Open Charm Tetraquarks
BES III Workshop

IHEP, 18/10/2024

Luciano Maiani, CERN

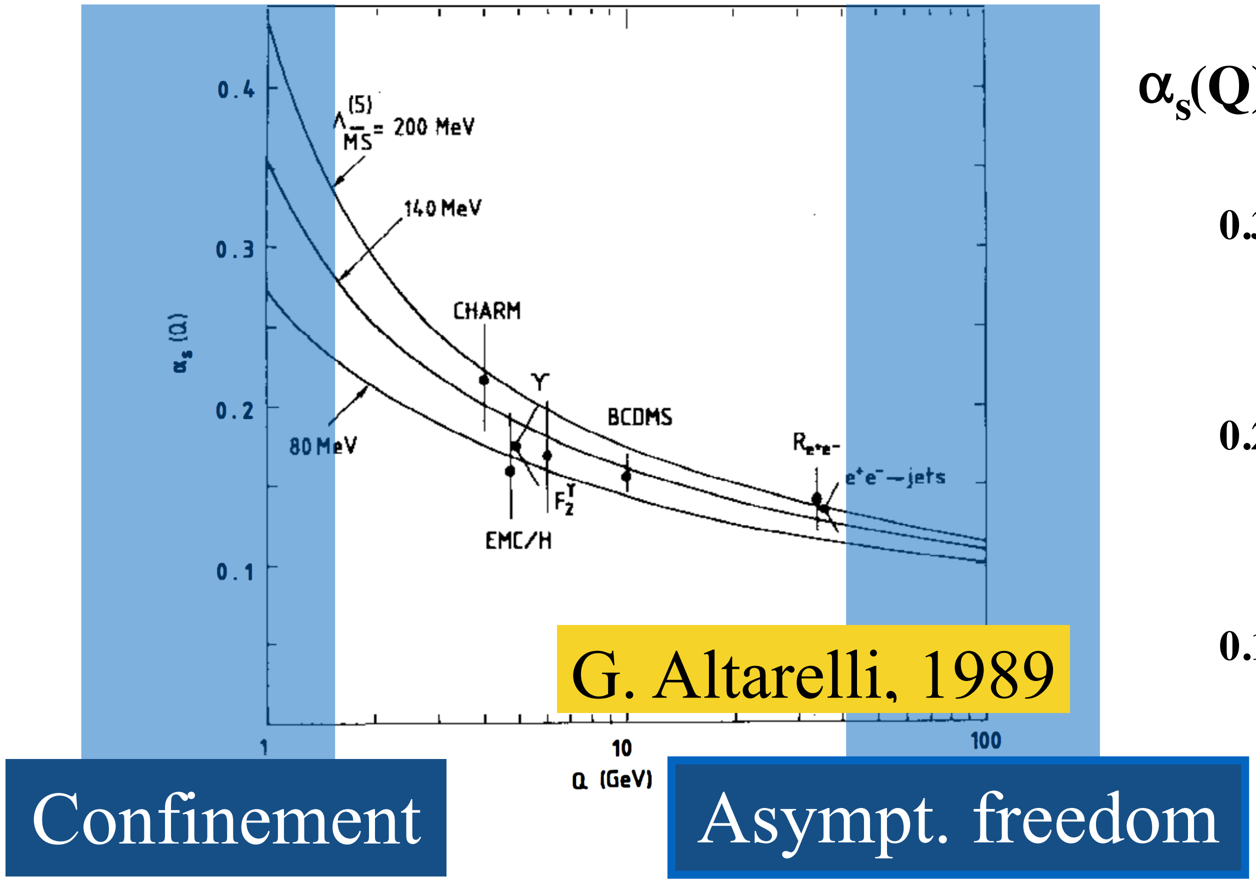
starring:

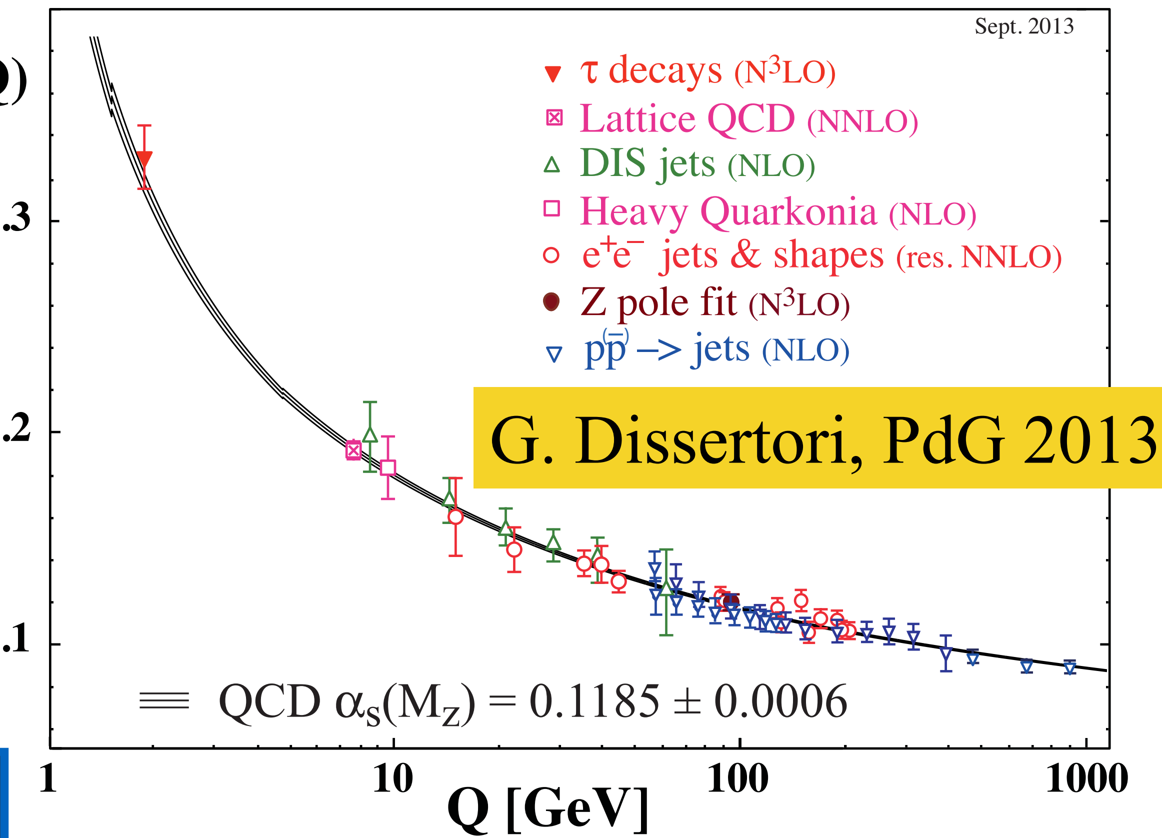
- BABAR, BELLE, LHCb, BES II/III
- theorists (not many) aficionados to hadron spectroscopy

***Abstract*.**

- Introducing quarks to describe the known mesons ($q\bar{q}$) and baryons (qqq) , Murray Gell-Mann, in 1964, suggested the existence of further $qq\bar{q}\bar{q}$ mesons (tetraquarks) and $qqqq\bar{q}$ baryons (pentaquarks). The first *unexpected hadron*, the $X(3872)$, was discovered by BELLE in 2003, confirmed by BABAR and seen in many other High Energy experiments. Since then, a wealth of Exotic Hadrons have been observed, mesons and baryons that cannot be described by the classical Gell-Mann configurations.
- Restricting to tetraquarks, I illustrate the attempts to understand their structure by the laws of Quantum Chromodynamics, QCD. Observations by BABAR, BELLE, LHCb, BES II/III of tetraquarks with different flavours lead to definite predictions about the missing particles needed to *complete SU(3) flavour mutiplets*, whose observation would provide a critical test of the present ideas about the dynamics of Exotic Hadrons.

1. QCD, Asymptotic Freedom and Heavy Quarks





- At distances of order $M_{c,b}^{-1}$, QCD interaction is already reduced, to allow perturbation theory to be, at least, a good guide.
- This is what makes Charmonium Spectrum calculable, as anticipated by Appelquist and Politzer

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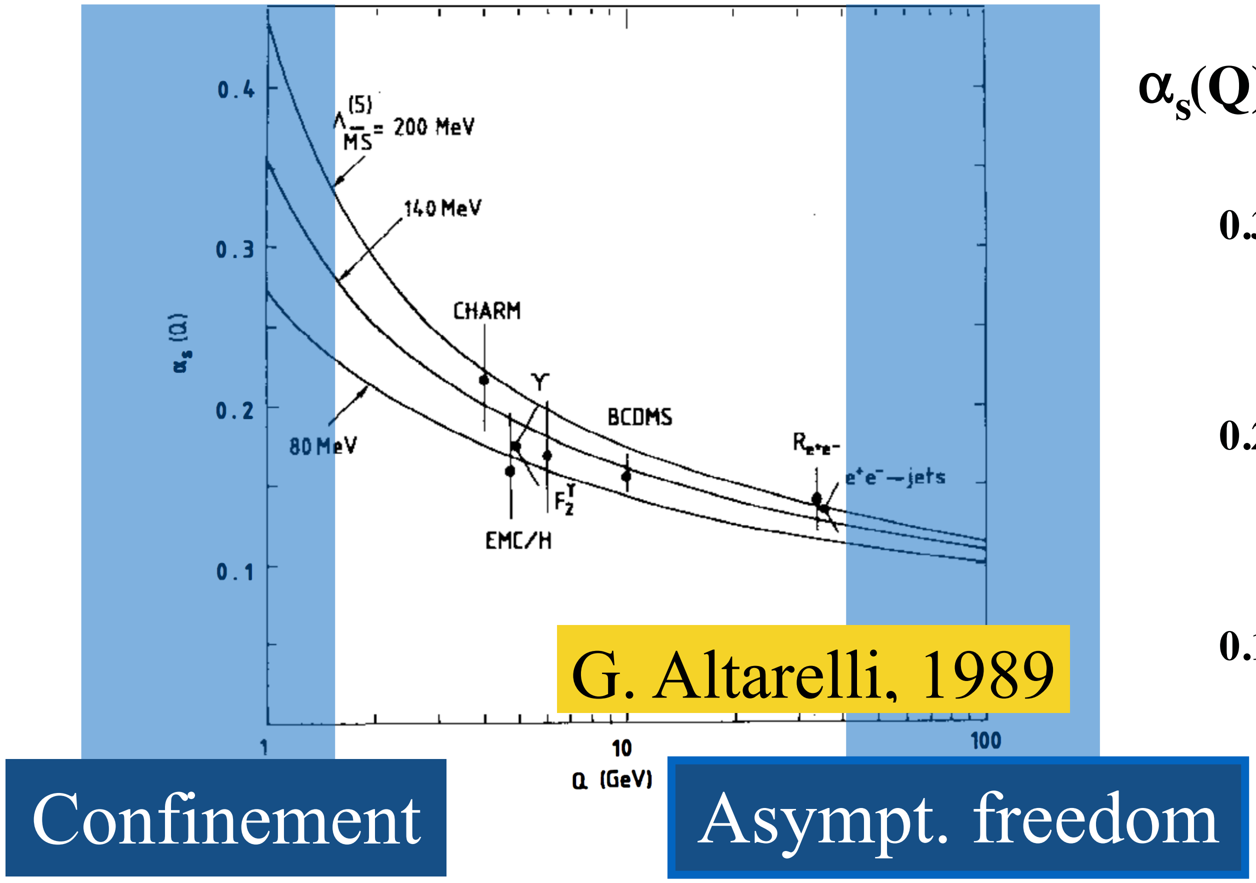

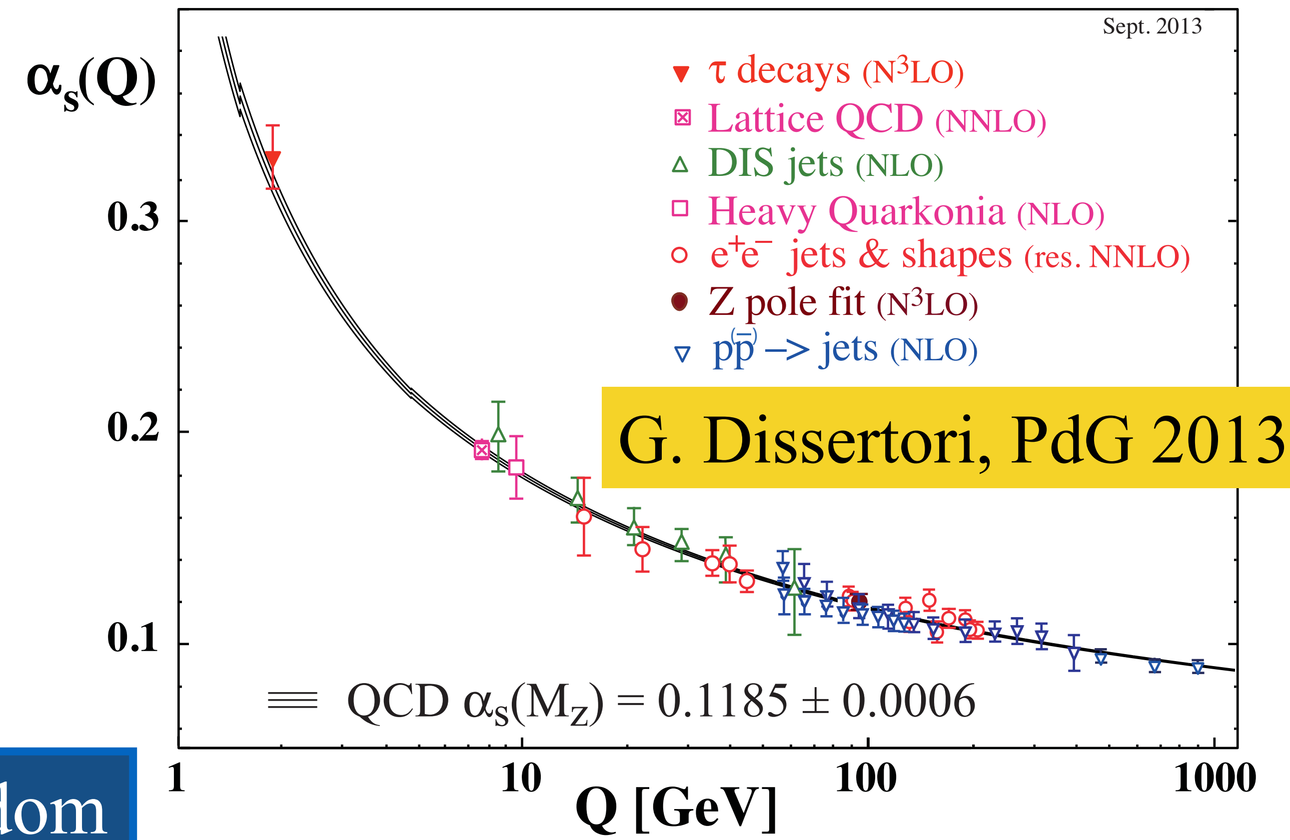

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Heavy quarks ***(*** $m_Q >> \Lambda_{QCD}$ ***)***:

- inclusive decays are calculable like deep inelastic processes;
- $c\bar{c}$ or $b\bar{b}$ bound states involve short distance forces: a calculable spectrum of charmonia/bottomonia;
- inside hadrons, $c\bar{c}$ or $b\bar{b}$ pairs are not easily created or destroyed:
- a hadron decaying into J/Ψ or $\Upsilon + \ldots$ indicates a valence $c\bar{c}$ or $b\bar{b}$ pair
- ***heavy-quark counting is possible***.

2. Unanticipated charmonia X, Y, Z… and more

- ***Unanticipated, hidden charm/beauty resonances not fitting in predicted charmonium/bottomonium spectra have been observed.***

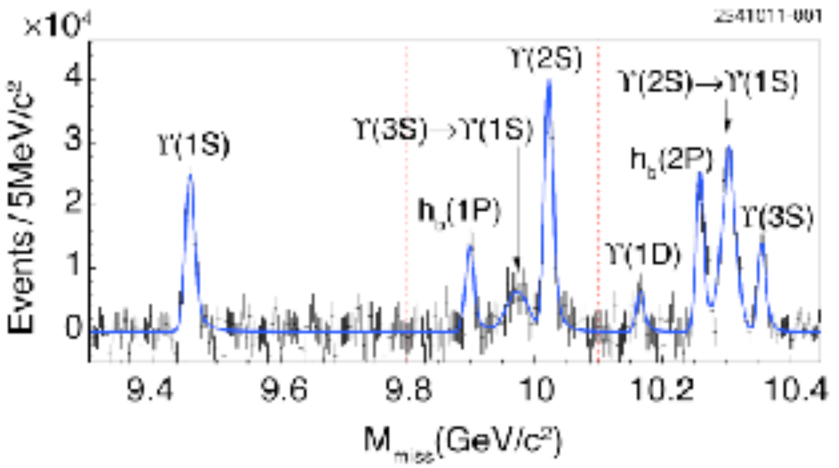

Figure 1: From Belle [31], the mass recoiling against $\pi^+\pi^-$ pairs, M_{miss}, in e^+e^- collision

- **X**, e.g. *X*(3872): neutral, typically seen in $\Psi + 2\ \pi$, positive parity, $J^{PC} = 0^{++}$, 1^{++}, 2^{++}
- **Y**, e.g. Y(4260): neutral, seen in e^+e^- annihilation with *Initial State Radiation* (ISR):

$$e^+e^- \to e^+e^- + \gamma_{ISR} \to Y + \gamma_{ISR}$$, therefore $J^{PC} = 1^{--}$,

- **Z**, e.g. Z(4430): typically $J^{PC} = 1^{+-}$, charged or neutral, 4 valence quarks manifest, mostly seen to decay in Ψ+ π and some in h_c(1P) +π (valence quarks: $c\bar{c}u\bar{d}$); Z_b observed ($b\bar{b}u\bar{d}$).

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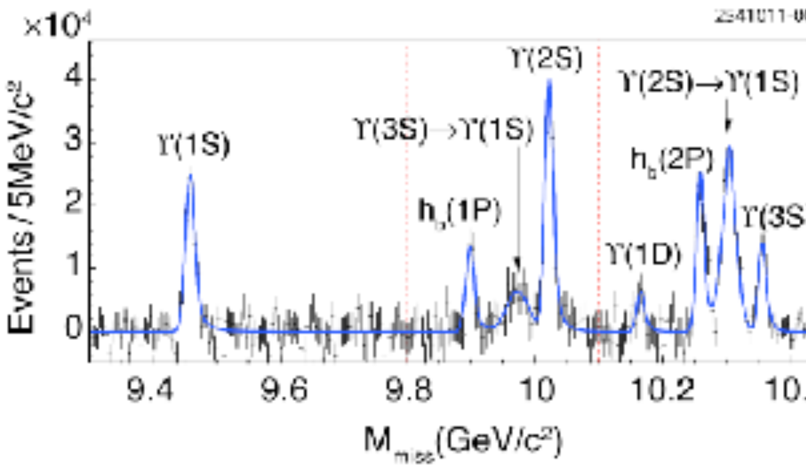

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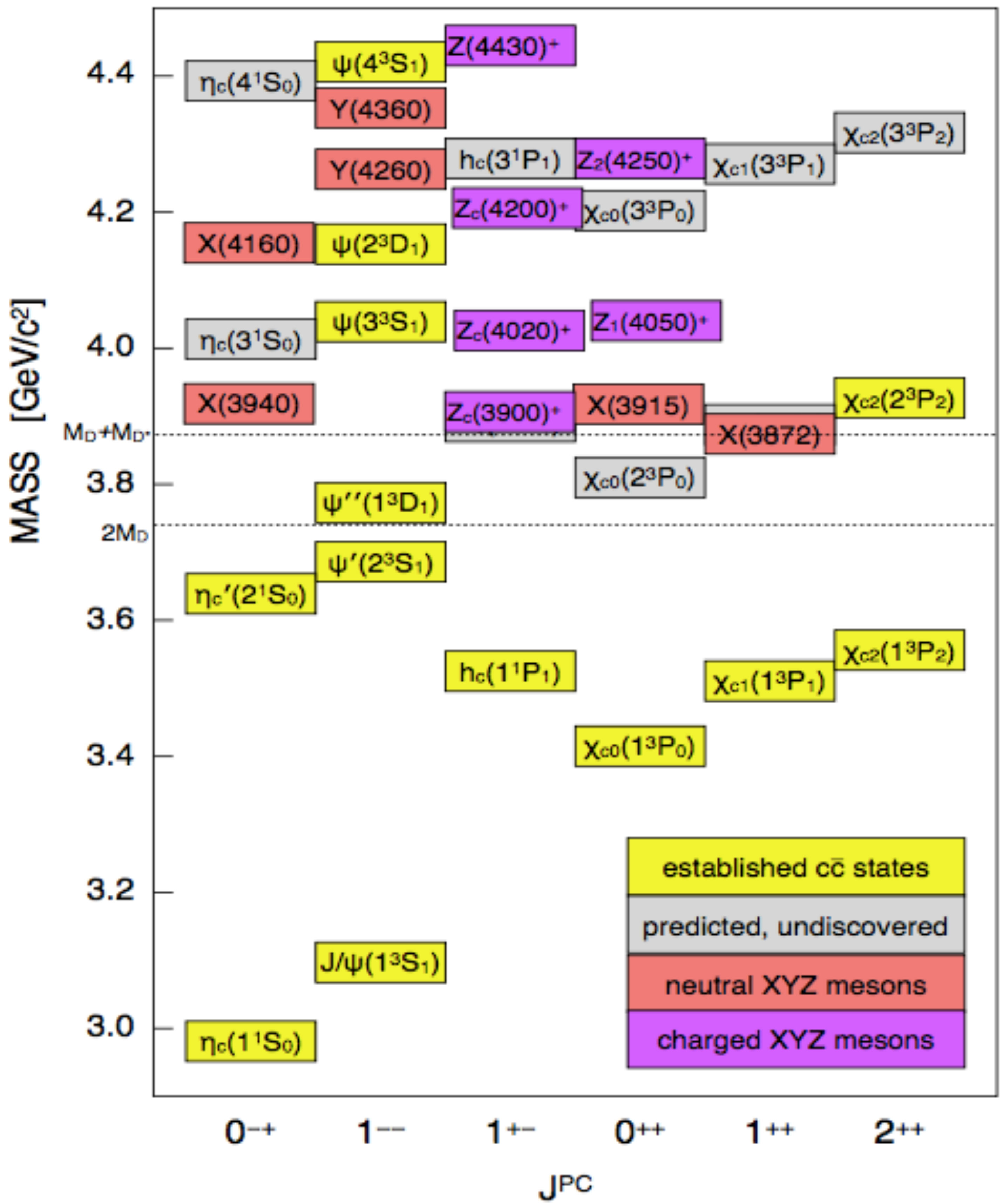
A new wave of Exotic Hadrons from 2015:

- Hidden charm pentaquarks, $\mathscr{P} \to J/\Psi + p$;
- Hidden charm and Hidden strangeness (LHCb), e.g. $X(4140) \to \Psi + \phi$, $J^{PC} = 1^{++}$
- 4 charm tetraquarks seen as di-Ψ resonances(LHCb), $X(6900) \to \Psi + \Psi \to 2(\mu^+\mu^-)$
- Hidden charm- Open strangeness ($c\bar{c}u\bar{s}$),: $Z_{cs}^+(3985) \to \Psi + K^+$ (BES III) and $Z_{cs}^+(4003) \to \Psi + K^+$ (LHCb).
- Double charm tetraquark: $\mathscr{T}_{cc}^+(3875) \to D^0 D^0 \pi^+$ (valence quarks: $cc\bar{u}\bar{d}$) by LHCb.

