Fully strange tetraquark resonant states as the cousins of X(6900)

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Mini-workshop on light QCD exotic states

Based on [arXiv:2408.00503](https://arxiv.org/abs/2408.00503) and papers in preparation **Together with** Wei-Lin Wu (PKU), Lu Meng (RUB), Yan-Ke Chen (PKU), and Shi-Lin Zhu (PKU)

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Introduction

- Multiquark states were predicted at the birth of quark model
-

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• Quark potential model——a useful theoretical tool to describe the interaction between quarks

Quark potential model

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• Semay-Silvestre-Brac Models

$$
V_{ij}(r) = \left[-\frac{\kappa}{r} + \lambda r^p - \Lambda + \frac{2\pi}{3m_i m_j} \kappa' \frac{1}{\pi^{3/2} r_0^3} e^{(-r^2/r_0^2)} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right] \lambda_i \cdot \lambda_j
$$

AL1: $p = 1$, API: $p = 2/3$

Semay:1994ht, Silvestre-Brac:1996myf

$$
V_{ij}(r) = \left[\frac{\alpha_s}{4}\left(\frac{1}{r} - \frac{1}{6m_im_j}\frac{e^{-r/r_0}}{r_0^2r}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right) + \left(-a_c(1 - e^{-\mu_c r}) + \Delta\right)\right]\lambda_i \cdot \lambda_j
$$

+
$$
V_{\pi} + V_K + V_{\eta} + V_{\sigma}
$$
Screened confinement

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Vijande:2004he, Segovia:2011dg

• Cornell model

$$
V_{ij}(r) = \left[\frac{\alpha_s}{r} - \frac{8\pi\alpha_s}{3m_i m_j} \frac{\tau^3}{\pi^{3/2}} e^{-\tau^2 r^2} \mathbf{s}_i \cdot \mathbf{s}_j + \left(-\frac{3b}{4}r + V_c\right)\right] \frac{\lambda_i \cdot \lambda_j}{4}
$$

OGE
Confinement

Eichten:1974af, Eichten:1978tg, Eichten:1979ms

• Chiral constituent quark model (*χ*CQM)

Complex scaling method (CSM)

 \bullet In CSM, the coordinate *r* and its conjugate momentum p are transformed as

• The complex-scaled Hamiltonian

$$
H(\theta) = \sum_{i=1}^{4} \left(m_i + \frac{p_i^2 e^{-2i\theta}}{2m_i} \right) + \sum_{i < j = 1}^{4} V_{ij} \left(r_{ij} e^{i\theta} \right)
$$

no longer hermitian, has complex eigenvalues

A method to obtain energies and wave functions of bound and resonant states.

 $U(\theta)$ r = r $e^{i\theta}$, $U(\theta)$ p = p $e^{-i\theta}$

 $Real (E)$

 The properties of solutions of the complex-scaled Schrödinger equation (the ABC theorem): F Resonance: $E_R = M_R - i \Gamma_R/2$ $\frac{r \rightarrow r e^{i\theta}}{r^2 \cdot r^2}$ square-integrable function 2θ > $\big| \text{Arg}(E_R) \big|$ Bound states: not change by scaling

Continuum states: start at the threshold, rotate clockwise by 2*θ*

 CSM was advocated to derive resonances in many-body systems. B. Simon, Communications in Mathematical Physics, 27(1): 1–9 (1972)

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Avoid mistaking scattering states as resonant states

Fully strange tetraquark system

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Motivation

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• Possible strange analogs as the cousins of *X*(6900)

- The strangenium-like state $Y(2175)$ was first reported in 2006 by the BaBar Collaboration in the process
	- BaBar:2006gsq, Belle:2008kuo, BES:2007sqy, BESIII:2014ybv, BESIII:2017qkh
- Other promising candidates: $X(2370)$, $X(2500)$, $X(2239)$, $X(2100)$, $X(2436)$,...... inspired by their many
	- BESIII:2010gmv, BESIII:2019wkp, BESIII:2016qzq, BESIII:2018ldc, BESIII:2018zbm, BaBar:2007ptr
		-

Recently, the LHCb Collaboration discovered a fully charmed tetraquark candidate $X(6900)$, and

strangeness decays.

confirmed by CMS and ATLAS.

LHCb:2020bwg, CMS:2023owd, ATLAS:2023bft

All candidates have negative parity. Do positive parity states exist?

• Experimental fully strange tetraquark state candidates

of $e^+e^- \to \phi(1020) f_0(980)$. Later it was confirmed by BES, BESIII and Belle Collaborations.

• AL1 model

Quark potential model

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Semay:1994ht, Silvestre-Brac:1996myf

$$
V_{ij} = -\frac{3}{16}\lambda_i^c \cdot \lambda_j^c \left(-\frac{\kappa}{r_{ij}} + \lambda r_{ij} - \Lambda \right.+ \frac{8\pi\kappa'}{3m_i m_j} \frac{\exp(-r_{ij}^2/r_0^2)}{\pi^{3/2}r_0^3} s_i \cdot s_j \right)
$$

TABLE I. The parameters in the AL1 quark potential model.

