Calculable neutrino Dirac mass matrix in the minimal left-right symmetric model

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based on Phys.Rev.D 110 (2024) 3, 035030
[arXiv:2404.16740] with Ding-Yi Luo and Xiang Zhao

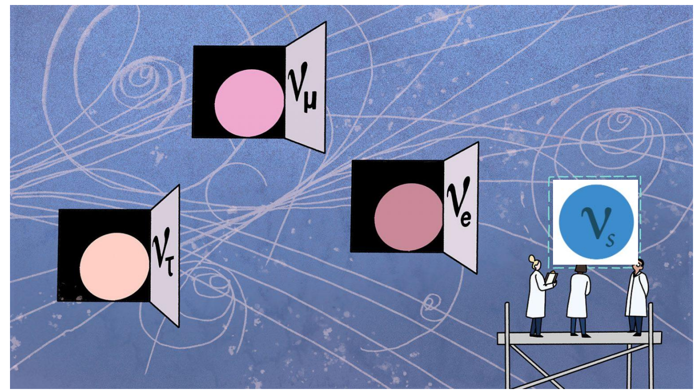
Lepton and Quark Flavour Structures

2024年10月18日，国科大杭州高等研究院

New physics

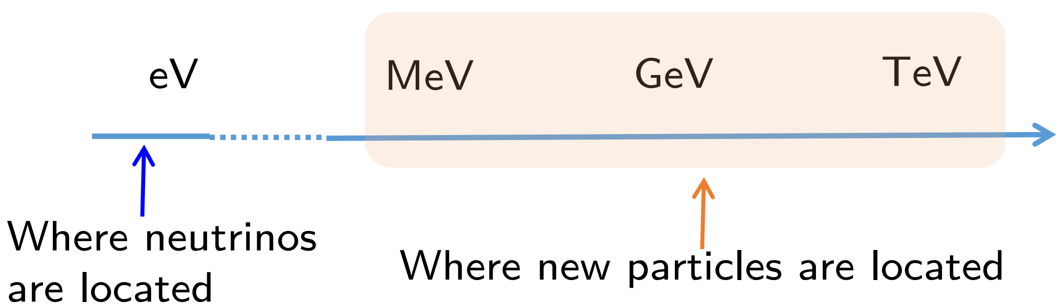
The Standard Model is successful but incomplete:

- neutrino masses
- baryon asymmetry
- dark matter
- strong CP problem
- ....



credit: FNAL “All Things Neutrino”

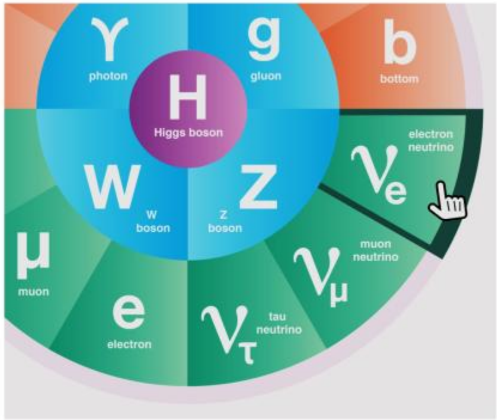
Mass scale



Neutrino physics

Open questions:

- Normal or Inverted (sign of Δm^2_{31}?)
- Leptonic CP Violation (δ = ?)
- Octant of θ_{23} (> or < 45°?)
- Absolute Neutrino Masses ($m_{\text{lightest}} = 0$?)
- Majorana or Dirac Nature ($\nu=\nu^C$?)
- Majorana CP-Violating Phases (how?)



- Extra Neutrino Species
- Exotic Neutrino Interactions
- Various LNV & LFV Processes
- Leptonic Unitarity Violation

- **Origin of Neutrino Masses**
- Flavor Structure (Symmetry?)
- Quark-Lepton Connection
- Relations to DM and/or BAU

credit: Shun Zhou

ν -New Physics connection

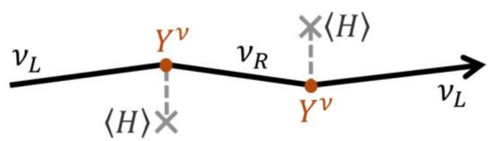
Overview

- Brief introduction of minimal left-right symmetric model
- Generalized Casas-Ibarra parametrization of neutrino Dirac mass matrix in the cases of
  - charge conjugation: fully solved
  - parity: partially solved
- Our parametrization of right-handed neutrino mass and lepton mixing matrices
- Phenomenological implication for the strong CP parameter