Expected and Unexpected Charmonia in 2015

figure by:
S. L. Olsen (2015)
arXiv:1511.01589



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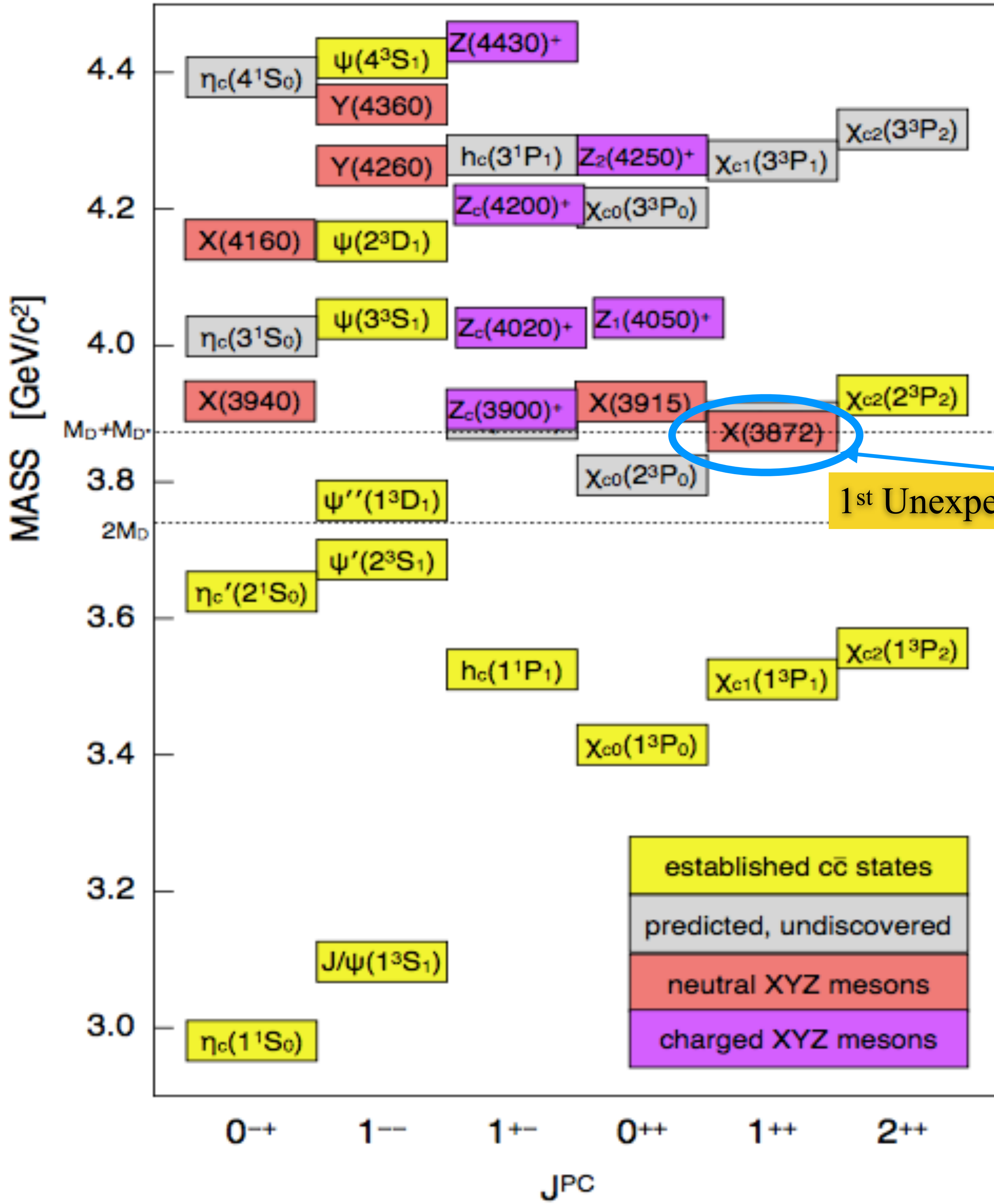

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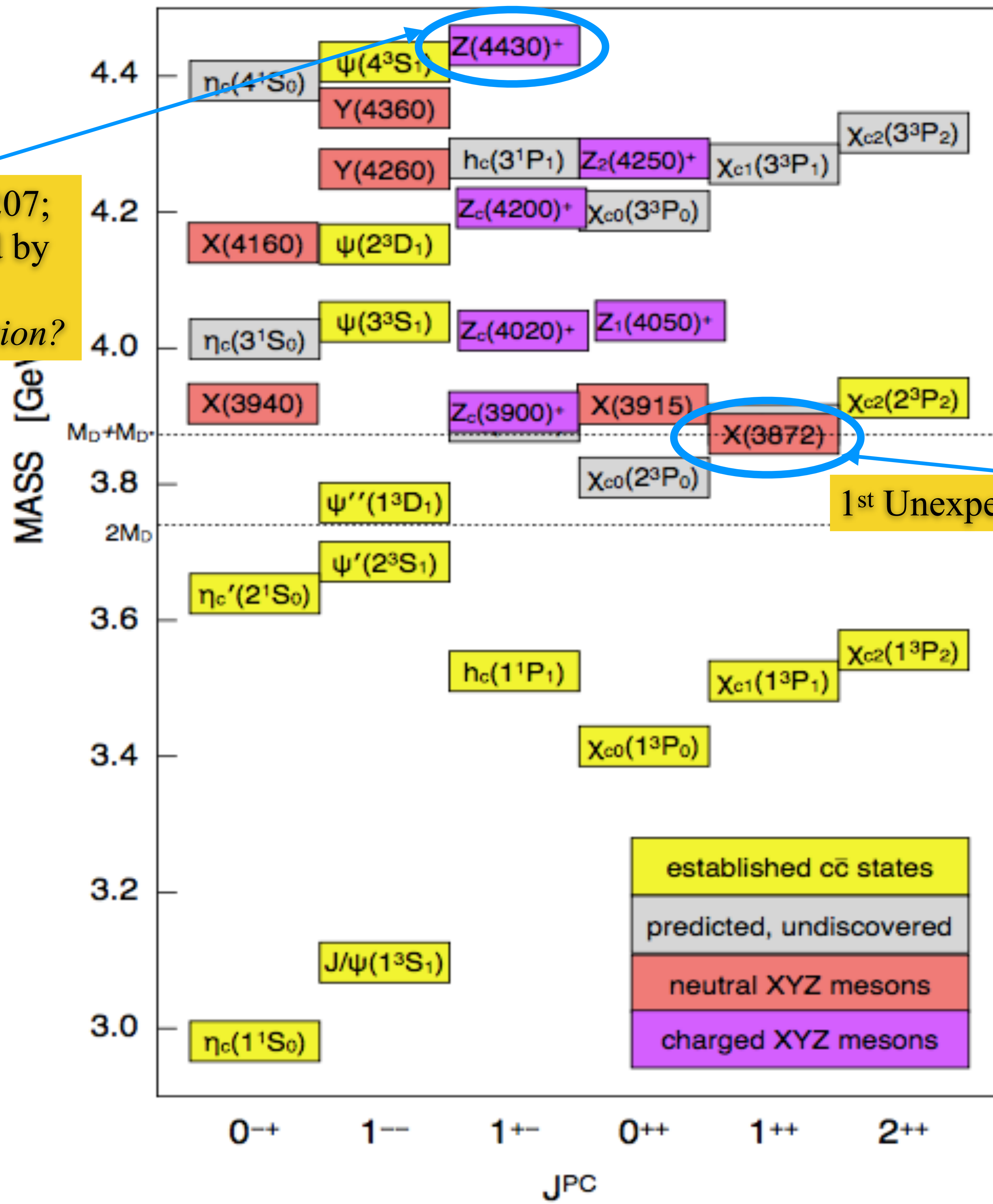

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first charged, a radial excitation?

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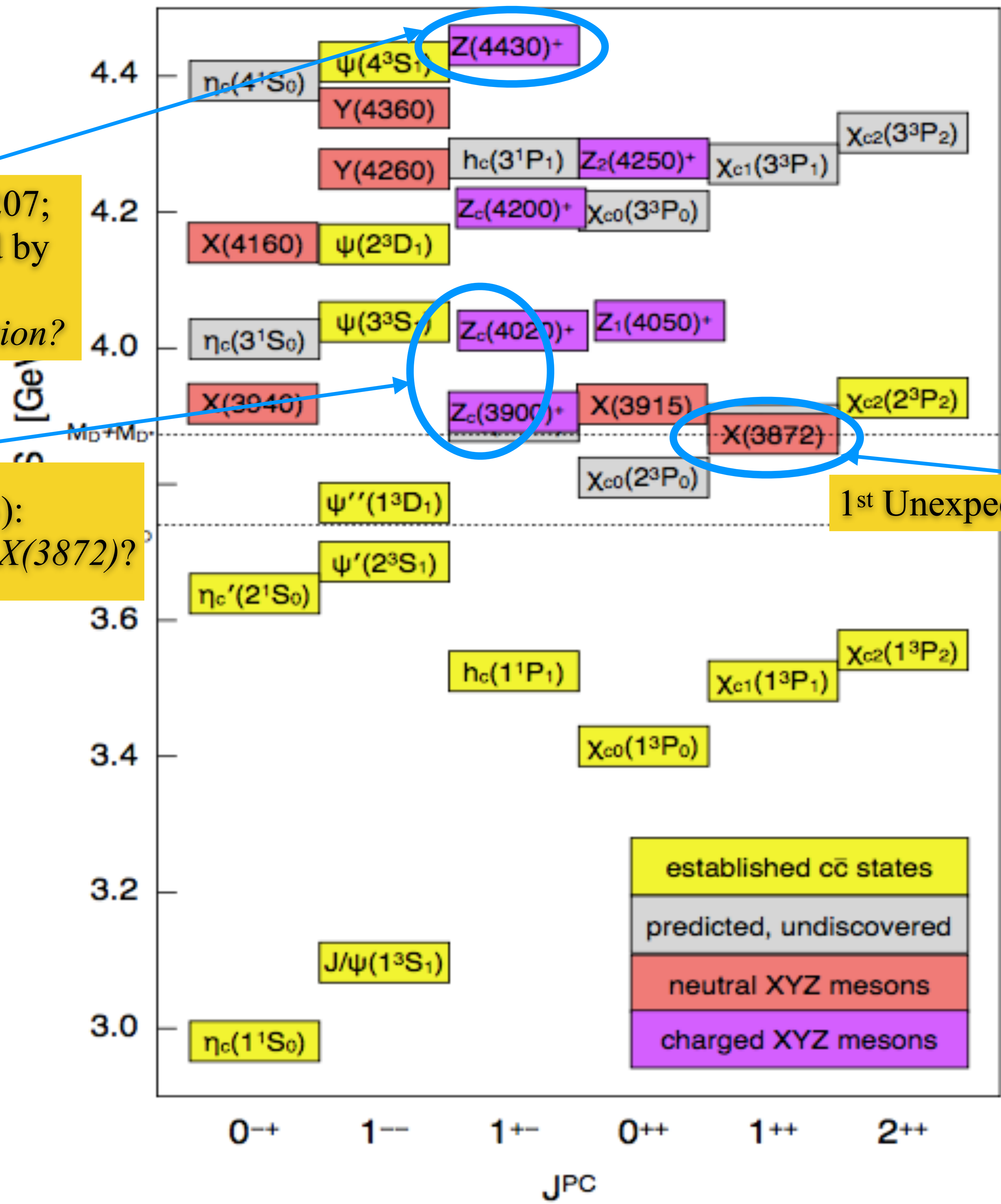

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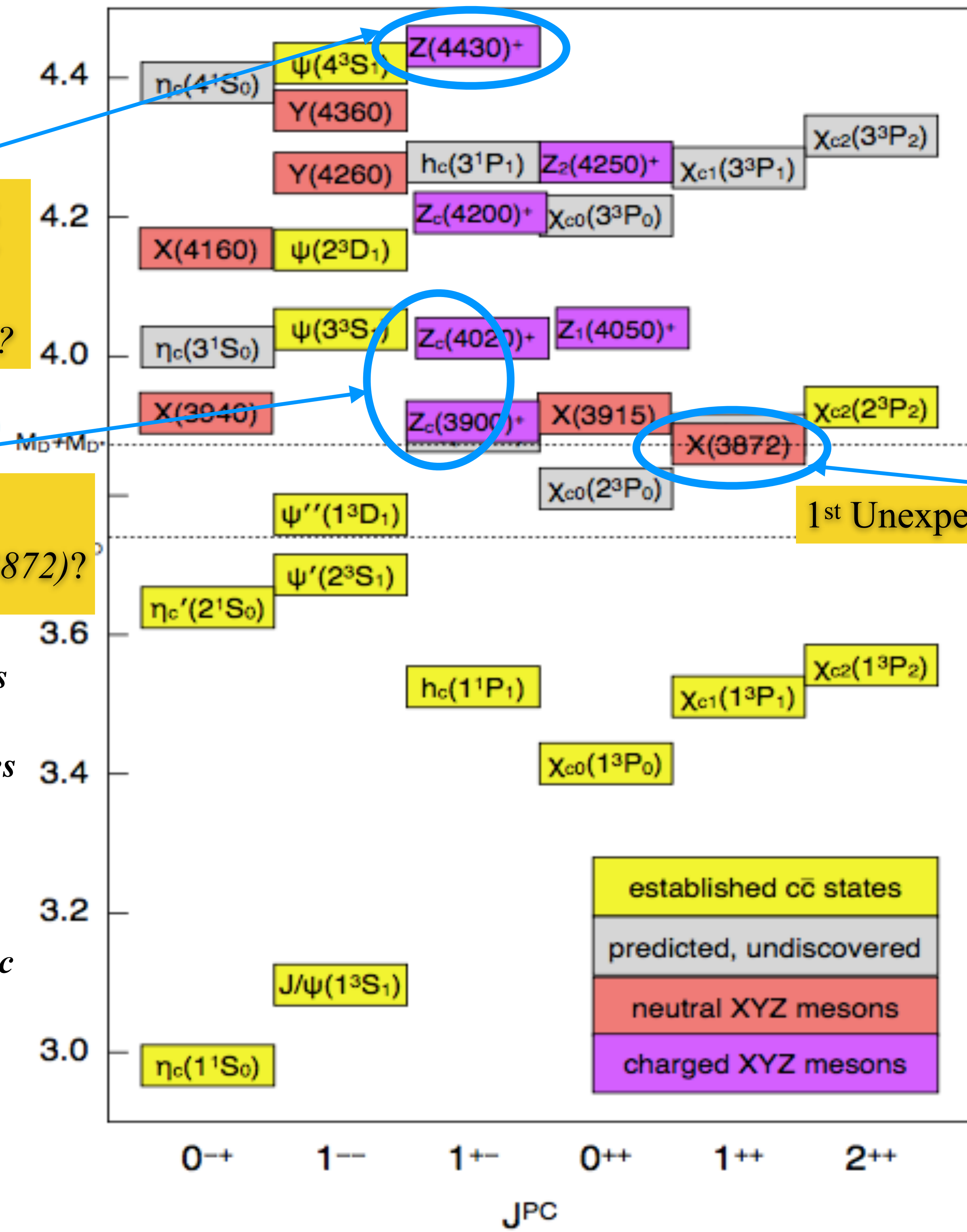

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recent additions:
more than coincidence?
or
an almost filled multiplet?

$\pi^+\chi_{c1}(1P)$

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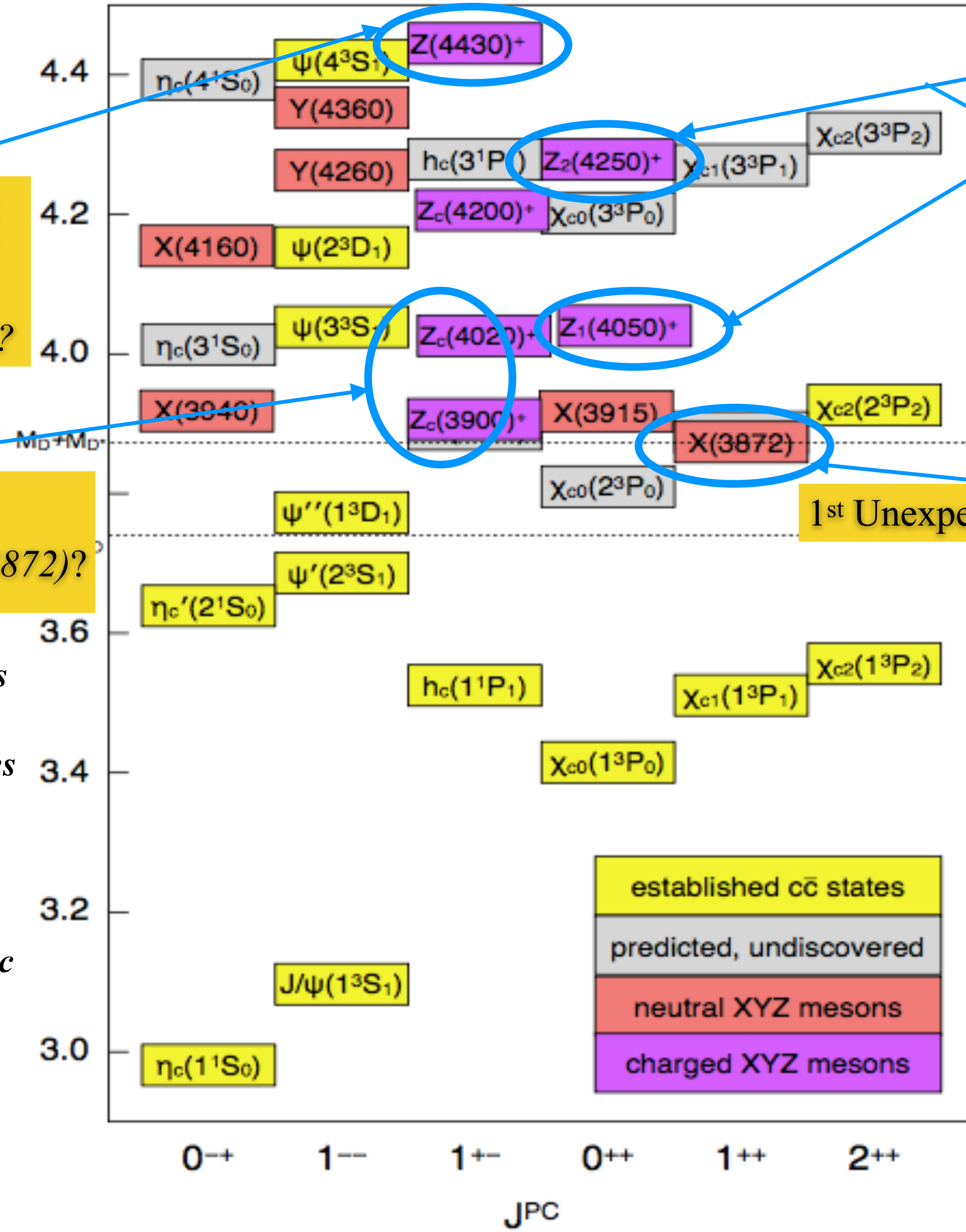

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Exotics: More flavours

Starting from 2016, new kinds of exotic hadrons have been discovered:

- $J/\Psi\ \phi$ resonances, $di - J/\Psi$ resonances,
- open strangeness Exotics: $Z_{cs}(3082)$ and $Z_{cs}(4003)$

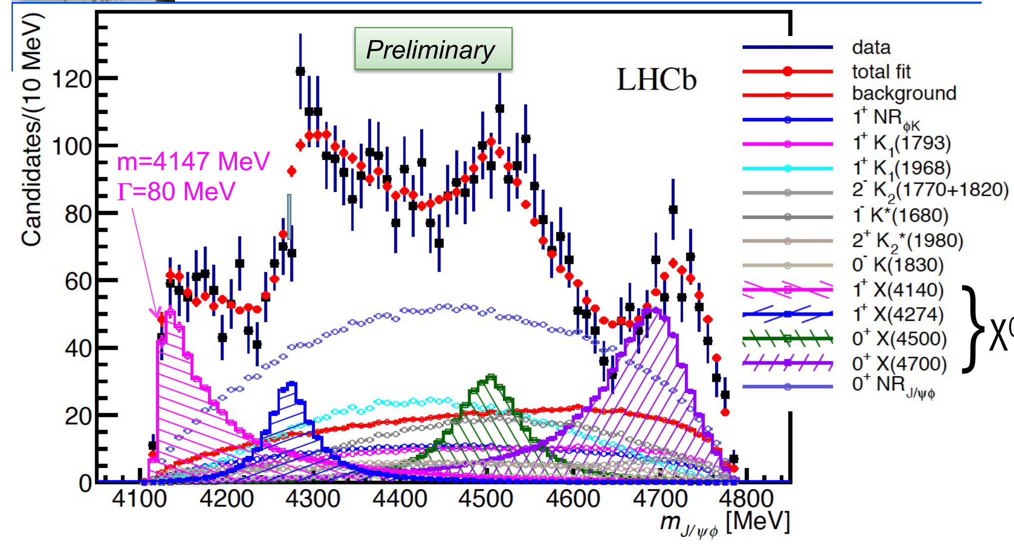
No pion exchange forces could bind them as hadron molecules made by color singlet mesons

molecular models applied to the new hadrons have to stand on the existence of "phenomenological forces" with undetermined parameters

The New Exotics arise very naturally as $([cq]^{\bar{\mathbf{3}}}[\bar{c}\bar{q}']_{\mathbf{3}})_{\mathbf{1}}$ bound in color singlet.

