Mini-workshop on light QCD exotic states

Fully strange tetraquark resonant states as the cousins of X(6900)

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Based on arXiv:2408.00503 and papers in preparation Together with Wei-Lin Wu (PKU), Lu Meng (RUB), Yan-Ke Chen (PKU), and Shi-Lin Zhu (PKU)



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Contents

- Quark potential model
- Complex scaling method (CSM)
- Fully strange tetraquark system
- Three- and four-lepton systems
- Summary

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Introduction



- Multiquark states were predicted at the birth of quark model

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• Quark potential model — — a useful theoretical tool to describe the interaction between quarks





Quark potential model

Cornell model

$$V_{ij}(r) = \left[rac{lpha_s}{r} - rac{8\pilpha_s}{3m_im_j}rac{ au^3}{\pi^{3/2}}e^{- au^2r^2}oldsymbol{s}_i\cdotoldsymbol{s}_j + \left(-rac{3b}{4}r+V_c
ight)
ight]rac{\lambda_i\cdot\lambda_j}{4}$$
OGE Confinement

Semay-Silvestre-Brac Models

$$V_{ij}(r) = igg[-rac{\kappa}{r} + \lambda r^p - \Lambda + rac{2\pi}{3m_im_j}\kappa'rac{1}{\pi^{3/2}r_0^3}e^{\left(-r^2/r_0^2
ight)}oldsymbol{\sigma}_i\cdotoldsymbol{\sigma}_jigg]\lambda_i\cdot\lambda_j$$

AL1: $p = 1$, AP1: $p = 2/3$

• Chiral constituent quark model (χ CQM)

$$egin{aligned} V_{ij}(r) = & [rac{lpha_s}{4} igg(rac{1}{r} - rac{1}{6m_im_j} rac{e^{-r/r_0}}{r_0^2 r} oldsymbol{\sigma}_i \cdot oldsymbol{\sigma}_jigg) + igg(-a_cig(1-e^{-\mu_c r}ig)+\Deltaig)]\lambda_i\cdot\lambda_j \ &+ V_\pi + V_K + V_\eta + V_\sigma \end{aligned}$$
 Screened confinement

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Eichten:1974af, Eichten:1978tg, Eichten:1979ms

Semay:1994ht, Silvestre-Brac:1996myf

Vijande:2004he, Segovia:2011dg







4

Complex scaling method (CSM)

 \bullet In CSM, the coordinate r and its conjugate momentum p are transformed as

The complex-scaled Hamiltonian

$$H(\theta) = \sum_{i=1}^{4} \left(m_i + \frac{p_i^2 e^{-2i\theta}}{2m_i} \right) + \sum_{i< j=1}^{4} V_{ij} \left(r_{ij} e^{i\theta} \right)$$

The properties of solutions of the complex-scaled Schrödinger equation (the ABC theorem): Bound states: not change by scaling Resonance: $E_R = M_R - i\Gamma_R/2$ $r \to re^{i\theta}$ square-integrable function

Continuum states: start at the threshold, rotate clockwise by 2θ

 CSM was advocated to derive resonances in many-body systems. B. Simon, Communications in Mathematical Physics, 27(1): 1–9 (1972)

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A method to obtain energies and wave functions of bound and resonant states.

 $U(\theta)\mathbf{r} = \mathbf{r}e^{i\theta}, \quad U(\theta)\mathbf{p} = \mathbf{p}e^{-i\theta}$

Avoid mistaking scattering states as resonant states

no longer hermitian, has complex eigenvalues





Real (E)



Fully strange tetraquark system

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Motivation

Experimental fully strange tetraquark state candidates

of $e^+e^- \rightarrow \phi(1020)f_0(980)$. Later it was confirmed by BES, BESIII and Belle Collaborations.

strangeness decays.

All candidates have negative parity. Do positive parity states exist?

• Possible strange analogs as the cousins of X(6900)

confirmed by CMS and ATLAS.

- The strangenium-like state Y(2175) was first reported in 2006 by the BaBar Collaboration in the process
 - BaBar:2006gsq, Belle:2008kuo, BES:2007sqy, BESIII:2014ybv, BESIII:2017qkh
- Other promising candidates: X(2370), X(2500), X(2239), X(2100), X(2436),.... inspired by their many
 - BESIII:2010gmv, BESIII:2019wkp, BESIII:2016qzq, BESIII:2018ldc, BESIII:2018zbm, BaBar:2007ptr

Recently, the LHCb Collaboration discovered a fully charmed tetraquark candidate X(6900), and

LHCb:2020bwg, CMS:2023owd, ATLAS:2023bft





Quark potential model

• AL1 model

Semay:1994ht, Silvestre-Brac:1996myf

$$V_{ij} = -\frac{3}{16}\lambda_i^c \cdot \lambda_j^c \left(-\frac{\kappa}{r_{ij}} + \lambda r_{ij} - \Lambda\right)$$
$$+ \frac{8\pi\kappa'}{3m_im_j} \frac{\exp(-r_{ij}^2/r_0^2)}{\pi^{3/2}r_0^3} \mathbf{s}_i \cdot \mathbf{s}_j\right)$$

TABLE I. The parameters in the AL1 quark potential model.