Neutrino masses

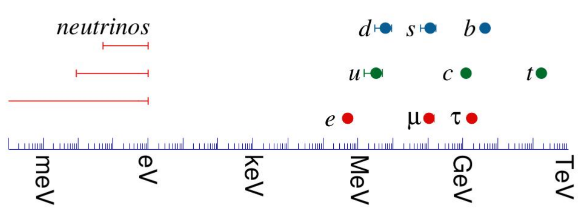
Two ways to generate netrino masses

Dirac mass:



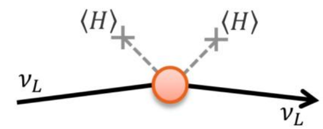
$$\mathcal{L}_D = -(Y^\nu \bar{L} H \nu_R + \text{h.c.})$$

Higgs mechanism



unnaturally small $Y_\nu < 10^{-13}$

Majorana mass:



$$\mathcal{L}_M = \frac{C_5}{\Lambda}(\bar{L}^c \tilde{H}^*)(\tilde{H}^\dagger L) + \text{h.c.}$$

seesaw mechanism

heavy new physics

Minimal left-right symmetric model

Gauge group: $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

Doublets:

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$\ell_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad \ell_R = \begin{pmatrix} \nu \\ e \end{pmatrix}_R$$

Bidoublet:

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \Rightarrow \langle\Phi\rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\alpha} \end{pmatrix} \qquad \tan\beta \equiv \frac{v_2}{v_1}$$

Triplets:

$$\Delta_{L,R} = \begin{pmatrix} \delta^+_{L,R}/\sqrt{2} & \delta^{++}_{L,R} \\ \delta^0_{L,R} & -\delta^+_{L,R}/\sqrt{2} \end{pmatrix}$$

$$\Rightarrow \langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, \qquad \langle\Delta_L\rangle = \begin{pmatrix} 0 & 0 \\ v_L e^{i\theta_L} & 0 \end{pmatrix}$$

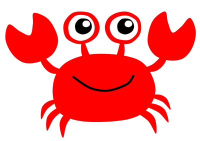
Mohapatra and Senjanovic, Phys.Rev.Lett. 44 (1980) 912, Phys.Rev.D 23 (1981) 165

Minimal left-right symmetric model

- Leptonic Yukawa interactions

$$\mathcal{L}_{\rm lep} = -\,\bar{\ell}_L\,(Y_1\Phi - Y_2\sigma_2\Phi^*\sigma_2)\ell_R - \frac{1}{2}(\ell_L^T C Y_L i\sigma_2\Delta_L\ell_L + \ell_R^T C Y_R i\sigma_2\Delta_R\ell_R) + \text{H.c.}$$

under left-right symmetry:



$$\mathcal{P}: \quad \{\ell_L, \Phi, \Delta_L\} \leftrightarrow \{\ell_R, \Phi^\dagger, \Delta_R\}$$

$$\mathcal{C}: \quad \{\ell_L, \Phi, \Delta_L\} \leftrightarrow \{(\ell_R)^c, \Phi^T, \Delta_R^*\}$$