$B^+ \to K^+ + X(4140) \to K^+ + \phi\ \Psi$, etc .

Results of fit: m(J/ψϕ)



- 4 visible structures fit with BW amplitudes

LHCb (2016): $\Psi\ \phi$ resonances (2016)

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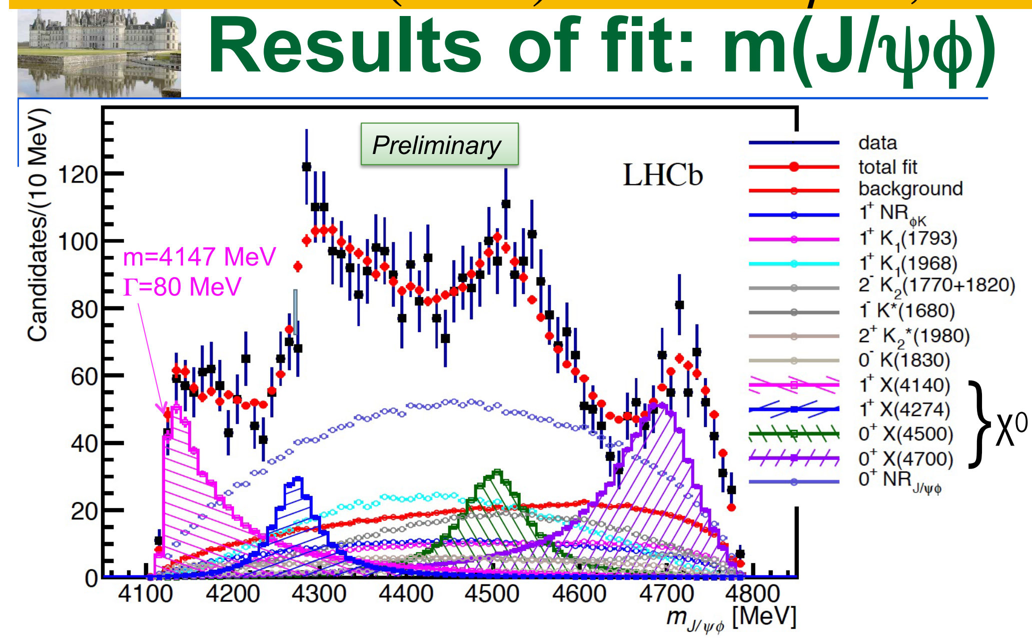

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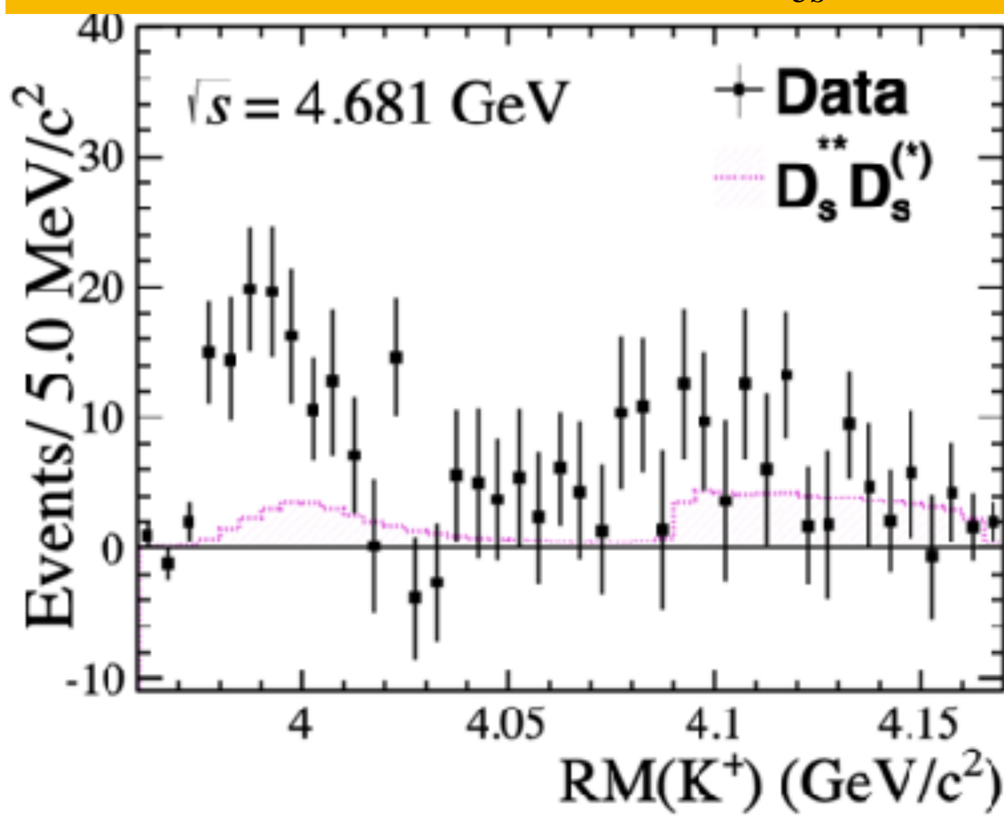

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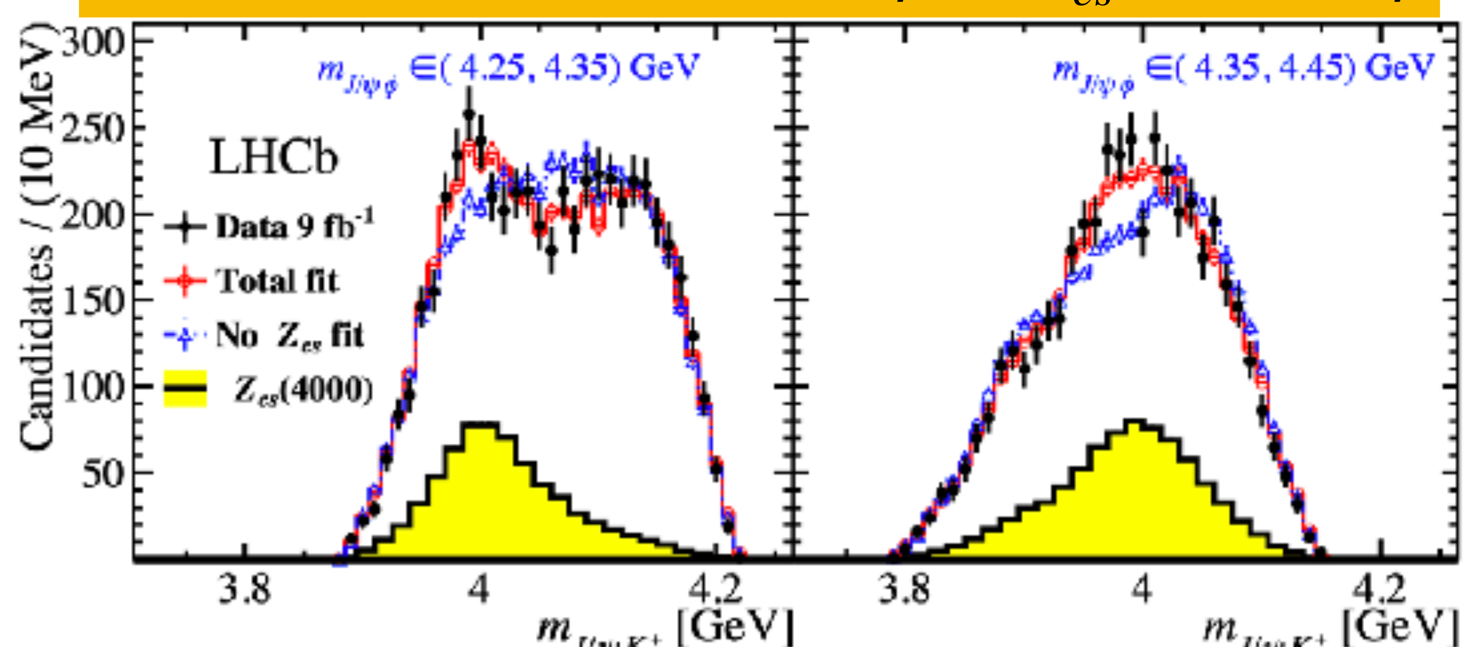

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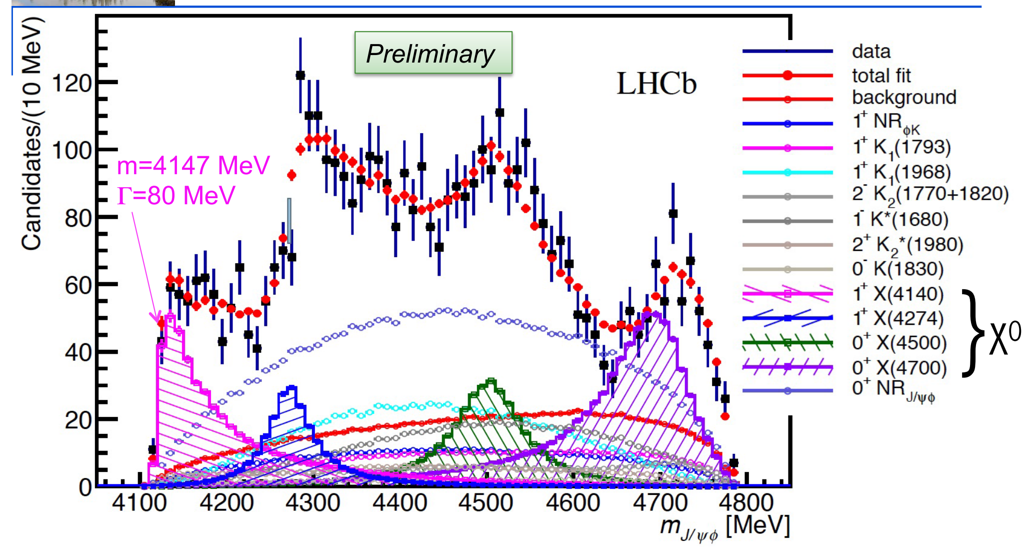

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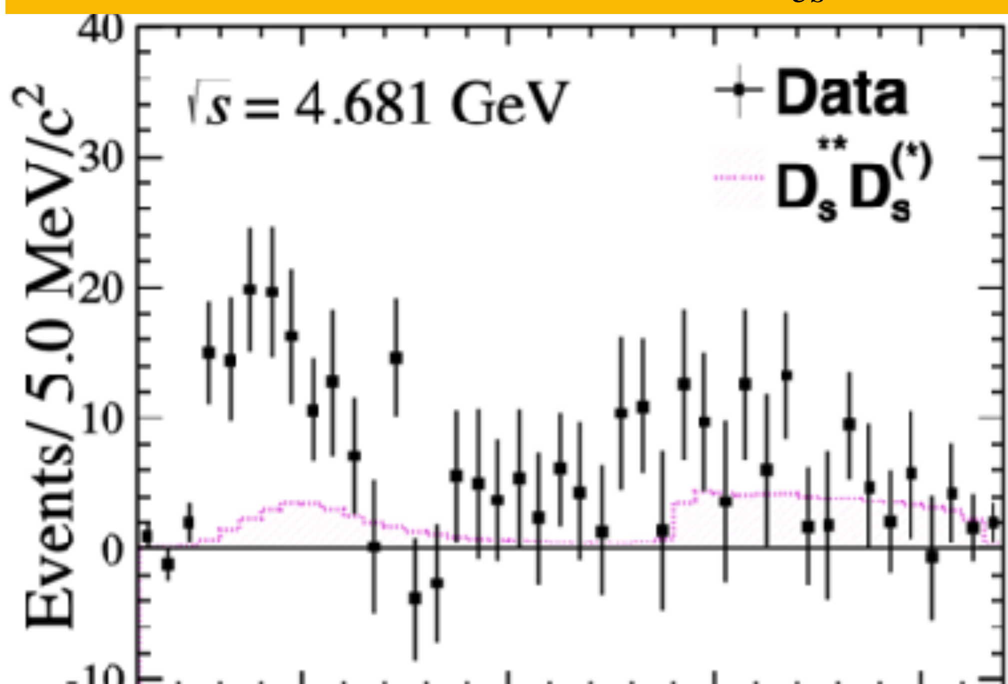

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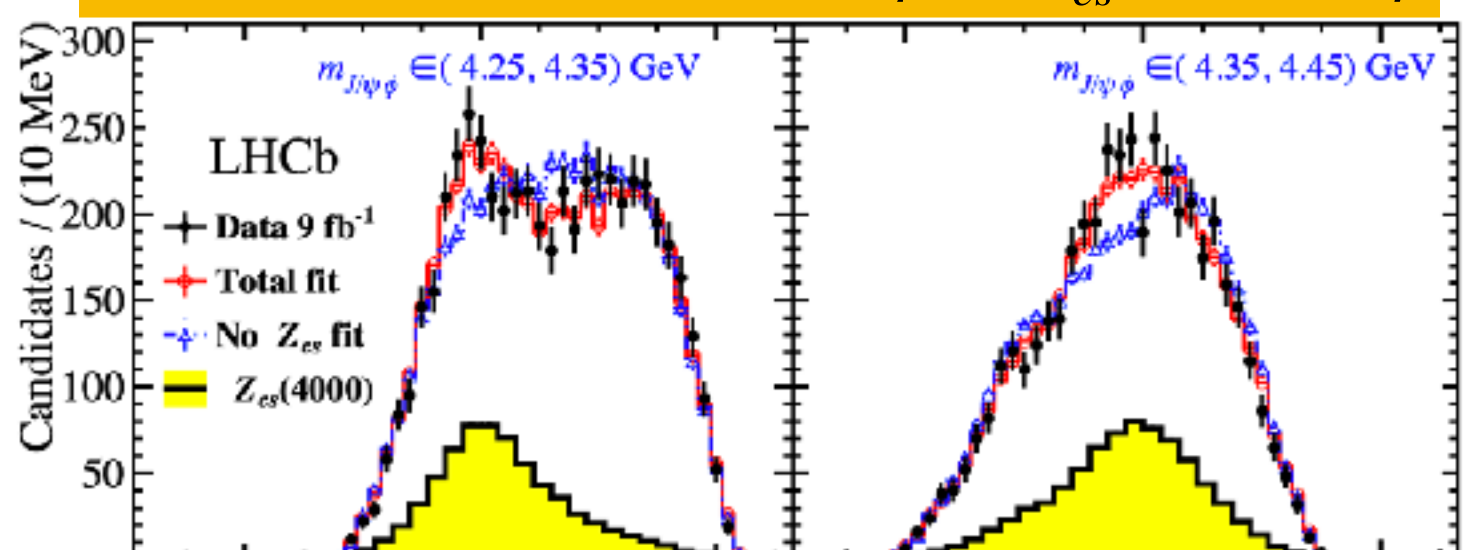
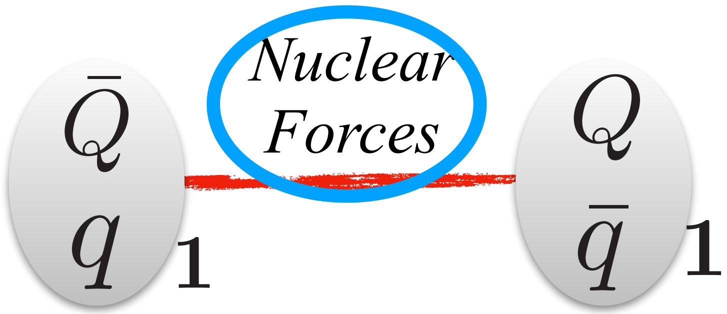


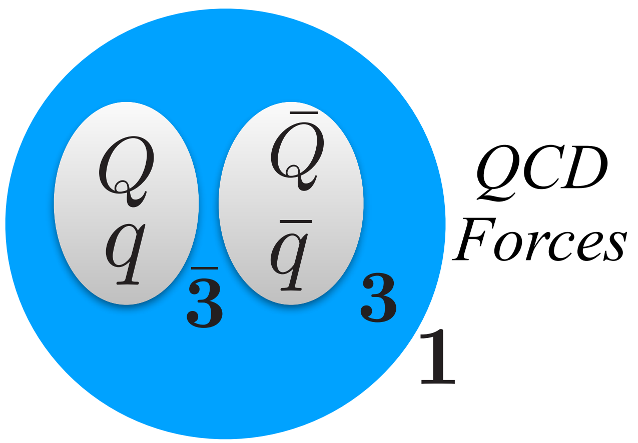
PdG: The incompatible values for the widths reported by AAIJ 2021E and ABLIKIM 2021G could either indicate the existence of two separate states or possibly be explained in a coupled channel model (see ORTEGA 2021

No consensus, yet



Hadron Molecule

F-K. Guo, C. Hanhart, U-G Meißner, Q. Wang, Q. Zhao, and B-S Zou, arXiv 1705.00141 (2017)



Compact Diquark-Antidiquark

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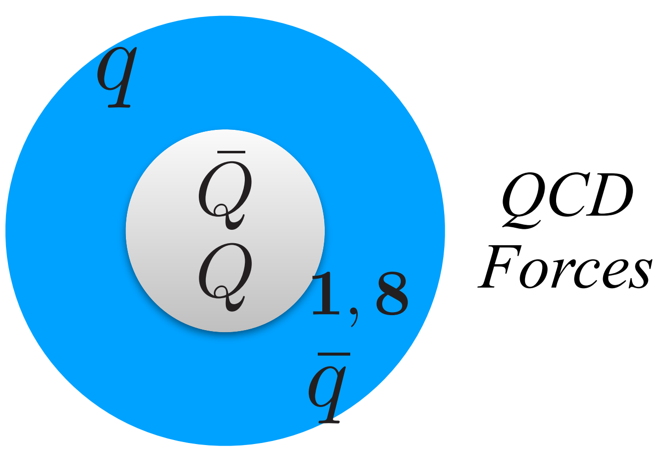

HadroCharmonium (1)
Quarkonium Adjoint Meson (8)