κ	$\lambda [{ m GeV}^2]$	$\Lambda[{ m GeV}]$	κ'	$m_s[{ m GeV}]$	$A[\operatorname{GeV}^{B-1}]$	B
0.5069	0.1653	0.8321	1.8609	0.577	1.6553	0.2204

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TABLE II. The theoretical masses (in MeV) and rms radii (in fm) of the $s\bar{s}$ mesons in the AL1 model, compared with the experimental results taken from Ref. [67].

$\overline{J^{PC}}$	Meson	$m_{ m Exp.}$	$m_{ m Theo.}$	$r_{ m Theo.}^{ m rms}$
0^{-+}	$\eta^{\prime {f a}}$	_	713.5	0.54
	$\eta'(2S)$	-	1565.2	1.17
	$\eta'(3S)$	-	2140.9	1.65
1	ϕ	1019.5	1021.0	0.70
	$\phi(2S)$	1680	1695.1	1.25
	$\phi(3S)$	2188	2231.6	1.70

^a For simplicity, we assume that there is no mixing effects between the $I = 0 \eta(n\bar{n})$ and $\eta'(s\bar{s})$.





Tetraquark wave function construction

$$\phi_{nlm}(\mathbf{r}) = \sqrt{\frac{2^{l+5/2}}{\Gamma\left(l+\frac{3}{2}\right)r_n^3}} \left(\frac{r}{r_n}\right)^l e^{-\frac{r^2}{r_n^2}} Y_{lm}(\hat{r})$$

Only S-wave is considered.



$$\begin{cases} r_0 = 0.4 \text{ fm}, r_{n_{\max}} = 2.0 \text{ fm } s - s \text{ or } \bar{s} - \bar{s} \\ r_0 = 0.4 \text{ fm}, r_{n_{\max}} = 2.0 \text{ fm } (ss) - (\bar{s}\bar{s}) \\ r_0 = 0.4 \text{ fm}, r_{n_{\max}} = 1.3 \text{ fm } s - \bar{s} \\ r_0 = 0.5 \text{ fm}, r_{n_{\max}} = 4.5 \text{ fm } (s\bar{s}) - (s\bar{s}) \end{cases}$$
 $n = 12$ for each

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 $\psi = \mathcal{A}\left(\phi \otimes \chi_s \otimes \chi_c\right)$

• Color wave function:
$$\operatorname{color-I:} \begin{cases} [(s_1s_2)_{\bar{3}} (\bar{s}_3\bar{s}_4)_3]_1 \\ [(s_1s_2)_6 (\bar{s}_3\bar{s}_4)_6]_1 \\ [(s_1s_2)_6 (\bar{s}_3\bar{s}_4)_6]_1 \\ [(s_1\bar{s}_3)_8 (s_2\bar{s}_4)_1]_1 \\ [(s_1\bar{s}_3)_8 (s_2\bar{s}_4)_8]_1 \\ [(s_1s_2)_0 (\bar{s}_3\bar{s}_4)_0]_6 \\ [(s_1s_2)_1 (\bar{s}_3\bar{s}_4)_1]_6 \\ [(s_1s_2)_1 (\bar{s}_3\bar{s}_4)_1]_2 \\ [(s_1s_2)_1 (\bar{s}_3\bar{s}_4)_1]_2 \\ [(s_1s_2)_1 (\bar{s}_3\bar{s}_4)_1]_2 \\ [(s_1s_2)_1 (\bar{s}_3\bar{s}_4)_1]_2 \end{cases}$$