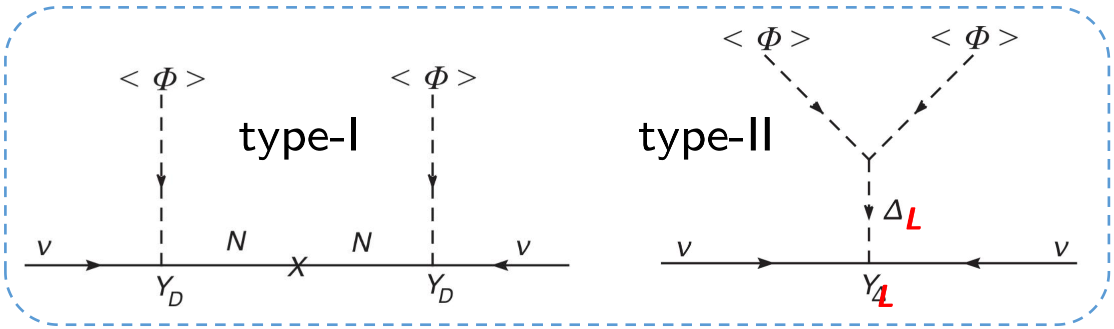
so that Yukawa matrices satisfy

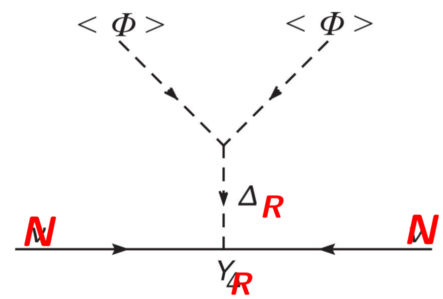
$$\mathcal{P}: \quad Y_L = Y_R\,, \quad Y_{1,2} = Y_{1,2}^\dagger \qquad \text{(Hermitian)}$$

$$\mathcal{C}: \quad Y_L = Y_R^*\,, \quad Y_{1,2} = Y_{1,2}^T \qquad \text{(symmetric)}$$

Minimal left-right symmetric model

- Type I+II seesaw:





In the flavor basis,

$$\mathcal{L}_\nu = -\frac{1}{2}\left(\bar{\nu}_L^c, \bar{N}_L^c\right) M_n \begin{pmatrix} \nu_L \\ N_L \end{pmatrix} + \text{H.c.} \qquad N_L \equiv \nu_R^c$$

The full neutrino mass matrix

$$M_n \equiv \begin{pmatrix} M_L & M_D^T \\ M_D & M_N \end{pmatrix}$$

Type I+II seesaw:

$$M_\nu = M_L - M_D^T \frac{1}{M_N} M_D$$

Charge conjugation

- Mass matrices satisfy

$$M_D = M_D^T, \quad M_L = \frac{v_L}{v_R} M_N \qquad \theta_L = 0$$

- The neutrino Dirac mass matrix is

$$M_\nu = M_L - M_D^T \frac{1}{M_N} M_D \quad \Rightarrow \quad M_D = M_N \sqrt{\frac{v_L}{v_R} - \frac{1}{M_N} M_\nu}$$

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- In terms of physical masses

$$M_\nu = V_L^* m_\nu V_L^\dagger$$

$$M_N = V_R m_N V_R^T$$

The right-handed lepton mixing matrix V_R is unknown (input)

Generalized Casas-Ibarra parametrization of the neutrino Dirac mass matrix

J.A. Casas, A. Ibarra, hep-ph/0103065 (NPB)

Charge conjugation

- For vanishing v_L, the neutrino Dirac mass matrix is

$$M_D = iV_L^* \sqrt{m_\nu} O \sqrt{m_N} V_R^T$$

The orthogonal matrix

$$O^T = \sqrt{m_N} \sqrt{m_N^{-1} V_R^\dagger V_L^* m_\nu V_L^\dagger V_R^*} V_R^T V_L \sqrt{m_\nu^{-1}} \qquad OO^T = \mathbb{1}$$

If one assumes $V_R = V_L^*$

$$M_D = iV_L^* m_N \sqrt{\frac{m_\nu}{m_N}} V_L^\dagger \qquad O = \mathbb{1}$$

M. Nemevsek, G. Senjanovic, V. Tello, 1211.2837 (PRL)

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$$M_D = V_L^* m_N \sqrt{\frac{v_L}{v_R} - \frac{m_\nu}{m_N}} V_L^\dagger$$

NEW! The general case with non-zero v_L and generic V_R was recently solved

J. Kriewald, M. Nemevsek, F. Nesti, 2403.07756

Parity

- Mass matrices satisfy

$$M_D = M_D^\dagger, \quad M_L = \frac{v_L}{v_R} M_N^* \qquad \theta_L = 0$$

- A ***different*** method is needed

G. Senjanovic, V. Tello, 1612.05503 (PRL); 1812.03790 (PRD)
J. Kiers, K. Kiers, A. Szynkman, T. Tarutina, 2212.14837 (PRD)