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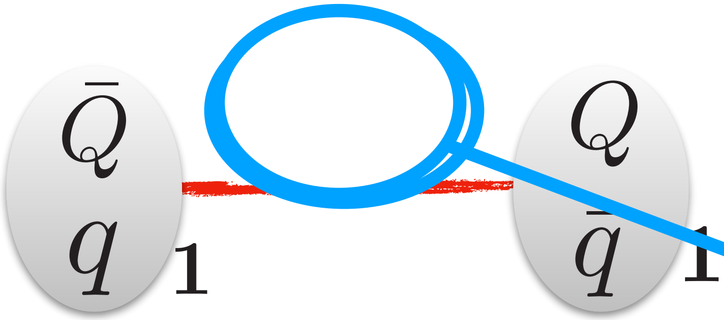

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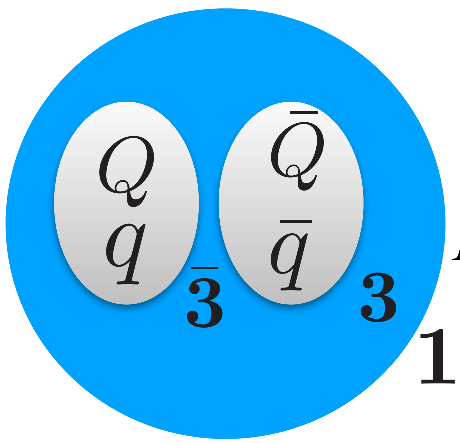
Z. H. Zhang *et al.*, arXiv:2404.11215 [hep-ph]

see later



QCD Forces

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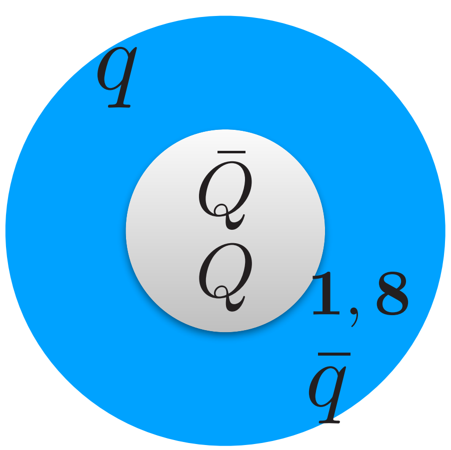

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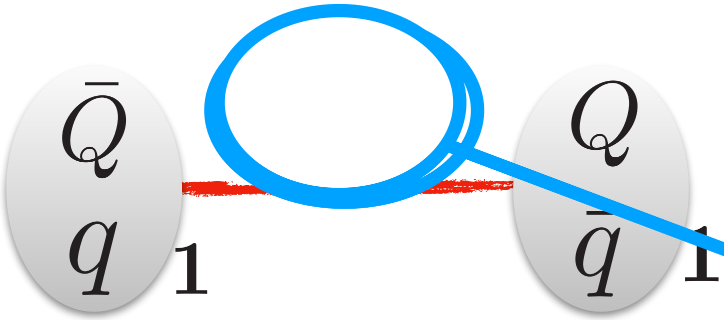

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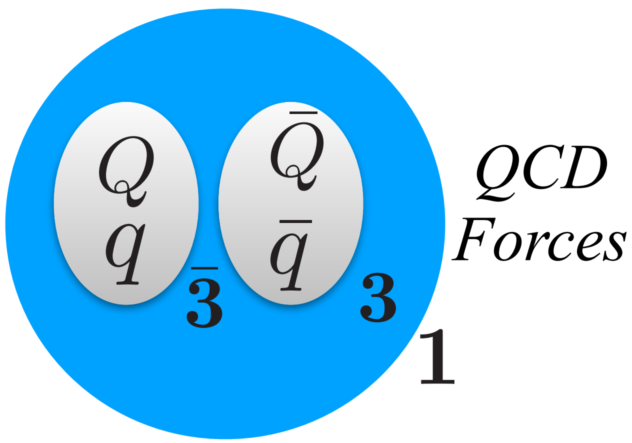

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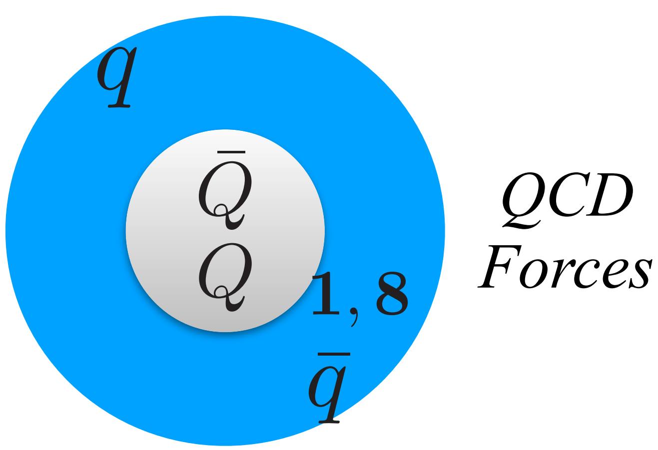

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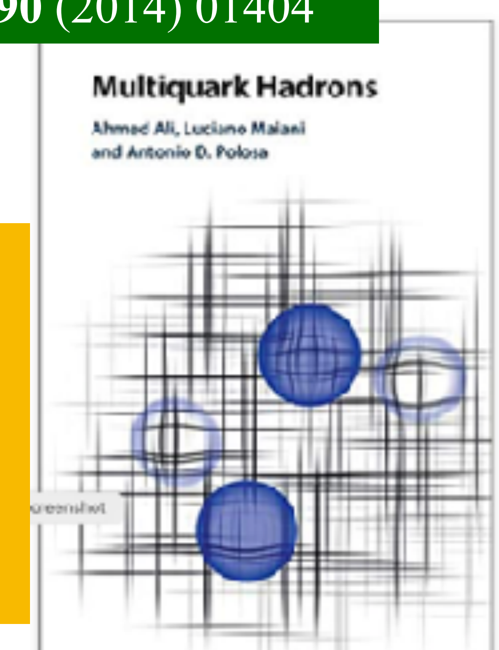
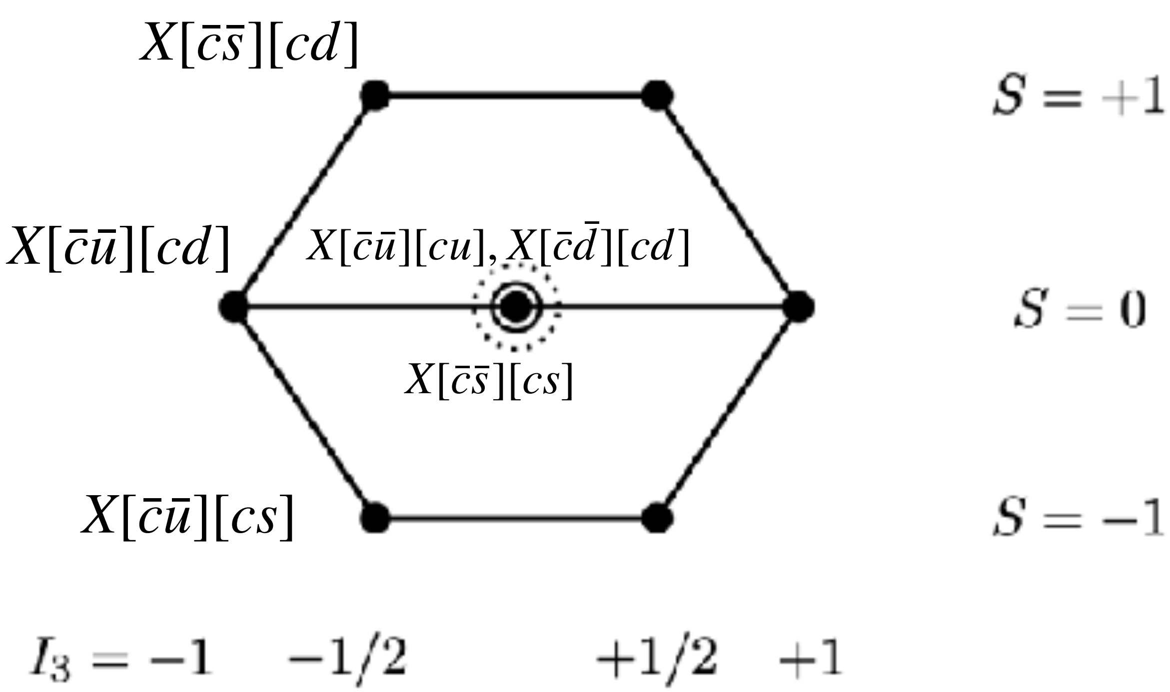
For a review, see:
A. Ali, L. Maiani and A.D. Polosa, *Multiquark Hadrons*, Cambridge University Press (2019)



3. Hidden Charm Tetraquarks form nonets of flavor SU(3) with mass differences determined by the quark mass difference $m_s - m_{u,d}$

with Z_{cs}(3082), Z_{cs}(4003), Z_{cs}(4220) we can almost fill three tetraquark nonets with the expected scale of mass differences



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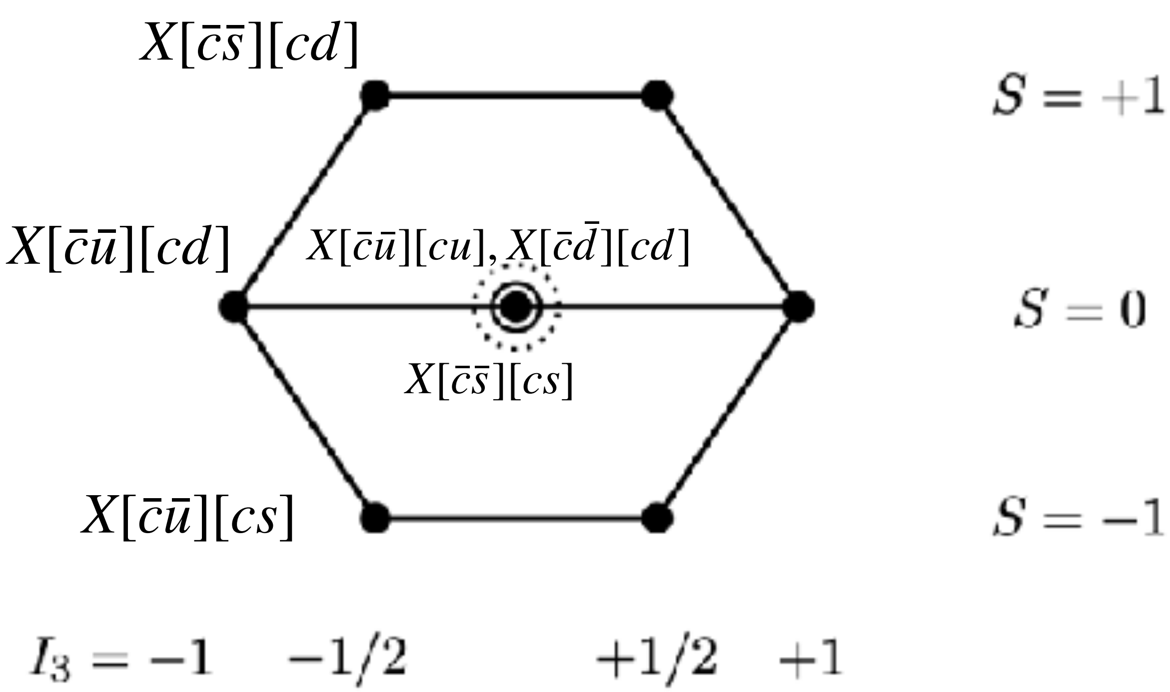

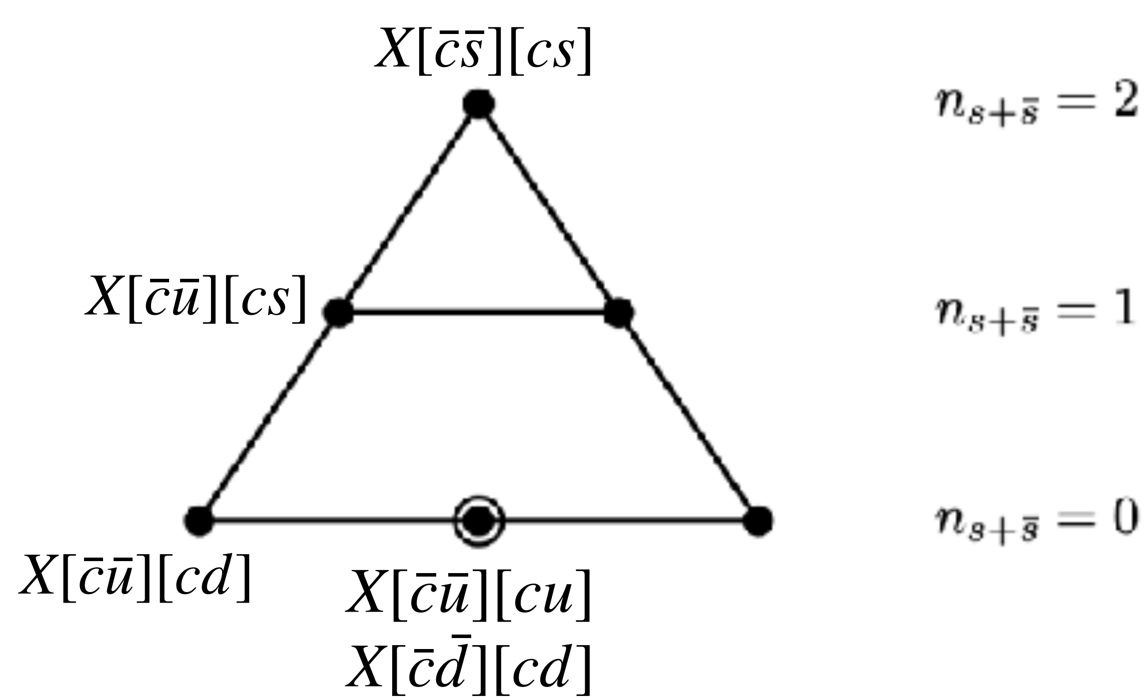

- Octet particles can be also represented in function of the total number of s or $\bar{s}$ quarks;
- octet breaking implies ***the equal spacing rule*** of the masses in the ladder

3. Three nonets: 2 solutions

L. Maiani, A. D. Polosa and V. Riquer, Sci. Bull. **66** (2021), 1616, arXiv:2103.08331

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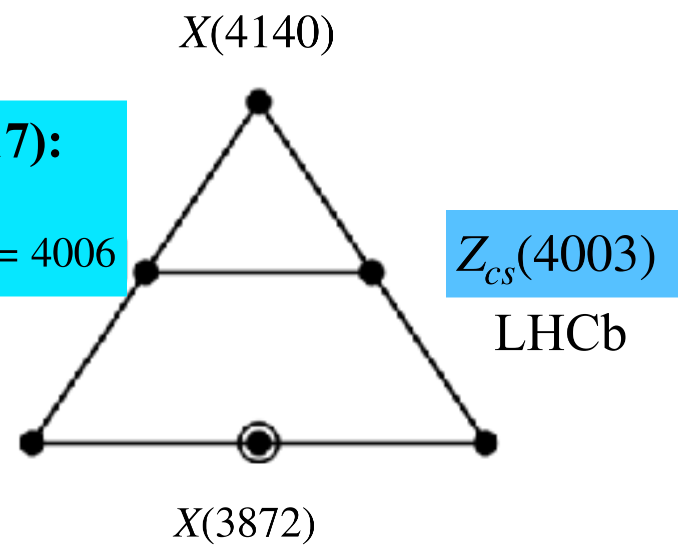
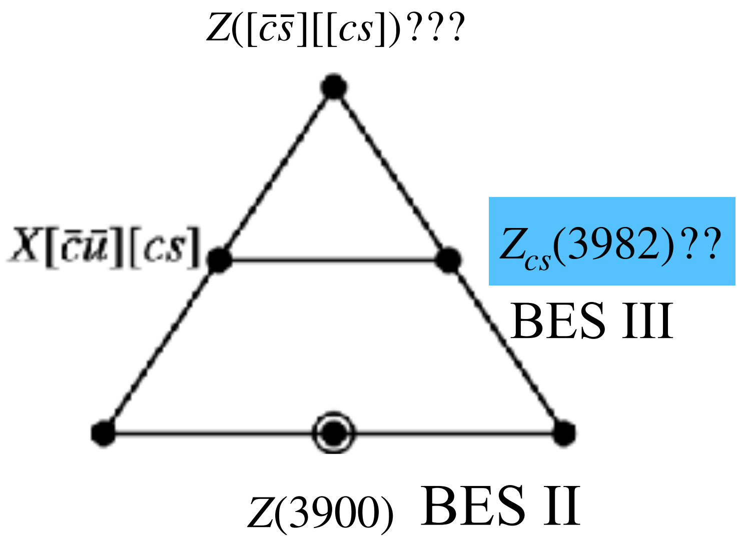
Solution 1

Predicted (2017):

$$\frac{X(4140) + X(3872)}{2} = 4006$$





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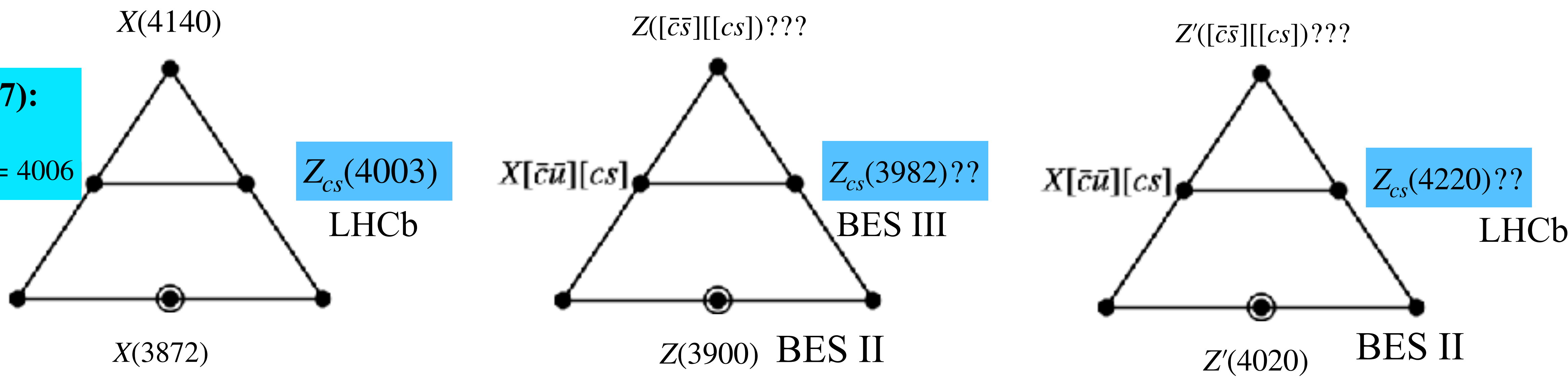

- There is a ***third nonet associated with*** $Z_c(4020)$, $J^{PC} = 1^{+-}$: a third Z_{cs} is required, with Mass=4150 - 4170
- LHCb sees a $Z_{cs}(4220)$, $J^P = 1^+$***: is it too heavy ?***
- A bold proposal: ***two nonets mixing***