9



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Spatial structure

rms radius→reflect spatial structure

 Newly defined rms radius Conventional definition of the rms radius under CSM

$$r_{ij}^{\text{rms,C}} \equiv \text{Re} \left[\sqrt{\frac{\left(\Psi(\theta) \left| r_{ij}^2 e^{2i\theta} \right| \Psi(\theta) \right)}{(\Psi(\theta) \mid \Psi(\theta))}} \right]$$

c-product:
 $(\phi_n \mid \phi_m) \equiv \int \phi_n(\mathbf{r}) \phi_m(\mathbf{r}) \phi_m$

generally not real, real part can still reflect the clustering behavior

Fail to identify the molecular structure when containing identical quarks

a new definition

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$$\begin{split} \Psi(\theta) &= \sum_{s_1 \leq s_2} \left[\left[(q_1 \bar{q}_3')_{1_c}^{s_1} (q_2 \bar{q}_4')_{1_c}^{s_2} \right]_{1_c}^S \phi\left(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \boldsymbol{r}_4; \theta\right) \right. \\ &- \left[(q_2 \bar{q}_3')_{1_c}^{s_1} (q_1 \bar{q}_4')_{1_c}^{s_2} \right]_{1_c}^S \phi\left(\boldsymbol{r}_2, \boldsymbol{r}_1, \boldsymbol{r}_3, \boldsymbol{r}_4; \theta\right) \right. \\ &- \left[(q_1 \bar{q}_4')_{1_c}^{s_1} (q_2 \bar{q}_3')_{1_c}^{s_2} \right]_{1_c}^S \phi\left(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_4, \boldsymbol{r}_3; \theta\right) \right. \\ &+ \left[(q_2 \bar{q}_4')_{1_c}^{s_1} (q_1 \bar{q}_3')_{1_c}^{s_2} \right]_{1_c}^S \phi\left(\boldsymbol{r}_2, \boldsymbol{r}_1, \boldsymbol{r}_4, \boldsymbol{r}_3; \theta\right) \right] \\ &= \mathcal{A} \left[\sum_{s_1 \leq s_2} \left[(q_1 \bar{q}_3')_{1_c}^{s_1} (q_2 \bar{q}_4')_{1_c}^{s_2} \right]_{1_c}^S \phi\left(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \boldsymbol{r}_4; \theta\right) \right] \\ &= \mathcal{A} \Psi_{13,24}(\theta), \end{split}$$

$$d^3 r$$

$$= \mathcal{A} \left[\sum_{s_1 \leq s_2} \left[(q_1 \bar{q}'_3)^{s_1}_{1_c} (q_2 \bar{q}'_4)^{s_2}_{1_c} \right]^S_{1_c} \phi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4; \\ \equiv \mathcal{A} \Psi_{13, 24}(\theta), \right] \right]$$

$$r_{ij}^{\text{rms,M}} \equiv \text{Re}\left[\sqrt{\frac{\left(\Psi_{13,24}(\theta) \left| r_{ij}^2 e^{2i\theta} \right| \Psi_{13,24}(\theta)\right)}{(\Psi_{13,24}(\theta) \mid \Psi_{13,24}(\theta))}}\right]$$

The impact of identical particle exchange on rms radius is removed.





							I					
	$M = i\Gamma/2$	24 -		24	2/2 - 2	$r_{ij}^{r_1}$	$_{\rm ms,C}$			$r_{ij}^{ m rms,M}$		structure
J	NI = iI / 2	$\chi_{\overline{3}_c \otimes 3_c}$	$\chi_{6_c \otimes \overline{6}_c}$	$\chi_{1_c\otimes 1_c}$	$\chi_{8_c\otimes 8_c}$	$r_{ss}^{ m rms}$	$r_{s\bar{s}}^{ m rms}$	$r^{\rm rms}_{s_1\bar{s}_3}$	$r^{\rm rms}_{s_2\bar{s}_4}$	$r_{s_{1}s_{2}}^{\rm rms} = r_{\bar{s}_{3}\bar{s}_{4}}^{\rm rms}$	$r_{s_1\bar{s}_4}^{\mathrm{rms}} = r_{s_2\bar{s}_3}^{\mathrm{rms}}$	Suuciuie
0^{++}	2852 - 40i	86%	14%	38%	62%	0.95	1.16	1.20	1.20	0.91	1.19	C.
	2917 - 9i	40%	60%	53%	47%	1.23	1.21	1.12	1.12	1.18	1.35	C.
	3133 - 7i	58%	42%	47%	53%	1.51	1.44	1.27	1.27	1.48	1.66	C.
	3175 - 4i	46%	54%	51%	49%	1.30	1.27	1.14	1.14	1.28	1.39	C.
	3248 - 10i	35%	65%	55%	45%	1.37	1.36	1.31	1.31	1.32	1.49	C.
1^{+-}	2819 - 3i	63%	37%	46%	54%	1.01	1.11	1.04	1.05	1.00	1.18	C.
	2940 - 46i	87%	13%	38%	62%	1.02	1.12	1.18	1.13	1.03	1.15	C.
	3142 - 12i	77%	23%	41%	59%	1.15	1.43	1.51	1.28	1.10	1.49	C.
	3228 - 2i	66%	34%	45%	55%	1.22	1.37	1.27	1.29	1.22	1.47	C.
	3237 - 4i	64%	36%	45%	55%	1.22	1.33	1.18	1.22	1.22	1.44	C.
2^{++}	2714 - 6i	75%	25%	42%	58%	1.06	1.09	1.11	1.11	0.98	1.14	C.
	2993 - 48i	85%	15%	38%	62%	1.00	1.03	1.14	1.14	1.03	1.03	C.
	3164 - 3i	92%	8%	36%	64%	0.94	1.47	1.47	1.47	0.93	1.50	C.
	3266 - 2i	66%	34%	45%	55%	1.29	1.38	1.22	1.22	1.28	1.50	C.