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From $$M_\nu = M_L - M_D^T \frac{1}{M_N} M_D$$

we have $$M_\nu = \frac{v_L}{v_R} M_N^* - M_D^T \frac{1}{M_N} M_D$$

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$$\frac{1}{\sqrt{M_N}} M_\nu^* \frac{1}{\sqrt{M_N}} = \frac{v_L}{v_R} \mathbb{1} - \frac{1}{\sqrt{M_N}} M_D \frac{1}{M_N} M_D^T \frac{1}{\sqrt{M_N}}$$

so that

$$HH^T = \frac{v_L}{v_R} - \frac{1}{\sqrt{M_N}} M_\nu^* \frac{1}{\sqrt{M_N}}$$

symmetric

$$H \equiv \frac{1}{\sqrt{M_N}} M_D \frac{1}{\sqrt{M_N^*}}$$

Hermitian

Parity

- Decompose the symmetric matrix HH^T

$$HH^T = OsO^T \qquad O \text{ is orthogonal and complex}$$

It was proved that

$$H = O\sqrt{s}EO^\dagger$$

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proof:

$$\frac{1}{\sqrt{s}}O^T HH^T O\frac{1}{\sqrt{s}} = \mathbb{1}$$

$$A \equiv \frac{1}{\sqrt{s}}O^T H \qquad A^T A = \mathbb{1}$$

$$H = O\sqrt{s}A \qquad E \equiv A\left(O^{-1}\right)^\dagger$$

then $H = O\sqrt{s}EO^\dagger$

The Hermicity of H gives

$$\sqrt{s}E = E\sqrt{s^*} \qquad E^T = E^* = E^{-1}$$

Parity

- The neutrino Dirac mass matrix is then

$$M_D = \sqrt{M_N} O \sqrt{s} E O^\dagger \sqrt{M_N^*}$$

Notice that O, s and E all follow from the knowledgeof M_ν and M_N

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- Since the matrix H is Hermitian, one must have

$$\mathrm{Im}\,\mathrm{Tr}\left(HH^T\right)^n = 0$$

proof: $\mathrm{Im}\,\mathrm{Tr}\,(HH^*)^n = \mathrm{Im}\,\mathrm{Tr}\,(H^*H)^n = \mathrm{Im}\,\mathrm{Tr}\,(HH^*)^{n*}$

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- To preserve the Hermicity of the neutrino Dirac mass matrix, the following condition must be satisfied

$$\mathrm{Im}\,\mathrm{Tr}\left[\frac{v_L^*}{v_R} - \frac{1}{M_N} M_\nu^*\right]^n = 0$$ constraint on M_N

General parametrization of M_N

- We propose a general parametrization

NEW!

$$M_N = P M_\nu^{-1} P^T \qquad V_R = \hat{P} V_L$$

$$\hat{P} \equiv P V_L \sqrt{m_N m_\nu}^{-1} V_L^\dagger$$

$\hat{P}$ is an arbitrary unitary matrix, and P can be Hermitian or anti-Hermitian

GL, Ding-Yi Luo, Xiang Zhao, 2404.16740 (PRD)

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proof: (1)

$$\left[\frac{v_L}{v_R} - \frac{1}{M_N} M_\nu^*\right]^n$$

$$= C_1 + C_2 \frac{1}{M_N} M_\nu^* + \cdots + C_n \left(\frac{1}{M_N} M_\nu^*\right)^n$$

Assume $M_N = P M_\nu^{-1} Q$, if $Q = \pm P^*$

$$\mathrm{Im}\ \mathrm{Tr}\left[\left(\frac{1}{M_N} M_\nu^*\right)^n\right] = \pm \mathrm{Im}\ \mathrm{Tr}\left[\left(P^{-1*} M_\nu P^{-1} M_\nu^*\right)^n\right]$$

$$= \mathrm{Im}\ \mathrm{Tr}\left[\left(\frac{1}{M_N} M_\nu^*\right)^{n*}\right]$$

General parametrization of M_N

proof: (2)

$$\begin{aligned} M_N &= P V_L m_\nu^{-1} V_L^T Q \\ &= V_R m_N V_R^T \end{aligned}$$