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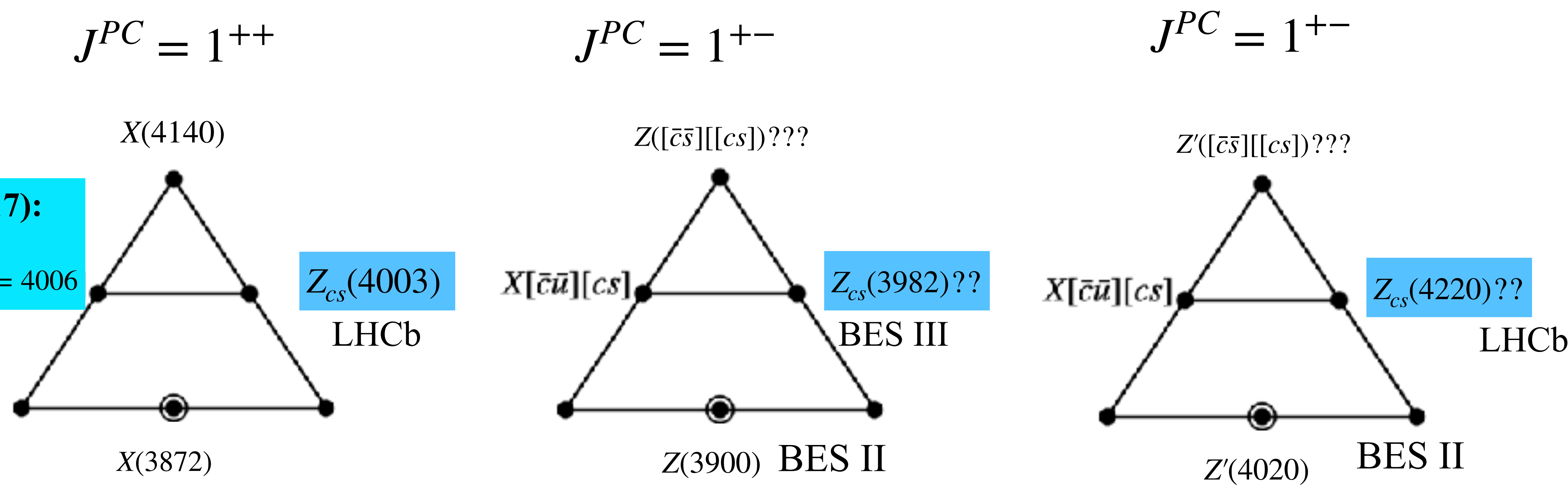
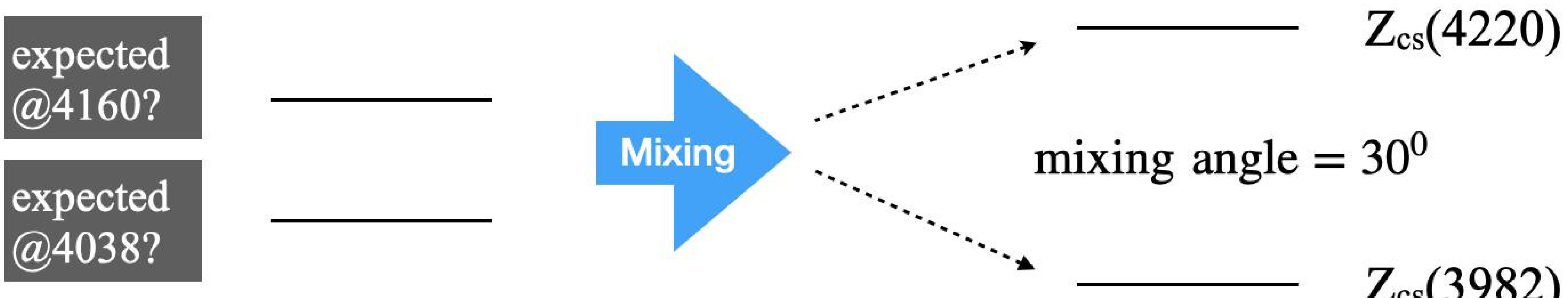
Predicted (2017):

$$\frac{X(4140)+X(3872)}{2} = 4006$$

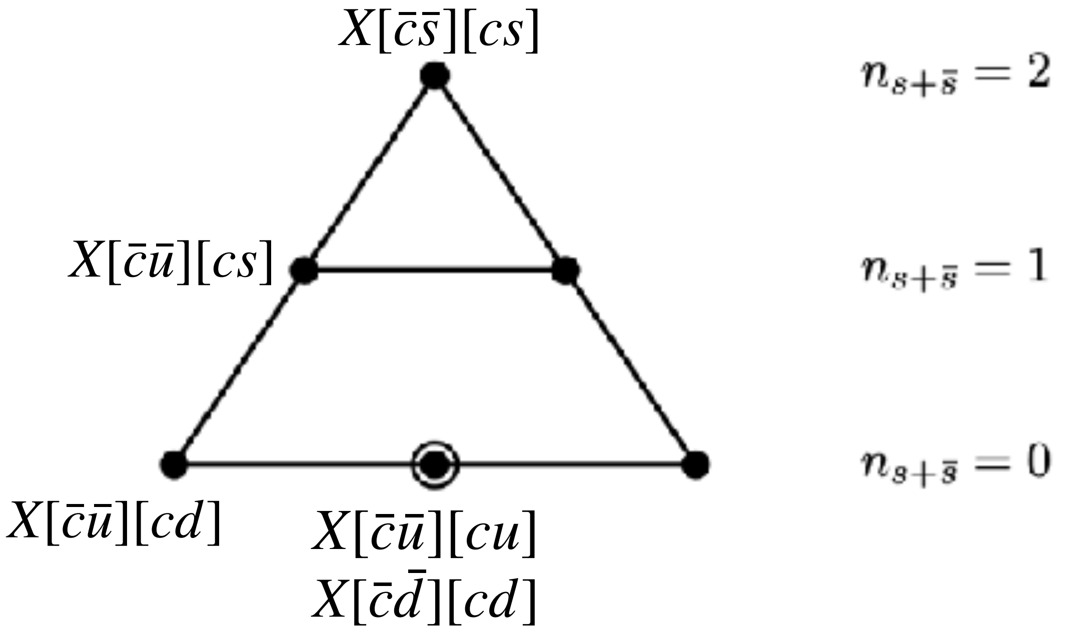


- There is a ***third nonet associated with*** $Z_c(4020)$, $J^{PC} = 1^{+-}$: a third Z_{cs} is required, with Mass=4150 - 4170
- LHCb sees a $Z_{cs}(4220)$, $J^P = 1^+$***: is it too heavy ?***
- A bold proposal: ***two nonets mixing***



Missing particles to complete the hidden charm nonets, $J^P = 1^+$

The shopping list towards completion of the ***hidden charm nonets*** (for ***Solution 1***)



- two $X_{[\bar{c}\bar{s}][cs]}$, expected at:

$M \sim 4170$ for $Z_c(3900)$ and $M \sim 4290$ for $Z_c(4020)$

with decays: $\eta\psi$, $\eta_c\ \phi$, $D_s^*\bar{D}_s$ (threshold: 4080 MeV)

- the I=1 partner of X(3872), with decay:

$$X^+ \to J/\psi\ \rho^\pm \to J/\psi\ \pi^+\pi^0 \text{ with the bounds:}$$

$$0.057 < R_{2\pi}^{(0+,00)} = \frac{\Gamma(B^0 \to K^+X^- \to K^+\psi\ \pi^0\pi^-)}{\Gamma(B^0 \to K^0X(3872) \to K^0\psi\ \pi^+\pi^-)} < 0.50$$

L. Maiani, A. D. Polosa and V. Riquer, Phys. Rev. D **102** (2020) 034017

- the I=0 partners of $Z_c(3900)$ and $Z_c(4020)$, possibly decaying into:

$$J/\psi + f_0(500) \text{ (aka } \sigma(500))$$

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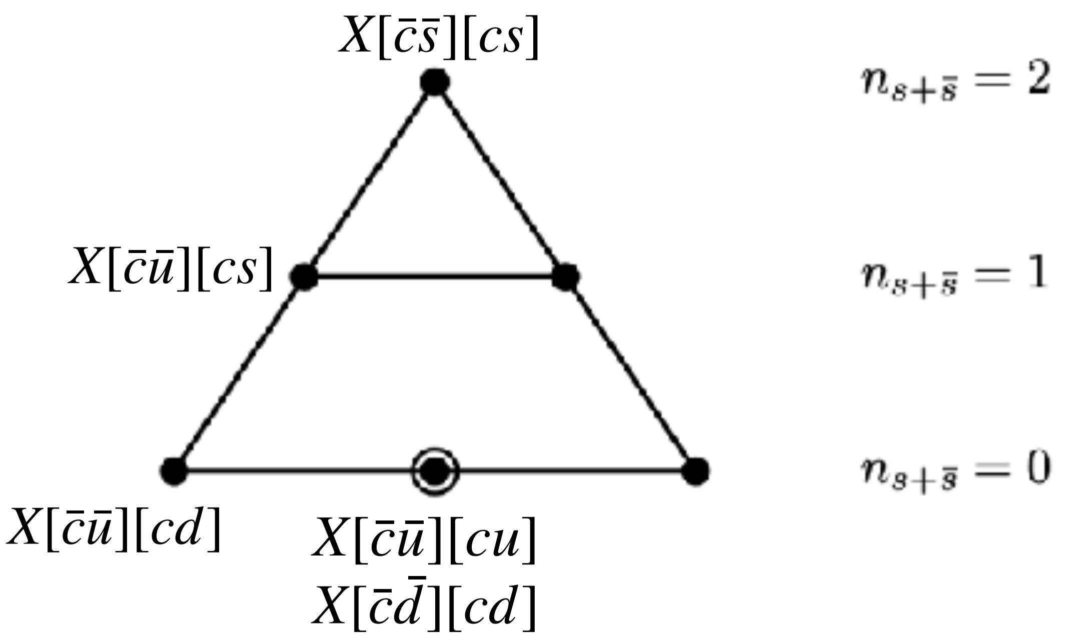

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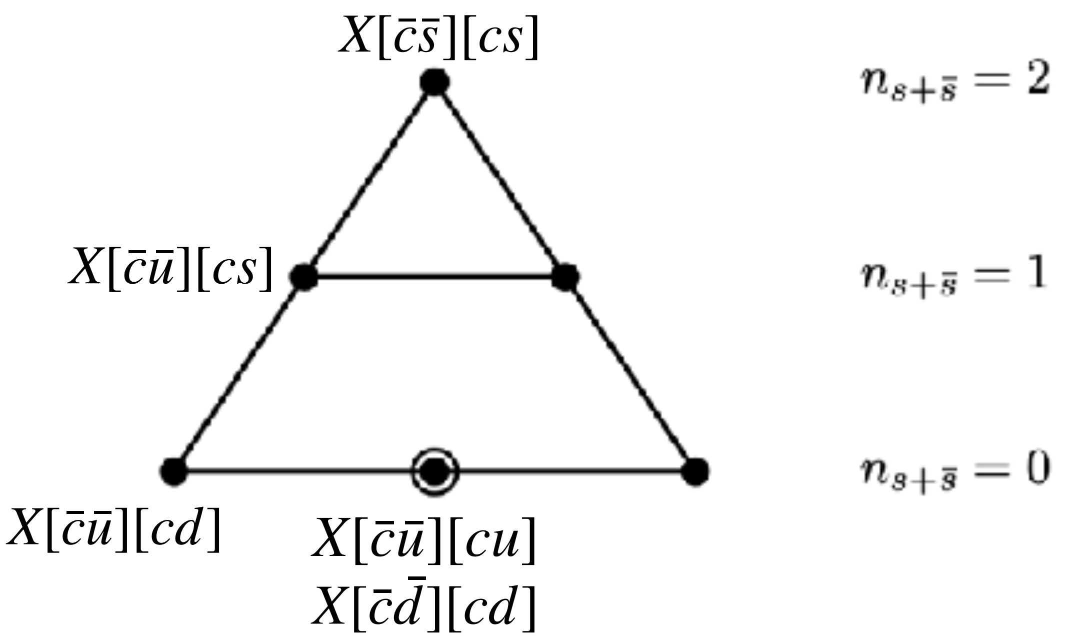

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Only few particles are missing, in well defined mass regions and with identified decay modes.

4. Single charm tetraquarks, with three SU(3)-flavour light mesons: the case of $J^P = 0^+$

L.Maiani, A. Polosa, V.Riquer, Phys. Rev. D **110** (2024) 034014

- In a recent lattice QCD calculation the $SU(3)_{flavor}$ configurations of possible bound states in the $\bar{D}K, J^P = 0^+,$ channel are studied;
- the allowed $SU(3)_{flavor}$ channels are those appearing as irreducible components of the tensor product

J. D. E.Yeo *et al.,* JHEP **07** (2024) 012

$$\bar{D}K = \mathbf{3} \otimes \mathbf{8} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15}$$

- Yeo *et al.* find attraction in $\mathbf{3}$ and $\bar{\mathbf{6}}$ but not in $\mathbf{15}$.

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- Yeo *et al.* find attraction in **3** and $\bar{\mathbf{6}}$ but not in **15**.

- Consider now four-quark mesons in the simplest diquark-antidiquark model restricting to all spin zero case:

$$[\bar{c}\bar{q}]^{\mathbf{3}}_{S_{c3}}[q_1 q_2]^{\bar{\mathbf{3}}}_{S_{12}}, \qquad S_{c3} = S_{12} = 0;\ \ J^P = S_{c3} + S_{12} = 0^+$$

- The product $[q_1 q_2]^{\bar{\mathbf{3}}}_{S_{12}=0}$ is antisymmetric in spin (to get total spin 0) and color (to obtain a $\bar{\mathbf{3}}_c$).

- Fermi statistics: *quarks in the light diquark must be antisymmetric in flavour, i.e must be in a* $\bar{\mathbf{3}}$ *of* $SU(3)_{flavor}$.
- combining with the light antiquark $\bar{q} \propto \bar{\mathbf{3}}$, the tetraquark must be in a $SU(3)_{flavor}$ multiplet:

$$\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \bar{\mathbf{6}}\text{ , no }\mathbf{15}$$

in agreement with the lattice indication.

$J^P = 0^+$, Open Charm and Strangeness Tetraquarks: what do we know ?

- We can restrict to particles with Charm C=-1(C=+1 particles obtained by CPT symmetry), i.e. with quark composition: $\bar{c}$+uncharmed quarks/antiquarks. PdG reports 4 entries with $J^P = 0^+$:
- $D^*_{s0}(2317)^\pm$, observed decay: $D^*_{s0}(2317)^- \to D_s^- + \pi^0$: I=0 ?
  quark composition: $D^{*-}_{s0} = (\bar{c}sq\bar{q})$, $(q = u, d)$
- $X(2900)^0$, required by LHCB for the full amplitude analysis of $B^+ \to D^+D^-K^+$

quark composition $X(2900)^0 = (\bar{c}\bar{s}ud)$

- $T^*_{c\bar{s}0}(2900)^{--,0}$, $I = 1$, $(I_3 = -1, +1)$, *alias*, in LHCb notation,
  $D^{--}_{s0}(2900)$, $D^0_{s0}(2900)$

Decays: $D^{--}_{s0}(2900) \to D_s^-\pi^-$ $(\bar{c}s\bar{u}d)$, $D^0_{s0}(2900) \to D_s^-\pi^+$ $(\bar{c}s\bar{d}u)$

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Decays: $D^{--}_{s0}(2900) \to D^-_s \pi^-\ (\bar{c}s\bar{u}d)$, $D^0_{s0}(2900) \to D^-_s \pi^+\ (\bar{c}s\bar{d}u)$

- The lightest particle $D^*_{s0}(2317)$ goes in the basic $\mathbf{3} \oplus \bar{\mathbf{6}}$ multiplet.
- However X and $T^*$are ***too heavy to be included in the same*** $\mathbf{3} \oplus \bar{\mathbf{6}}$ ***multiplet***:

$$M(2900) - M(2317) = 583\ \text{MeV}$$

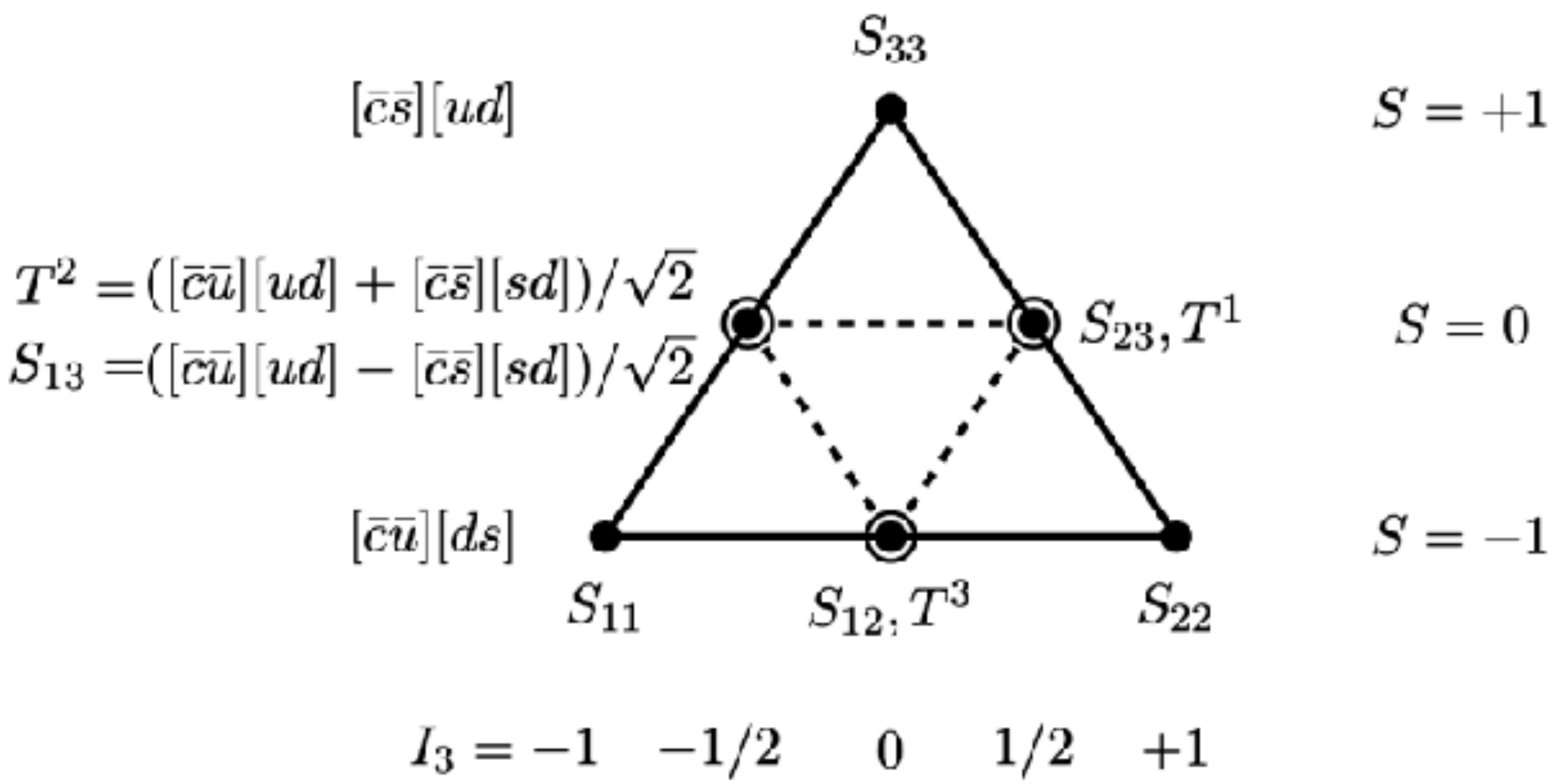
is similar to the mass gaps:

$$M(J/\Psi) - M(J/\Psi') = 590\ \text{MeV},\ M(X(3872) - M(Z(4430)) = 558\ \text{MeV}$$

- We interpret the LHCb resonances as the ***first radial excitations*** $(n = 2)$ of the basic $D^*_{s0}(2317)$ multiplet, to be allocated in a different $\mathbf{3} \oplus \bar{\mathbf{6}}$ multiplet.

