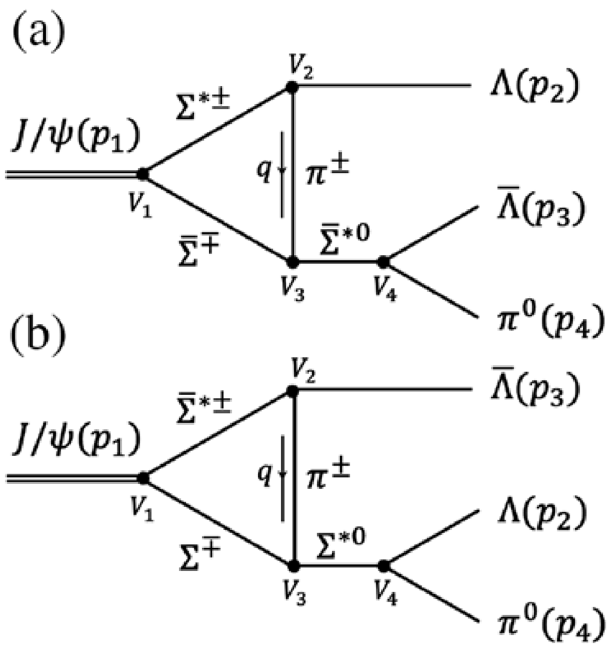
| Coupling constant | Branching ratio |
|---|---|
| $g_{\Lambda(1405)\Lambda\pi}$ | $\sim 1\%$ |
| $g_{J/\psi\Sigma(1670)\Sigma}$ | $\sim 6.2 \times 10^{-5}$ |
| $g_{J/\psi\Sigma(1620)\Sigma}$ | $\sim 6.2 \times 10^{-5}$ |
| $g_{\Sigma(1381)\Lambda\pi}$ | $\sim 85\%$ |
| $g_{\Sigma(1381)\Sigma\pi}$ | $\sim 15\%$ |

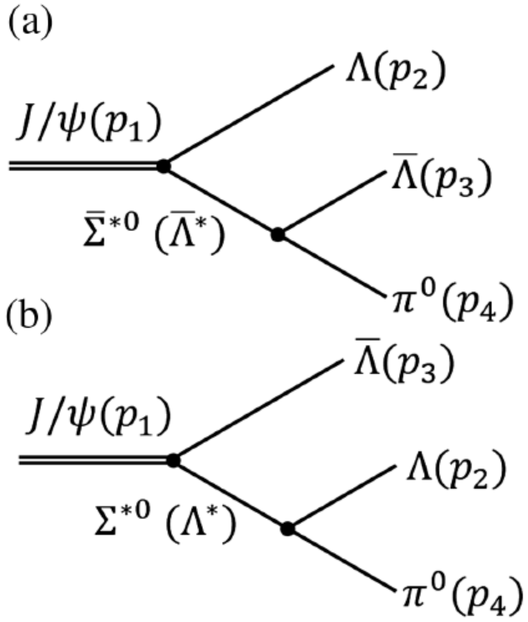
| Couplings | Branching ratio |
|---|---|
| $g_{J/\psi\Sigma(1385)\Sigma}$ | $(3.1 \pm 0.5) \times 10^{-4}$ |
| $g_{J/\psi\Sigma(1385)\Lambda}$ | $< 4.1 \times 10^{-6}$ |
| $g_{\Sigma(1385)\Lambda\pi}$ | $(87.0 \pm 1.5)\%$ |
| $g_{\Sigma(1385)\Sigma\pi}$ | $(11.7 \pm 1.5)\%$ |
| $g_{\Sigma(1620)\Lambda\pi}$ | $(9.0 \pm 3.0)\%$ |
| $g_{\Sigma(1620)\Sigma\pi}$ | $(17 \pm 5)\%$ |
| $g_{\Sigma(1670)\Lambda\pi}$ | $(10 \pm 5)\%$ |
| $g_{\Sigma(1670)\Sigma\pi}$ | $(45 \pm 15)\%$ |
| $g_{\Lambda(1405)\Sigma\pi}$ | $\sim 100\%$ |
| $g_{J/\psi\Lambda(1405)\Lambda}$ | $(8.3 \pm 0.7) \times 10^{-4}$ |

# TS in $J/\psi \to \bar{\Lambda}\Lambda\pi$

Two questions:

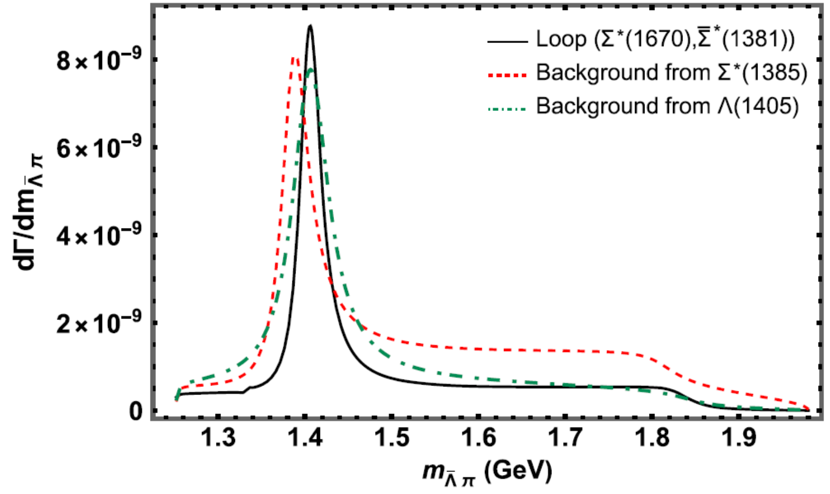
1. Other choices
2. Backgrounds

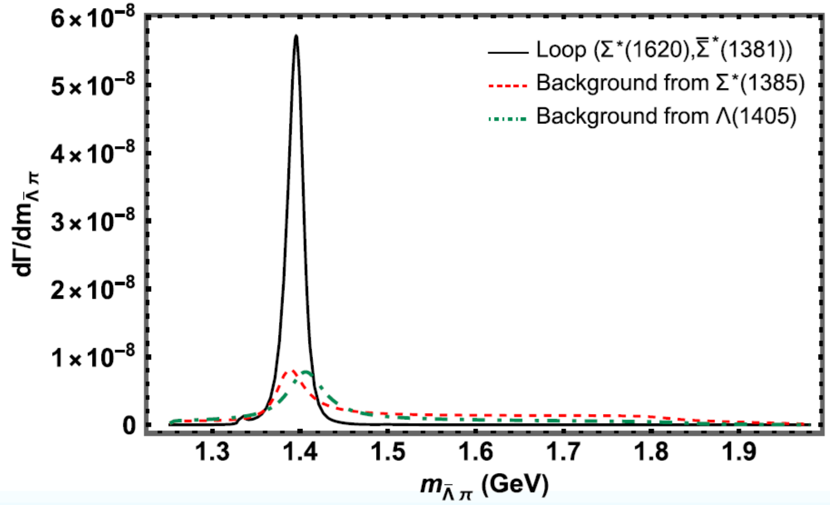



$$\Sigma^{*\pm} = \Sigma^*(1620)\frac{1}{2}^- , \Sigma^*(1670)\frac{3}{2}^-$$

$$\Sigma^{*0} = \Sigma^*(1385)\frac{3}{2}^+ , \Sigma^*(1380)\frac{1}{2}^-$$

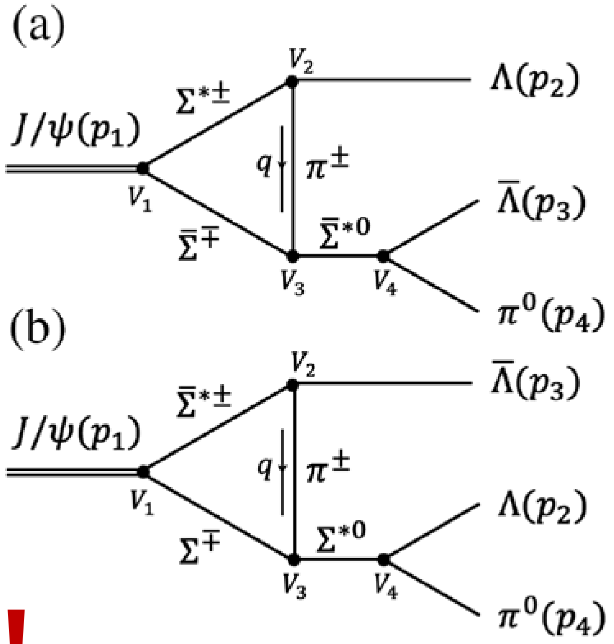


# Summary

We discuss the Triangle Singularity plus the iso-spin breaking, we find that the role of TS will be much more important in the iso-spin breaking process.

We propose the triangle loop in $J/\psi \to \bar{\Lambda}\Lambda\pi$, where **TS may play an important role.**




Once such TS peak can be confirmed, **it would provide the existence of the two $\Sigma\left(\frac{1}{2}^-\right)$ states around 1.4 GeV and 1.6 GeV.**

# Thanks for attention!