To find a possible form of V_R, we define

$$F = \sqrt{m_\nu} X \sqrt{m_N} \qquad X \text{ is orthogonal}$$

so that

$$m_\nu = F m_N^{-1} F^T$$

General parametrization of M_N

proof: (2)

$$M_N = PV_L m_\nu^{-1} V_L^T Q$$
$$= V_R m_N V_R^T$$

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Thus

$$PV_L \left(F^T\right)^{-1} m_N F^{-1} V_L^T Q = V_R m_N V_R^T$$

It is satisfied if

$$V_R = PV_L \left(F^T\right)^{-1}, \quad V_R^T = F^{-1} V_L^T Q$$

General parametrization of M_N

Combine (1) and (2), we have

$$Q = P^T = \pm P^* \qquad V_R = P V_L \sqrt{m_N m_\nu}^{-1}$$

Thus we have

$$M_N = P M_\nu^{-1} P^T \qquad V_R = \hat{P} V_L$$

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- For $V_R = V_L$,

$$\hat{P} = \mathbb{1} \qquad P = V_L \sqrt{m_N m_\nu} V_L^\dagger \qquad \text{Hermitian}$$

- Type-II seesaw scenario $M_N = v_R/v_L M_\nu^*$ can be obtained with

$$V_R = V_L \ , \quad m_N = v_R/v_L m_\nu$$

Solution of strong CP problem

In the SM,

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \theta \frac{g_s^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu} + \bar{q}\left(i\gamma^\mu D_\mu - M_q\right) q$$

$\bar{\theta} = \theta + \arg\det\left(M_u M_d\right)$ $\quad$ $\bar{\theta}_{\mathrm{exp}} < 10^{-10}$ $\quad$ unnaturally small

Solution of strong CP problem

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$$\bar{\theta} = \theta + \arg\det\left(M_u M_d\right) \qquad \bar{\theta}_{\rm exp} < 10^{-10}$$ unnaturally small

Beyond the SM,

- Peccei-Quinn solution: promote $\bar{\theta}$ to be a dynamic field (axion)

  Peccei, Quinn, 1977; Weinberg 1978; Wilczek 1978

- Parity solution: forbid θ at tree and one-loop levels in the left-right symmetric model

  Mohapatra, Senjanovic, 1978
  Babu, Mohapatra, 1989, 1990

$$\bar{\theta} = \arg\det(M_u M_d)$$

Solution of strong CP problem

In the left-right symmetric model,

Parity as the left-right symmetry: $Y_q = Y_q^\dagger$ (quark sector)

Quark mass matrix $M_q = Y_q \langle \Phi \rangle$ is generally complex

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 e^{i\alpha} \end{pmatrix}$$ spotaneous CP phase

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$$\bar{\theta} = \arg\det(M_u M_d) \simeq s_\alpha t_{2\beta} m_t / (2 m_b)$$

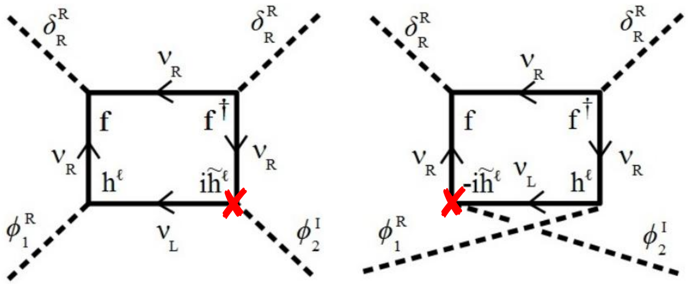
A. Maiezza and M. Nemevšek, 1407.3678 (PRD)

The stringent limit on $\bar{\theta}$ implies an extremely small CP phase α

Why is it so small?