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TABLE II. The theoretical masses (in MeV) and rms radii (in fm) of the $s\bar{s}$ mesons in the AL1 model, compared with the experimental results taken from Ref. [67].

J^{PC}	Meson	$m_{\rm Exp.}$	$m_{\rm Theo.}$	$r_{\rm Theo.}^{\rm rms}$
0^{-+}	η' ^a		713.5	0.54
	$\eta'(2S)$		1565.2	1.17
	$\eta'(3S)$		2140.9	1.65
	$\cal O$	1019.5	1021.0	0.70
	$\phi(2S)$	1680	1695.1	1.25
	$\phi(3S)$	2188	2231.6	1.70

^a For simplicity, we assume that there is no mixing effects between the $I = 0 \eta(n\bar{n})$ and $\eta'(s\bar{s})$.

Tetraquark wave function construction

• Spatial wave function:

$$
\phi_{nlm}(\boldsymbol{r}) = \sqrt{\frac{2^{l+5/2}}{\Gamma\left(l+\frac{3}{2}\right)r_n^3}} \left(\frac{r}{r_n}\right)^l e^{-\frac{r^2}{r_n^2}} Y_{lm}(\hat{r})
$$

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Only S-wave is considered.

• Color wave function:
\n
$$
\text{color-I: } \begin{cases}\n[(s_1s_2)_{\bar{3}} (\bar{s}_3\bar{s}_4)_{3}]_1 \\
[(s_1s_2)_{6} (\bar{s}_3\bar{s}_4)_{\bar{6}}]_1\n\end{cases}
$$
\n
$$
\text{color-II: } \begin{cases}\n[(s_1\bar{s}_3)_{1} (s_2\bar{s}_4)_{1}]_1 \\
[(s_1\bar{s}_3)_{8} (s_2\bar{s}_4)_{8}]_1\n\end{cases}
$$
\n• Spin wave function:
\n
$$
S = 0 : \begin{cases}\n[(s_1s_2)_{0} (\bar{s}_3\bar{s}_4)_{0}]_1 \\
[(s_1s_2)_{1} (\bar{s}_3\bar{s}_4)_{1}]_1 \\
[(s_1s_2)_{1} (\bar{s}_3\bar{s}_4)_{1}]_1\n\end{cases}
$$
\n
$$
S = 1 : \begin{cases}\n[(s_1s_2)_{1} (\bar{s}_3\bar{s}_4)_{1}]_1 \\
[(s_1s_2)_{1} (\bar{s}_3\bar{s}_4)_{1}]_2\n\end{cases}
$$
\n
$$
S = 2 : [(s_1s_2)_{1} (\bar{s}_3\bar{s}_4)_{1}]_2
$$

$$
\begin{cases}\nr_0 = 0.4 \text{ fm}, r_{n_{\text{max}}} = 2.0 \text{ fm } s - s \text{ or } \bar{s} - \bar{s} \\
r_0 = 0.4 \text{ fm}, r_{n_{\text{max}}} = 2.0 \text{ fm } (ss) - (\bar{s}\bar{s}) \\
r_0 = 0.4 \text{ fm}, r_{n_{\text{max}}} = 1.3 \text{ fm } s - \bar{s} \\
r_0 = 0.5 \text{ fm}, r_{n_{\text{max}}} = 4.5 \text{ fm } (s\bar{s}) - (s\bar{s})\n\end{cases}
$$

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 $\psi = \mathcal{A}\left(\phi\otimes\chi_{s}\otimes\chi_{c}\right)$

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Spatial structure

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rms radius→reflect spatial structure

$$
r_{ij}^{\text{rms},\text{C}} \equiv \text{Re}\left[\sqrt{\frac{\left(\Psi(\theta)\left|r_{ij}^{2}e^{2i\theta}\right|\Psi(\theta)\right)}{\left(\Psi(\theta)\mid\Psi(\theta)\right)}}\right]
$$
\n
$$
\text{c-product:}
$$
\n
$$
(\phi_n \mid \phi_m) \equiv \int \phi_n(\mathbf{r})\phi_m(\mathbf{r})
$$