5. Quantum numbers and Mass Formulae for $[\bar{c}\bar{q}]^{\mathbf{3}}_{S_{c3}=0}[q_1q_2]^{\bar{\mathbf{3}}}_{S_{12}=0}$

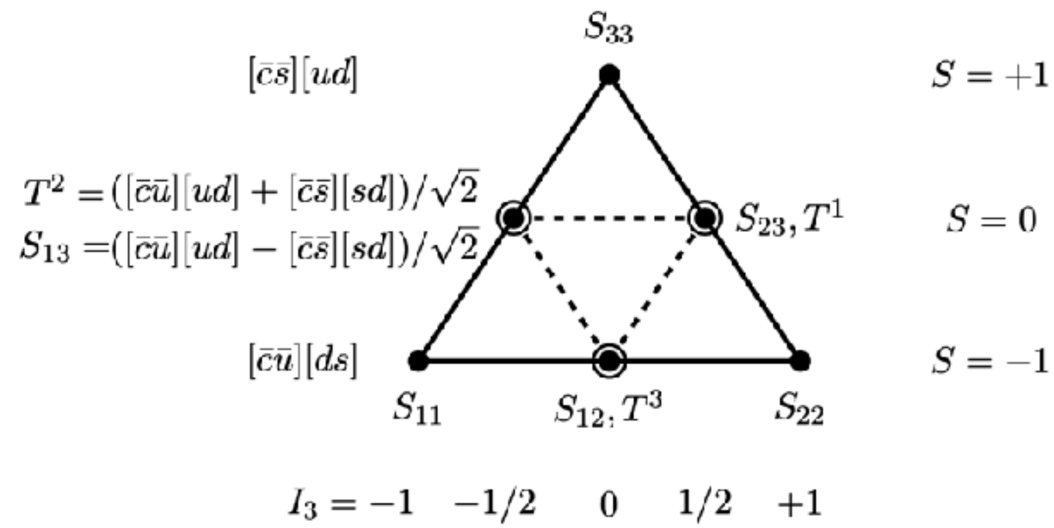
The $\mathbf{3} \oplus \bar{\mathbf{6}}$ representation in the $I_3 - S$ plane:



Mass formulae with octet symmetry breaking of $SU(3)_{flavor}$

- $\mathbf{6} \otimes \bar{\mathbf{6}}$ contains rep. **8** only once and the same for $\bar{\mathbf{3}} \otimes \mathbf{3}$
- in both representations, octet ***symmetry breaking*** is ***proportional to Strangeness*** and the mass formulae are:

$$M_{\bar{\mathbf{6}}} = m_{\bar{\mathbf{6}}} + \alpha(S+1);$$
$$M_{\mathbf{3}} = m_{\mathbf{3}} + \beta(S+1);$$



- octet breaking produces a mixing $S_{13} - T^2$ with a matrix:
- the same matrix for $S_{23} - T^1$ mixing.

$$\mathcal{M} = \begin{pmatrix} m_{\mathbf{3}} + \beta & \delta \\ \delta & m_{\bar{\mathbf{6}}} + \alpha \end{pmatrix}$$

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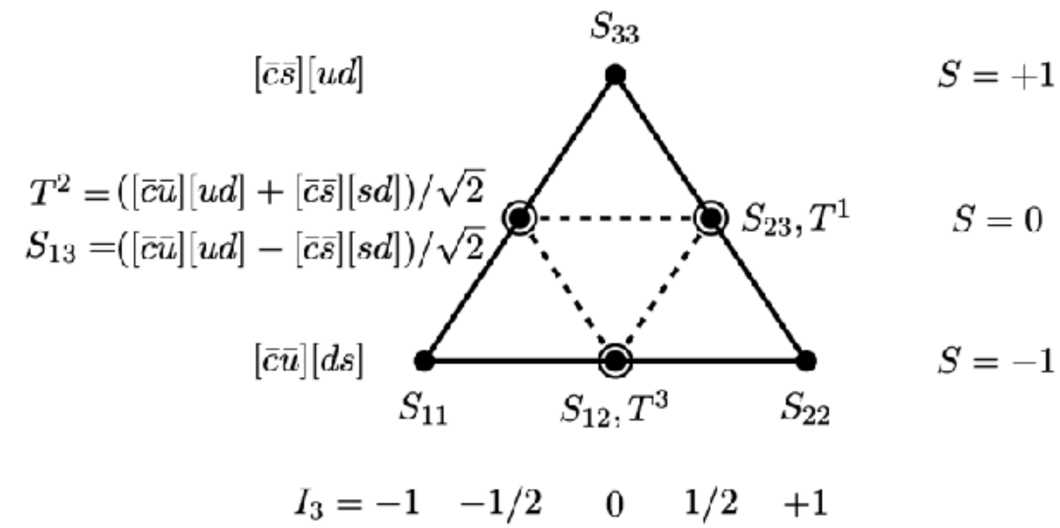

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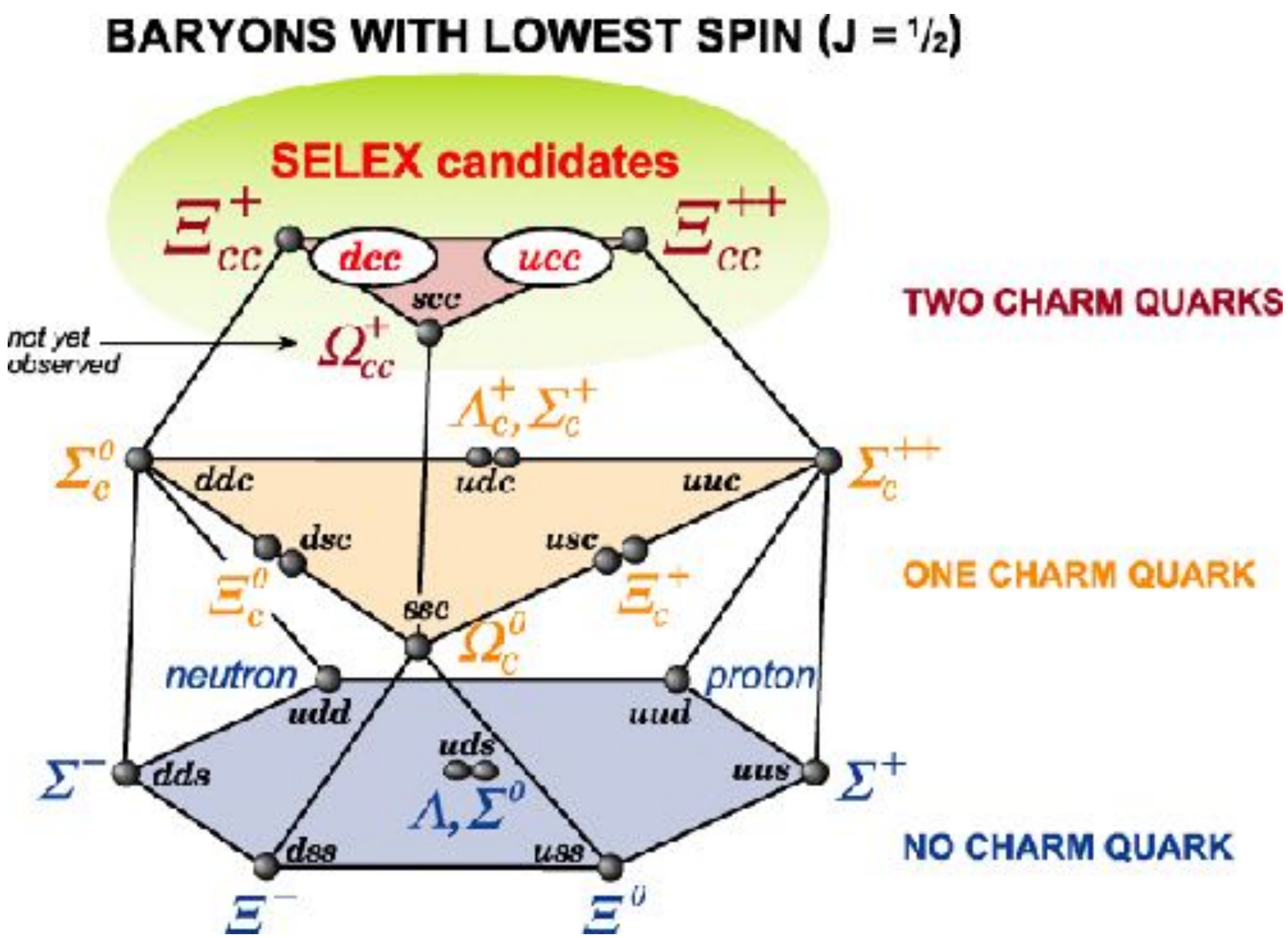
- Quark model requires:

(i) all S=-1 states to have about the same mass ($m_{\bar{\mathbf{6}}} = m_{\mathbf{3}} = M$): they have the same quark composition (like $\rho - \omega$ mesons)
(ii) $\alpha = \beta$: for the mixing matrix to be diagonalized by $(S_{13} \pm T^2)/\sqrt{2}$ (analogous to $\omega - \phi$ mixing)

$6 \oplus \bar{3}$ & Fermi Statistics work in Single Charm Baryons



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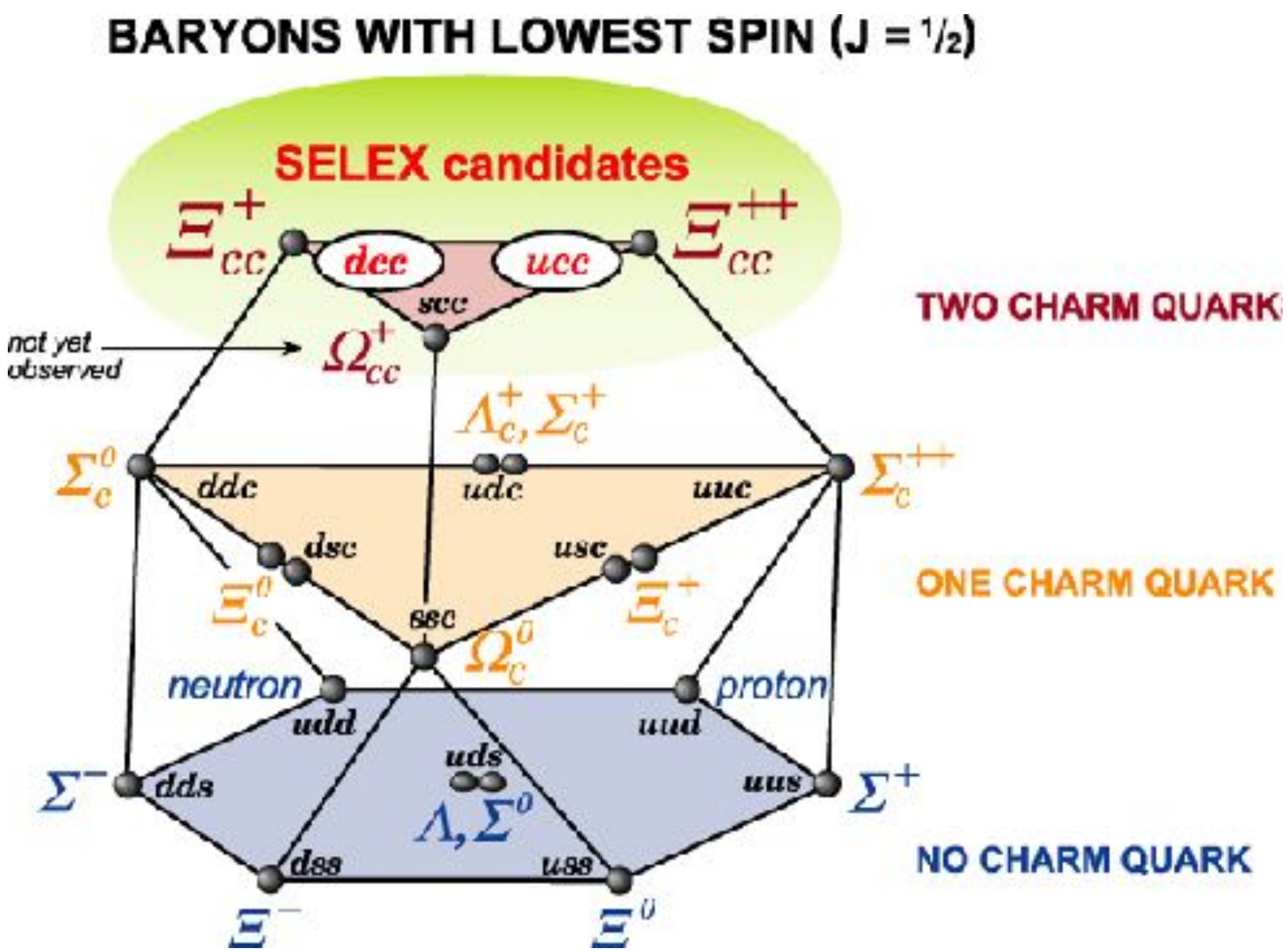


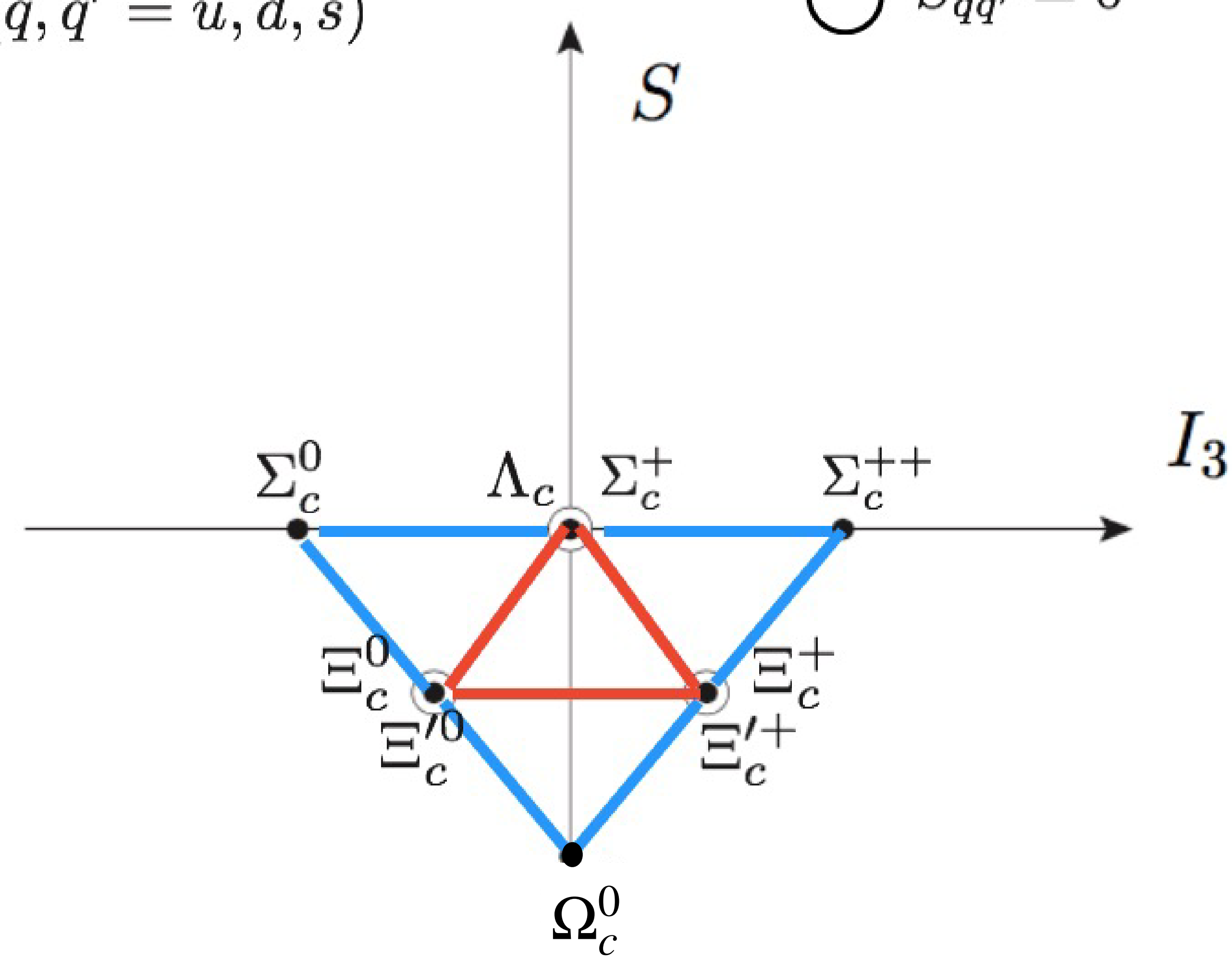
$q,\ q'$: color $= \mathbf{3} \otimes \mathbf{3} \to \bar{\mathbf{3}}$

$\mathbf{J} = 1/2:\ \ \mathbf{6} \oplus \bar{\mathbf{3}}$

$(q, q' = u, d, s)$

● $S_{qq'} = 1$

○ $S_{qq'} = 0$



$$I = 1 \ \leftrightarrow \ spin = 1,\ (\Sigma_c^{0,+,++})$$

$$I = 0 \ \leftrightarrow \ spin = 0 \ \ (\Lambda_c^+)$$

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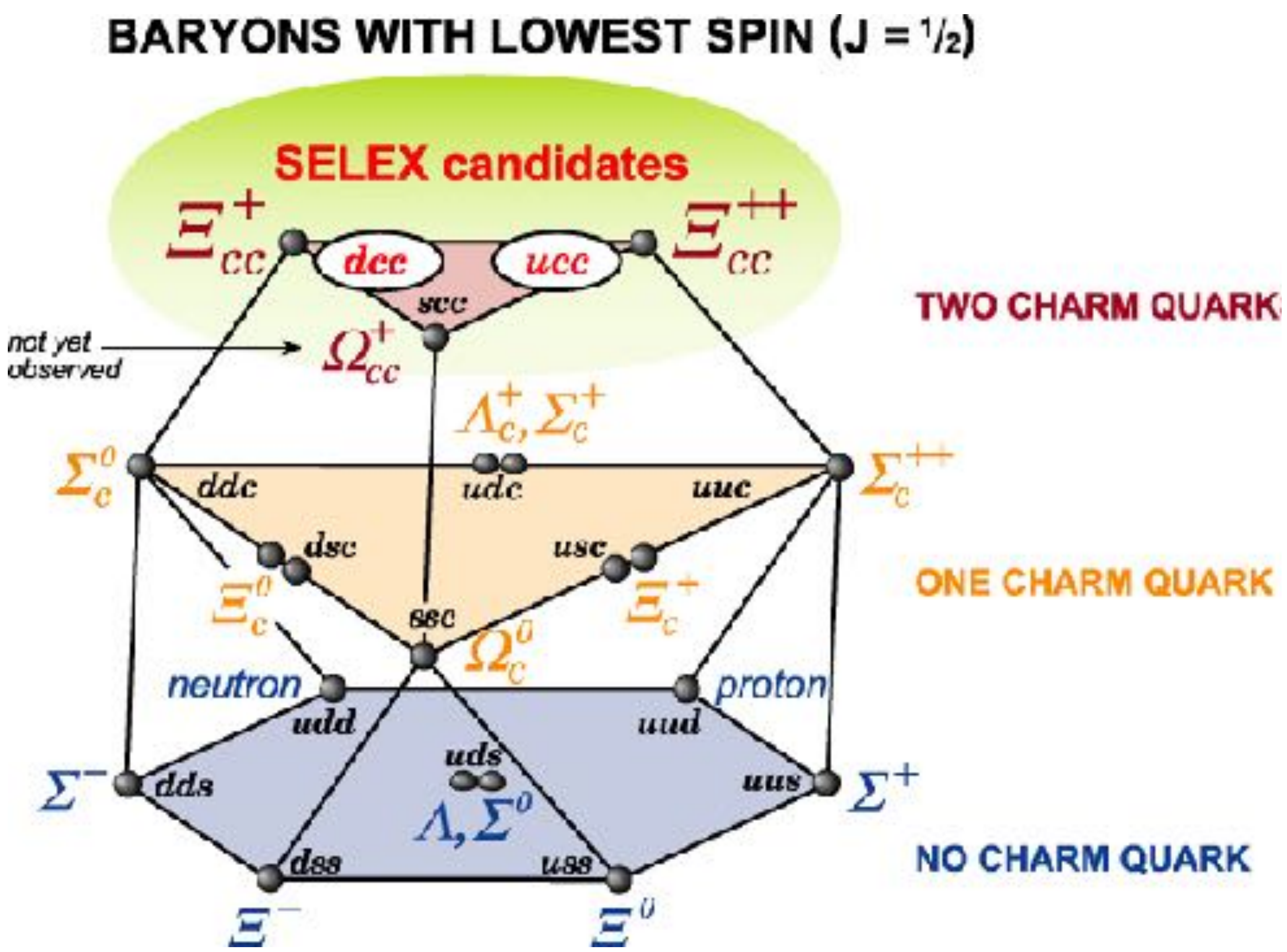

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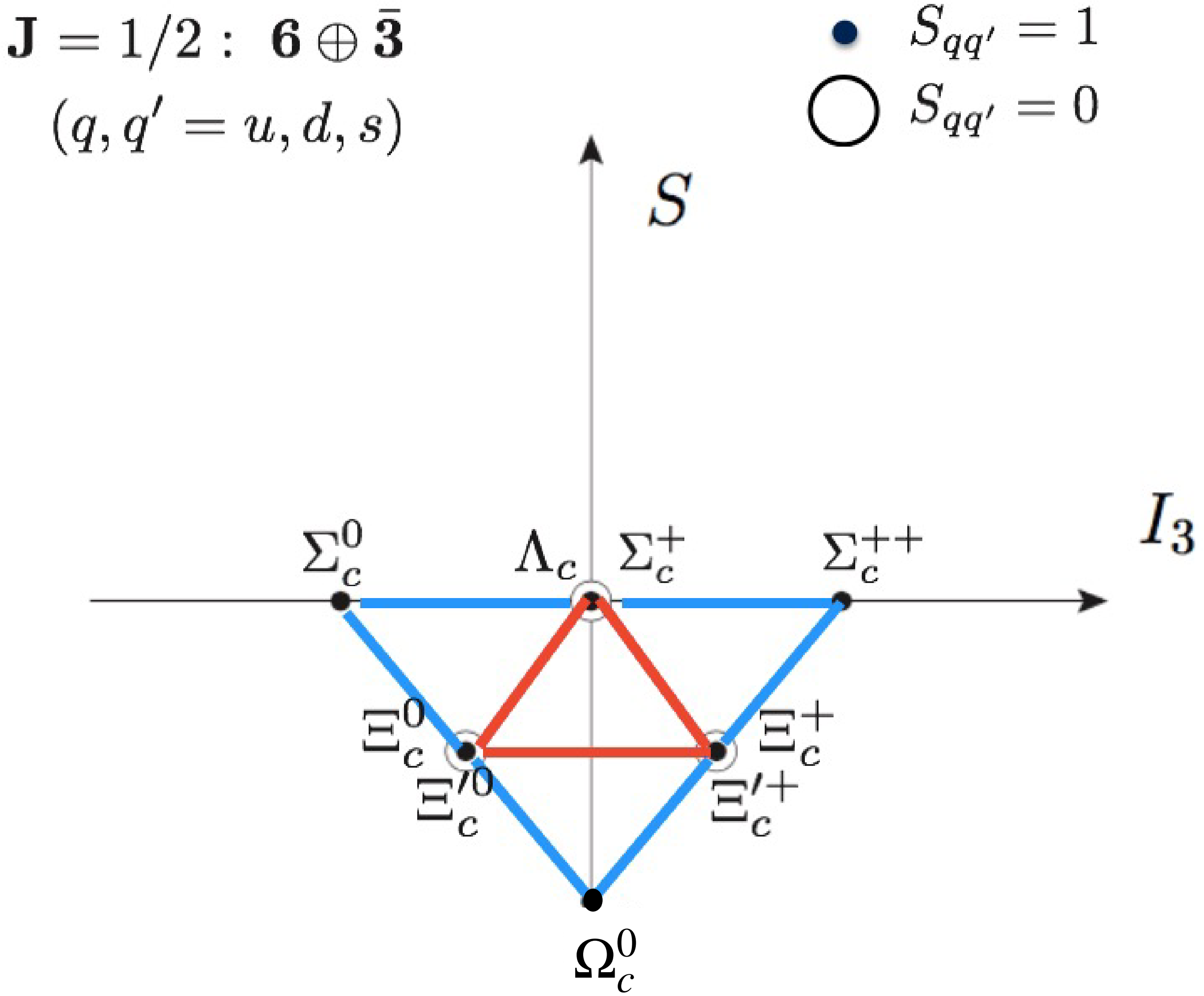
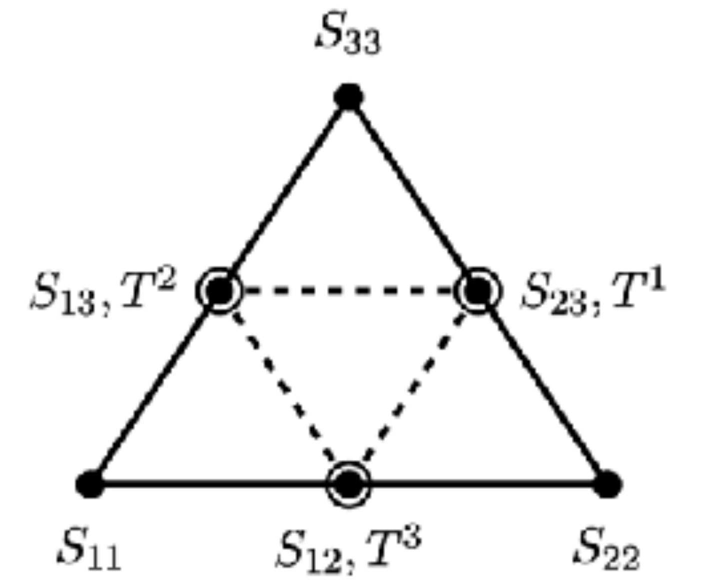


The masses of spin 1 diquarks in the **6** increase linearly with Strangeness, i.e. with the number of strange quarks: $M(\Omega_c) - M(\Sigma_c) \simeq 270$ MeV

$$I = 1 \leftrightarrow spin = 1,\ (\Sigma_c^{0,+,++})$$

$$I = 0 \leftrightarrow spin = 0 \quad (\Lambda_c^+)$$

A remarkable regularity



- Like the masses of single charm mesons, the masses of single charm tetraquarks are equally spaced in Strangeness, with a slope given by the parameter α.
- However, ***unlike charmed baryons,*** the lower indices in S_{11} correspond to the quark-diquark antisymmetric configuration $\bar{u} \otimes [ds]_A$, while the lower indices in S_{33} correspond to $\bar{s} \otimes [ud]_A$,

which have obviously the same content in quark masses, two light and one heavy.