Sterile neutrinos and strong CP problem

- Strong and leptonic CP violation



Necessary conditions:

- sterile neutrinos in the loop
- leptonic CP violation

R. Kuchimanchi, 1408.6382 (PRD)

$$V \supset \left[\alpha_2 \,\mathrm{Tr}\left(\tilde{\Phi}\Phi^\dagger\right) + \text{ h.c. } \right] \mathrm{Tr}\left(\Delta_R \Delta_R^\dagger\right) + \alpha_3 \,\mathrm{Tr}\left(\Phi^\dagger \Phi \Delta_R \Delta_R^\dagger\right).$$

complex scalar potential

Sterile neutrinos and strong CP problem

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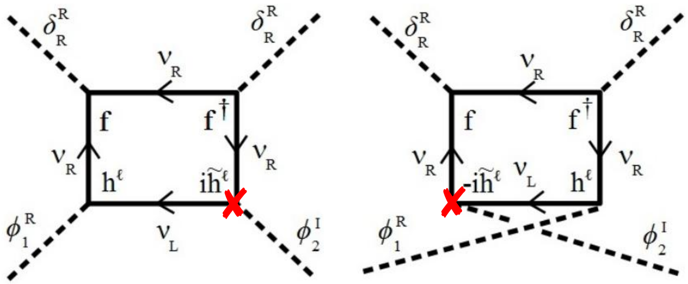

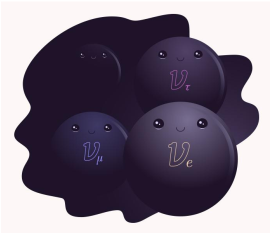

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$$s_\alpha t_{2\beta} = -4 \frac{\operatorname{Im}\alpha_2}{\alpha_3}$$

$$\bar{\theta} \simeq s_\alpha t_{2\beta} m_t / \left(2 m_b\right)$$

complex scalar potential

spotaneous symmetry breaking

complex vacuum

strong CP violation

Sterile neutrinos and strong CP problem

- In terms of mass parameters

One-loop strong CP parameter

$$\bar{\theta}_{\text{loop}} \simeq \frac{1}{16\pi^2}\frac{m_t}{m_b}\frac{1}{v_R^2 v^2}\mathrm{Im}\,\mathrm{Tr}\big(M_N^T M_N^*[M_D, M_\ell]\big)\ln\frac{M_{\text{Pl}}}{v_R}$$

The neutrino Dirac mass matrix

$$M_D = \sqrt{M_N} O \sqrt{s} E O^\dagger \sqrt{M_N^*}$$

$$V_R = \hat{P} V_L \qquad M_N = P M_\nu^{-1} P^T \qquad \hat{P} \equiv P V_L \sqrt{m_N m_\nu}^{-1} V_L^\dagger$$

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$$M_D = \sqrt{M_N} O \sqrt{s} E O^\dagger \sqrt{M_N^*}$$

$$V_R = \hat{P} V_L \qquad M_N = P M_\nu^{-1} P^T \qquad \hat{P} \equiv P V_L \sqrt{m_N m_\nu}^{-1} V_L^\dagger$$

In the case of $\hat{P} = \mathbf{1}$, which was widely assumed in previous studies, we find

$$[M_D, M_\ell] = 0 \quad \Rightarrow \quad \bar{\theta}_{\rm loop} = 0$$

Sterile neutrinos and strong CP problem

- Upper bound on the heaviest sterile neutrino mass

One can estimate that

$$\bar{\theta}_{\rm loop} \propto (m_{\rm N\,max})^{5/2}$$

G. Senjanovic, V. Tello, 2004.04036 (IJMPA)

Non-trivial cases:

$$\hat{P}_1 = i\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad \hat{P}_2 = i\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\hat{P}_3 = i\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{\sqrt{2}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{pmatrix}.$$

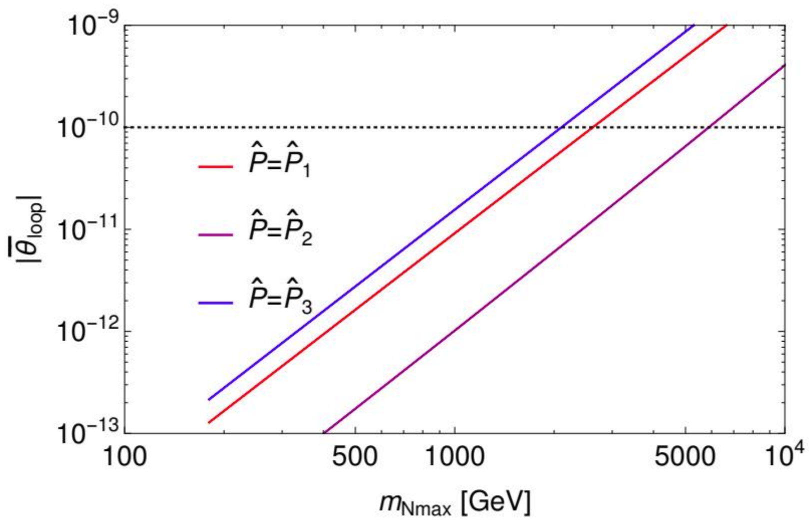
$$\hat{P} \equiv P V_L \sqrt{m_N m_\nu}^{-1} V_L^\dagger$$