 Conventional definition of the rms radius under CSM Newly defined rms radius

generally not real, real part can still reflect the clustering behavior

Fail to identify the molecular structure when containing identical quarks

a new definition

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$$
\Psi(\theta) = \sum_{s_1 \leq s_2} \left[\left[(q_1 \bar{q}'_3)_{1_c}^{s_1} (q_2 \bar{q}'_4)_{1_c}^{s_2} \right]_{1_c}^{S} \phi\left(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \boldsymbol{r}_4; \theta\right) \right. \\ \left. - \left[(q_2 \bar{q}'_3)_{1_c}^{s_1} (q_1 \bar{q}'_4)_{1_c}^{s_2} \right]_{1_c}^{S} \phi\left(\boldsymbol{r}_2, \boldsymbol{r}_1, \boldsymbol{r}_3, \boldsymbol{r}_4; \theta\right) \right. \\ \left. - \left[(q_1 \bar{q}'_4)_{1_c}^{s_1} (q_2 \bar{q}'_3)_{1_c}^{s_2} \right]_{1_c}^{S} \phi\left(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_4, \boldsymbol{r}_3; \theta\right) \right] \\ \left. + \left[(q_2 \bar{q}'_4)_{1_c}^{s_1} (q_1 \bar{q}'_3)_{1_c}^{s_2} \right]_{1_c}^{S} \phi\left(\boldsymbol{r}_2, \boldsymbol{r}_1, \boldsymbol{r}_4, \boldsymbol{r}_3; \theta\right) \right] \right] \\ = \mathcal{A} \left[\sum_{s_1 \leq s_2} \left[(q_1 \bar{q}'_3)_{1_c}^{s_1} (q_2 \bar{q}'_4)_{1_c}^{s_2} \right]_{1_c}^{S} \phi\left(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \boldsymbol{r}_4; \theta\right) \right]
$$

$$
\equiv \mathcal{A} \Psi_{13,24}(\theta),
$$

$$
\text{d}^3\bm{r}
$$

 $\mathbf{r} \setminus \mathcal{A}$

$$
r_{ij}^{\text{rms},\text{M}} \equiv \text{Re}\left[\sqrt{\frac{\left(\Psi_{13,24}(\theta)\left|r_{ij}^{2}e^{2i\theta}\right|\Psi_{13,24}(\theta)\right)}{\left(\Psi_{13,24}(\theta)\right)\Psi_{13,24}(\theta)\right)}}
$$

The impact of identical particle exchange on rms radius is removed.

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No molecular spatial characteristic. All states are compact.

No molecular spatial characteristic.

Lowest S-wave state: ~2.7 GeV
All states are compact. The compact P-wave states are expected to be heavier. \rightarrow $Y(2175)$ and $X(2370)$ are unlikely to be compact tetraquark states.

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Three- and four-lepton systems - Preliminary results

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Three- and four-lepton systems

Motivation

- limited to the di-positronium system $Ps_{2}.$
- If resonant states such as $\mu^+\mu^+e^-e^-$ exist, they may be detectable in future experiments.

• Theoretical research on three-lepton resonant states is scarce. Research on four-lepton resonant states is

QED Coulomb potential

Wave function construction

- No color wave function
- No coupling between spin channels.

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$$
V_{ij}(r) = \frac{Q_i Q_j}{r_{ij}}
$$

• 3-lepton systems with bound state or resonant state solutions

-
-
- No bound states or resonant states in $\mu^+ \mu^- e^+$ and $e^+ e^- \mu^+$ systems.

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• Only the S=1/2 systems have bound and resonant states. \rightarrow $\llbracket ll\rrbracket_{s=0}$ component is necessary. • Consistent with the previous calculations [\[Ho:1979zz\]](https://doi.org/10.1103/PhysRevA.19.2347) and [\[liverts2013three](https://doi.org/10.1016/j.cpc.2013.06.013)]. We obtain more states.

• 4-lepton systems with bound state or resonant state solutions

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• S=2 system: pure $[(\mu^+\mu^+)_1(e^-e^-)_1]_{S=2}$ component, higher energy, more difficult to form bound states and resonant states.

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• 4-body systems with bound state or resonant state solutions

Summary

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Fully strange tetraquark system

- around 50 MeV.
- All these states are compact tetraquark states.
- Since the lowest S-wave state is already as high as 2.7 GeV, $Y(2175)$ and $X(2370)$ are unlikely to be compact tetraquark states.

Three- and four-lepton systems

- *r*esonant states in $\mu^+\mu^-e^+$ and $e^+e^-\mu^+$ systems.
- In the three-lepton systems, only the S=1/2 systems have bound and resonant states.
- or resonant states are found in $\mu^+\mu^-e^+e^-$ system.

• We calculate the mass spectrum of the S-wave fully strange tetraquark systems with $(J^{PC}=0^{++},1^{+-},2^{++})$ and $U^{PC} = 0^{+-}, 1^{++}, 2^{+-}$ using AL1 quark potential model and complex scaling method. We obtain a series of resonant and zero-width states in the mass region $(2.7,3.3)$ GeV, with widths ranging from less than 1 MeV to

• We obtain a series of bound and resonant states in $e^+e^+e^-, e^+e^+\mu^-$ and $\mu^+\mu^+e^-$ systems, and no bound or

• In the four-lepton systems, $e^+e^+e^-e^-$ and $\mu^+\mu^+e^-e^-$ have a series of bound and resonant states, and no bound

Thanksforyourattention!

Backup

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Three- and four-lepton systems

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