- Exact equality $M(S_{33}) = M(S_{11})$ corresponds to $\alpha = 0$: same masses at the upper vertex and lower corners of the triangle in the figure.
- In this case, symmetry breaking is restricted to the mass difference between the two $S = 0$, $I = 1/2$ multiplets induced by the $\mathbf{3} - \bar{\mathbf{6}}$ mixing and of order $\delta \sim 2(m_s - m_q)$, with all other masses degenerate at M.
- A small, non vanishing value of α arises from differences in the hyperfine interactions, which are between different pairs in the two cases.

Note: *In charmed baryons, two light quarks in spin one are in a* **6** *representation. In this case, indices 1 or 3 correspond univocally to u or s quarks. Group theory disentangles efficiently the ambiguity in these two* **6** *representations* ***making use of the parameter*** α allowed by the Wigner-Eckart theorem.

A "Constituent Diquark-Antidiquark Model"

- We define ``complete diquark masses" which include the hyperfine interaction appropriate to diquarks with spin =0, e.g.

$$\overline{M}_{cq} = M_{cq} - \frac{3}{2}\kappa_{cq}, \text{ etc.}; \quad \overline{M}_{cq} = \overline{M}_{\bar{c}\bar{q}}$$

- Charmed diquark masses and hyperfine interactions are taken from the masses of hidden charm (X(3872), etc.), hidden charm and strangeness (X(4140), etc.)

L. Maiani et al., Phys. Rev. D 89 (2014), 114010 [arXiv:1405.1551 [hep-ph]]; Phys. Rev. D 94 (2016) , 054026 [arXiv:1607.02405 [hep-ph]].

- and the masses of uncharmed, spin 0 diquarks from the, not so well determined, masses of the light scalar mesons $f_0(500)$ and $f_0(980)$ (see errors in Tab).

R. L. Jaffe, Phys. Rev. D 15 (1977), 281

quark	q	s	c
q	300 ± 100	490 ± 10	1877
s	– – -	–	2035
c	– – -	– – -	–

Table 1: Complete diquark masses, $\overline{M}_{ij}$, in MeV.

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$$M(S_{12}) = M(T_3) = \overline{M}_{cu} + \overline{M}_{sd} = 2367 \pm 10 \text{ MeV} \longleftrightarrow D^*_{cs0}(2317)$$

$$M(T_-) = \overline{M}_{cu} + \overline{M}_{ud} = 2177 \pm 100 \text{ MeV}$$

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In red the missing S=0 states and their estimated masses. Expected decay modes:

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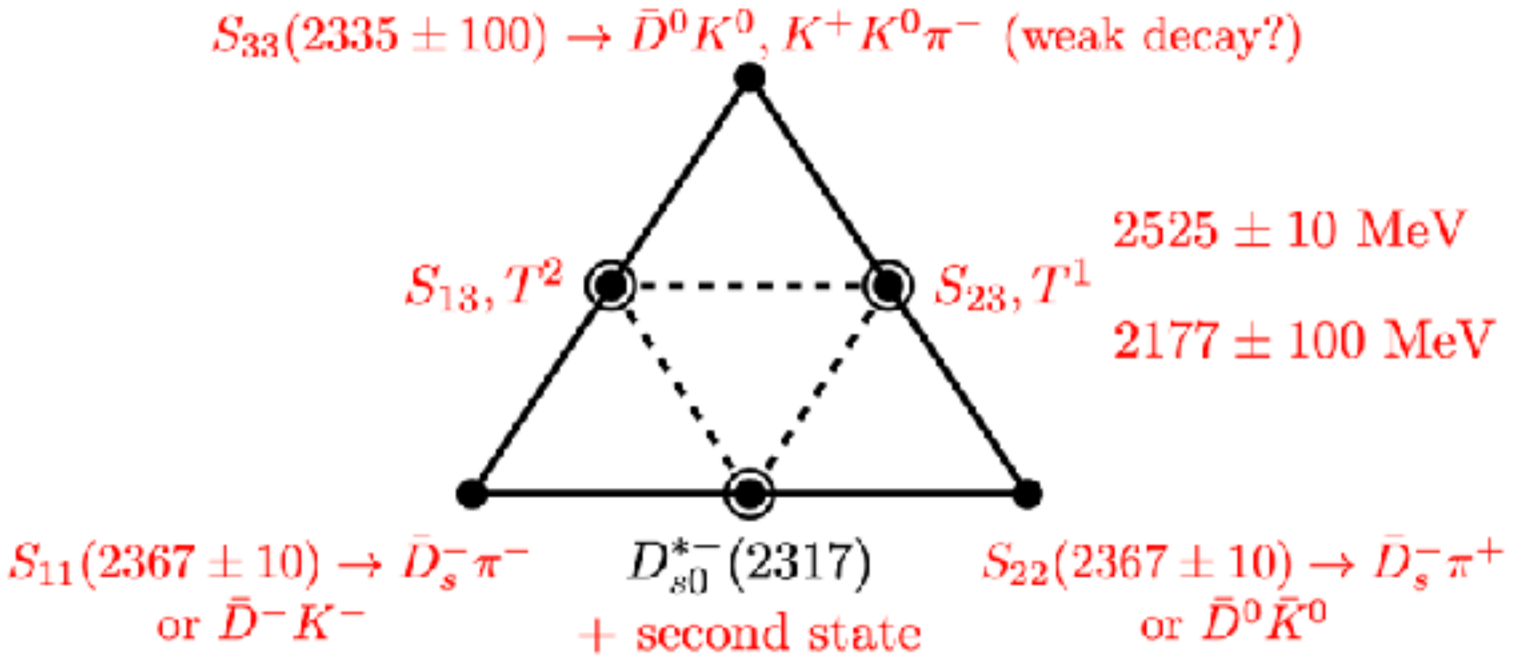

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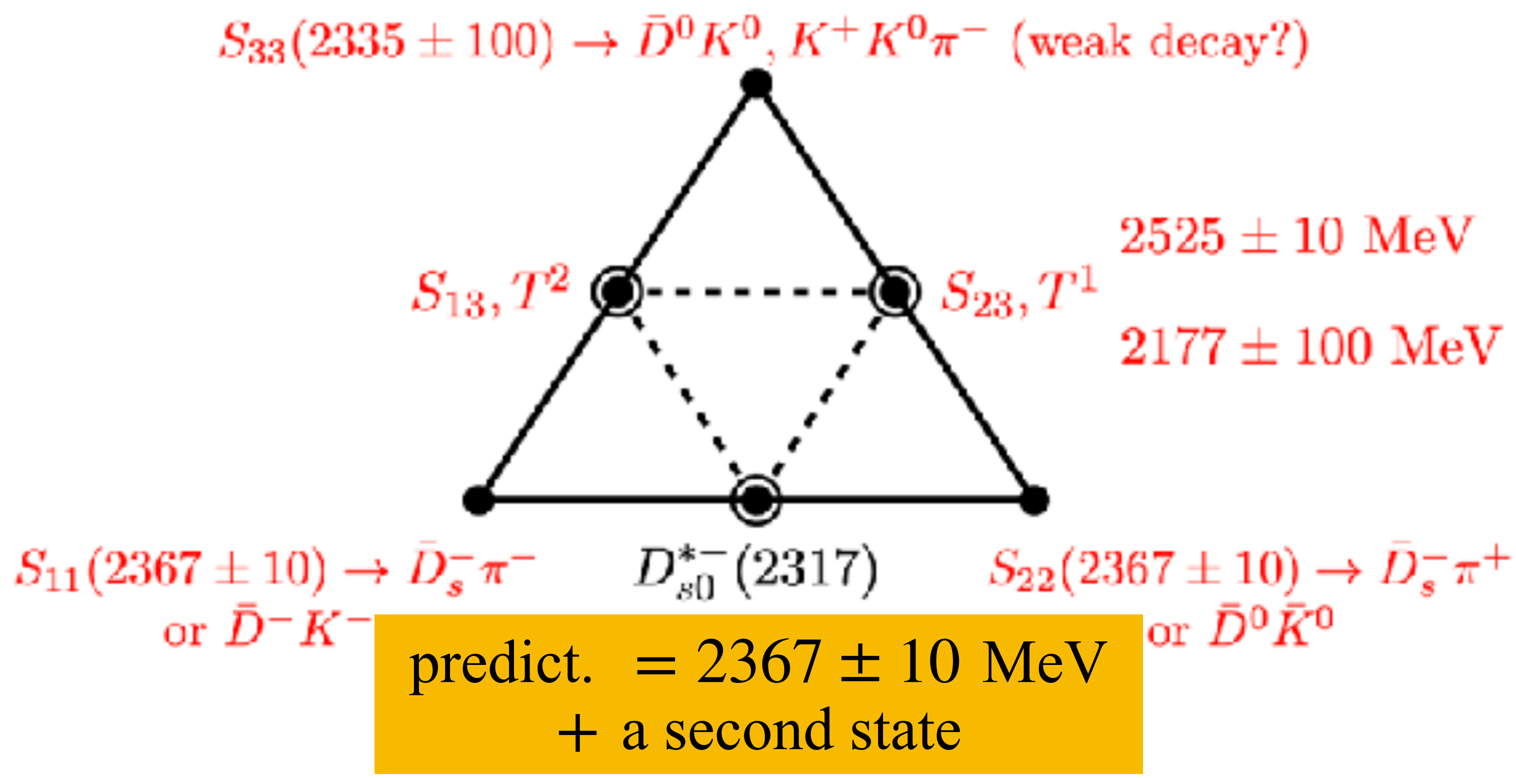

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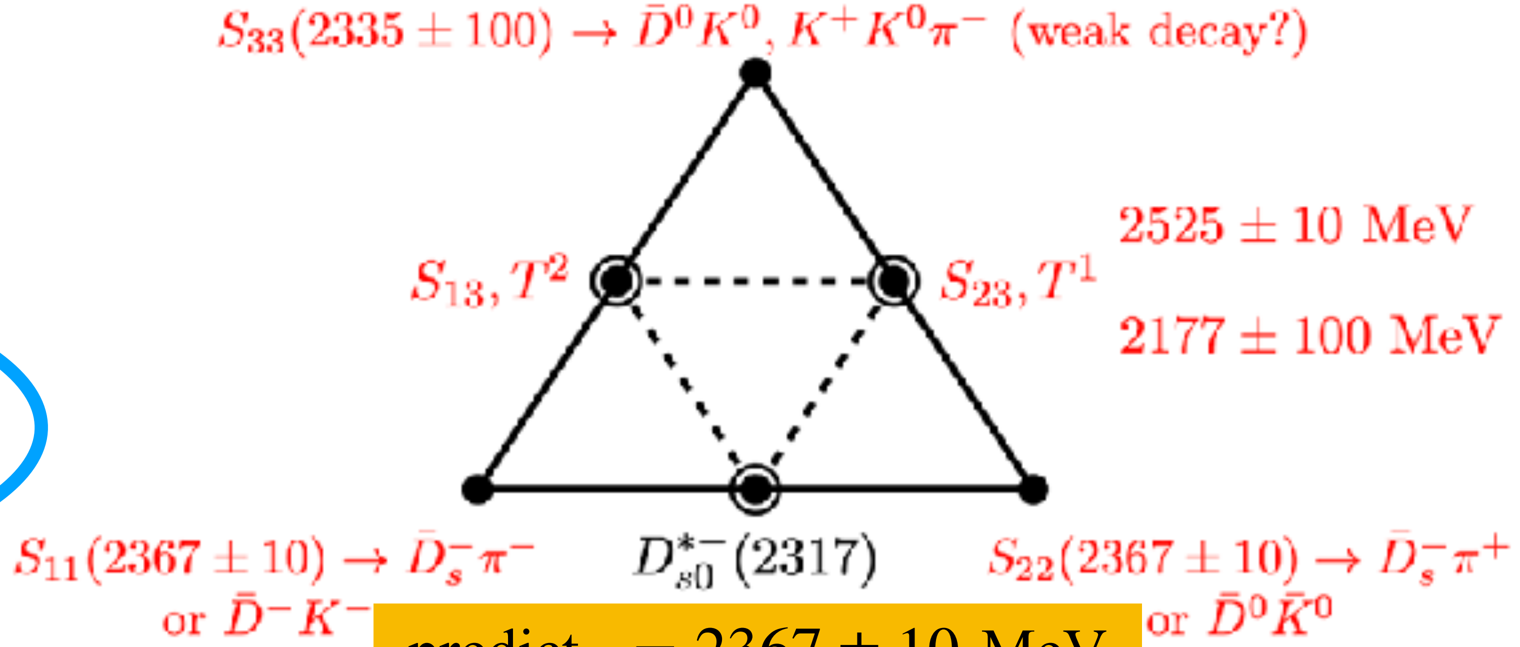

predict. $= 2367 \pm 10$ MeV
+ a second state

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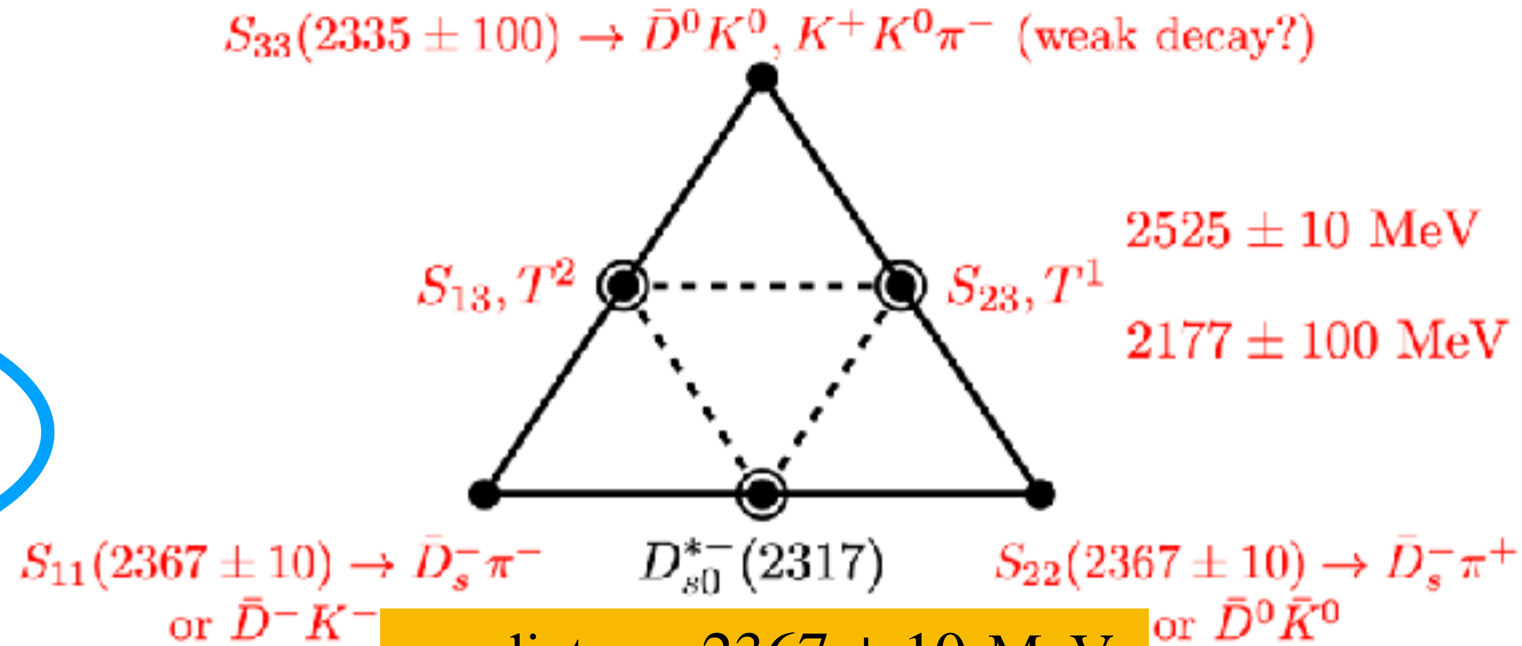

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- $D^{*-}_{s0}(2317)$ has I=0 (PdG) and it should decay into $\bar{D}_s^- \eta$, which however is forbidden by phase space.