The resulting forms of P are all anti-Hermitian

GL, Ding-Yi Luo, Xiang Zhao, 2404.16740 (PRD)

Sterile neutrinos and strong CP problem

- TeV-scale sterile neutrino



GL, Ding-Yi Luo, Xiang Zhao, 2404.16740 (PRD)

Type I+II seesaw scenario:

$$m_1 = 10^{-3}\ \text{eV},\ v_L = 1\ \text{eV},\ v_R = 15\ \text{TeV}$$

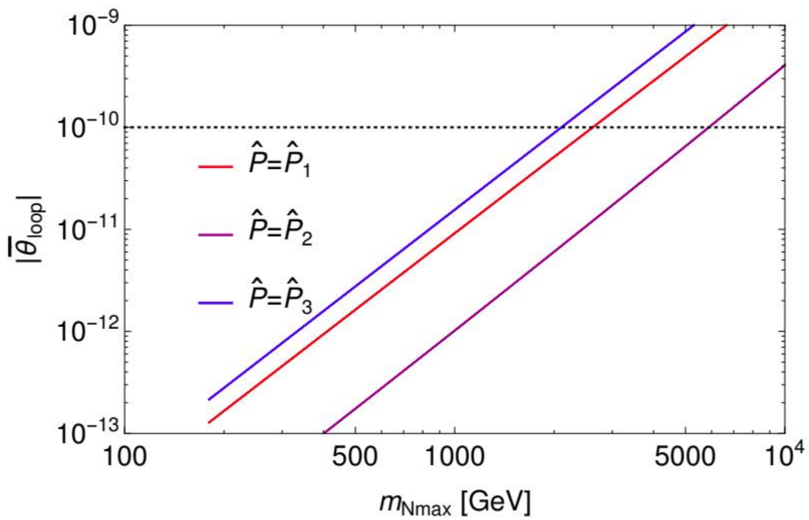
For $\hat{P} = \hat{P}_1$,

$$m_N = \begin{pmatrix} 2.86\ \text{TeV} & 0 & 0 \\ 0 & 3.32\ \text{GeV} & 0 \\ 0 & 0 & 57.2\ \text{MeV} \end{pmatrix}$$

$$\Rightarrow \quad \bar{\theta}_{\rm loop} = -1.241335 \times 10^{-10}$$

Sterile neutrinos and strong CP problem

- TeV-scale sterile neutrino



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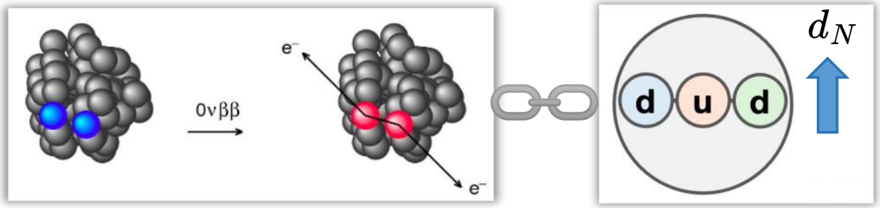
The sterile neutrino can have mass as light as $\sim$ MeV or as heavy as $\sim$ TeV



Summary

- Neutrinos serve as an intriguing window to new physics
- In the context of minimal left-right symmetric model, we have
  - proposed a general parametrization of the right-handed neutrino matrix in the case of parity
  - calculated the neutrino Dirac mass matrix in the generalized Casas-Ibarra parametrization
- We have applied it to calculate the one-loop strong parameter, arising from sterile neutrinos and leptonic CP violation

Thank you