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No molecular spatial characteristic. All states are compact.



J^{PC}	$M - i\Gamma/2$	$\gamma_{\overline{2}}$ or	$\gamma_{c} \overline{c}$	$\gamma_1 \otimes 1$	γ of	$r_{ij}^{\rm rms,C}$		$r_{ij}^{\rm rms,M}$				structure
		$\Lambda 3_c \otimes 3_c$	$\Lambda_{0_c}\otimes_{0_c}$	$\Lambda^{1}c \otimes^{1}c$	$\Lambda^{o_c \otimes o_c}$	$r_{ss}^{ m rms}$	$r^{ m rms}_{sar{s}}$	$r^{\rm rms}_{s_1\bar{s}_3}$	$r^{ m rms}_{s_2ar{s}_4}$	$r_{s_1s_2}^{\mathrm{rms}} = r_{\bar{s}_3\bar{s}_4}^{\mathrm{rms}}$	$r_{s_1\bar{s}_4}^{\mathrm{rms}}=r_{s_2\bar{s}_3}^{\mathrm{rms}}$	
0^{+-}	2725	33%	67%	56%	44%	1.13	0.96	0.96	0.96	1.10	0.96	C.
	2873	65%	35%	45%	55%	1.17	1.03	1.02	1.02	1.13	1.02	C.
	3148	21%	79%	60%	40%	1.44	1.20	1.20	1.20	1.41	1.20	C.
	3285	78%	22%	41%	59%	1.30	1.28	1.28	1.28	1.24	1.28	C.
1^{++}	2723 - 0.5i	59%	41%	47%	53%	1.14	1.09	0.90	1.03	0.99	1.13	C.
	2863 - 4i	99%	1%	34%	66%	1.07	1.01	1.05	0.95	1.07	1.01	C.
	3151 - 0.1i	66%	34%	45%	55%	1.17	1.31	1.14	1.23	1.18	1.42	C.
2^{+-}	2896	100%	0%	33%	67%	1.10	1.02	1.02	1.02	1.10	1.02	C.

Lowest S-wave state: ~2.7 GeV The compact P-wave states are expected to be heavier. \rightarrow Y(2175) and X(2370) are unlikely to be compact tetraquark states.

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No molecular spatial characteristic. All states are compact.







Three- and four-lepton systems - Preliminary results

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Three- and four-lepton systems

Motivation

- limited to the di-positronium system Ps_2 .
- If resonant states such as $\mu^+\mu^+e^-e^-$ exist, they may be detectable in future experiments.

QED Coulomb potential

$$V_{ij}(r) = \frac{Q_i Q_j}{r_{ij}}$$



Wave function construction

- No color wave function
- No coupling between spin channels.

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• Theoretical research on three-lepton resonant states is scarce. Research on four-lepton resonant states is

0	System	CSM	
1	$\mathrm{Ps}(1S)$	$-6.80 { m eV}$	
	Ps(2S)	$-1.70 \mathrm{eV}$	
	$\mathrm{Ps}(3S)$	-0.76 eV	
	$\mu^+\mu^-(1S)$	-1.41 keV	Spin and C-parity degene
	$\mu^+\mu^-(2S)$	-0.35 keV	\rightarrow threshold degenerate
	$\mu^+\mu^-(3S)$	-0.16 keV	\rightarrow intestiona degenerate
	$\mu^+ e^- (1S)$	$-13.6 { m eV}$	
	$\mu^+ e^- (2S)$	-3.4 eV	
	$\mu^+e^-(3S)$	-1.5 eV	







3-lepton systems with bound state or resonant state solutions



- No bound states or resonant states in $\mu^+\mu^-e^+$ and $e^+e^-\mu^+$ systems.