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In red the missing S=0 states and their estimated masses. Expected decay modes:

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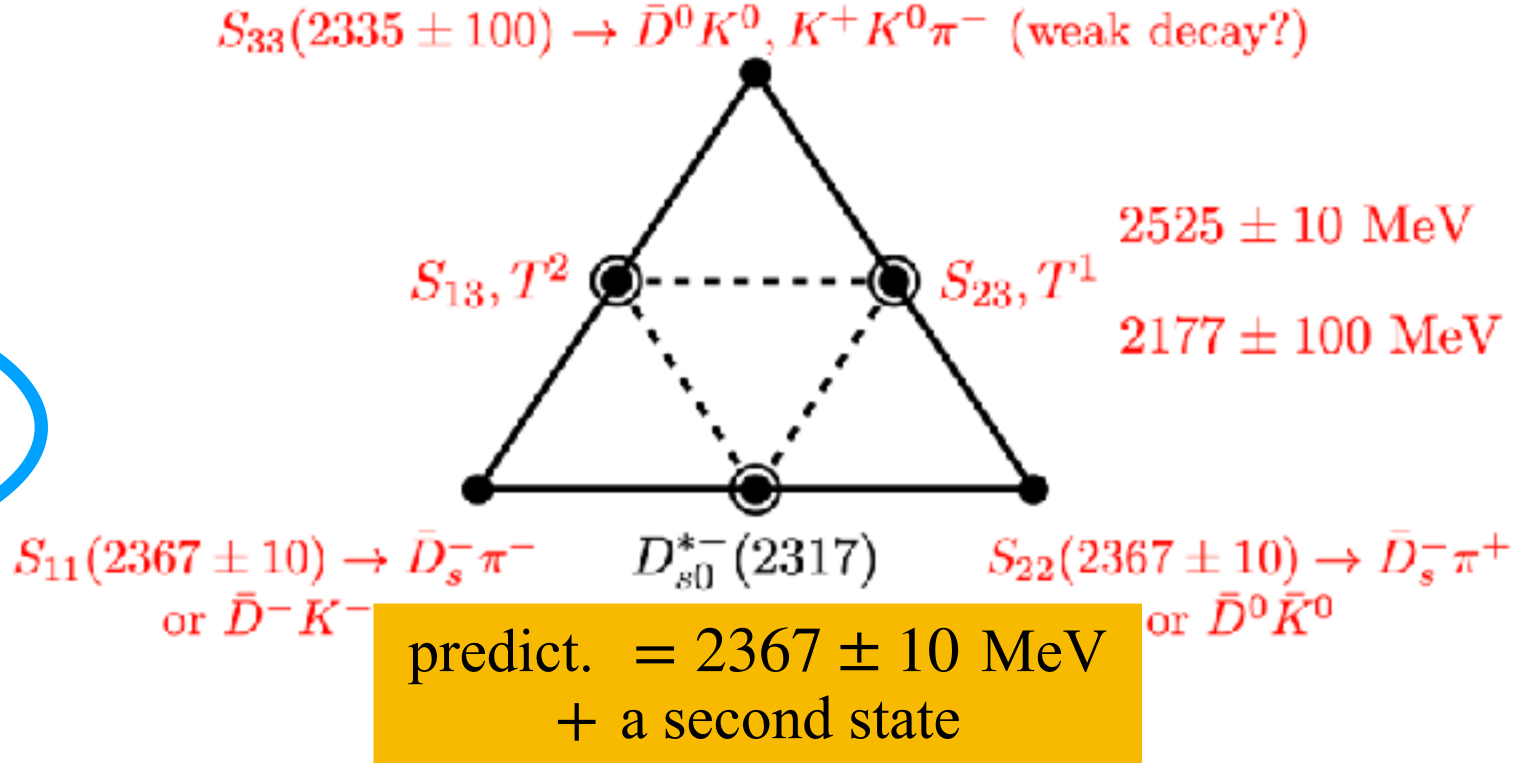
$[\bar{c}\bar{d}][ud](2177 \pm 100) \to \bar{D}\pi$



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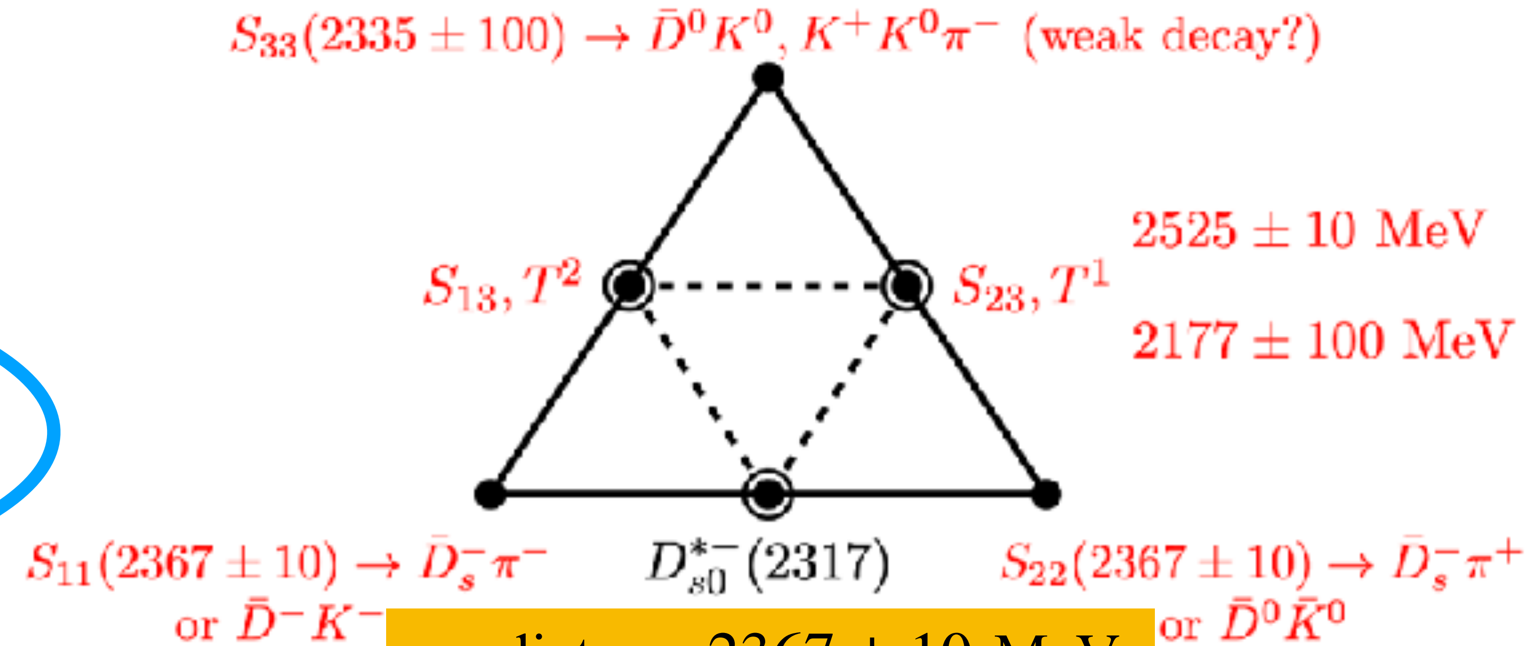

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- ***Interesting*** to observe the decay $D^*_{s0} \to \bar{D}_s \gamma\gamma$, quoted in PdG with the upper bound $B(\gamma\gamma) < 0.18$, to compare with $D^*_{s0}(2317) \to D_s^- \eta^* \to D_s^- \gamma\gamma$ via a virtual η.

6. The $\mathbf{3} \oplus \bar{\mathbf{6}}$ (n=2) radially excited multiplet

- $X_0(2900)$ and $D^0_{s0}(2900)$, $D^{++}_{s0}(2900)$ observed by LHCb

$X_0(2900)$ R. Aaij et al. [LHCb], Phys. Rev. D 102 (2020), 112003 | $D^{0,++}_{s0}(2900)$ R. Aaij et al. [LHCb], Phys. Rev. Lett. 131 (2023) 041902;

- We interpret the LHCb resonances as the ***first radial excitations*** ($n = 2$) of the basic $D^*_{s0}(2317)$ multiplet.

The n=2 multiplet:

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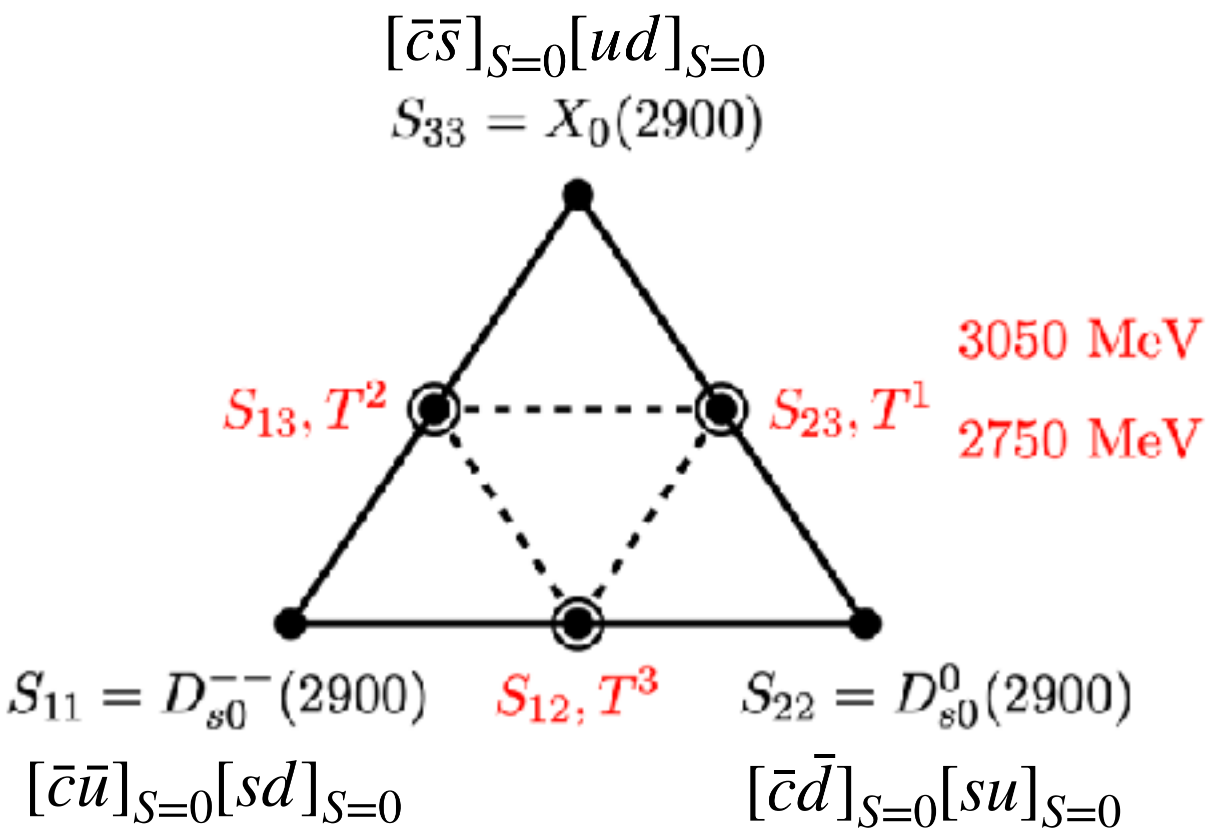

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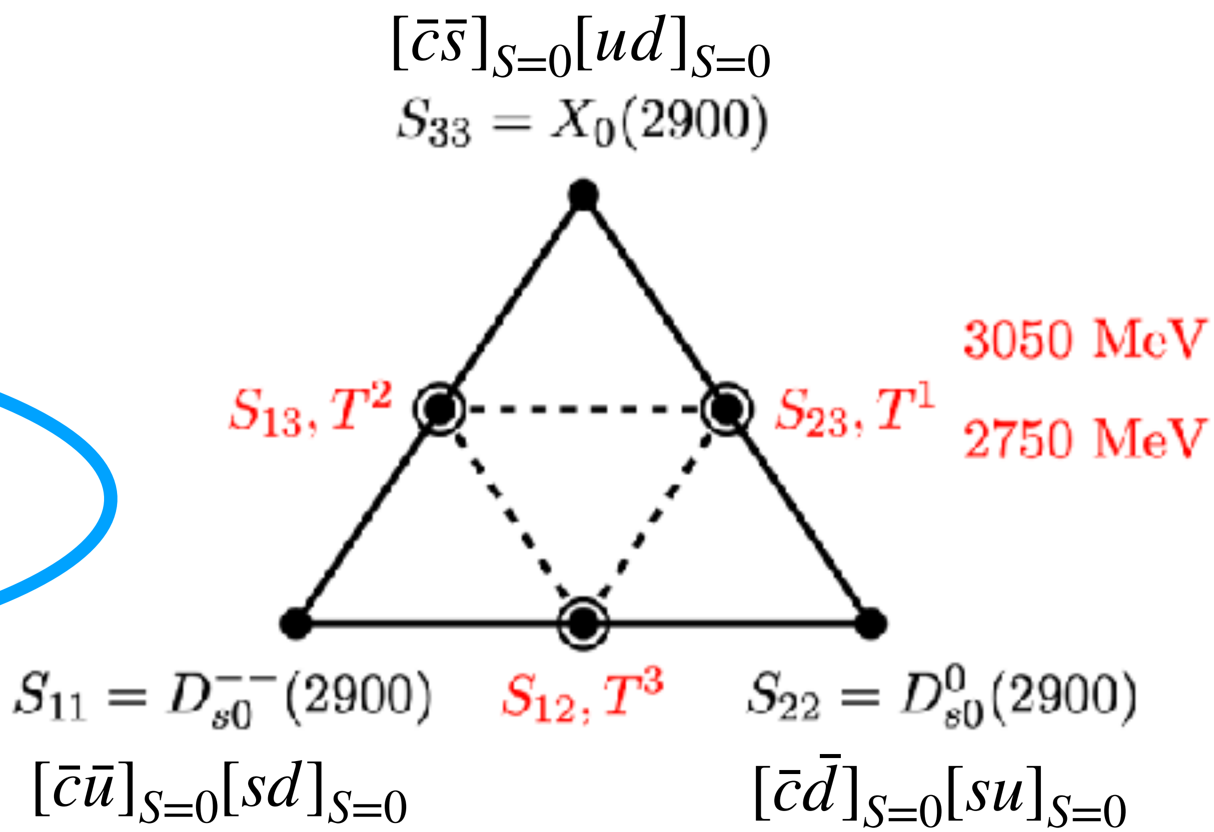

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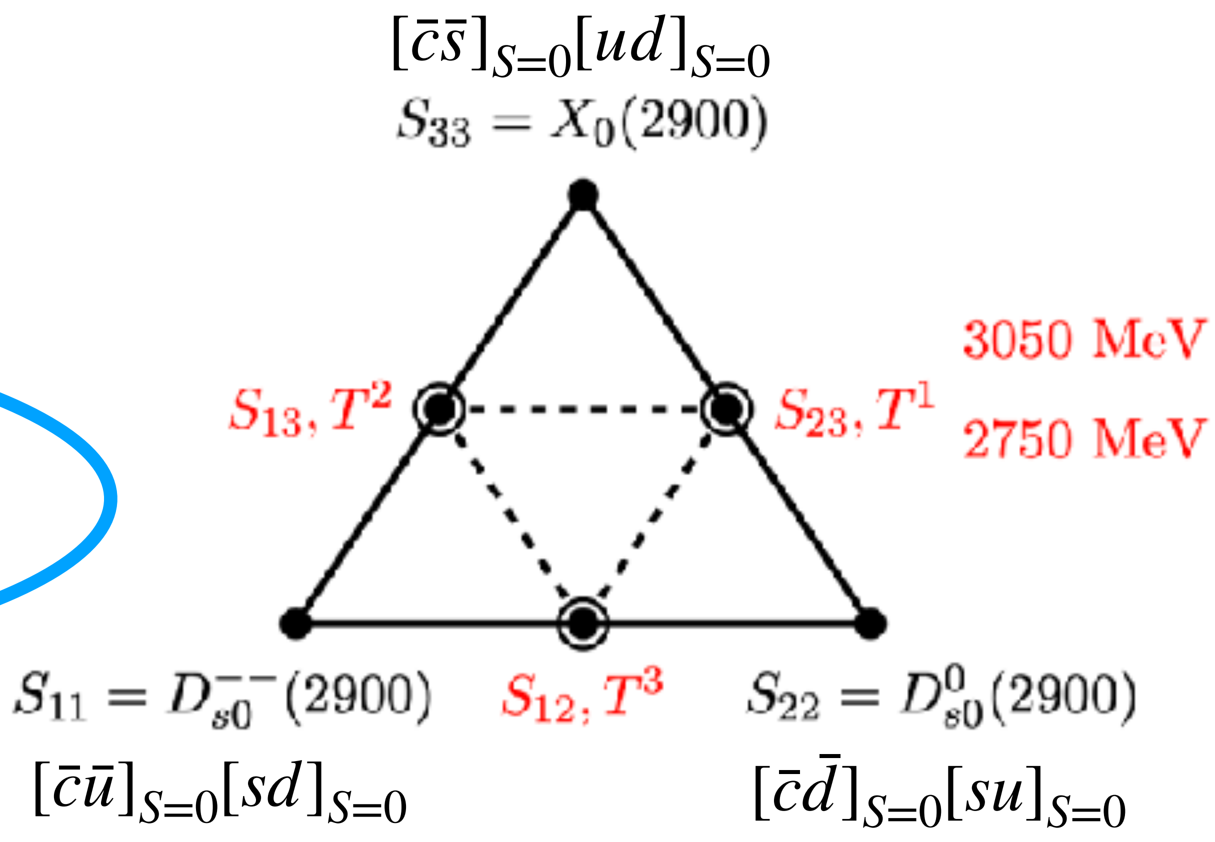

- ***Mass degeneracy between*** **X_0(2900)** ***(S=+1) and*** **$D_{s0}^{--,0}$(2900)** ***(S=-1) is the footprint of the tetraquark compositions:***

$[\bar{c}\bar{s}]_0[ud]_0$ and $[\bar{c}\bar{u}]_0[sd]_0$

n=1 and 2 multiplets (summary)

- The resonances $D_{s0}^{--,0}(2900)$ and $X_0(2900)$ recently discovered by LHCb nicely fit in a $\bar{\mathbf{6}} \oplus \mathbf{3}$ representation of $SU(3)_{flavor}$.
- ***Missing states***:

(i) the very likely $D_{s0}^{-}(2900)(I_3 = 0,\ I = 1)$, to fill an isotriplet with $D_{s0}^{--,0}(2900)$

(ii) its (almost degenerate) $(I_3 = 0,\ I = 0)$ partner;

(iii) two other I=1/2 tetraquark multiplets of composition:

$[\bar{c}\bar{s}][sd] + (d \to u)$ and $[\bar{c}\bar{u}][ud] + (\bar{u} \to \bar{d})$ of predicted masses and decay modes.

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- The observation that

$$M(2900) - M(2317) = 583\ \text{MeV} \simeq M(\psi(2S)) - M(\psi(1S))$$

suggests that the multiplet we discuss could be the radial excitation of the lower multiplet containing the $D_{s0}^{*}(2317)$, in a similar way in which $Z(4430)$ is interpreted as a radial excitation of $X(3872)$.

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- Our results are in agreement with a recent lattice calculation showing that in the $\bar{D}K$ scattering there is no attraction in the **15** representation, something which is expected in the quark model description we present here.

7. Heavy particle spin conservation and Fermi Statistics of light quark pairs: QCD tetraquarks vs hadron molecules

- For molecular tetraquarks *treated in Chiral Perturbation Theory*, the light quark total spin is a separatly conserved quantity in the limit of very massive charm quark (this is the *light quark spin symmetry in the static quark approximation* introduced by Isgur and Wise).
- For hidden charm molecules $(\bar{c}q)(\bar{q}'c)$, flavour symmetry, e.g. Isospin, is also an independent (commuting) conserved quantity. The possible combinations of light and heavy spin generate six states with definite Isospin, total angular momentum and charge conjugation:

Z. H. Zhang *et al*., JHEP **08** (2024) 130

$$J_I^{PC} = 0_I^{++},\ 1_I^{+-},\ 1_I^{\prime +-},\ 1_I^{++},\ 0_I^{\prime ++},\ 2_I^{++}$$

- These are the same J_I^{PC} states predicted for diquark-antidiquark tetraquarks of the form $[cq]^{\bar{\mathbf{3}}}[\bar{c}\bar{q}']^{\mathbf{3}}$. Noticeably, they include the I=1 partner of $X(3872)$, i.e. X^+

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- Concerning Fermi Statistics, the situation for the molecular structure $(\bar{c}q_1)(\bar{s}q_2)$ is different with respect to diquark-antidiquark situation.
- q_1 and q_2, sit in different color singlets and the color of the pair $q_1 \otimes q_2$ is not determined (in fact it is a superposition of $\bar{\mathbf{3}}_c$ and $\mathbf{6}_c$). There is no definite restriction to their behaviour under flavor exchange and no forbidden **15**.

A message in the bottle (following interesting ongoing discussions…)

$$J_I^{PC} = 0_I^{++},\ 1_I^{+-},\ 1_I^{'+-},\ 1_I^{++},\ 0_I^{'++},\ 2_I^{++}$$

- The I in the subscript of the spin states of hidden charm "molecules" can be extended, in general, to SU(3)-flavour

- If so, the $X^+, J^{PC} = 1^{++}$ should belong to the same octet as $Z_{cs}(4003)$ or $X(4140)$.

- Can the X^+ be a "virtual state pole of the scattering matrix" (Zhang et al.) and at the same time make an SU(3) octet together with normal resonances like $X(4140)$ (a $J/\Psi - \phi$ resonance) or $Z_{cs}(4003)$(a $J/\Psi - K$ resonance)

????

- Alternatively: are all exotic states just "cusps" at the meson-meson thresholds?
- What about Z(4430), far from any threshold, with its Argand plot observed by LHCb ?

8. Final questions (to LHCb and BESIII)

- Are $Z_{cs}(3986)$ and $Z(4003)$ two different states? is there a third $Z_{cs}(4220)$?
- Can X^+ near $X(3872)$ be found in B decays?
- can we find the missing partners of the $\bar{\mathbf{6}} \oplus \mathbf{3}$, (n=2) multiplet:

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Open charm and strangeness: Complete $\bar{\mathbf{3}} \oplus \mathbf{6}$?

- LHCb has used efficiently the channel $B \to (J/\Psi)\phi K + \ldots$to study $X_{ss}(4140)$ etc.., and Z_{cs} etc. of SU(3) nonet tetraquarks…
- Can the study of $B \to \bar{D}_s D\phi$ channel be similarly used to study single charm $[\bar{c}\bar{s}]_{S=1}[ss]_{S=0}$, $J^P = 1^+$ tetraquarks of the interesting $\mathbf{15} \oplus \mathbf{3}$, $J^P = 1^+$ multiplet ?
- Reconsider K-like states which decay into $K\phi$ (e.g. $K_1(2650)$), therefore unlikely to be $(s\bar{q})$ excited Kaons: could they be zero-charm $[\bar{u}\bar{s}][ss]$ tetraquarks?

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Open strangeness ?

- ***Tough orders***: more luminosity, better energy definition, detectors with exceptional qualities… a lot of work…
- ***Close exchange between theory and experiments*** is essential and it has to continue.