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• Only the S=1/2 systems have bound and resonant states. $\rightarrow [ll]_{s=0}$ component is necessary. • Consistent with the previous calculations [Ho:1979zz] and [liverts2013three]. We obtain more states.



• 4-lepton systems with bound state or resonant state solutions



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4-body systems with bound state or resonant state solutions



resonant states.

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• S=2 system: pure $[(\mu^+\mu^+)_1(e^-e^-)_1]_{S=2}$ component, higher energy, more difficult to form bound states and





Summary

Fully strange tetraquark system

- around 50 MeV.
- All these states are compact tetraquark states.
- tetraquark states.

Three- and four-lepton systems

- resonant states in $\mu^+\mu^-e^+$ and $e^+e^-\mu^+$ systems.
- In the three-lepton systems, only the S=1/2 systems have bound and resonant states.
- or resonant states are found in $\mu^+\mu^-e^+e^-$ system.

• We calculate the mass spectrum of the S-wave fully strange tetraquark systems with $(J^{PC} = 0^{++}, 1^{+-}, 2^{++})$ and $(J^{PC} = 0^{+-}, 1^{++}, 2^{+-})$ using AL1 quark potential model and complex scaling method. We obtain a series of resonant and zero-width states in the mass region (2.7,3.3) GeV, with widths ranging from less than 1 MeV to

• Since the lowest S-wave state is already as high as 2.7 GeV, Y(2175) and X(2370) are unlikely to be compact

• We obtain a series of bound and resonant states in $e^+e^+e^-$, $e^+e^+\mu^-$ and $\mu^+\mu^+e^-$ systems, and no bound or

• In the four-lepton systems, $e^+e^+e^-e^-$ and $\mu^+\mu^+e^-e^-$ have a series of bound and resonant states, and no bound







19

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Backup

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Three- and four-lepton systems

	-
system	$\Delta E - i\Gamma/2$
$e^+e^+e^-$	-7.12
	-2.01
	-1.15 - 0.01i
	-0.98
$e^+e^+\mu^-$	-14.29
	-4.03 - 0.02i
	-1.86 - 0.02i
$\mu^+\mu^+e^-$	-15.61
	-14.92
	-14.37
	-13.95
	-13.67
	-3.63
	-3.55
	-3.48

$\overline{J^{PC}}$	$\Delta E - i\Gamma/2$	component	$r_{e^+e^+} = r_{e^-e^-}$	$r_{e^+e^-}$
0++	-14.00	$[00]_{0}$	0.37	0.29
	-8.80 - 0.07i	$[00]_{0}$	0.63	0.56
	-7.59 - 0.03i	$[00]_{0}$	1.30	1.16

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J^P	$\Delta E - i\Gamma/2$	component	$r_{\mu^+\mu^+}$	$r_{e^-e^-}$	1
0+	-30.30	[00]0	0.08	0.14	
	-29.01	$[00]_{0}$	0.11	0.16	
	-28.01	$[00]_{0}$	0.13	0.18	
	-27.34	$[00]_{0}$	0.18	0.22	
	-18.55	$[00]_{0}$	0.12	0.41	
	-17.96	$[00]_{0}$	0.16	0.41	
	-17.61	$[00]_{0}$	0.21	0.41	
	-17.34	$[00]_{0}$	0.26	0.44	
	-17.12	$[00]_{0}$	0.32	0.48	
	-16.98	$[00]_{0}$	0.41	0.56	
	-16.33 - 0.03i	$[00]_{0}$	0.12	1.27	
	-16.22 - 0.01i	$[00]_{0}$	0.15	0.91	
	-15.72 - 0.01i	$[00]_{0}$	0.18	0.86	
	-15.60 - 0.02i	$[00]_{0}$	0.16	1.30	
	-15.33 - 0.01i	$[00]_{0}$	0.24	0.85	
1 ⁺	-18.20	$[01]_1$	0.13	0.39	
	-17.53	$[01]_1$	0.16	0.41	
	-17.07	$[10]_1$	0.35	0.48	
	-17.04	$[01]_1$	0.16	0.66	
	-17.03	$[01]_1$	0.17	0.64	
	-16.95	$[10]_1$	0.46	0.58	
	-16.42 - 0.01i	$[01]_1$	0.12	1.22	
	-16.95 - 0.01i	$[01]_1$	0.14	0.95	
	-15.75 - 0.01i	$[01]_1$	0.17	1.00	
	-15.58 - 0.02i	$[01]_1$	0.17	1.18	
	-15.30	$[01]_1$	0.23	0.88	



